

Today: 11.2

L2

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Wednesday: 11.2

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HW#1 Due Friday

L2

From last time :

$$V = \frac{dx}{dt}$$

From last time :

$$v = \frac{dx}{dt} , a = \frac{dv}{dt}$$

From last time :

$$v = \frac{dx}{dt} \quad , \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

From last time :

$$v = \frac{dx}{dt} , a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Going backwards :

From last time :

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Going backwards :

Start with $a(t) \rightarrow$

From last time :

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Going backwards :

Start with $a(t) \rightarrow a = \frac{dv}{dt}$

From last time :

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Going backwards :

Start with $a(t) \rightarrow a = \frac{dv}{dt} \Rightarrow$

$$\int a dt = \int \frac{dv}{dt} dt$$

From last time :

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Going backwards :

Start with $a(t) \rightarrow a = \frac{dv}{dt} \Rightarrow$

$$\int a dt = \int \frac{dv}{dt} dt \Rightarrow \int a dt = \int dv$$

From last time :

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Going backwards :

Start with $a(t) \rightarrow a = \frac{dv}{dt} \Rightarrow$

$$\int a dt = \int \frac{dv}{dt} dt \Rightarrow \int a dt = \int dv$$

$$\Rightarrow v - v_0 = \int_0^t a dt$$

Constant acceleration

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if $a = \text{constant}$ $\frac{dv}{dt} = a \Rightarrow$

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$$\int_{v_0}^v dv = \int_0^t a dt$$

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$$v = at + v_0$$

Constant acceleration

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$$\int_{v_0}^v dv = \int_0^t a dt \Rightarrow v - v_0 = at \Rightarrow$$

$$v = at + v_0 \quad \& \quad \frac{dx}{dt} = v \Rightarrow$$

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$$v = at + v_0 \quad \& \quad \frac{dx}{dt} = v \Rightarrow$$

$$\int_{x_0}^x dx = \int_0^t v dt$$

Constant acceleration

$$\text{if } a = \text{constant} \quad \frac{dv}{dt} = a \Rightarrow$$

$$\int_{v_0}^v dv = \int_0^t a dt \Rightarrow v - v_0 = at \Rightarrow$$

$$\underline{v = at + v_0} \quad \& \quad \frac{dx}{dt} = v \Rightarrow$$

$$\int_{x_0}^x dx = \int_0^t \underline{v} dt \Rightarrow x - x_0 = \int_0^t \underline{(at + v_0)} dt$$

Constant acceleration

$$\text{if } a = \text{constant} \quad \frac{dv}{dt} = a \Rightarrow$$

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$$\int_{x_0}^x dx = \int_0^t \underline{v} dt \Rightarrow x - x_0 = \int_0^t \underline{(at + v_0)} dt$$

$$\Rightarrow x = \frac{1}{2} at^2 + v_0 t + x_0$$

Constant acceleration

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Constant acceleration

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$$\int_{x_0}^x dx = \int_0^t \underline{v} dt \Rightarrow x - x_0 = \int_0^t \underline{(at + v_0)} dt$$

$$\Rightarrow x = \frac{1}{2}at^2 + v_0t + x_0 \quad \underline{\underline{\text{If } a = -g}}$$

$$\text{then } x = -\frac{1}{2}gt^2 + v_0t + x_0$$

Section 11.2

Section 11.2

Special cases of Relative motion

Section 11.2

Special cases of Relative motion

$v = \text{constant}$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$
$$a = \text{constant}$$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$

$$a = \text{constant} \Rightarrow v = at + v_0 \quad \& \quad x = \frac{1}{2}at^2 + v_0t + x_0$$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$

$$a = \text{constant} \Rightarrow v = at + v_0 \quad \& \quad x = \frac{1}{2}at^2 + v_0t + x_0$$

Also

$$a = v \frac{dv}{dx} \Rightarrow a \int_{x_0}^x dx = \int_{v_0}^v v dv$$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$

$$a = \text{constant} \Rightarrow v = at + v_0 \quad \& \quad x = \frac{1}{2}at^2 + v_0t + x_0$$

$$\text{Also} \quad a = v \frac{dv}{dx} \Rightarrow a \int_{x_0}^x dx = \int_{v_0}^v v dv$$

$$\Rightarrow a(x - x_0) = \frac{1}{2}(v^2 - v_0^2)$$

Section 11.2

Special cases of Relative motion

$$v = \text{constant} \Rightarrow a = 0 \quad \& \quad x = vt + x_0$$

$$a = \text{constant} \Rightarrow v = at + v_0 \quad \& \quad x = \frac{1}{2}at^2 + v_0t + x_0$$

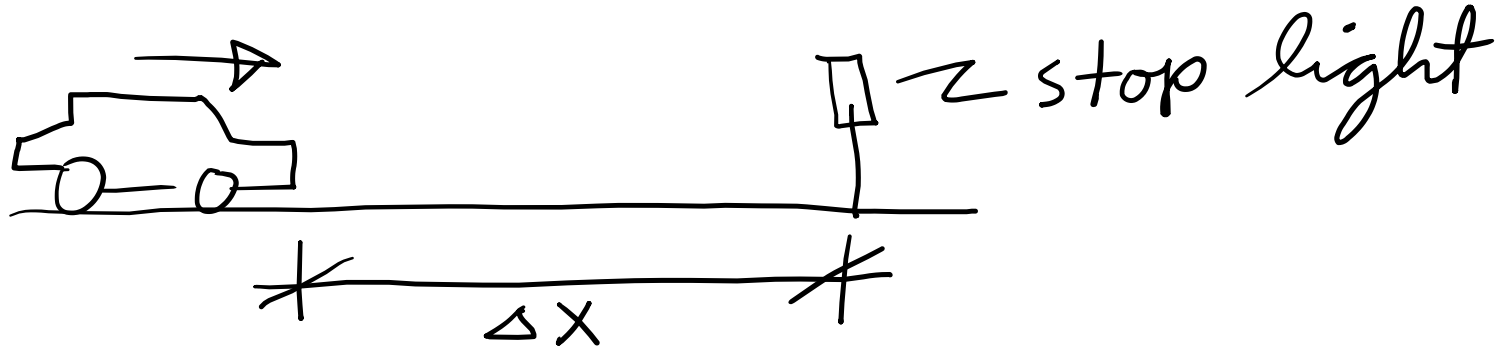
$$\text{Also} \quad a = v \frac{dv}{dx} \Rightarrow a \int_{x_0}^x dx = \int_{v_0}^v v dv$$

$$\Rightarrow a(x - x_0) = \frac{1}{2}(v^2 - v_0^2) \quad \underline{\text{or}}$$

