Today: Section 11.5

$\boxed{L5}$

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Wednesday: Section 11.5

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Curvilinear motion of particles

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Curvilinear motion of particles

We will learn what these mean

$\vec{v} = \rho\dot{\theta}\hat{e}_t$ & $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$ | Today

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Wednesday: Section 11.5

Curvilinear motion of particles

We will learn what these mean

$\vec{v} = \rho\dot{\theta}\hat{e}_t$ & $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$	Today
$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = [\ddot{r} - r\dot{\theta}^2] + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$	Wednesday

L5

Today: Section 11.5

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$\boxed{L5}$

HW#1 Due Wednesday

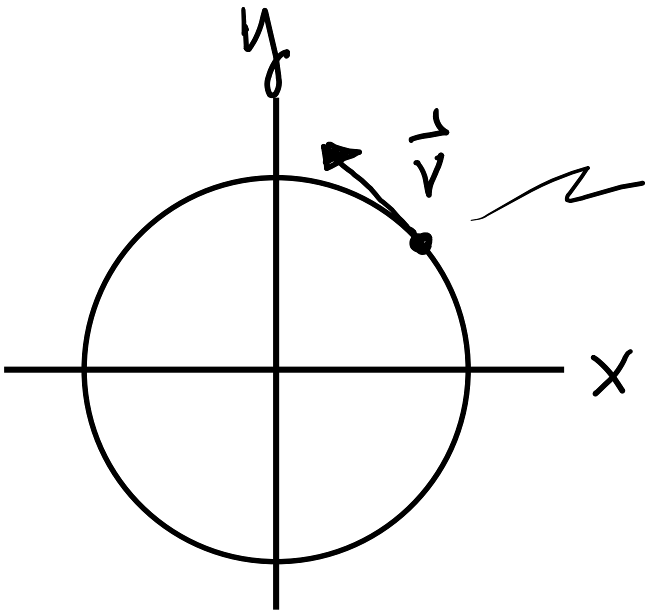
Today: Section 11.5 $\boxed{L5}$

Wednesday: Section 11.5

HW#1 Due Wednesday

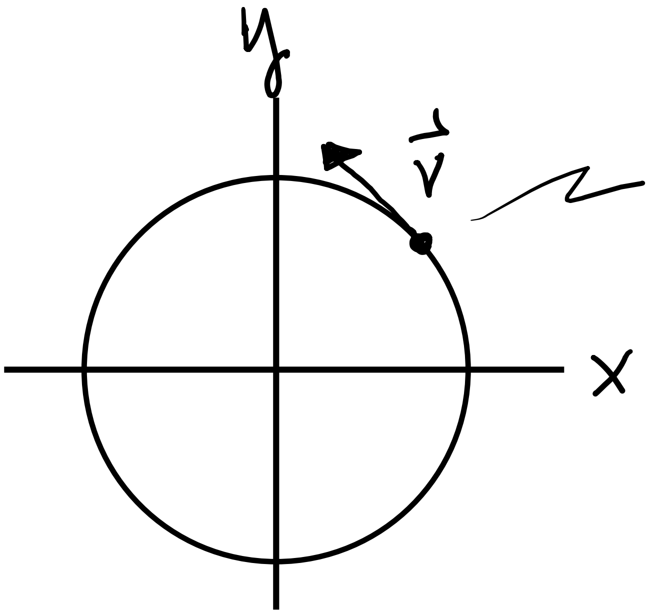
HW#2 Due Friday

Uniform circular motion



particle moving at constant speed

Uniform circular motion



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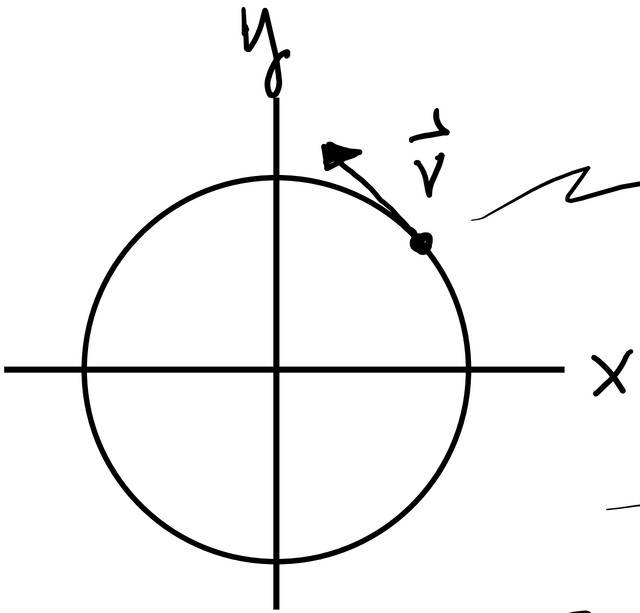
$|\vec{v}| = \text{const.}$

so $\dfrac{d|\vec{v}|}{dt} = 0$

Uniform circular motion



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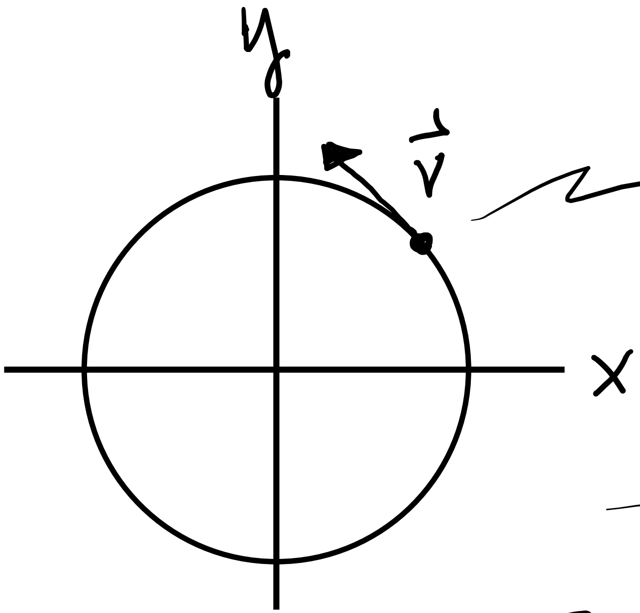
But the direction **does** change.

