Today: Section 11.5

$\boxed{26}$

Today: Section 11.5 $\boxed{L6}$

Curvlinear motion of particles

Today: Section 11.5

Friday: Section 12.1

momentum & forces

26

Today: Section 11.5

$\boxed{26}$

Friday: Section 12.1

HW#1 Due Today

Today: Section 11.5

$\boxed{26}$

Friday: Section 12.1

HW#1 Due Today

HW#2 Due Friday

26

Today: Section 11.5

Friday: Section 12.1

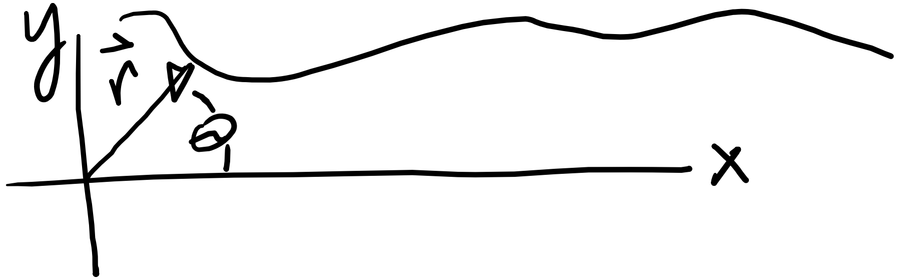
HW#1 Due Today

HW#2 Due Friday

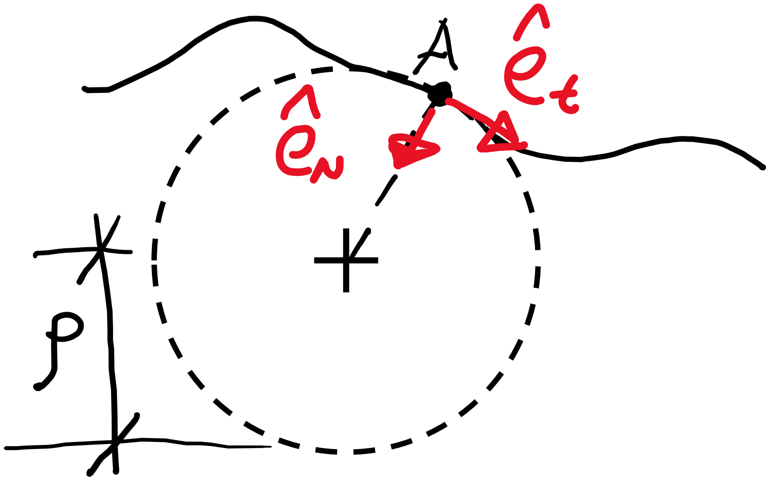
Monday: Holiday ☺

ASU

Fixed coordinates in polar form

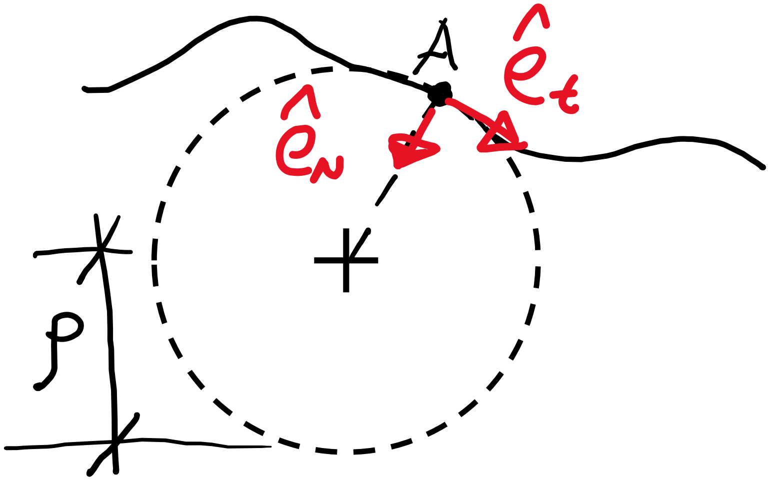


