Today: 12.2

$\boxed{L8}$

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$\boxed{L8}$

HW#3: Due Friday

$\boxed{L8}$

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Friday : Review

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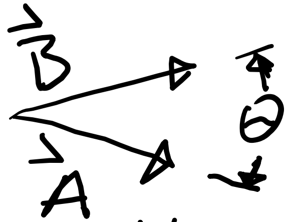
$\boxed{L8}$

HW#3 : Due Friday

Friday : Review

Monday : Exam #1

Cross product:



$\vec{A}$ & $\vec{B}$ define a plane. I will call that plane the AB-plane

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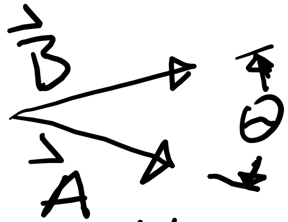

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$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector orthogonal to the AB-plane & points in direction according to the right hand rule.

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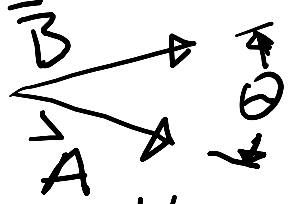

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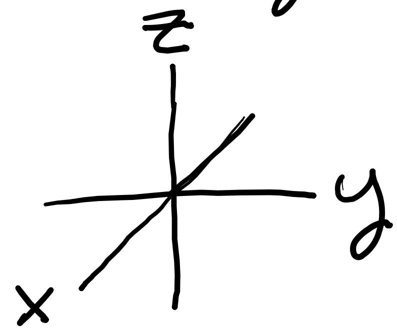
For rectangular coordinates



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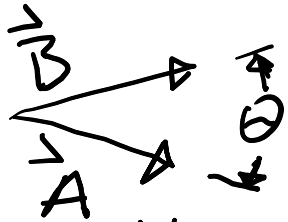

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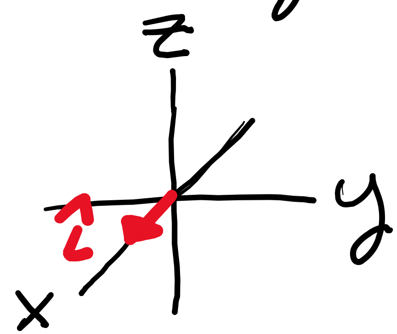

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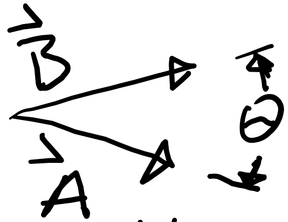

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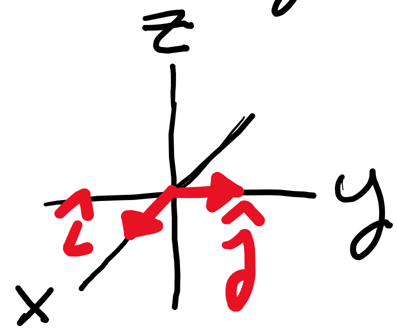

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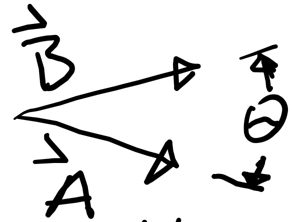

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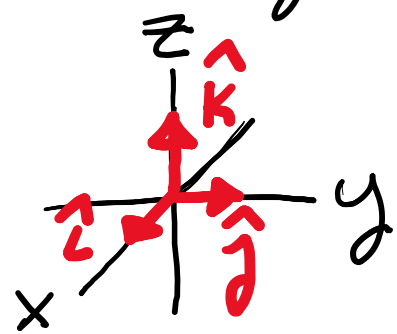

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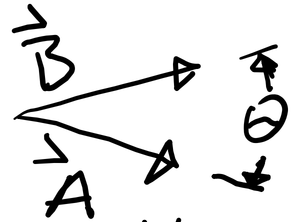

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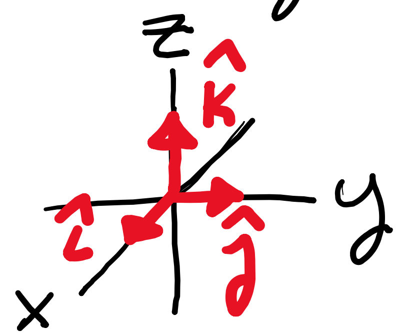

$\hat{\imath}\times\hat{\jmath} = \hat{k}$, $\hat{\jmath}\times\hat{k} = \hat{\imath}$, $\hat{k}\times\hat{\imath} = \hat{\jmath}$

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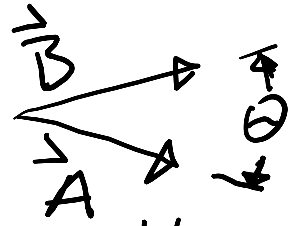

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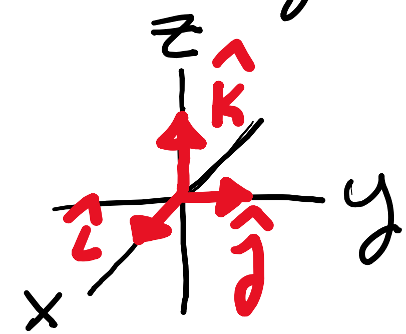

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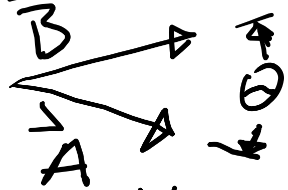

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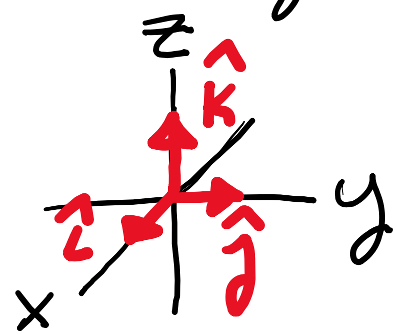

$\hat i \times \hat j = \hat k$, $\hat j \times \hat k = \hat i$, $\hat k \times \hat i = \hat j$

& $\hat j \times \hat i = -\hat k$, $\hat k \times \hat j = -\hat i$, $\hat i \times \hat k = -\hat j$

