

Today: 11.4 → Curvilinear
Motion of
particles

L4

Today: 11.4

Monday: 11.5 → Non-Rectangular
Components

L4

Today: 11.4

L4

Monday: 11.5

HW#1 I will accept as late as
Wednesday

Today: 11.4

L4

Monday: 11.5

HW#1 I will accept as late as
Wednesday

HW#2: Due Friday Sept. 4th

Today: 11.4

L4

Monday: 11.5

HW#1 I will accept as late as
Wednesday

HW#2: Due Friday Sept. 4th

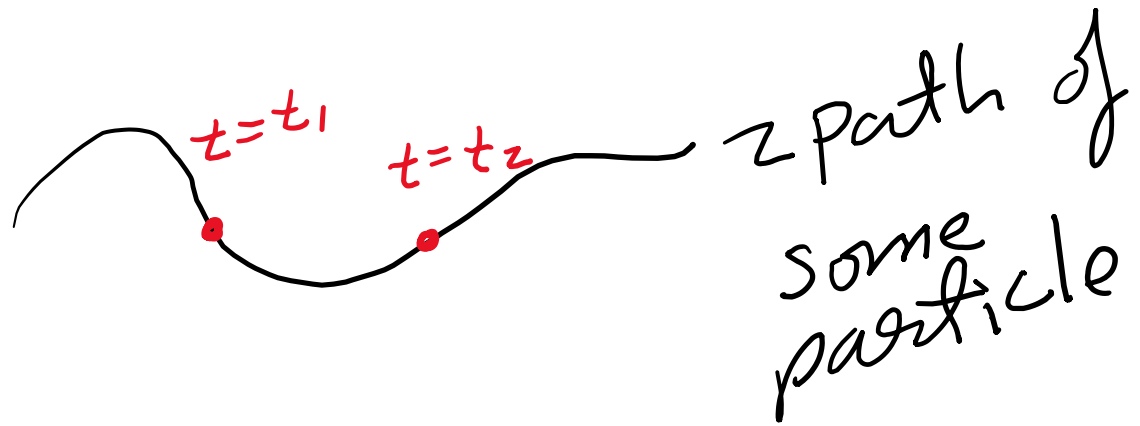
11.89, 11.97, 11.105, 11.119	§11.4
11.133, 11.134, 11.138, 11.139, 11.145	§11.5

Now moving from 1d to 3d

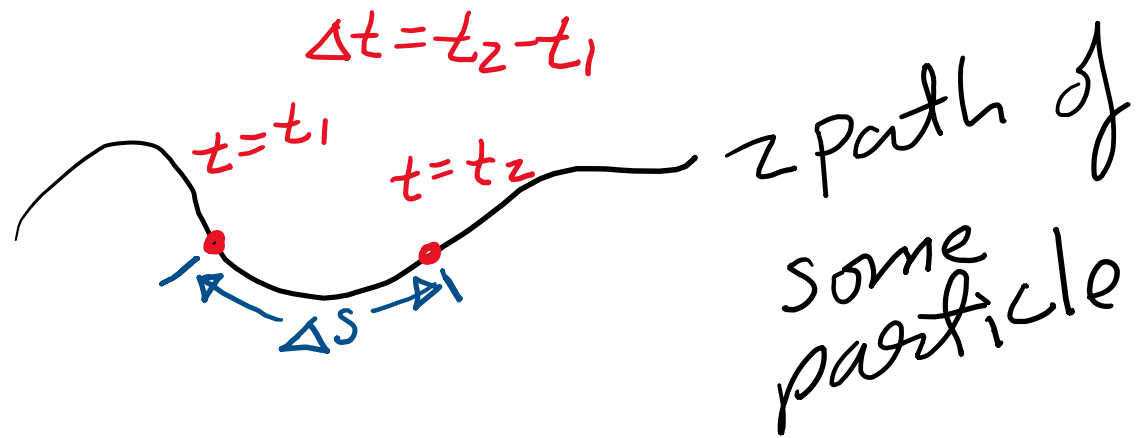
Now moving from 1d to 3d

 z path of
some
particle

Now moving from 1d to 3d

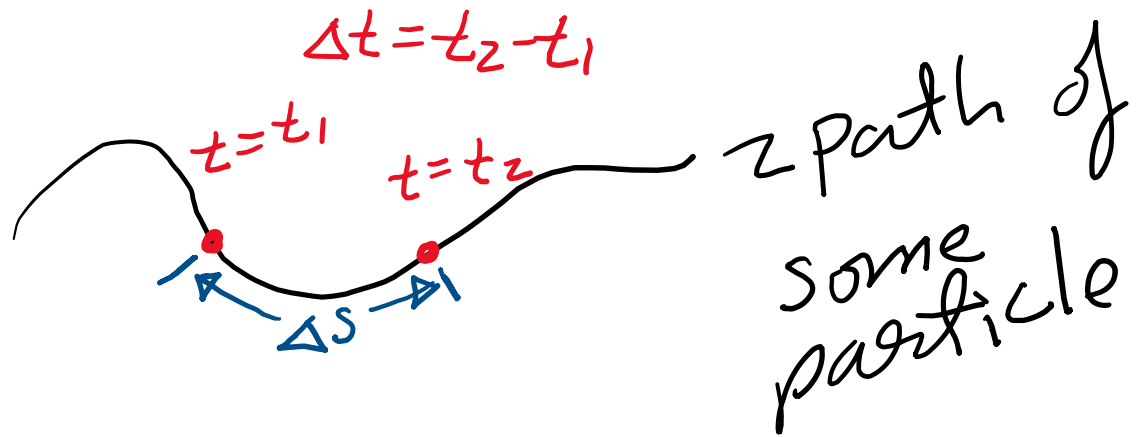


Now moving from 1d to 3d



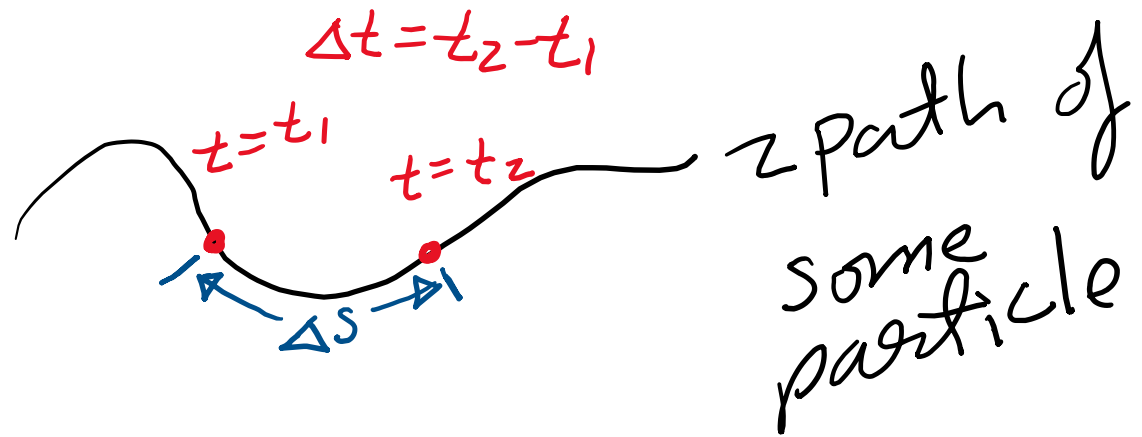
Now moving from 1d to 3d

$$v_{\text{ave}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Now moving from 1d to 3d

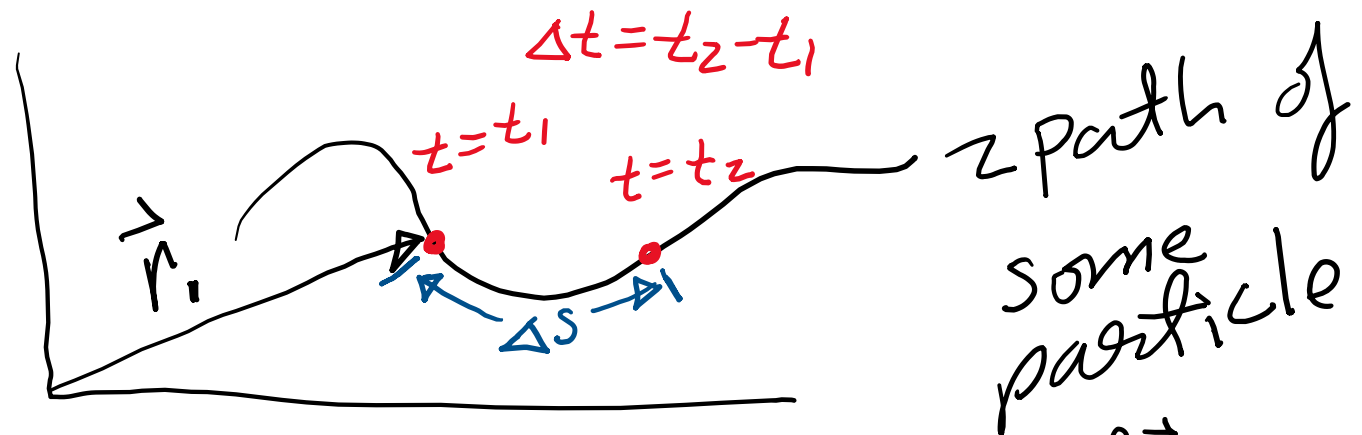
$$v_{\text{ave}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$

Now moving from 1d to 3d

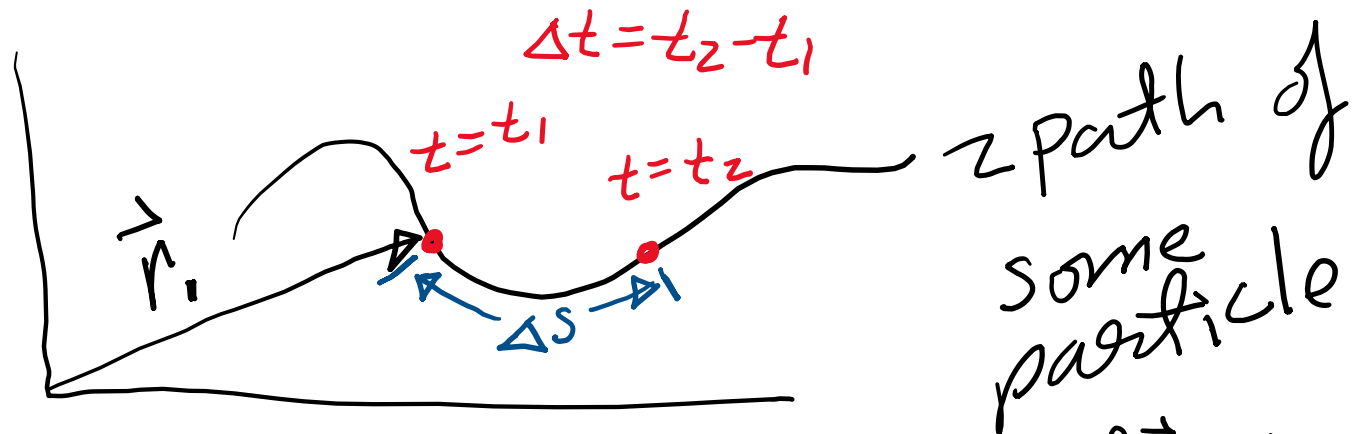
$$v_{\text{ave}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt}$

Now moving from 1d to 3d

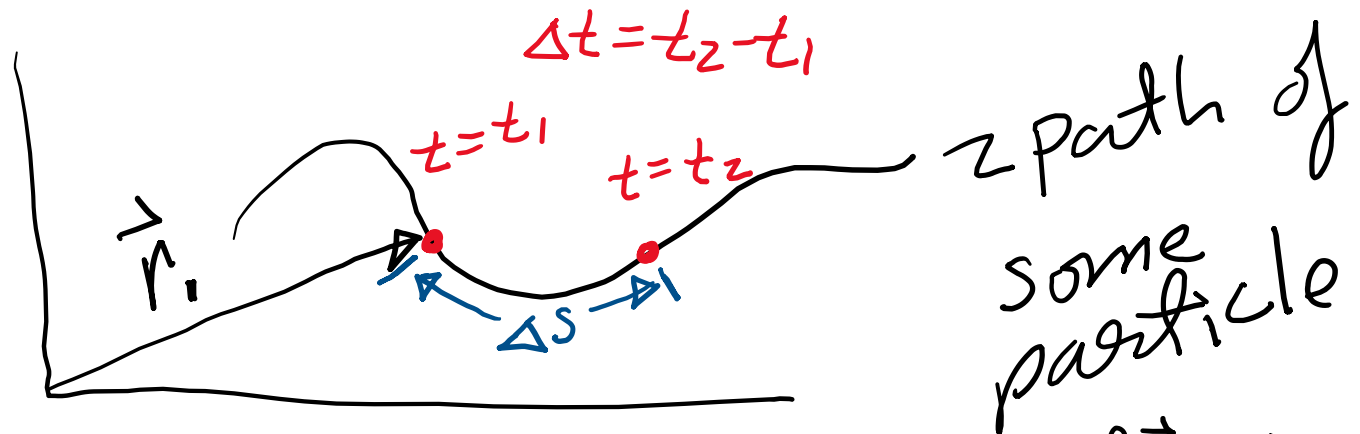
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