Uniform circular motion



particle moving at constant speed

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So $\dfrac{d\vec{v}}{dt} \neq 0$

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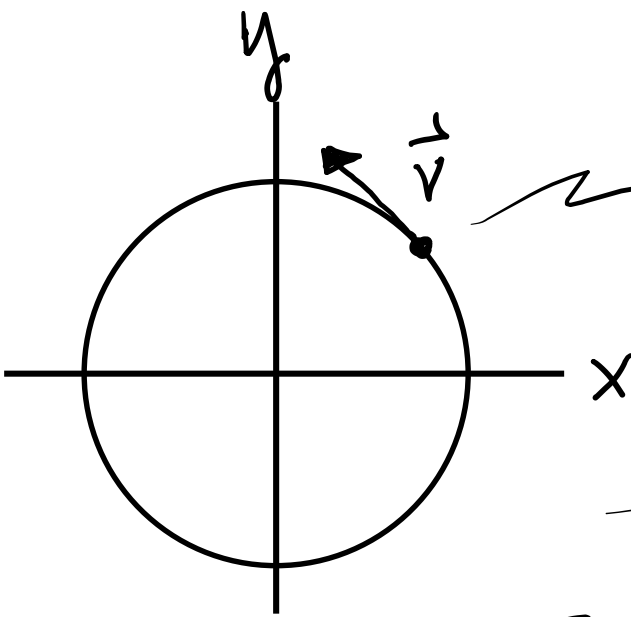

particle moving at constant speed

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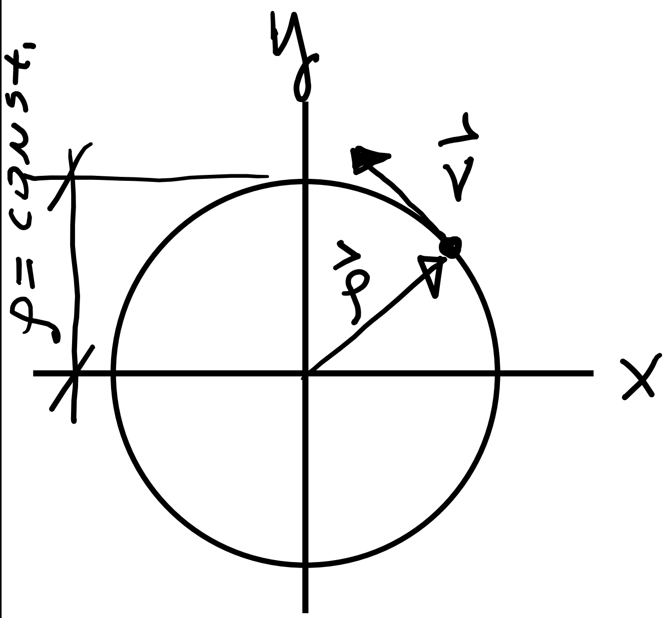
so $\dfrac{d|\vec{v}|}{dt} = 0$

<u>But</u> the direction <u>does</u> change.

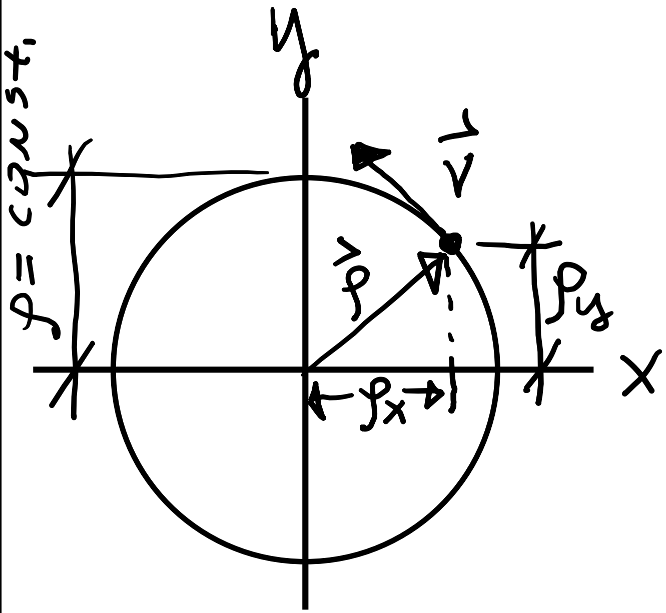
So $\dfrac{d\vec{v}}{dt} \neq 0$. We want to find an expression $\vec{a} = \dfrac{d\vec{v}}{dt}$ for uniform circular motion

Uniform circular motion



We can construct a position vector $\vec{\rho}$

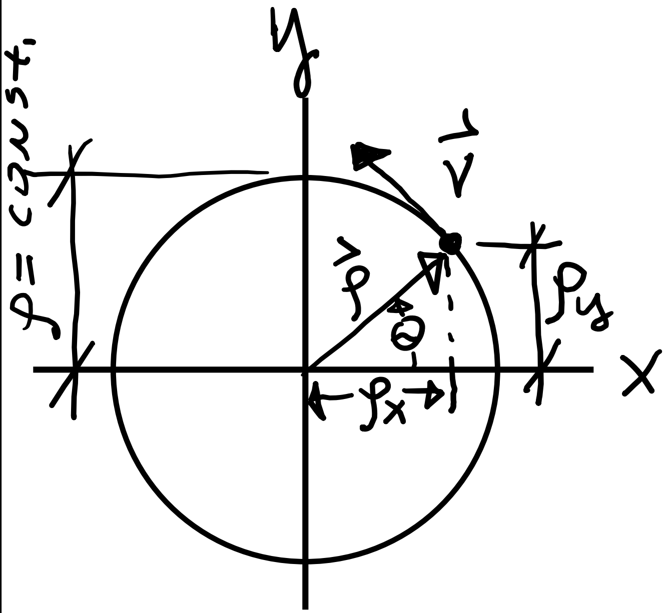
Uniform circular motion



We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j}$$

Uniform circular motion



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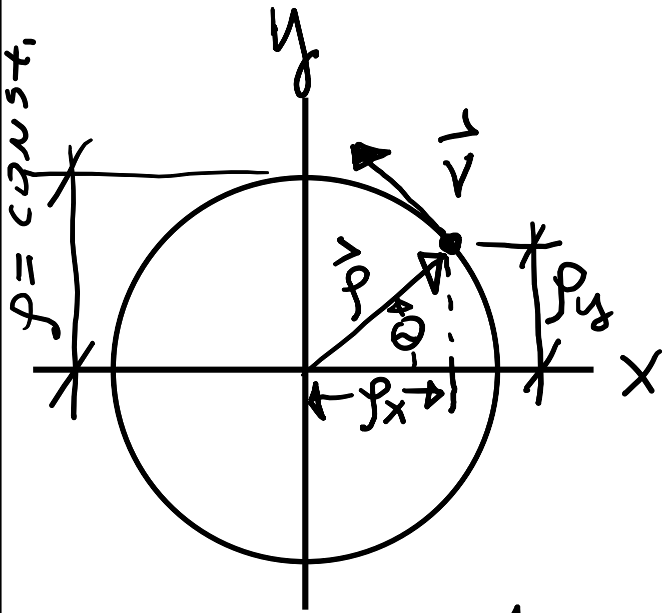
$$\rho_x = \rho \cos\theta \ \& \ \rho_y = \rho \sin\theta$$

