Today : 11.2

$\boxed{L3}$

$\boxed{L3}$

Today : 11.2

Friday : 11.4

L3

Today: 11.2
Friday: 11.4

HW#1: Due Friday

Previously:

Previously: $v = \frac{dx}{dt}$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt}$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

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If $a = \text{const.}$

<u>Previously:</u> $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\quad \frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt}\,dt = a\int dt$

<u>Previously</u>: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\quad \frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

$\Rightarrow \int dv = a \int dt$

Previously: $V = \frac{dx}{dt}$ & $a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\frac{dV}{dt} = a \Rightarrow \int \frac{dV}{dt} dt = a \int dt$

$\Rightarrow \int dV = a \int dt \Rightarrow V - V_0 = at$

Previously: $V = \frac{dx}{dt}$ & $a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\frac{dV}{dt} = a \Rightarrow \int \frac{dV}{dt} dt = a \int dt$

$\Rightarrow \int dV = a \int dt \Rightarrow V - V_0 = at \Rightarrow V = at + V_0$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt}dt = a\int dt$

$\Rightarrow \int dv = a\int dt \Rightarrow v - v_0 = at \Rightarrow v = at + v_0$

But $v = \frac{dx}{dt}$

Previously: $V = \frac{dx}{dt}$ & $a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\quad \frac{dV}{dt} = a \Rightarrow \int \frac{dV}{dt} dt = a \int dt$

$\Rightarrow \int dV = a \int dt \Rightarrow V - V_0 = at \Rightarrow V = at + V_0$

But $V = \frac{dx}{dt}$ so $\quad \int \frac{dx}{dt} dt = \int (at + V_0) dt$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $a = $ const. $\frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

$\Rightarrow \int dv = a \int dt \Rightarrow v - v_0 = at \Rightarrow v = at + v_0$

But $v = \frac{dx}{dt}$ so $\int \frac{dx}{dt} dt = \int (at + v_0) dt$

$\Rightarrow \int dx = \int (at + v_0) dt$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\quad \frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

$$\Rightarrow \int dv = a \int dt \Rightarrow v - v_0 = at \Rightarrow v = at + v_0$$

But $v = \frac{dx}{dt}$ so $\quad \int \frac{dx}{dt} dt = \int (at + v_0) dt$

$$\Rightarrow \int dx = \int (at + v_0) dt \Rightarrow x - x_0 = \frac{1}{2} a t^2 + v_0 t$$

Previously: $V = \frac{dx}{dt}$ & $a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

If $a = \text{const.}$ $\quad \frac{dV}{dt} = a \Rightarrow \int \frac{dV}{dt}\,dt = a\int dt$

$\Rightarrow \int dV = a\int dt \Rightarrow V - V_0 = at \Rightarrow V = at + V_0$

But $V = \frac{dx}{dt}$ so $\quad \int \frac{dx}{dt}\,dt = \int (at + V_0)\,dt$

$\Rightarrow \int dx = \int (at + V_0)\,dt \Rightarrow X - X_0 = \frac{1}{2}at^2 + V_0 t$

$$\Rightarrow \quad X = \frac{1}{2}at^2 + V_0 t$$

Previously: $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If $\boxed{a = \text{const.}}$ $\quad \frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt}\,dt = a\int dt$

$\Rightarrow \int dv = a\int dt \Rightarrow v - v_0 = at \Rightarrow \boxed{v = at + v_0}$

But $v = \frac{dx}{dt}$ so $\int \frac{dx}{dt}\,dt = \int (at + v_0)\,dt$

$\Rightarrow \int dx = \int (at + v_0)\,dt \Rightarrow x - x_0 = \frac{1}{2}at^2 + v_0 t$

$$\Rightarrow \boxed{x = \frac{1}{2}at^2 + v_0 t}$$

Also $a = \frac{dv}{dt}$ $\&$

Also $a = \frac{dv}{dt}$ & $\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)$

&

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Also $a = \frac{dv}{dt}$ & $\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)$

& $v = \frac{dx}{dt} \Rightarrow a = v\frac{dv}{dx}$

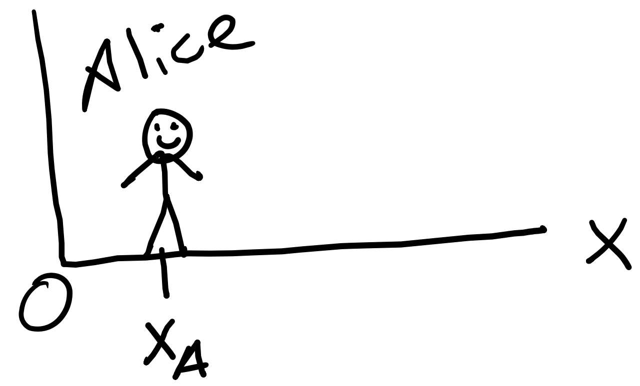
Also $a = \frac{dv}{dt}$ & $\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)$

& $v = \frac{dx}{dt}$ $\Rightarrow$ $a = v\frac{dv}{dx}$ → Important

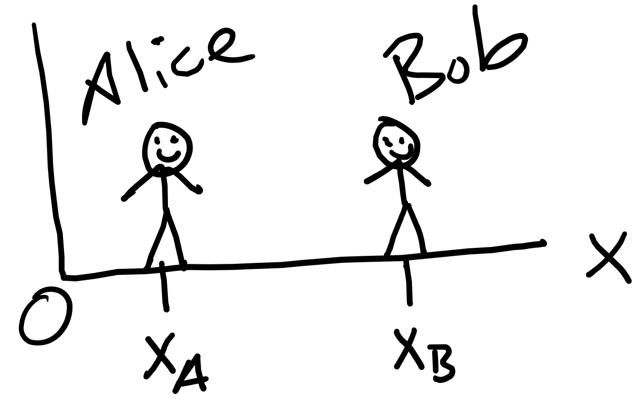
Form for work/energy relation

Relative motion

Relative motion



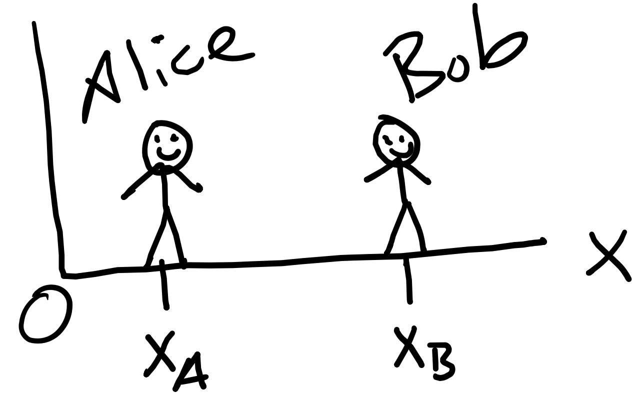
Relative motion



Relative motion

position:

$x_{B/A} \equiv x$ of B relative to A

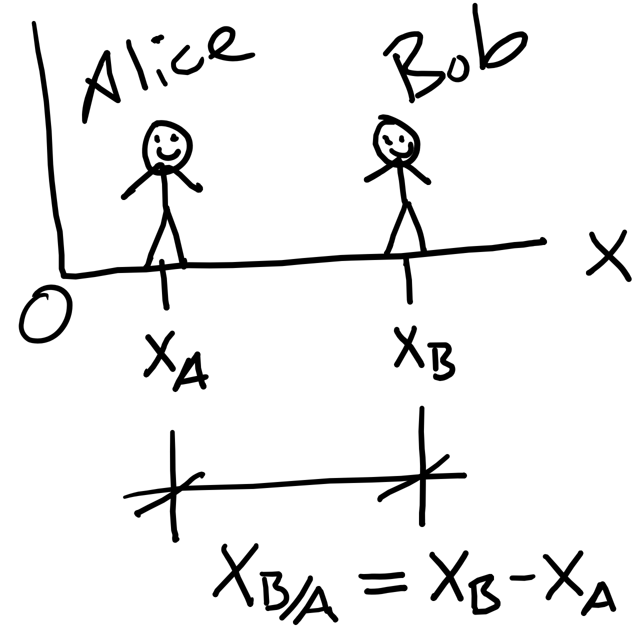


Relative motion

position:

$x_{B/A} \equiv$ x of B relative to A

$x_{B/A} \equiv x_B - x_A$



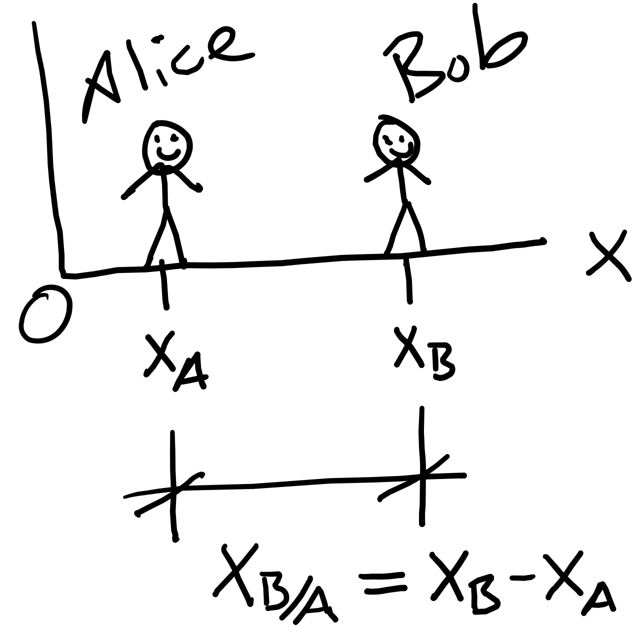
Relative motion

position:

$x_{B/A} \equiv$ x of B relative to A

$x_{B/A} \equiv x_B - x_A$

$x_{B/A}$ is position of Bob as measured by Alice



Relative motion

position:

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$x_{B/A} \equiv x_B - x_A$

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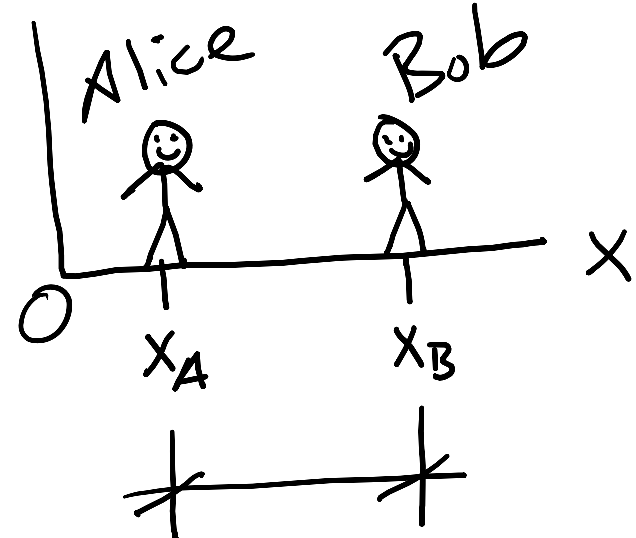


$x_{B/A} = x_B - x_A$

Velocity: $v_{B/A} = v_B - v_A$

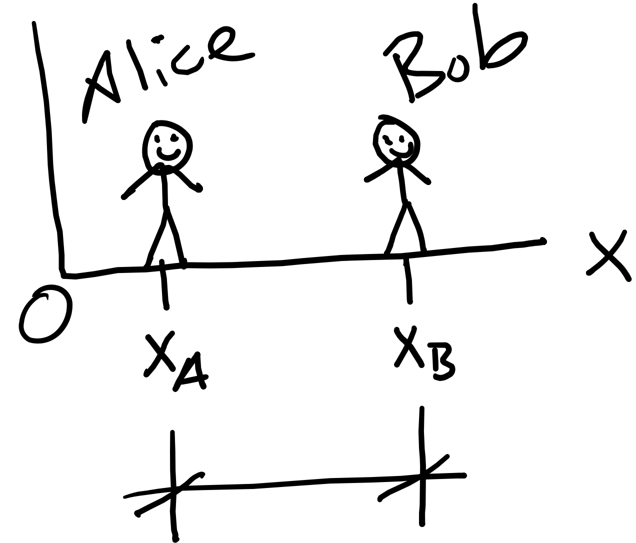
Relative motion

position:

$x_{B/A} \equiv$ x of B relative to A

$x_{B/A} \equiv x_B - x_A$

$x_{B/A}$ is position of Bob as measured by Alice

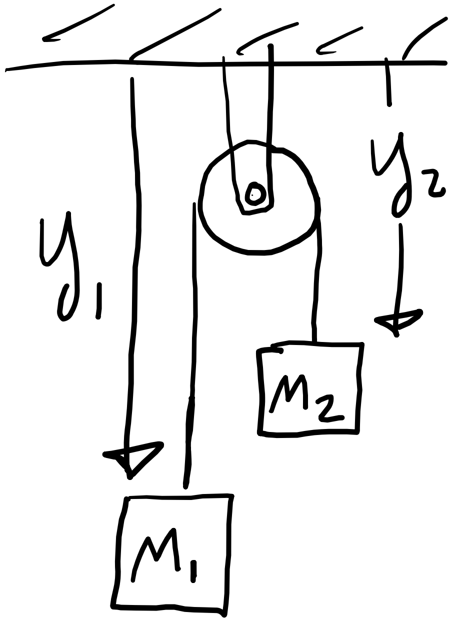


$x_{B/A} = x_B - x_A$

Velocity: $v_{B/A} = v_B - v_A$

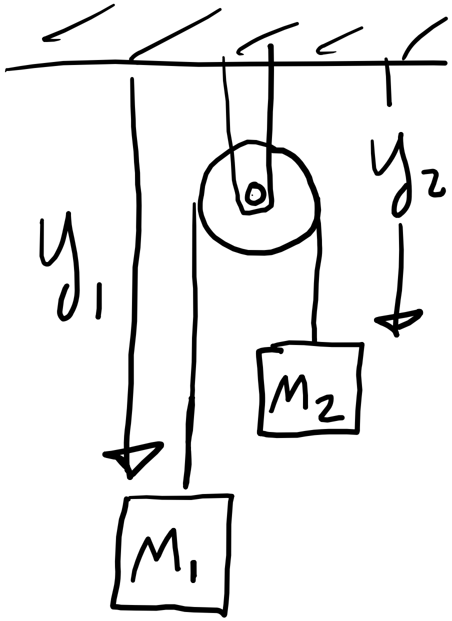
acceleration: $a_{B/A} = a_B - a_A$

Pulley problems



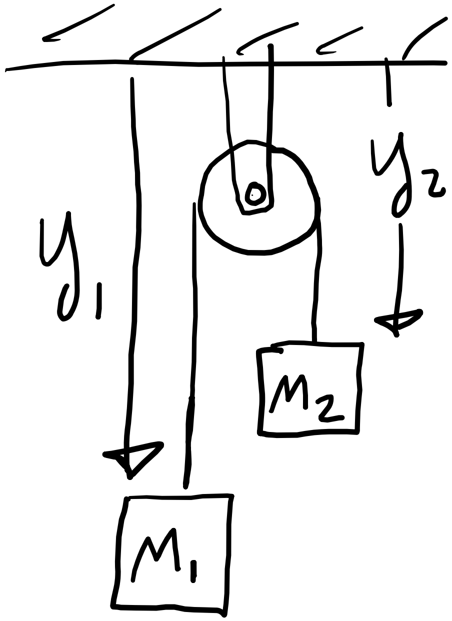
← Super simple example

Pulley problems



For these problems we tend to want the velocity $\frac{dy_i}{dt}$ and/or acceleration $\frac{d^2y_i}{dt^2}$

Pulley problems

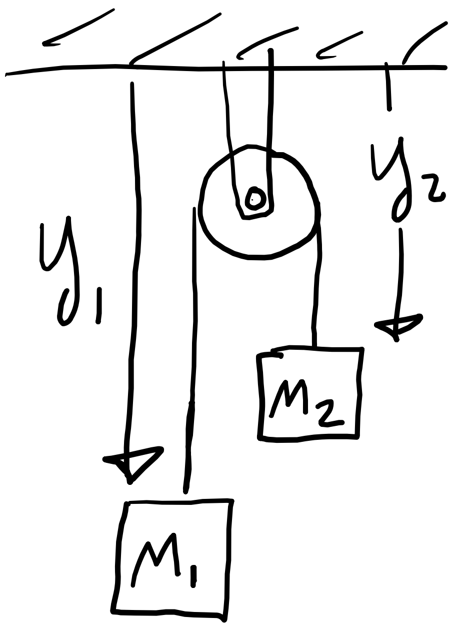


For these problems we tend to want the velocity $\frac{dy_i}{dt}$ and/or acceleration $\frac{d^2y_i}{dt^2}$

Note: If length of rope $\equiv \ell$

& $\ell = y_1 + y_2 + \text{constant}$

Pulley problems



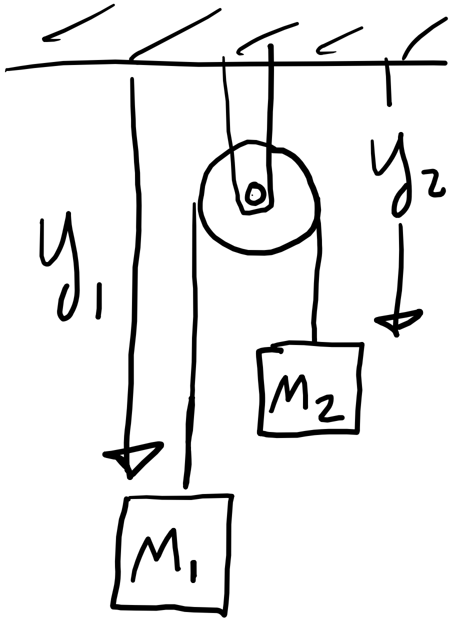
For these problems we tend to want the velocity $\frac{dy_i}{dt}$ and/or acceleration $\frac{d^2y_i}{dt^2}$

Note: If length of rope $\equiv l$ & $l = y_1 + y_2 + \text{constant}$ then

$$\frac{d}{dt} l = \frac{d}{dt}\left[y_1 + y_2 + \text{constant}\right] \Rightarrow$$

$$0 = V_1 + V_2$$

Pulley problems



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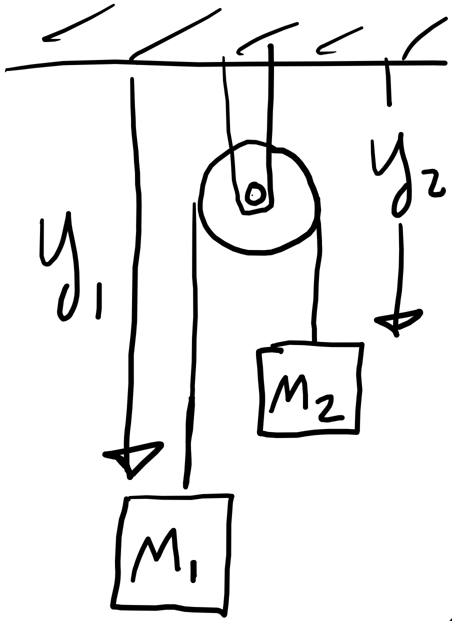
Note: If length of rope $\equiv l$ & $l = y_1 + y_2 + \text{constant}$ then

$$\frac{d}{dt} l = \frac{d}{dt}[y_1 + y_2 + \text{constant}] \Rightarrow$$

$$0 = V_1 + V_2 \Rightarrow V_1 = -V_2$$

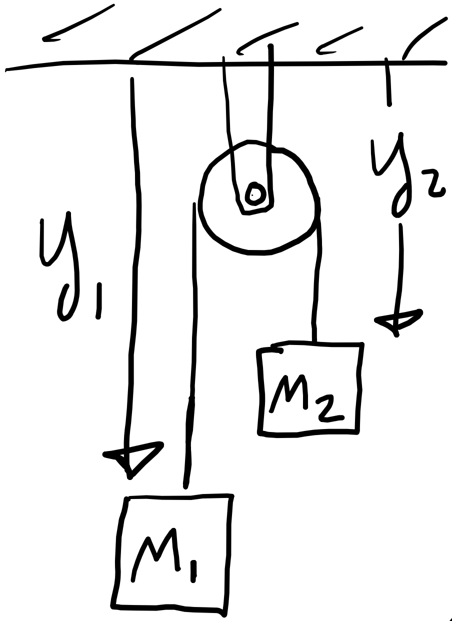
As expected ☺

Pulley problems



We can relate y_1 & y_2 to length of rope (ℓ) & take time derivitive to get rid of any constant lengths [like length ℓ of rope]

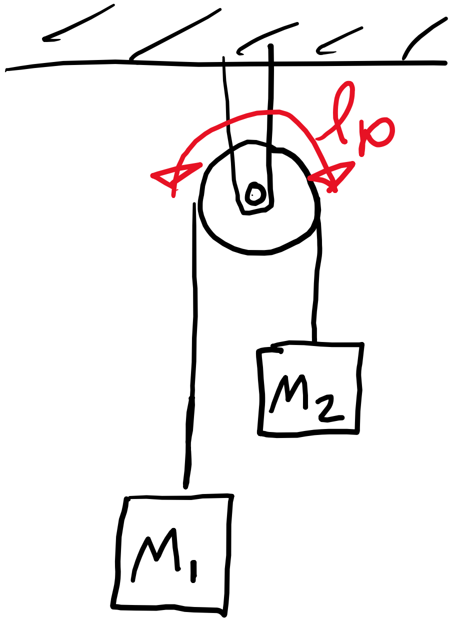
Pulley problems



We can relate y_1 & y_2 to length of rope (ℓ) & take time derivitive to get rid of any constant lengths [like length ℓ of rope].