$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$



Previously we found that

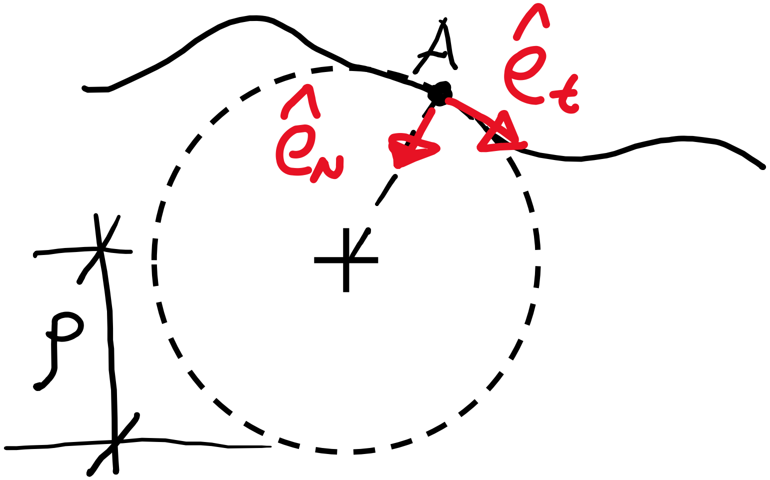
$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t$$



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

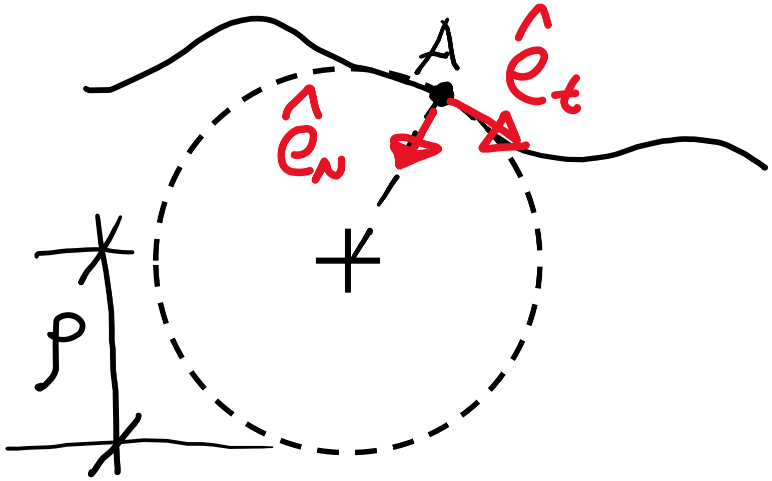
$$\vec{a} = \frac{v^2}{\rho}\hat{e}_N + \dot{v}\,\hat{e}_t$$



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

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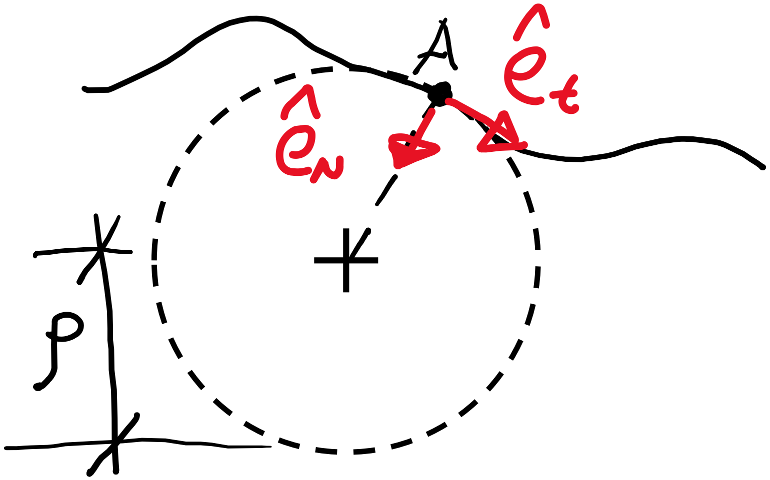
$\hat{e}_t \equiv$ unit vector in tangential direction