Uniform circular motion



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So $\vec{\rho} = \rho\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$

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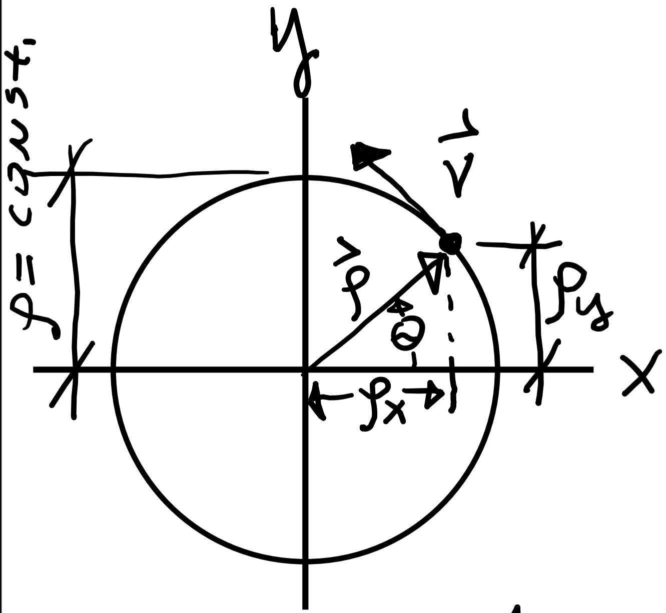

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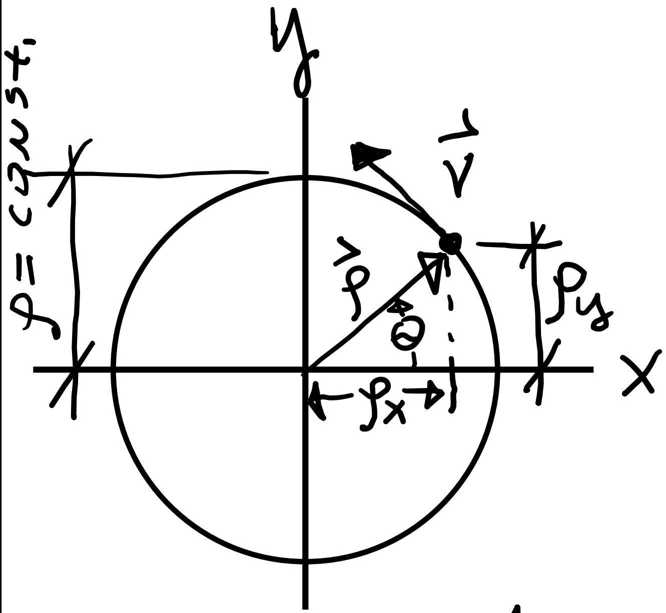
Uniform circular motion



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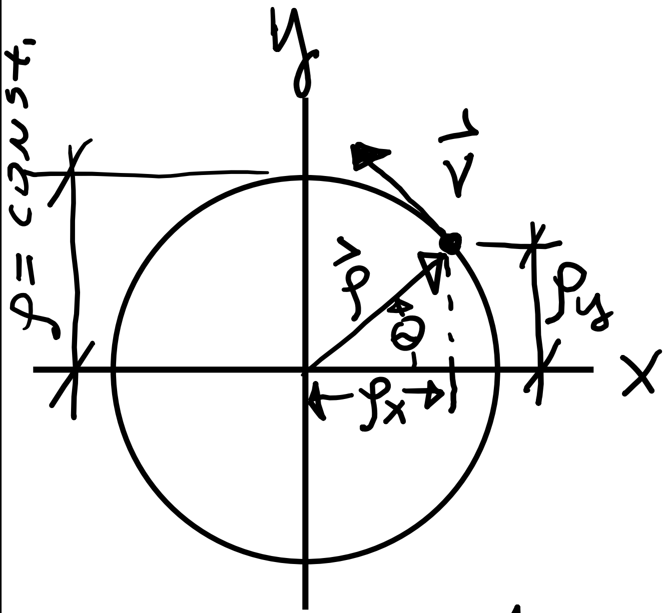
Now $\vec{v} = \rho[\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}]$