Now moving from 1d to 3d

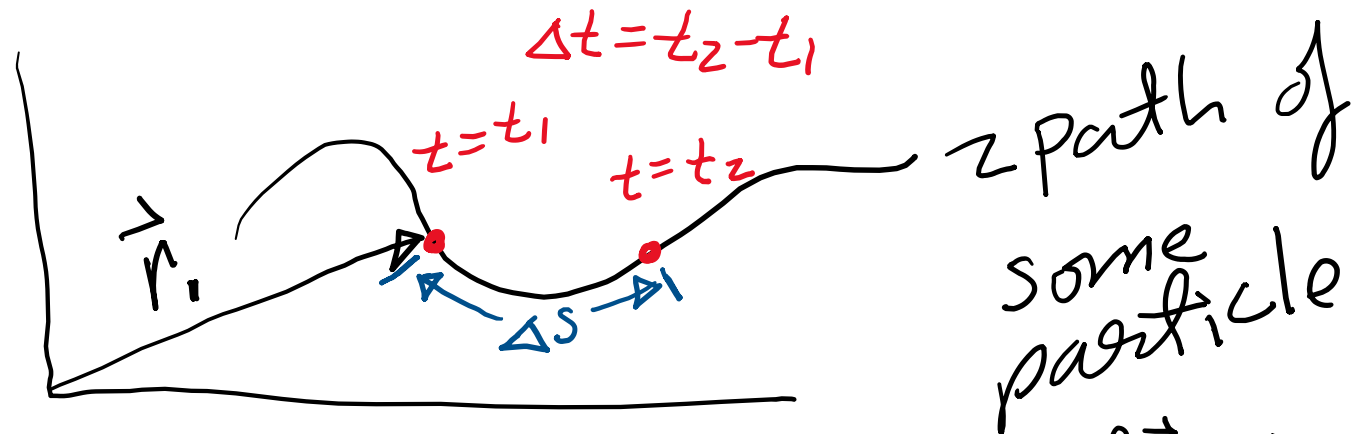
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$
make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different

Now moving from 1d to 3d

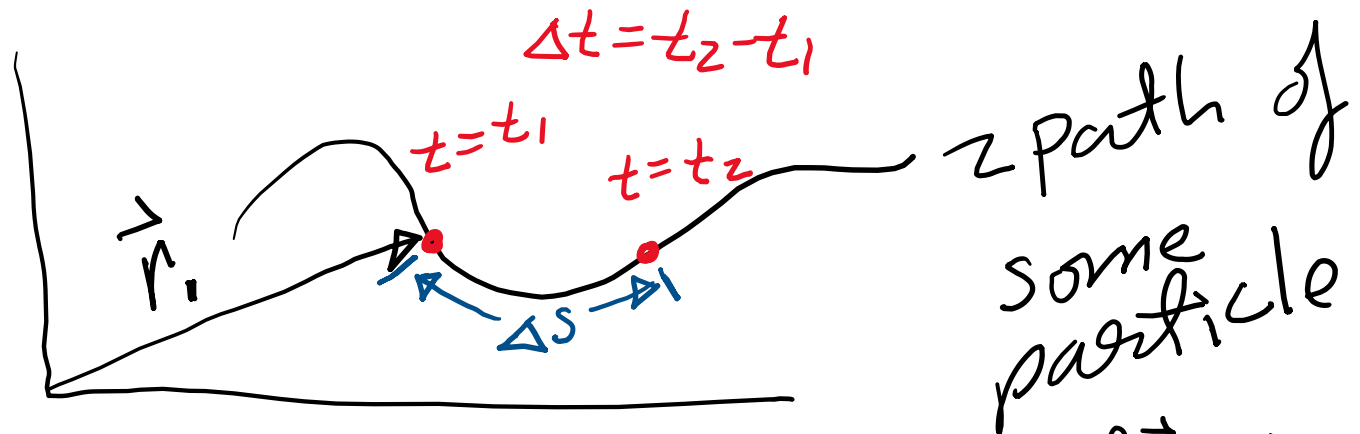
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$
make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar

Now moving from 1d to 3d

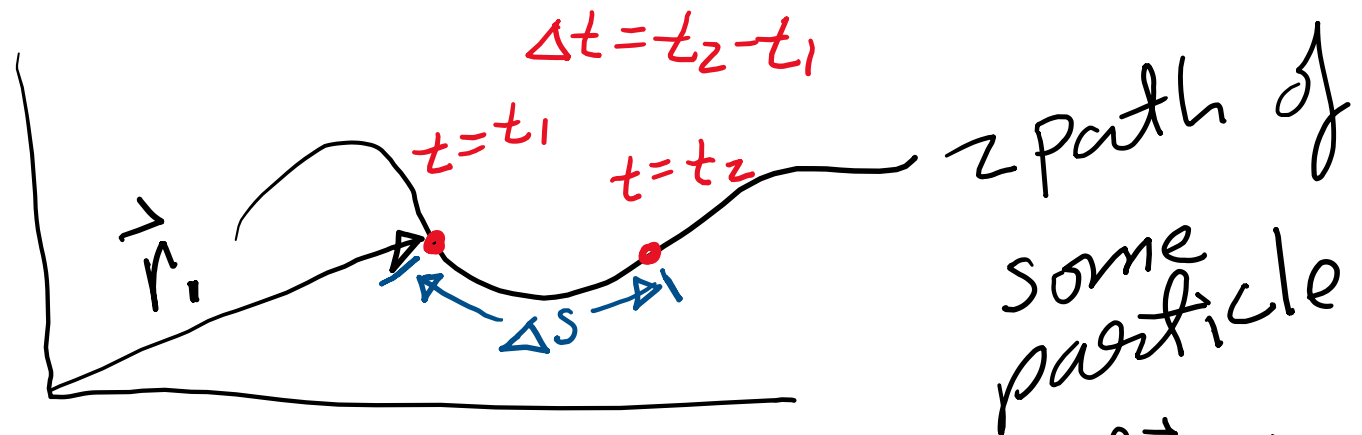
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$
make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Now moving from 1d to 3d

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$
$$\Rightarrow v = \frac{ds}{dt}$$



Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$
make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Unit vectors:

- $\hat{i} \equiv$ one unit in x-direction
- $\hat{j} \equiv$ one unit in y-direction
- $\hat{k} \equiv$ one unit in z-direction



Other common notation

$$\begin{aligned}\hat{L} &= \hat{X} = X_1 \\ \hat{J} &= \hat{Y} = X_2 \\ \hat{K} &= \hat{Z} = X_3\end{aligned}$$

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\begin{aligned}\vec{r} &= r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \\ \vec{r} &= x \hat{i} + y \hat{j} + z \hat{k}\end{aligned} \quad \text{or}$$

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or}$$

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\begin{aligned}\vec{r} &= r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or} \\ \vec{r} &= x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or} \\ \vec{r} &= r_x \hat{x} + r_y \hat{y} + r_z \hat{z}\end{aligned}$$

Just
Needs to be
clear and understandable

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or}$$

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

Velocity vector: $\vec{v} = \frac{dr_x}{dt} \hat{i} + \frac{dr_y}{dt} \hat{j} + \frac{dr_z}{dt} \hat{k}$

Other common notation

$$\begin{aligned}\hat{i} &= \hat{x} = \hat{x}_1 \\ \hat{j} &= \hat{y} = \hat{x}_2 \\ \hat{k} &= \hat{z} = \hat{x}_3\end{aligned}$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or}$$

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

Velocity vector:

$$\vec{v} = \frac{dr_x}{dt} \hat{i} + \frac{dr_y}{dt} \hat{j} + \frac{dr_z}{dt} \hat{k}$$

Acceleration vector:

$$\vec{a} = \frac{d^2 r_x}{dt^2} \hat{i} + \frac{d^2 r_y}{dt^2} \hat{j} + \frac{d^2 r_z}{dt^2} \hat{k}$$

Let time derivative be given
by an over-dot

Let time derivative be given
by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k}$$

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

& similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Two
over-dots
for 2nd
time derivatives

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product:

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared:

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $r^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $r^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude:

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $r^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r} \cdot \vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $r^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r} \cdot \vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

Cross product:



Let time derivative be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} + \dot{z} \hat{k}$$

‡ similarly $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{z} \hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

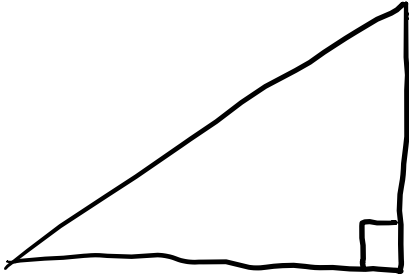
Magnitude-squared: $r^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r} \cdot \vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

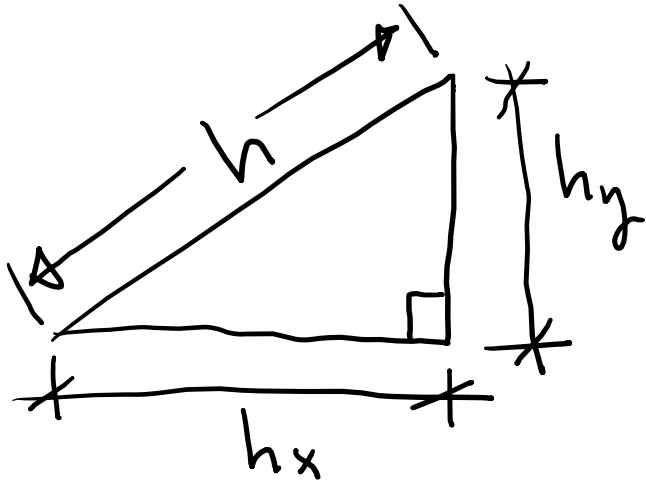
Cross product: $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$

Super quick trig review:

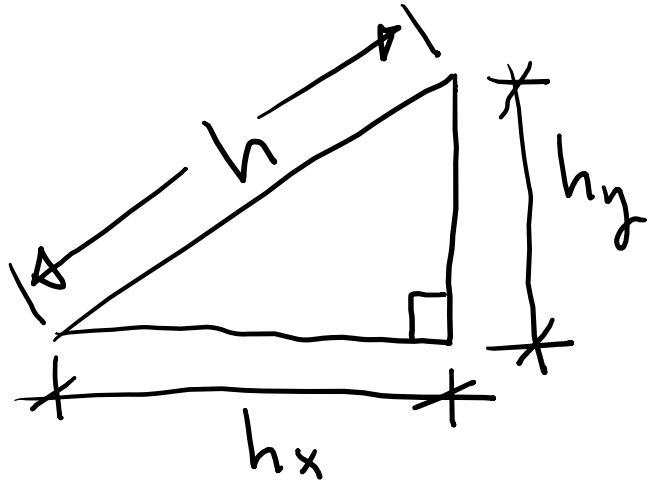
Super quick trig review:



Super quick trig review:

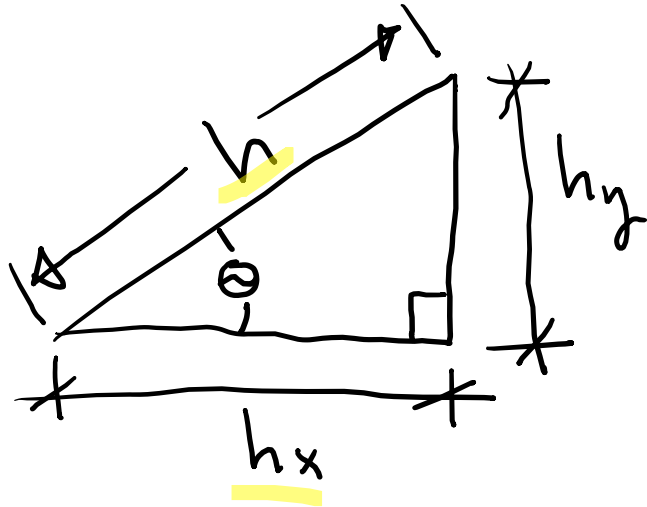


Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

Super quick trig review:

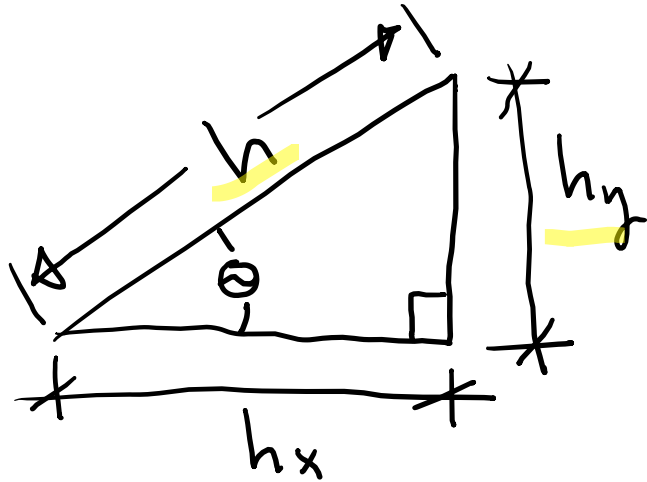


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{h_x}{h}$$

Super quick trig review:

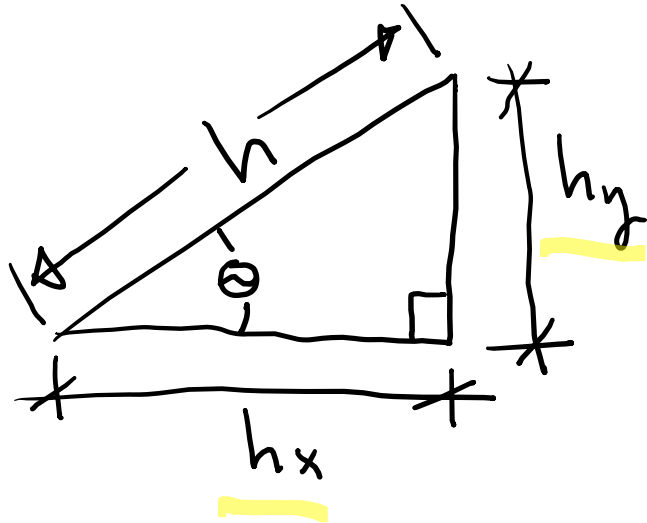


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}$$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{h_y}{h}$$

Super quick trig review:

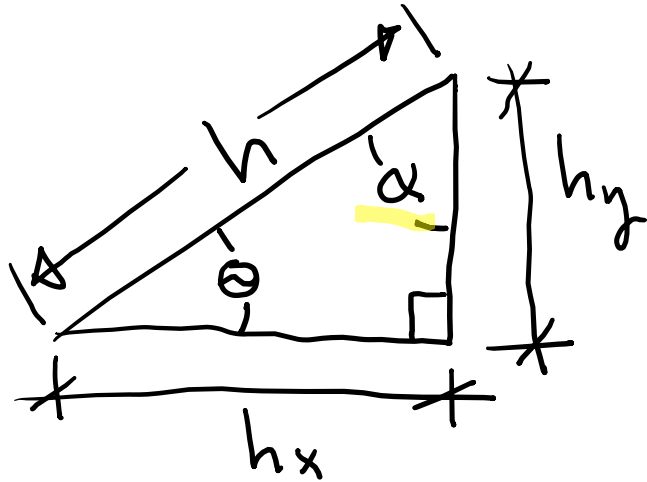


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}} = \frac{h_y}{h_x}$$

Super quick trig review:

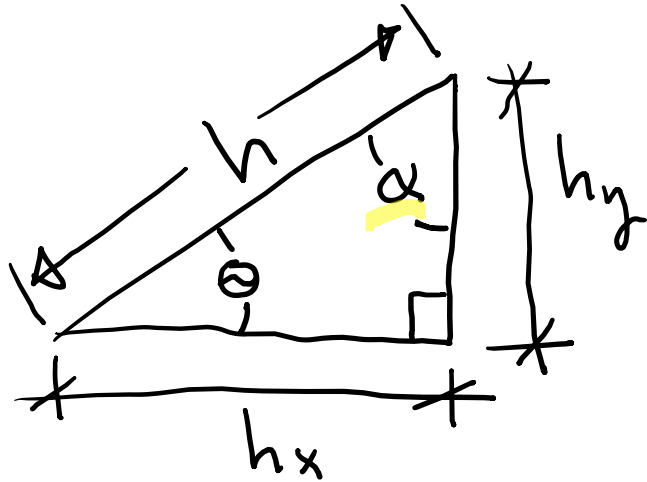


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}$$

Super quick trig review:

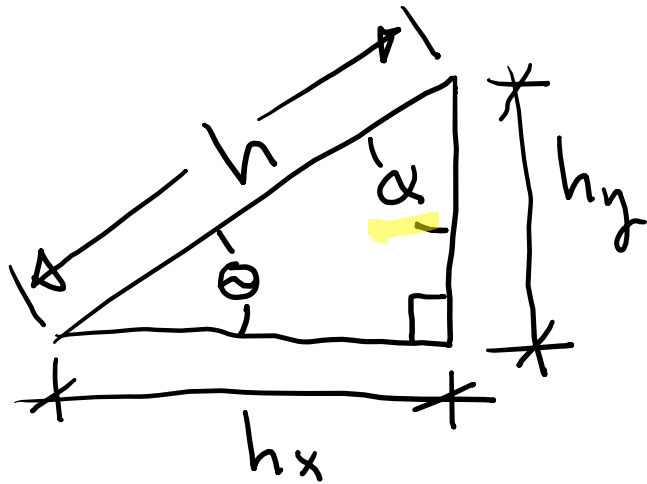


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}$$

Super quick trig review:

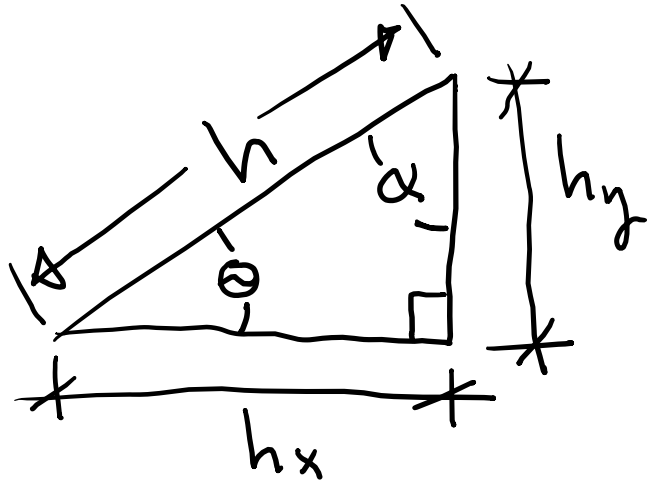


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

Super quick trig review:



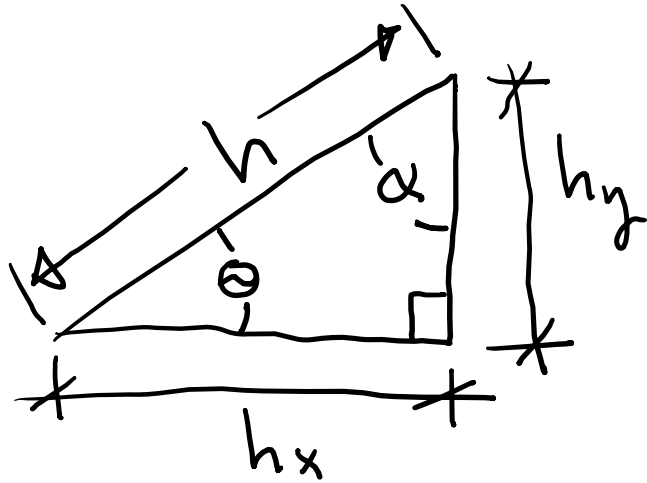
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ$$

Super quick trig review:



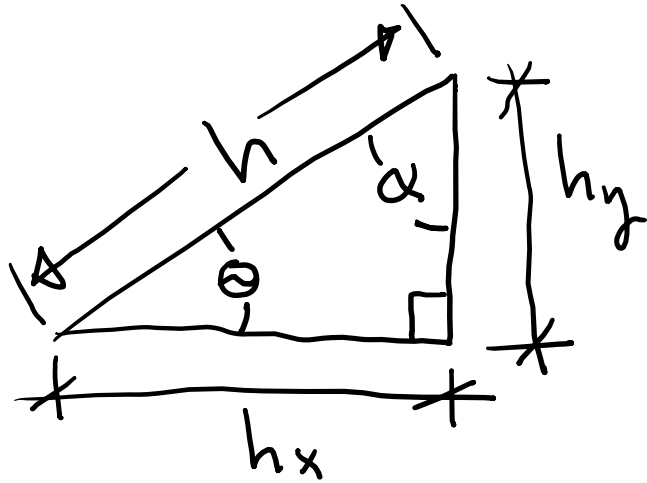
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

Super quick trig review:



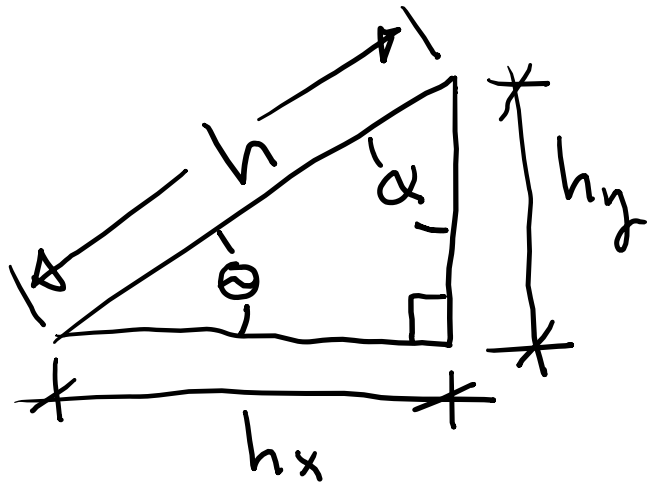
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$
$$\cos(90^\circ - \theta) = \sin\theta$$
$$\sin(90^\circ - \theta) = \cos\theta$$

Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

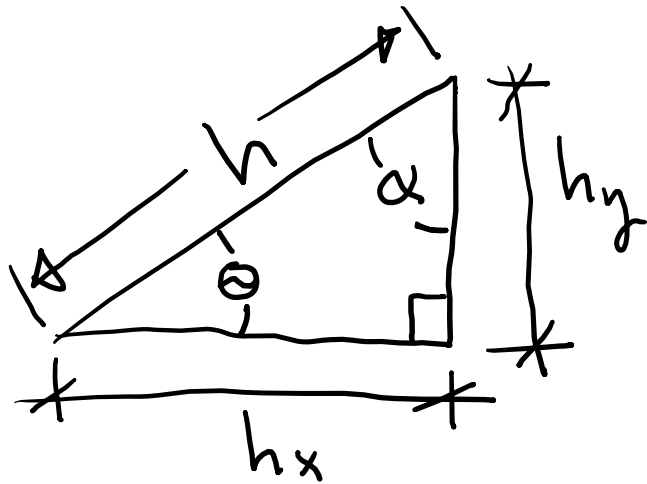
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\sin(90^\circ - \theta) = \cos\theta$$

Super quick trig review:



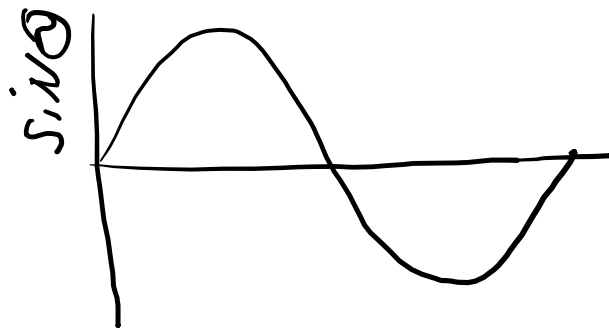
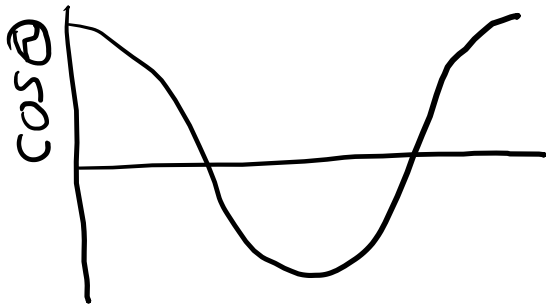
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \sin\theta = \frac{h_y}{h}, \tan\theta = \frac{h_y}{h_x}$$