Also $\hat i \times \hat i = \hat j \times \hat j = \hat k \times \hat k = \vec 0$

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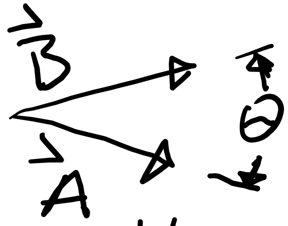

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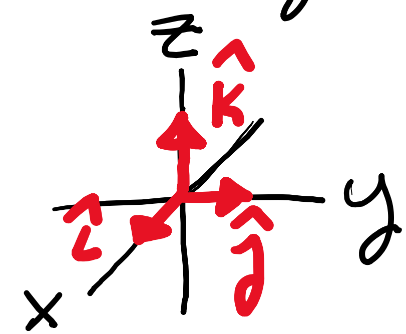

$$\left.\begin{aligned} &\hat{\imath} \times \hat{\jmath} = \hat{k}, \quad \hat{\jmath} \times \hat{k} = \hat{\imath}, \quad \hat{k} \times \hat{\imath} = \hat{\jmath} \\ &\text{\&} \ \hat{\jmath} \times \hat{\imath} = -\hat{k}, \quad \hat{k} \times \hat{\jmath} = -\hat{\imath}, \quad \hat{\imath} \times \hat{k} = -\hat{\jmath} \end{aligned}\right\}$$

Also $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$

Notice the structure

$\Rightarrow \quad \hat{i} \times \hat{j} =$

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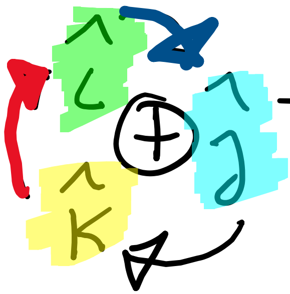

$\Rightarrow$ $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$

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$$\Rightarrow \quad \hat{i}\times\hat{j}=\hat{k},\quad \hat{j}\times\hat{k}=\hat{i},\quad \hat{k}\times\hat{i}=\hat{j}$$

& if going in opposite direction, we change the sign

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$\hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -\hat{i}, \quad \hat{i} \times \hat{k} = -\hat{j}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - A_z B_y) + \hat{j}(A_z B_x - A_x B_z) + \hat{k}(A_x B_y - A_y B_x)$$

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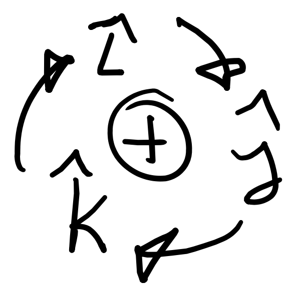
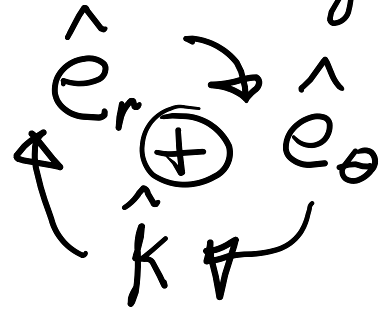
Do you see the



structure?

Same thing holds for cylindrical coordinates, with



$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{k} \\ A_r & A_\theta & A_z \\ B_r & B_\theta & B_z \end{vmatrix}$$

$$= \hat{e}_r (A_\theta B_z - A_z B_\theta) + \hat{e}_\theta (A_z B_r - A_r B_z) + \hat{k} (A_r B_\theta - A_\theta B_r)$$

From last lecture: $\vec{F} = \frac{d}{dt}\vec{L}$.

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Using vector calculus we are going to see that, by taking cross product on both sides with the position vector $\vec{r}$, From last lecture : $\vec{F} = \frac{d}{dt}\vec{L}$.

Using vector calculus we are going to see that, by taking cross product on both sides with the position vector $\vec{r}$, we will obtain

Torque = time rate of change of angular momentum

Create a coordinate system with the origin denoted as O

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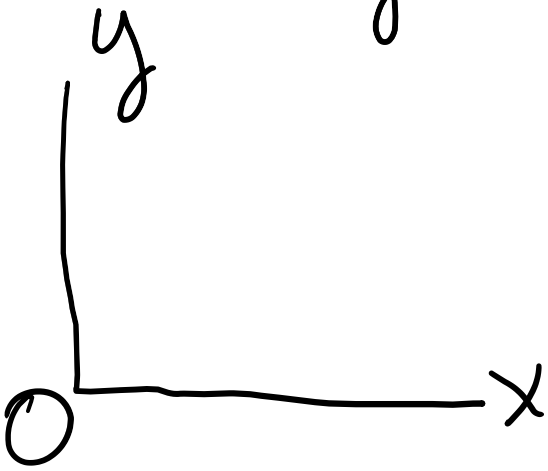
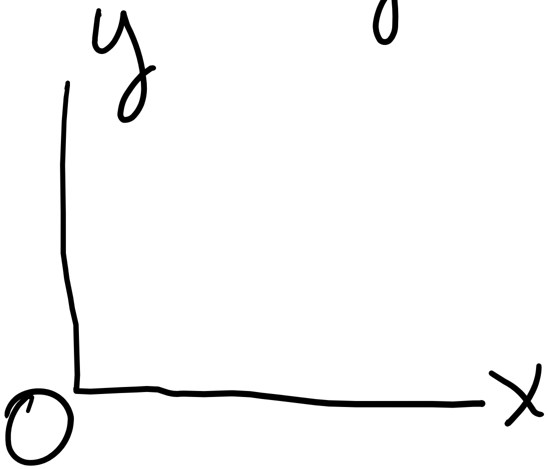


Create a coordinate system with the origin denoted as O. Place an object in our coordinate system thas has linear momentum $\vec{L}$



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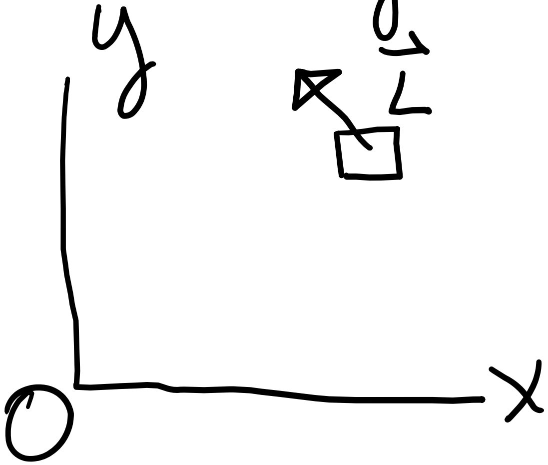