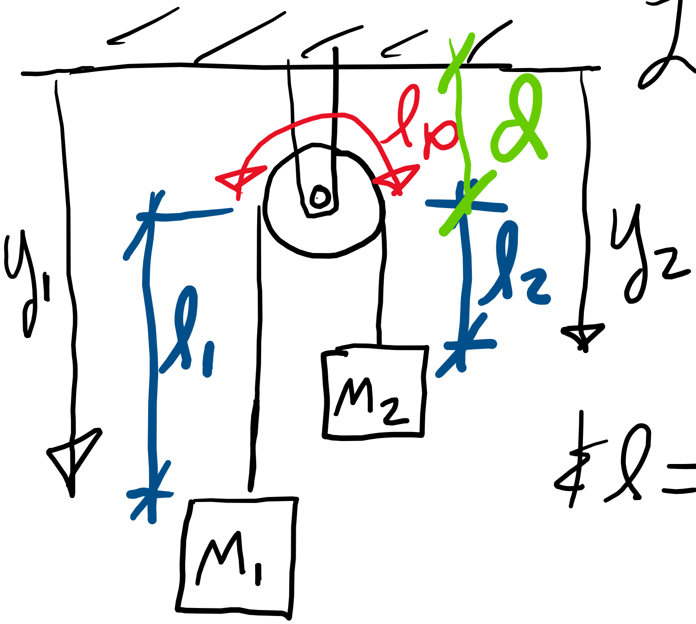
With some practice, we can get fairly quick at these kind of calculations ☺

Pulley problems



Let $l_p \equiv$ length of rope along pulley

Pulley problems

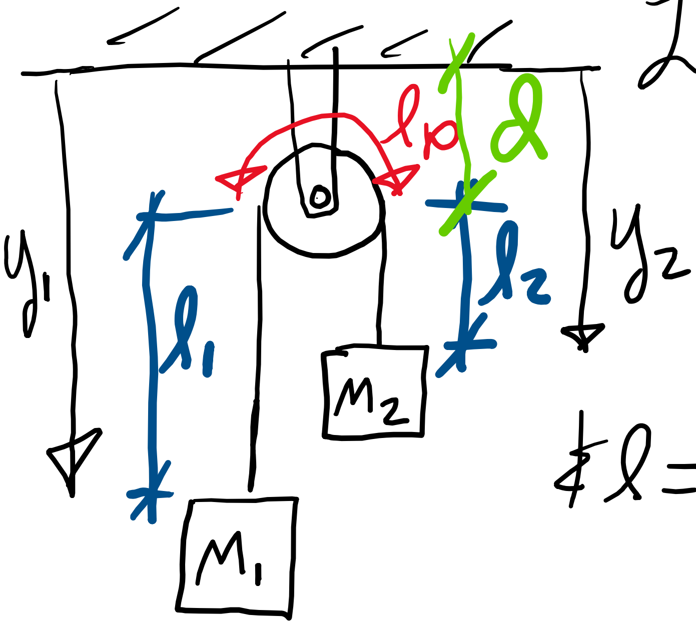


Let $l_p \equiv$ length of rope along pulley

Total length of rope $\equiv l$

$\therefore l = l_1 + l_p + l_2$

Pulley problems

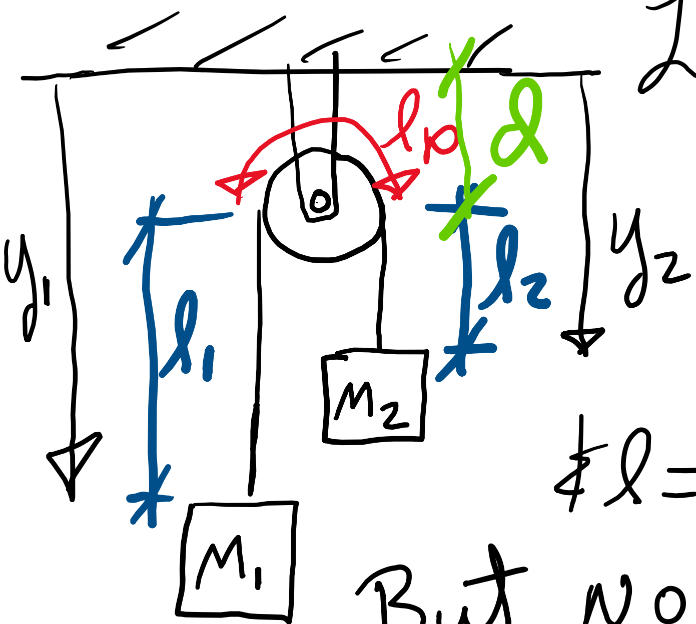


Let $l_p \equiv$ length of rope along pulley

Total length of rope $\equiv l$

& $l = l_1 + l_p + l_2$ <u>or</u> $l = (y_1 - d) + l_p + (y_2 - d)$

Pulley problems



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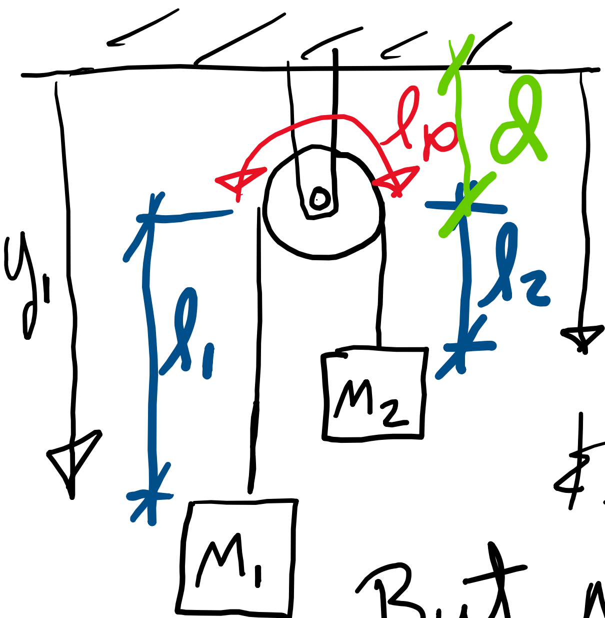
& $l = l_1 + l_p + l_2$ or $l = (y_1 - d) + l_p + (y_2 - d)$

But no matter how m_1 & M_2 move

l_p & d do not change

Pulley problems



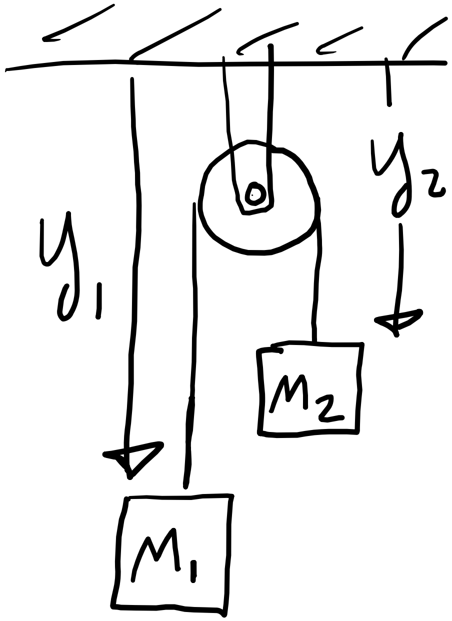
Let $l_p \equiv$ length of rope along pulley

Total length of rope $\equiv l$

$\&\ l = l_1 + l_p + l_2$ **or** $l = (y_1 - d) + l_p + (y_2 - d)$

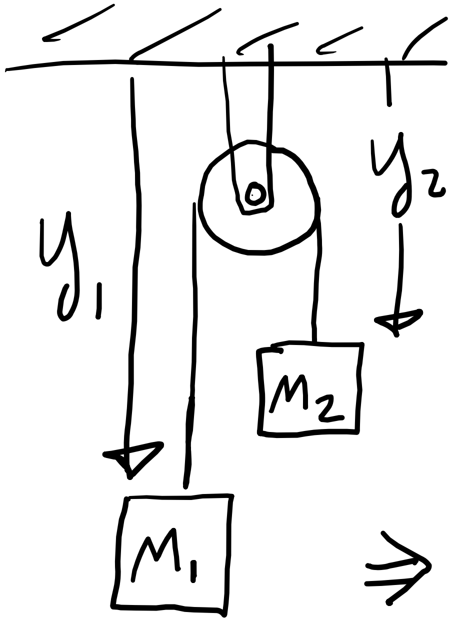
But no matter how m_1 & M_2 move l_p & d do not change. Could simply write $y_1 + y_2 = \text{constant}$ ☺

Pulley problems



We simply neglect those parts of rope that are constant.

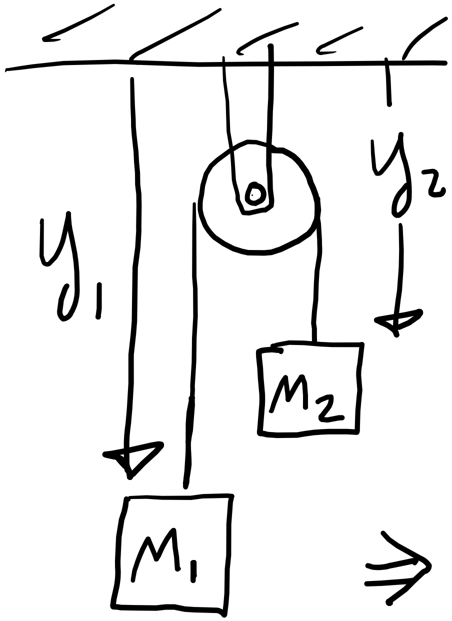
Pulley problems



We simply neglect those parts of rope that are constant. As stated before $\frac{d}{dt}[y_1+y_2] = \frac{d}{dt}[\text{const}]$

$\Rightarrow$ $V_1 = -V_2$

Pulley problems



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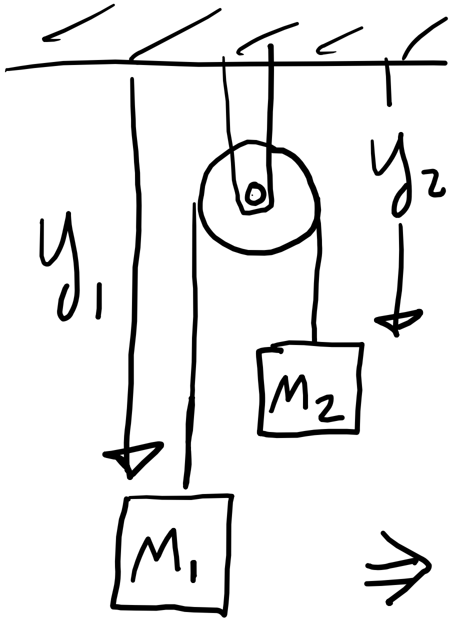
$\Rightarrow \quad V_1 = -V_2$

But we already knew that by just looking at the picture

Pulley problems



We simply neglect those parts of rope that are constant. As stated before $\frac{d}{dt}[y_1+y_2] = \frac{d}{dt}[\text{const}]$

