

Today: Syllabus & section 11.1

21



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Monday: sections 11.1-11.2

21

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Monday: sections 11.1-11.2

HW#1: Due August 28<sup>th</sup> {Next Friday}

11.2, 11.3, 11.4a, 11.16a, 11.23a, 11.28a	§11.1
11.34, 11.36, 11.44, 11.47, 11.51, 11.57	§11.2

Today: Syllabus & section 11.1

Monday: sections 11.1-11.2

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11.2, 11.3, 11.4a, 11.16a, 11.23a, 11.28a	§11.1
11.34, 11.36, 11.44, 11.47, 11.51, 11.57	§11.2

\* Syllabus

Who are we?

1 Applied mathematics

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- 1 Applied mathematics
- 13 Engineering - Auto systems

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- 13 Engineering - Auto systems
- 21 Engineering - Mechanical systems

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- 1 Applied mathematics
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Academic level

# Who are we?

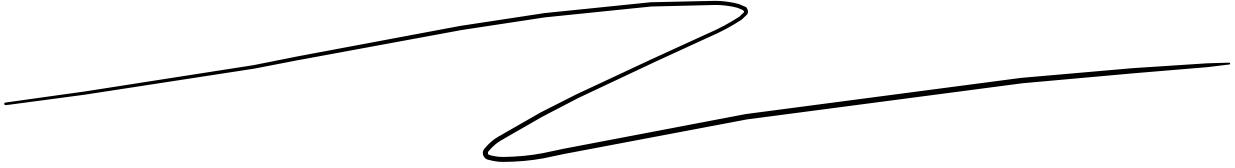
- 1 Applied mathematics
- 13 Engineering - Auto systems
- 21 Engineering - Mechanical systems
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## Academic level

Even  
split

{ 20 Junior  
20 Senior

Locally sourced  
hand-crafted  
Lectures



# Kinematics

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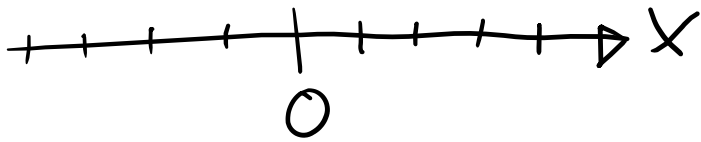
1d motion [Rectilinear motion]: Just need  
time and position along a line

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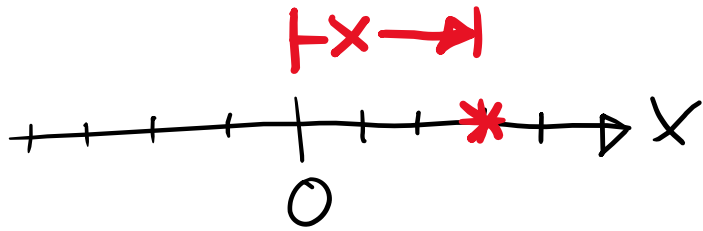


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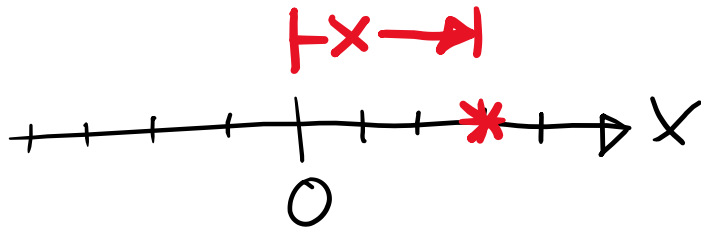
$x = +3$  units

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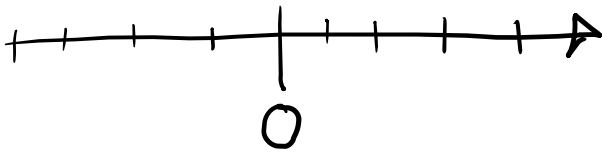
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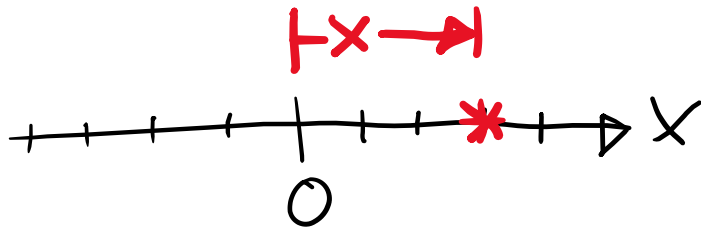


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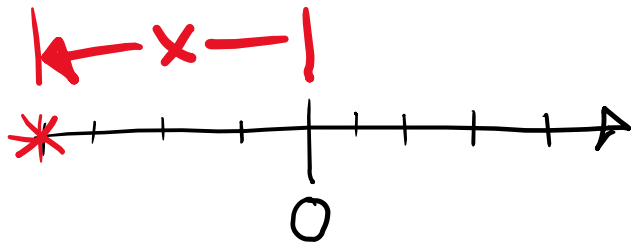
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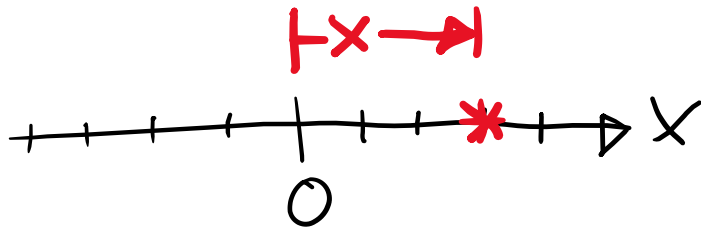


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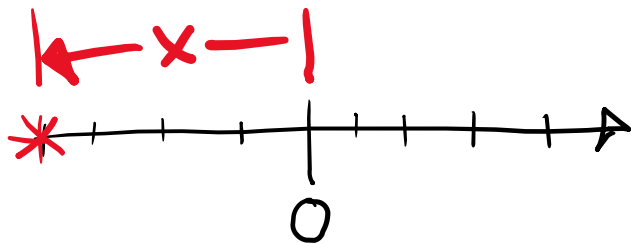
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$$x = +3 \text{ units}$$



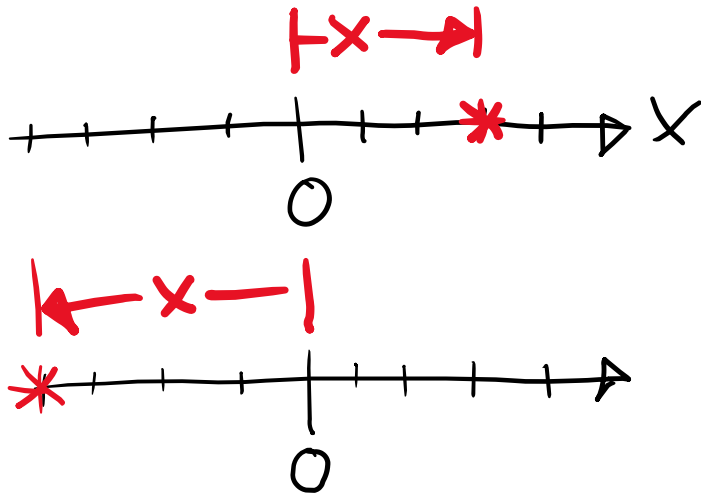
$$x = -4 \text{ units}$$

# Kinematics

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1d motion [Rectilinear motion]: Just need time and position along a line



$x = +3$  units

$x = -4$  units

Units  
could be

ft, m, km,  
Miles, cm, ...