Uniform circular motion



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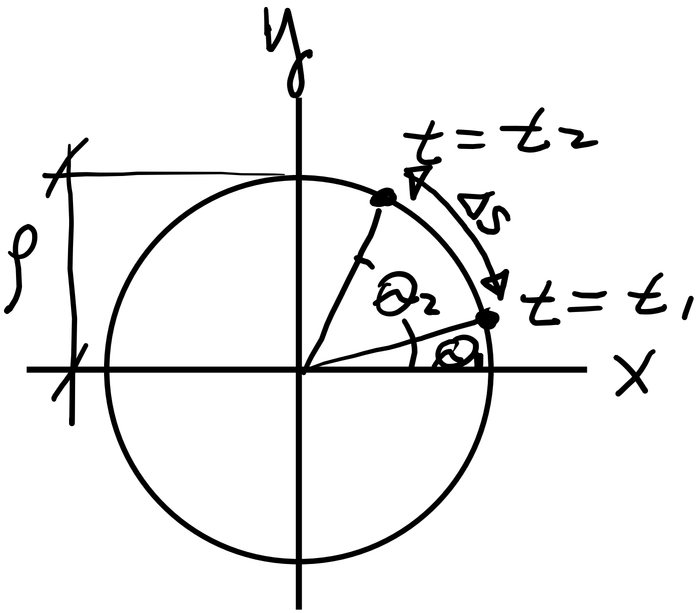
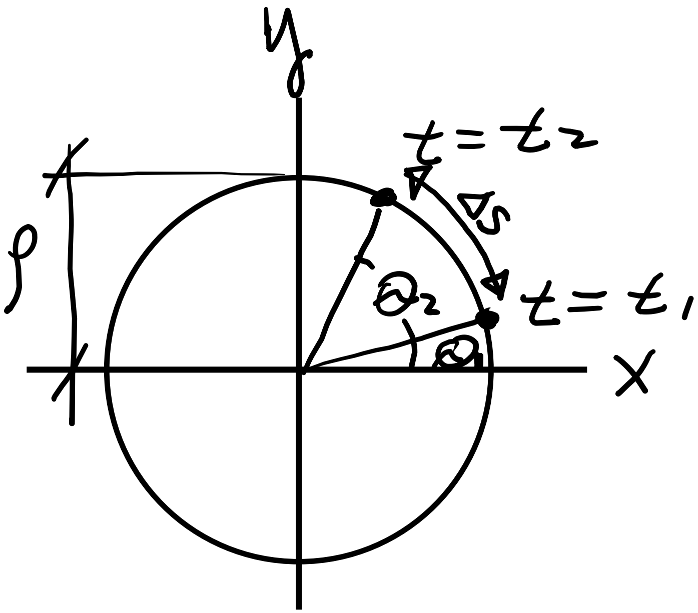
Uniform circular motion



Since $\Delta s = \rho \Delta \theta$

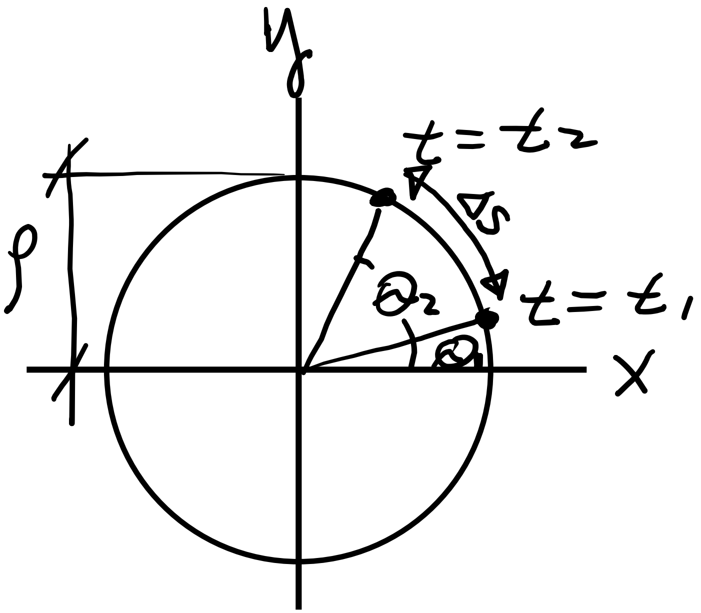
Uniform circular motion



Since $\Delta s = \rho\,\Delta\theta$ and

$$|\vec{v}|_{AVE} = \frac{\Delta s}{\Delta t} = \rho\,\frac{\Delta\theta}{\Delta t}$$

Uniform circular motion



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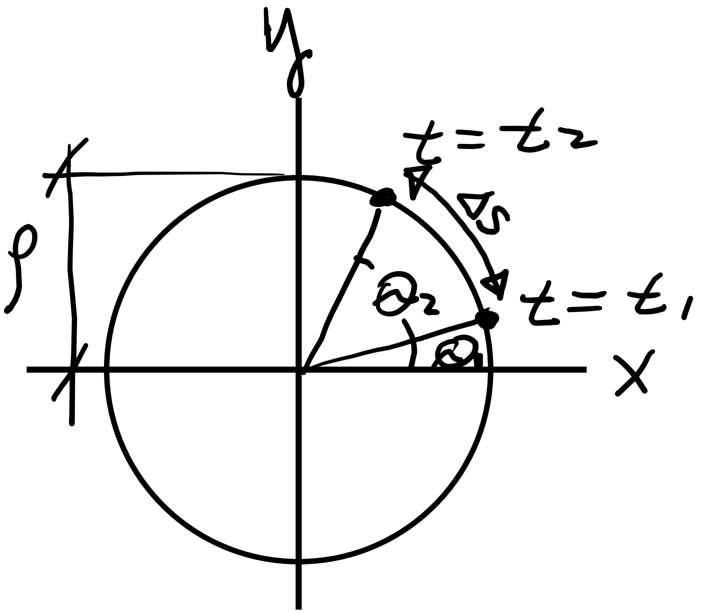
Uniform circular motion



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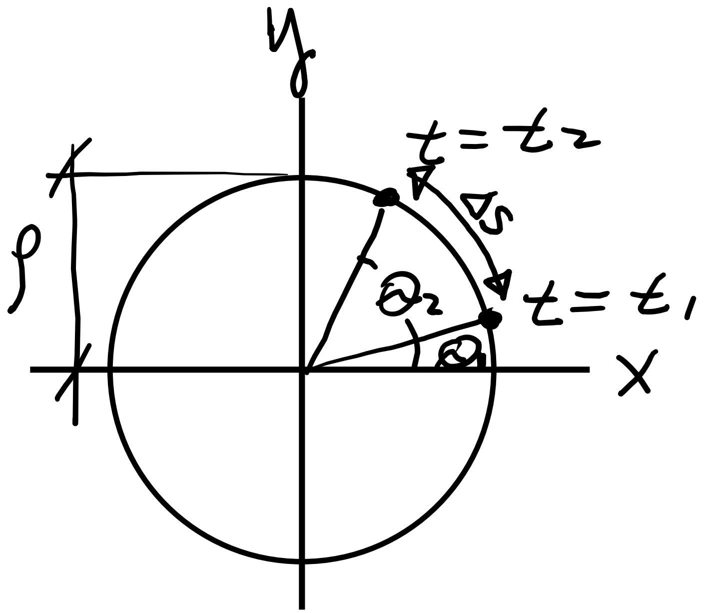
Now $\vec{v}=\rho\dot{\theta}[-\hat{i}\sin\theta+\hat{j}\cos\theta]$
{from previous slide}

Uniform circular motion



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Now $\vec{v} = \rho\dot{\theta}\,[-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta]$

{from previous slide} becomes

$$\vec{v} = |\vec{v}|\,[-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta],$$ where

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For position we have $\vec{p} = |\vec{p}|\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$

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What does the dot product tell us?

