Today: 13.1

$\boxed{L10}$

Today: 13.1 $\boxed{L10}$

work & kinetic energy

Today: 13.1

Friday: 13.2

L10

Today: 13.1 $\boxed{L10}$

Friday: $\underbrace{13.2}$

Conservation of energy

ASU

Today: 13.1 $\boxed{L10}$

Friday: 13.2

HW #4:

13.2a, 13.11, 13.19a, 13.45	§13.1
13.62a, 13.64, 13.72, 13.100	§13.2
13.120, 13.134	§13.3

Today: 13.1

L10

Friday: 13.2

HW #4:

13.2a, 13.11, 13.19a, 13.45	§13.1
13.62a, 13.64, 13.72, 13.100	§13.2
13.120, 13.134	§13.3

Due Wednesday Sept. 23rd

ASU

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ASU

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$$\text{Work} = \left\{\begin{matrix}\text{Change in}\\ \text{kinetic energy}\end{matrix}\right\}$$

ASU

$\boxed{\text{1-d case}}$ $\quad \Sigma F = ma$

$\boxed{\text{1-D case}}$ $\Sigma F = ma$ But $a = v\frac{\partial v}{\partial x}$

$\boxed{\text{1-d case}}$ $\Sigma F = ma$ But $a = v\frac{dv}{dx}$

So $\Sigma F = m v \frac{dv}{dx}$

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so $\quad \Sigma\int F\,dx = m\int v\frac{dv}{dx}\,dx$

$\boxed{\text{1-D case}}$ $\quad \sum F = ma \quad$ But $\quad a = v\frac{dv}{dx}$

So $\quad \sum\int F\,dx = m\int v\frac{dv}{dx}\,dx \Rightarrow \sum\int F\,dx = m\int v\,dv$

$\boxed{\text{1-d case}}$ $\sum F = ma$ But $a = v\frac{dv}{dx}$

So $\sum \int F\,dx = m\int v\frac{dv}{dx}dx \Rightarrow \sum\int_{x_1}^{x_2} F\,dx = m\int_{v_1}^{v_2} v\,dv$

$$\Rightarrow \sum\int_{x_1}^{x_2} F\,dx = \frac{1}{2}mv^2\Big|_{v_1}^{v_2}$$

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<u>Define</u>: $U_{1\to2} \equiv$ Work from point 1 to point 2

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& $T \equiv$ Kinetic energy

& $T \equiv \frac{1}{2}mv^2$

Now $U_{1\to 2} = T_2 - T_1$ or $U_{1\to 2} = \Delta T$ or $T_1 + U_{1\to 2} = T_2$

ASU

For 3-d case $U_{1\to2}$ generalizes as

$$U_{1\to2} = \sum \int \vec{F} \cdot d\vec{r}$$

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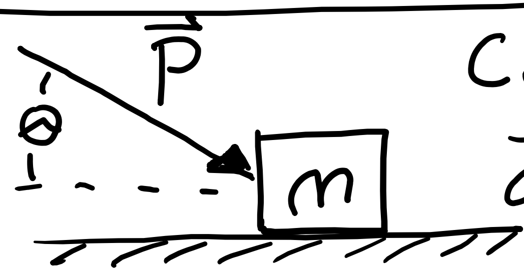
<u>Example</u>:



Coefficient of static friction $= \mu_k$

Box starts at rest & moves distance L. Find T_2

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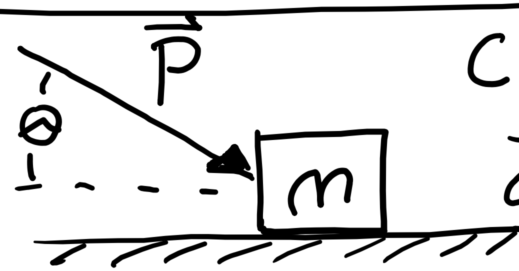

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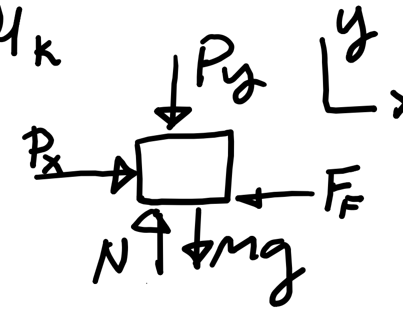

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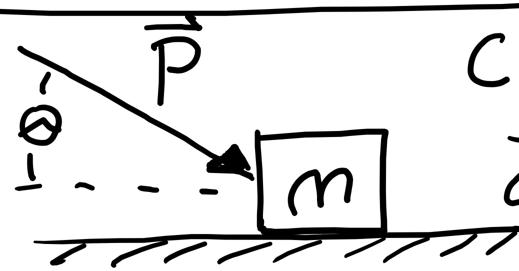

