review #1

Kinematics 1D: General case

$a = \frac{dv}{dt}$, $v = \frac{dx}{dt} \Rightarrow \int v\,dt = \int dx$ & $\int a\,dt = \int dv$

Also $a = \frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx}$ & $\int a\,dx = \int v\,dv$

For these kind of problems, you are given some functional form $a(x)$ or $v(x)$ or $a(t)$ or $v(t)$ or $x(t)$ and are to determine, through calculus, other expressions. For example, you could be given $x = At + B\sin(\omega t)$ & asked for $v(t)$ & $a(t) \Rightarrow v = \frac{dx}{dt} = A + B\omega\cos(\omega t)$ $\Rightarrow a = \frac{dv}{dt} = -B\omega^2\sin(\omega t)$. At this point we have 3 equations, so we could have 3 unknown quantities determined. Of course, we could have been given $a(t)$ & asked for $v(t)$ & $x(t)$. In that case, we would integrate instead of using derivitives.

The expression $a = v\frac{dv}{dx}$ looks different but is important for spring-type problems and for energy relationships we will explore in the next chapter. For this sort of problem, we could be given something like $a = -kx$ & asked for v_F at $x = x_F$ given that $v = v_I$ at $x = x_I$

$$\Rightarrow a = v\frac{dv}{dx} \Rightarrow \int_{x_I}^{x_F} a\,dx = \int_{v_I}^{v_F} v\,dv \Rightarrow -k\int_{x_I}^{x_F} x\,dx = \frac{1}{2}(v_F^2 - v_I^2)$$

$$\Rightarrow -k(x_F^2 - x_I^2) = \frac{1}{2}(v_F^2 - v_I^2) \Rightarrow v_F = \left[-2k(x_F^2 - x_I^2) + v_I^2\right]^{1/2}$$

Kinematics 1d (continued)

As you can see, this is mostly a mathematical endeavor concentrating on calculus & systems of equations. In physics, there are special cases that often come up:

constant position: $x = x_0$ & $V = 0$ & $a = 0$
constant velocity: $x = x_0 + V_0 t$ & $V = V_0$ & $a = 0$
constant acceleration: $x = \frac{1}{2}at^2 + V_0 t + x_0$ & $V = a_0 t + V_0$ & $a = a_0$

For these kind of problems we can memorize the above equations & skip the calculus & go straight to solving a system of equations, where we have, at most, 3 unknown quantities.

We could also talk about relative motion of two particles. Let me call one particle Bob & another particle Alice. Relative position of Bob with respect to Alice $\equiv x_{B/A}$ & $x_{B/A} \equiv x_B - x_A$

For velocity we have $V_{B/A} = V_B - V_A$ &

for acceleration $a_{B/A} = a_B - a_A$

Kinematics 2d

We can easily push into higher dimension:

position → $\vec{r} = x\hat{i} + y\hat{j}$ & $\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\hat{i} + \dot{y}\hat{j}$

& $\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ we could also write $\vec{r} = r_x\hat{i} + r_y\hat{j}$ & $\vec{v} = V_x\hat{i} + V_y\hat{j}$ & $\vec{a} = a_x\hat{i} + a_y\hat{j}$

For each dimension we have a set of equations, so the problems can get more complex. There is a special class of 2-d problems we did work on:

Trajectories

Here we have an object thrown near earth where $|a_y| = g$, with $g \equiv$ acceleration due to gravity near earth & $g = 9.81\,m/s^2$ or $g = 32.2\,ft/s^2$. If we set the coordinate system with the y-axis pointing up away from earth, then

$$y = -\tfrac{1}{2}gt^2 + V_{oy} + y_o \quad \& \quad x = V_{ox}t + x_o .$$

Moreover, we could set the origin of our coordinate system at $(x_o, y_o) = (0, 0)$ so that $y = -\frac{1}{2}gt^2 + V_{oy}t$ & $x = V_{ox}t$

Often we are given a problem without time information and it is useful to have y as a function of x. To get such an expression we notice that $x = V_{ox}t \Rightarrow t = \frac{x}{V_{ox}}$ and put that into $y \Rightarrow y(x) = -\frac{1}{2}g\left(\frac{x}{V_{ox}}\right)^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$.