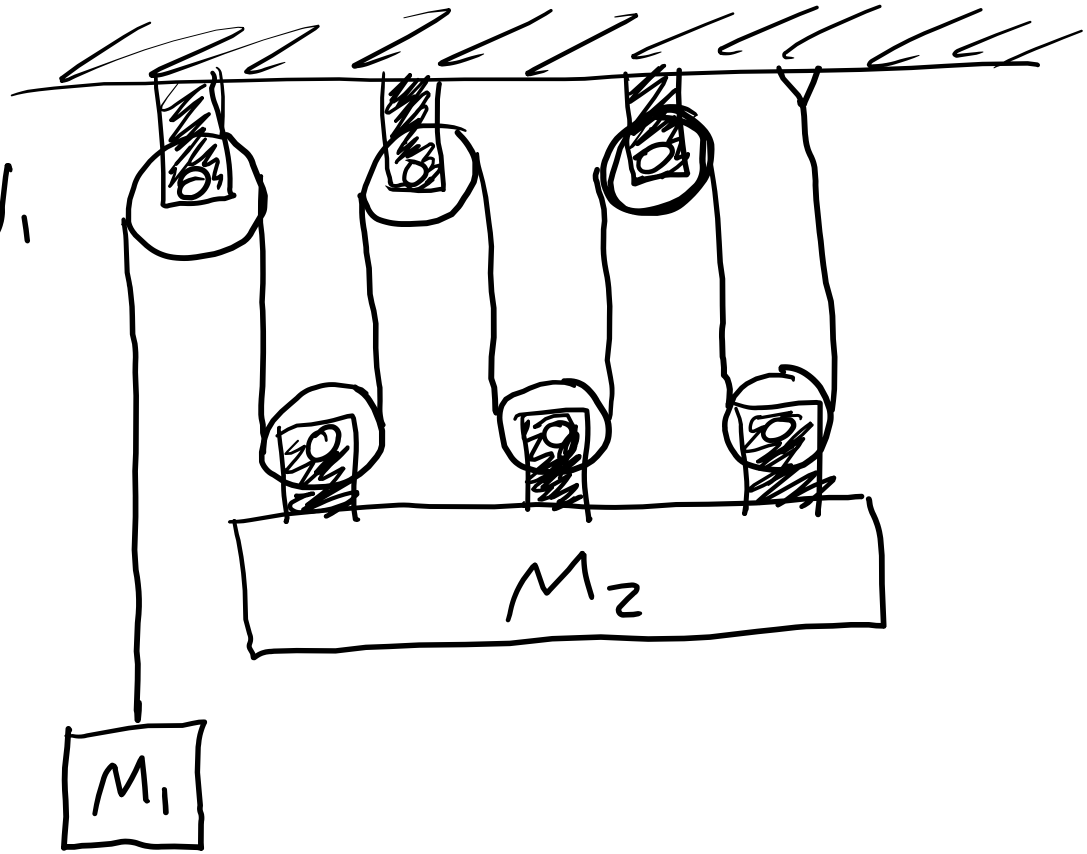
$\Rightarrow V_1 = -V_2$

But we already knew that by just looking at the picture

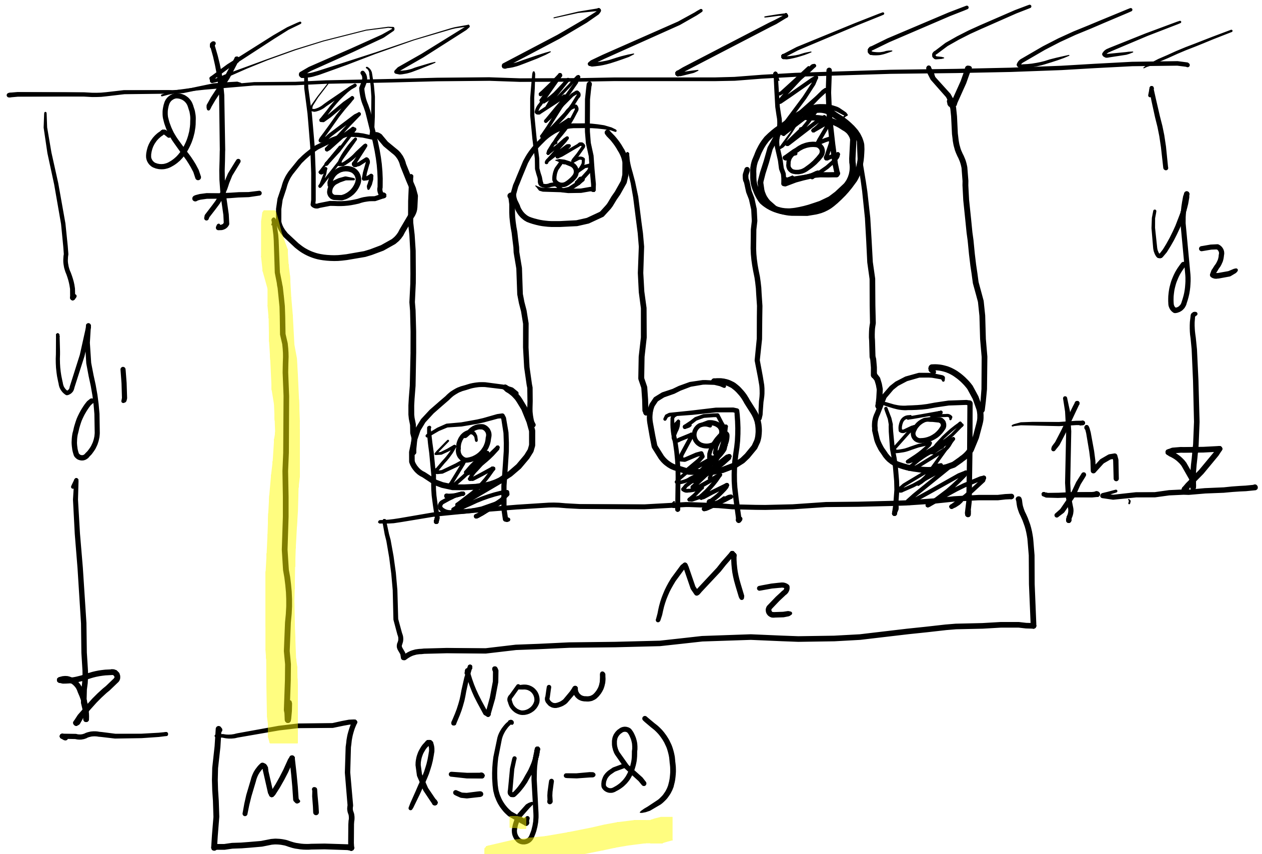
More difficult problem →

Another example

Find
v_2 given v_1



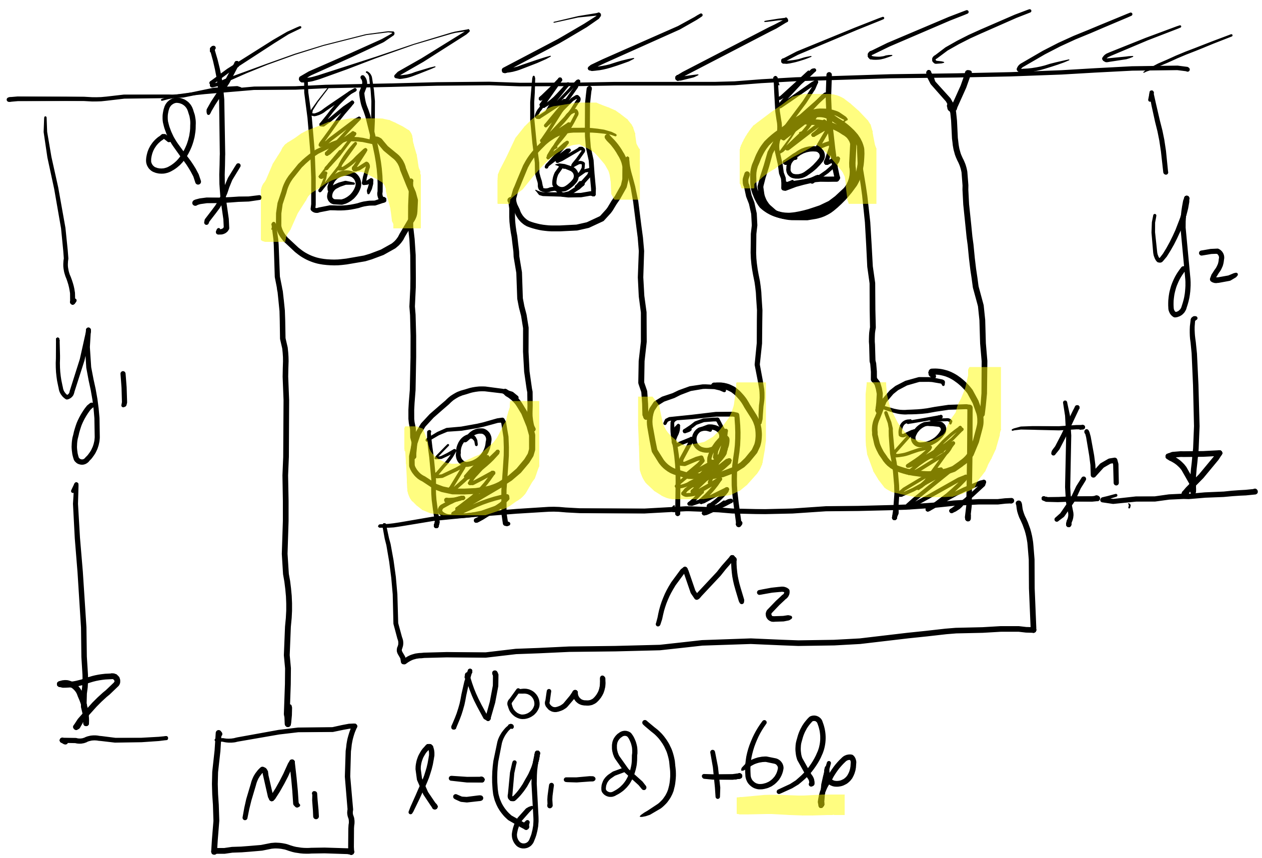
Another example



Now

$l = (y_1 - d)$

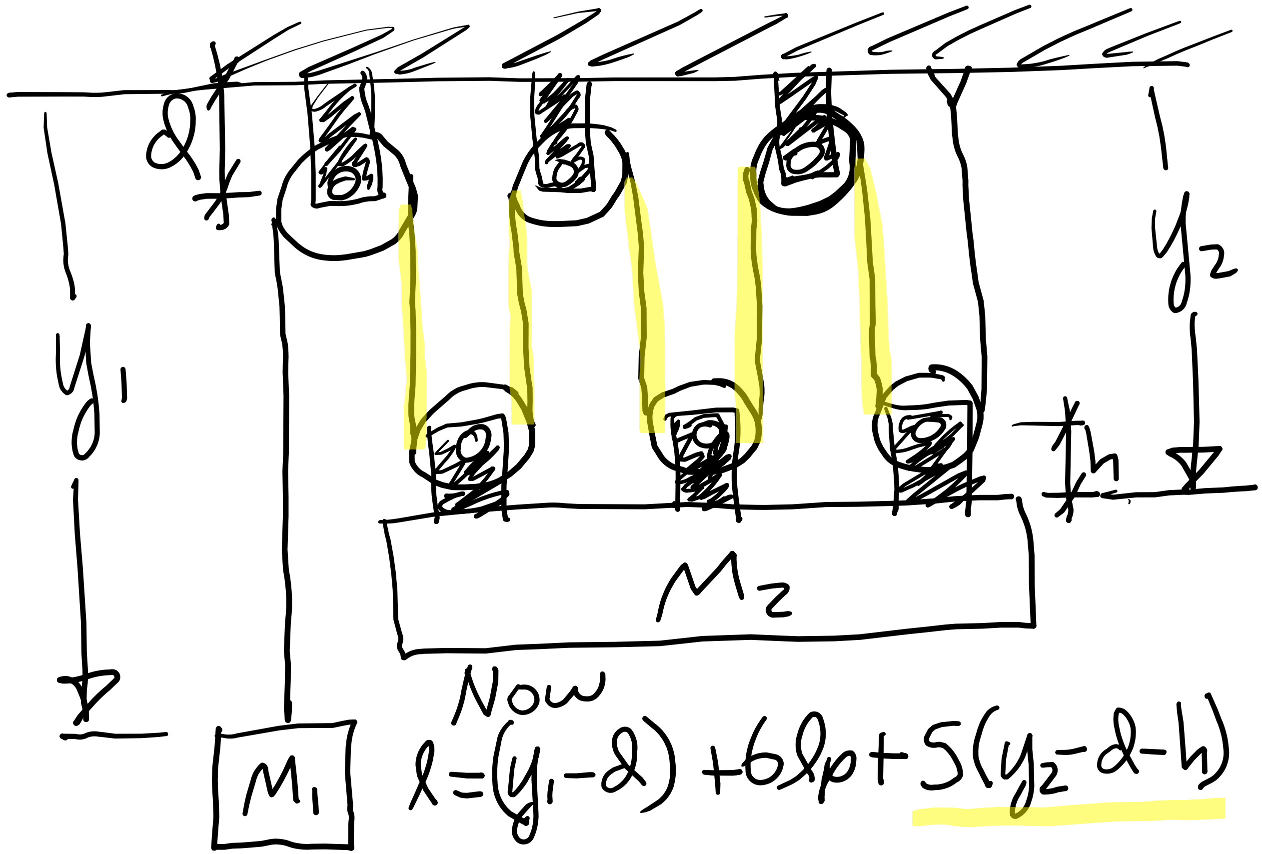
Another example



Now

$l = (y_1 - d) + 6 l_p$

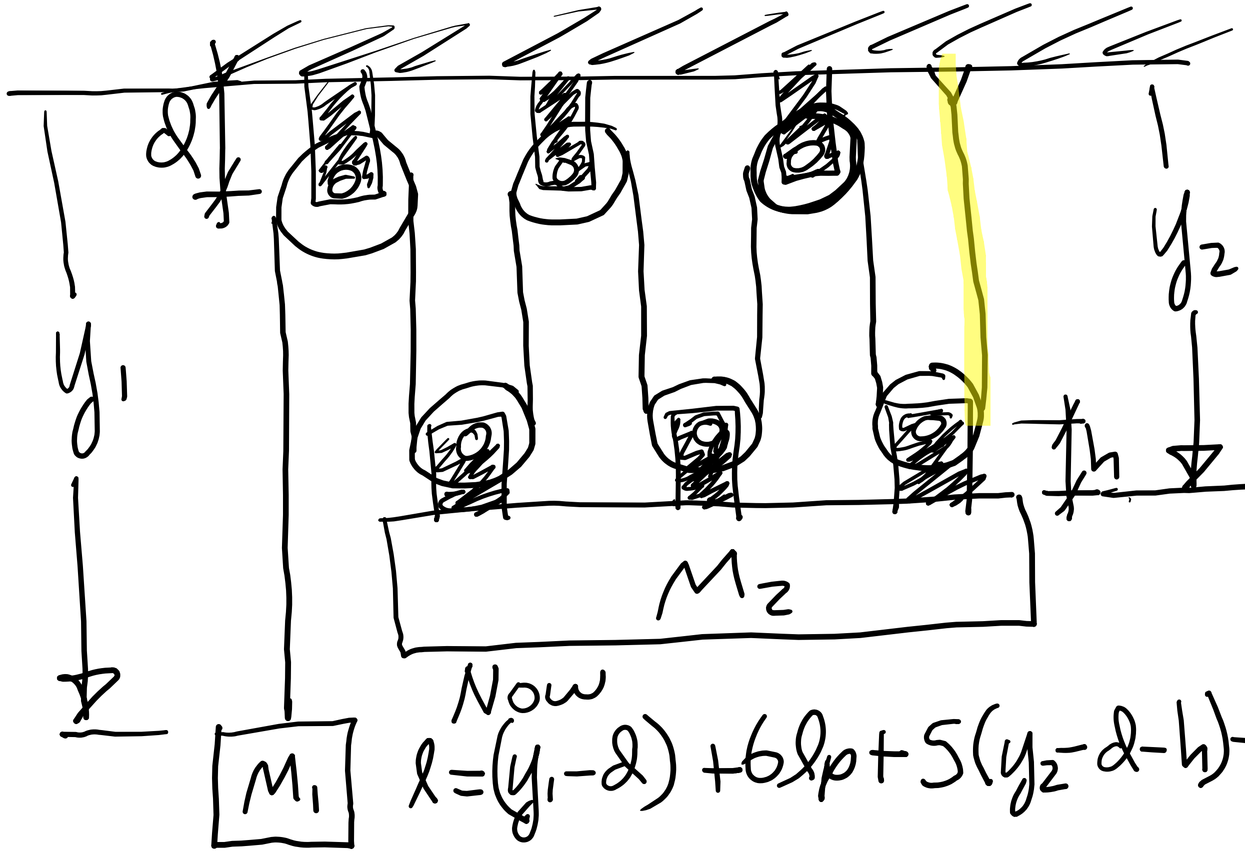
Another example



Now

$$l = (y_1 - d) + 6l_p + \underline{5(y_2 - d - h)}$$

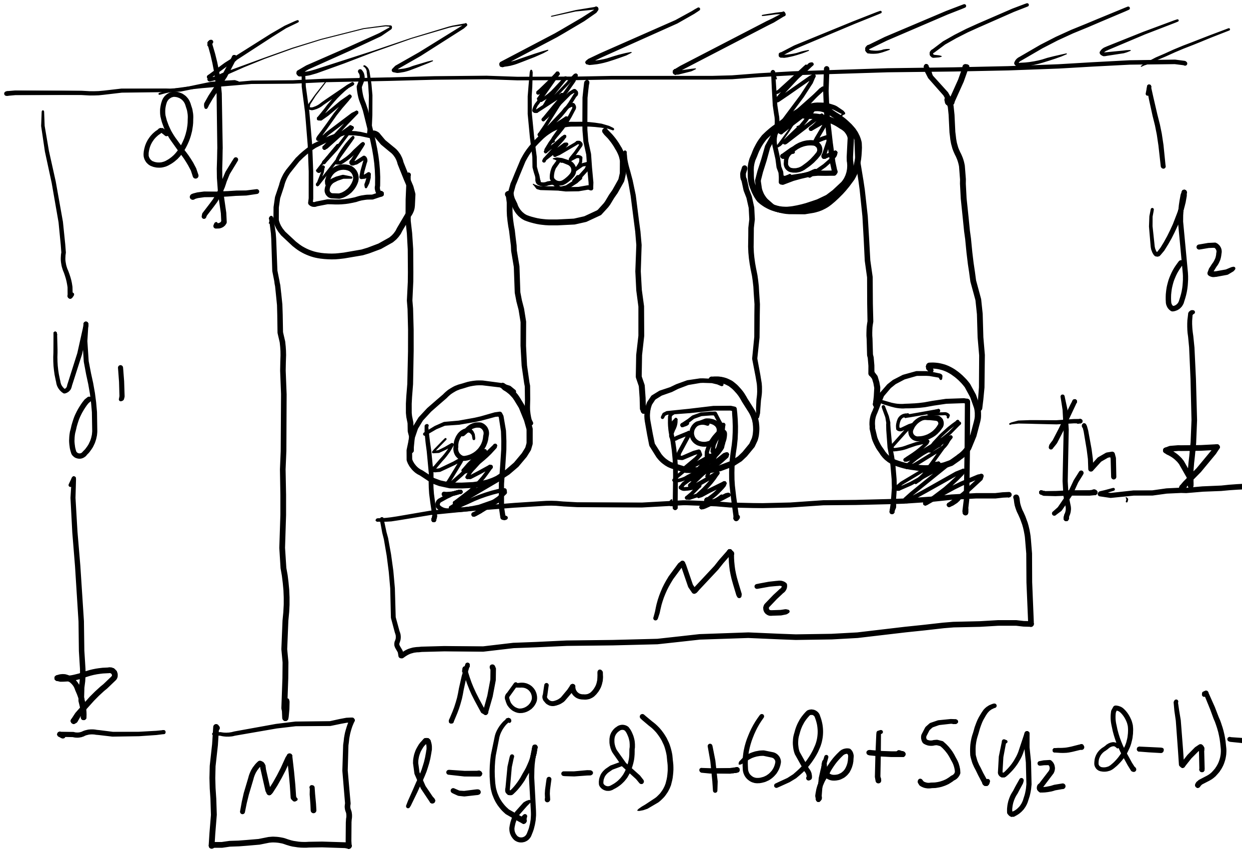
Another example



Now

$$l = (y_1 - d) + 6 l_p + 5(y_2 - d - h) + (y_2 - h)$$

Another example

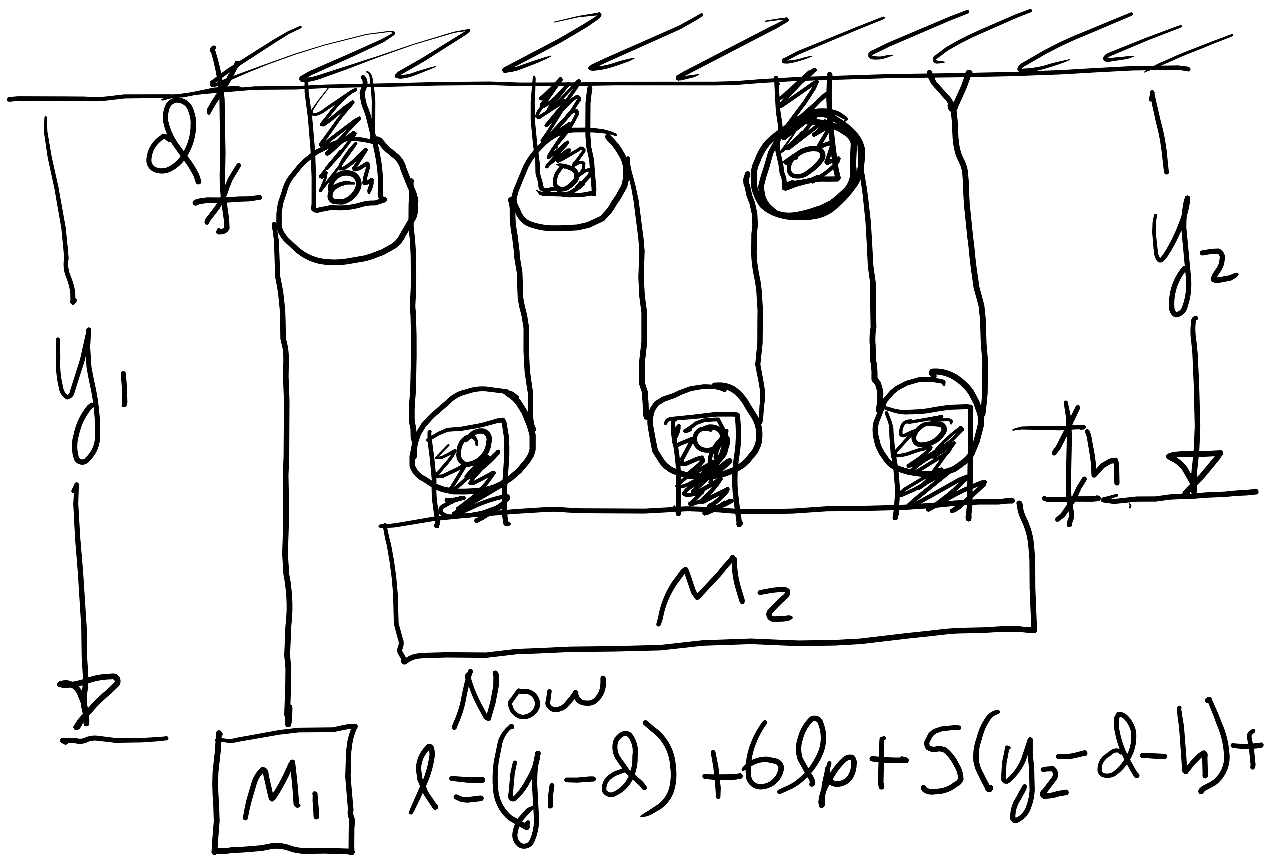


Now

$$l = (y_1 - d) + 6 l_p + 5(y_2 - d - h) + (y_2 - h)$$

OR simply $y_1 + 6 y_2 = \text{const}$

Another example



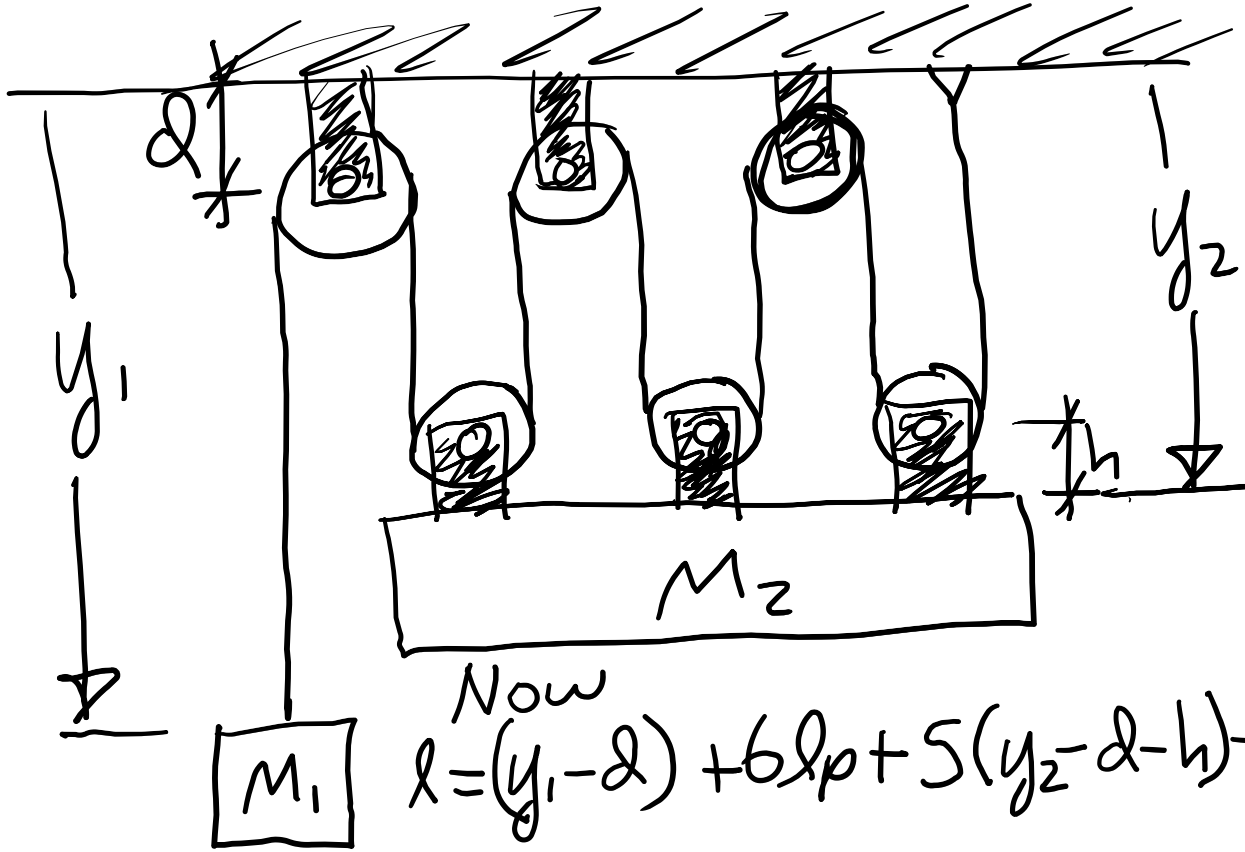
Now

$$l = (y_1 - d) + 6l_p + 5(y_2 - d - h) + (y_2 - h)$$

OR simply $y_1 + 6y_2 = \text{const} \Rightarrow$

$$V_1 + 6V_2 = 0$$

Another example



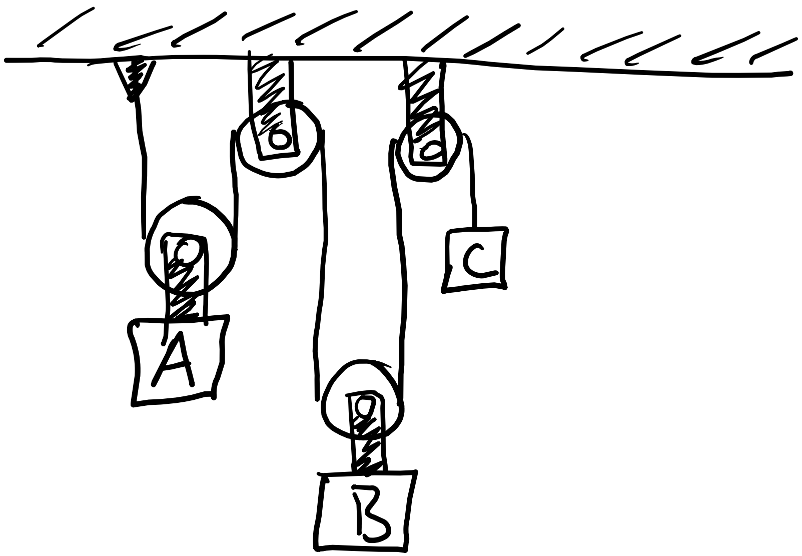
Now $l = (y_1 - d) + 6l_p + 5(y_2 - d - h) + (y_2 - h)$

OR simply $y_1 + 6y_2 = const \Rightarrow$

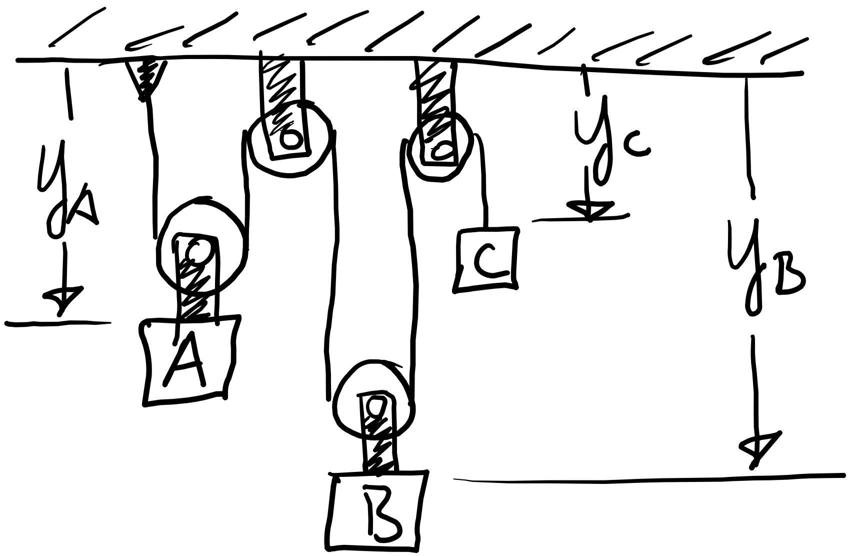
$V_1 + 6V_2 = 0 \Rightarrow V_2 = -\frac{V_1}{6}$

ASU

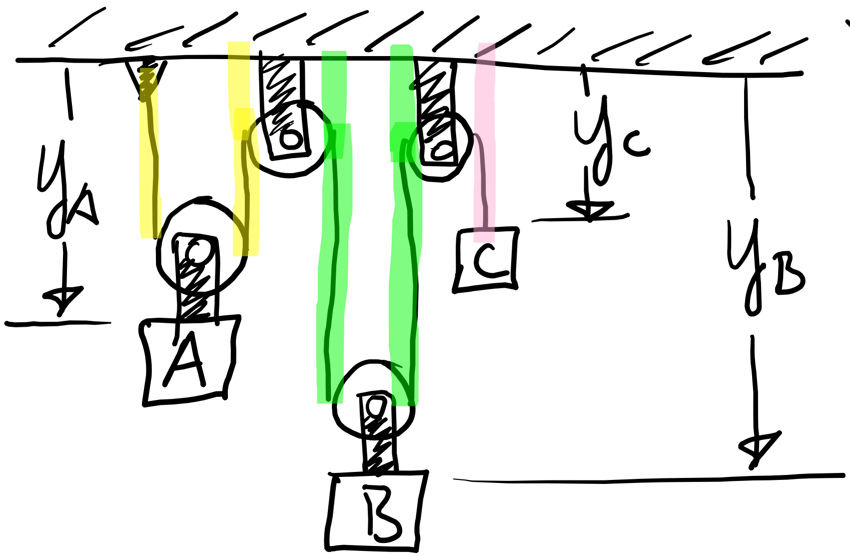
One more example



One more example



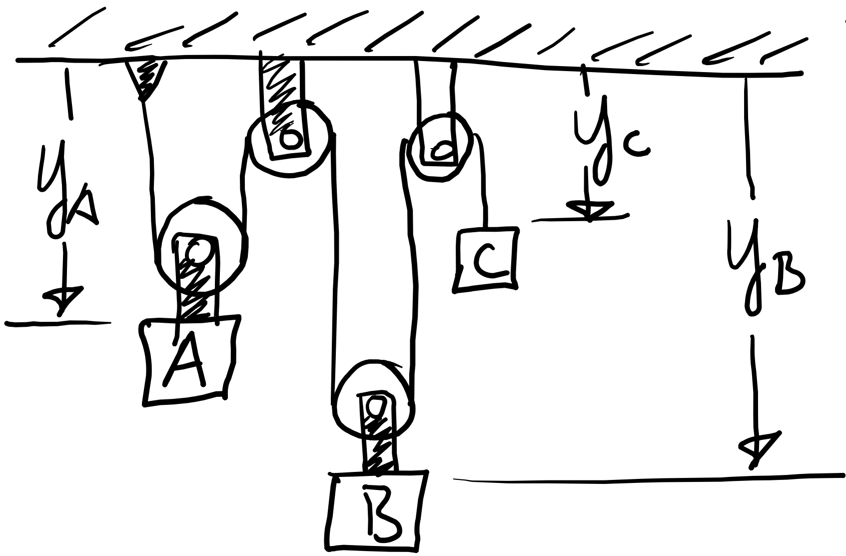
One more example



So

$$2y_A + 2y_B + y_C = \text{Const}$$

One more example

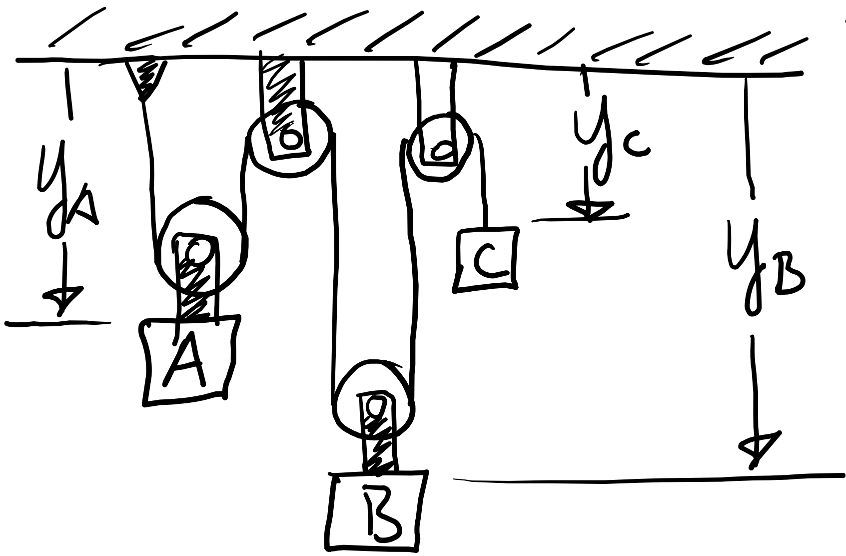


Do

$$2y_A + 2y_B + y_C = \text{Const}$$

$$\Rightarrow 2v_A + 2v_B + v_C = 0$$

One more example



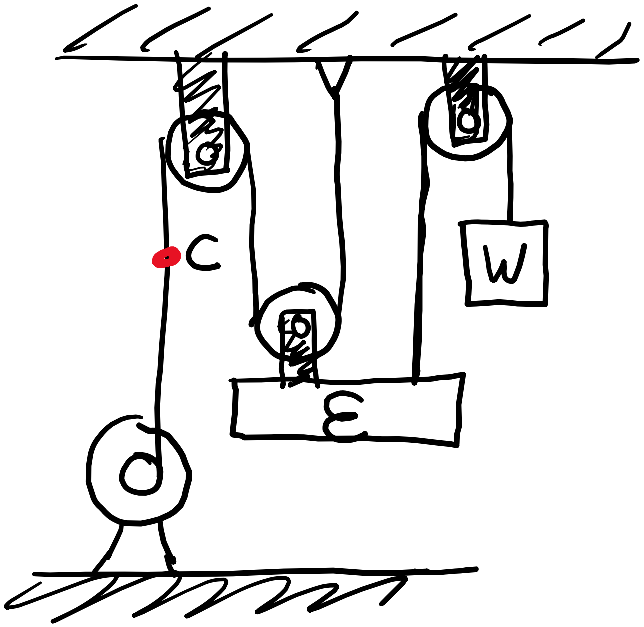
So

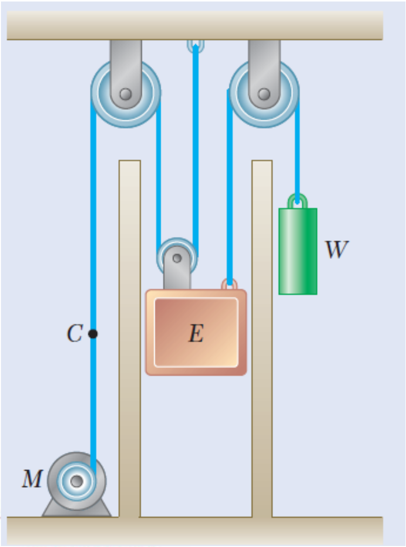
$$2y_A + 2y_B + y_C = \text{Const}$$

$$\Rightarrow 2v_A + 2v_B + v_C = 0$$

$$\& \quad 2a_A + 2a_B + a_C = 0$$

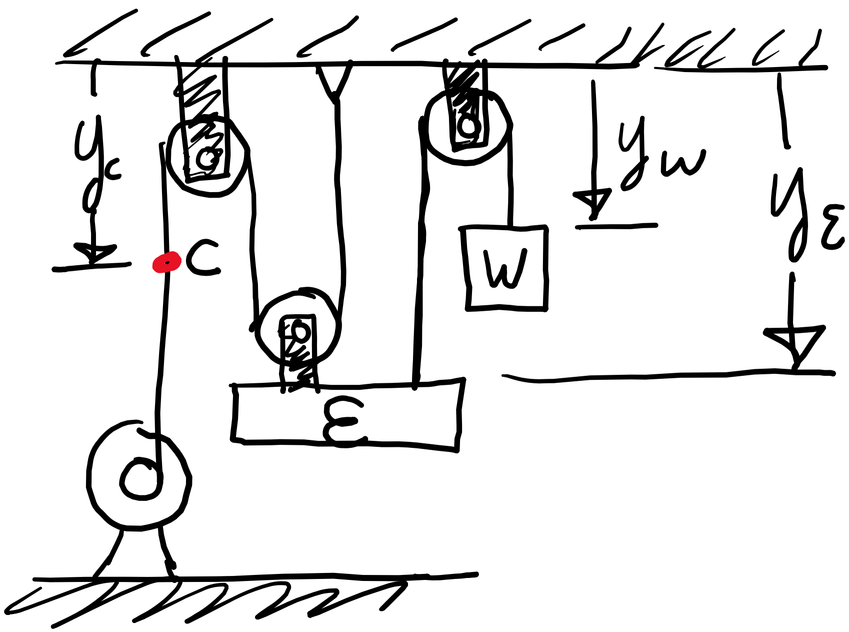
Notes on problems: $\boxed{11.47}$

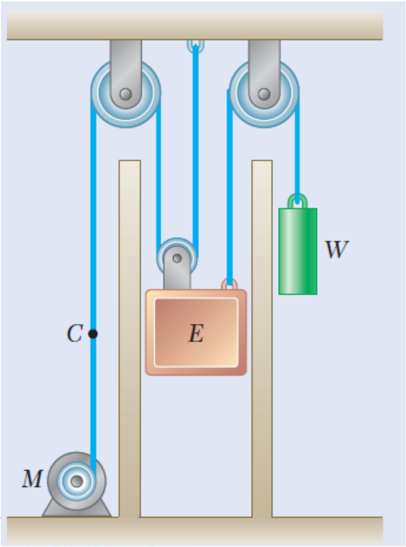




11.47 The elevator E shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable C, (*b*) the velocity of the counterweight W, (*c*) the relative velocity of the cable C with respect to the elevator, (*d*) the relative velocity of the counterweight W with respect to the elevator.

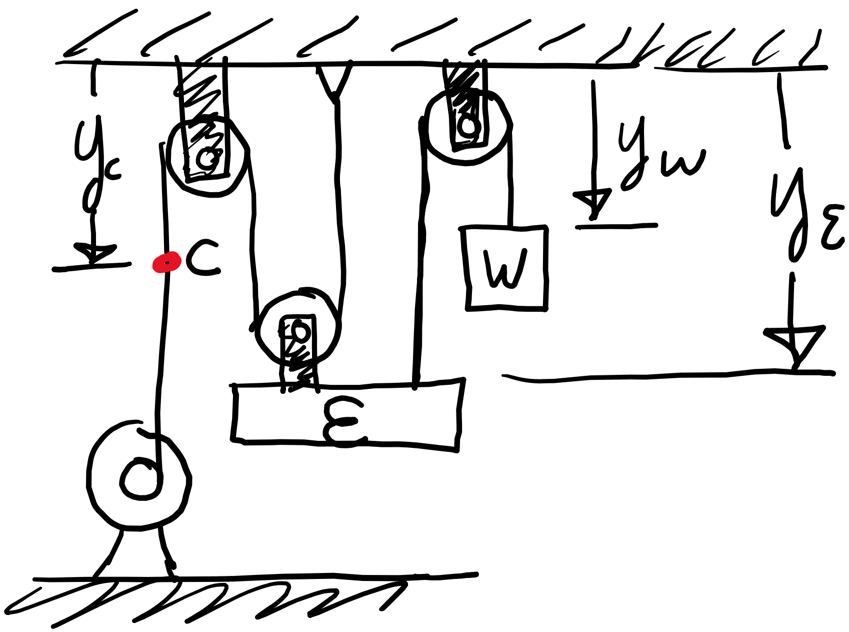
Notes on problems: $\boxed{11.47}$



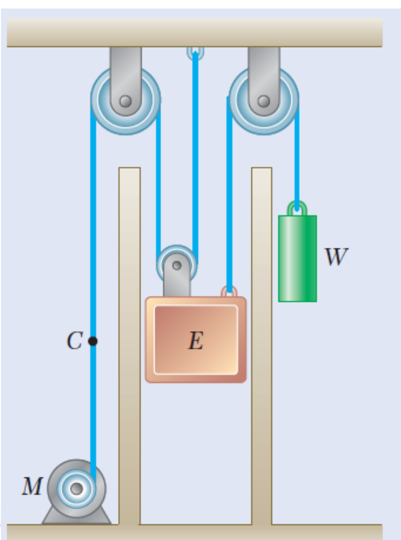


11.47 The elevator E shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable C, (*b*) the velocity of the counterweight W, (*c*) the relative velocity of the cable C with respect to the elevator, (*d*) the relative velocity of the counterweight W with respect to the elevator.