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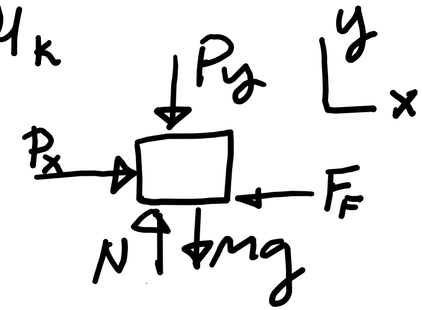

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$$\sum F_x = P_x - F_F = P_x - N\mu_k$$

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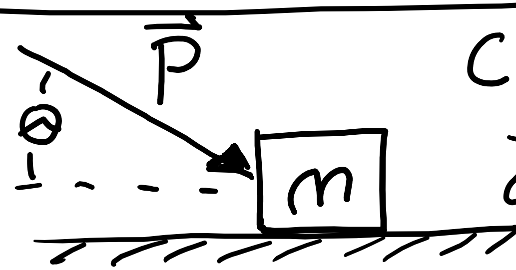

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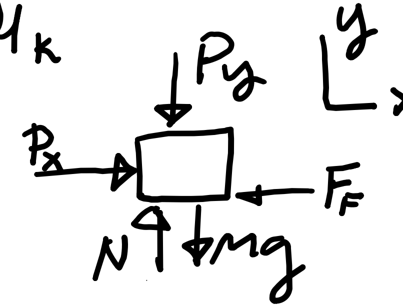

Box starts at rest & moves distance L. Find T_2

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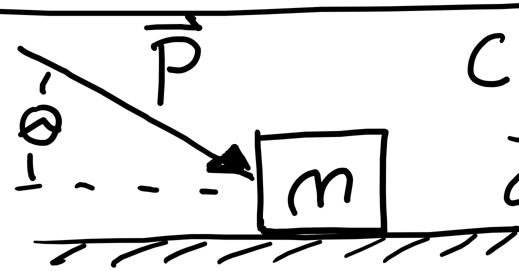

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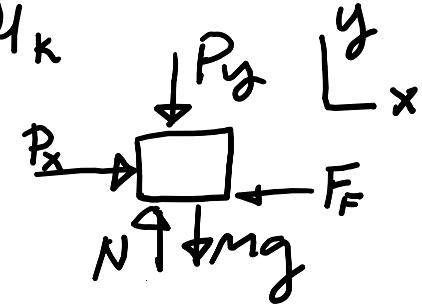

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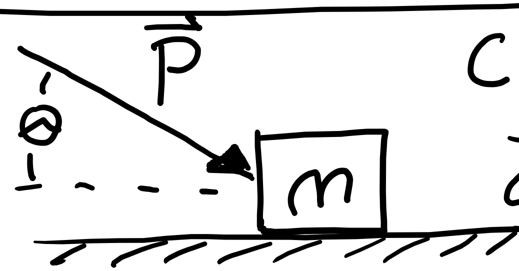

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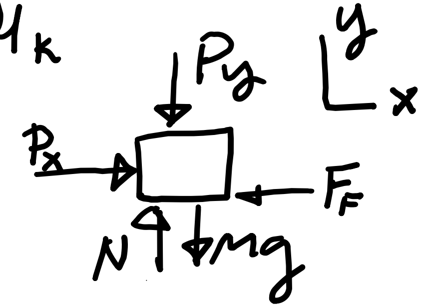

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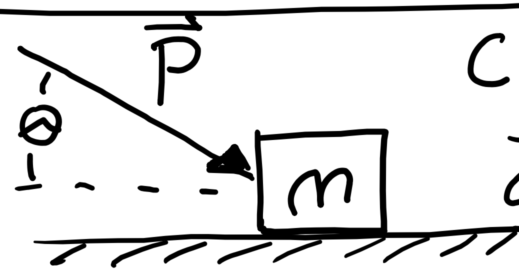

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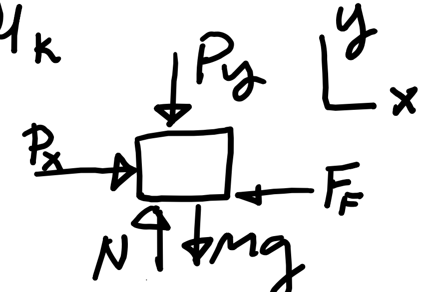