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$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$

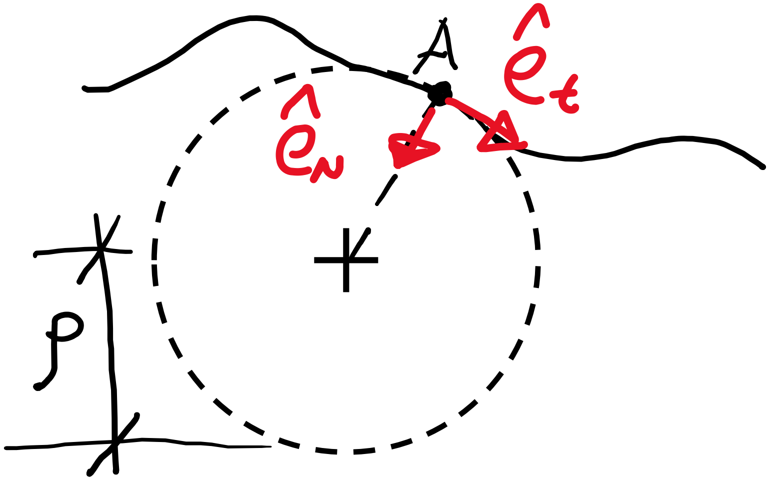
$\hat{e}_t \equiv$ unit vector in tangential direction