$$v^2 = 2a(x - x_0) + v_0^2$$

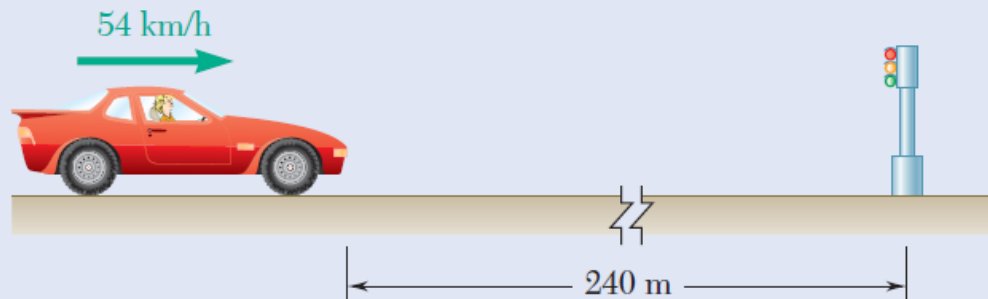
Notes on problems

Notes on problems : 11.34

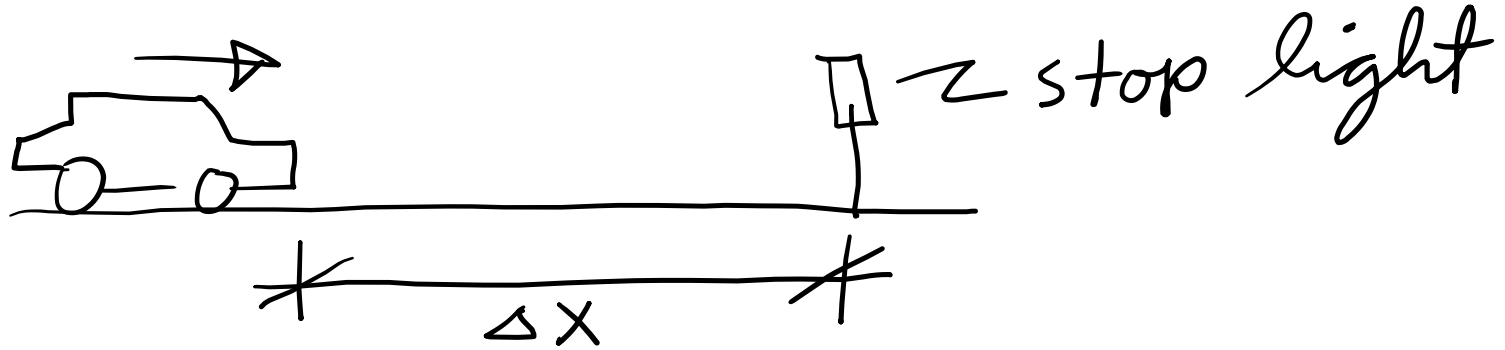


Given: $V_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240 \text{m}$, $\Delta t = 24 \text{s}$ & $a = \text{const.}$

11.34 A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (a) the required uniform deceleration of the car, (b) the speed of the car as it passes the light.



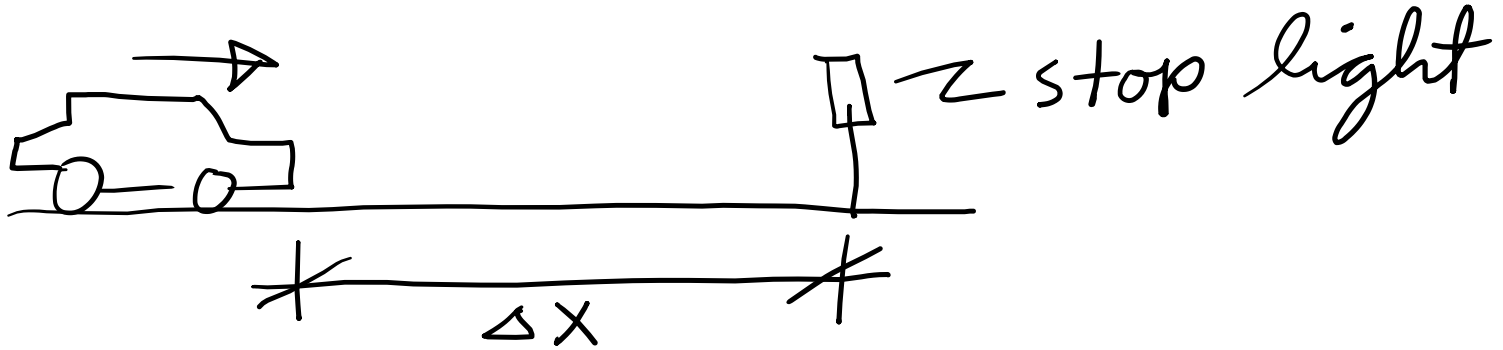
Notes on problems : 11.34



Given: $v_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240 \text{m}$, $\Delta t = 24 \text{s}$ & $a = \text{const.}$

Note units $\downarrow \downarrow$

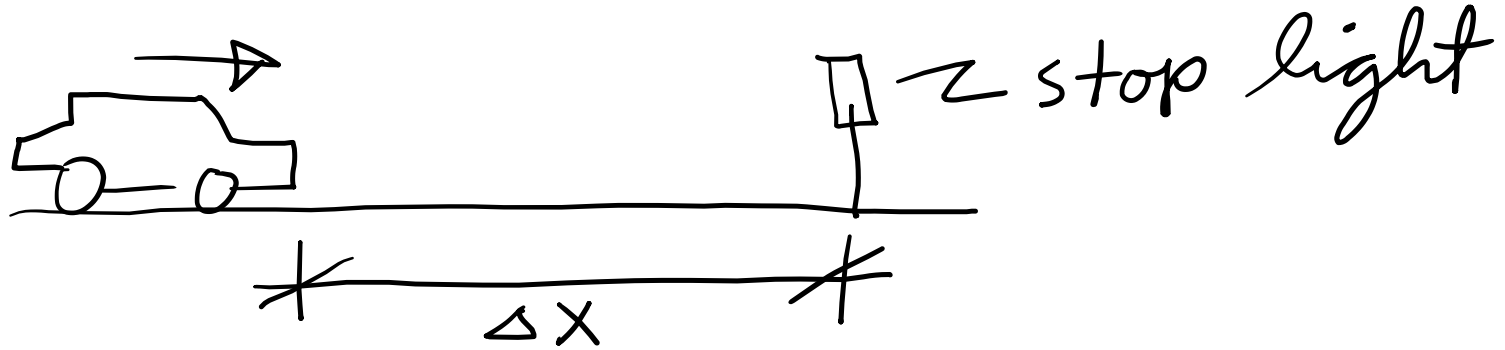
Notes on problems : 11.34



Given: $v_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240\text{m}$, $\Delta t = 24\text{s}$ & $a = \text{const.}$

(a) Find a :

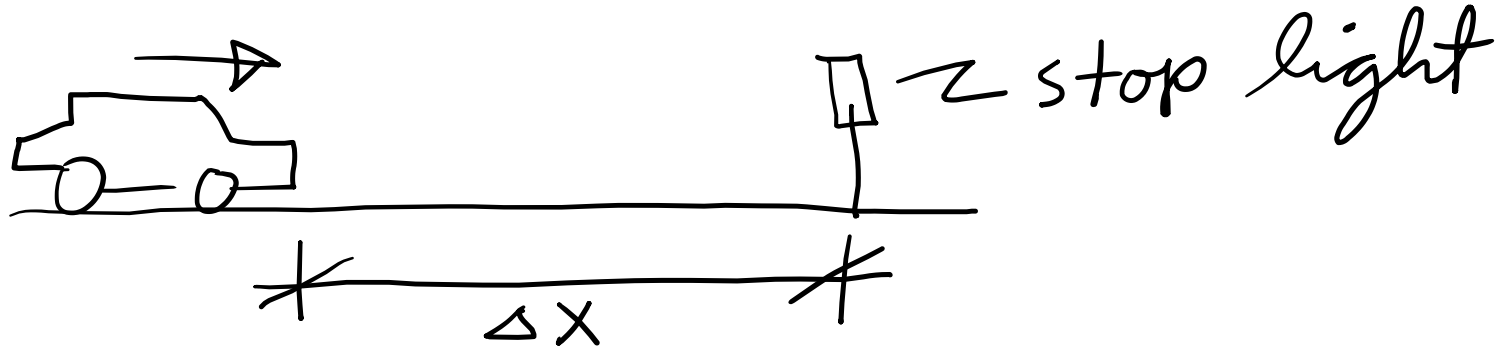
Notes on problems : 11.34



Given: $v_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240\text{m}$, $\Delta t = 24\text{s}$ & $a = \text{const.}$