Perhaps we are given the angle of trajectory at time $= 0 \Rightarrow \tan\theta_o = \frac{V_{oy}}{V_{ox}}$ & $V_{ox} = V_o\cos\theta_o$

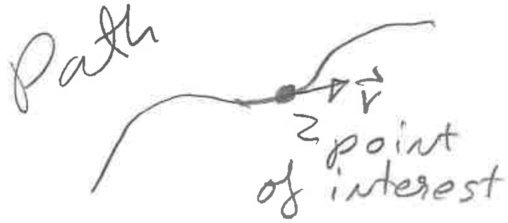
$$\Rightarrow y(x) = \left(-\frac{1}{2}\right)\frac{g\,x^2}{V_o^2\cos^2\theta_o} + x\tan\theta_o \text{, where } V_o = \left[V_{ox}^2 + V_{oy}^2\right]^{1/2}$$

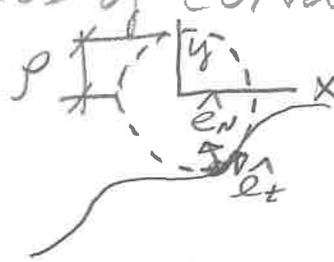
As you can imagine (and have seen in HW problems) there are many ways one could set up a trajectory problem.

We are not forced to choose rectangular components. In fact we looked at two other choices.

Tangential & Normal components

For any smooth curved path of some particle, we could, at some instance in time, create a coordinate system such that the distance from the origin is constant at that moment. That distance is called the radius of curvature ρ





The unit vector pointing from the point to the origin is $\hat{e}_n$ & the unit vector pointing in the direction of $\vec{v}$ is $\hat{e}_t$

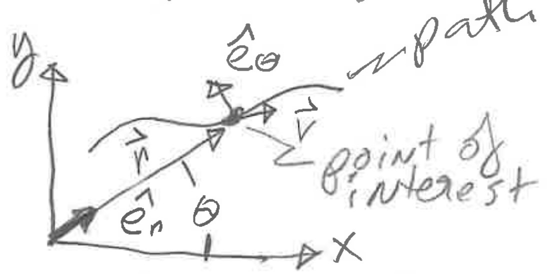
$\Rightarrow \quad \vec{v} = v\hat{e}_t$ For acceleration we can obtain $\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

$\Rightarrow \quad \vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$, where

$a_t \equiv \dot{v}$ & $a_n = \frac{v^2}{\rho}$

Another choice of components is <u>Radial and transverse</u>

In this case we lock down the coordinate system for all time and allow for the distance from origin to point of interest to vary with time.



For this choice, we get expressions that look more complicated:

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$\Rightarrow$ $\vec{v} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$, where $v_r = \dot{r}$ & $v_\theta = r\dot{\theta}$

& $\vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta$, where $a_r = \ddot{r} - r\dot{\theta}^2$ & $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Of course, this should look similar to the previous case when $\dot{r} = \ddot{r} = 0 \Rightarrow r = \text{const.}$

For $r = \text{const.}$ we have $\vec{v} = r\dot{\theta}\hat{e}_\theta$

& $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$. Note $v = r\dot{\theta} \Rightarrow r\dot{\theta}^2 = v^2/r$

& $r\ddot{\theta} = \dot{v}$ so we could write this special case ($r = \text{const}$) as $\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

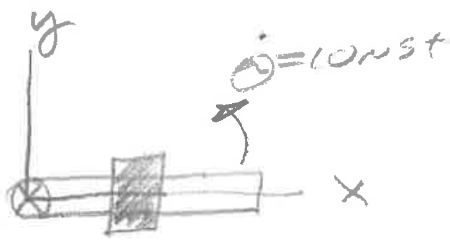
and this looks like $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$, as it should