$\hat{e}_n \equiv$ unit vector in normal direction

Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$

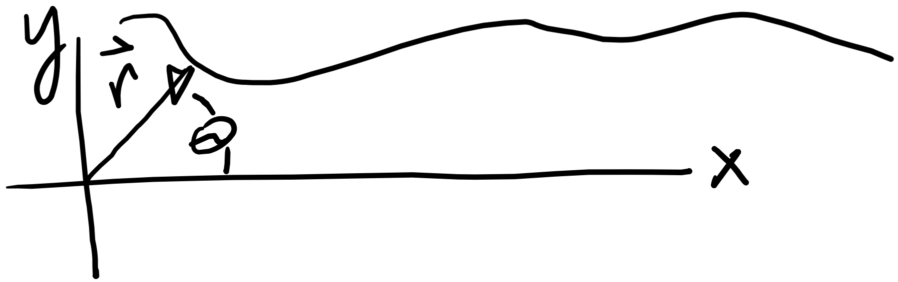


$\hat{e}_t \equiv$ unit vector in tangential direction

$\hat{e}_n \equiv$ unit vector in normal direction

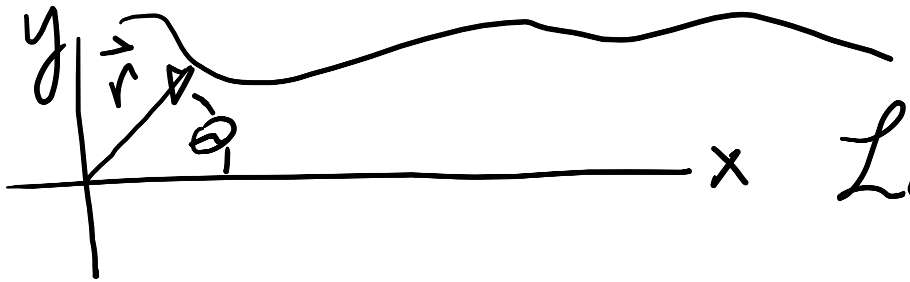
Normal & tangential components

Fixed coordinates in polar form



$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

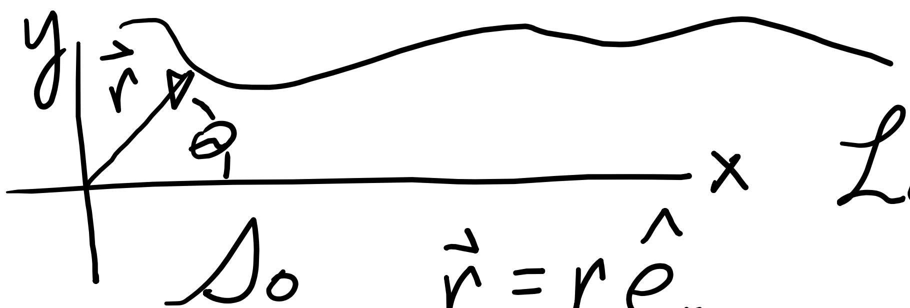
Fixed coordinates in polar form



$\vec{r} = r[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta]$

Fixed coordinates in polar form

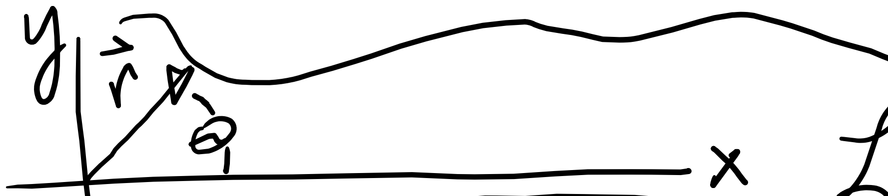


$$\vec{r} = r\left[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$$

Let $\hat{e}_r \equiv \left[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$

$$\vec{r} = r\,\hat{e}_r$$

Fixed coordinates in polar form



S_o

$$\boxed{\vec{r} = r\hat{e}_r}$$

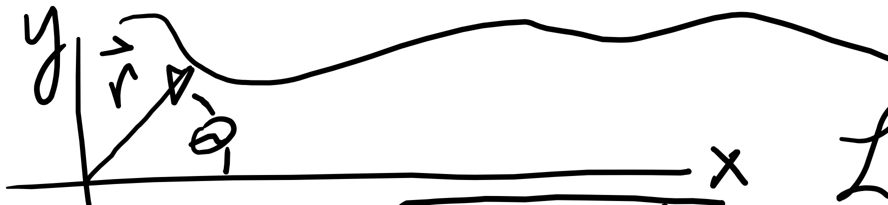
$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

<u>Note:</u>

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}]$$

Fixed coordinates in polar form



$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

S_0 $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Fixed coordinates in polar form

S_o

$$\boxed{\vec{r} = r\hat{e}_r}$$

$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$

Fixed coordinates in polar form

$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}\underline{[-\hat{i}\sin\theta + \hat{j}\cos\theta]}$$

Let $\hat{e}_\theta = \underline{[-\hat{i}\sin\theta + \hat{j}\cos\theta]}$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\underline{\hat{e}_\theta}$

Fixed coordinates in polar form

S_0

$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

$\boxed{\vec{r} = r\hat{e}_r}$

Note:

$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Note: $\frac{d}{dt}\hat{e}_\theta = [-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}]$

Fixed coordinates in polar form

$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Note: $\frac{d}{dt}\hat{e}_\theta = [-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = -\dot{\theta}\hat{e}_r$

Fixed coordinates in polar form

$\vec r = r[\hat i\cos\theta + \hat j\sin\theta]$

Let $\hat e_r \equiv [\hat i\cos\theta + \hat j\sin\theta]$

So $\boxed{\vec r = r\hat e_r}$

Note:

$$\frac{d}{dt}\hat e = [\hat i(-\sin\theta)\dot\theta + \hat j(\cos\theta)\dot\theta] = \dot\theta[-\hat i\sin\theta + \hat j\cos\theta]$$

Let $\hat e_\theta = [-\hat i\sin\theta + \hat j\cos\theta]$ so that $\frac{d}{dt}\hat e_r = \dot\theta\hat e_\theta$

Note: $\frac{d}{dt}\hat e_\theta = [-\hat i(\cos\theta)\dot\theta - \hat j(\sin\theta)\dot\theta] = -\dot\theta\hat e_r$

So $\boxed{\frac{d}{dt}\hat e_\theta = -\dot\theta\hat e_r}$

Fixed coordinates in polar form

$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Note: $\frac{d}{dt}\hat{e}_\theta = [-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = -\dot{\theta}\hat{e}_r$