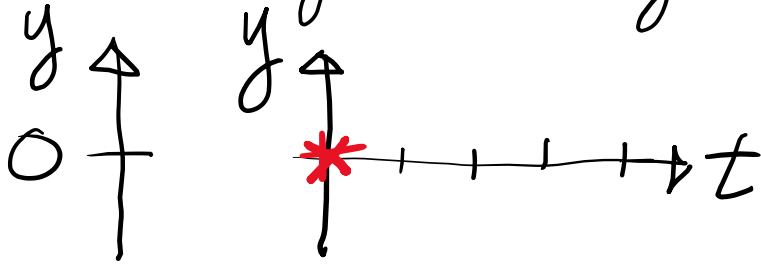
Imagine the motion of a flashing object  
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$$\text{Average velocity} = \frac{\Delta y}{\Delta t}$$

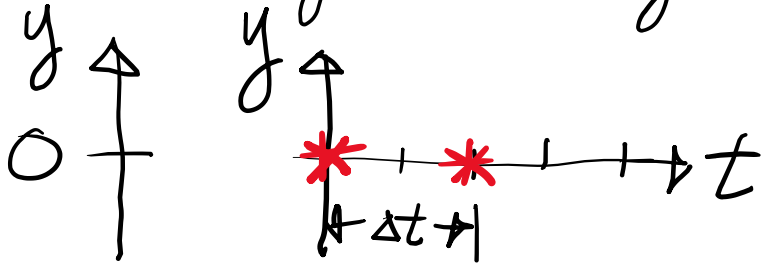
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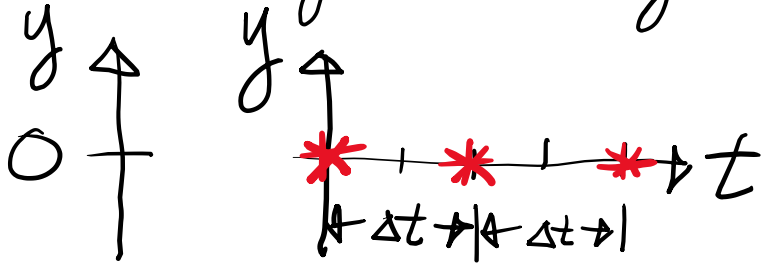
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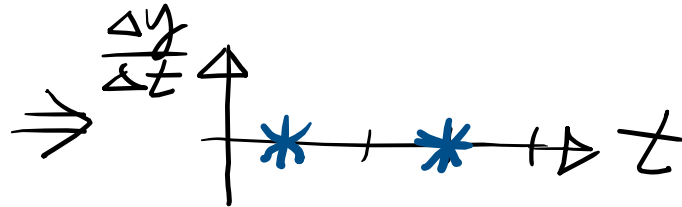
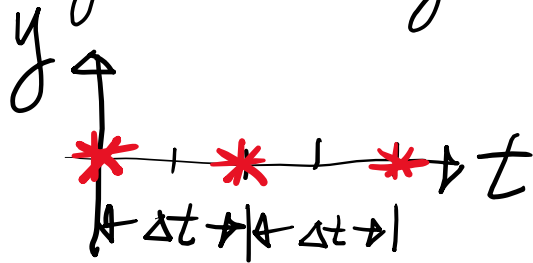
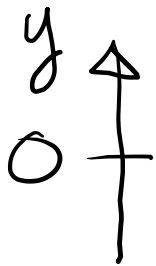
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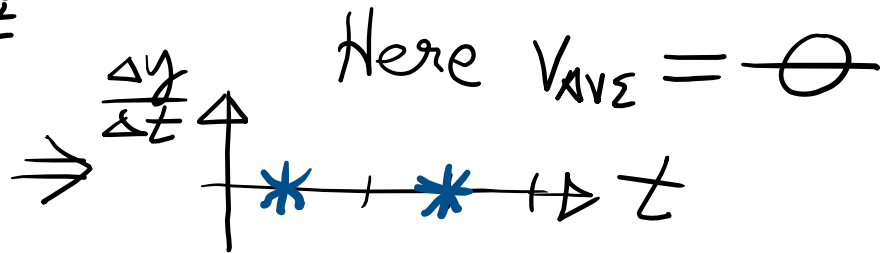
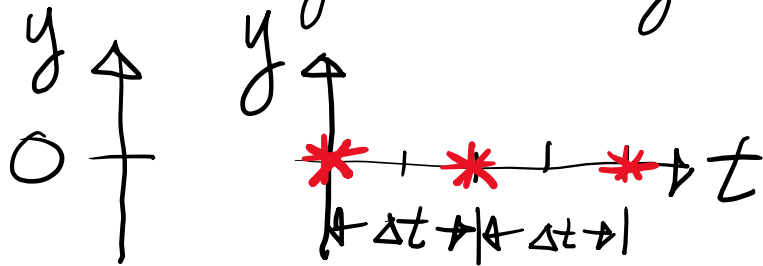
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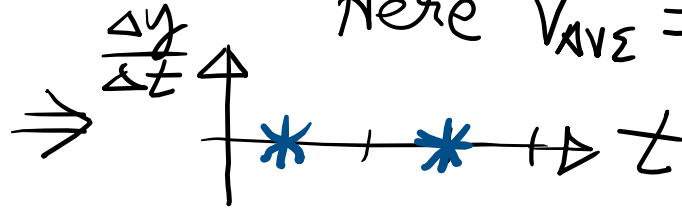
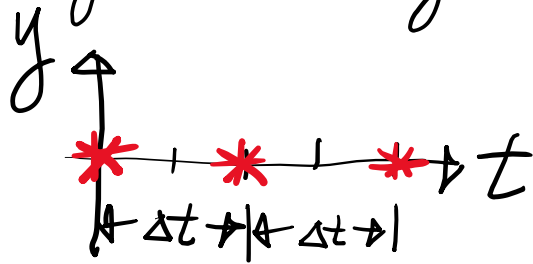
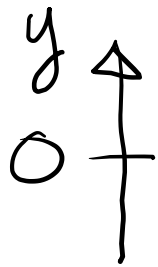
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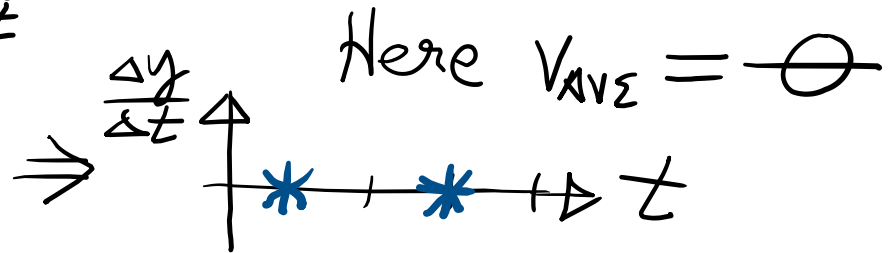
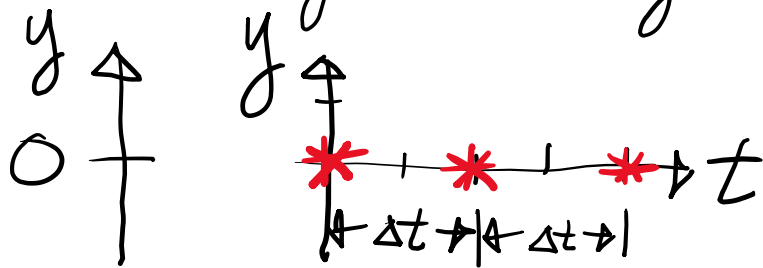


Here  $v_{\text{avg}} = 0$

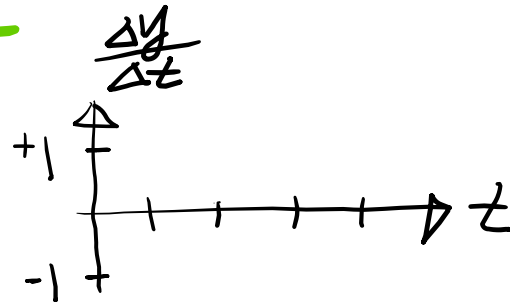
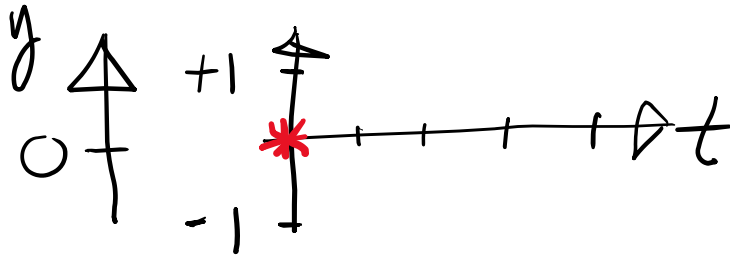
Now double the rate

Imagine the motion of a flashing object constrained to move in the vertical direction