(a) Find a : We know $x = \frac{1}{2}at^2 + v_0t + x_0$
& Δx & Δt

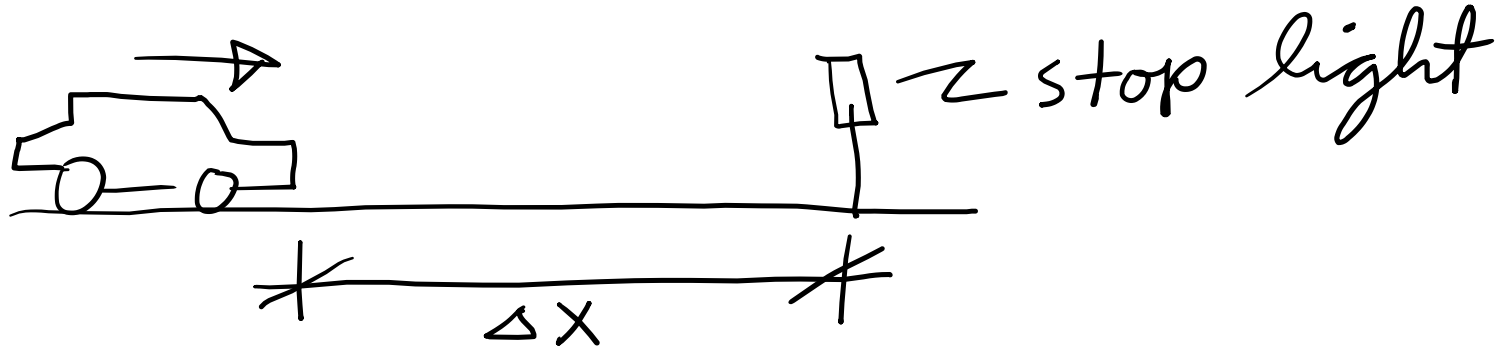
Notes on problems : 11.34



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(a) Find a : We know $x = \frac{1}{2}at^2 + v_0t + x_0$
& Δx & $\Delta t \Rightarrow$ just need to dig
out a

Notes on problems : 11.34

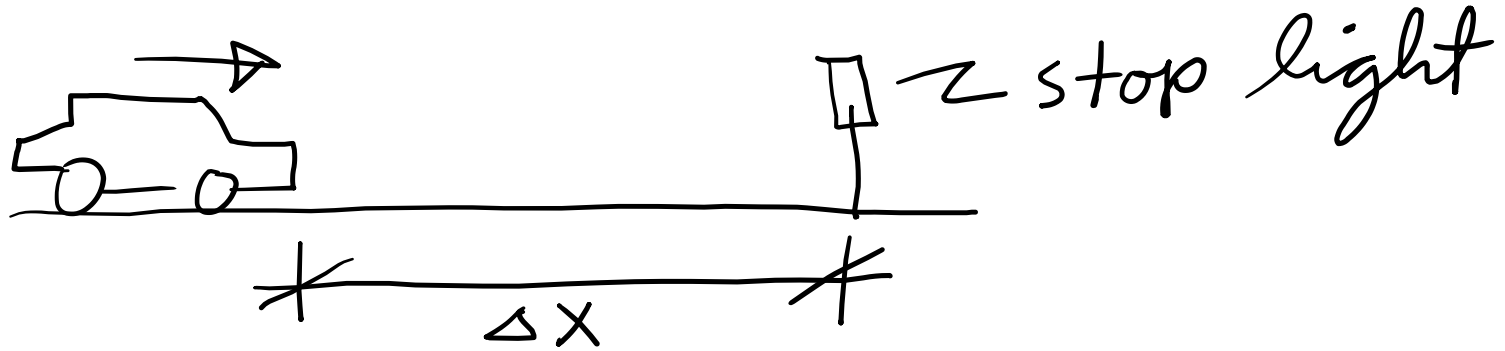


Given: $v_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240\text{m}$, $\Delta t = 24\text{s}$ & $a = \text{const.}$

(a) Find a : We know $x = \frac{1}{2}at^2 + v_0t + x_0$
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(b) Find v at $t = 24\text{s}$:

Notes on problems : 11.34

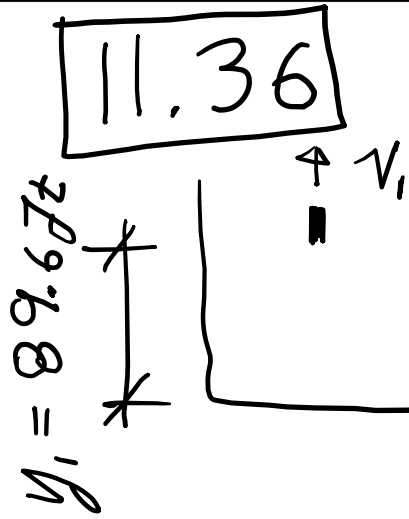


Given: $v_0 = 54 \frac{\text{km}}{\text{hr}}$, $\Delta x = 240\text{m}$, $\Delta t = 24\text{s}$ & $a = \text{const.}$

(a) Find a : We know $x = \frac{1}{2}at^2 + v_0t + x_0$
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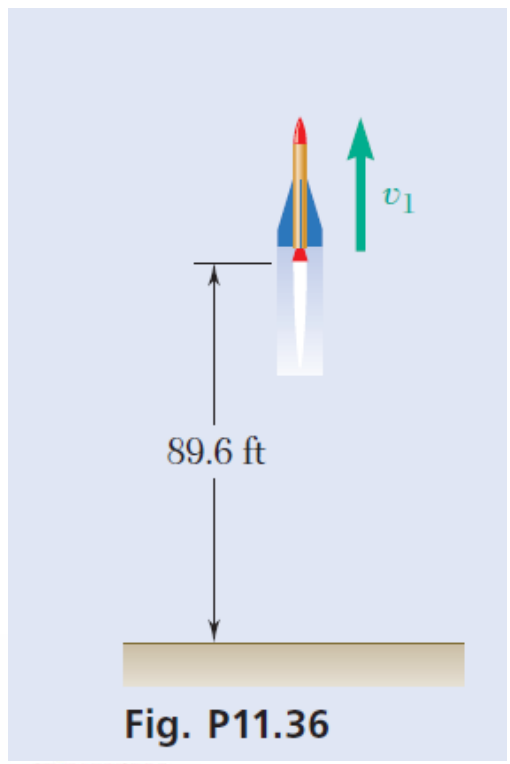
(b) Find v at $t = 24\text{s}$: $v = \frac{dx}{dt} \Rightarrow$