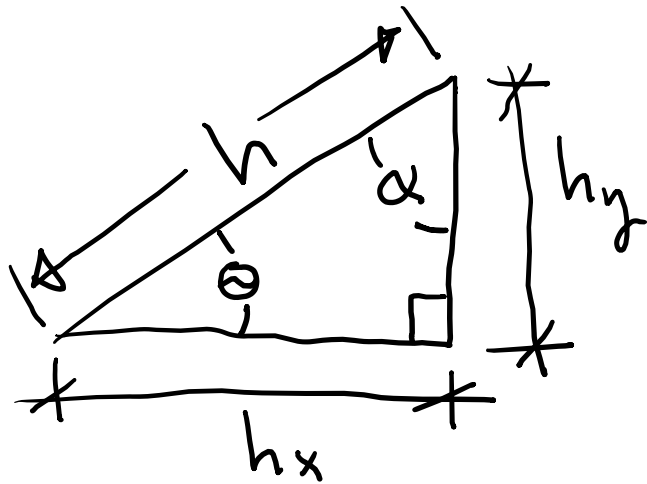
$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \quad \cos(90^\circ - \theta) = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \quad \& \sin(90^\circ - \theta) = \cos\theta$$



Super quick trig review:



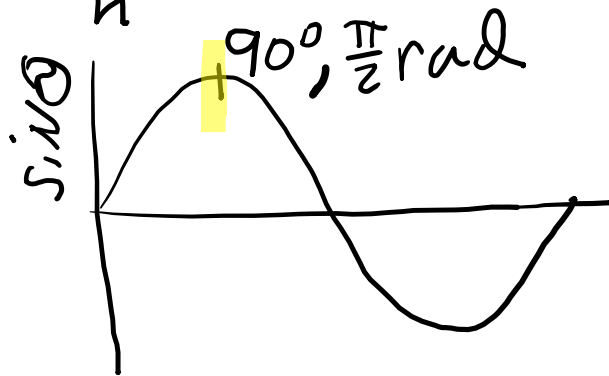
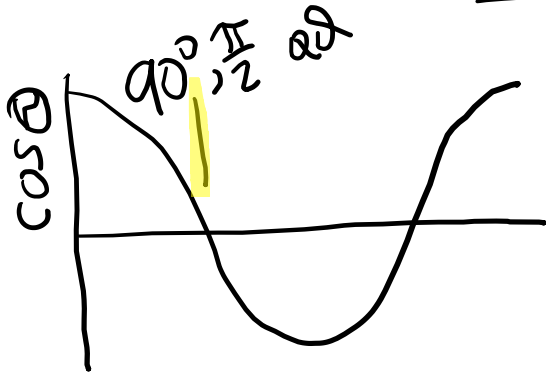
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \sin\theta = \frac{h_y}{h}, \tan\theta = \frac{h_y}{h_x}$$

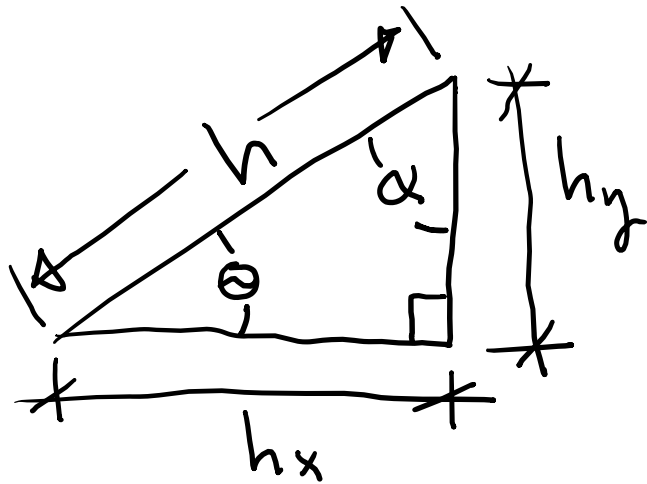
$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \quad \cos(90^\circ - \theta) = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \quad \& \sin(90^\circ - \theta) = \cos\theta$$



Super quick trig review:



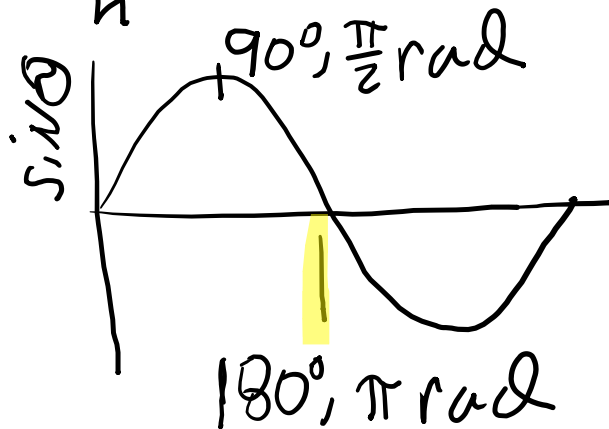
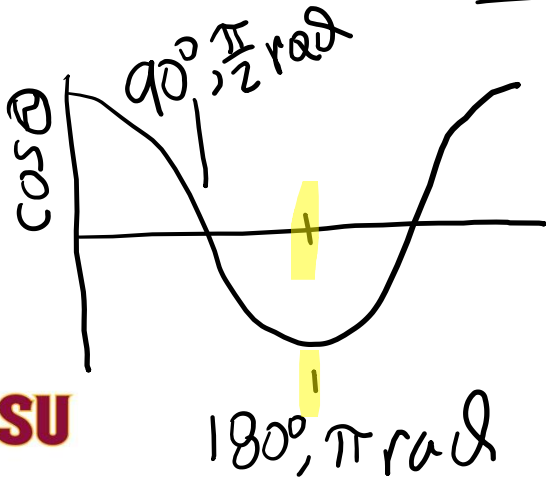
$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \sin\theta = \frac{h_y}{h}, \tan\theta = \frac{h_y}{h_x}$$

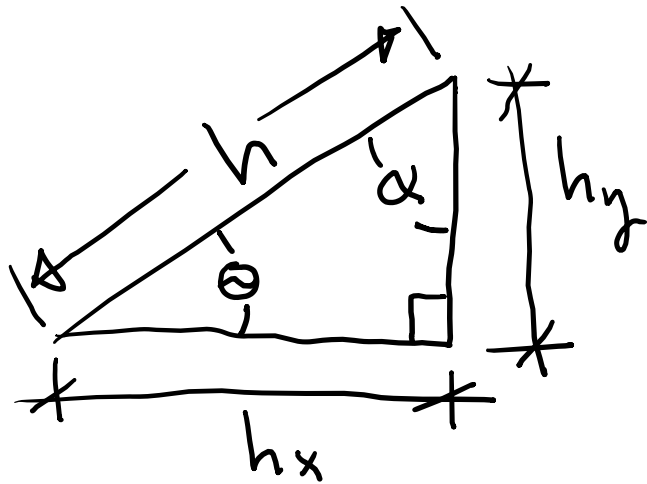
$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \quad \cos(90^\circ - \theta) = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \quad \& \sin(90^\circ - \theta) = \cos\theta$$



Super quick trig review:



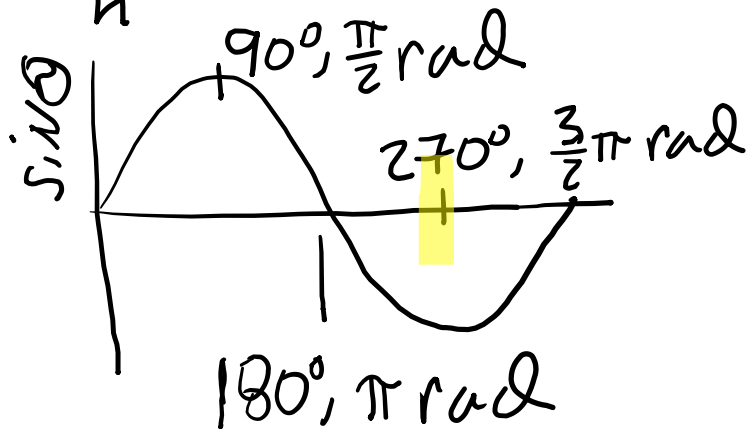
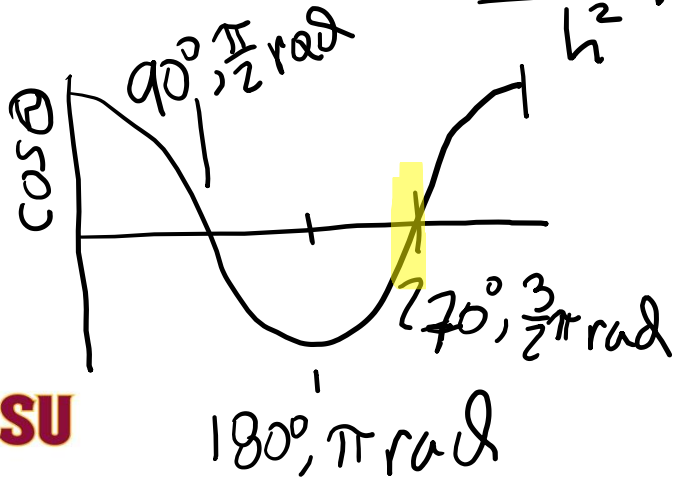
$$h^2 = h_x^2 + h_y^2$$

$$\cos \theta = \frac{h_x}{h}, \sin \theta = \frac{h_y}{h}, \tan \theta = \frac{h_y}{h_x}$$

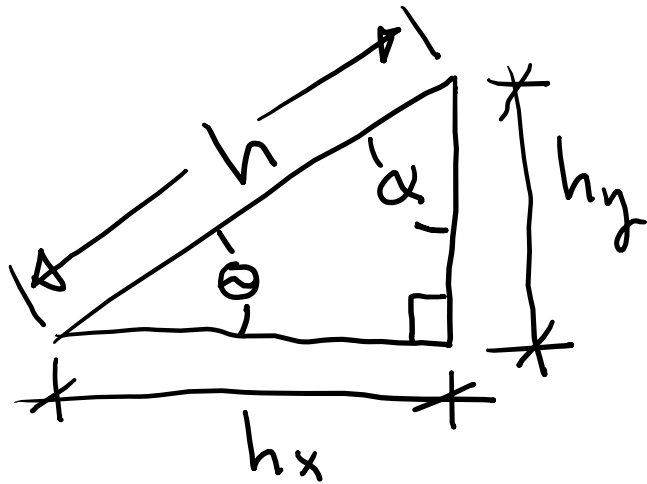
$$\cos \alpha = \frac{h_y}{h}, \sin \alpha = \frac{h_x}{h}, \tan \alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \quad \cos(90^\circ - \theta) = \sin \theta$$

$$\cos^2 \theta + \sin^2 \theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \quad \& \sin(90^\circ - \theta) = \cos \theta$$



Super quick trig review:



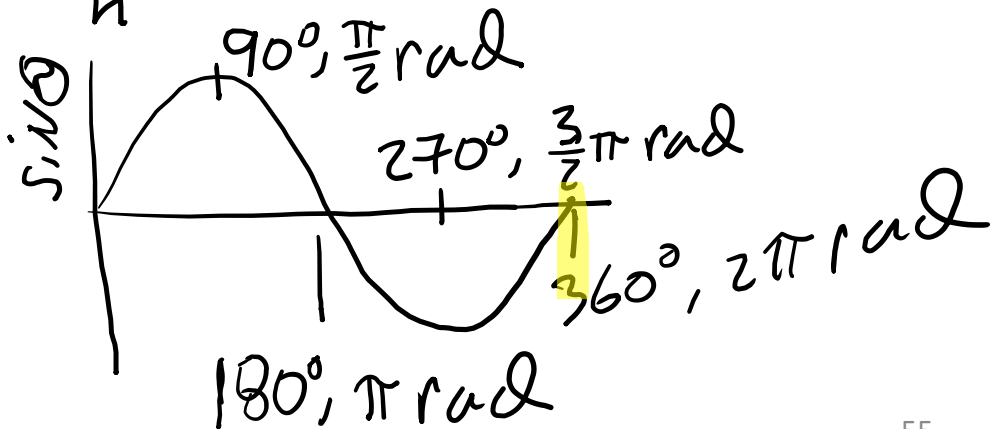
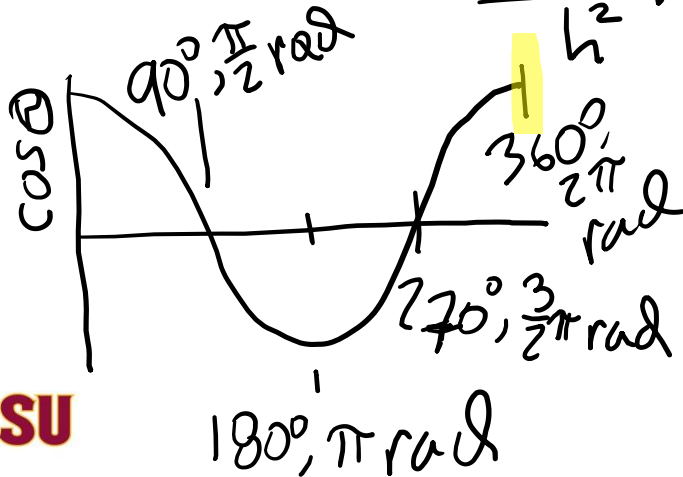
$$h^2 = h_x^2 + h_y^2$$

$$\cos \theta = \frac{h_x}{h}, \sin \theta = \frac{h_y}{h}, \tan \theta = \frac{h_y}{h_x}$$

$$\cos \alpha = \frac{h_y}{h}, \sin \alpha = \frac{h_x}{h}, \tan \alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta \quad \cos(90^\circ - \theta) = \sin \theta$$

$$\cos^2 \theta + \sin^2 \theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1 \quad \& \sin(90^\circ - \theta) = \cos \theta$$