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

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$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.

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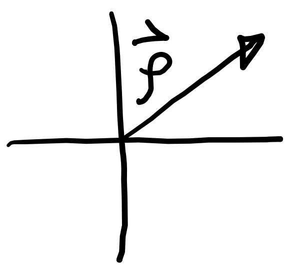

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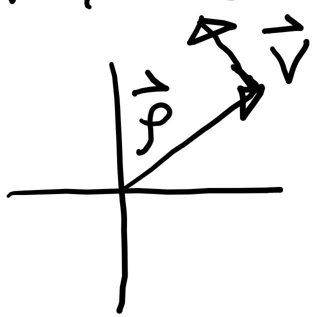

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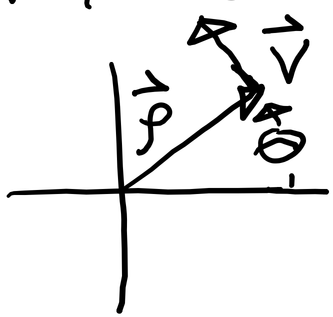

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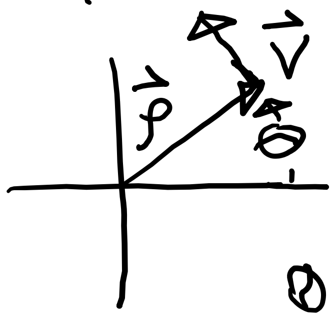
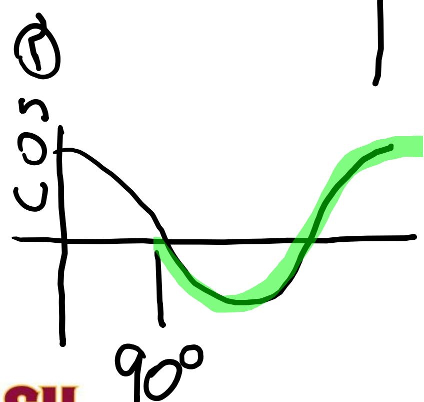
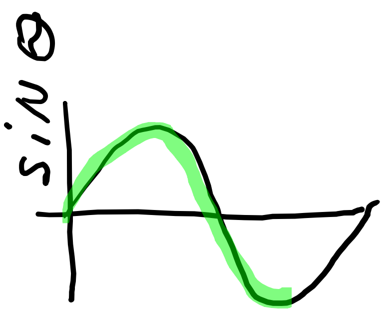


Notice that

$\cos(\theta + 90^\circ) = -\sin\theta$





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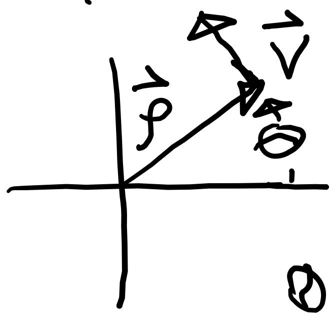

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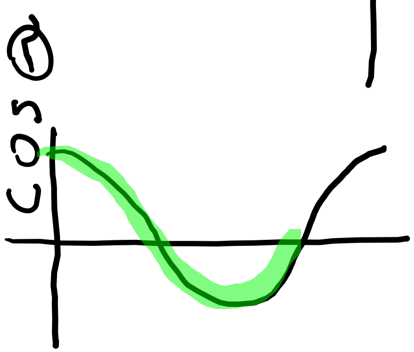

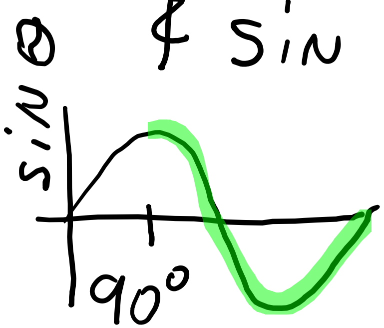

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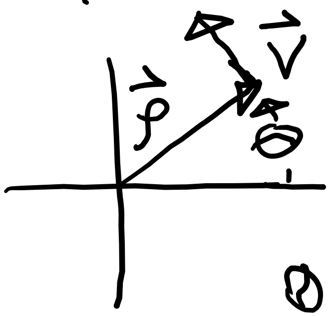

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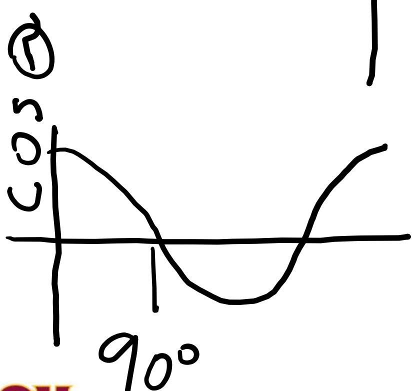

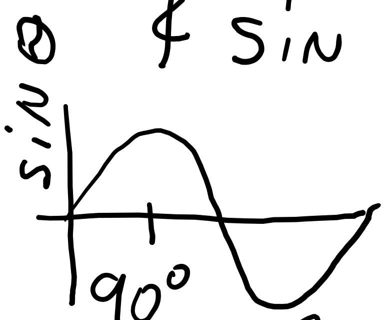

It looks like our expression for $\vec{v}$ makes sence ☺

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

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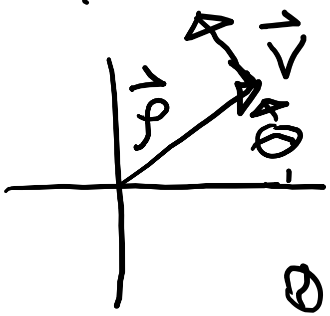

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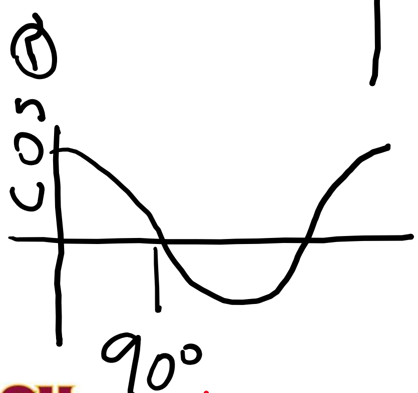