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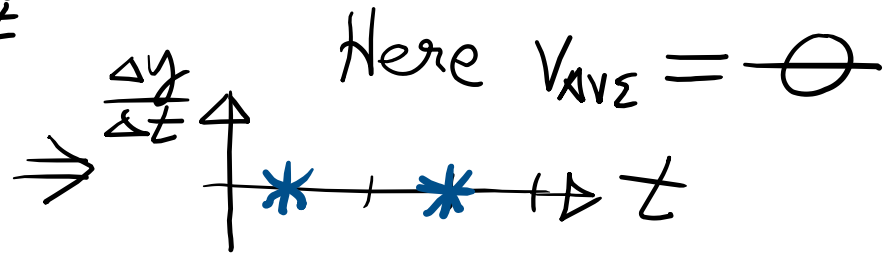
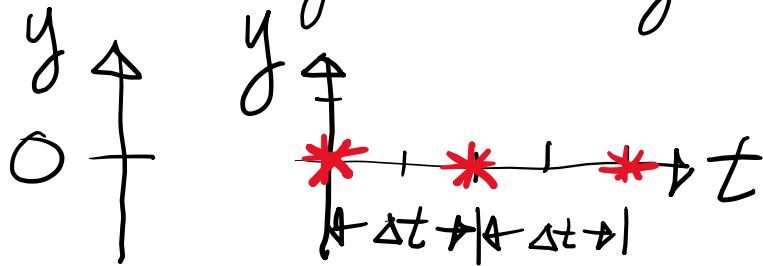


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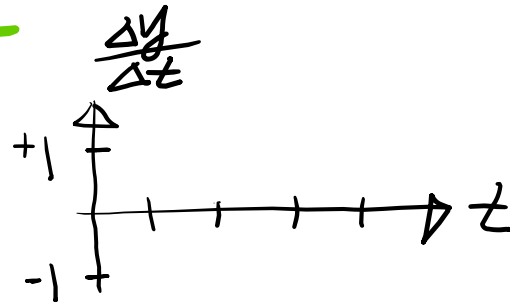
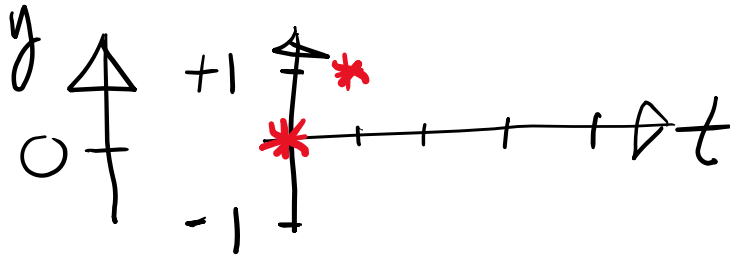


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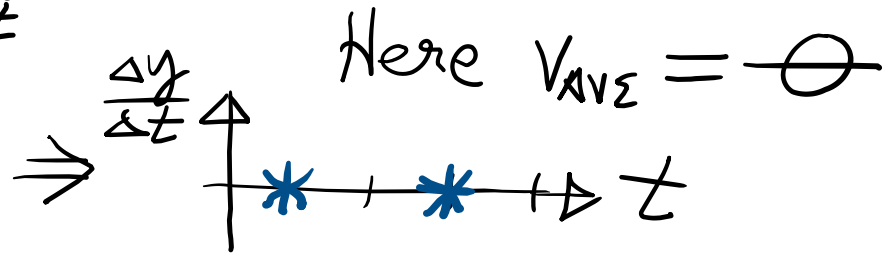
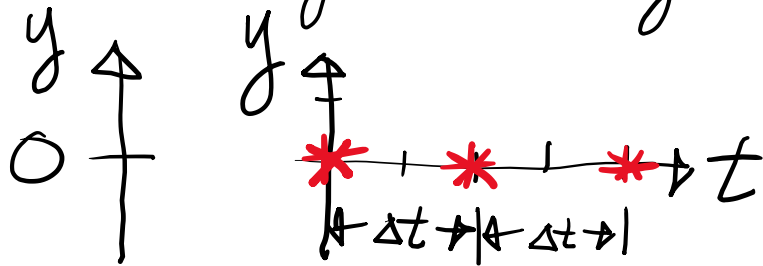


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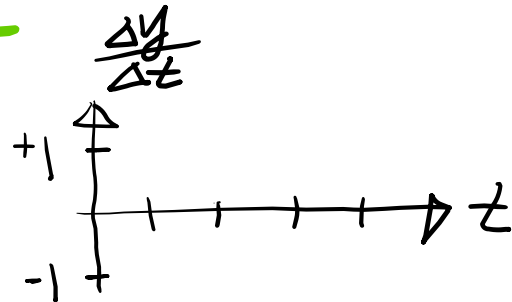
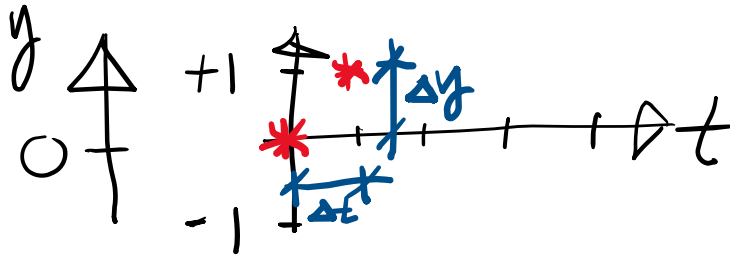


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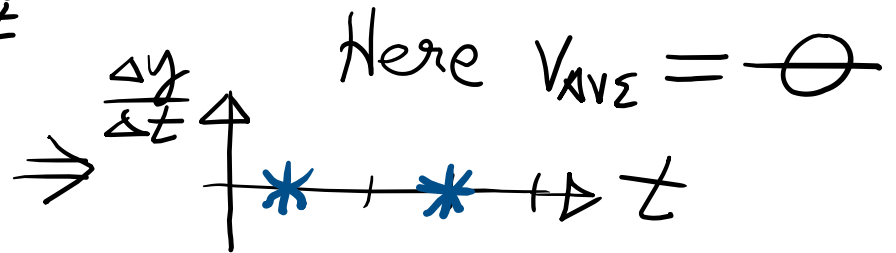
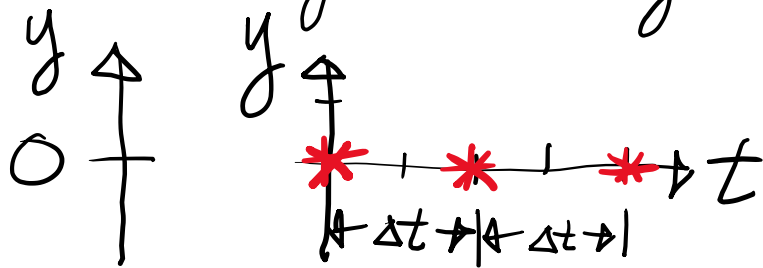


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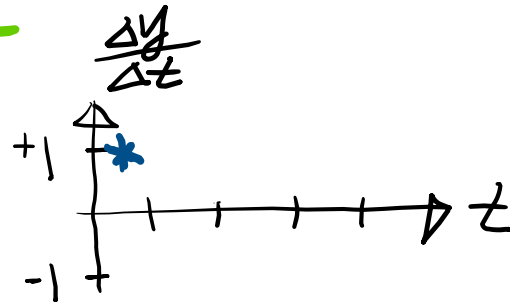
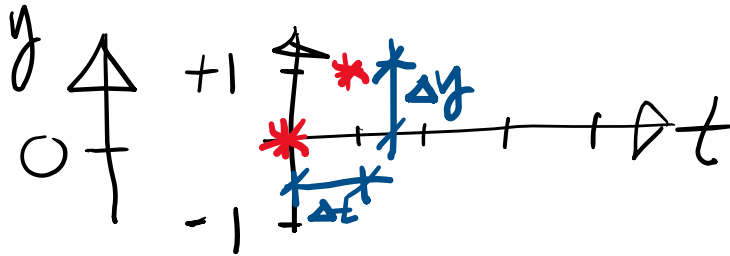


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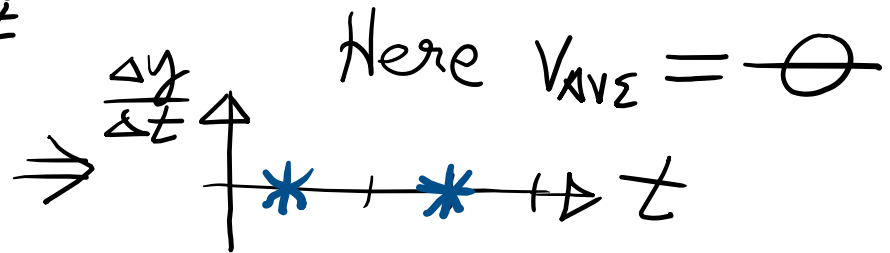
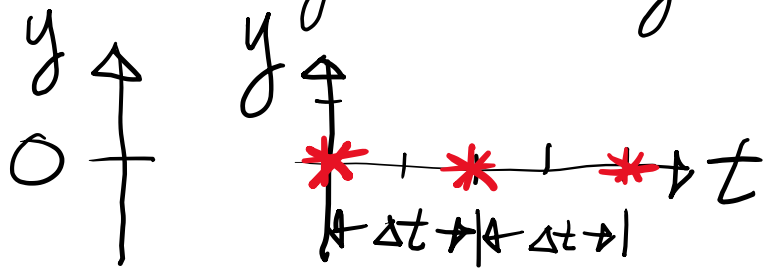