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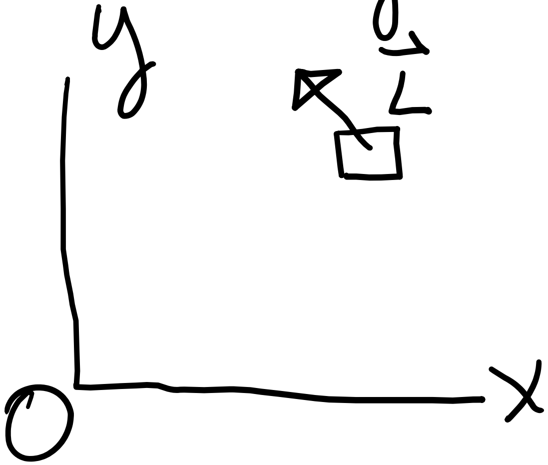

object in our coordinate system thas has linear momentum $\vec{L}$ & force $\vec{F}$ acting on it

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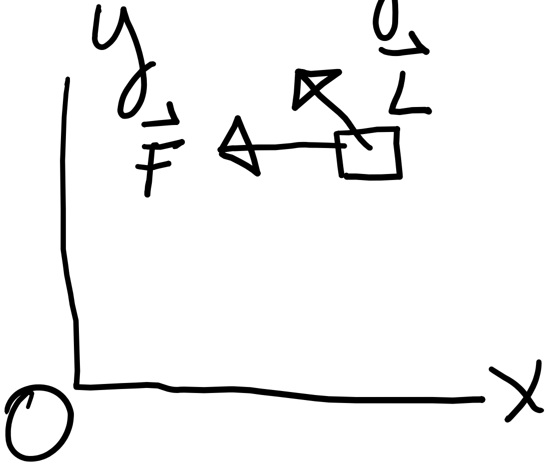

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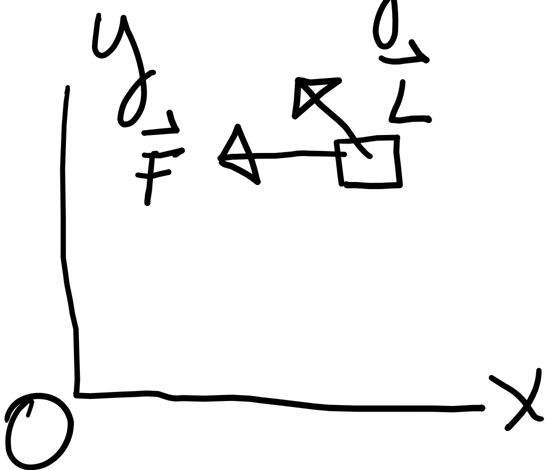

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Now draw a position vector $\vec{r}$ from origin to object

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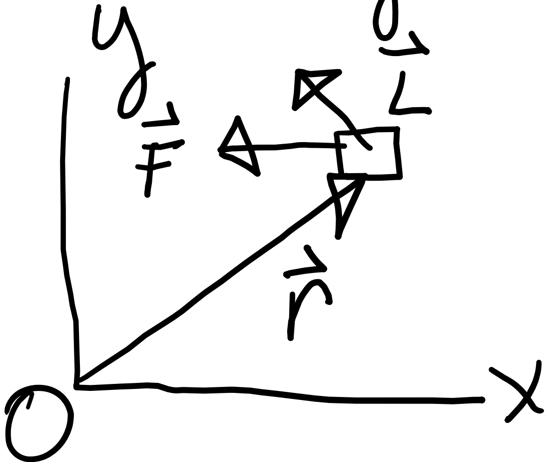

object in our coordinate system that has linear momentum $\vec{L}$ & force $\vec{F}$ acting on it

Now draw a position vector $\vec{r}$ from origin to object. <u>Now, pure mathematics:</u>

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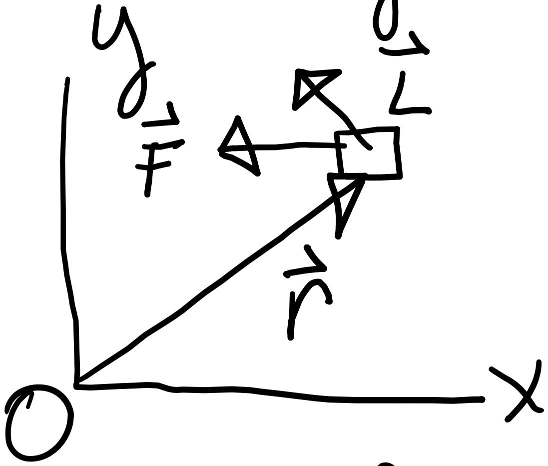

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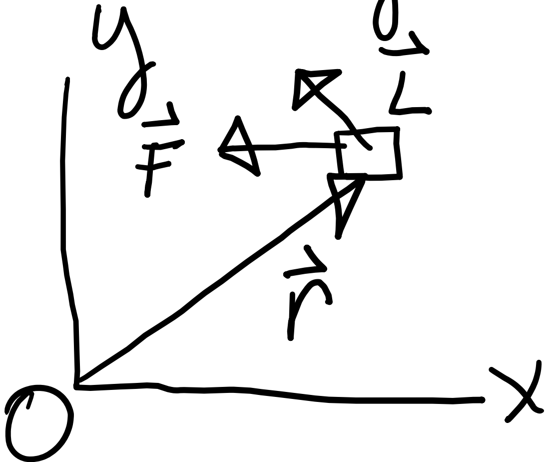

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Note: $\frac{d}{dt}(\vec{r} \times \vec{L}) = \dot{\vec{r}} \times \vec{L} + \vec{r} \times \dot{\vec{L}}$, But $\dot{\vec{r}} \times \vec{L} = \dot{\vec{r}} \times m\dot{\vec{r}} = 0$