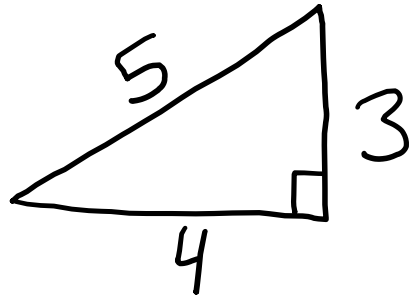
Special triangle [my favorite]

Special triangle [my favorite]

3, 4, 5 triangle

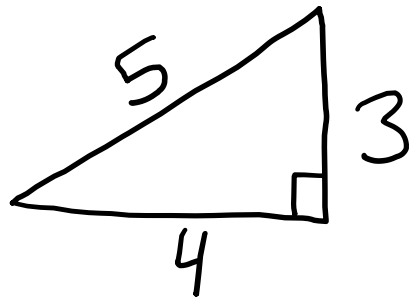
Special triangle [my favorite]

3, 4, 5 triangle



Special triangle [my favorite]

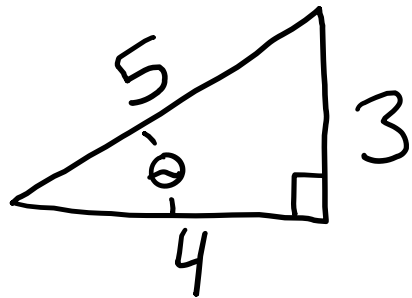
3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$
 $\Rightarrow h = 5$



Special triangle [my favorite]

3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

$$\Rightarrow h = 5$$

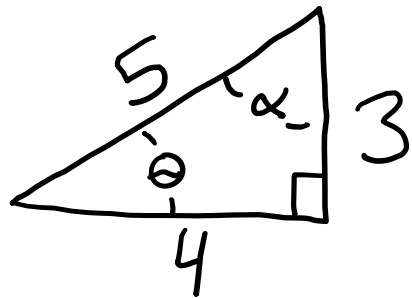


$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}, \tan \theta = \frac{3}{4}$$

Special triangle [my favorite]

3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

$$\Rightarrow h = 5$$



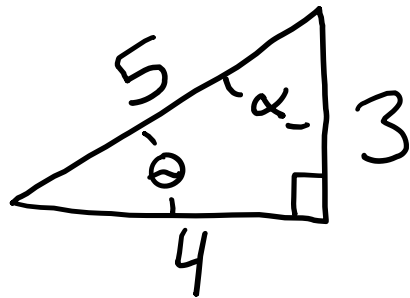
$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}, \tan \theta = \frac{3}{4}$$

$$\cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5}, \tan \alpha = \frac{4}{3}$$

Special triangle [my favorite]

3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

$$\Rightarrow h = 5$$



$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}, \tan \theta = \frac{3}{4}$$

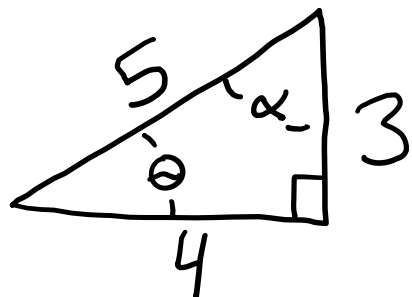
$$\cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5}, \tan \alpha = \frac{4}{3}$$

Vector derivatives

Special triangle [my favorite]

3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

$$\Rightarrow h = 5$$



$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}, \tan \theta = \frac{3}{4}$$

$$\cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5}, \tan \alpha = \frac{4}{3}$$

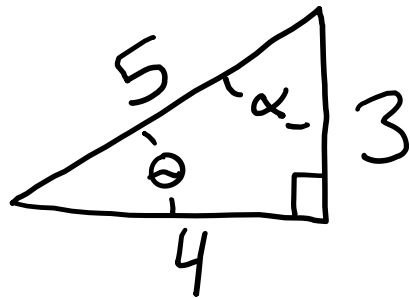
Vector derivatives

If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u

Special triangle [my favorite]

3, 4, 5 triangle : $h = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

$$\Rightarrow h = 5$$



$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}, \tan \theta = \frac{3}{4}$$

$$\cos \alpha = \frac{3}{5}, \sin \alpha = \frac{4}{5}, \tan \alpha = \frac{4}{3}$$

Vector derivatives

If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u , then

$$\frac{d}{du} [\vec{P} + \vec{Q}] = \frac{d}{du} \vec{P} + \frac{d}{du} \vec{Q}$$

Vector Derivatives [continued]

If $f(u)$ is a scalar function of variable u

Vector derivatives [continued]

If $f(u)$ is a scalar function of variable u , then

$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

Vector Derivatives [continued]

If $f(u)$ is a scalar function of variable u , then

$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

$$\& \frac{d}{du} (\vec{P} \cdot \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \cdot \vec{Q} + \vec{P} \cdot \left(\frac{d\vec{Q}}{du} \right)$$

Vector Derivatives [continued]

If $f(u)$ is a scalar function of variable u , then

$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \cdot \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \cdot \vec{Q} + \vec{P} \cdot \left(\frac{d\vec{Q}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \times \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \times \vec{Q} + \vec{P} \times \left(\frac{d\vec{Q}}{du} \right)$$

Vector derivatives [continued]

If $f(u)$ is a scalar function of variable u , then

$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \cdot \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \cdot \vec{Q} + \vec{P} \cdot \left(\frac{d\vec{Q}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \times \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \times \vec{Q} + \vec{P} \times \left(\frac{d\vec{Q}}{du} \right)$$

with $\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$

Vector Derivatives [continued]

If $f(u)$ is a scalar function of variable u , then

$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \cdot \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \cdot \vec{Q} + \vec{P} \cdot \left(\frac{d\vec{Q}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \times \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \times \vec{Q} + \vec{P} \times \left(\frac{d\vec{Q}}{du} \right)$$

With $\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k} \Rightarrow$

$$\frac{d\vec{P}}{du} = \left(\frac{dP_x}{du} \right) \hat{i} + \left(\frac{dP_y}{du} \right) \hat{j} + \left(\frac{dP_z}{du} \right) \hat{k}$$

Vector derivatives [continued]

If $f(u)$ is a scalar function of variable u , then


$$\frac{d}{du} (f \vec{P}) = \left(\frac{df}{du} \right) \vec{P} + f \left(\frac{d\vec{P}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \cdot \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \cdot \vec{Q} + \vec{P} \cdot \left(\frac{d\vec{Q}}{du} \right)$$

$$\nabla \frac{d}{du} (\vec{P} \times \vec{Q}) = \left(\frac{d\vec{P}}{du} \right) \times \vec{Q} + \vec{P} \times \left(\frac{d\vec{Q}}{du} \right)$$

With $\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k} \Rightarrow$

$$\frac{d\vec{P}}{du} = \left(\frac{dP_x}{du} \right) \hat{i} + \left(\frac{dP_y}{du} \right) \hat{j} + \left(\frac{dP_z}{du} \right) \hat{k}$$

 Notation: $\frac{d}{dt} \vec{P} \equiv \dot{\vec{P}} = \dot{P}_x \hat{i} + \dot{P}_y \hat{j} + \dot{P}_z \hat{k}$

Relative motion

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0$$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$
 $\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$
 $\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t \quad \& \quad x = v_{ox}t$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t \quad \& \quad x = v_{ox}t$$

We have y as a function of t

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \quad \& \quad x = v_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t \quad \& \quad x = v_{ox}t$$

We have y as a function of t & x as a function of t .

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
 $\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t + y_0 \quad \& \quad x = v_{0x}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t \quad \& \quad x = v_{0x}t$$

We have y as a function of t & x as a function of t . We can get rid of the t variable & obtain an equation of y as a function of x .

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

To get rid of t : First obtain t as a function of x

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

To get rid of t : First obtain t as a
function of x : $t = \frac{x}{v_{0x}}$

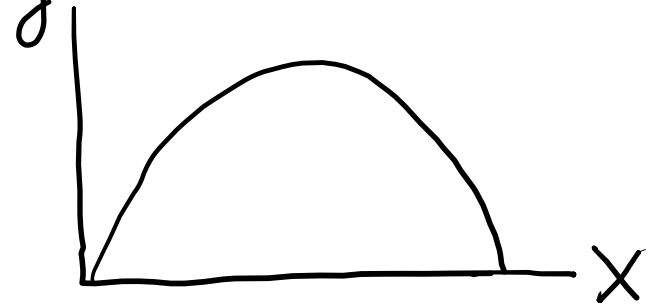
We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

To get rid of t : First obtain t as a function of x : $t = \frac{x}{v_{0x}}$. Now substitute into equation for y :

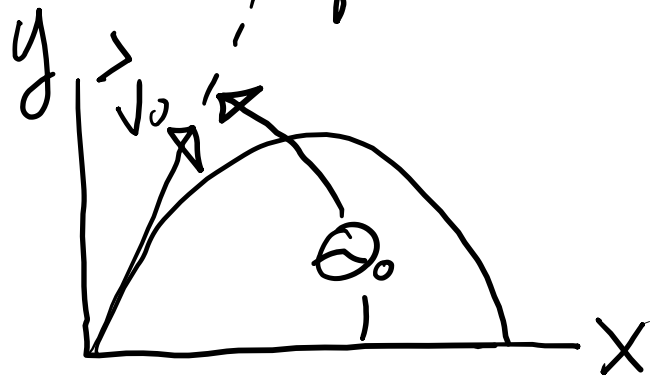
We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$
To get rid of t : First obtain t as a
function of x : $t = \frac{x}{v_{0x}}$. Now substitute
into equation for y : $y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

To get rid of t : First obtain t as a function of x : $t = \frac{x}{v_{0x}}$. Now substitute into equation for y : $y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x$



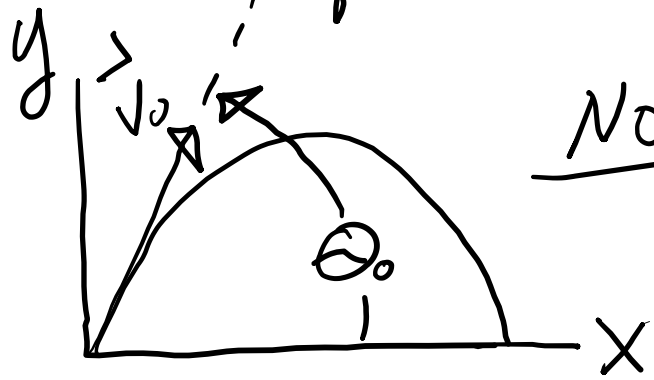
We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$
To get rid of t : First obtain t as a
function of x : $t = \frac{x}{v_{0x}}$. Now substitute
into equation for y : $y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x$



We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t$ & $x_0 = v_{0x}t$

To get rid of t : First obtain t as a function of x : $t = \frac{x}{v_{0x}}$. Now substitute into equation for y : $y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x$