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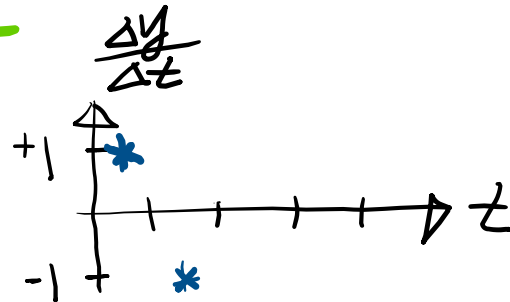
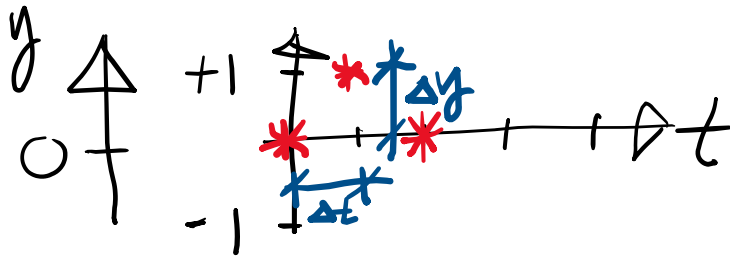


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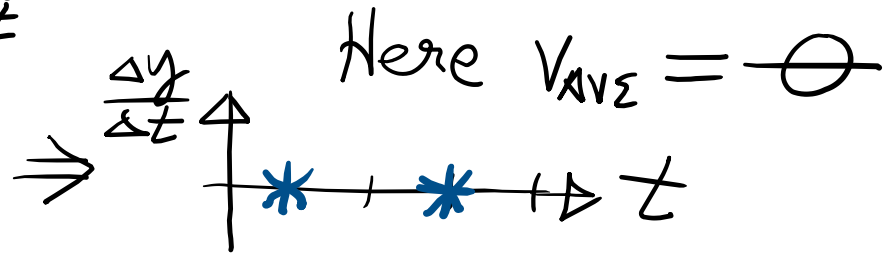
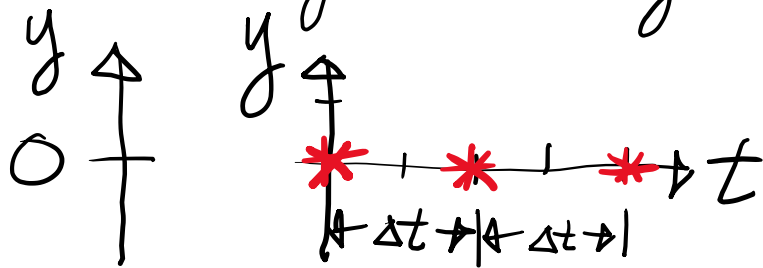


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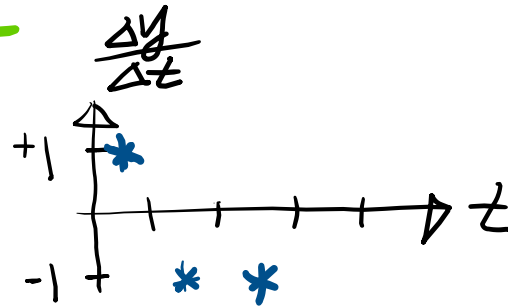
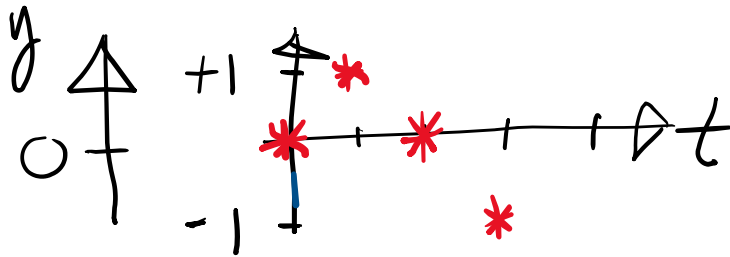


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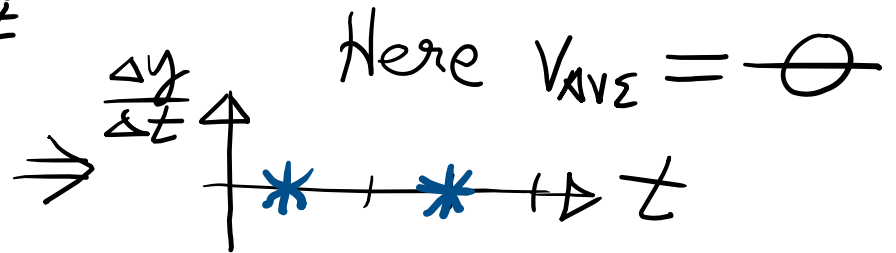
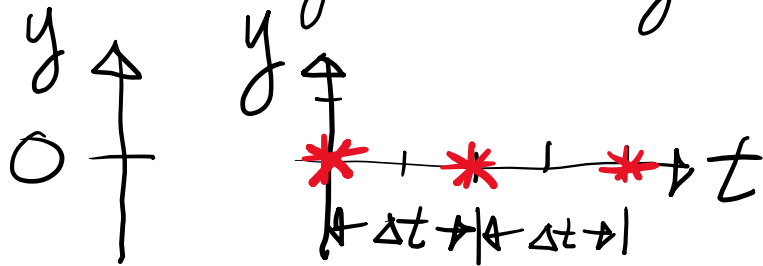


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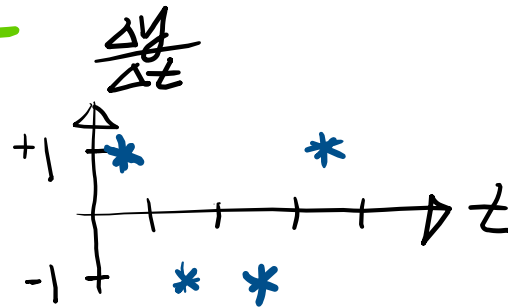
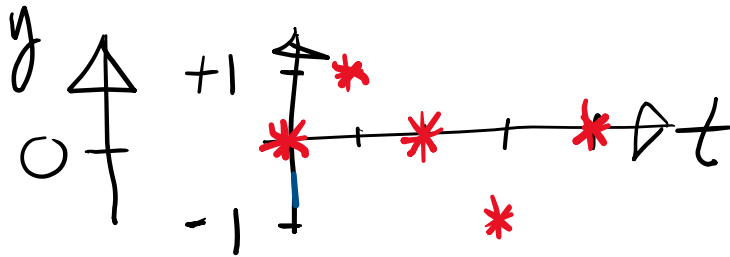


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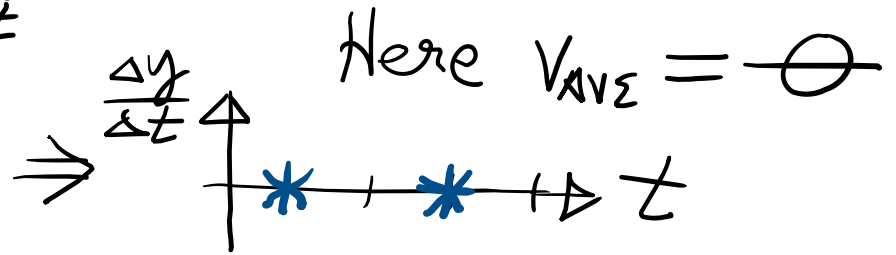
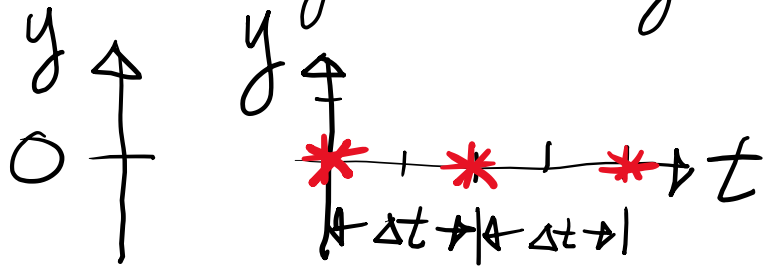