So $\boxed{\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r}$

With this bit of Vector calculus out of the way, we can now easily determine $\vec{v}$ & $\vec{a}$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r}$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r]$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\,\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\,\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta]$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r +$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r +$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta +$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$
$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$-r\dot{\theta}^2\hat{e}_r$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \underline{\ddot{r}\hat{e}_r} + \underline{\underline{\dot{r}\dot{\theta}\hat{e}_\theta}} + \underline{\underline{\dot{r}\dot{\theta}\hat{e}_\theta}} + \underline{\underline{r\ddot{\theta}\hat{e}_\theta}}$$

$$\underline{-r\dot{\theta}^2\hat{e}_r}$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}e_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}e_\theta$$

$$- r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}e_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}e_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $$\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$ &

$$\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

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$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$
$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

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Note: If $\dot{r} = 0$, then

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

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so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

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so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

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$$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta} \ \& \ \vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}e_\theta$$

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$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta}$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}e_\theta \Rightarrow$
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ASU

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So

polar with $\dot{r} = \cancel{0}$

$\vec{V} = r\dot{\theta}\hat{e}_\theta$

Normal & tangential

$\vec{V} = \rho\dot{\theta}\hat{e}_t$

So

polar with $\dot{r} = 0$

$$\vec{v} = r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$$

Normal & tangential

$$\vec{v} = \rho\dot{\theta}\hat{e}_t$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$$

So

polar with $\dot{r} = 0$

$$\vec{V} = r\dot{\theta}\hat{e}_\theta$$

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Very Similar when $\dot{r} = 0$

So

polar with $\dot{r} = 0$

$\vec{V} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{V^2}{r}\hat{e}_r + \dot{V}\hat{e}_\theta$

Normal & tangential

$\vec{V} = \rho\dot{\theta}\hat{e}_t$

$\vec{a} = \frac{V^2}{\rho}\hat{e}_n + \dot{V}\hat{e}_t$

Making sense of $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

So

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Goal: To understand each term

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polar with $\dot{r} = 0$

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Goal: To understand each term

Solution: Look at special cases

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polar with $\dot{r} = 0$

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Goal: To understand each term

Solution: Look at special cases

CASE I: $\theta = \text{constant}$

So

polar with $\dot{r} = 0$

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

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Normal & tangential

$\vec{v} = \rho\dot{\theta}\hat{e}_t$

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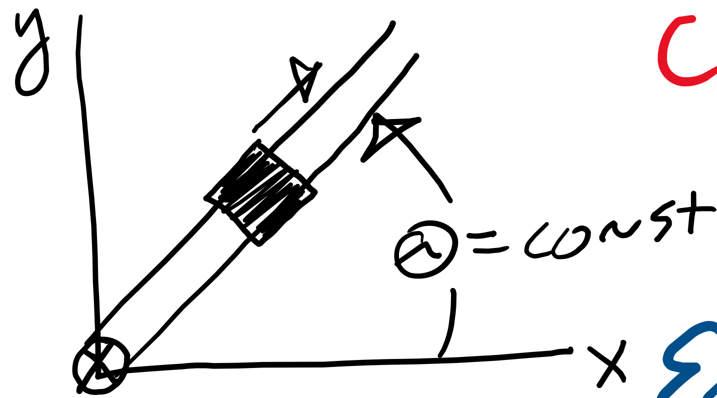
Goal: To understand each term

Solution: Look at special cases

CASE I: θ = constant $\Rightarrow \vec{a} = \ddot{r}\hat{e}_r$



Collar on shaft fixed at constant angle θ

Easiest to understand

CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

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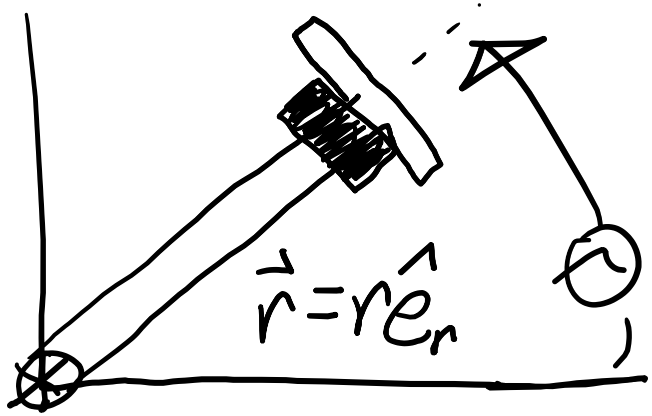
[uniform circular motion]