Note: The initial velocity



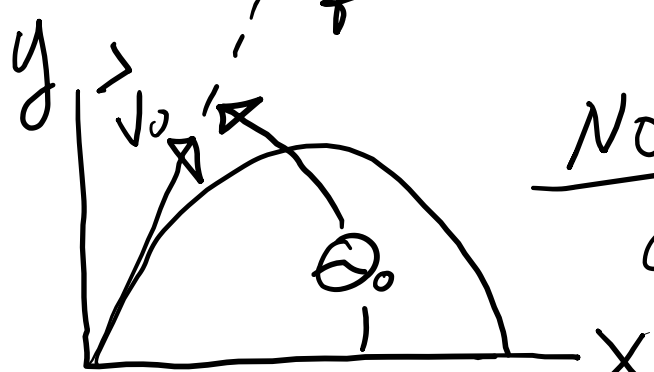
We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$
To get rid of t : First obtain t as a
function of x : $t = \frac{x}{V_{0x}}$. Now substitute
into equation for y : $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$



Note: The initial velocity
given by $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x = V_{0x}t$

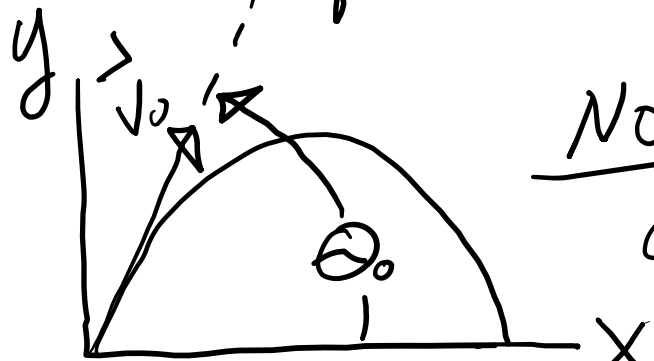
To get rid of t : First obtain t as a function of x : $t = \frac{x}{V_{0x}}$. Now substitute into equation for y : $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$



Note: The initial velocity given by $\vec{v}_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$

starts out at an angle θ_0 with respect to the x-axis

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$
 To get rid of t : First obtain t as a function of x : $t = \frac{x}{V_{0x}}$. Now substitute into equation for y : $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$



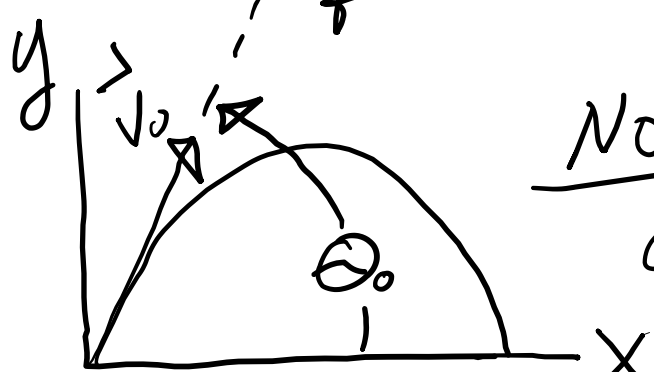
Note: The initial velocity given by $\vec{v}_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$ starts out at an angle

θ_0 with respect to the x-axis

$$\Rightarrow V_{0x} = V_0 \cos \theta_0 \quad \& \quad V_{0y} = V_0 \sin \theta_0$$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t : First obtain t as a function of x : $t = \frac{x}{V_{ox}}$. Now substitute into equation for y : $y = \left(-\frac{g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$



Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$

starts out at an angle

θ_0 with respect to the x -axis

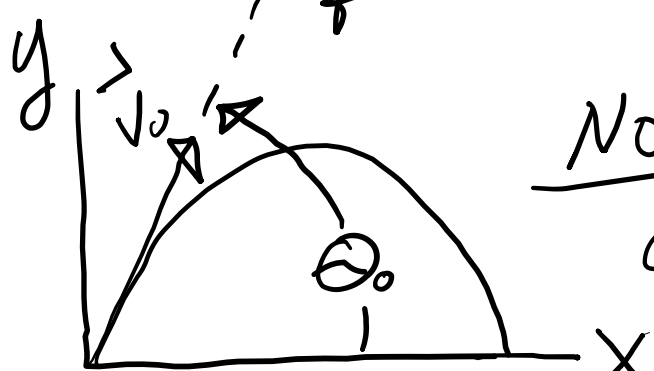
$$\Rightarrow V_{ox} = V_0 \cos \theta_0 \quad \& \quad V_{oy} = V_0 \sin \theta_0 \Rightarrow$$

$$\frac{V_{oy}}{V_{ox}} = \frac{\sin \theta_0}{\cos \theta_0} = \tan \theta_0$$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

To get rid of t : First obtain t as a function of x : $t = \frac{x}{V_{0x}}$. Now substitute

into equation for y : $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$



Note: The initial velocity given by $\vec{v}_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$

starts out at an angle

θ_0 with respect to the x -axis

$$\Rightarrow V_{0x} = V_0 \cos \theta_0 \quad \& \quad V_{0y} = V_0 \sin \theta_0 \Rightarrow$$

$$\frac{V_{0y}}{V_{0x}} = \frac{\sin \theta_0}{\cos \theta_0} = \tan \theta_0 \Rightarrow y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + x \tan \theta_0$$

We have $y = \left(\frac{-g}{2v_{0x}} \right) x^2 + x \tan \theta_0$

We have $y = \left(\frac{-g}{2v_{0x}}\right)x^2 + x \tan \theta_0$, but

$$\frac{1}{v_{0x}} = \left(\frac{1}{v_0}\right) \left(\frac{1}{\cos^2 \theta_0}\right)$$

We have $y = \left(\frac{-g}{2v_{0x}}\right)x^2 + x \tan \theta_0$, but

$$\frac{1}{v_{0x}} = \left(\frac{1}{v_0}\right)\left(\frac{1}{\cos^2 \theta_0}\right) \text{ so } y = \left[\frac{-g}{2v_0^2 \cos^2 \theta_0}\right]x^2 + x \tan \theta_0$$

We have $y = \left(\frac{-g}{2v_{0x}}\right)x^2 + x \tan \theta_0$, but

$$\frac{1}{v_{0x}} = \left(\frac{1}{v_0}\right)\left(\frac{1}{\cos^2 \theta_0}\right) \text{ so } y = \left[\frac{-g}{2v_0^2 \cos^2 \theta_0}\right]x^2 + x \tan \theta_0$$

We can write $\frac{1}{\cos^2 \theta_0} = \tan^2 \theta_0 + 1$ by

noticing that $\sin^2 \theta_0 + \cos^2 \theta_0 = 1 \Rightarrow$

$$\frac{1}{\cos^2 \theta_0} = \frac{\sin^2 \theta_0 + \cos^2 \theta_0}{\cos^2 \theta_0} = \frac{\sin^2 \theta_0}{\cos^2 \theta_0} + 1 = \tan^2 \theta_0 + 1$$

We have $y = \left(\frac{-g}{2v_{0x}^2}\right)x^2 + x \tan \theta_0$, but

$$\frac{1}{v_{0x}^2} = \left(\frac{1}{v_0^2}\right)\left(\frac{1}{\cos^2 \theta_0}\right) \text{ so } y = \left[\frac{-g}{2v_0^2 \cos^2 \theta_0}\right]x^2 + x \tan \theta_0$$

We can write $\frac{1}{\cos^2 \theta_0} = \tan^2 \theta_0 + 1$ by

noticing that $\sin^2 \theta_0 + \cos^2 \theta_0 = 1 \Rightarrow$

$$\frac{1}{\cos^2 \theta_0} = \frac{\sin^2 \theta_0 + \cos^2 \theta_0}{\cos^2 \theta_0} = \frac{\sin^2 \theta_0}{\cos^2 \theta_0} + 1 = \tan^2 \theta_0 + 1$$

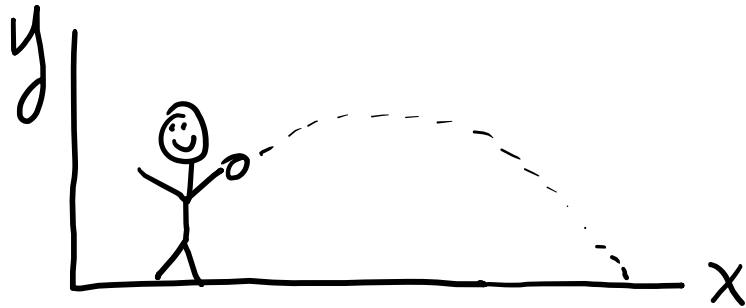
So $y = \left(\frac{-g}{2v_0^2}\right)x^2 (\tan^2 \theta_0 + 1) + x \tan \theta_0$

Notes on problem 11.89

Ball thrown
such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1\text{s}$



11.89 A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (a) the velocity at $t = 1\text{ s}$, (b) the horizontal distance the ball travels before hitting the ground.

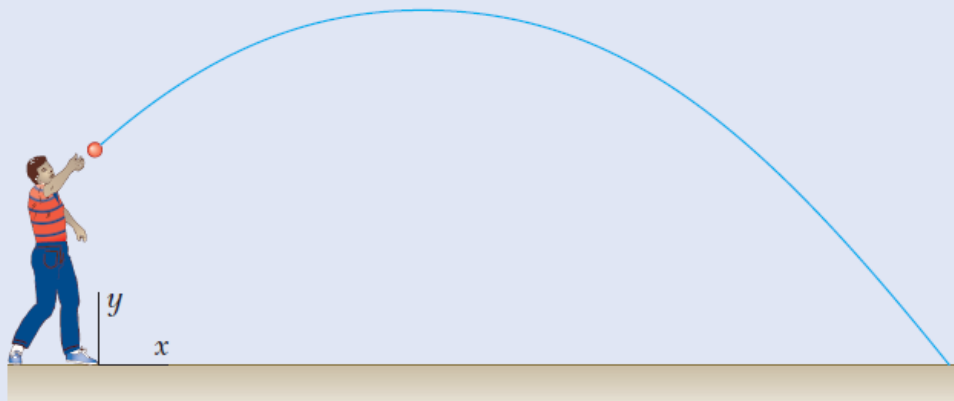


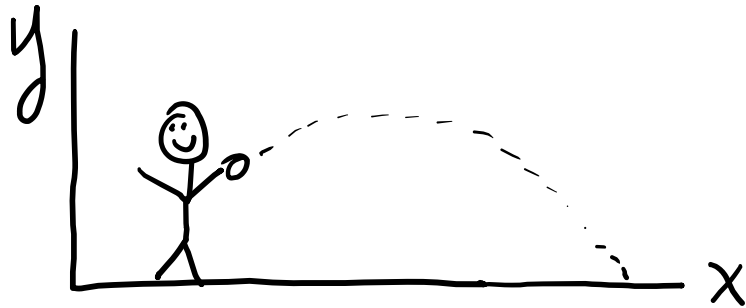
Fig. P11.89

Notes on problem 11.89

Ball thrown
such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



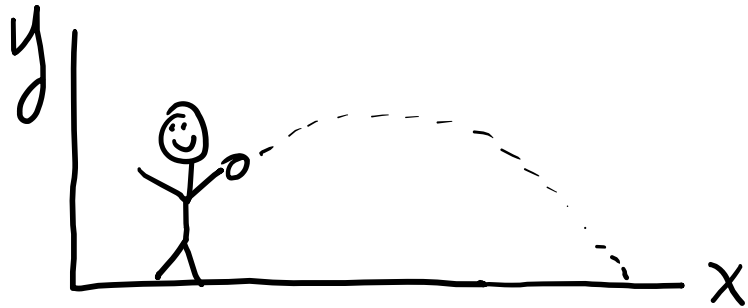
Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$
 $B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$.