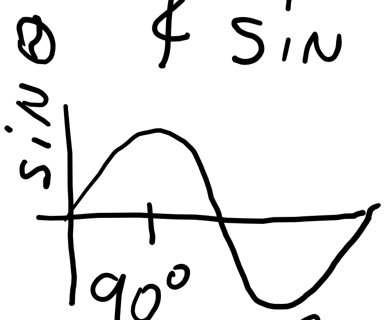

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Now lets find $\vec{a} = d\vec{v}/dt$ →

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]\right\}$$

$$= \rho\ddot{\theta} + \rho\dot{\theta}\left[-\hat{i}\,(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\,\dot{\theta}\right]$$

$$= \rho\ddot{\theta} - \rho\dot{\theta}^2\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

But we have <u>uniform</u> circular motion : $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

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$$a = \rho\dot{\theta}\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

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Compare to $\rho = \rho\left[+\hat{i}\cos\theta + \hat{j}\sin\theta\right]$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta\right]\right\}$$

$$= \rho\ddot{\theta} + \rho\dot{\theta}\left[-\hat{\imath}(\cos\theta)\dot{\theta} - \hat{\jmath}(\sin\theta)\dot{\theta}\right]$$

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Compare to $\rho = \rho\left[+\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$ So

$\vec{\rho}$ points from center to position

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]\right\}$$

$$= \rho\ddot{\theta} + \rho\dot{\theta}\left[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}\right]$$

$$= \rho\ddot{\theta} - \rho\dot{\theta}^2\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

But we have uniform circular motion : $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$\Rightarrow \ddot{\theta} = \text{zero}$ So

$$a = \rho\dot{\theta}\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

Compare to $\rho = \rho\left[+\hat{i}\cos\theta + \hat{j}\sin\theta\right]$ So

$\vec{\rho}$ points from center to position &

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Let $\hat{e}_N \equiv \left[-\hat{i}\cos\theta - \hat{i}\sin\theta\right]$

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Note: Since $a = \rho\dot{\theta}^2$ & $V = \rho\dot{\theta}$

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Note: Since $a = \rho\dot{\theta}^2$ & $v = \rho\dot{\theta}$ then we can write $a = \dfrac{v^2}{\rho}$

So, for uniform circular motion $v = \text{const.}$ & $\rho = \text{const.}$ But

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$\vec{a} = \frac{v^2}{\rho}\hat{e}_n$, where $\hat{e}_n$ points towards center and is orthogonal to $\vec{v} = \rho\dot{\theta}\hat{e}_t$

Non-Uniform circular motion

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We now allow $\dot{\theta} \neq$ const. $\Rightarrow \frac{d|\vec{v}|}{dt} \neq 0$

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ASU

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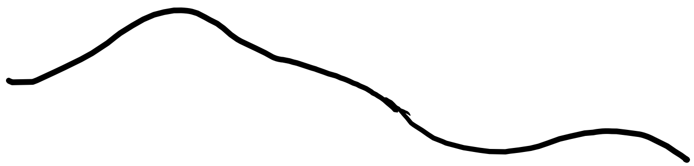
Or $\boxed{\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\,\hat{e}_n}$

Now generalize to some arbitrary smooth path



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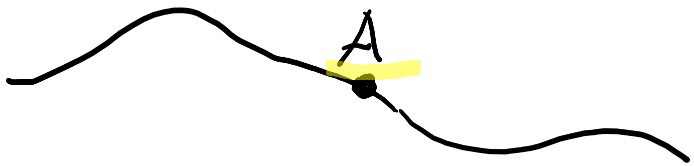
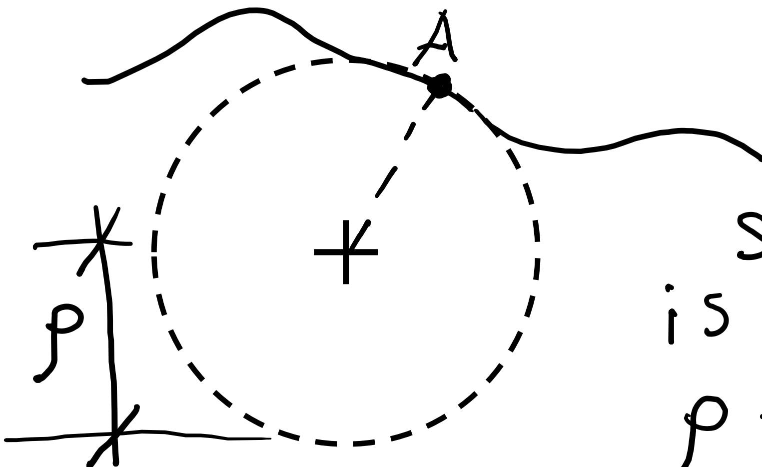


For some point A on some smooth curve

ASU

Now generalize to some arbitrary smooth path



For some point A on some smooth curve, there is a radius of curvature $\rho = \text{constant}$ $\left\{ \begin{array}{c} \text{could be} \\ \rho \to \infty \end{array} \right\}$

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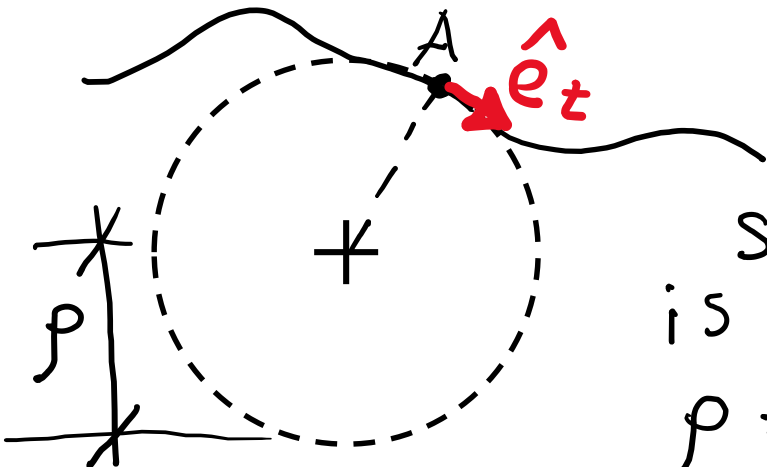
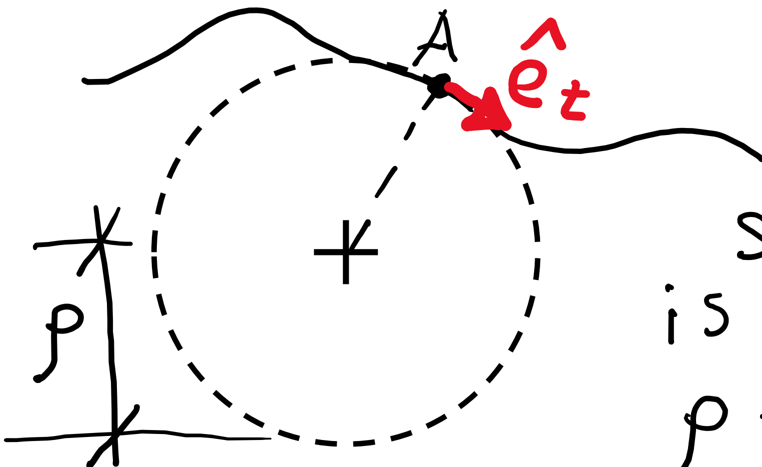