$\dot{\theta} = \text{constant}$

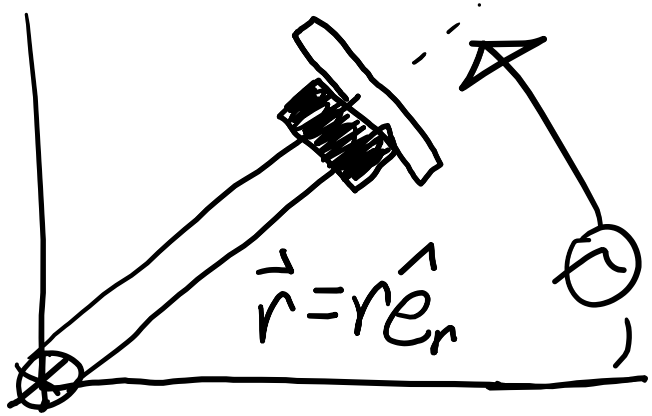


CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$

$\Delta s = r\Delta\theta$: Distance traveled



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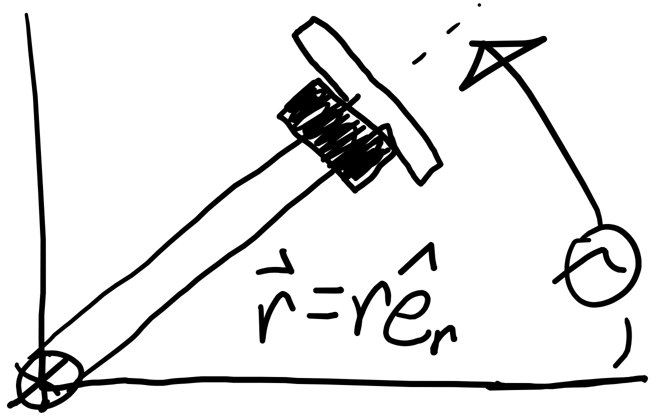

$\Delta s = r\Delta\theta$: Distance traveled

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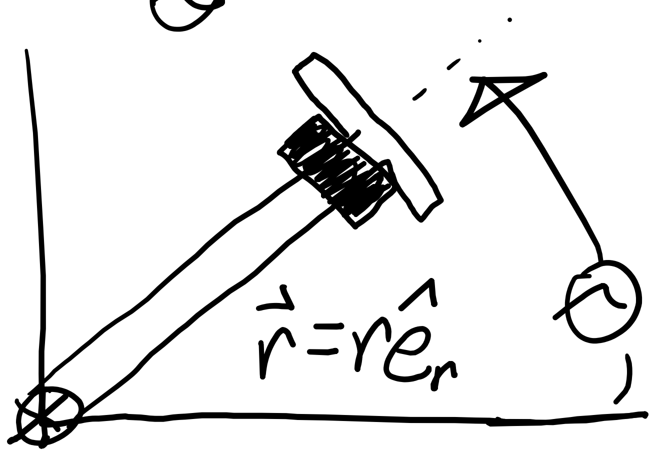

$\Delta s = r\Delta\theta$: Distance traveled

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[uniform circular motion]