Notes on problem 11.89

Ball thrown
such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

Notes on problem 11.89

Ball thrown
such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$

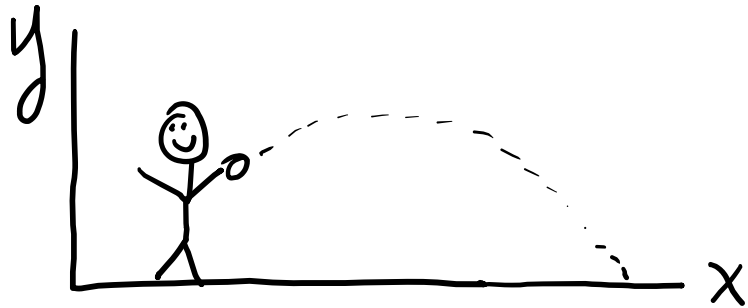
Notes on problem 11.89

Ball thrown

such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$
just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

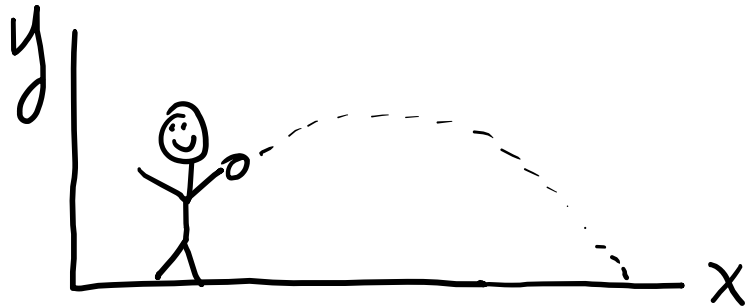
Notes on problem 11.89

Ball thrown

such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$
just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

(b) Find $x(t_h)$, where $t_h \equiv$ time of hit

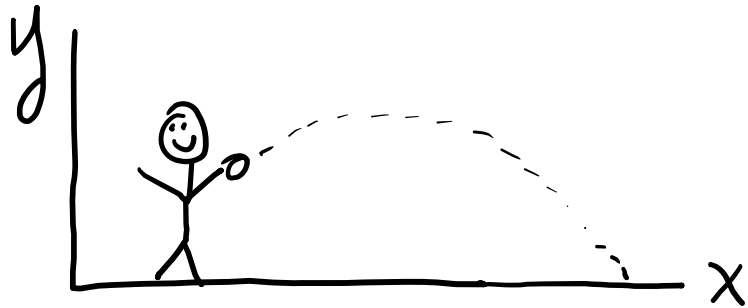
Notes on problem 11.89

Ball thrown

such that

$$y = 2 + 6t - 4.9t^2 \quad \& \quad x = 5t$$

(a) Find $|\vec{v}|$ at $t = 1s$



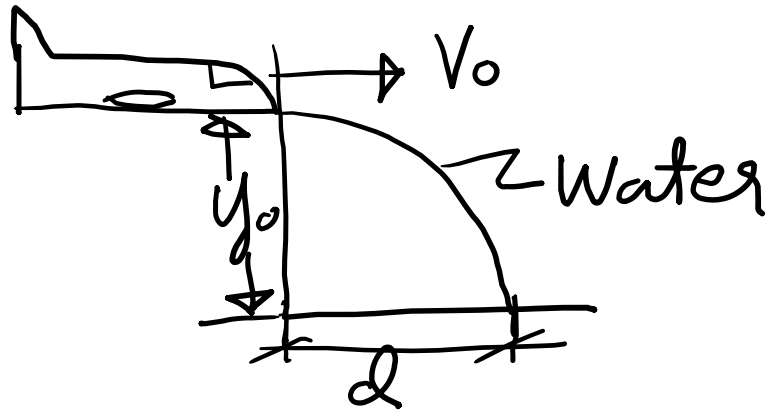
Note: For some vector $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$
just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

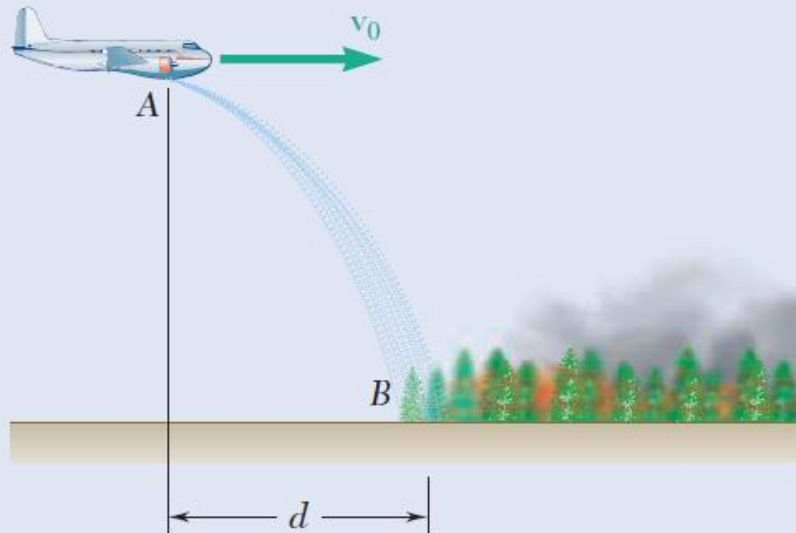
(b) Find $x(t_h)$, where $t_h \equiv$ time of hit: Couple of ways to do this. One way is to set $y(t_h) = 0$ & get t_h [quadratic formula] & then find $x(t_h)$

Notes on 11.97

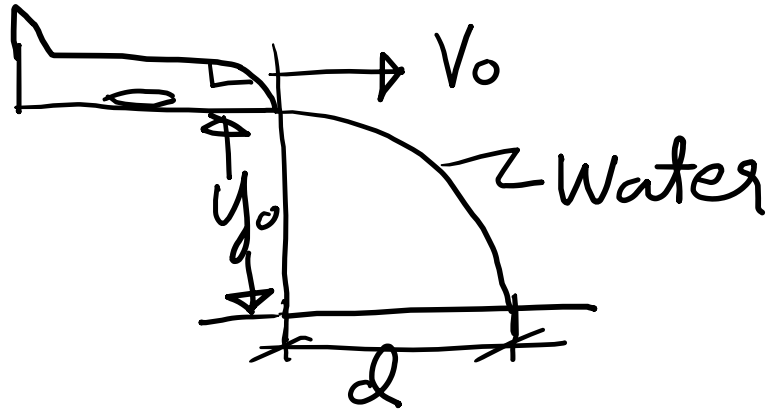


Given:
$$\begin{cases} V_0 = 180 \frac{\text{Mi}}{\text{hr}} \\ y_0 = 300 \text{ ft} \\ V_{y_0} = 0 \end{cases}$$

11.97 An airplane used to drop water on brushfires is flying horizontally in a straight line at 180 mi/h at an altitude of 300 ft. Determine the distance d at which the pilot should release the water so that it will hit the fire at B .



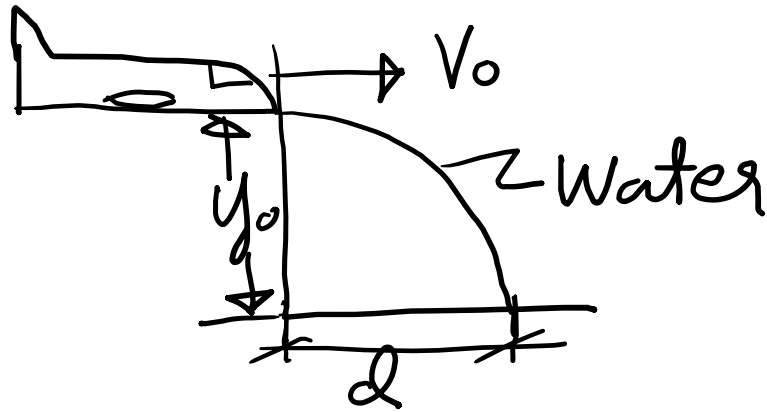
Notes on 11.97



Given:
$$\begin{cases} V_0 = 180 \frac{\text{Mi}}{\text{hr}} \\ y_0 = 300 \text{ ft} \\ V_{y_0} = 0 \end{cases}$$

$$X = V_0 t \Rightarrow t = \frac{X}{V_0}$$

Notes on 11.97

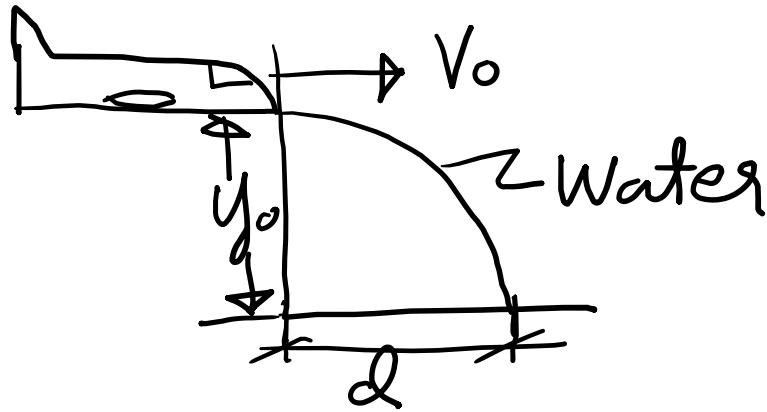


Given:

$$\begin{cases} V_0 = 180 \frac{\text{Mi}}{\text{hr}} \\ y_0 = 300 \text{ ft} \\ V_{y_0} = 0 \end{cases}$$

$x = V_0 t \Rightarrow t = \frac{x}{V_0}$ so when water hits ground $x = d \Rightarrow t_{\text{hit}} = \frac{d}{V_0}$

Notes on 11.97



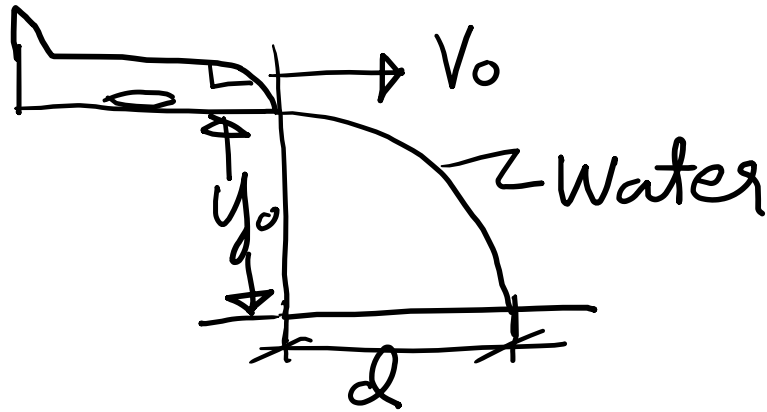
Given:

$$\begin{cases} V_0 = 180 \frac{\text{Mi}}{\text{hr}} \\ y_0 = 300 \text{ ft} \\ V_{y_0} = 0 \end{cases}$$

$x = V_0 t \Rightarrow t = \frac{x}{V_0}$ so when water hits ground $x = d \Rightarrow t_{\text{hit}} = \frac{d}{V_0}$

Also $y = -\frac{g}{2} t^2 + V_{0y} t + y_0$

Notes on 11.97



Given:

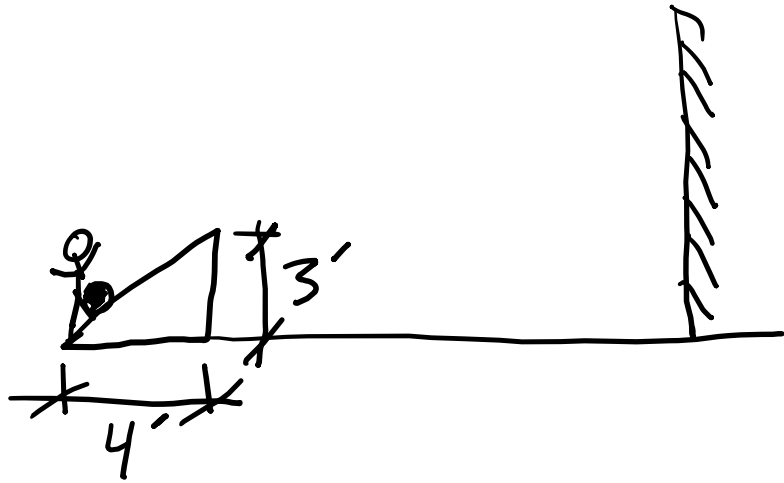
$$\begin{cases} v_0 = 180 \frac{\text{mi}}{\text{hr}} \\ y_0 = 300 \text{ ft} \\ v_{y_0} = 0 \end{cases}$$

$x = v_0 t \Rightarrow t = \frac{x}{v_0}$ so when water hits ground $x = d \Rightarrow t_{\text{hit}} = \frac{d}{v_0}$

Also $y = -\frac{g}{2}t^2 + v_{y_0}t + y_0 \Rightarrow y_{\text{hit}} = -\frac{g}{2}t_{\text{hit}}^2 + y_0$