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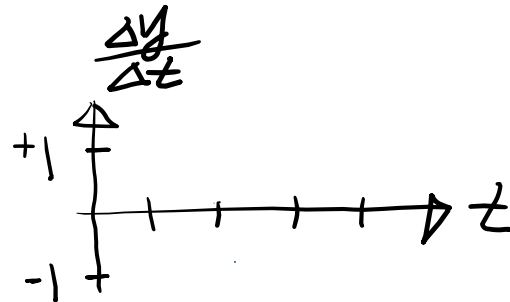
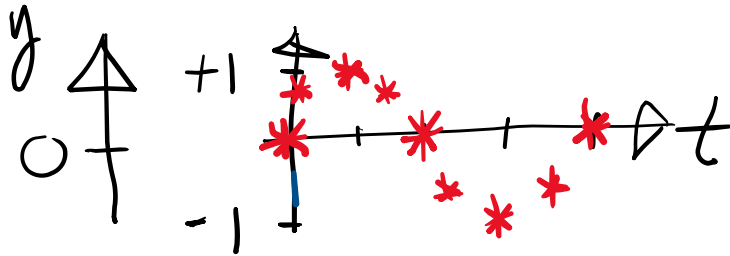


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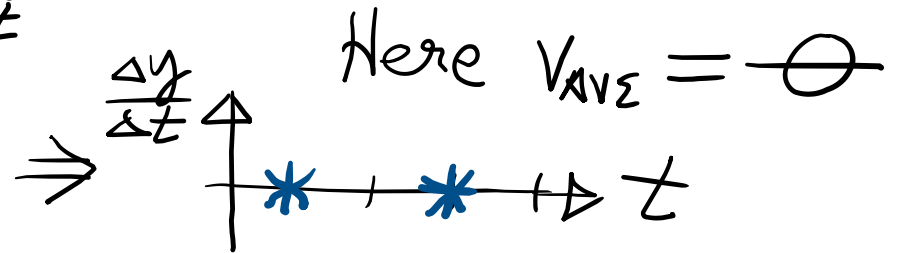
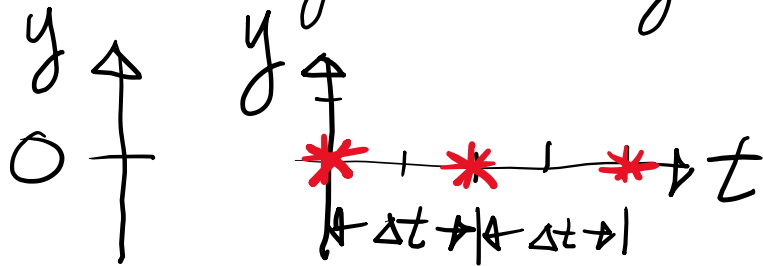


## QUAD RATE

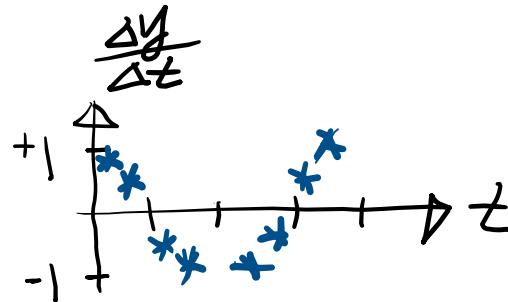
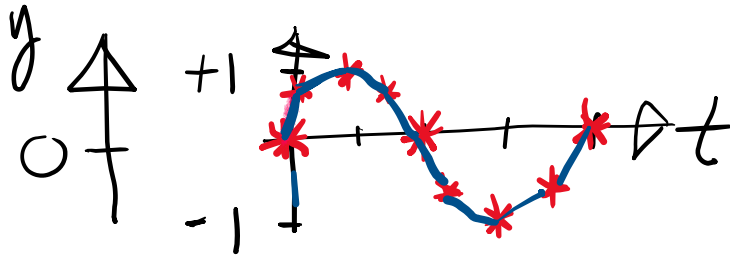


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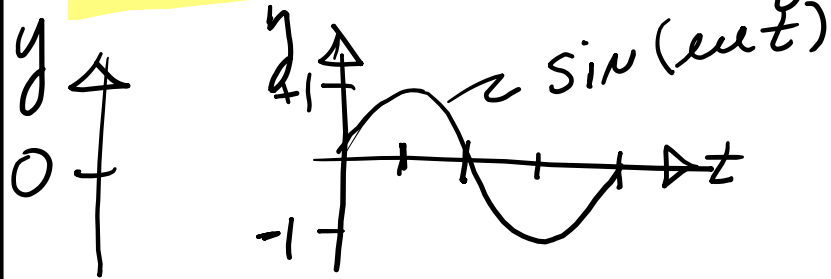


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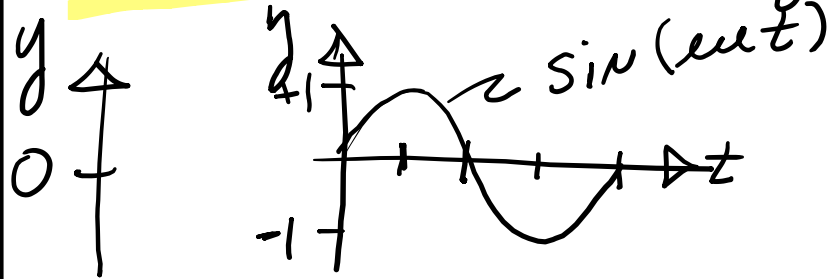


# Continuous light

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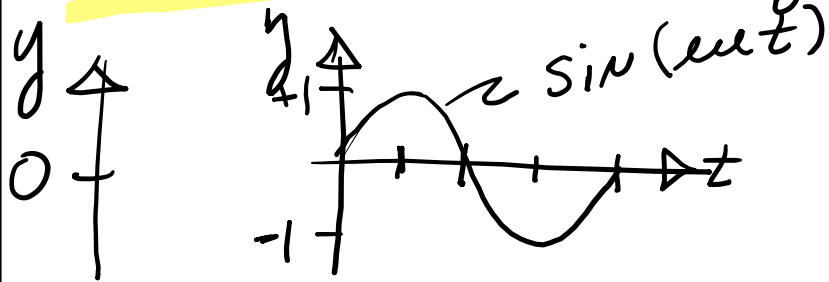


# Continuous light



Now there is no nice  $\Delta t$  between flashes of light

# Continuous light

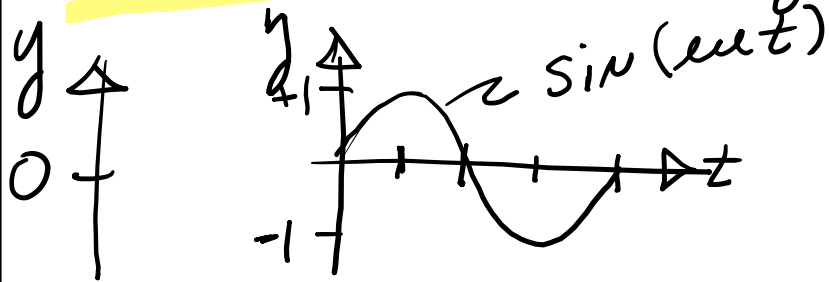


Now there is no nice  $\Delta t$  between flashes of light

Solution: Take limit as  $\Delta t \rightarrow 0$ :

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta t} \equiv \frac{dy}{dt}$$

# Continuous light



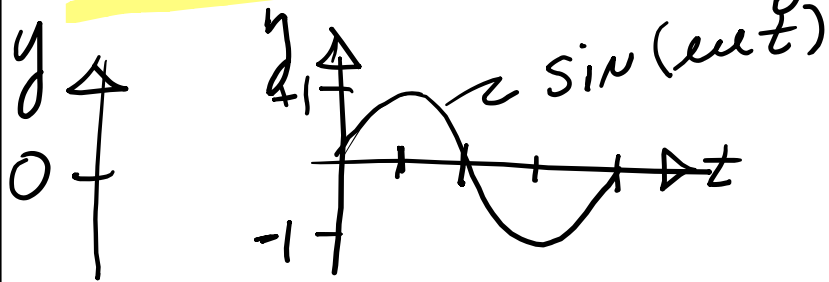
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$$v = \frac{d}{dt} \sin(\omega t) = \omega \cos(\omega t)$$