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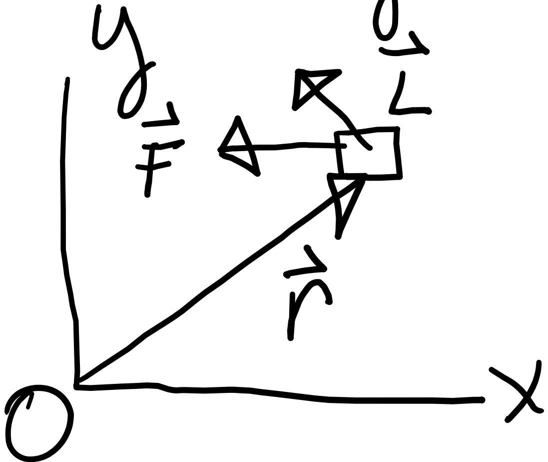

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So $\vec{r} \times \vec{F} = \frac{d}{dt}\left[\vec{r} \times \vec{L}\right]$

We have $\vec{r} \times \vec{F} = \frac{d}{dt}(\vec{r} \times \vec{L})$

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Notation & terminology:

We have $\vec{r} \times \vec{F} = \frac{d}{dt}(\vec{r} \times \vec{L})$

Notation & terminology: Let $\vec{H}_o \equiv \vec{r} \times \vec{L}$

& $\vec{M}_o \equiv \vec{r} \times \vec{F}$

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& $\vec{M}_0 \equiv \vec{r} \times \vec{F}$, where

$\vec{M}_0 \equiv$ Moment [A.K.A. Torque] about O

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For single moment: $\vec{M}_O = \frac{d}{dt} \vec{H}_O$

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For single moment: $\vec{M}_O = \frac{d}{dt}\vec{H}_O$ or

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For multiple moments:

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For single moment: $\vec{M}_o = \frac{d}{dt}\vec{H}_o$ or

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For multiple moments: $\Sigma\vec{M}_o = \frac{d}{dt}H_o$

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For multiple moments: $\Sigma \vec{M}_o = \frac{d}{dt} H_o$ or

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ASU

Central Force

Central Force

If $\vec{F}$ is in same or opposite direction as $\vec{r}$

Central Force

If $\vec{F}$ is in same or opposite direction as $\vec{r}$, then $\vec{M}_o = \vec{r} \times \vec{F} = 0$

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Central force

If $\vec{F}$ is in same or opposite direction as $\vec{r}$, then $\vec{M}_o = \vec{r} \times \vec{F} = 0$ & $\vec{F}$ is called a central force &

since $\vec{M}_o = \dot{\vec{H}}$

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$\vec{H} = \text{const.}$ and is conserved

A funny example of conserved angular momentum

A funny example of conserved angular momentum

particle in straight line at constant speed v.

A funny example of conserved angular momentum

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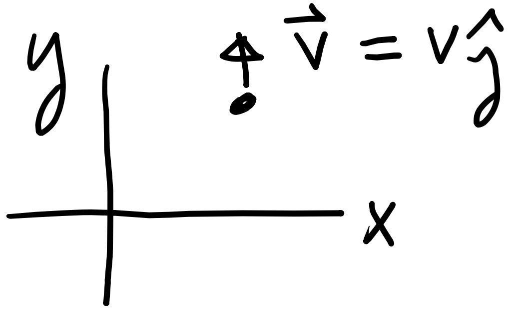

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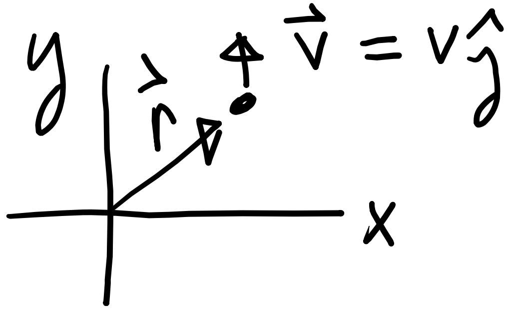

A funny example of conserved angular momentum

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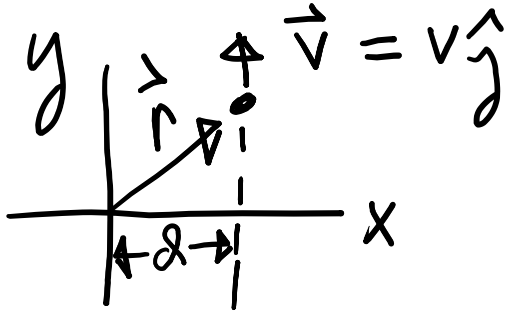
$$\vec{r} = d\hat{i} + (vt + y_0)\hat{j}$$



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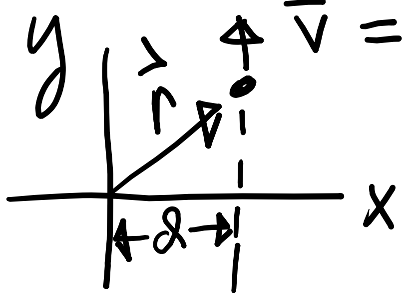

$\vec{v} = v\hat{j}$

$$\vec{r} = d\hat{i} + (vt + y_0)\hat{j} \quad \text{So}$$

$$\vec{H}_0 = \vec{r} \times \vec{L}$$

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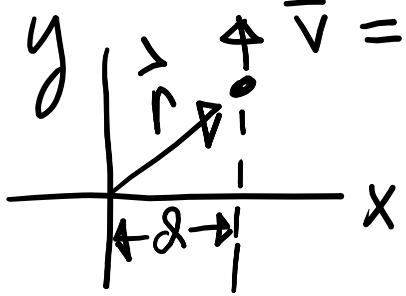

$$\vec{r} = d\hat{i} + (vt + y_0)\hat{j} \quad \text{So}$$

$$\vec{H}_o = \vec{r} \times \vec{L} = m\vec{r} \times \vec{v}$$

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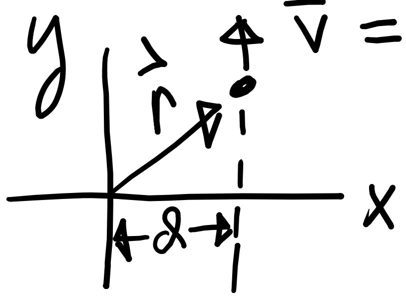