Notes on problems: 11.47

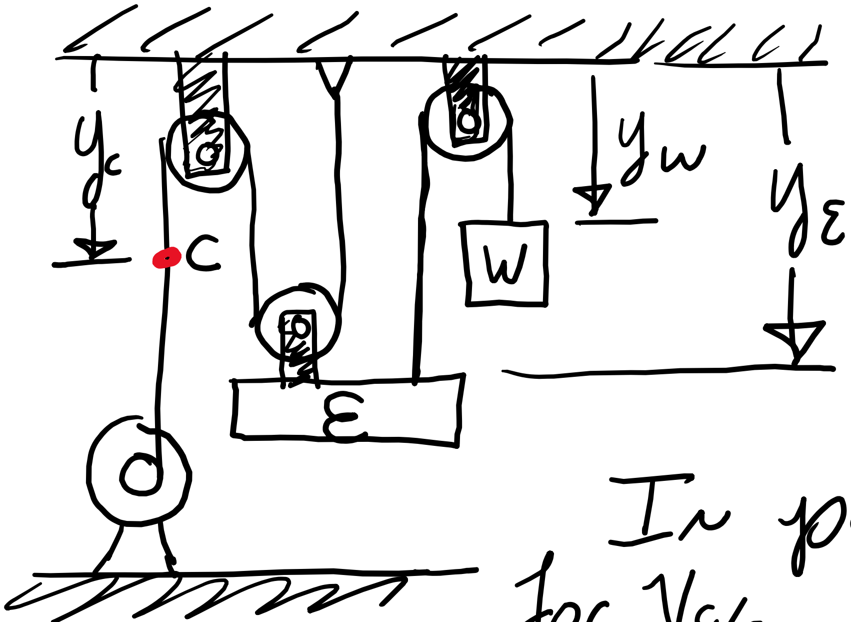


Notice the <u>two</u> ropes ⇒ two equations



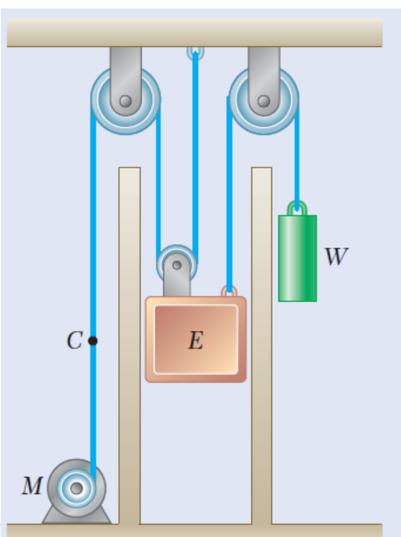
11.47 The elevator E shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable C, (*b*) the velocity of the counterweight W, (*c*) the relative velocity of the cable C with respect to the elevator, (*d*) the relative velocity of the counterweight W with respect to the elevator.

Notes on problems: $\boxed{11.47}$



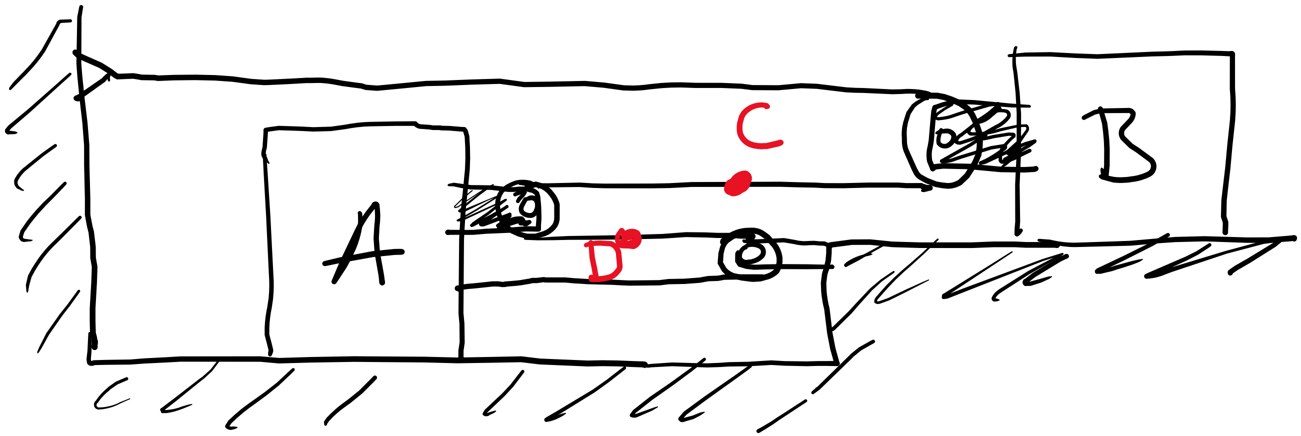
Notice the two ropes $\Rightarrow$ two equations

In part (c) you are asked for $V_{C/E}$. Make sure to include direction in calculation !!



11.47 The elevator E shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable C, (*b*) the velocity of the counterweight W, (*c*) the relative velocity of the cable C with respect to the elevator, (*d*) the relative velocity of the counterweight W with respect to the elevator.

Notes on problem $\boxed{11.51}$



Given

$V_B = 300 \frac{mm}{s}$

11.51 Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (*a*) the velocity of slider block A, (*b*) the velocity of portion C of the cable, (*c*) the velocity of portion D of the cable, (*d*) the relative velocity of portion C of the cable with respect to slider block A.

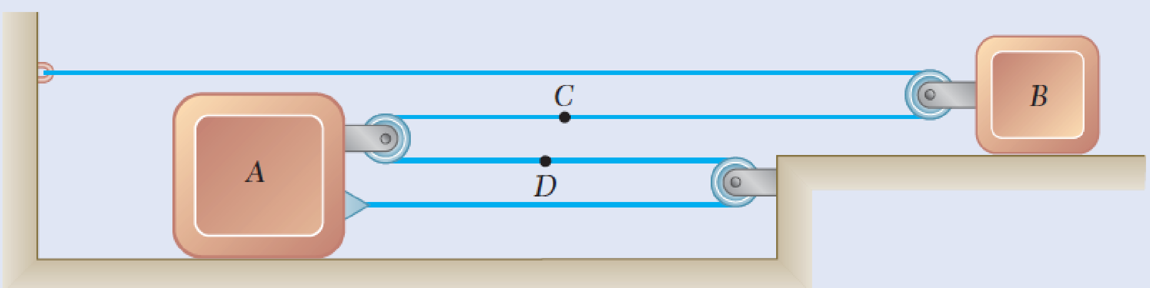
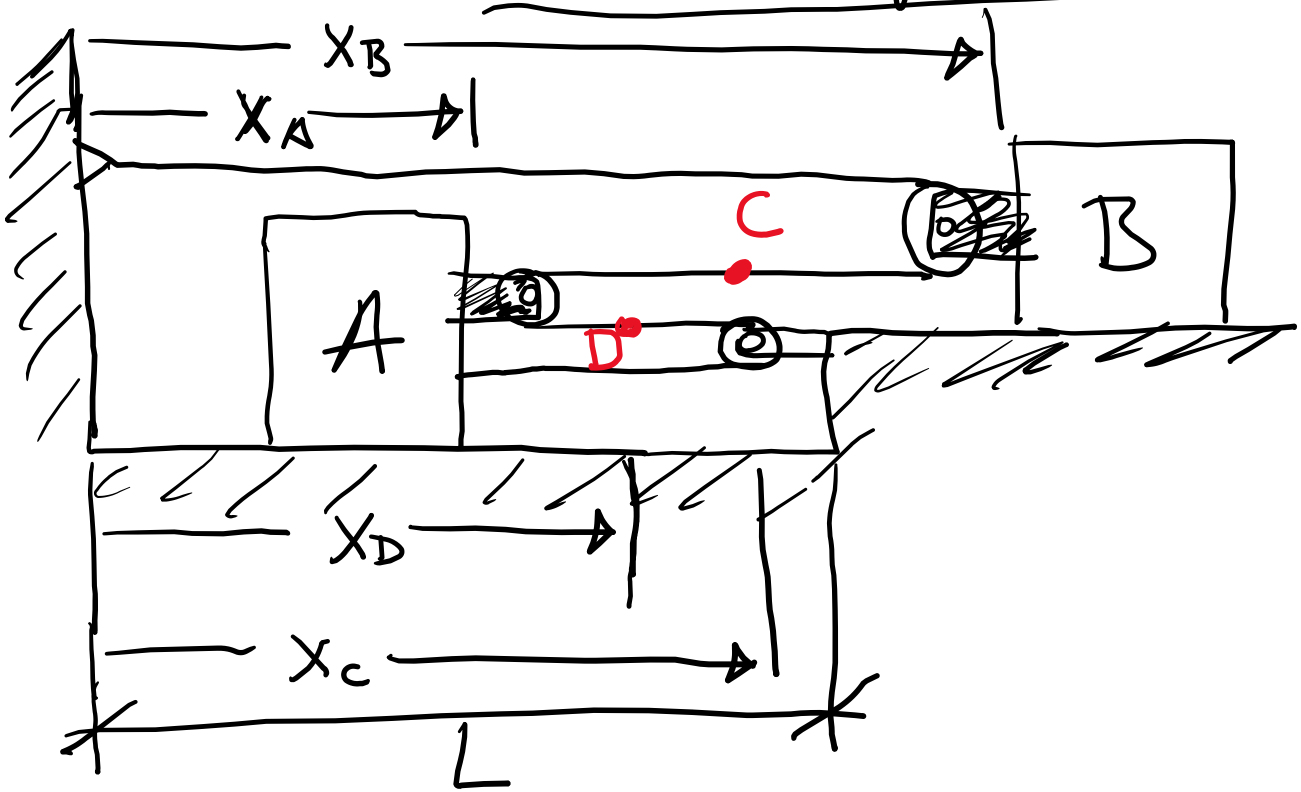


Fig. P11.51 and P11.52

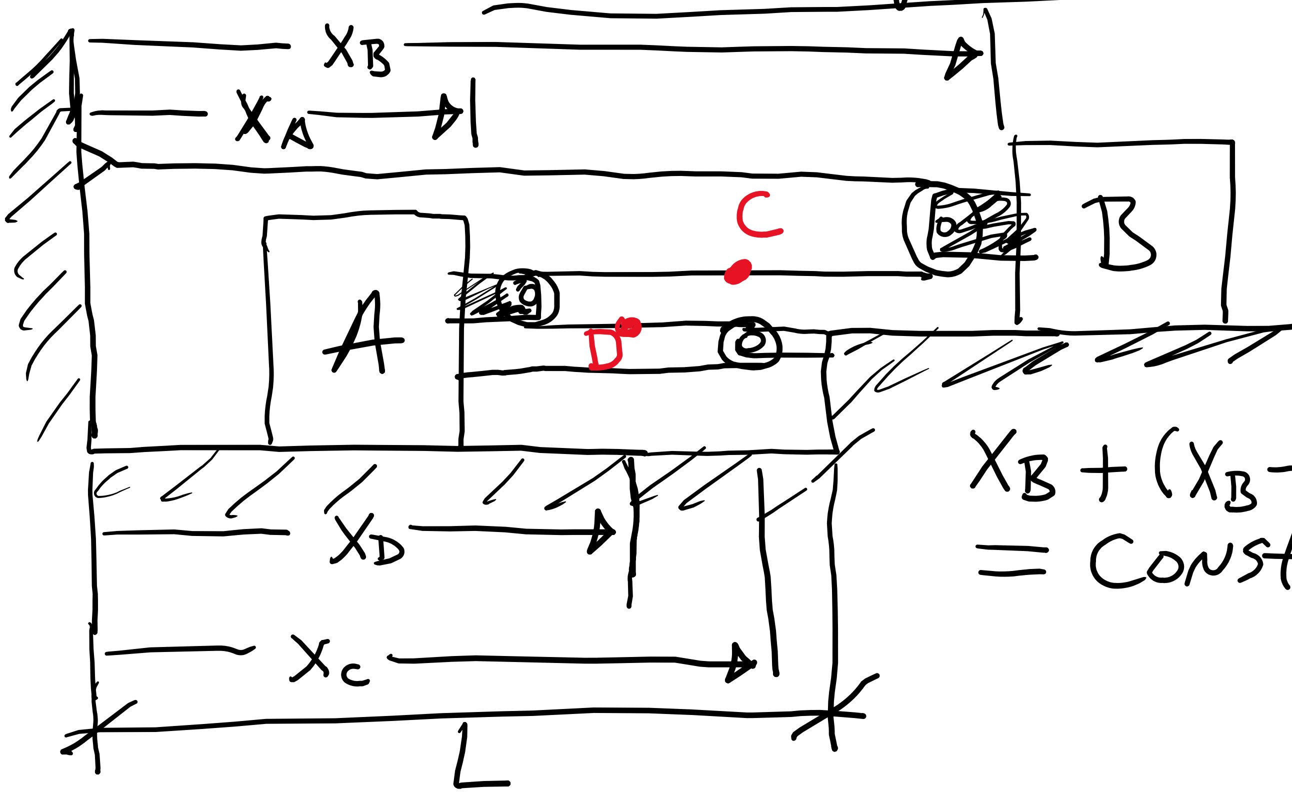
Notes on problem $\boxed{11.51}$



Given

$v_B = 300\frac{mm}{s}$

Notes on problem $\boxed{11.51}$

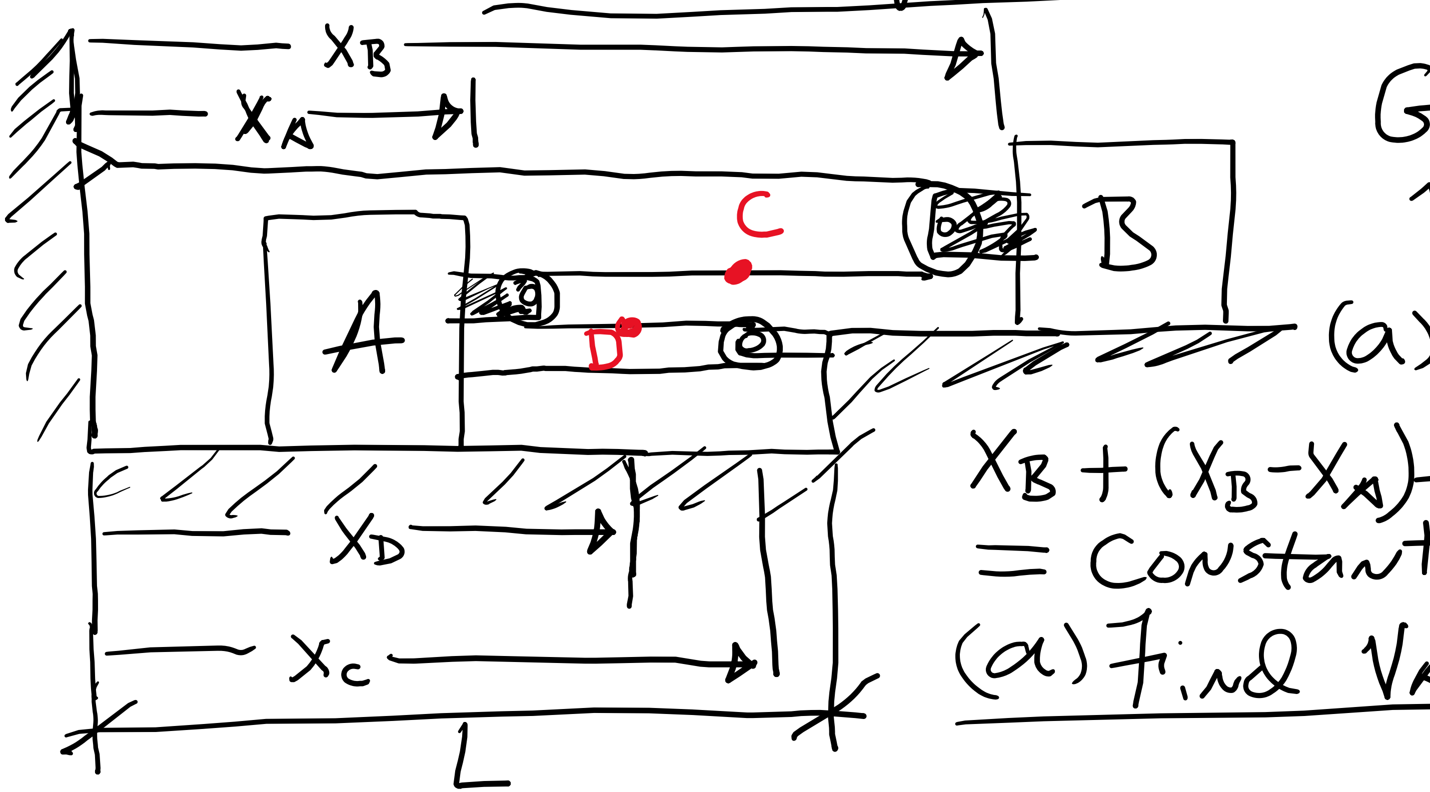


Given

$v_B = 300 \frac{mm}{s}$

$X_B + (X_B - X_A) + (L - X_A)2$
$= \text{constant}.$

Notes on problem $\boxed{11.51}$



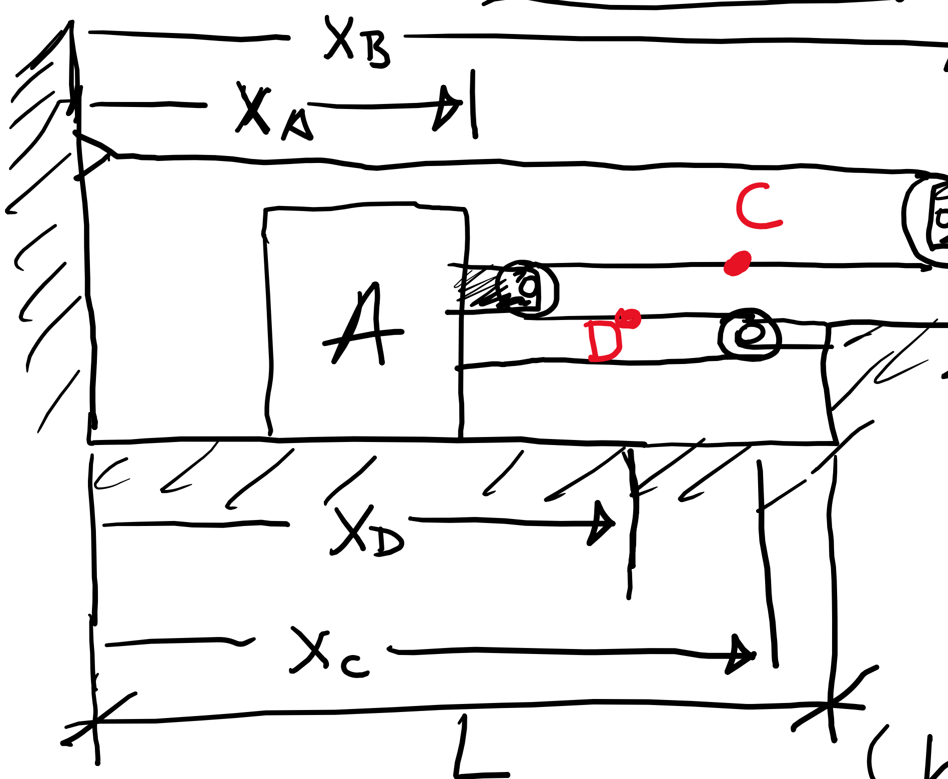
Given

$v_B = 300 \frac{mm}{s}$

$x_B + (x_B - x_A) + (L - x_A)2 = \text{constant}$

(a) find v_A: just take derivitives.

Notes on problem $\boxed{11.51}$



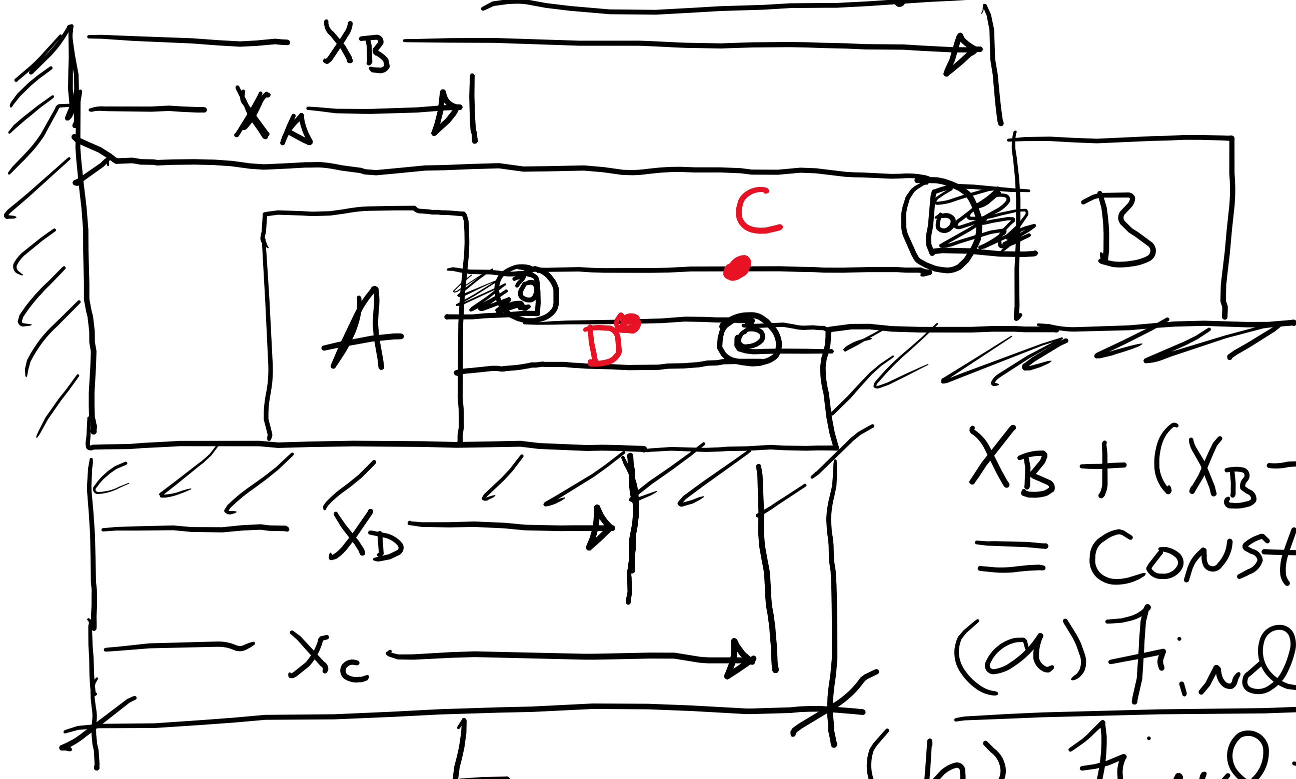
Given
$v_B = 300\frac{mm}{s}$

$X_B + (X_B - X_A) + (L - X_A)2 = \text{Constant}$

(a) find v_A: just take derivitives.

(b) find v_C
(c) find v_D
} follow rope from one end to point of interest.