# Continuous light

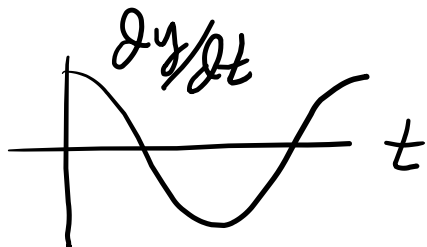


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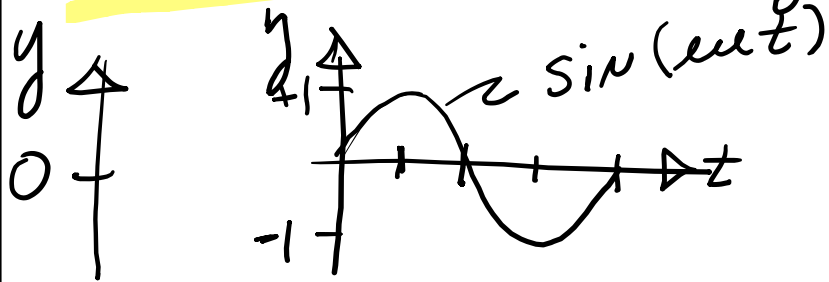
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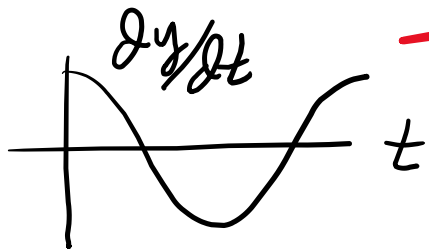


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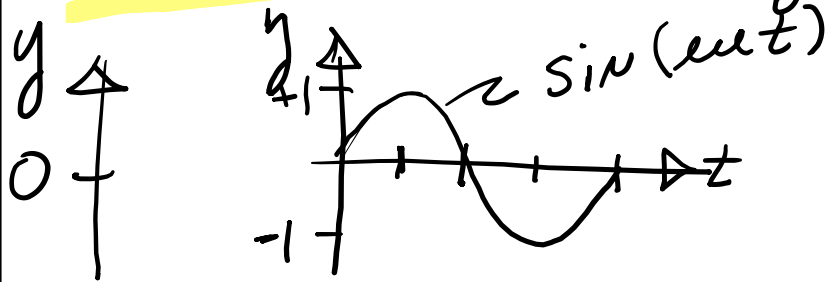
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→ We can see that the velocity is never constant (in this example)

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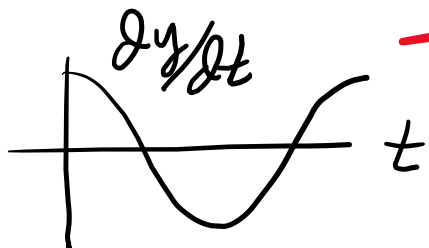


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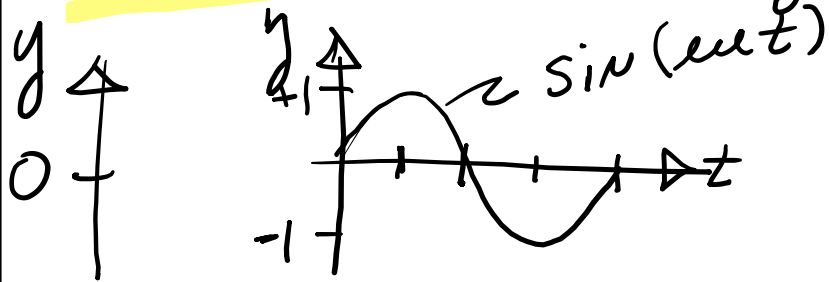
$$v = \frac{d}{dt} \sin(elt) = el \cos(elt)$$



We can see that the velocity is never constant (in this example)

Define acceleration:  $a_{\text{AVE}} \equiv \frac{\Delta v}{\Delta t}$

# Continuous light

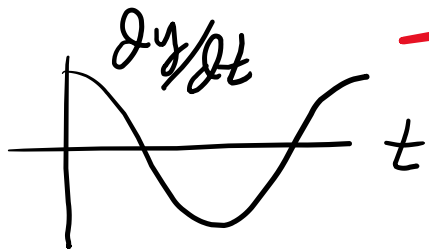


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Solution: Take limit as  $\Delta t \rightarrow 0$ :

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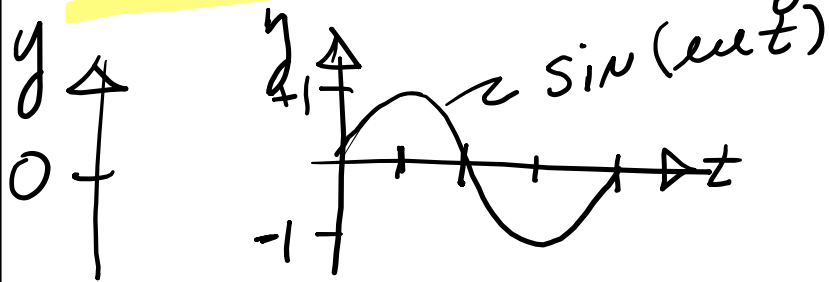
$$v = \frac{d}{dt} \sin(elt) = el \cos(elt)$$



→ We can see that the velocity is never constant (in this example)

Define acceleration:  $a_{\text{ave}} \equiv \frac{\Delta v}{\Delta t}$  &  $a = \frac{dv}{dt}$

# Continuous light

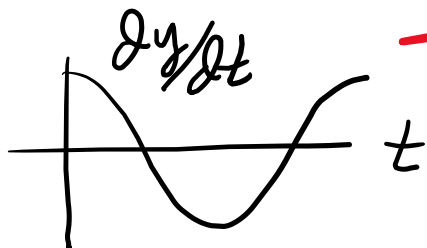


Now there is no nice  $\Delta t$  between flashes of light

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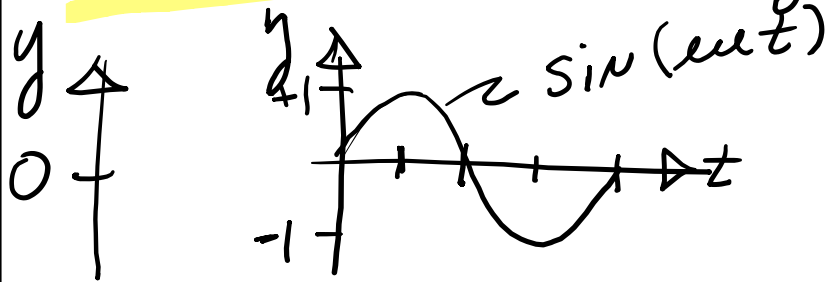
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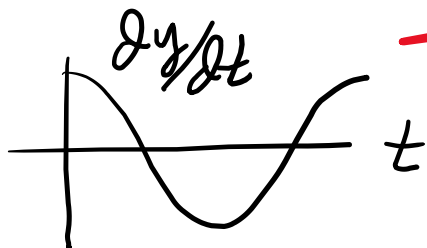


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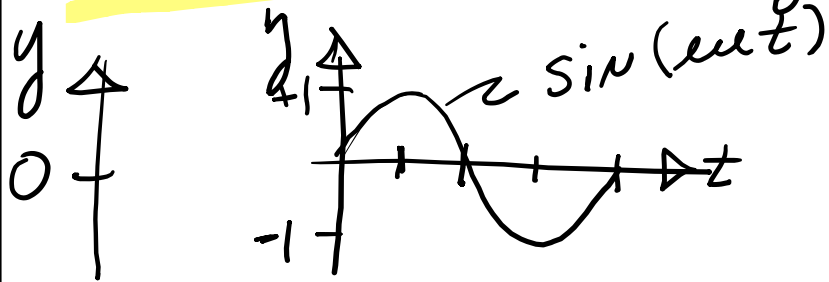
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But  $v = \frac{dy}{dt}$  so  $a = \frac{d}{dt} \left( \frac{dy}{dt} \right)$

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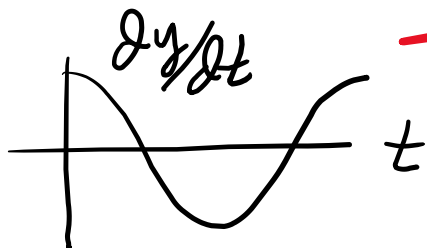


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But  $v = \frac{dy}{dt}$

$$\text{so } a = \frac{d}{dt} \left( \frac{dy}{dt} \right) = \frac{d^2 y}{dt^2}$$