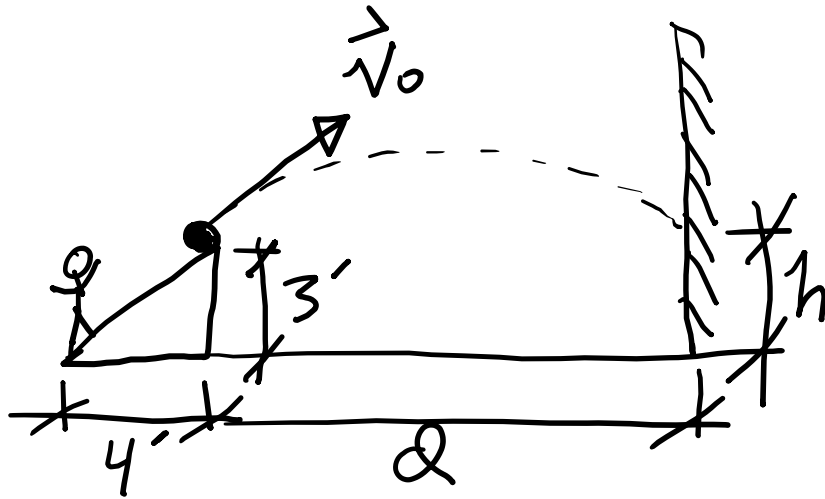
But $y_{\text{hit}} = 0$

Example similar to problem 11.105



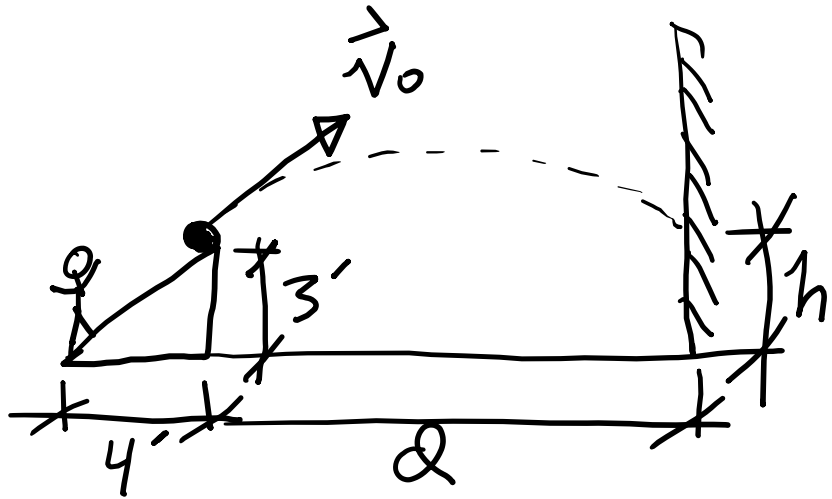
Kid kicks ball
off ramp. Ball hits
wall at height h
a distance d from
ramp

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Example similar to problem 11.105

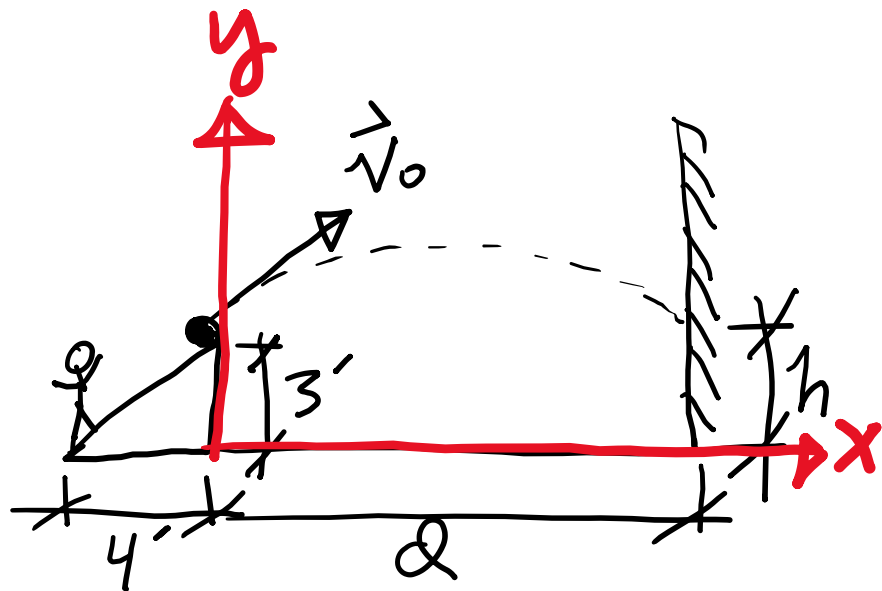


Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32 \text{ ft/s}$, $d = 16 \text{ ft}$ & taking $g = 32 \text{ ft/s}^2$

Instead of 32.2 ft/s^2
[to make life easier]

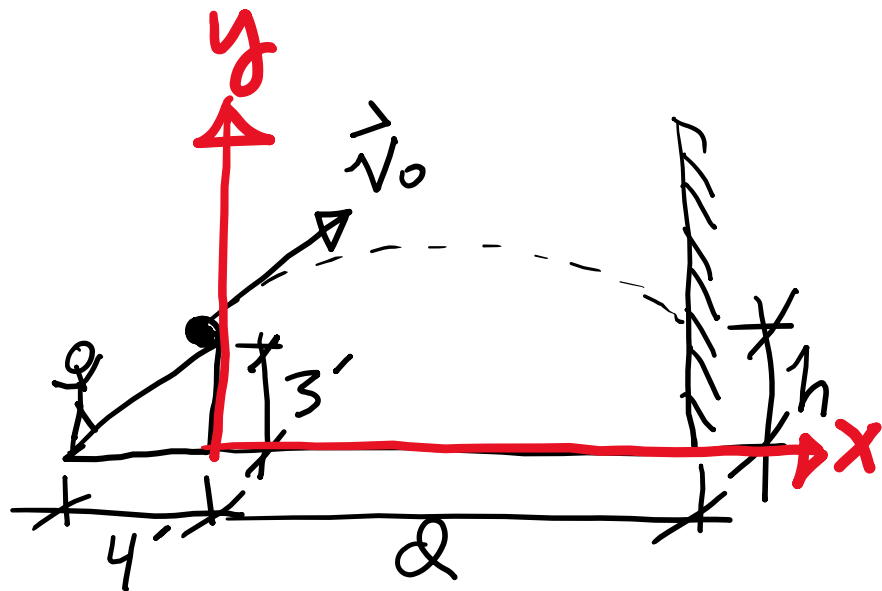
Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32 \frac{\text{ft}}{\text{s}}$, $d = 16 \text{ft}$ & taking $g = 32 \frac{\text{ft}}{\text{s}^2}$
Using coordinate system shown above

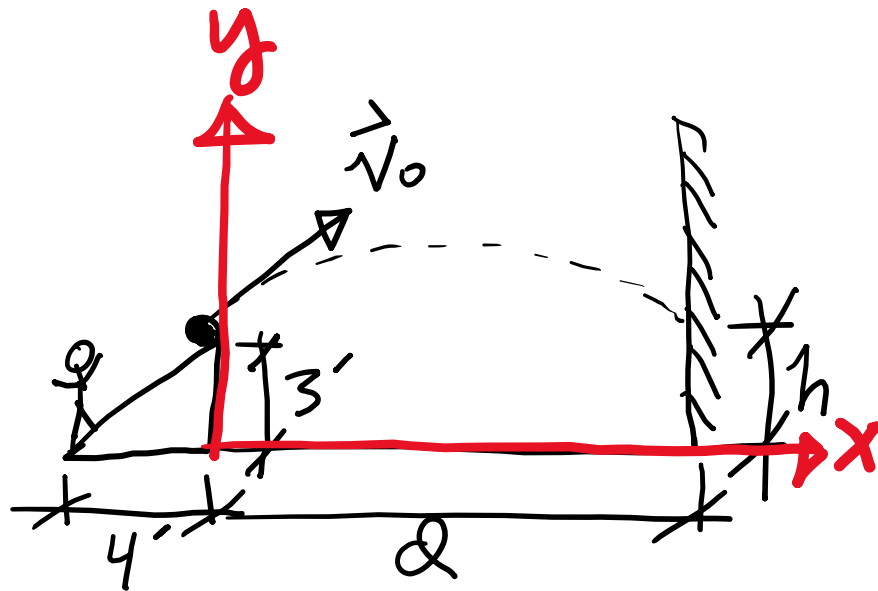
Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32 \frac{\text{ft}}{\text{s}}$, $d = 16 \text{ft}$ & taking $g = 32 \frac{\text{ft}}{\text{s}^2}$
We have $x = v_{x_0} t \Rightarrow t_h = \frac{d}{v_{x_0}}$

Example similar to problem 11.105



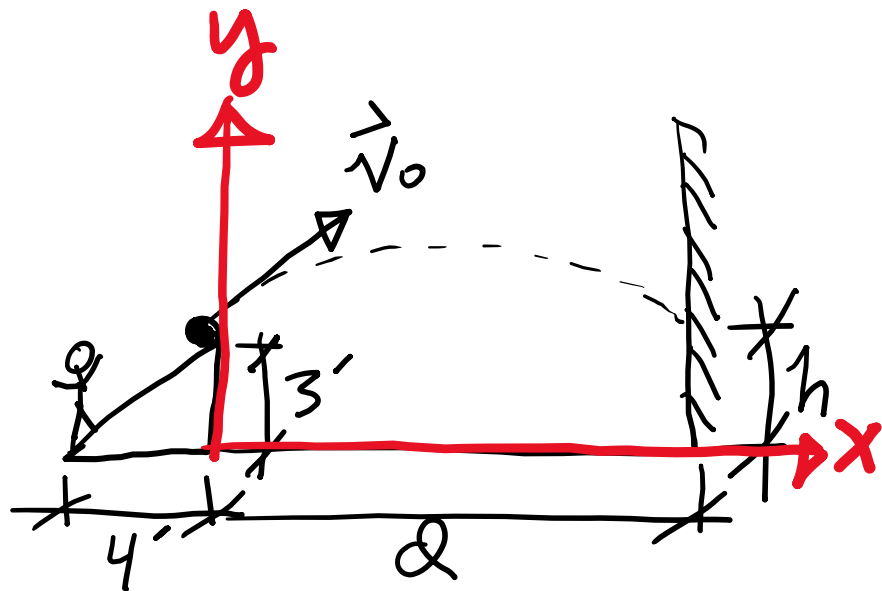
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32 \frac{\text{ft}}{\text{s}}$, $d = 16 \text{ft}$ & taking $g = 32 \frac{\text{ft}}{\text{s}^2}$

We have $x = v_{x_0} t \Rightarrow t_h = \frac{d}{v_{x_0}}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{y_0}t + y_0$$

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32 \frac{ft}{s}$, $d = 16 ft$ & taking $g = 32 \frac{ft}{s^2}$

We have $x = v_{x0}t \Rightarrow t_h = d/v_{x0}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0 \Rightarrow h = \left(-\frac{g}{2}\right)t_h^2 + v_{oy}t_h + y_0$$



$$\Rightarrow h = \left(-\frac{g}{2V_{x_0}^2} \right) d^2 + \left(\frac{V_{0y}}{V_{0x}} \right) d + y_0$$

$$\Rightarrow h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0, \text{ but}$$

$$V_{0x} = V_0 \cos \theta_0 \Rightarrow V_{0x} = V_0 \left(\frac{4}{5}\right) = (32 * \frac{4}{5}) \frac{ft}{s}$$

$$\Rightarrow h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0, \text{ but}$$

$$V_{0x} = V_0 \cos \theta_0 \Rightarrow V_{0x} = V_0 \left(\frac{4}{5}\right) = (32 * \frac{4}{5}) \frac{ft}{s}$$

Note: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin \theta_0}{V_0 \cos \theta_0} = \tan \theta_0 = \frac{3}{4}$

$$\Rightarrow h = \left(-\frac{g}{2V_{x_0}^2} \right) d^2 + \left(\frac{V_{0y}}{V_{0x}} \right) d + y_0, \text{ but}$$

$$V_{0x} = V_0 \cos \theta_0 \Rightarrow V_{0x} = V_0 \left(\frac{4}{5} \right) = \left(32 * \frac{4}{5} \right) \frac{ft}{s}$$

Note: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin \theta_0}{V_0 \cos \theta_0} = \tan \theta_0 = \frac{3}{4}$

$$\text{So } h = \left\{ \left[\frac{-32 * 5^2}{2 * (32 * 4)^2} \right] 16^2 + \frac{3}{4} * 16 + 3 \right\} ft$$

$$\Rightarrow h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0, \text{ but}$$

$$V_{0x} = V_0 \cos \theta_0 \Rightarrow V_{0x} = V_0 \left(\frac{4}{5}\right) = (32 * \frac{4}{5}) \frac{\text{ft}}{\text{s}}$$

Note: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin \theta_0}{V_0 \cos \theta_0} = \tan \theta_0 = \frac{3}{4}$

$$\text{So } h = \left\{ \left[\frac{-32 * 5^2}{2 * (32 * 4)^2} \right] 16^2 + \frac{3}{4} * 16 + 3 \right\} \text{ft}$$

$$\Rightarrow h = \left\{ \left[\frac{-25}{2 * 32 * 16} \right] 16^2 + 12 + 3 \right\} \text{ft}$$

$$\Rightarrow h = \left\{ -\frac{25}{4} + 15 \right\} \text{ft} = 8.75 \text{ft}$$