Notes on problem 11.51



(a)

Given
$v_B = 300 \frac{mm}{s}$

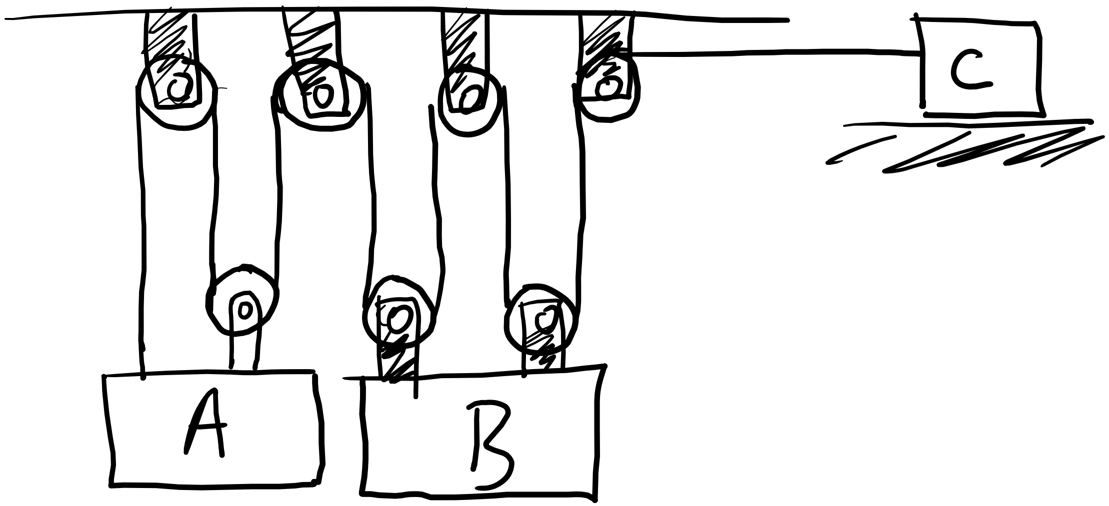
$x_B + (x_B - x_A) + (L - x_A)2$
$= \text{constant}$

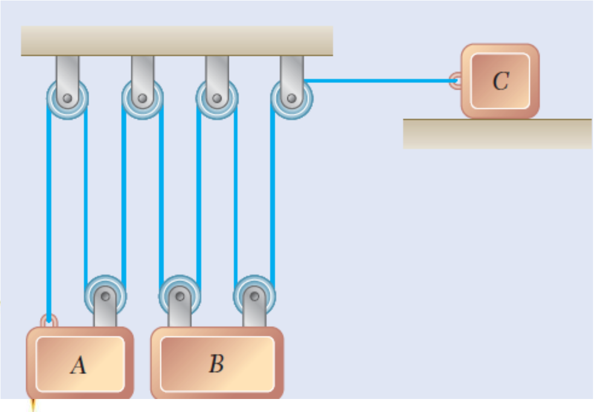
(a) Find v_A: just take derivitives.

(b) Find v_C
(c) Find v_D
} follow rope from one end to point of interest.

(d) Find $v_{C/A}$: Make sure to note direction

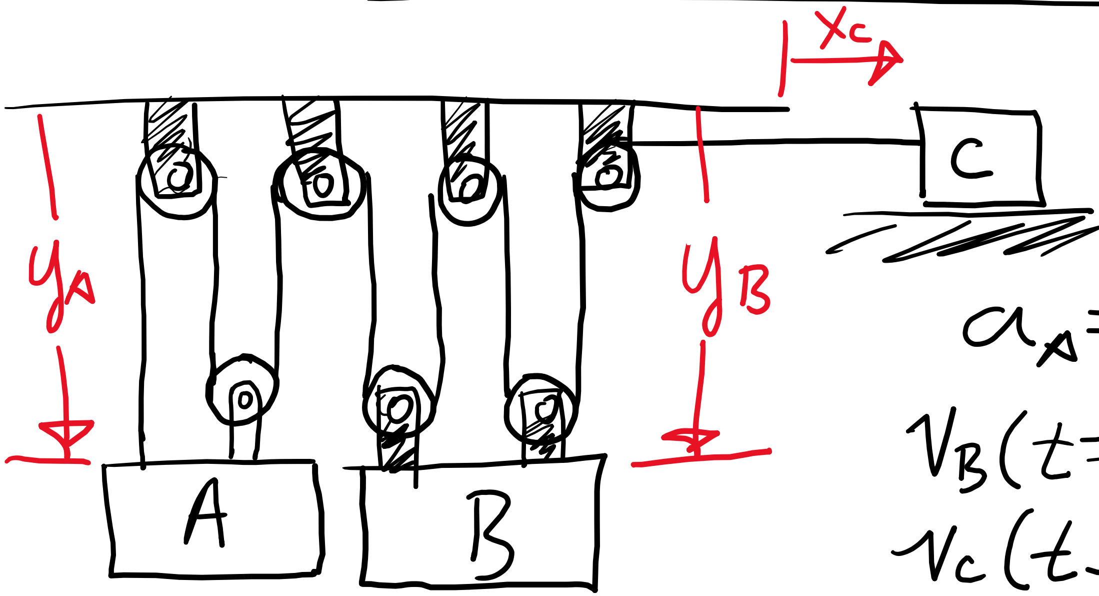
Notes on problem $\boxed{11.57}$





11.57 Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

Notes on problem $\boxed{11.57}$

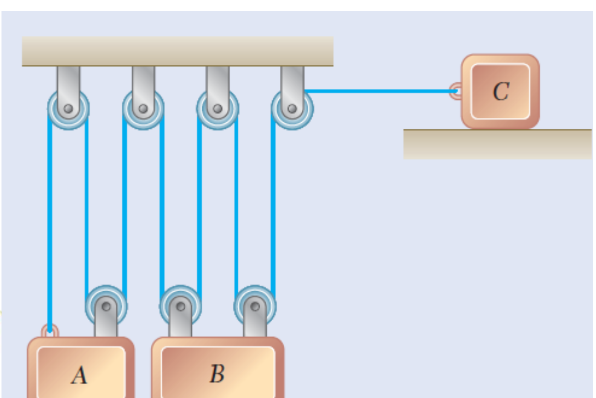


Given: $v_{0B} = 0$

$a_A = \text{const.}, \; a_C = 75 \frac{mm}{s^2}$

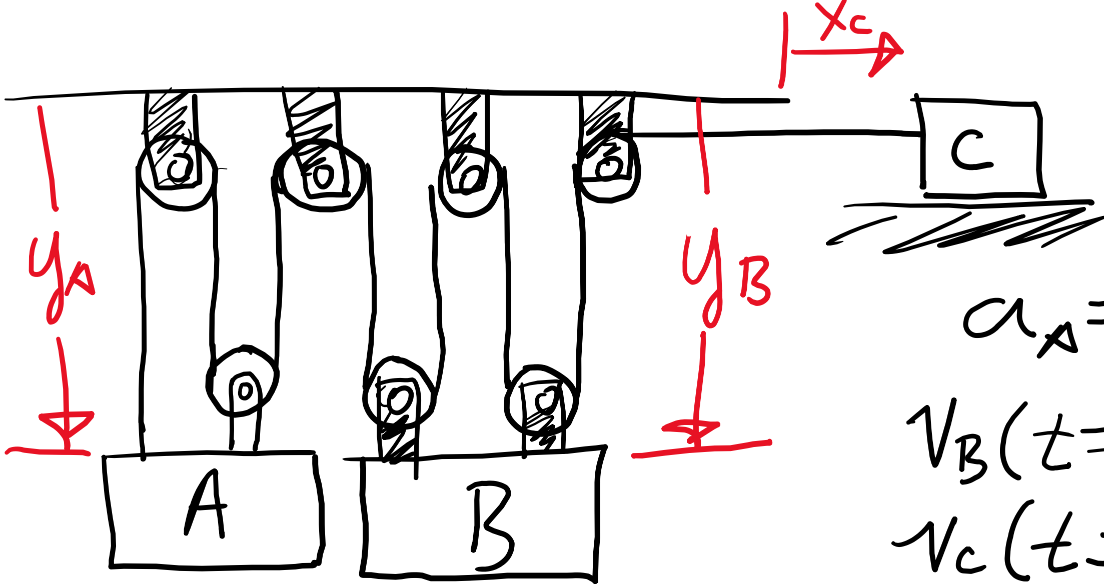
$v_B(t=2s) = 480 \frac{mm}{s}$

$v_C(t=2s) = 280 \frac{mm}{s}$



11.57 Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

Notes on problem 11.57



Given: $v_{0B} = 0$

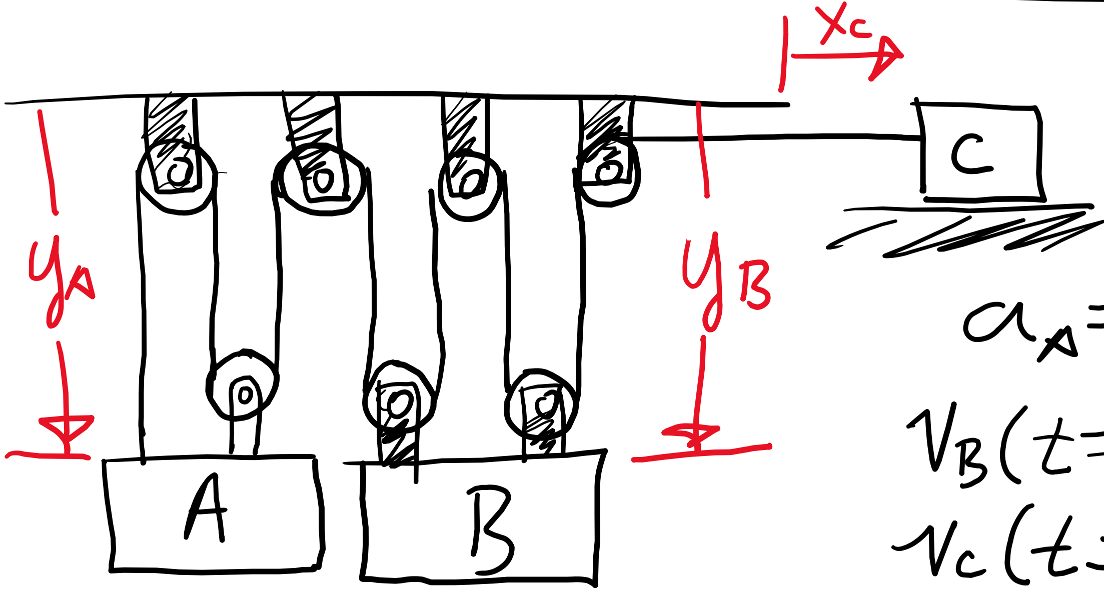
$a_A = \text{const.}$, $a_C = 75\,\frac{mm}{s^2}$

$v_B(t=2s) = 480\,\frac{mm}{s}$

$v_C(t=2s) = 280\,\frac{mm}{s}$

(a) Find a_A & a_B : Get equation relating y_A, y_B & x_C, then v_A, v_B & v_C

Notes on problem $\boxed{11.57}$



Given: $v_{0B} = 0$

$a_A = \text{const.}$, $a_C = 75\,\frac{mm}{s^2}$

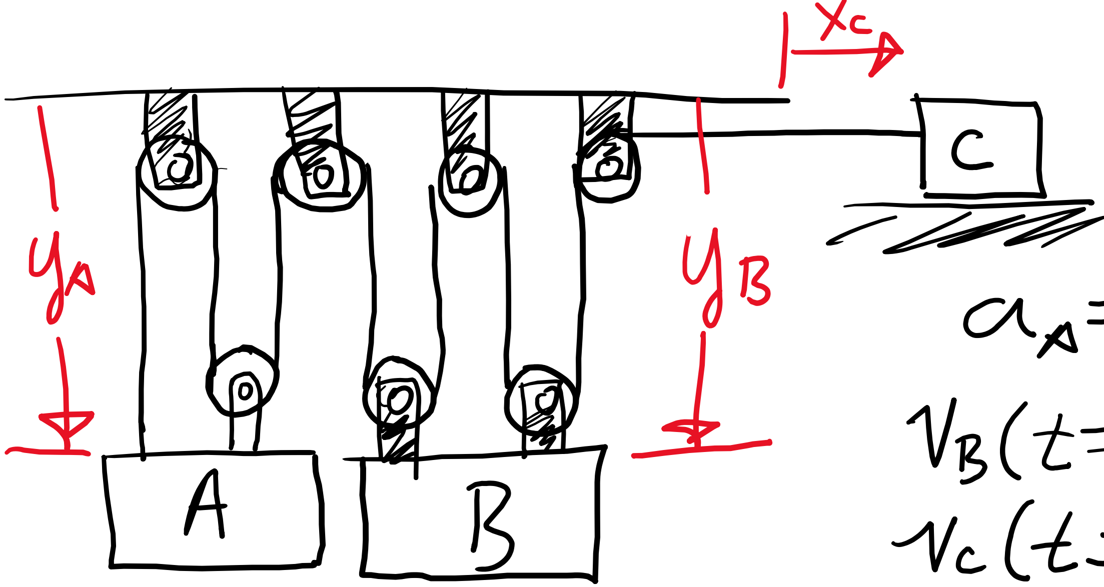
$v_B(t=2s) = 480\,\frac{mm}{s}$

$v_C(t=2s) = 280\,\frac{mm}{s}$

(a) Find a_A & a_B: Get equation relating y_A, y_B & x_C, then v_A, v_B & v_C

Note: $a_A = \text{const.}$ & $a_C = \text{const} \Rightarrow a_B = \text{const.}$

Notes on problem $\boxed{11.57}$



Given: $v_{OB} = 0$

$a_A = \text{const.}$, $a_C = 75 \frac{mm}{s^2}$

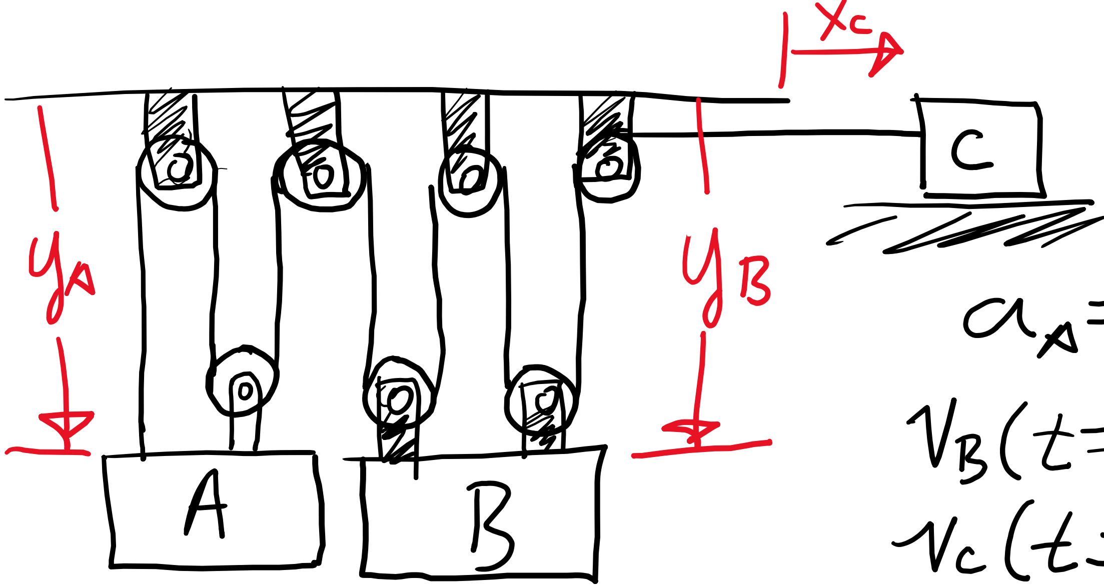
$v_B(t=2s) = 480 \frac{mm}{s}$

$v_C(t=2s) = 280 \frac{mm}{s}$

(a) Find a_A & a_B : Get equation relating y_A, y_B & x_C, then v_A, v_B & v_C

Note: $a_A = \text{const.}$ & $a_C = \text{const} \Rightarrow a_B = \text{const.}$ & since $v_{BO} = 0$ then $v_B = a_B t$

Notes on problem $\boxed{11.57}$



Given: $v_{0B} = 0$

$a_A = \text{const.}$, $a_C = 75\,\frac{mm}{s^2}$

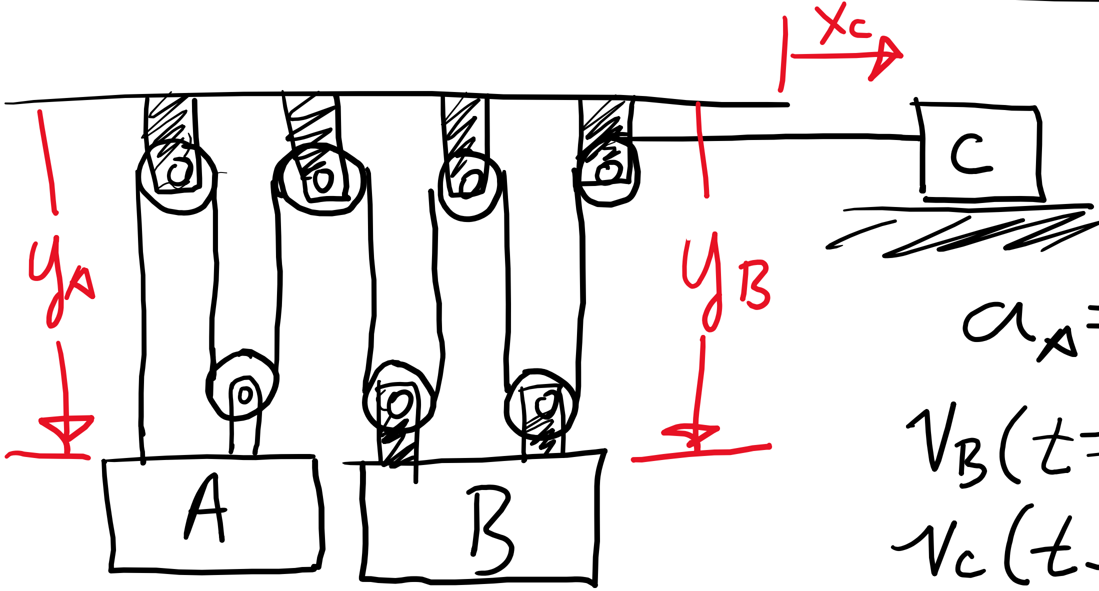
$v_B(t=2s) = 480\,\frac{mm}{s}$

$v_C(t=2s) = 280\,\frac{mm}{s}$

(a) Find a_A & a_B : Get equation relating y_A, y_B & x_C, then v_A, v_B & v_C

Note: $a_A = \text{const.}$ & $a_C = \text{const} \Rightarrow a_B = \text{const.}$ & since $v_{B0} = 0$ then $v_B = a_B t$. $v_B(t=2s)$ is given. Use system of equations to get a_A & a_B

Notes on problem $\boxed{11.57}$



Given: $v_{0B} = 0$

$a_A = \text{const.}$, $a_C = 75\,\frac{mm}{s^2}$

$v_B(t=2s) = 480\,\frac{mm}{s}$

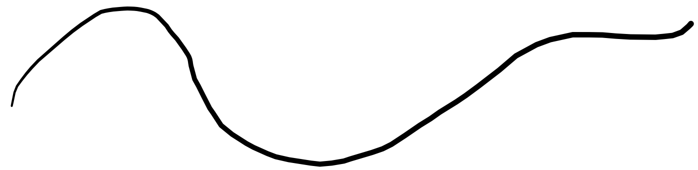
$v_C(t=2s) = 280\,\frac{mm}{s}$

(a) Find a_A & a_B : Get equation relating y_A, y_B & x_C, then v_A, v_B & v_C

Note: $a_A = \text{const.}$ & $a_C = \text{const} \Rightarrow a_B = \text{const.}$ & since $v_{B0} = 0$ then $v_B = a_B t$. $v_B(t=2s)$ is given. Use system of equations to get a_A & a_B. For rest: Kinematics & system of equations

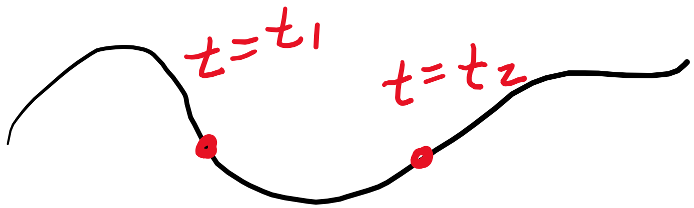
Now moving from 1d to 3d

Now moving from 1d to 3d



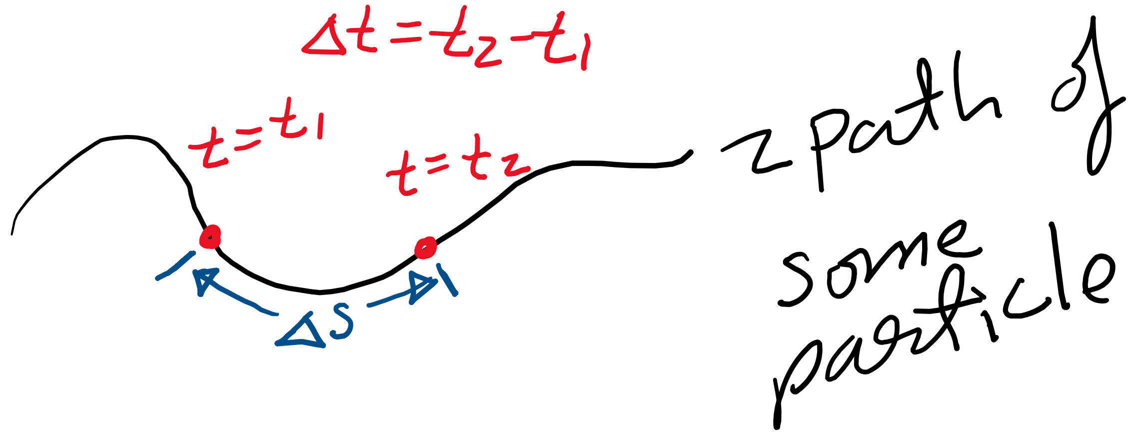
path of some particle

Now moving from 1d to 3d



path of some particle

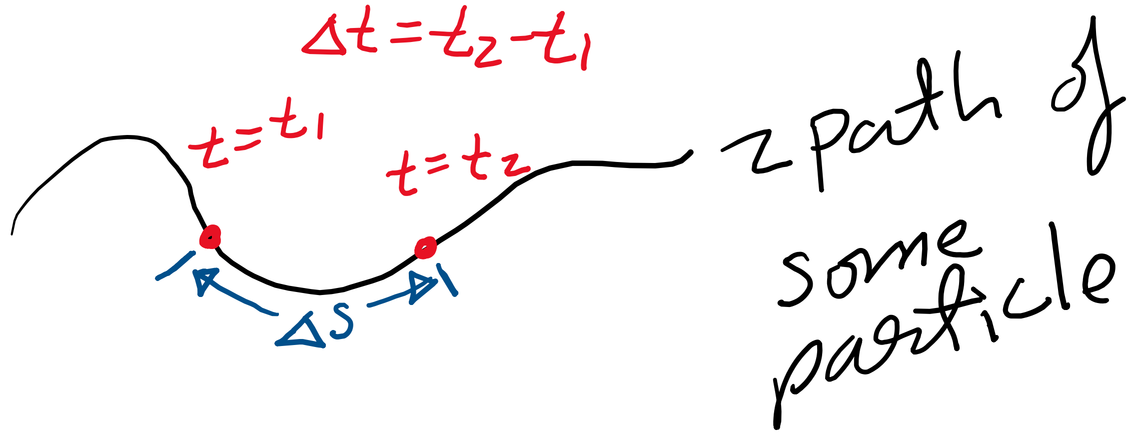
Now moving from 1d to 3d



Now moving from 1d to 3d

$$v_{AVE} = \frac{\Delta s}{\Delta t}$$

$$\Rightarrow v = \frac{ds}{dt}$$



Now moving from 1d to 3d

$$v_{AVE} = \frac{\Delta s}{\Delta t}$$

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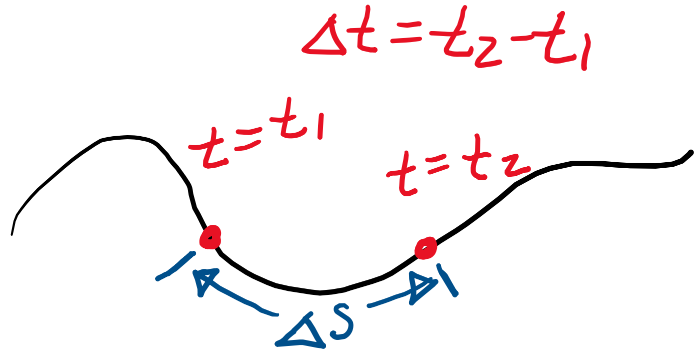