2nd derivative

Starting with  $x(t)$

Starting with  $x(t)$

$x$  as a function of  $t$

Starting with  $x(t)$ :  $v = \frac{dx}{dt}$  &  $a = \frac{d^2x}{dt^2}$

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Differential elements

Starting with  $x(t)$ :  $v = \frac{dx}{dt}$  &  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

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If  $y$  is a function of  $x$ , the differential element of  $y$  is  $dy = \frac{dy}{dx} dx$

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Differential element  $dv$

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IF  $t_I = 0$  &  $t_F = t$  &  $v_I = v_0$  &  $v_F = v$

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IF  $t_I = 0$  &  $t_F = t$  &  $v_I = v_0$  &  $v_F = v$

Then  $v = v_0 + \int_0^t a dt$

Example:  $a = -9.8 \frac{m}{s^2}$

Find  $v$  &  $x$

Example:  $a = -9.8 \frac{m}{s^2} \Rightarrow$

$$v = v_0 + \int_0^t (-9.8 \frac{m}{s^2}) dt = v_0 - (9.8 \frac{m}{s^2})t$$

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$$\text{so } x = x_0 + \int_0^t v dt = \int_0^t 9.8 \frac{m}{s^2} t dt$$

$$\Rightarrow x = x_0 + v_0 t - 9.8 \frac{m}{s^2} \left( \frac{t^2}{2} \right) \text{ or rearrange}$$

to get

$$x = -\frac{1}{2} (9.8 \frac{m}{s^2}) t^2 + v_0 t + x_0$$

What if  $x$  is a function of  $t$   
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Chain  
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Now  $a = \frac{dv}{dt} = \left( \frac{dv}{dx} \right) \left( \frac{dx}{dt} \right)$  ← Chain rule

§ since  $\frac{dx}{dt} = v$ , then  $a = \left( \frac{dv}{dx} \right) v$

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 $\int_{x_0}^x a dx = \int_{v_0}^v v dv \Rightarrow$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{x_0}^x a dx$$

What if  $a(v)$  ?

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---

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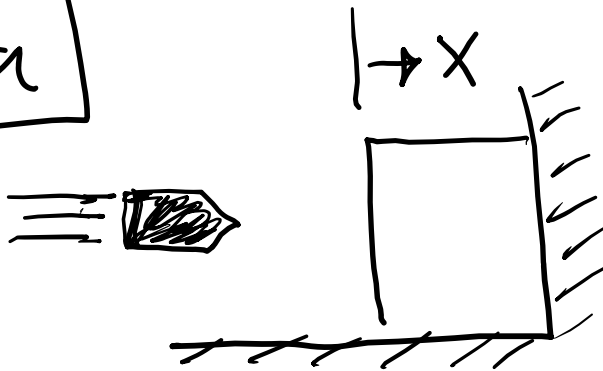
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# Notes on problems:

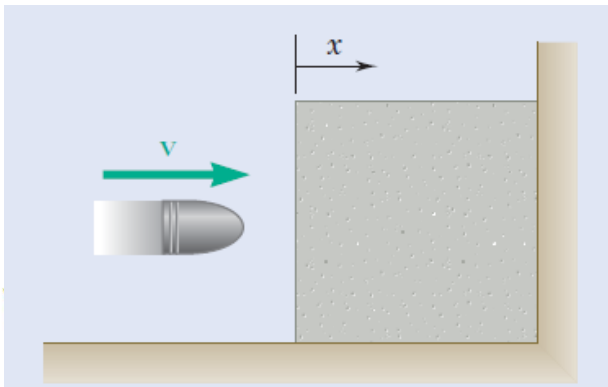
11.16a



Given:

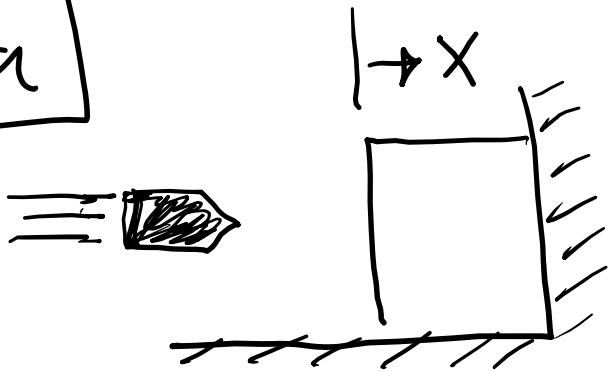
$$\left\{ \begin{array}{l} v_0 = 900 \frac{\text{ft}}{\text{s}} \\ \quad \text{at } x_0 = 0 \\ \\ v_1 = 0 \text{ at} \\ \quad x_1 = 4 \text{ in} \\ \& v = v_0 - kx \end{array} \right.$$

**11.16** A projectile enters a resisting medium at  $x = 0$  with an initial velocity  $v_0 = 900$  ft/s and travels 4 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation  $v = v_0 - kx$ , where  $v$  is expressed in ft/s and  $x$  is in feet, determine (a) the initial acceleration of the projectile, (b) the time required for the projectile to penetrate 3.9 in. into the resisting medium.



# Notes on problems:

11.16a



Given:

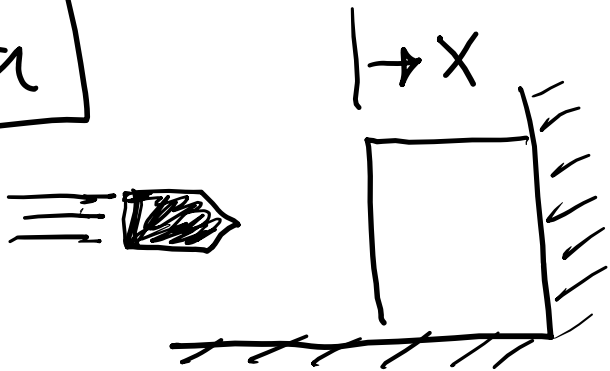
$$v_0 = 900 \frac{\text{ft}}{\text{s}} \text{ at } x_0 = 0$$
$$v_1 = 0 \text{ at } x_1 = 4 \text{ in}$$
$$v = v_0 - kx$$

A red arrow points from the circled equation  $x_1 = 4 \text{ in}$  to the text "note unit change".

note unit  
change

# Notes on problems:

11.16a



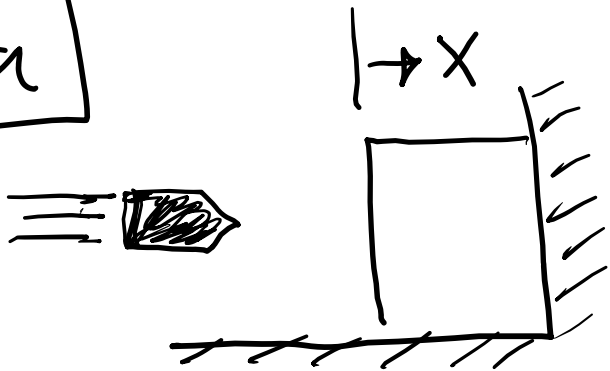
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Find  $a_0 = a(t=0)$

# Notes on problems:

11.16a



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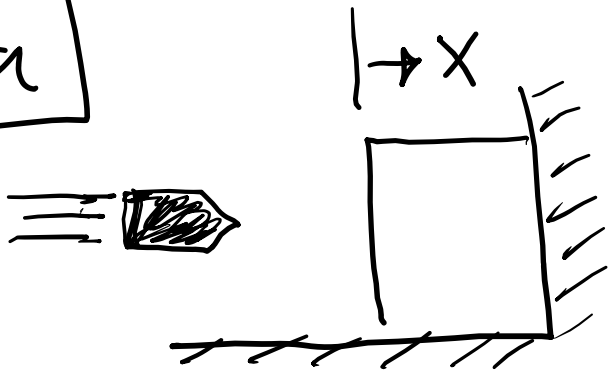
Find  $a_0 = a(t=0)$

$v$  as a function of  $x$ .

We have

# Notes on problems:

11.16a



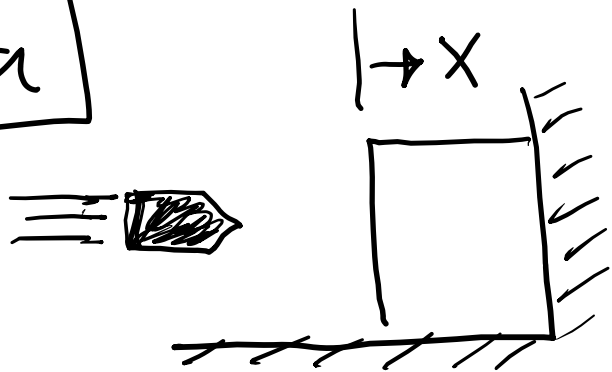
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Find  $a_0 = a(t=0)$  We have  
 $v$  as a function of  $x$ . This  
suggests we use  $a = v \frac{dv}{dx}$ .