$$v(t=24\text{s}) = \left. \frac{dx}{dt} \right|_{t=24\text{s}}$$



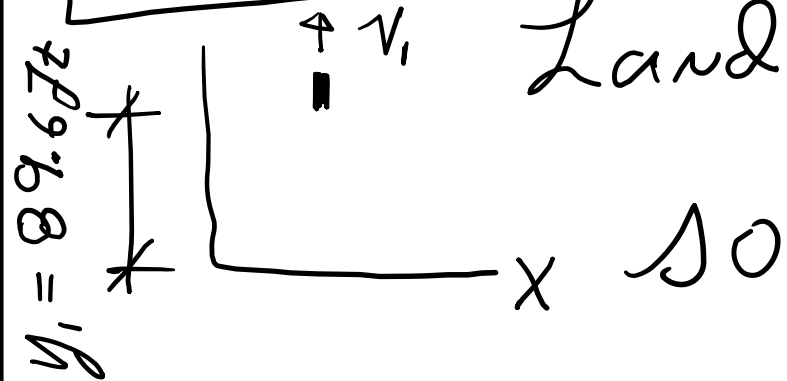
Lands 16 s later

$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$



11.36 A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 32.2 \text{ ft/s}^2$, determine (a) the speed v_1 of the rocket at the end of powered flight, (b) the maximum altitude reached by the rocket.

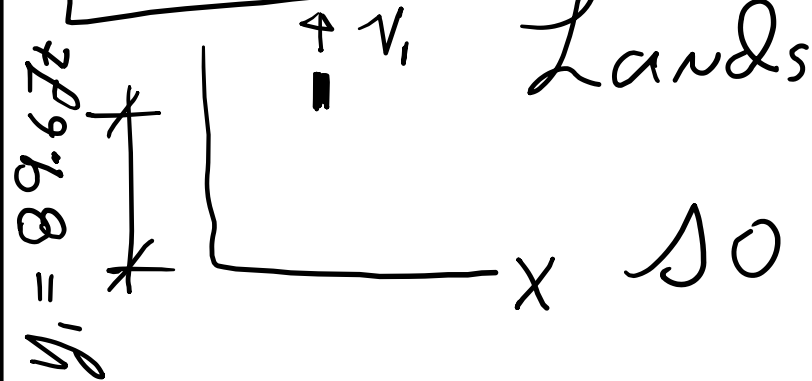
11.36



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

11.36



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

$$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$$

11.36

$$y_i = 89.6 \text{ ft}$$

$\uparrow v_i$

Lands 16s later

so

$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$ we know $t_f, y_f \neq y_i$
so just need to dig out v_i

11.36

$y_i = 89.6 \text{ ft}$

$\uparrow v_i$

Lands 16s later

so

$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

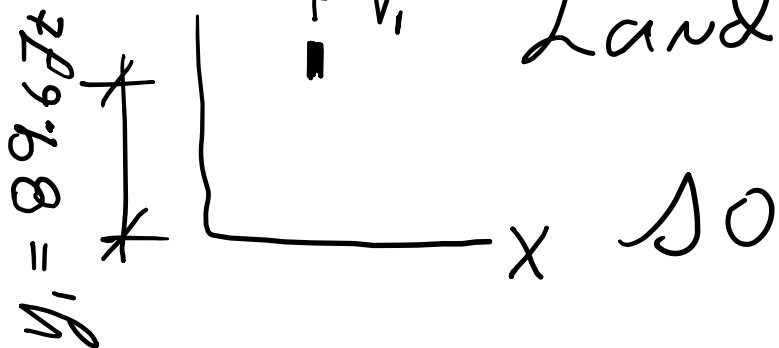
$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$ we know $t_f, y_f \neq y_i$
so just need to dig out v_i

(b) Find y_{max} :

11.36

$\uparrow v_i$

Lands 16s later



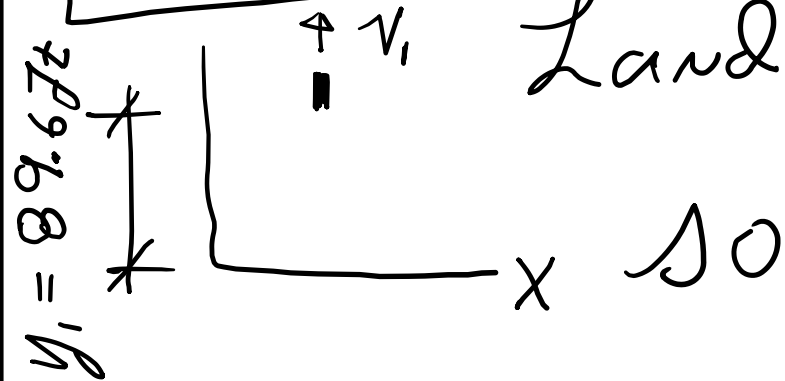
$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$ we know $t_f, y_f \neq y_i$
so just need to dig out v_i

(b) Find y_{max} : 2 ways

11.36



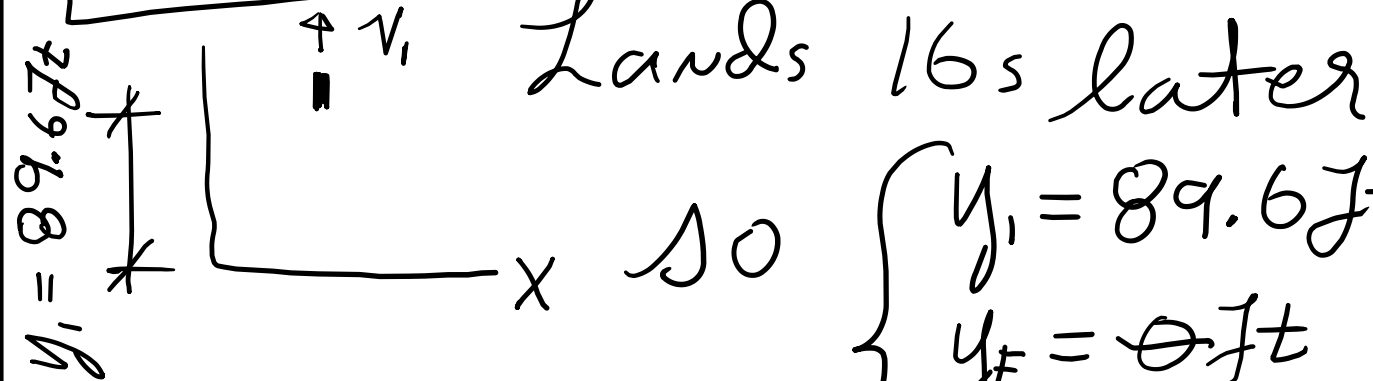
$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$ we know $t_f, y_f \neq y_i$
so just need to dig out v_i

(b) Find y_{\max} : 2 ways: ① y_{\max} when $v = 0$
 \Rightarrow Find t at $v = 0$ then use t to get y_{\max}

11.36



$$\begin{cases} y_1 = 89.6 \text{ ft}, t_1 = 0 \\ y_F = 0 \text{ ft}, t_F = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_1