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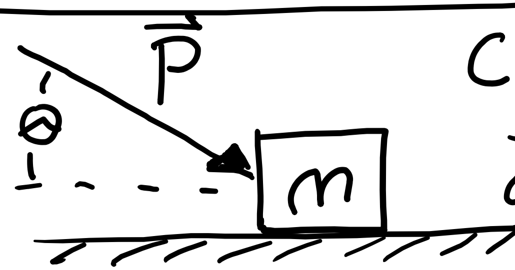

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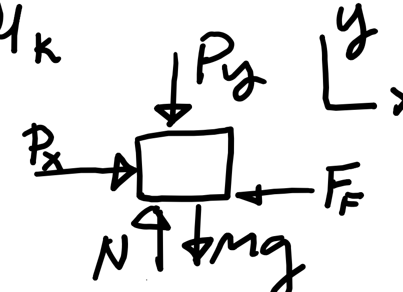

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ASU

For 3-d case $U_{1\to 2}$ generalizes as

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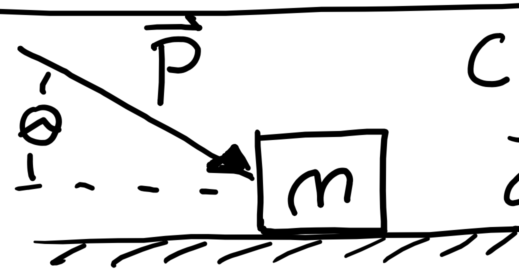

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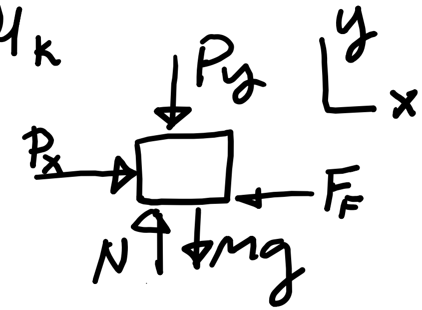

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If no friction $\mu_k = 0$

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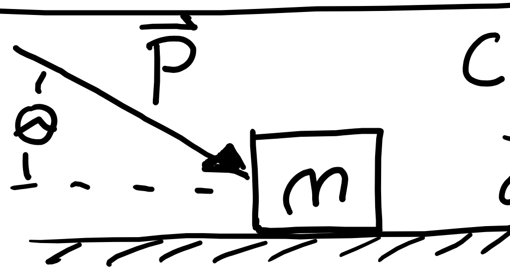

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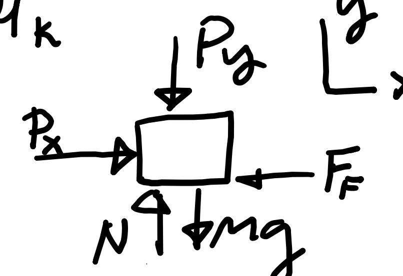

Box starts at rest & moves distance L. Find T_2

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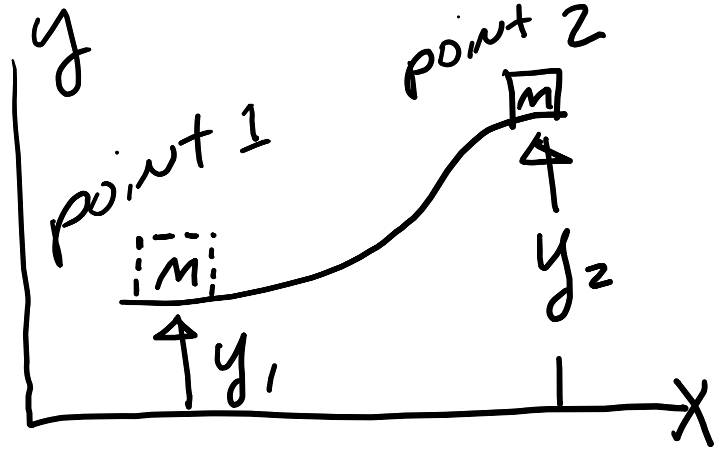
If no friction $\mu_k = 0$ & $T_2 = PL\cos\theta$