For some point A on some smooth curve, there is a radius of curvature ρ = constant $\{$could be $\rho \to \infty\}$

For a particle located at A & moving along the path

Now generalize to some arbitrary smooth path



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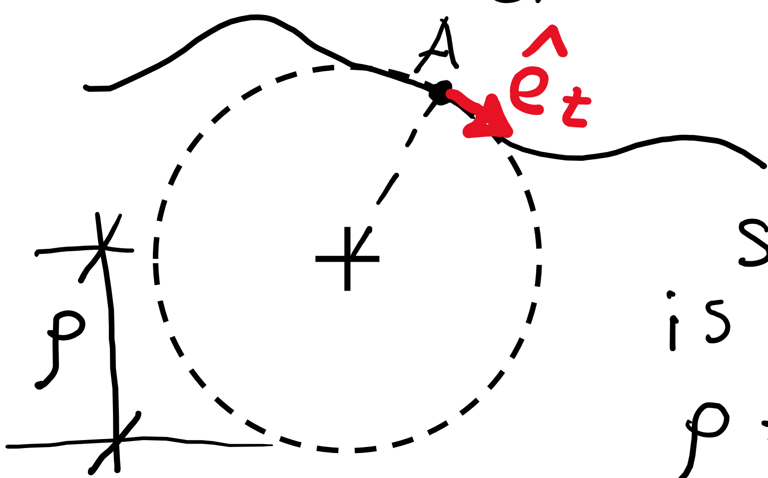
For a particle located at A & moving along the path

$$\vec{V} = V\hat{e}_t$$

Now generalize to some arbitrary smooth path



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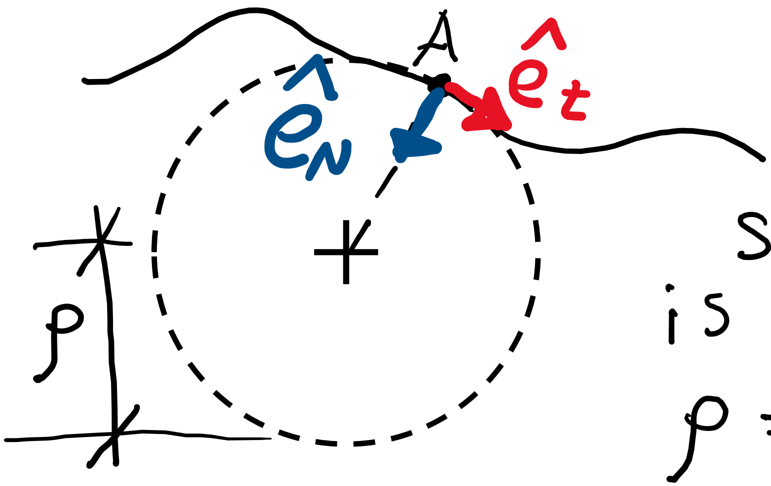
For a particle located at A & moving along the path

$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t +$$

Now generalize to some arbitrary smooth path



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$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_N$$

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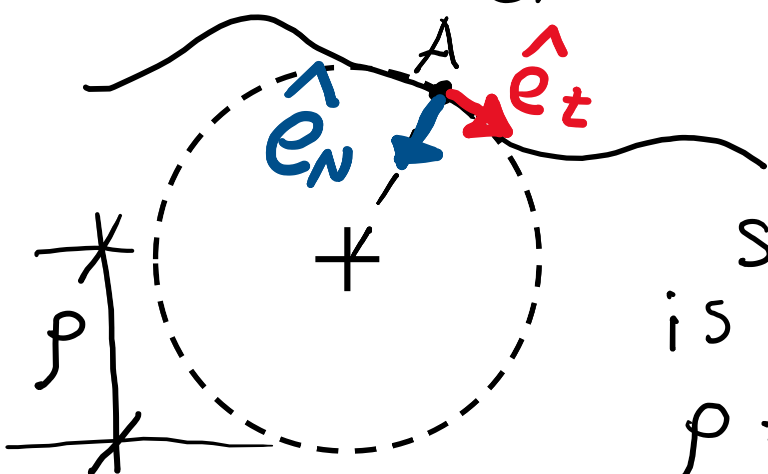

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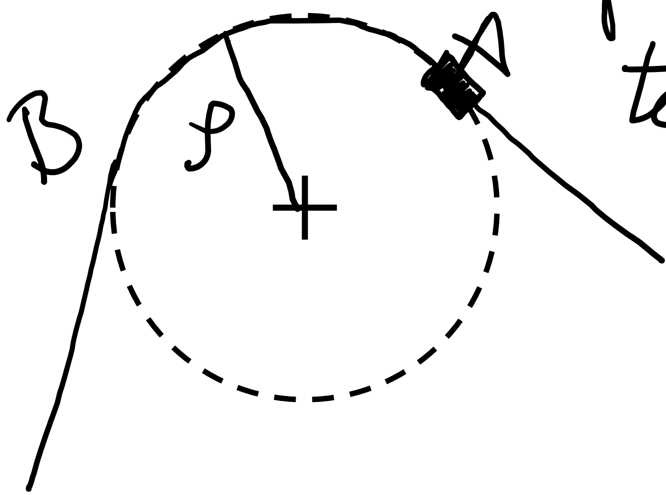
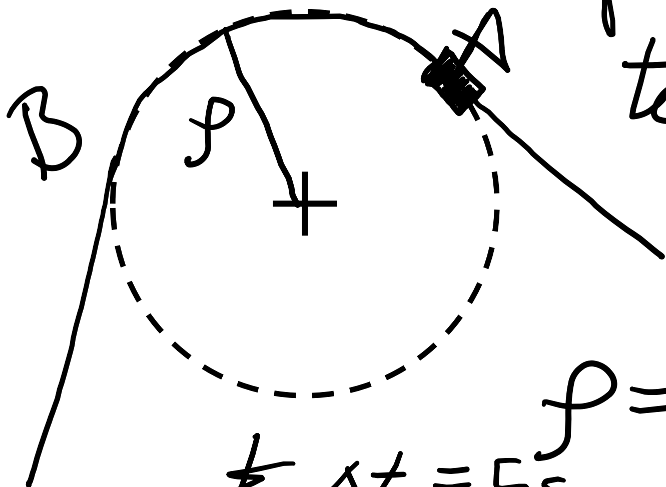
$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_N$$