$y_F = -\frac{1}{2}gt^2 + v_1 t_F + y_1$ we know $t_F, y_F \neq y_1$
so just need to dig out v_1

(b) Find y_{max} : 2 ways: ① y_{max} when $v = 0$

\Rightarrow Find t at $v = 0$ then use t to get y_{max} or

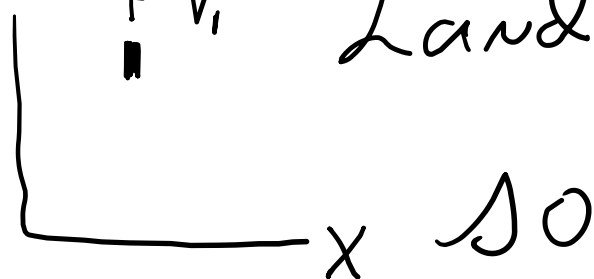
② Use $a = v \frac{dv}{dy} \Rightarrow -g = v \frac{dv}{dy}$

11.36

$y_i = 89.6 \text{ ft}$

$\uparrow v_i$

Lands 16s later



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find v_i

$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$ we know $t_f, y_f \neq y_i$
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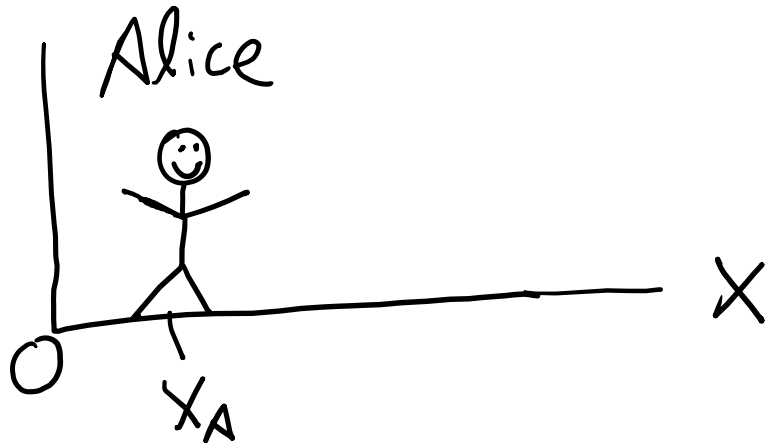
② Use $a = v \frac{dv}{dy} \Rightarrow -g = v \frac{dv}{dy} \Rightarrow -g dy = v dv$

$$\Rightarrow -g \int_{y_i}^{y_{\text{max}}} dy = \int_{v_i}^{v=0} v dv$$

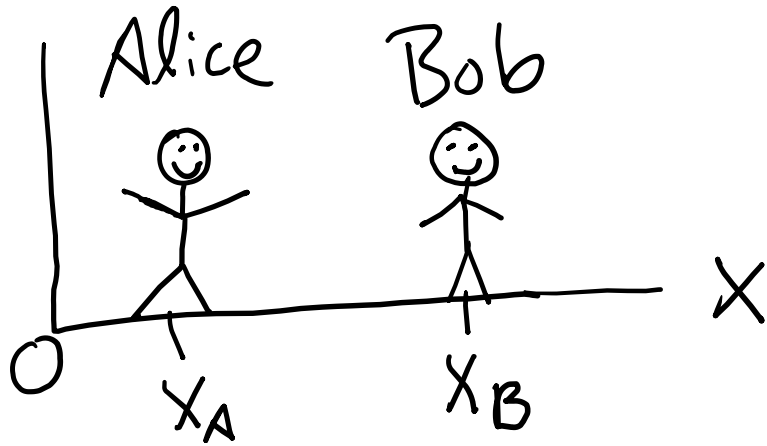


Relative motion

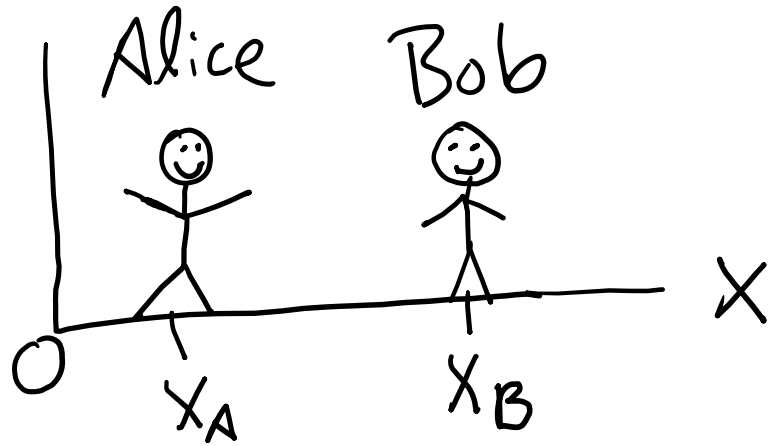
Relative motion



Relative motion



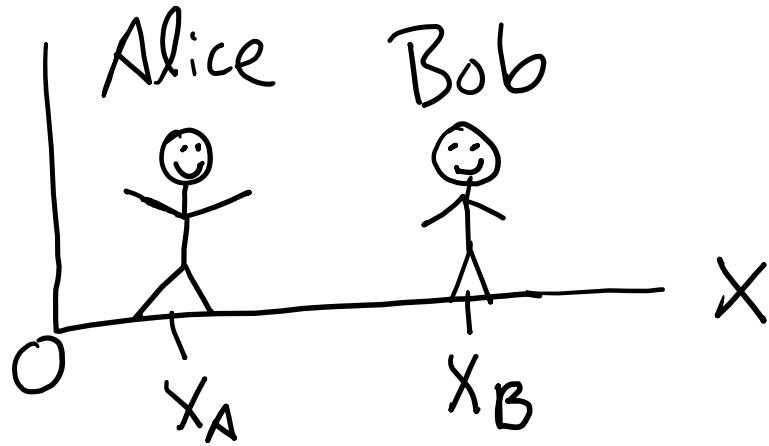
Relative motion



position:

$x_{B/A} \equiv x$ of B relative to A

Relative motion

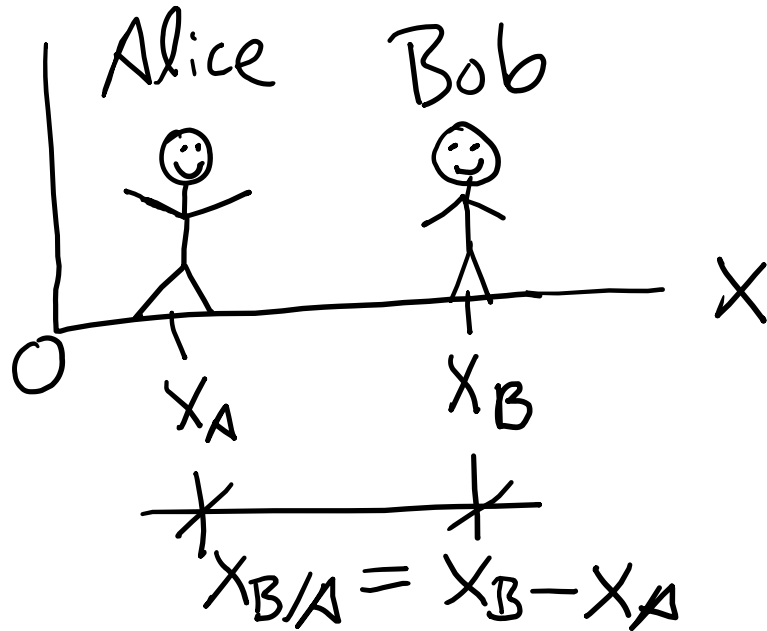


position:

$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Relative motion

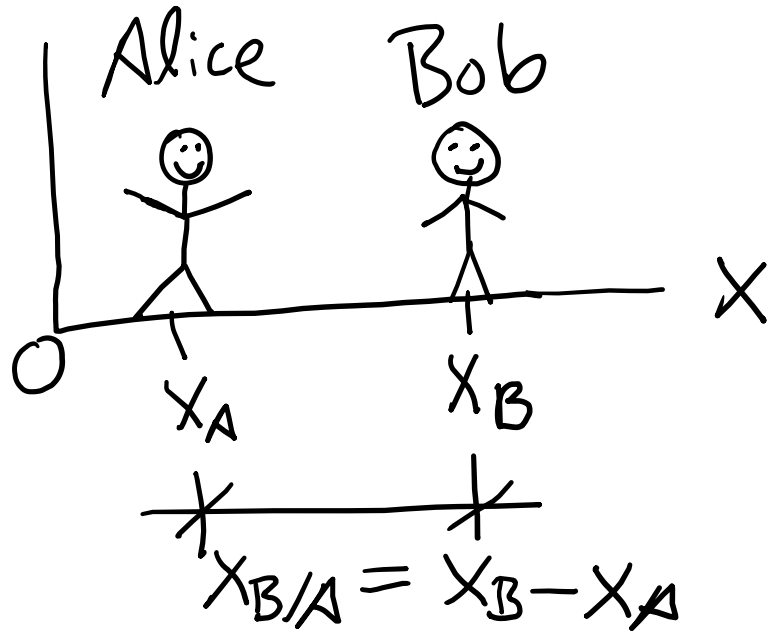


position:

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Relative motion



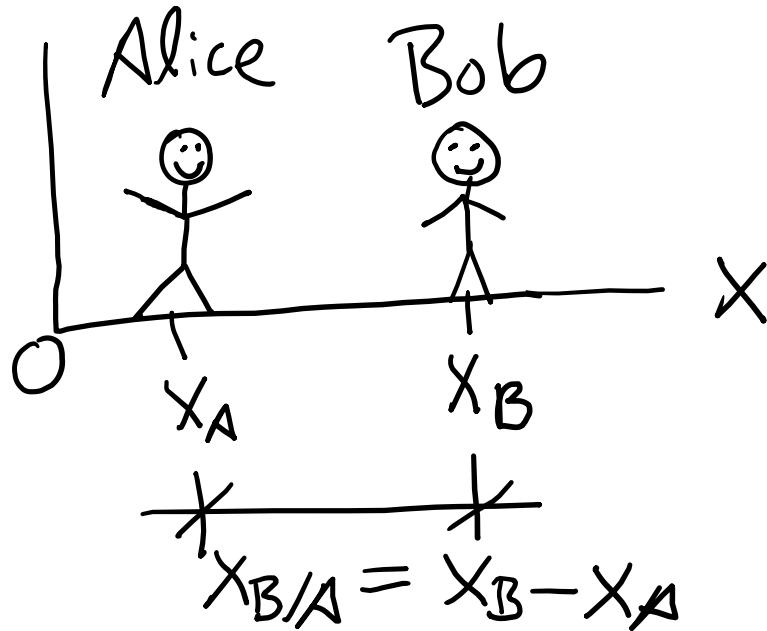
position:

$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:

Relative motion



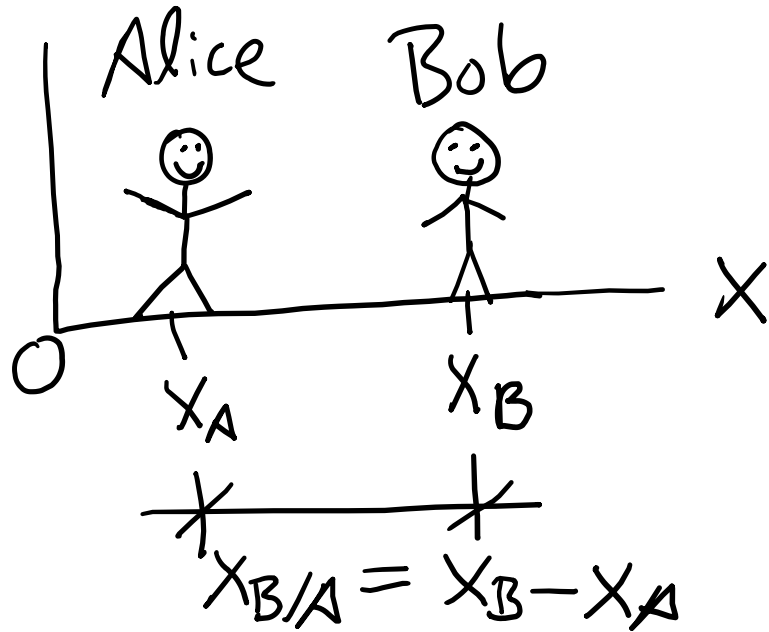
position:

$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity: $v_{B/A} = v_B - v_A$

Relative motion



position:

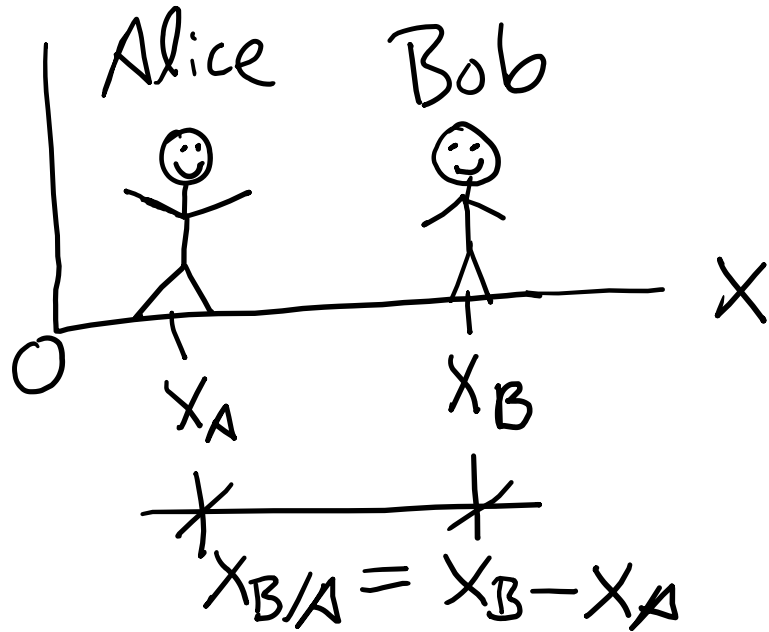
$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity: $v_{B/A} = v_B - v_A$

acceleration:

Relative motion



position:

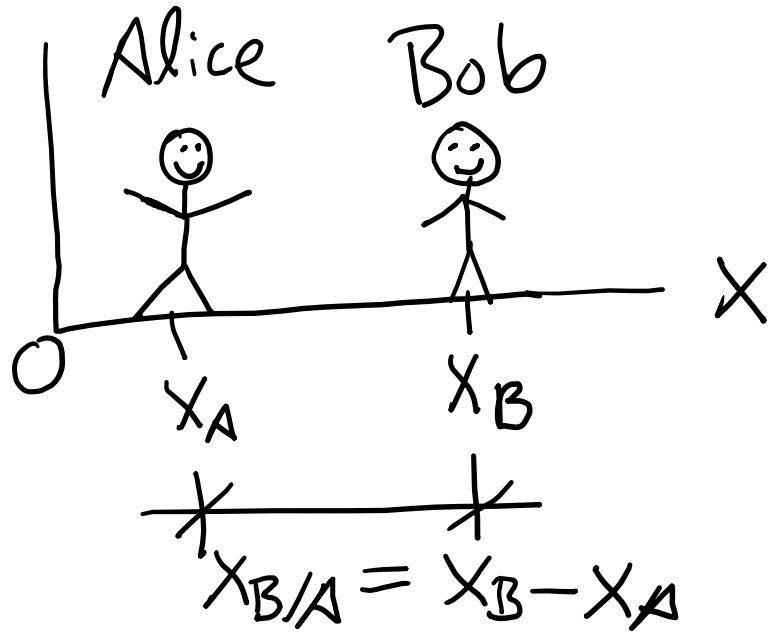
$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity: $v_{B/A} = v_B - v_A$

acceleration: $a_{B/A} = a_B - a_A$

Relative motion



position:

$x_{B/A} \equiv x$ of B relative to A

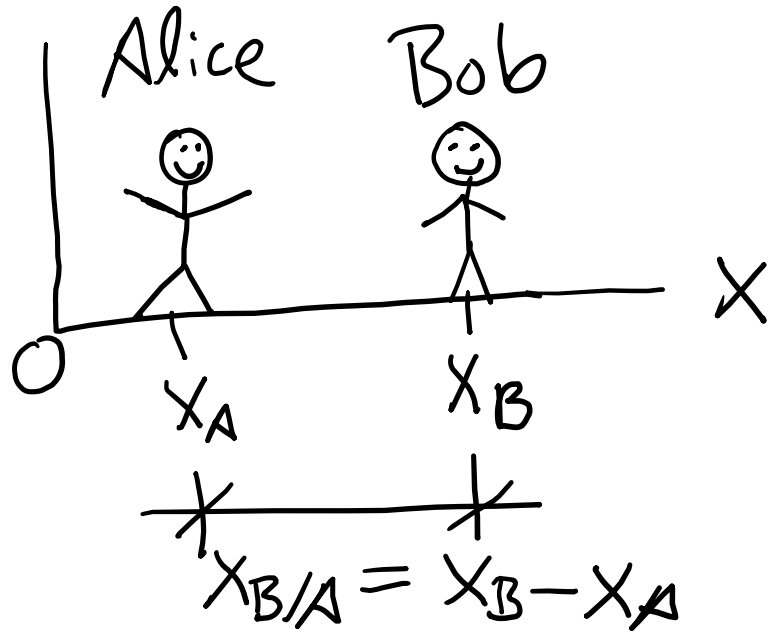
$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity: $v_{B/A} = v_B - v_A$

acceleration: $a_{B/A} = a_B - a_A$

Note: $v_{B/A} = \frac{dx_{B/A}}{dt}$

Relative motion



position:

$x_{B/A} \equiv x$ of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity: $v_{B/A} = v_B - v_A$

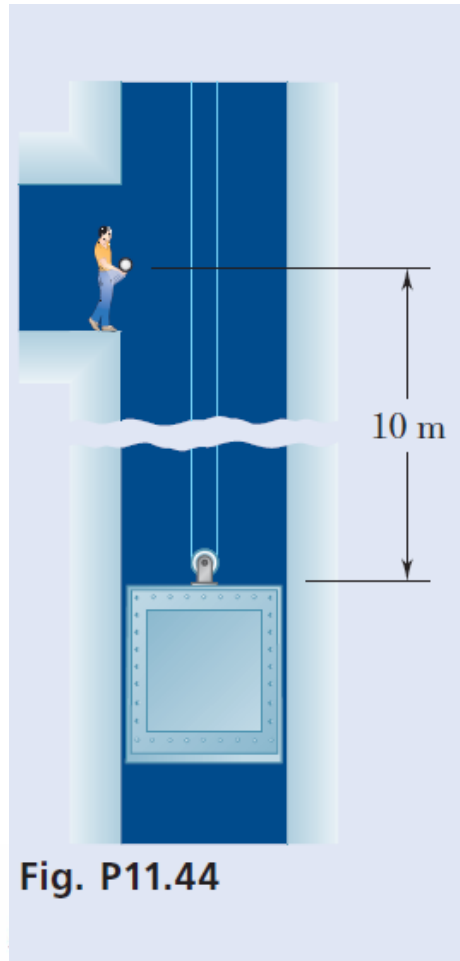
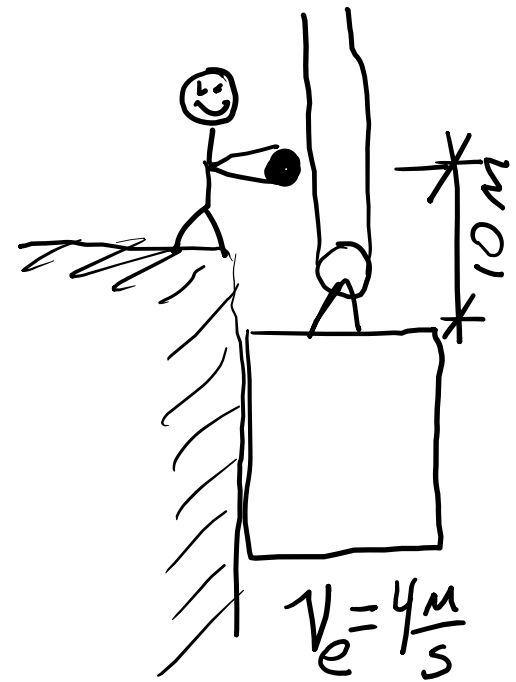
acceleration: $a_{B/A} = a_B - a_A$

Note: $v_{B/A} = \frac{dx_{B/A}}{dt}$ $\&$ $a_{B/A} = \frac{dv_{B/A}}{dt}$

Notes on problem 11.44:

Notes on problem 11.44:

Man throws ball up with initial velocity of $v_{bI} = 3 \frac{m}{s}$



- 11.44** An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (a) when the ball will hit the elevator, (b) where the ball will hit the elevator with respect to the location of the man.