Gravity Near earth

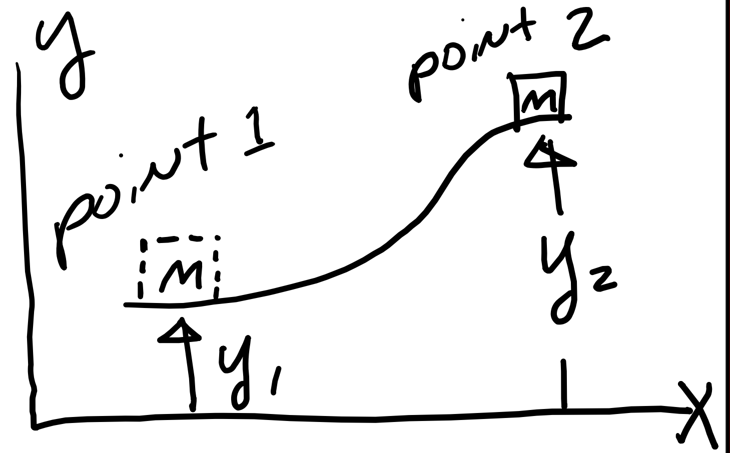
Gravity Near earth



Gravity Near earth

We want the worke **due to gravity**

$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$



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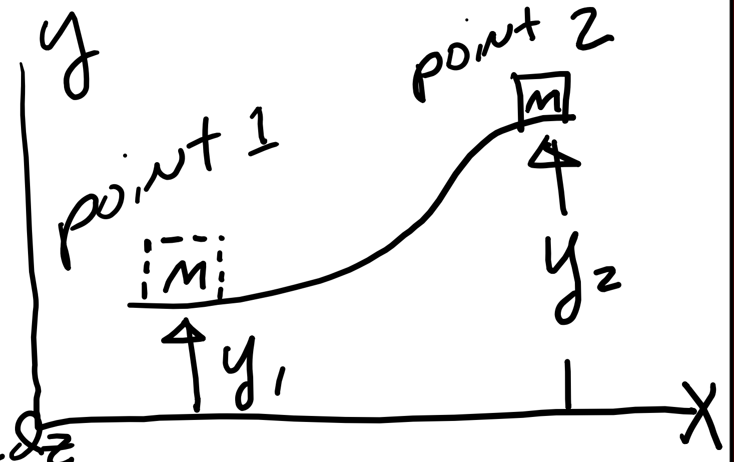

$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

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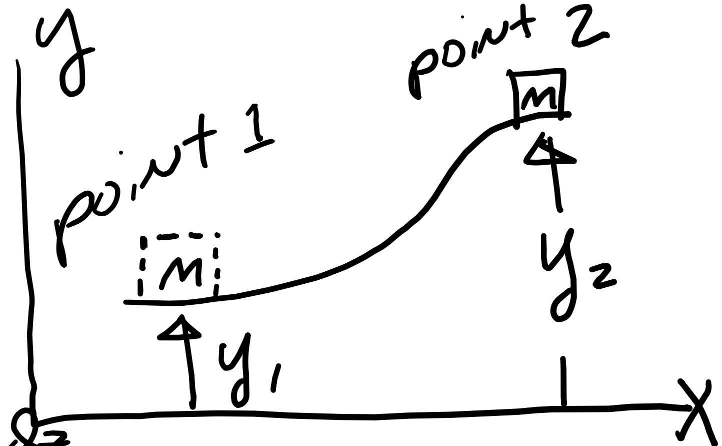

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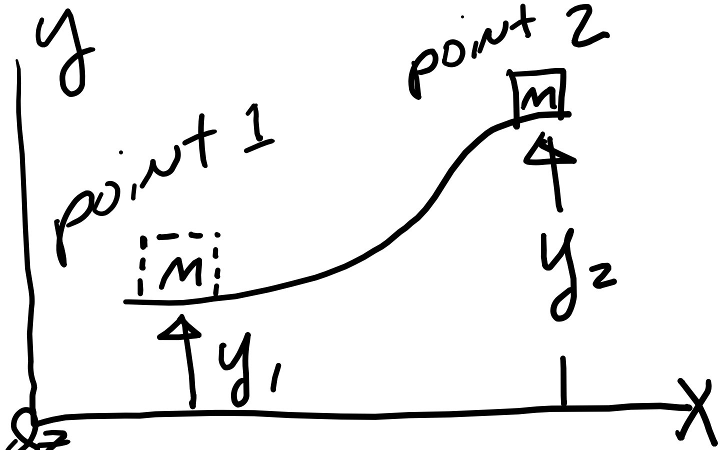
At any point along curve $F_x = 0, F_y = -mg, F_z = 0$