# Notes on problems:

11.16a



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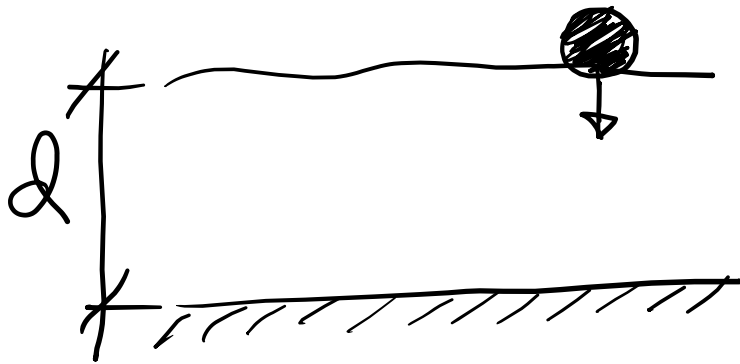
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We have

$v$  as a function of  $x$ . This suggests we use  $a = v \frac{dv}{dx}$ .

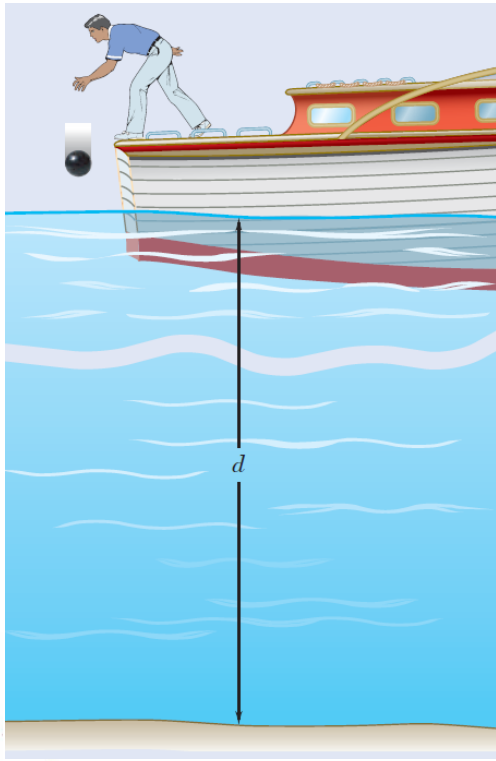
Will need to find  $k$  first

11.23



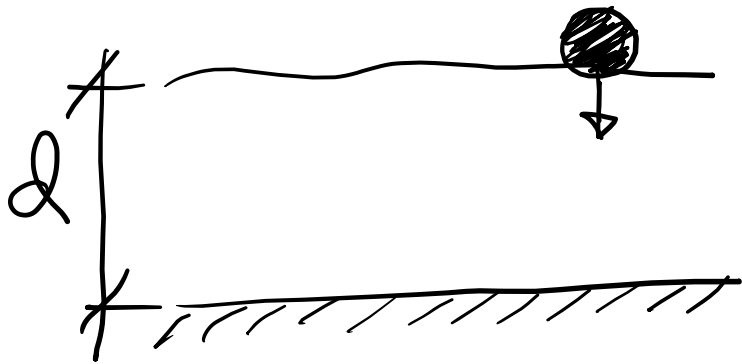
Given: 
$$\left\{ \begin{array}{l} v_0 = 16.5 \frac{\text{ft}}{\text{s}} \\ a = 10 - 0.8v \\ \Delta t = 3 \text{ s to} \\ \text{hit bottom} \end{array} \right.$$

Find  $d$



**11.23** A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of  $a = 10 - 0.8v$ , where  $a$  and  $v$  are expressed in  $\text{ft/s}^2$  and  $\text{ft/s}$ , respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (a) the depth of the lake, (b) the speed of the ball when it hits the bottom of the lake.

11.23

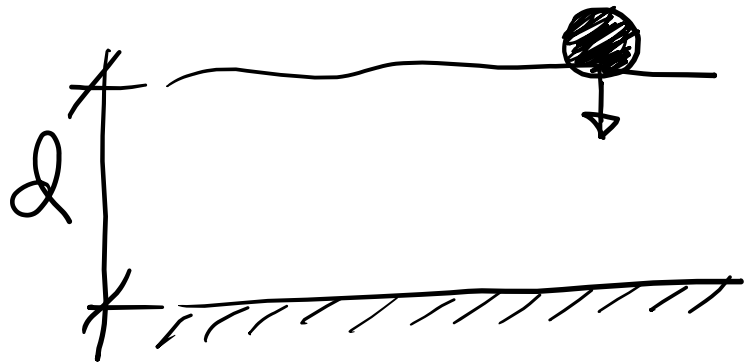


Given: 
$$\left\{ \begin{array}{l} v_0 = 16.5 \frac{\text{ft}}{\text{s}} \\ a = 10 - 0.8v \\ \Delta t = 3 \text{ s to hit bottom} \end{array} \right.$$

Find  $d$

Here we have  $a$  as a function of  $v$

11.23



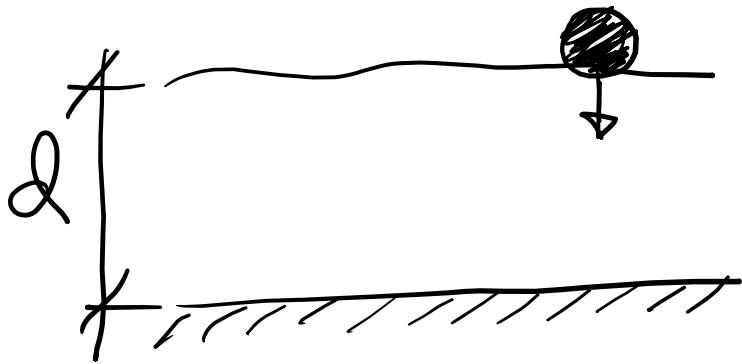
Given:

$$\begin{cases} v_0 = 16.5 \frac{\text{ft}}{\text{s}} \\ a = 10 - 0.8v \\ \Delta t = 3 \text{ s to hit bottom} \end{cases}$$

Find  $d$

Here we have  $a$  as a function of  $v$ . One way to solve is to note that  $a = \frac{dv}{dt} \Rightarrow \int_0^t dt = \int_{v_0}^v \frac{dv}{a}$ , where  $a = 10 - 0.8v$ .

11.23



Given: 
$$\left\{ \begin{array}{l} v_0 = 16.5 \frac{\text{ft}}{\text{s}} \\ a = 10 - 0.8v \\ \Delta t = 3 \text{ s to} \\ \text{hit bottom} \end{array} \right.$$

Find  $d$

Here we have  $a$  as a function of  $v$ . One way to solve is to note that  $a = \frac{dv}{dt} \Rightarrow \int_0^t dt = \int_{v_0}^v \frac{dv}{a}$ , where  $a = 10 - 0.8v$ . Solve for  $v$  then use  $\frac{dx}{dt} = v \Rightarrow \int dx = \int v dt$

11.28a

$$\text{Given: } \begin{cases} v = 7.5(1 - 0.04x)^{0.3} \\ x = 0 \text{ at } t = 0 \end{cases}$$

Find  $x(t=1h)$

**11.28** Based on observations, the speed of a jogger can be approximated by the relation  $v = 7.5(1 - 0.04x)^{0.3}$ , where  $v$  and  $x$  are expressed in mi/h and miles, respectively. Knowing that  $x = 0$  at  $t = 0$ , determine (a) the distance the jogger has run when  $t = 1$  h, ~~(b) the jogger's acceleration in  $\text{ft/s}^2$  at  $t = 0$ , (c) the time required for the jogger to run 6 mi.~~

11.28a

$$\text{Given: } \begin{cases} v = 7.5(1 - 0.04x)^{0.3} \\ \& x = 0 \text{ at } t = 0 \end{cases}$$

Find  $x(t=1h)$

We have  $v$  as a  
function of  $x$

11.28a

$$\text{Given: } \begin{cases} v = 7.5(1 - 0.04x)^{0.3} \\ \& x = 0 \text{ at } t = 0 \end{cases}$$

Find  $x(t=1h)$

We have  $v$  as a function of  $x$  & know

$$v = \frac{dx}{dt} \Rightarrow \int dt = \int \frac{dx}{v}$$