Radial & transverse (continued)

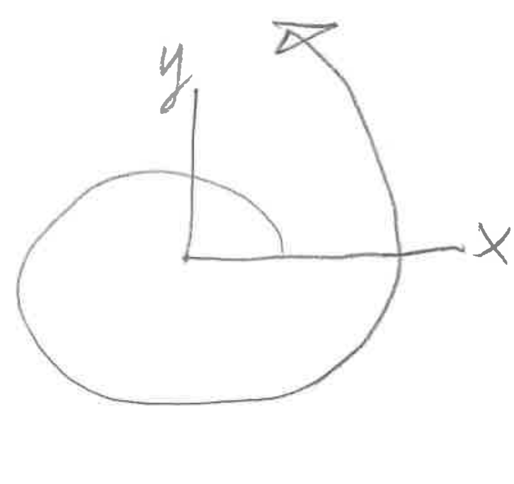
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

We saw how, for a special case where $r = \text{const}$, that $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta \Rightarrow \vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$ and is very similar to the case where tangential & normal components are used. But what about the other terms in $\vec{a} = (\underline{\underline{\ddot{r}}} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + \underline{\underline{2\dot{r}\dot{\theta}}})\hat{e}_\theta$?

For a special case where the particle is accelerating directly towards or away from the origin, $\dot{\theta} = 0 \Rightarrow \vec{a} = \ddot{r}\hat{e}_r$. Now we have 3 of the 4 terms figured out. What about the funny looking term: $2\dot{r}\dot{\theta}\hat{e}_\theta$? We can isolate this term with the following example: A frictionless collar on rod rotating at constant angular speed $\dot{\theta}$.



Collar free to move in $\hat{e}_r$ direction so $a_r = 0$ & since $\dot{\theta} = \text{const}$, then $\ddot{\theta} = 0$. Now $\vec{a} = 2\dot{r}\dot{\theta}\hat{e}_\theta$ & motion looks like spiral:

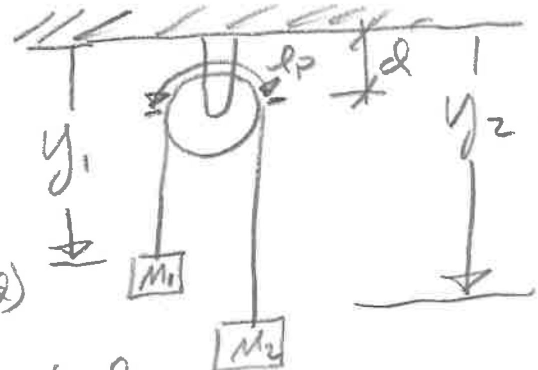


I had a difficult time figuring out where best to talk about pulley problems, so I left that for the end of our discussion of kinematics. RI, p7

pulleys

The idea is to determine the length of rope, where all constant lengths are thrown into a "bucket" called const.

Simple example:

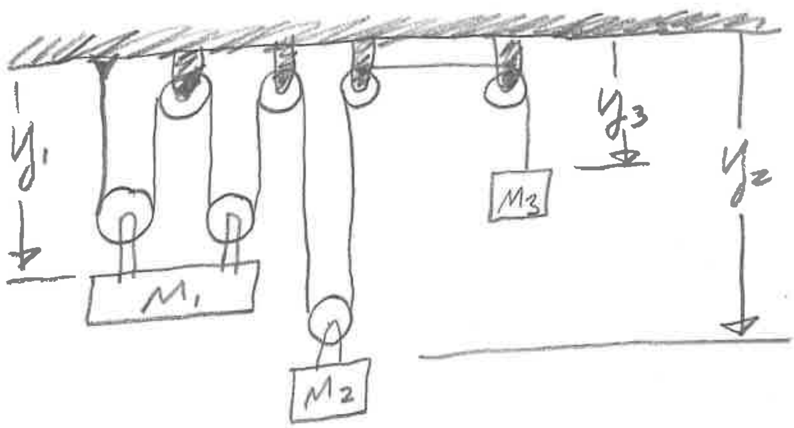


length of rope $= (y_1 - d) + l_p + (y_2 - d)$

But length of rope & l_p & d are all constant so $y_1 + y_2 = \text{const.}$

$\Rightarrow \quad V_1 + V_2 = 0 \Rightarrow V_1 = -V_2$

Now a more complex case where we simply neglect the constant terms

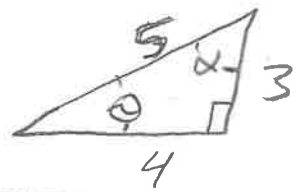


Here $4y_1 + 2y_2 + y_3 = \text{const.} \Rightarrow 4V_1 + 2V_2 + V_3 = 0$

& $4a_1 + 2a_2 + a_3 = 0$

3, 4, 5 Triangle

My favorite triangle:



$\cos\theta = \frac{4}{5}, \cos\alpha = \frac{3}{5}$

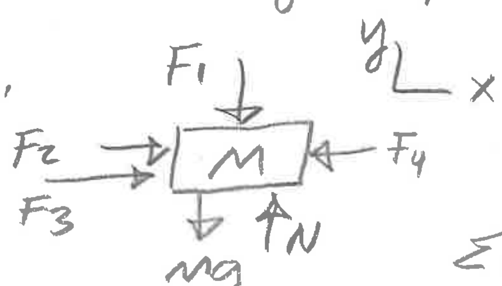
$\sin\theta = \frac{3}{5}, \sin\alpha = \frac{4}{5}$

$\tan\theta = \frac{3}{4}, \tan\alpha = \frac{4}{3}$

$\sum \vec{F} = m\vec{a}$

There are too many possible problems for me to do any kind of justice to this subject matter on a review. With that statement out of the way, lets look at how we tend to go about solving problems.

The 1ST order of business is to determine the forces on our object & draw a free body diagram.



Now add up components

$\sum F_x = ma_x \Rightarrow F_2 + F_3 - F_4 = Ma_x$

& $\sum F_y = ma_y \Rightarrow N - mg - F_1 = Ma_y$

Many times, one or more components of acceleration equals zero

Friction

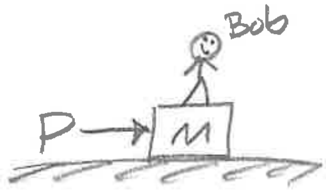
$\mu_s \equiv$ coefficient of static friction

$\mu_k \equiv$ coefficient of kinetic friction

$F_F \leq \mu_s N$ for static case

& $F_F = \mu_k N$ for kinetic case

Example



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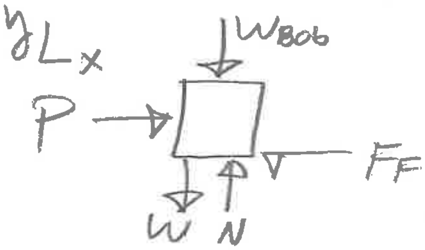
Bob weighs 180 lb & stands on a box that weighs 20 lb. Alice pushes with 100 lb of force. Given that $\mu_s = 0.4$ & $\mu_k = 0.1$ & box is initially at rest, does the box move? If so, what is the acceleration?



$\sum F_x = Ma_x \Rightarrow P - F_F = Ma_x$

$\sum F_y = M\cancel{a_y}^{0} \Rightarrow N - W_{box} - W_{Bob} = 0$

so $N = W_{Box} + W_{Bob} = 200\,lb$.