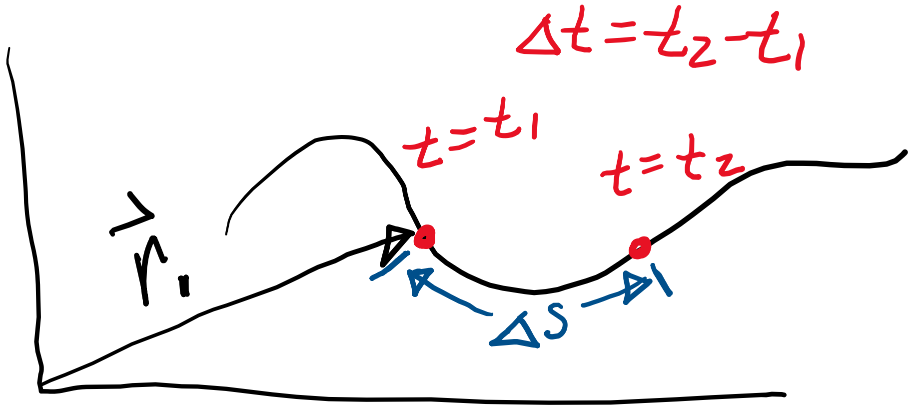
path of some particle

Note: scalar $v = |\vec{v}|$

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



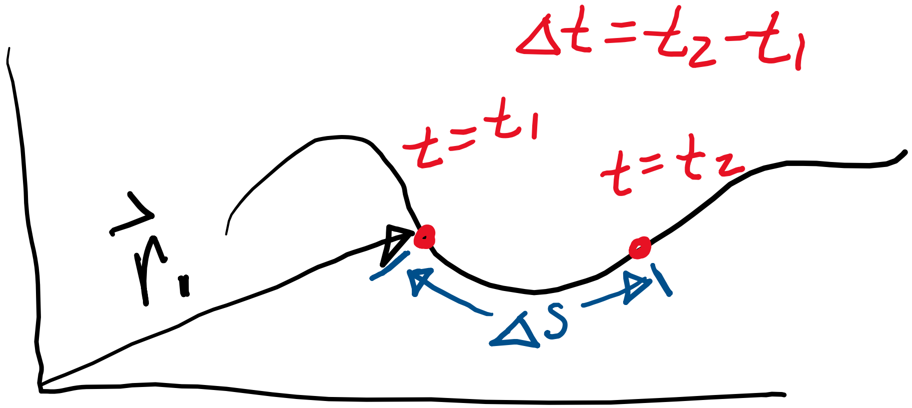
← path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt}$

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



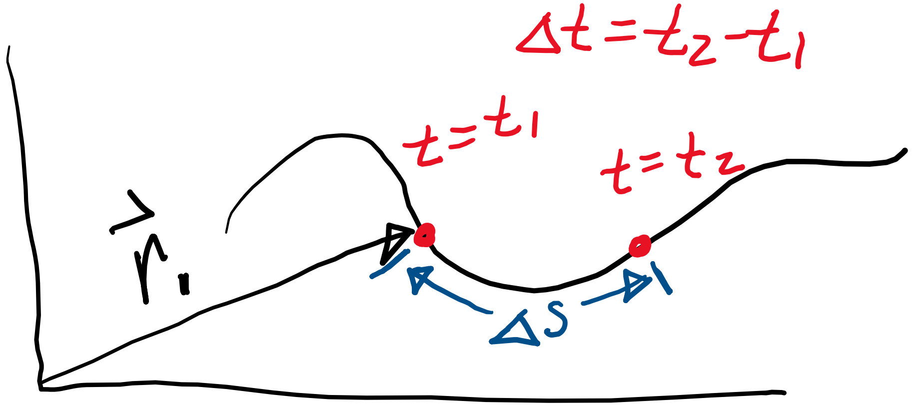
← path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



← path of some particle

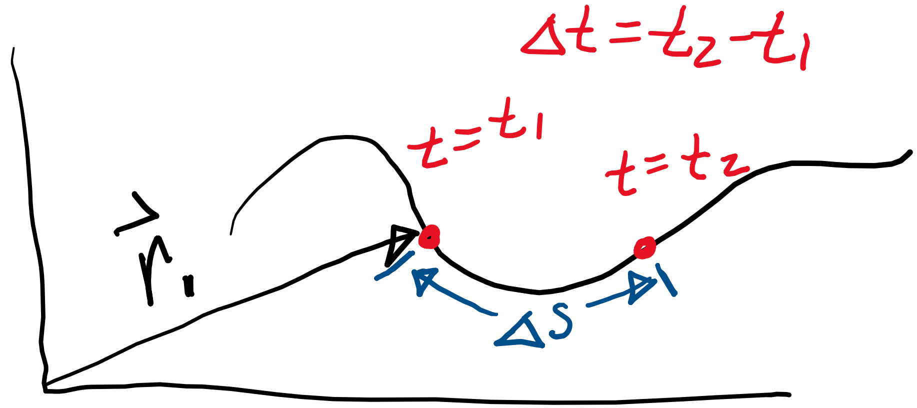
Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



← path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

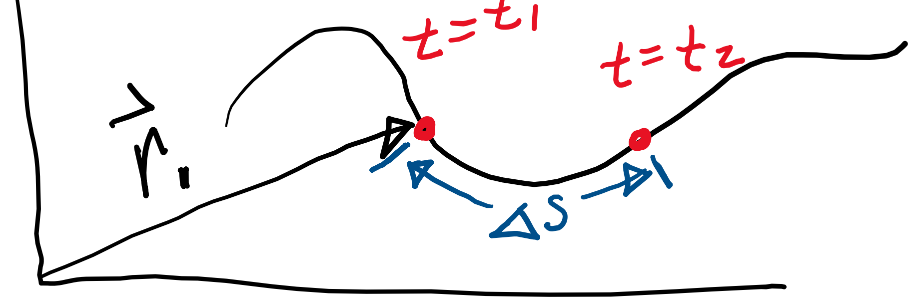
Make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar.

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$

$\Delta t = t_2 - t_1$



← path of some particle

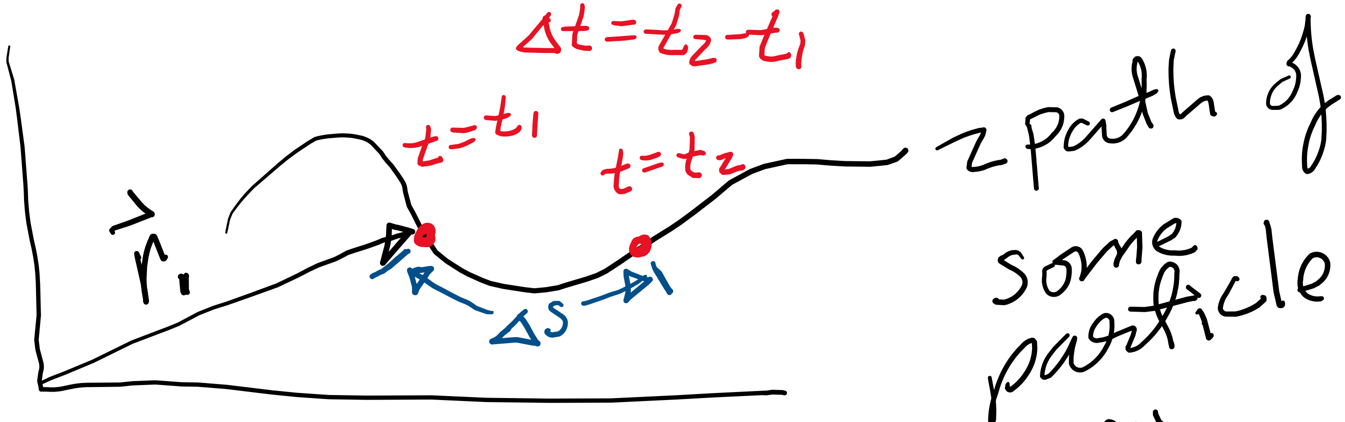
Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Now moving from 1d to 3d

$v_{ave} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Unit vectors:

$\hat{i} \equiv$ one unit in x-direction

$\hat{j} \equiv$ one unit in y-direction

$\hat{k} \equiv$ one unit in z-direction

Other common notation

$$\hat{i} = \hat{x} = \hat{x}_1$$

$$\hat{j} = \hat{y} = \hat{x}_2$$

$$\hat{k} = \hat{z} = \hat{x}_3$$

Other common notation

$$\hat{i} = \hat{x} = \hat{x}_1$$
$$\hat{j} = \hat{y} = \hat{x}_2$$
$$\hat{k} = \hat{z} = \hat{x}_3$$

Position vector:

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or

$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or

$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ or

$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$

ASU

Other common notation

$$\hat{i} = \hat{x} = \hat{x}_1$$
$$\hat{j} = \hat{y} = \hat{x}_2$$
$$\hat{k} = \hat{z} = \hat{x}_3$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or}$$
$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or}$$
$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

Just Needs to be clear and understandable

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ or

$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$

Velocity vector:

$$\vec{V} = \frac{dr_x}{dt}\hat{i} + \frac{dr_y}{dt}\hat{j} + \frac{dr_z}{dt}\hat{k}$$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$
$\hat{j} = \hat{y} = \hat{x}_2$
$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or
$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ or
$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$

Velocity vector: $\vec{V} = \frac{dr_x}{dt} \hat{i} + \frac{dr_y}{dt} \hat{j} + \frac{dr_z}{dt} \hat{k}$

Acceleration vector:

$$\vec{a} = \frac{d^2 r_x}{dt^2} \hat{i} + \frac{d^2 r_y}{dt^2} \hat{j} + \frac{d^2 r_z}{dt^2} \hat{k}$$

Let time derivitive be given by an over-dot

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k}$$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x\hat{i} + \dot{r}_y\hat{j} + \dot{r}_z\hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Let time derivitive be given by an over-dot such that

$\vec{v} = \dot{r}_x\hat{i} + \dot{r}_y\hat{j} + \dot{r}_z\hat{k}$ or $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Two over-dots for 2nd time derivitives

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \dot{y}\hat{j} + \ddot{z}\hat{k}$

<u>Dot product:</u>

ASU

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared:

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $\vec{r}^2 = \vec{r}\cdot\vec{r} = r_x^2 + r_y^2 + r_z^2$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

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Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $\vec{r}^2 = \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude:

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $\vec{r}^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r} \cdot \vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

ASU

Let time derivitive be given by an over-dot such that

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Cross product:

Let time derivitive be given by an over-dot such that

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& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

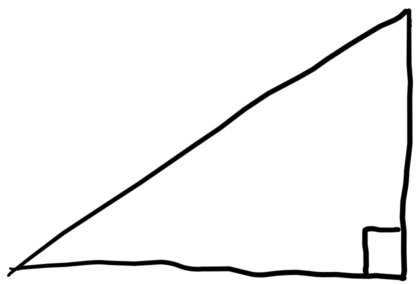
Magnitude-squared: $\vec{r}^{\,2} \equiv \vec{r}\cdot\vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r}\cdot\vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

Cross product: $\vec{A}\times\vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

Super quick trig review:

Super quick trig review:



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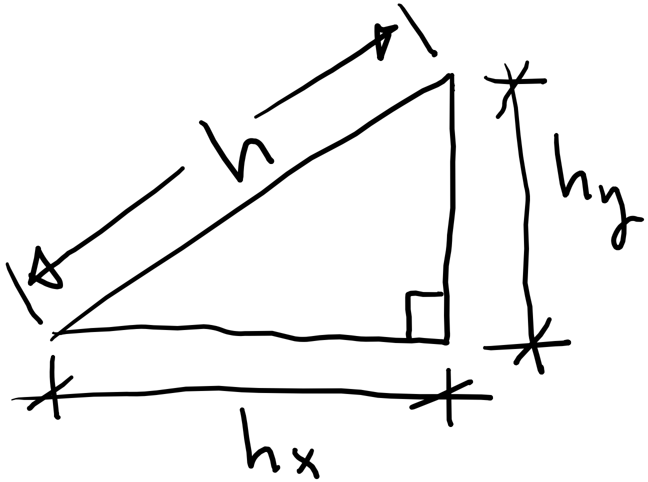

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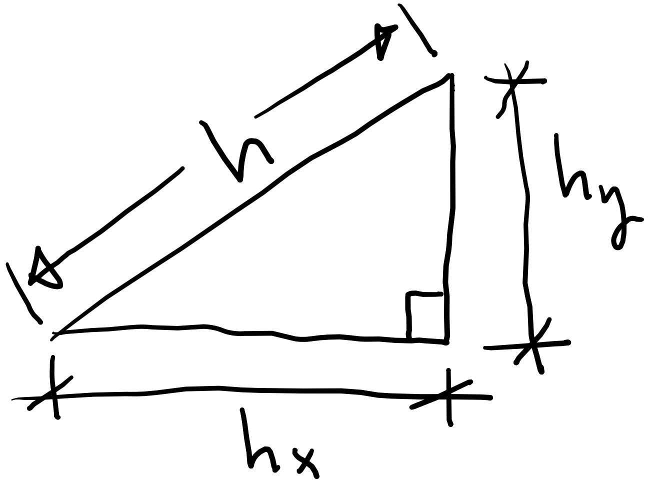
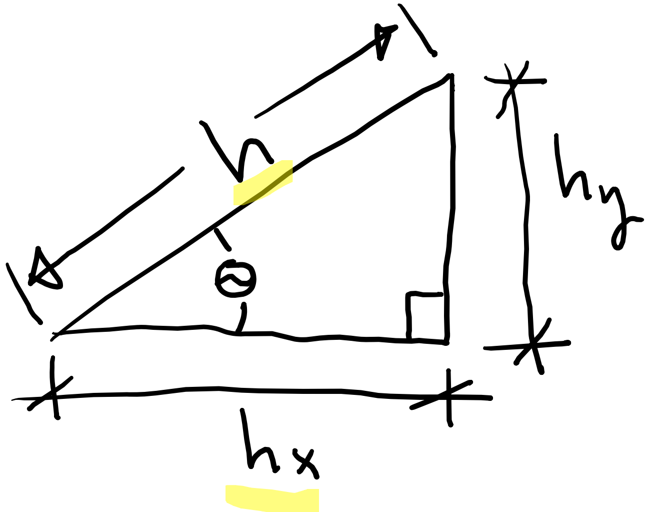


$$h^2 = h_x^2 + h_y^2$$

Super quick trig review:

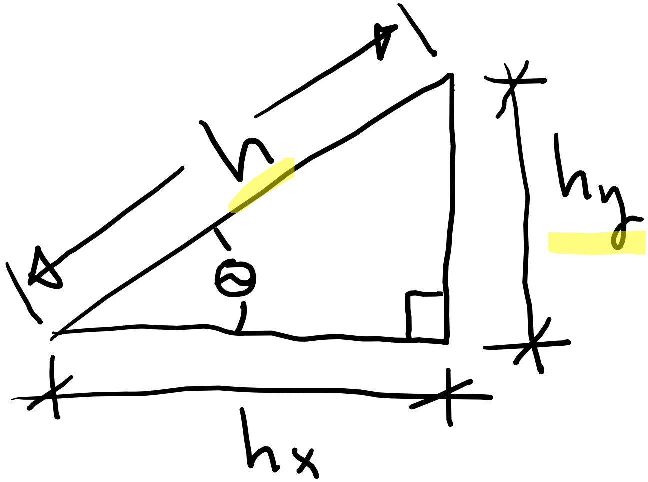


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{h_x}{h}$$

Super quick trig review:

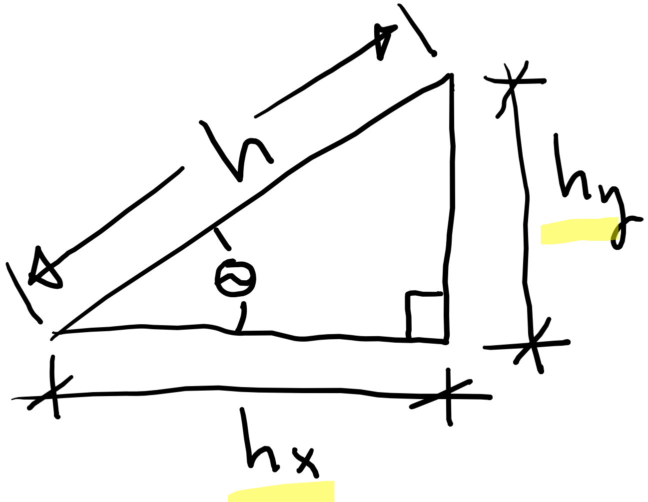


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}$$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{h_y}{h}$$

Super quick trig review:

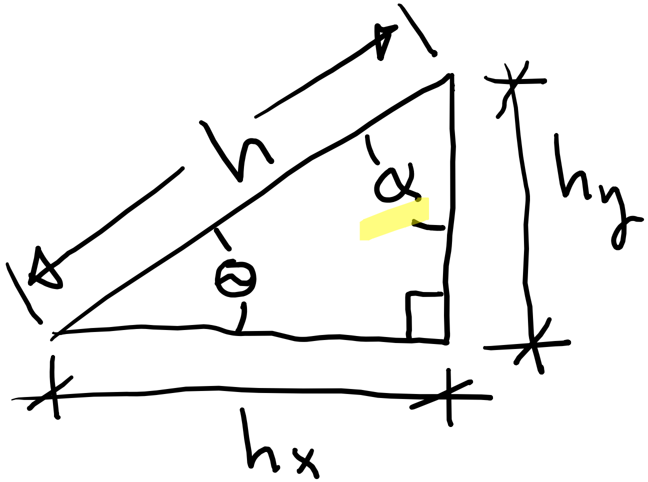


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}} = \frac{h_y}{h_x}$$

Super quick trig review:



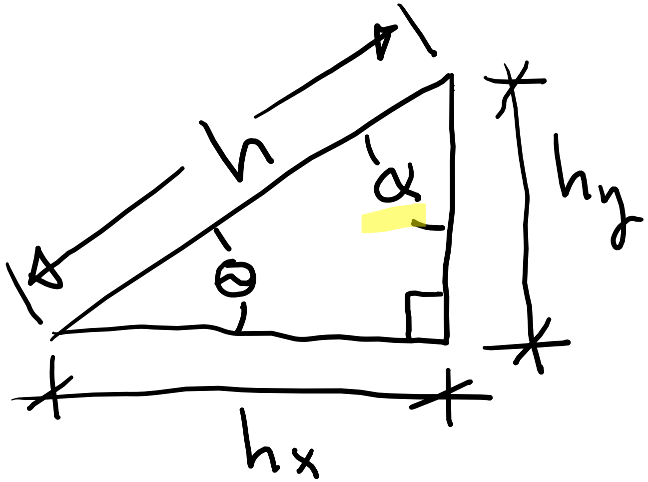
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}$$

ASU

Super quick trig review:



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$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}$$

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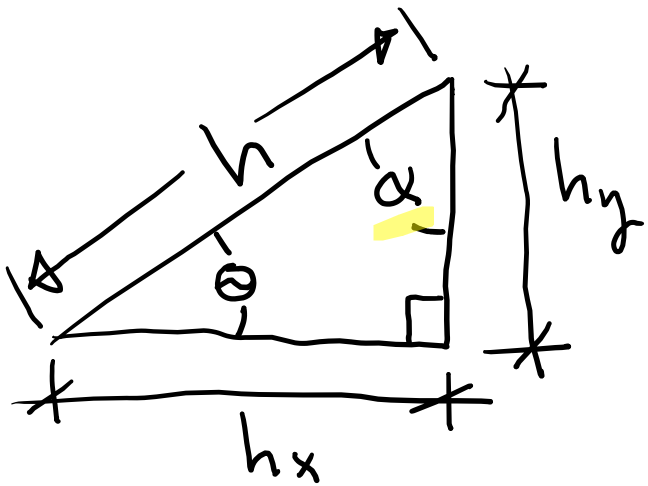

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$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

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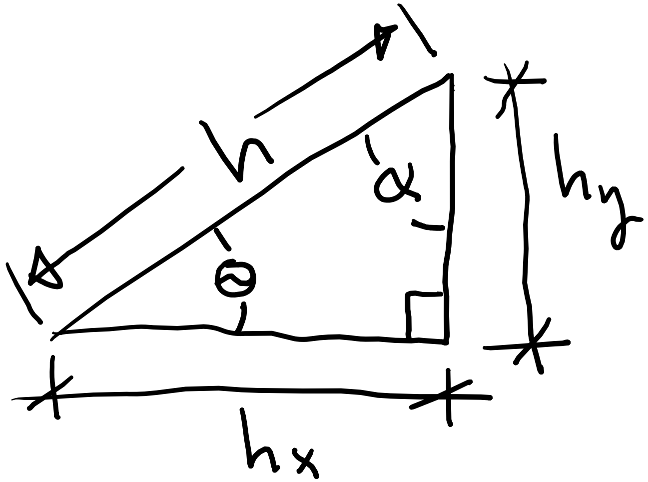


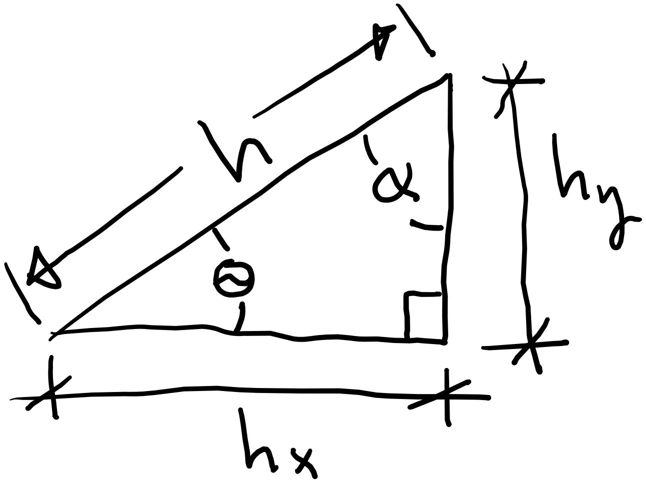
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h},\ \sin\theta = \frac{h_y}{h},\ \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h},\ \sin\alpha = \frac{h_x}{h},\ \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ$$

Super quick trig review:



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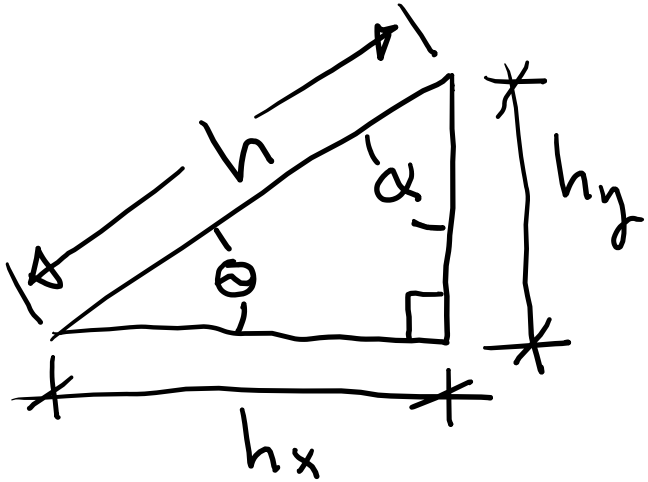
$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

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$$\Theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \Theta$$

$$\cos(90^\circ - \Theta) = \sin\Theta$$

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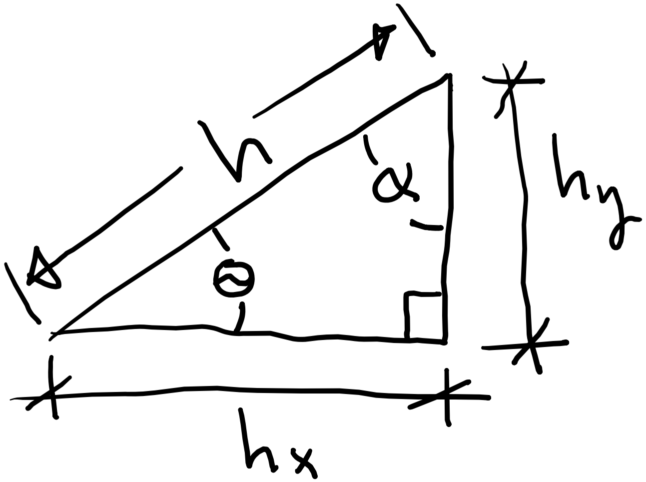

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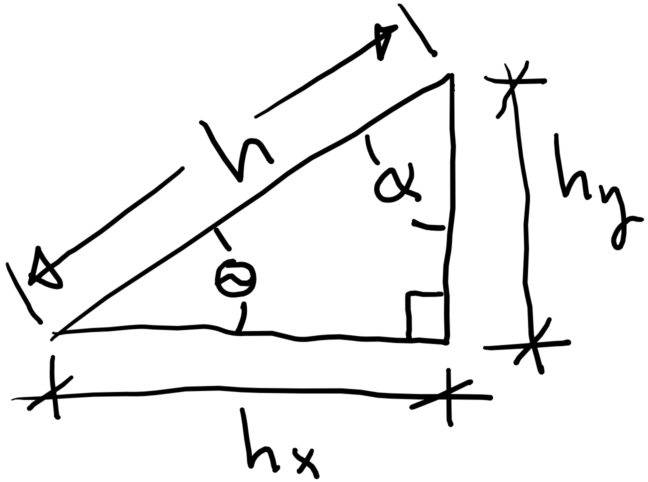
$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$

Super quick trig review:



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$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