It is like having an instantaneous coordinate system for each point along the path

Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B.





Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given

$\rho = 100m$, $v_A = 20 m/s$, $v_B = 5 \frac{m}{s}$

& $\Delta t = 5s$

Find $|\vec{a}|$ at point A:

$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

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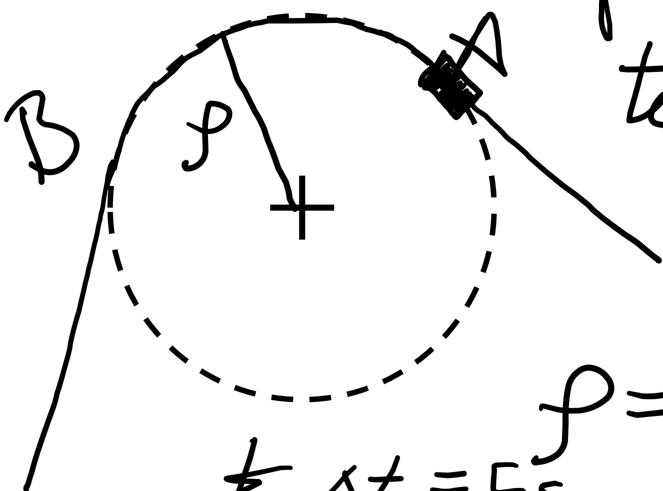

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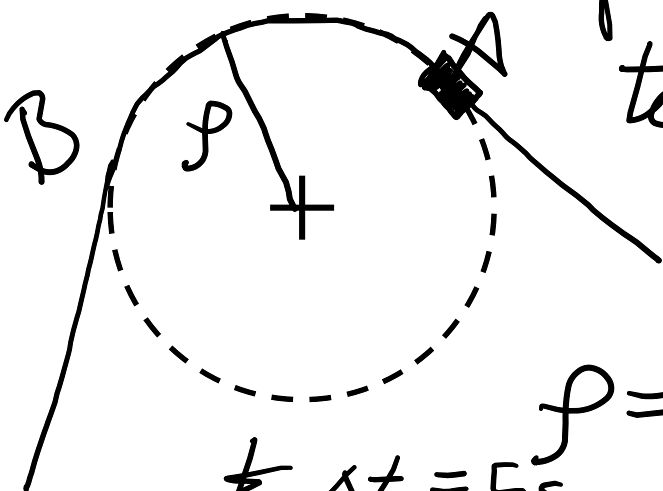

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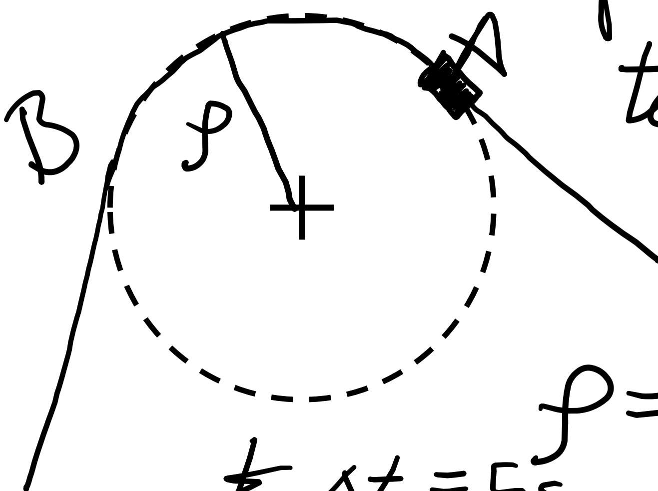

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& $\frac{v^2}{\rho} = \frac{400\,m^2/s^2}{100m} = 4\,m/s^2$ so $\vec{a} = 3\frac{m}{s^2}\hat{e}_t + 4\frac{m}{s^2}\hat{e}_n$

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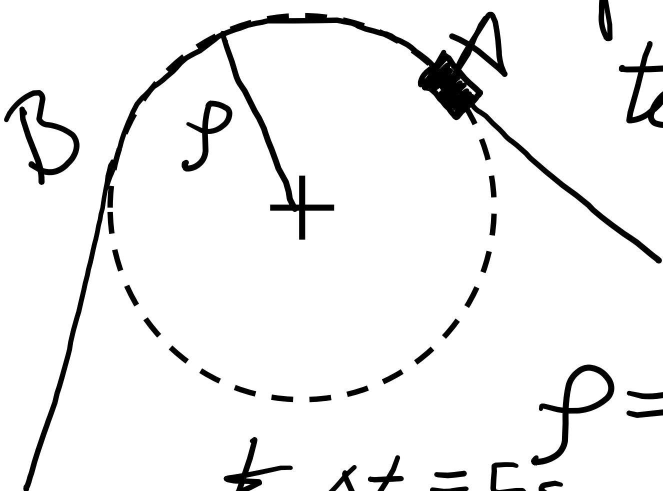

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& $\frac{v^2}{\rho} = \frac{400 m^2/s^2}{100m} = 4 m/s^2$ so $\vec{a} = 3\frac{m}{s^2}\hat{e}_t + 4\frac{m}{s^2}\hat{e}_n$

$\Rightarrow |\vec{a}| = \sqrt{3^2+4^2}\left(\frac{m}{s^2}\right) = 5\frac{m}{s^2}$