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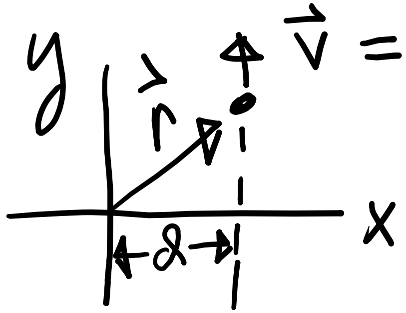

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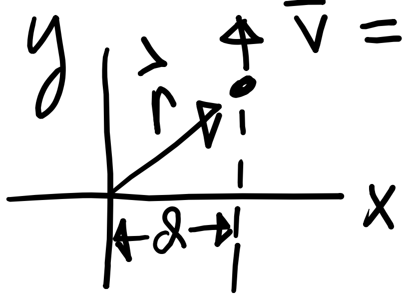

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particle in straight line at constant speed v.

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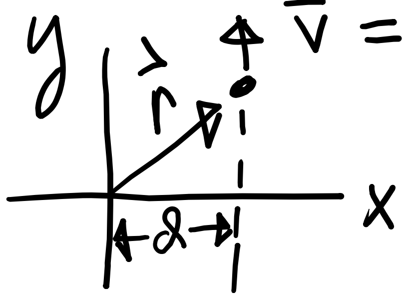

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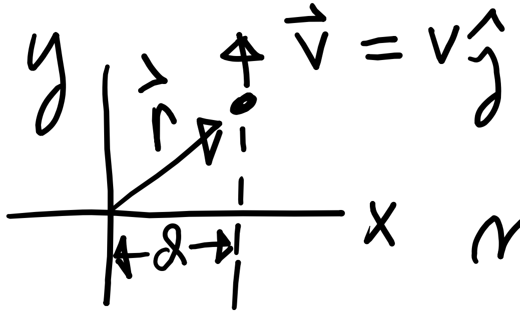
particle in straight line at constant speed v.



$\vec r = d\hat i + (vt + y_0)\hat j$ So

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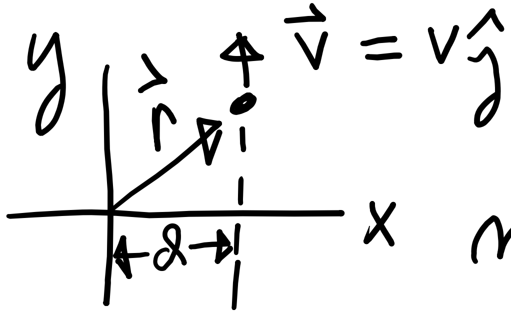
A funny example of conserved angular momentum

particle in straight line at constant speed v.



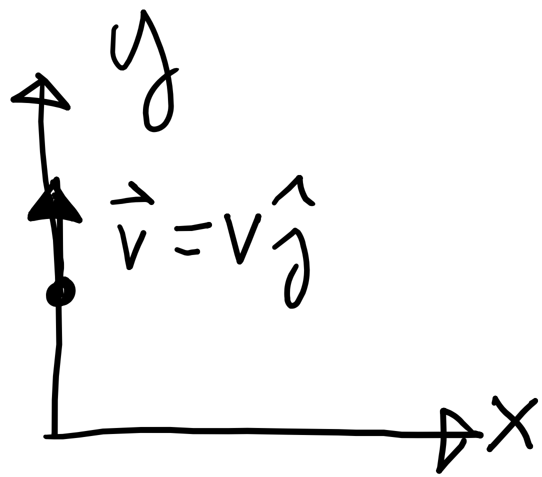
$\vec{r} = d\hat{i} + (vt + y_0)\hat{j}$ So

$$\vec{H}_0 = \vec{r} \times \vec{L} = m\vec{r} \times \vec{v} = m[d\hat{i} + (vt+y_0)\hat{j}] \times v\hat{j} = mdv(\hat{i} \times \hat{j}) + m(vt+y_0)v(\hat{j} \times \hat{j})$$

But $\hat{j} \times \hat{j} = 0$ & $\hat{i} \times \hat{j} = \hat{k}$

So $\vec{H}_0 = dmv\hat{k}$ We have angular momentum = const $\neq 0$, but NO rotation is happening.

Note: We can get rid of the angular momentum by choosing a different coordinate system:



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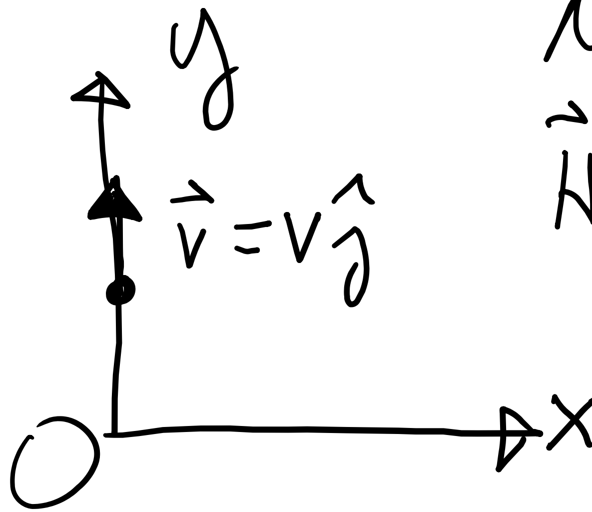

Now $r = (vt + y_0)\hat{j}$ &

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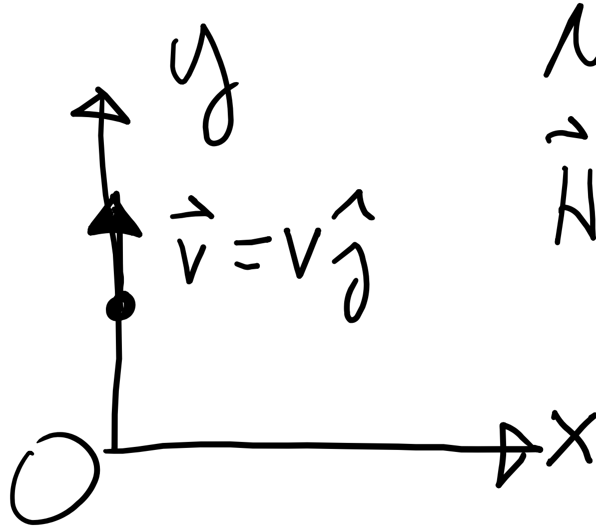