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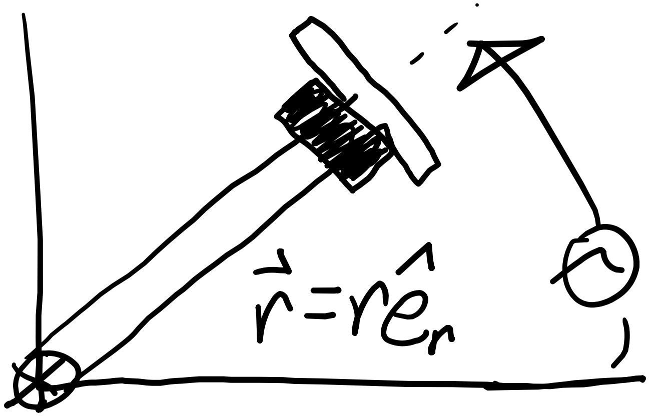

$\Delta s = r\Delta\theta$: Distance traveled

$\vec{v} = r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r$

acceleration to stay in circular motion

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

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$\Rightarrow \vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$

$\underline{\text{CASE III}}$: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

$$\Rightarrow \vec{a} = \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\substack{\text{keeps} \\ \text{motion circular}}} + r\ddot{\theta}\hat{e}_\theta$$

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, $r\ddot{\theta}$ is the increase (or decrease) in $|\vec{V}|$

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, $r\ddot{\theta}$ is the increase (or decrease) in $|\vec{V}|$ since

$$|\vec{V}| = r\dot{\theta} \Rightarrow \frac{dv}{dt} = r\ddot{\theta}$$

particle moves in fixed circle but goes faster (or slower) as time passes

CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.

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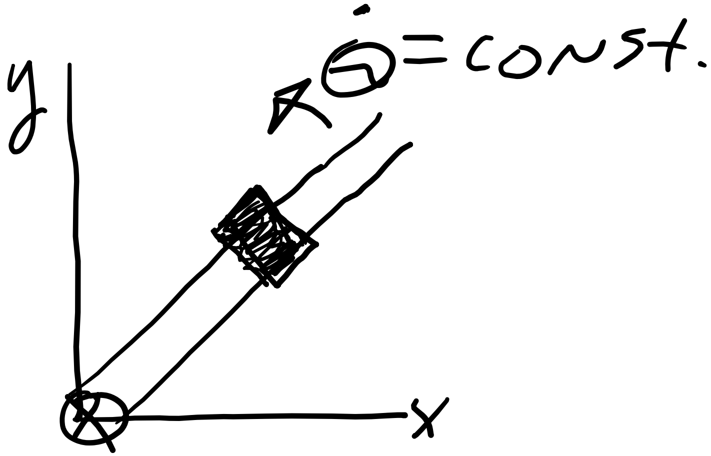

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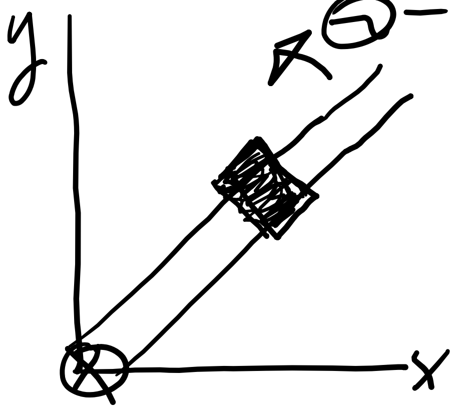
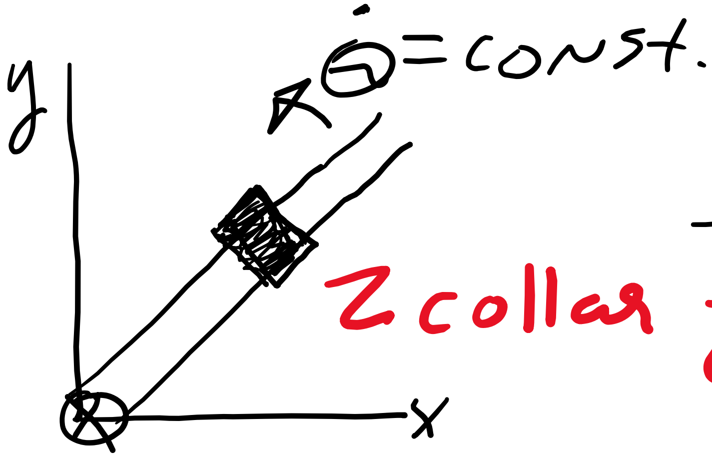


Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

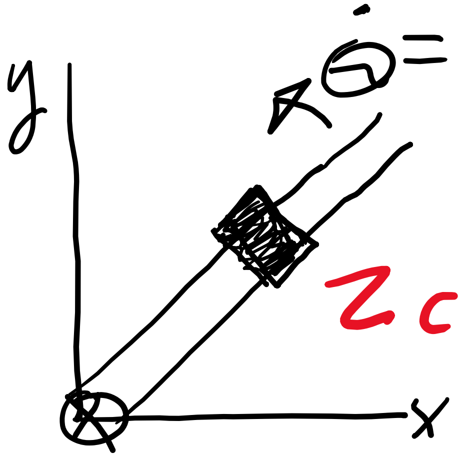
<u>CASE IV</u>: $\dot{\theta} = \text{const.}$ & <u>NO</u> acceleration in $\hat{e}_r$ direction.



Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

Z collar free to move on shaft

CASE IV: $\dot{\theta}$ = const. & NO acceleration in $\hat{e}_r$ direction.



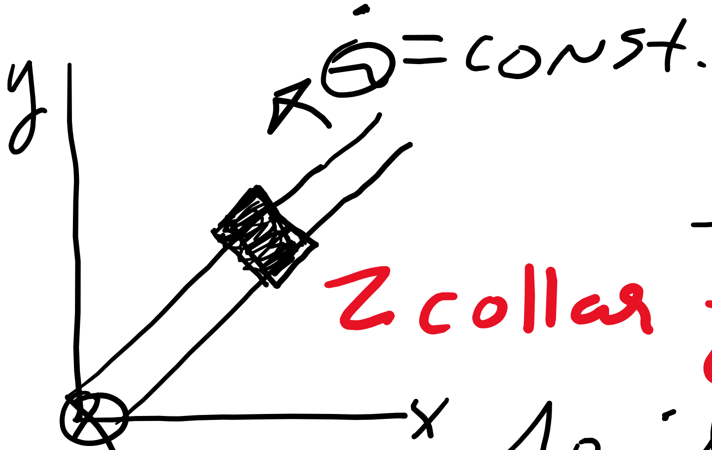
Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

So, if no force in $\hat{e}_r$ then

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{zero}}\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$r\ddot{\theta}$: zero since $\dot{\theta}$ = const

CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.



Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

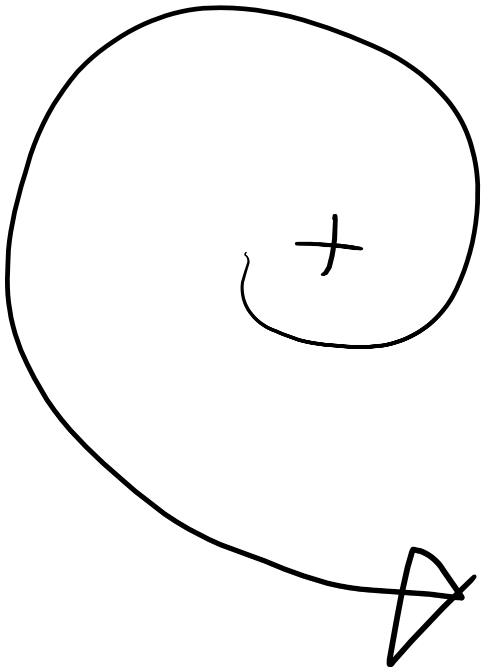
collar free to move on shaft

So, if no force in $\hat{e}_r$ then

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{zero}}\hat{e}_r + (\underbrace{r\ddot{\theta}}_{\text{zero since } \dot{\theta} = \text{const}} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$\Rightarrow \boxed{\vec{a} = 2\dot{r}\dot{\theta}\hat{e}_\theta}$$

Motion looks like



Just as you would expect for the motion of a collar that is free to move on a rotating shaft.