Gravity Near earth

We want the work due **to gravity**



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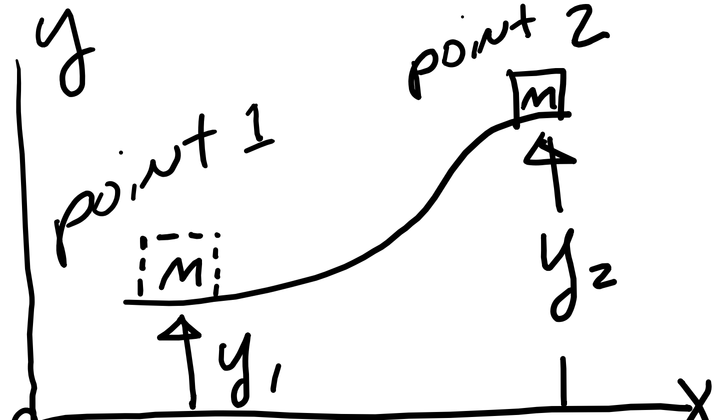
At any point along curve $F_x = 0,\ F_y = -mg,\ F_z = 0$

So $U_{1\to 2} = \int_{y_1}^{y_2} F_y\,dy$

Gravity Near earth



We want the worke **due to gravity**

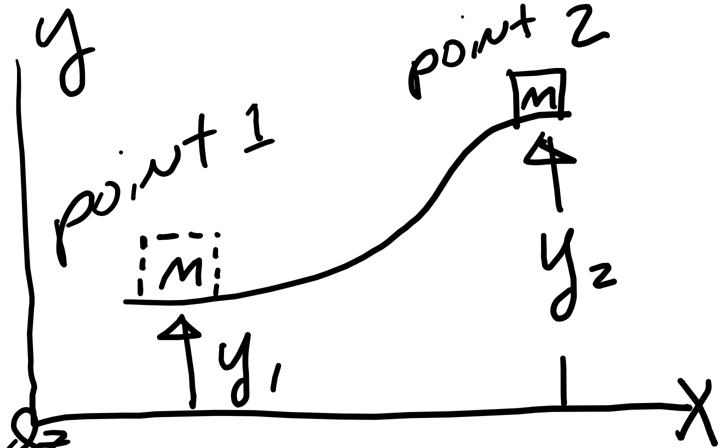
$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

At any point along curve $F_x = 0, F_y = -mg, F_z = 0$

So $U_{1\to 2} = \int_{y_1}^{y_2} F_y\,dy = -mg\int_{y_1}^{y_2} dy = \underbrace{-mg\Delta y}_{\text{decrease in kinetic energy}}$

Gravity Near earth

We want the work **due to gravity**



$$U_{1\to 2}^{(g)} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

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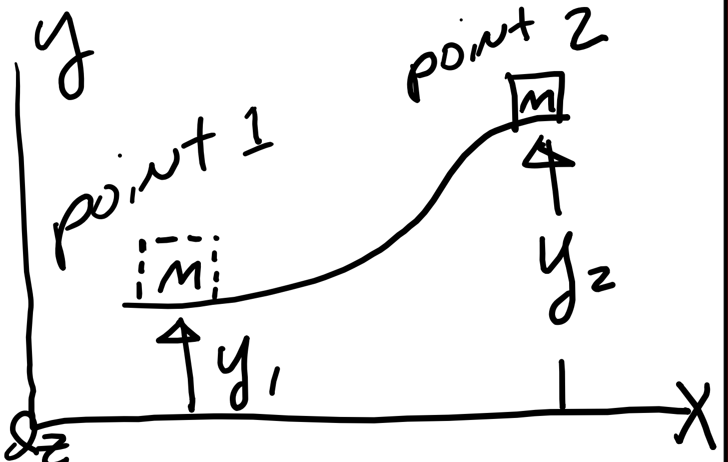
Perhaps a bit more natural to go from point 2 to point 1

Gravity Near earth

We want the worke **due to gravity**



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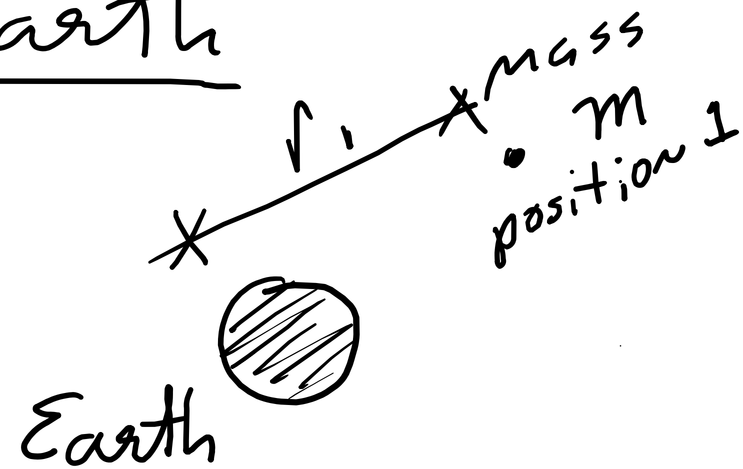
At any point along curve $F_x = 0,\ F_y = -mg,\ F_z = 0$

$$\text{So } U_{1\to 2} = \int_{y_1}^{y_2} F_y\,dy = -mg\int_{y_1}^{y_2} dy = \underbrace{-mg\Delta y}_{\text{decrease in kinetic energy}}$$