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Choice of coordinate system can be helpful in solving problems

Newtonian gravity

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$$F = G\frac{m_1 m_2}{r^2}$$

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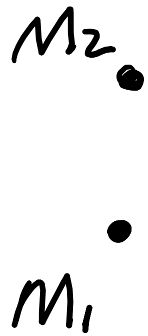

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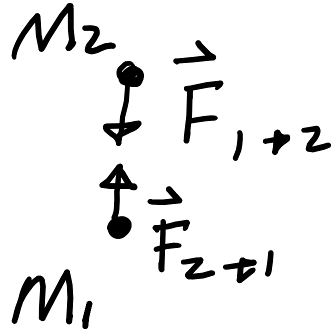

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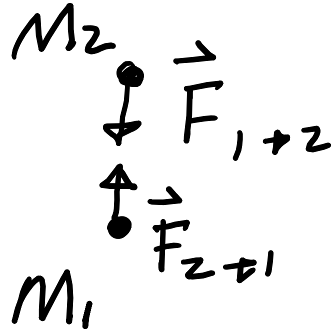

$$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$$

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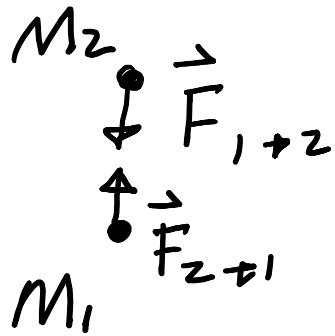

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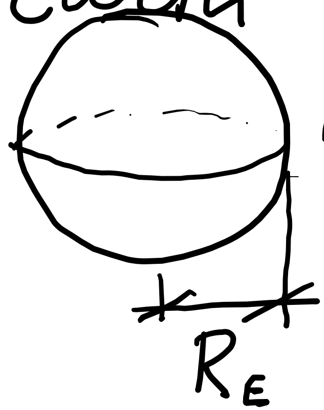
Earth



Mass $= M_E$

Newtonian gravity

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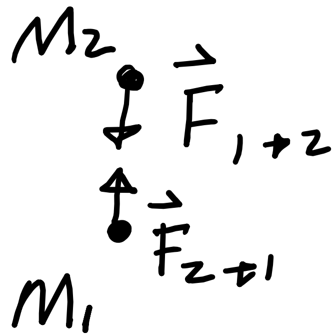

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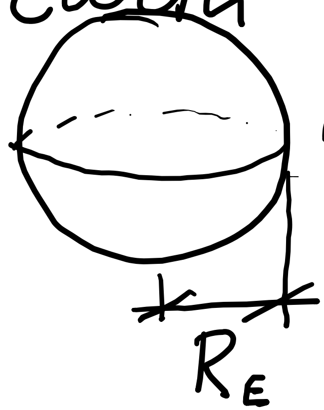

Mass $= M_E$

Let $r = R_E + h$, where $h \ll R_E$

Newtonian gravity

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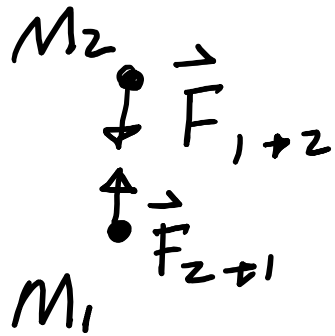

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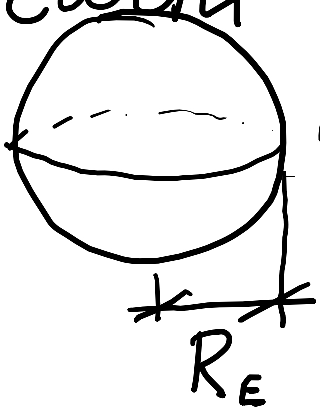

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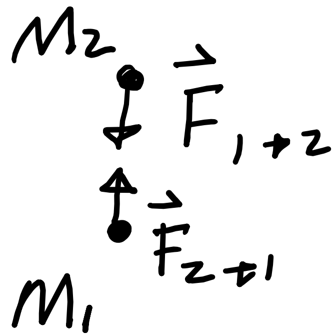

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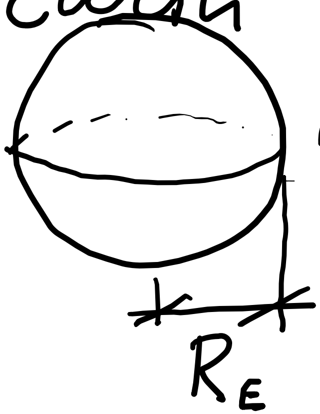

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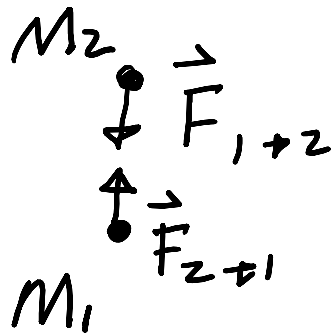

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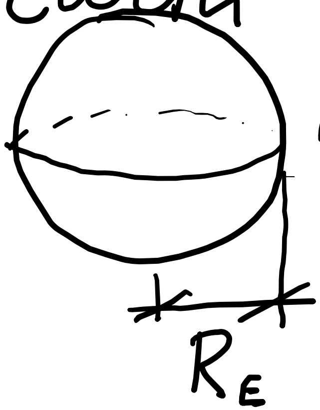

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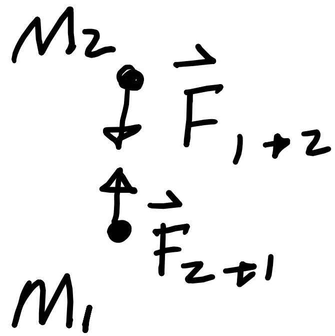