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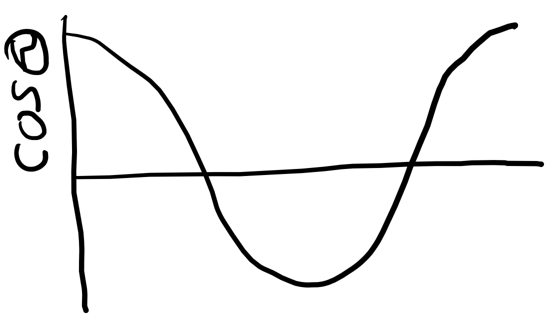
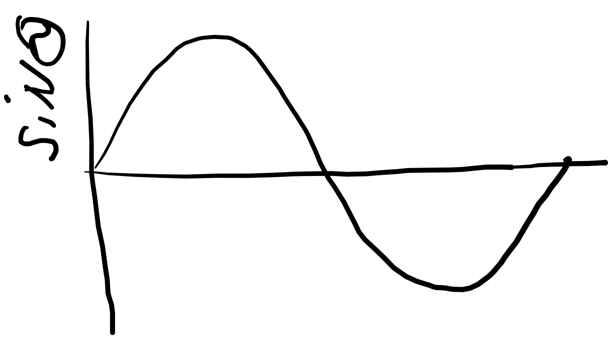
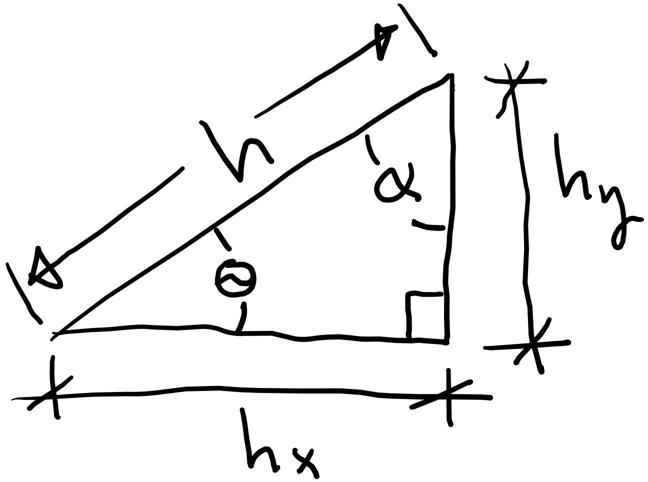




Super quick trig review:



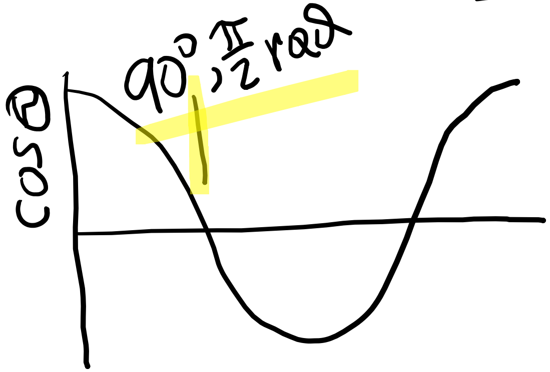
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \ \sin\theta = \frac{h_y}{h}, \ \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \ \sin\alpha = \frac{h_x}{h}, \ \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \qquad \& \ \sin(90^\circ - \theta) = \cos\theta$$



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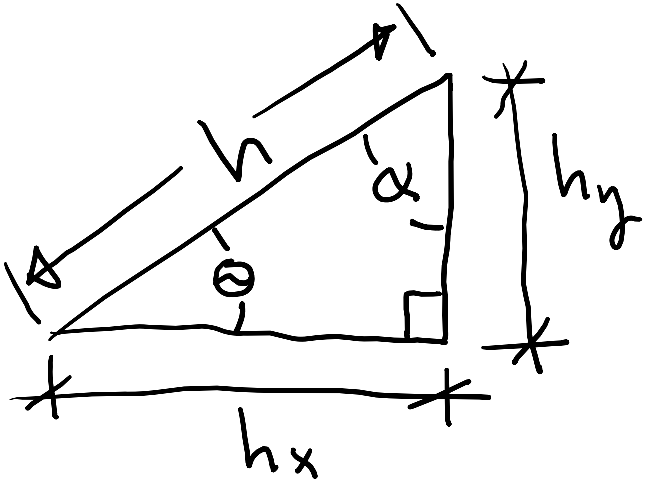

$$h^2 = h_x^2 + h_y^2$$

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$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

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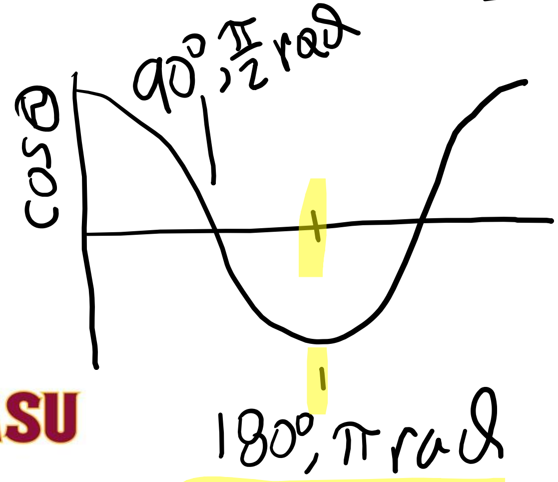
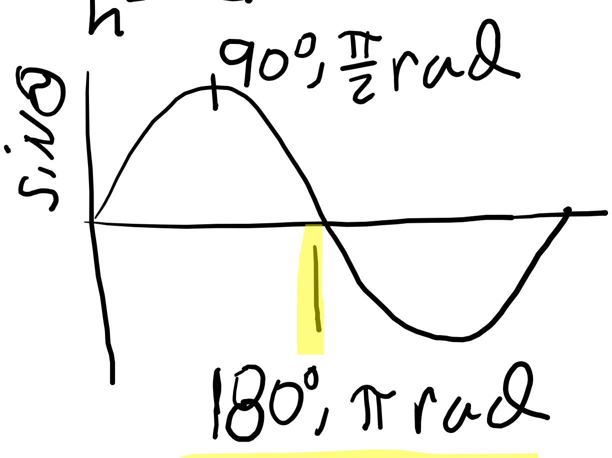
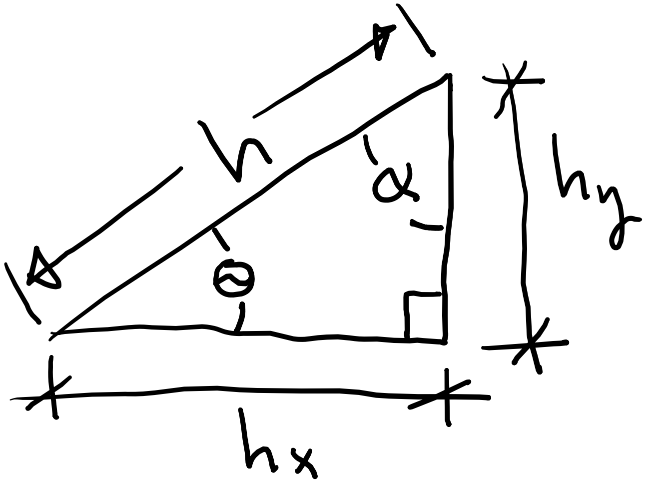




Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

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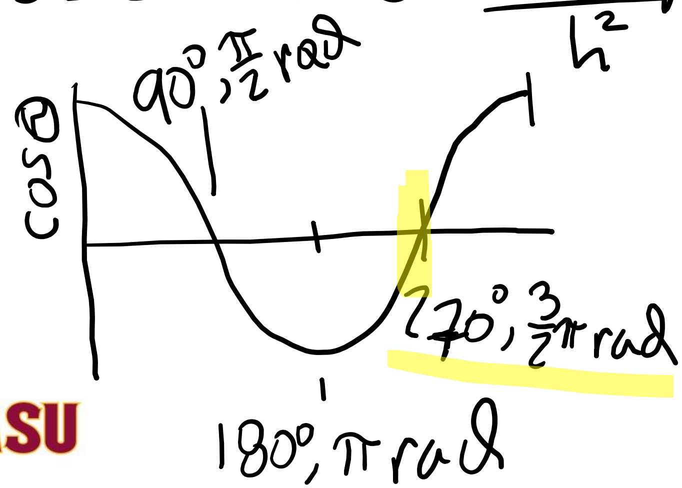

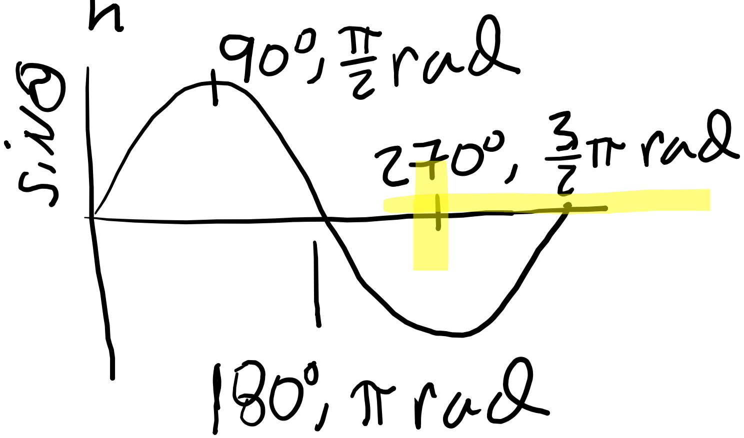

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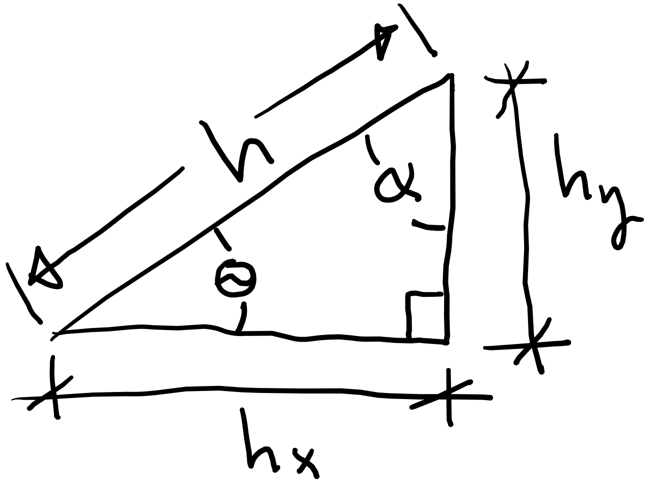

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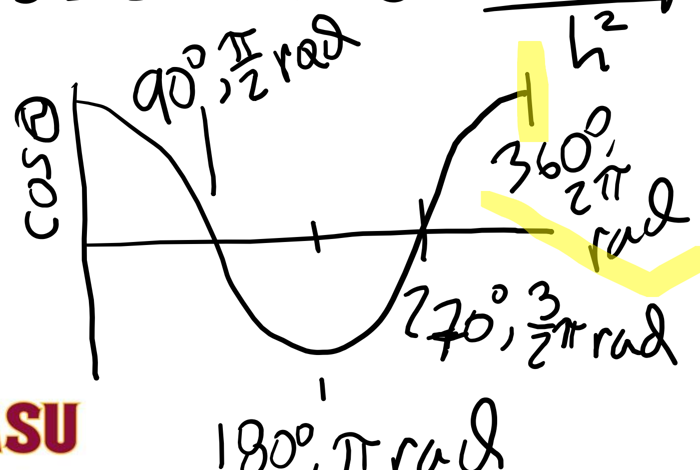

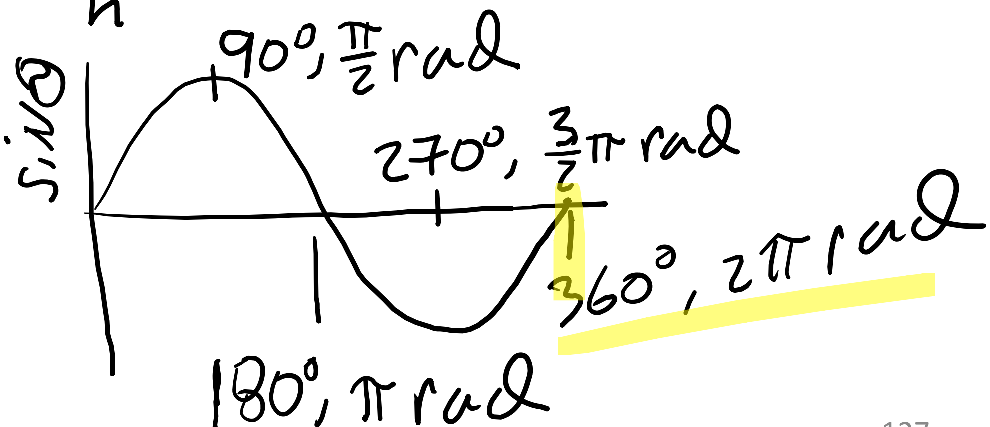

Special triangle [my favorite]

Special triangle [my favorite]

3, 4, 5 triangle

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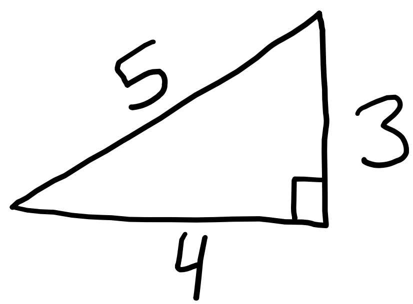
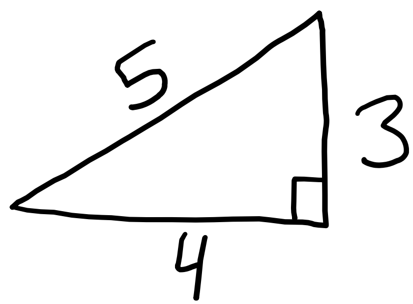


Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



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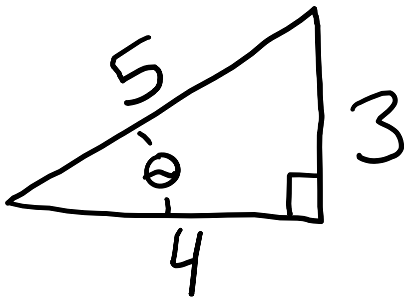

$\Rightarrow h = 5$

$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}, \tan\theta = \frac{3}{4}$

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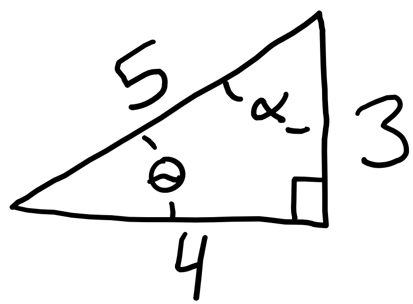

$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}, \tan\theta = \frac{3}{4}$

$\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}, \tan\alpha = \frac{4}{3}$

ASU

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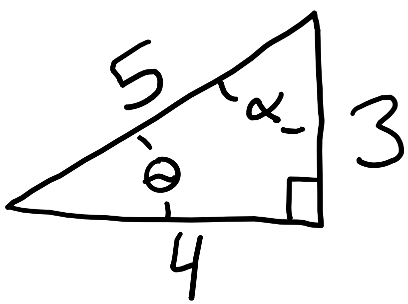

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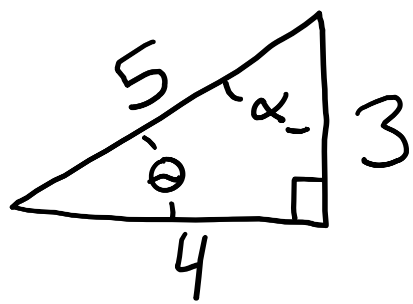
Vector derivitives

Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}, \tan\theta = \frac{3}{4}$

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If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u

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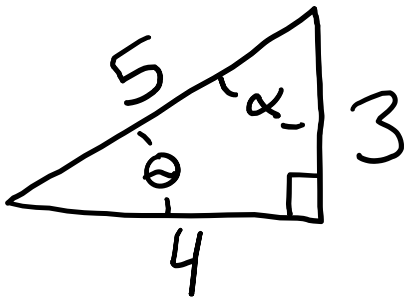

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Vector derivitives

If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u, then

$$\frac{d}{du}\left[\vec{P} + \vec{Q}\right] = \frac{d}{du}\vec{P} + \frac{d}{du}\vec{Q}$$

Vector derivitives [continued]

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$$\frac{d}{du}\left(f\vec{P}\right) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}(f\vec{P}) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}(\vec{P}\cdot\vec{Q}) = \left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q} + \vec{P}\cdot\left(\frac{dQ}{du}\right)$$

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with $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

Vector derivitives [continued]

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with $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k} \Rightarrow$

$$\frac{d\vec{P}}{du} = \left(\frac{dP_x}{du}\right)\hat{i} + \left(\frac{dP_y}{du}\right)\hat{j} + \left(\frac{dP_z}{du}\right)\hat{k}$$

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Notation: $\frac{d}{dt}\vec{P} \equiv \dot{\vec{P}} = \dot{P}_x\hat{i} + \dot{P}_y\hat{j} + \dot{P}_z\hat{k}$

Relative motion

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

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ASU

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Special case of projectile motion

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Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_o$$

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We have y as a function of t

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ASU

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We have y as a function of t & x as a function of t. We can get rid of the t variable & obtain an equation of y as a function of x.

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

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To get rid of t: First obtain t as a function of x

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To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$

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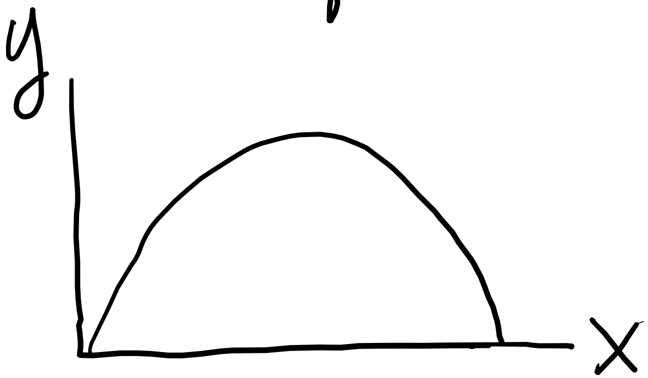

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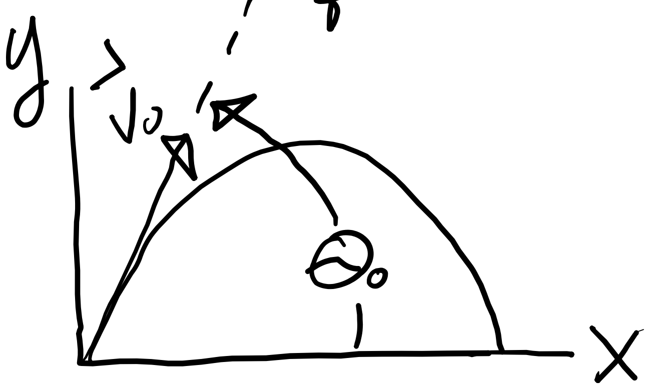

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Note: The initial velocity

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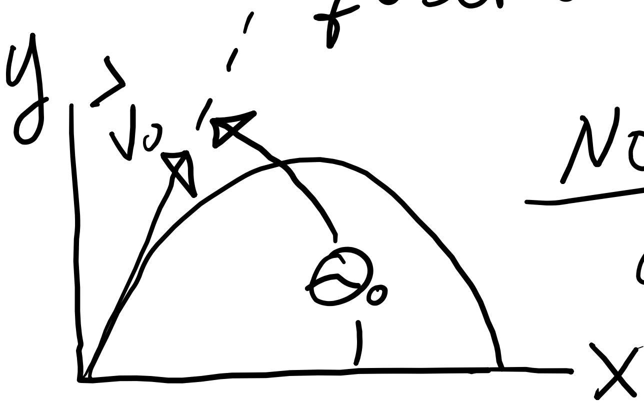

Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

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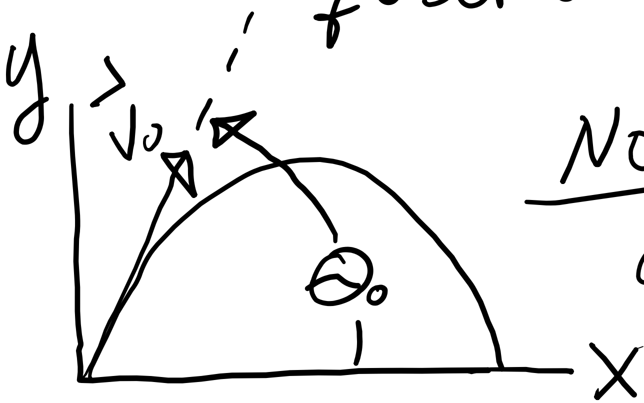

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$\Rightarrow V_{ox} = V_0 \cos\theta_0$ & $V_{oy} = V_0 \sin\theta_0 \Rightarrow$

$$\frac{V_{oy}}{V_{ox}} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0$$

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To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{0x}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$

Note: The initial velocity given by $\vec{V}_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

$\Rightarrow V_{0x} = V_0 \cos\theta_0$ & $V_{0y} = V_0 \sin\theta_0 \Rightarrow$

$$\frac{V_{0y}}{V_{0x}} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0 \Rightarrow y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + x\tan\theta_0$$

We have $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + x\tan\theta_0$

We have $y = \left(\frac{-g}{2V_{0x}^2}\right)x^2 + x\tan\theta_0$, but

$$\frac{1}{V_{0x}^2} = \left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$$

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$\frac{1}{V_{0x}^2} = \left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$ so $y = \left[\frac{-g}{2V_0^2\cos^2\theta_0}\right]x^2 + x\tan\theta_0$

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We can write $\frac{1}{\cos^2\theta_0} = \tan^2\theta_0 + 1$ by

Noticing that $\sin^2\theta_0 + \cos^2\theta_0 = 1 \Rightarrow$

$$\frac{1}{\cos^2\theta_0} = \frac{\sin^2\theta_0 + \cos^2\theta_0}{\cos^2\theta_0} = \frac{\sin^2\theta_0}{\cos^2\theta_0} + 1 = \tan^2\theta_0 + 1$$

ASU

We have $y = \left(\frac{-g}{2V_{0x}^2}\right)x^2 + x\tan\theta_0$, but

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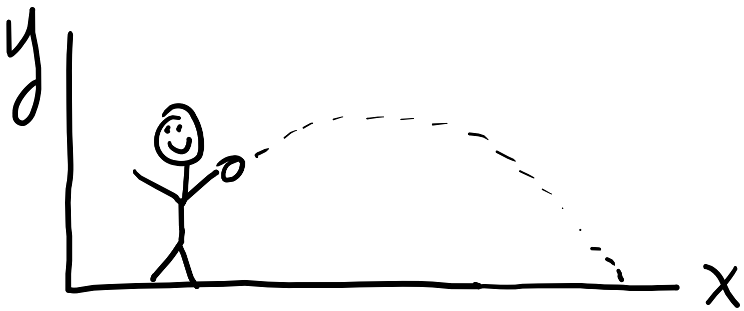
So $y = \left(\frac{-g}{2V_0^2}\right)x^2\left(\tan^2\theta_0 + 1\right) + x\tan\theta_0$

Notes on problem $\boxed{11.89}$

Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$



11.89 A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.

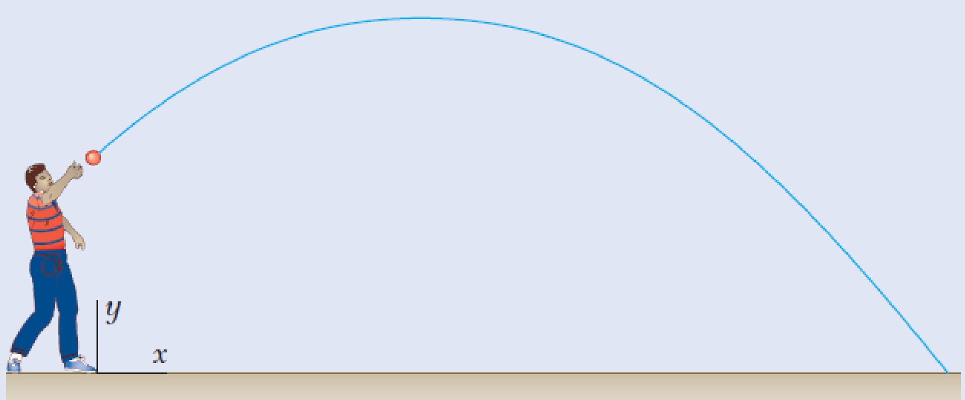
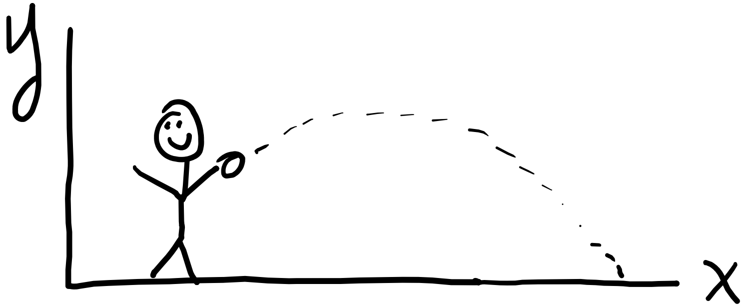


Fig. P11.89

Notes on problem $\boxed{11.89}$



Ball thrown such that

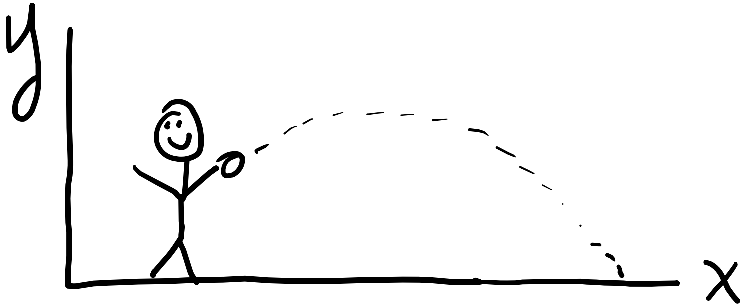
$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$.