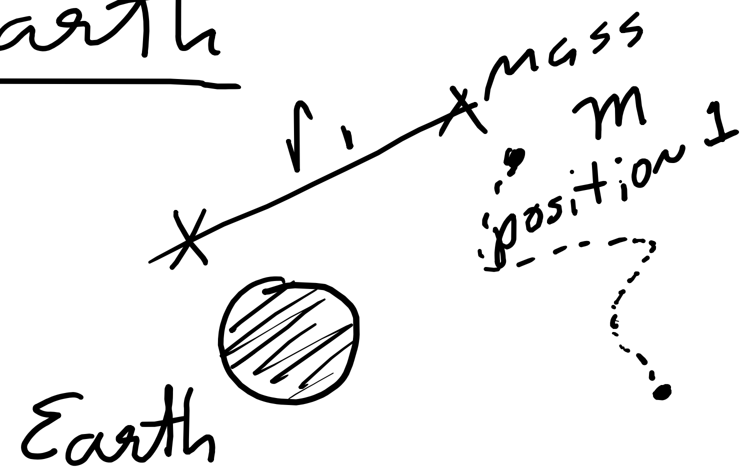
Perhaps a bit more natural to go from point 2 to point 1. In that case

$U_{2\to 1} = +mg\Delta y \Rightarrow$ Increase in Kinetic energy

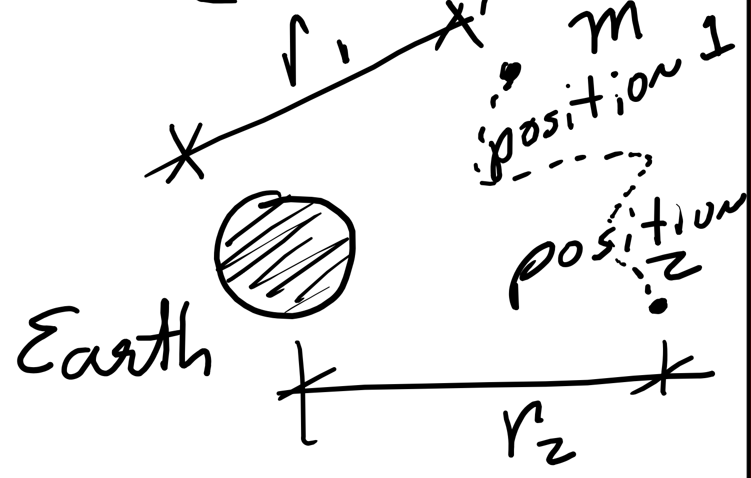
Far from earth



Far from earth

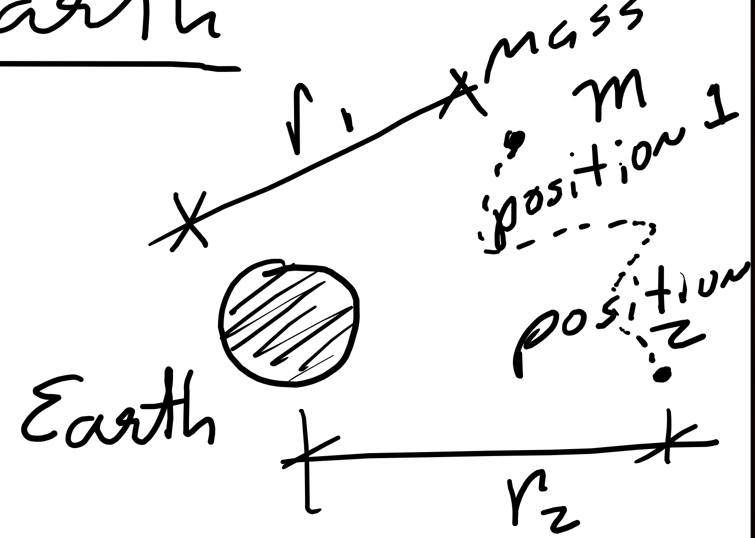


Far from earth



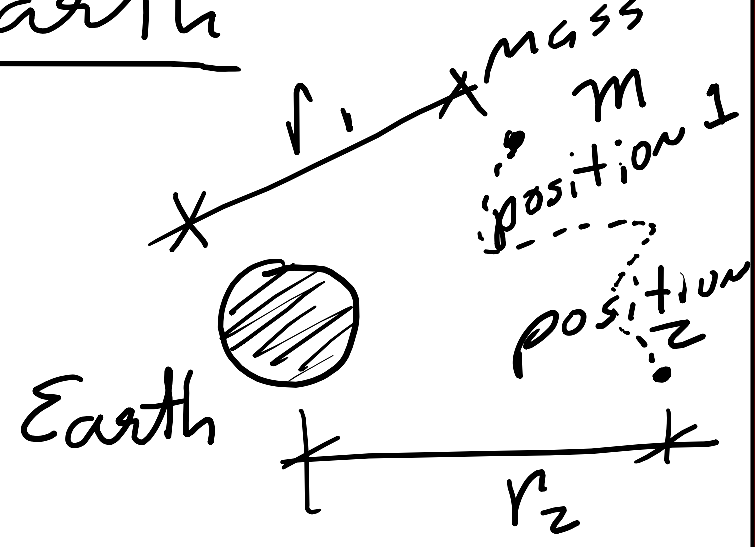
Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$



Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_{r_1}^{r_2} F_r \, dr$$

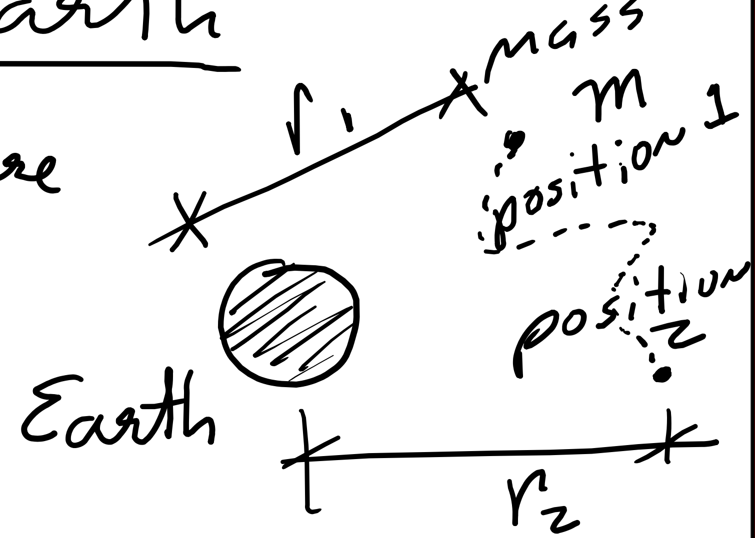


Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_{r_1}^{r_2} F_r\, dr, \text{ where}$$

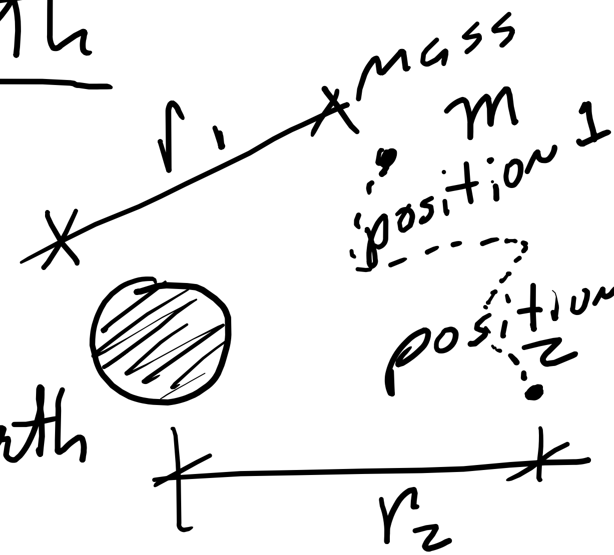
$$F_r = -G\frac{M_E m}{r^2} \Rightarrow$$

$$U_{1\to 2} = -GM_E m\int_{r_1}^{r_2}\frac{dr}{r^2}$$



Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_{r_1}^{r_2} F_r\, dr, \text{ where}$$



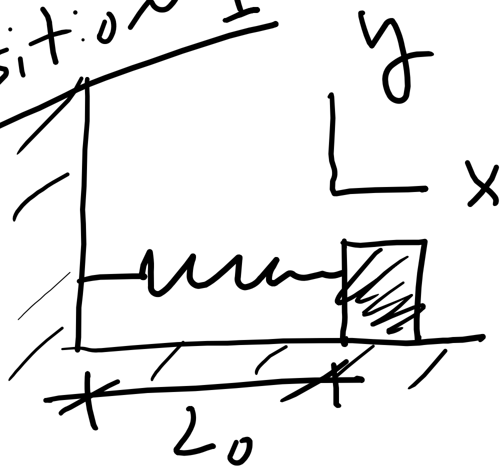
$$F_r = -G\frac{M_E m}{r^2} \Rightarrow$$

$$U_{1\to 2} = -GM_E m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$\Rightarrow \quad U_{1\to 2} = (GM_E m)\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