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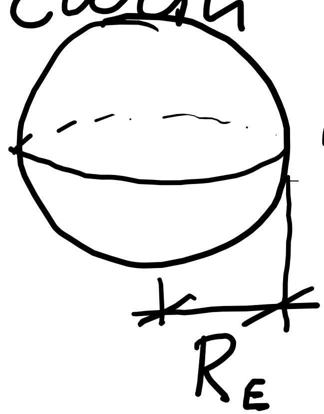

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So $$F(r) \cong \left[\frac{GmM_E}{R_E^2}\right]\left[1 - \left(\frac{2}{R_E}\right)(r - R_E)\right]$$ Near Earth

but $r - R_E = h$

Newtonian gravity

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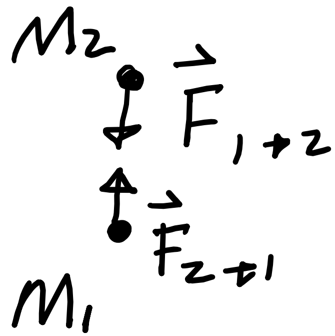

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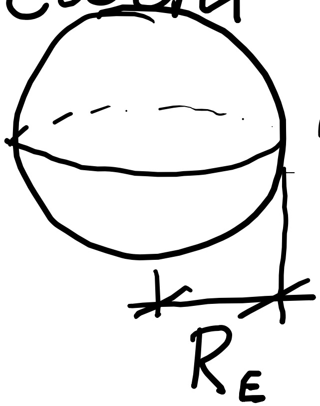

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Near Earth

but $r - R_E = h \Rightarrow F(h) = \left[\frac{GmM_E}{R_E^2}\right]\left[1 - \frac{2h}{R_E}\right]$

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ASU

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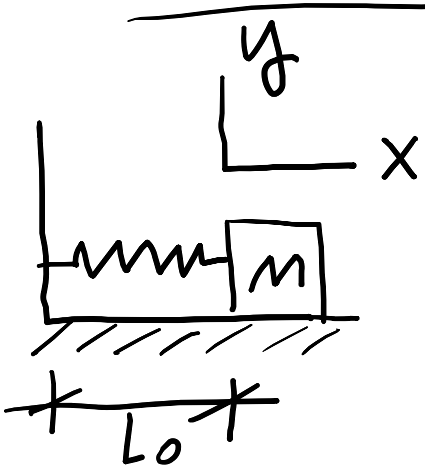
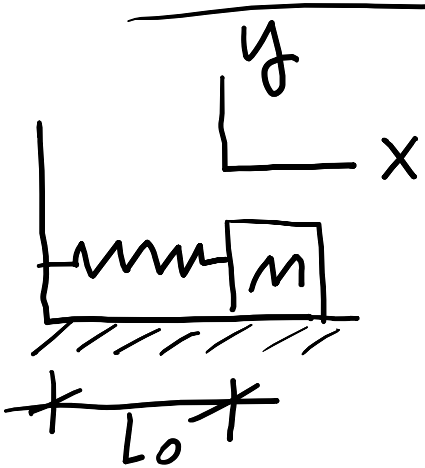
Force due to spring (Need for problem 12.90)



Let natural unstretched [nor compressed] length be L_0.

Force due to spring (Need for problem 12.90)



Let natural unstretched [nor compressed] length be L_0. In this case, no spring force F_{sp} on box

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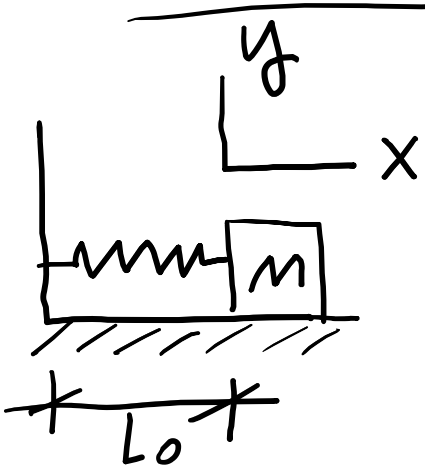

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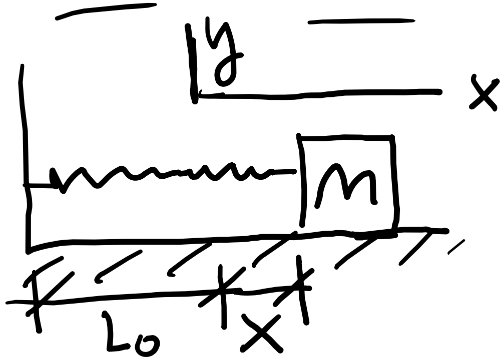

In this case, the spring is stretched and has length $L = L_0 + x$

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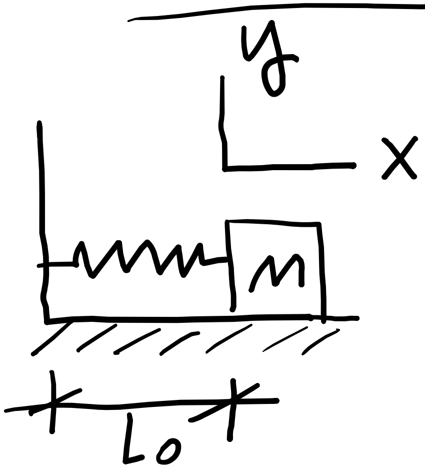