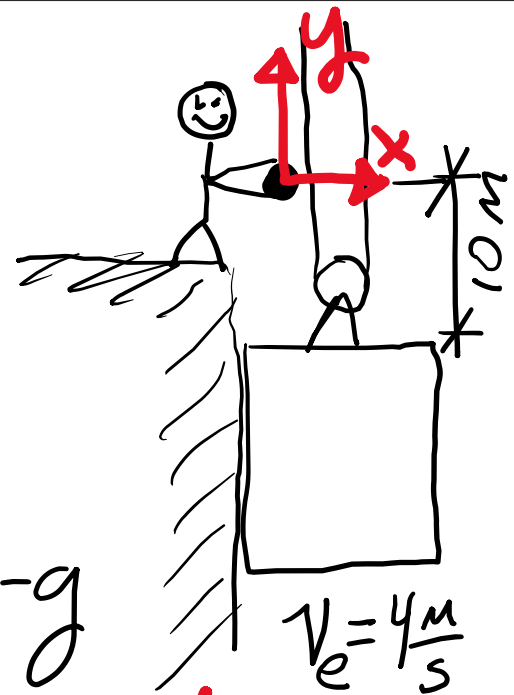
Fig. P11.44

Notes on problem 11.44:

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$$\text{so } \begin{cases} v_e = 4 \frac{m}{s}, & y_{e0} = -10 \text{ m} \\ v_{b0} = 3 \frac{m}{s}, & y_{b0} = 0 \text{ m} \end{cases} \quad \& \quad a_b = -g$$

Note: Setting coordinate system at initial position of ball system at

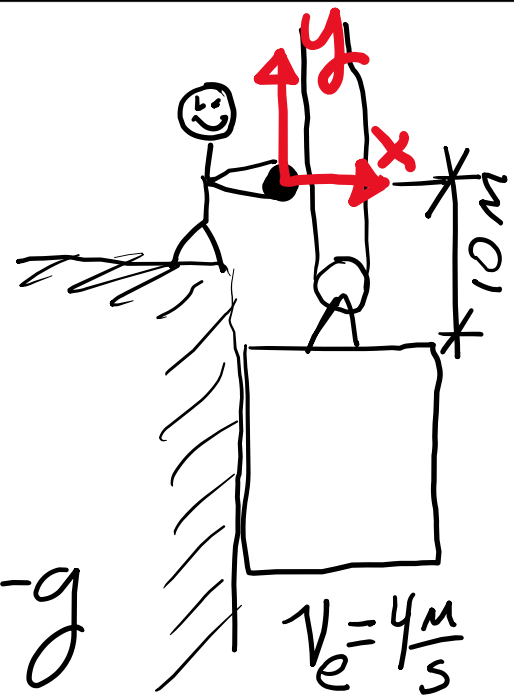


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Can write $y_b = -\frac{1}{2}gt^2 + v_{b0}t + y_{b0}$

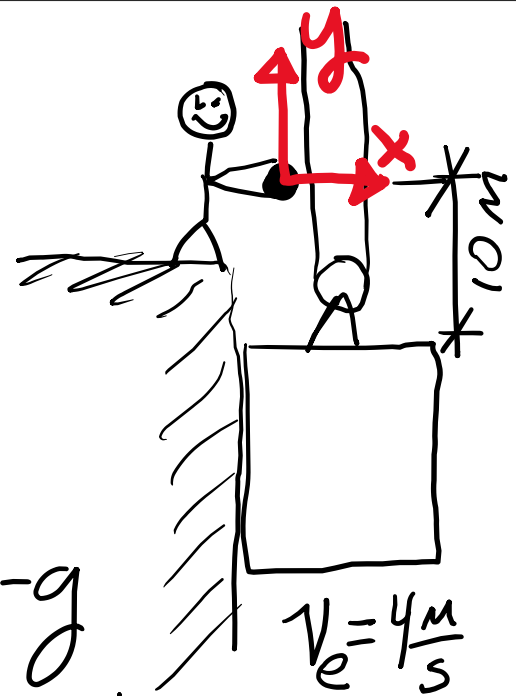


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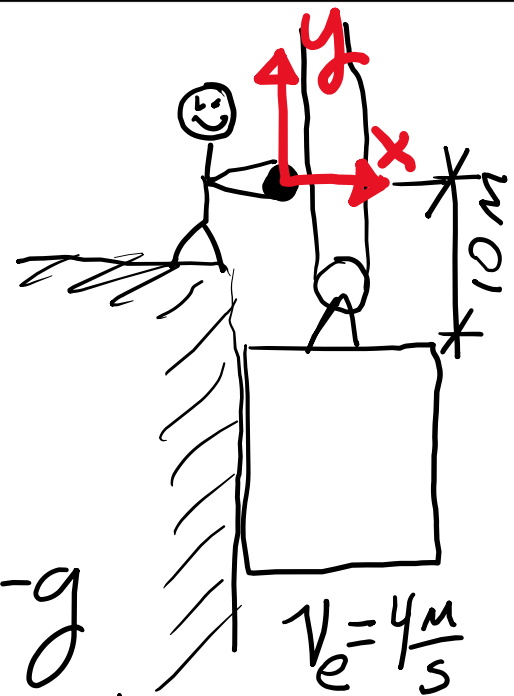
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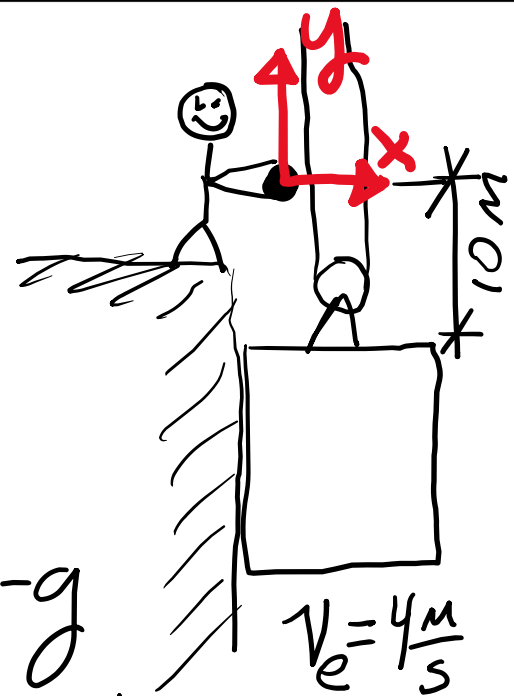
(a) Find t at hit $\equiv t_h$:



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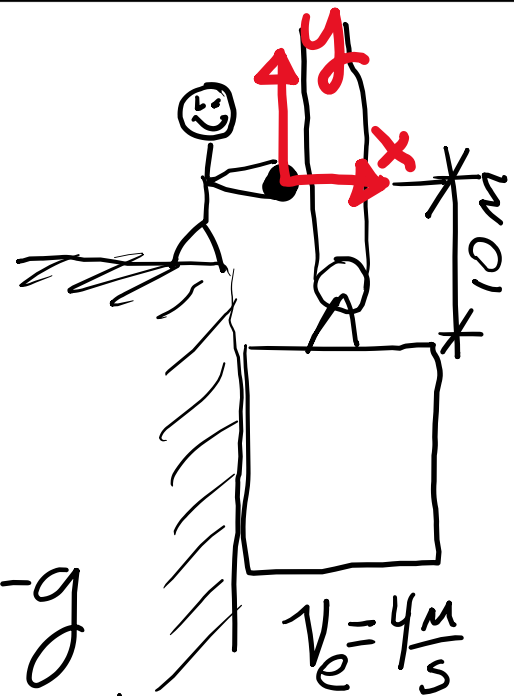
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(b) Find $y_e(t_h)$ or $y_b(t_h)$: Since $y_{b|e} = 0$
 $\Rightarrow y_b = y_e$