Notes on problem $\boxed{11.89}$



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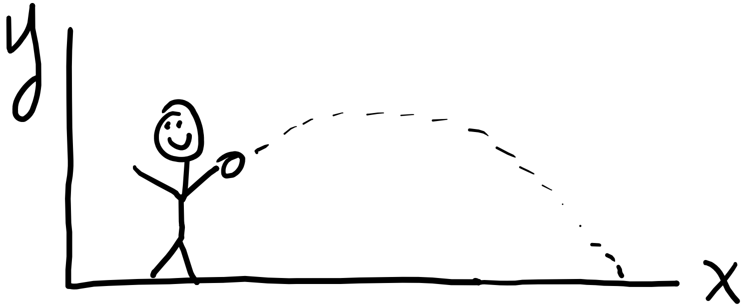
Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

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$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{V}| = [V_x^2 + V_y^2]^{1/2}$, where $V_x = \dot{x}$ & $V_y = \dot{y}$

Notes on problem $\boxed{11.89}$

Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{V}|$ at $t = 1s$

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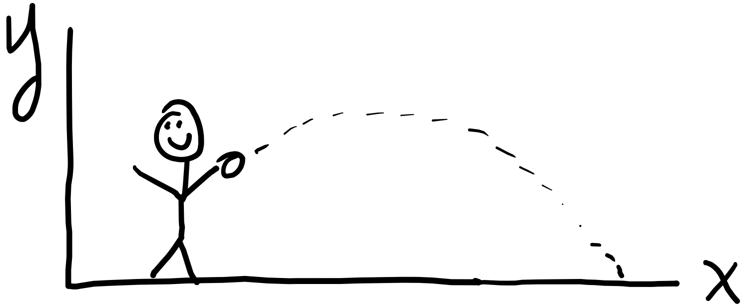
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{V}| = [V_x^2 + V_y^2]^{1/2}$, where $V_x = \dot{x}$ & $V_y = \dot{y}$

just need to find $V_x(t=1s)$ & $V_y(t=1s)$ to determine $|\vec{V}|$ at 1s

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{V}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

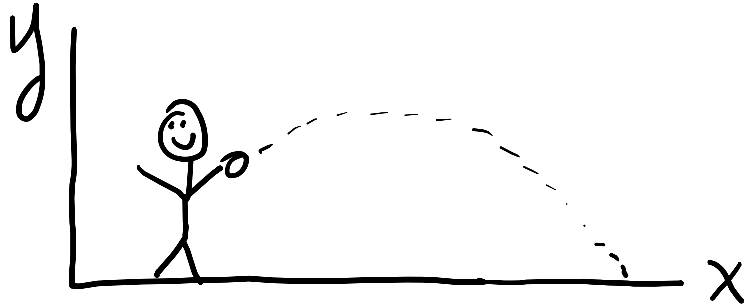
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{V}| = [V_x^2 + V_y^2]^{1/2}$, where $V_x = \dot{x}$ & $V_y = \dot{y}$

just need to find $V_x(t=1s)$ & $V_y(t=1s)$ to determine $|\vec{V}|$ at 1s

(b) Find $x(t_h)$, where $t_h \equiv$ time of hit

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

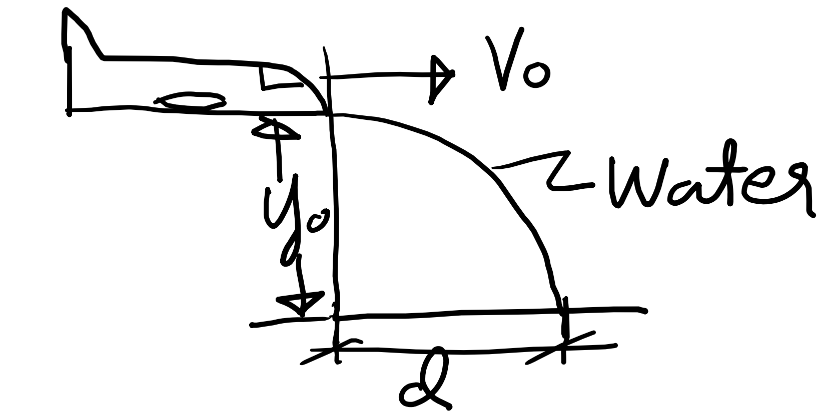
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$

just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

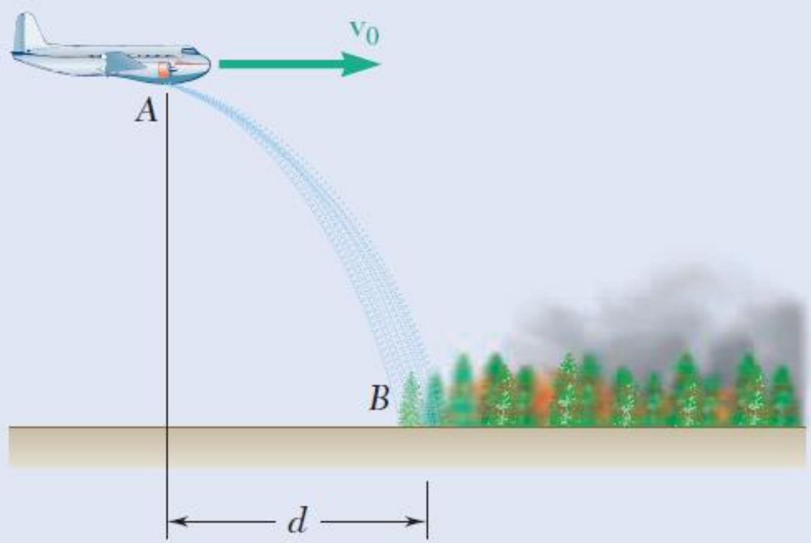
(b) Find $x(t_h)$, where $t_h \equiv$ time of hit: Couple of ways to do this. One way is to set $y(t_h) = 0$ & get t_h [quadratic formula] & then find $x(t_h)$

Notes on $\boxed{11.97}$

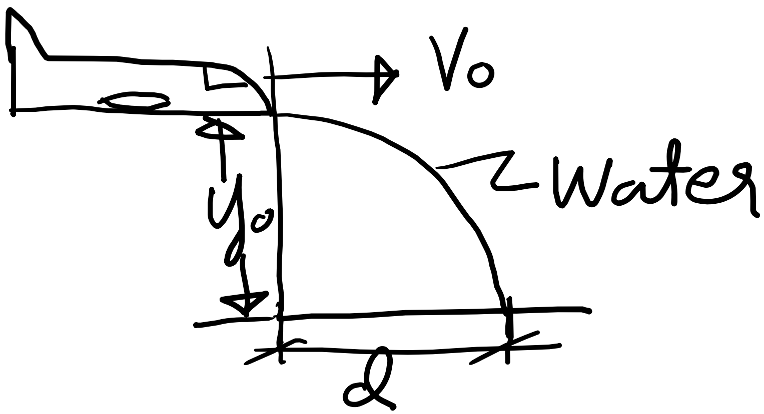


Given:
$$\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$$

11.97 An airplane used to drop water on brushfires is flying horizontally in a straight line at 180 mi/h at an altitude of 300 ft. Determine the distance *d* at which the pilot should release the water so that it will hit the fire at *B*.



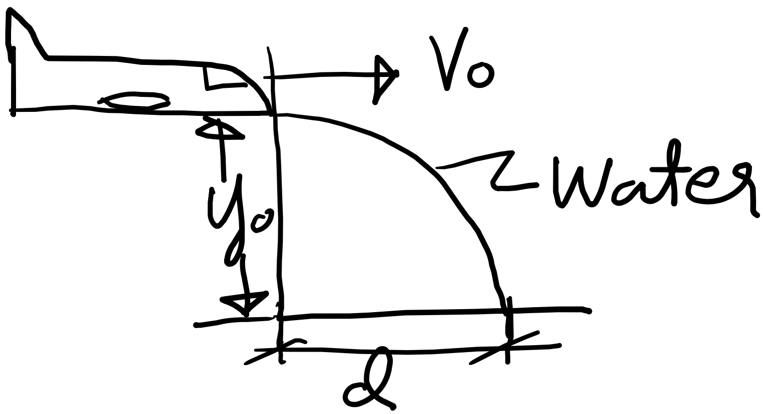
Notes on $\boxed{11.97}$



Given: $\begin{cases} V_0 = 180\,\frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$

$$x = V_0 t \Rightarrow t = \frac{x}{V_0}$$

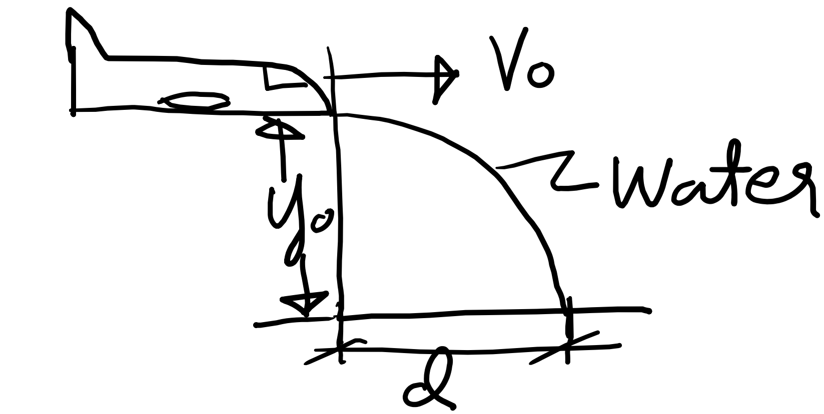
Notes on $\boxed{11.97}$



Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$

$X = V_0 t \Rightarrow t = \frac{X}{V_0}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_0}$

Notes on $\boxed{11.97}$

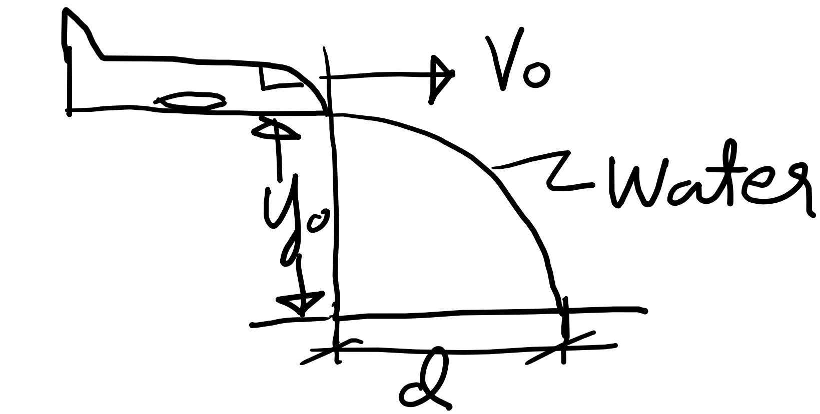


Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\, ft \\ V_{y_0} = 0 \end{cases}$

$x = V_0 t \Rightarrow t = \frac{x}{V_0}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_0}$

Also $y = -\frac{g}{2} t^2 + V_{0y} t + y_0$

Notes on $\boxed{11.97}$



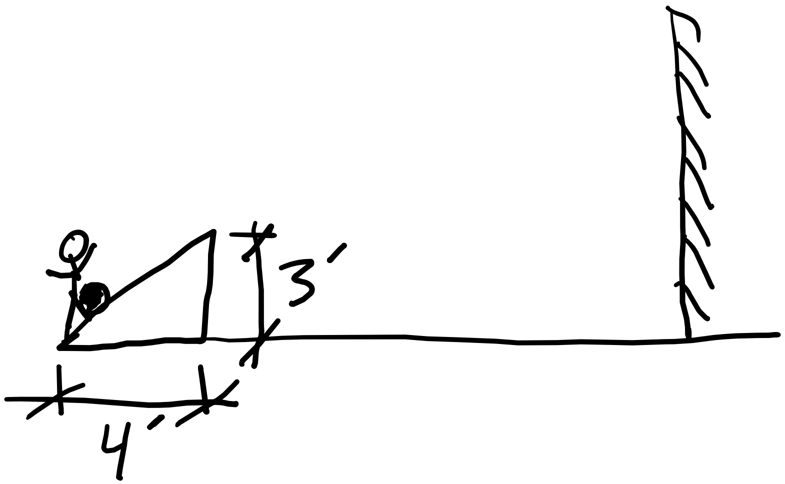
Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$

$x = V_0 t \Rightarrow t = \frac{x}{V_0}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_0}$

Also $y = -\frac{g}{2}t^2 + \cancel{V_{0y}}^{\,0}\,t + y_0 \Rightarrow y_{hit} = -\frac{g}{2}t_{hit}^2 + y_0$

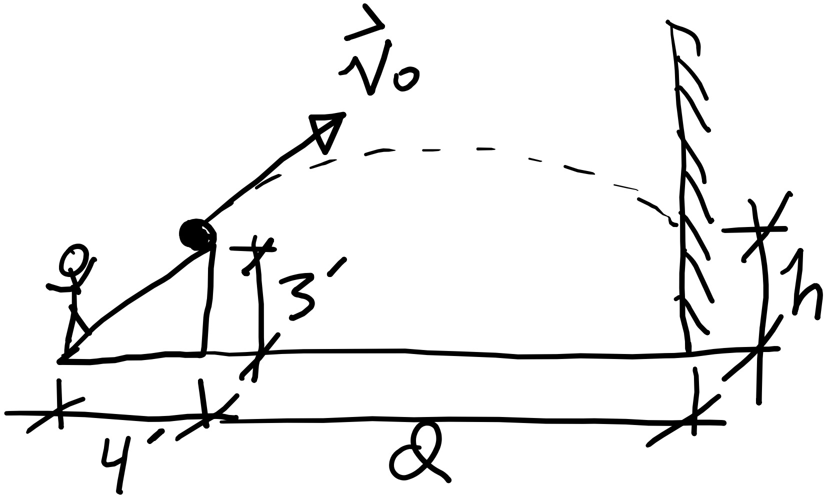
But $y_{hit} = 0$

Example similar to problem 11.105



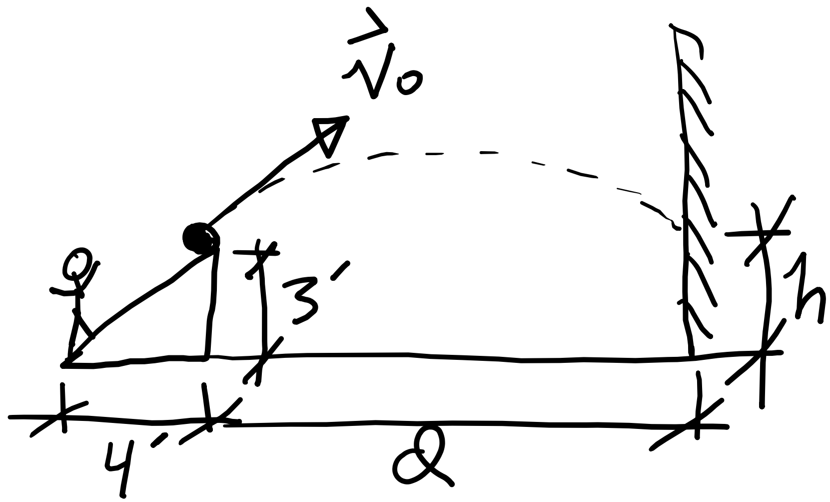
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Example similar to problem 11.105



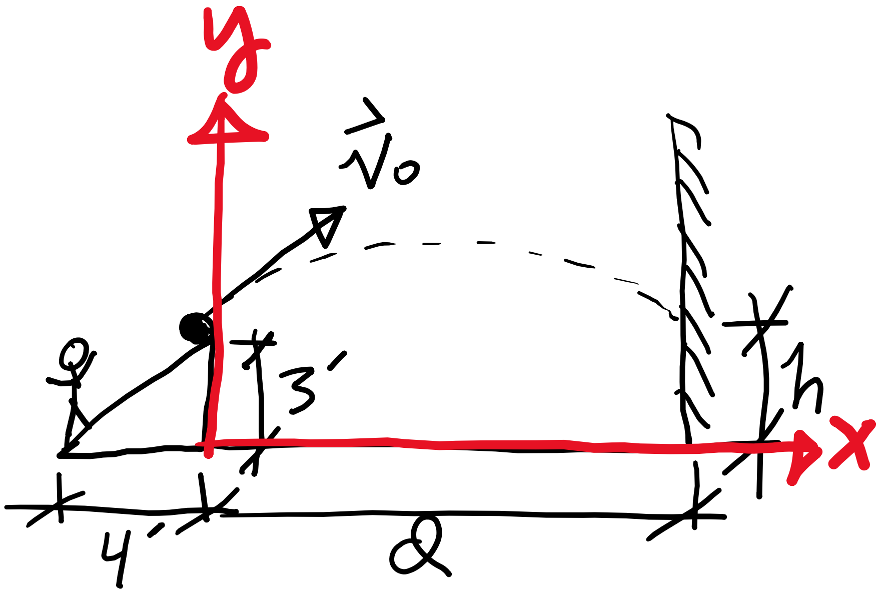
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\ \text{ft/s}$, $d = 16\ \text{ft}$ & taking $g = 32\ \text{ft/s}^2$

Instead of $32.2\ \text{ft/s}^2$

[to make life easier]

Example similar to problem 11.105

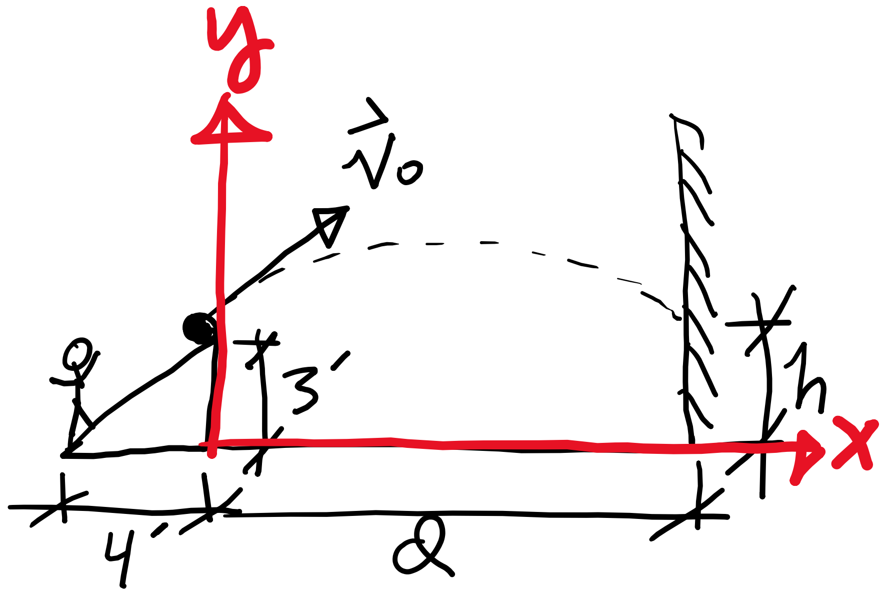


Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\ ft/s$, $d = 16\ ft$ & taking $g = 32\ ft/s^2$

Using coordinate system shown above

Example similar to problem 11.105

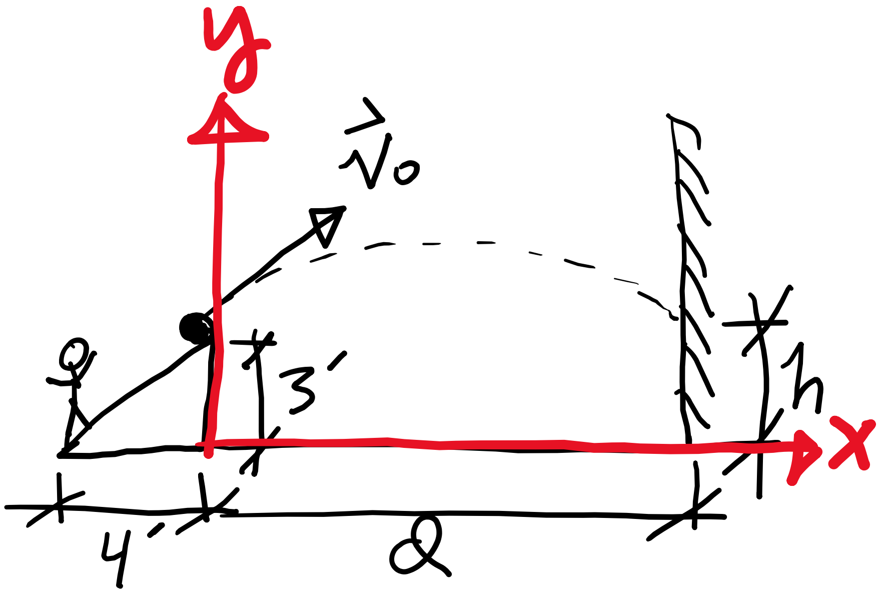


Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\ ft/s$, $d = 16\ ft$ & taking $g = 32\ ft/s^2$

We have $\quad x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$

Example similar to problem 11.105



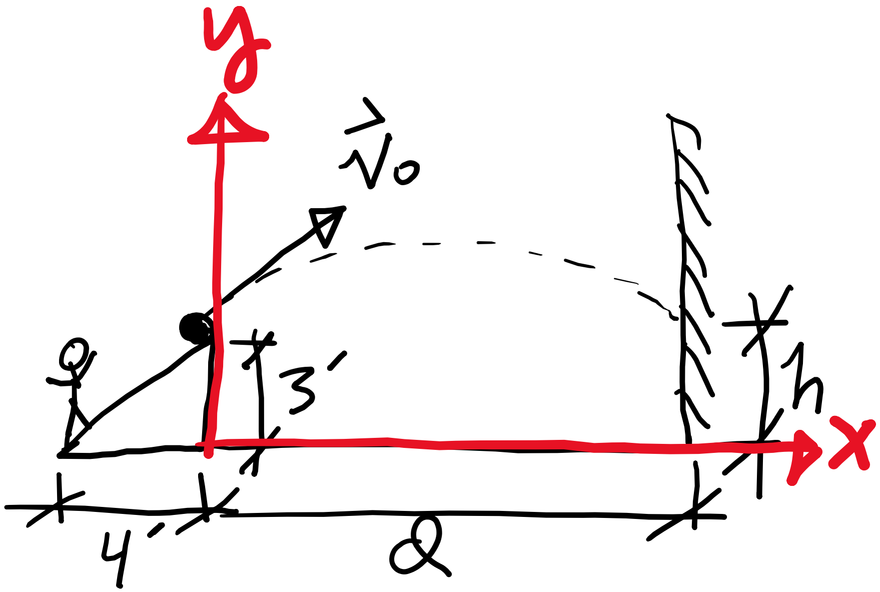
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\ ft/s$, $d = 16\ ft$ & taking $g = 32\ ft/s^2$

We have $x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0$$

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\,\text{ft/s}$, $d = 16\,\text{ft}$ & taking $g = 32\,\text{ft/s}^2$

We have $\quad x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy} t + y_o \Rightarrow h = \left(-\frac{g}{2}\right)t_h^2 + v_{oy} t_h + y_o$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right) d^2 + \left(\frac{V_{0y}}{V_{0x}}\right) d + y_0 \text{ , but}$$

$$V_{0x} = V_0 \cos \theta_0 \Rightarrow V_{0x} = V_0 \left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right) \text{ft/s}$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0 \text{ , but}$$

$$V_{0x} = V_0 \cos\theta_0 \Rightarrow V_{0x} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right) ft/s$$

Note: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin\theta_0}{V_0 \cos\theta_0} = \tan\theta_0 = \frac{3}{4}$

ASU

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0 \text{ , but}$$

$$V_{0x} = V_0 \cos\theta_0 \Rightarrow V_{0x} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right) ft/s$$

<u>Note</u>: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin\theta_0}{V_0 \cos\theta_0} = \tan\theta_0 = \frac{3}{4}$

So $h = \left\{\left[\frac{-32 * 5^2}{2*(32*4)^2}\right]16^2 + \frac{3}{4}*16 + 3\right\} ft$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{oy}}{V_{ox}}\right)d + y_0 \text{ , but}$$

$$V_{ox} = V_0 \cos\theta_0 \Rightarrow V_{ox} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right) ft/s$$

Note: $\frac{V_{oy}}{V_{ox}} = \frac{V_0 \sin\theta_0}{V_0 \cos\theta_0} = \tan\theta_0 = \frac{3}{4}$

$$\text{So} \quad h = \left\{\left[\frac{-32 * 5^2}{2*(32*4)^2}\right]16^2 + \frac{3}{4}*16 + 3\right\} ft$$

$$\Rightarrow \quad h = \left\{\left[\frac{-25}{2*32*16}\right]16^2 + 12 + 3\right\} ft$$

$$\Rightarrow \quad h = \left\{-\frac{25}{4} + 15\right\} ft = 8.75 ft$$