Static case: $P - F_{F_{MAX}} = (100 - 0.4 * 200)\,lb = 20\,lb$

so $P > F_{F_{MAX}}$ so NOT STATIC

Thus $P - F_F = (100 - 0.1 * 200)\,lb = 80\,lb$

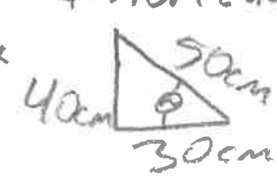
$\Rightarrow 80\,lb = (M_{Box} + M_{Bob}) * a_x \Rightarrow a_x = \left(\frac{80\,lb}{200\,lb}\right) g$

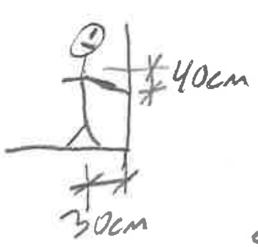
Note: used $(M_{Box} + M_{Bob}) \frac{g}{g} = \frac{W_{Box} + W_{Bob}}{g} = \frac{200\,lb}{g}$

so $a_x = 0.4g$

Child pushes against wall without slipping. The child's arm is 50 cm long and is 30 cm away from the wall & pushes with 10 N of force. Find the normal force against her hand in vertical & horizontal directions.

Free body diagram:





$\vec{P} = P\cos\theta\,\hat{i} - P\sin\theta\,\hat{j}$

& $\sum F_x = M\cancel{a_x}^{0}$ & $\sum F_y = M\cancel{a_y}^{0}$

$\Rightarrow P\cos\theta + N_x = 0$ & $-P\sin\theta + N_y = 0$

So $N_x = 10N * \frac{3}{5} = 6N$ & $N_y = 10N * \frac{4}{5} = 8N$

Example

Uniform circular motion with gravity: A planet revolves about a star on a circular path of radius R. The planet has a mass m_p & the star has a mass M_s. How fast is the planet moving?

Taking $M_s >> M_p$ such that star can be taken as stationary.

Free body diagram of planet: $F_{S \to P} \leftarrow \bigcirc \Rightarrow \Sigma F_N = m_p a_N$

$\Rightarrow F_{S\to P} = M_p a_N$ but $F_{S \to P} = G\dfrac{M_s M_p}{R^2}$ & $a_N = \dfrac{V^2}{R}$

so $G\dfrac{M_s M_p}{R^2} = m_p \dfrac{V^2}{R} \Rightarrow V = \left[\dfrac{G M_s}{R}\right]^{1/2}$

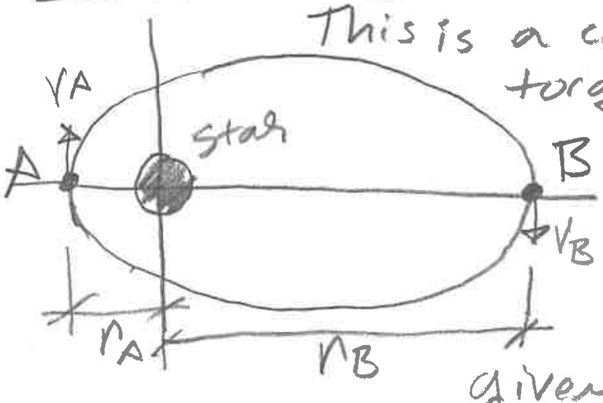
Angular momentum & Torque (Moment)

$\vec{H}_o = \vec{r} \times \vec{L}$ & $\Sigma \vec{M}_o = \dot{\vec{H}}_o$ **Note**: If no torque then

$\Sigma \vec{M}_o = \vec{0} \Rightarrow \dot{\vec{H}}_o = \vec{0} \Rightarrow \vec{H}_o = \text{const.}$ So

No torque $\Rightarrow$ Angular momentum is constant.

Example: **Eliptical orbit of planet about star**



This is a central force problem. There is **no** torque: $\vec{M}_o = \vec{0}$. So $\vec{H}_o$ is constant. At position A we have $H_{OA} = M_p r_A V_A$ & at position B we have $H_{OB} = M_p r_B V_B$ and these must be equal: $H_{OA} = H_{OB}$

$\Rightarrow M_p r_A V_A = M_p r_B V_B \Rightarrow r_A/r_B = V_B/V_A$

& since $r_A < r_B$ then $V_A > V_B$. If given some numbers, we could figure it out more explicitly.