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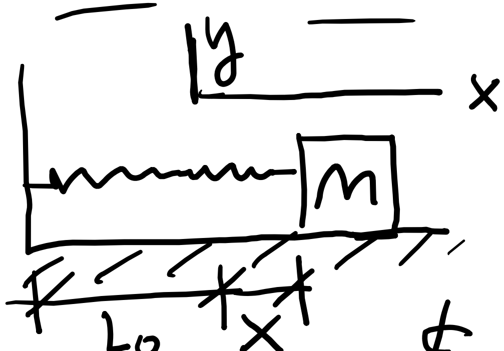
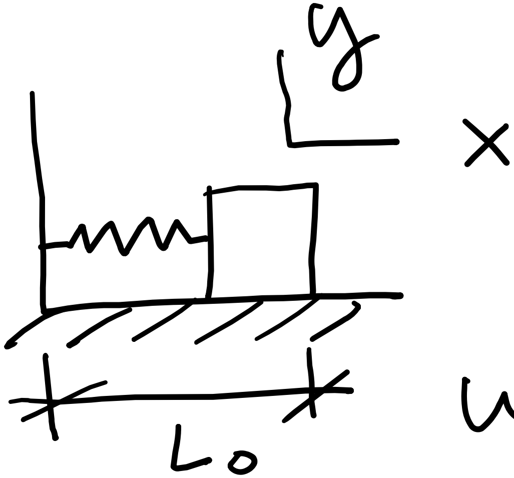
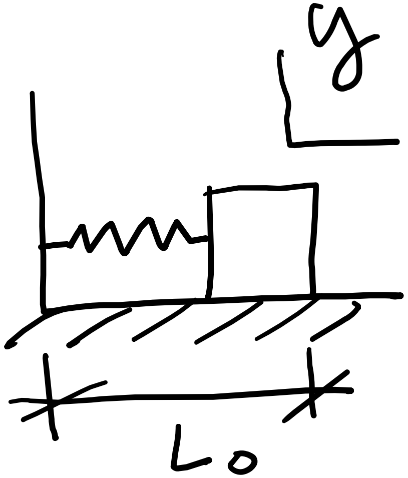


In this case, the spring is stretched and has length $L = L_0 + x$. So $x = L - L_0$ & the force is $F_{sp} = -Kx$, where K is the spring constant.



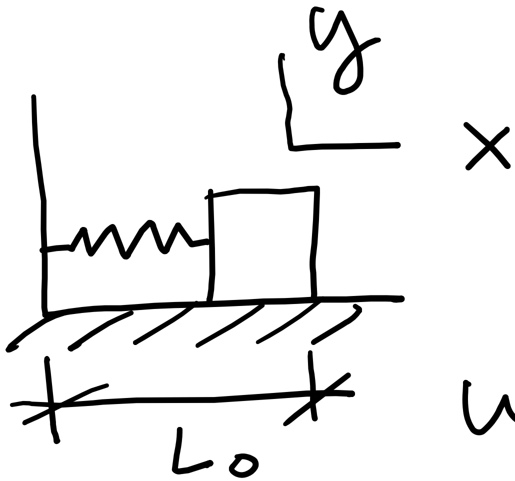
In this case the spring is compressed and has length $L = L_0 + x$, where in this situati

$x < 0$



In this case the spring is compressed and has length $L = L_0 + x$, where in this situati

$x < 0$ & $x = L - L_0$ as before



In this case the spring is compressed and has length $L=L_0+X$, where in this situati[on] $X<0$ & $X=L-L_0$ as before

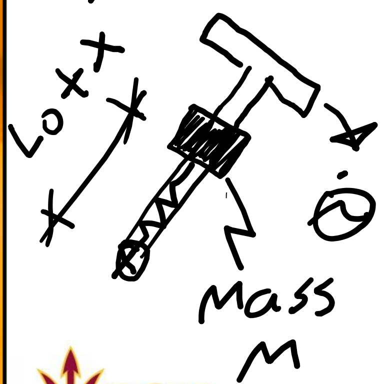
The force is $F_{sp}=-kx$ as before

Example: Massless rod is free to rotate and has a collar with no friction.

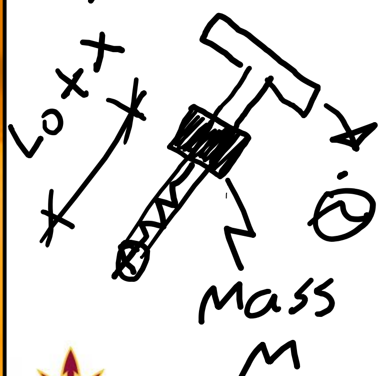
Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation.

Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0

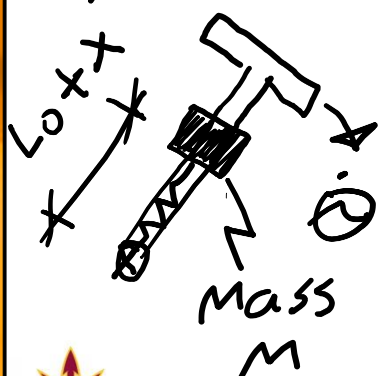
Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_o. Collar starts at position A that is distance $L_o + X$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c:



Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + X$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + X$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



$$\Rightarrow -kx = m(\ddot{r}_A - r_A \dot{\theta}^2)$$

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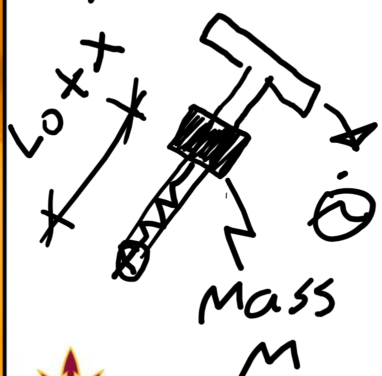
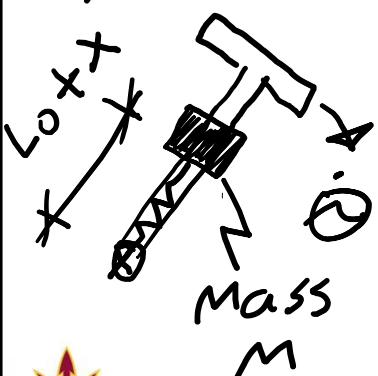


$$\Rightarrow -kx = m(\ddot{r}_A - r_A \dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A \dot{\theta}^2$$

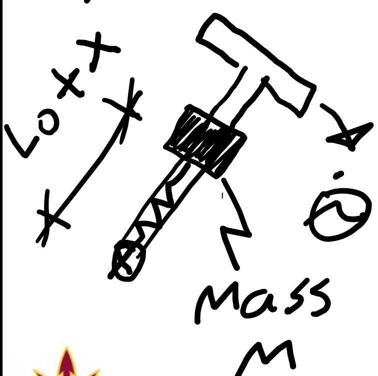
Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance L_0+x from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A\dot{\theta}^2$$

But $x = r_A - L_0$

Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + x$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A\dot{\theta}^2$$

But $x = r_A - L_0$ So

$$\ddot{r}_A = \frac{-k(r_A - L_0)}{m} + r_A\dot{\theta}^2$$