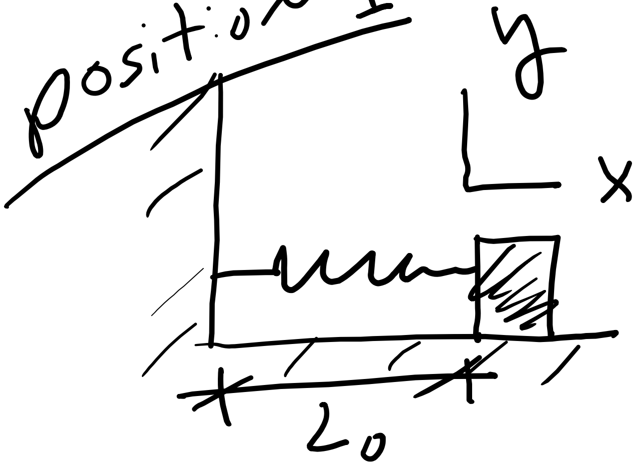
Spring

position 1

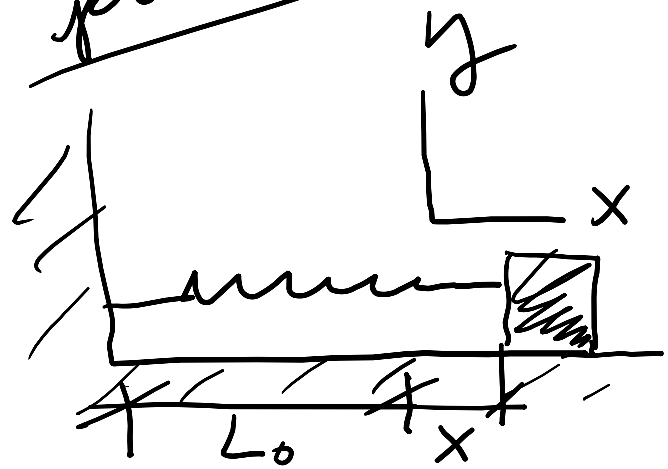


Spring

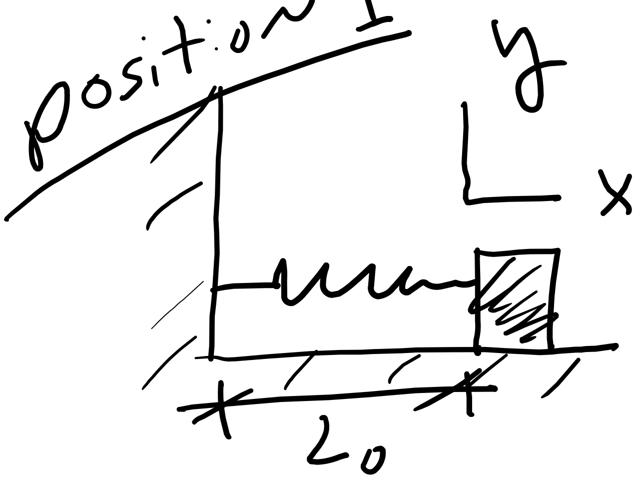
position 1

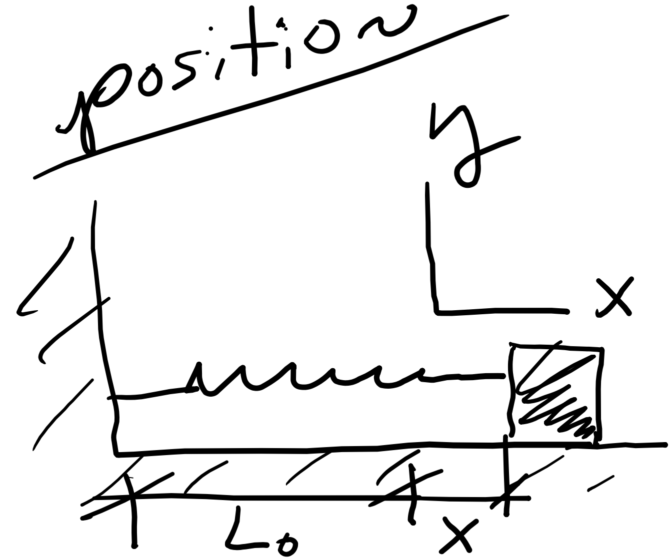


position



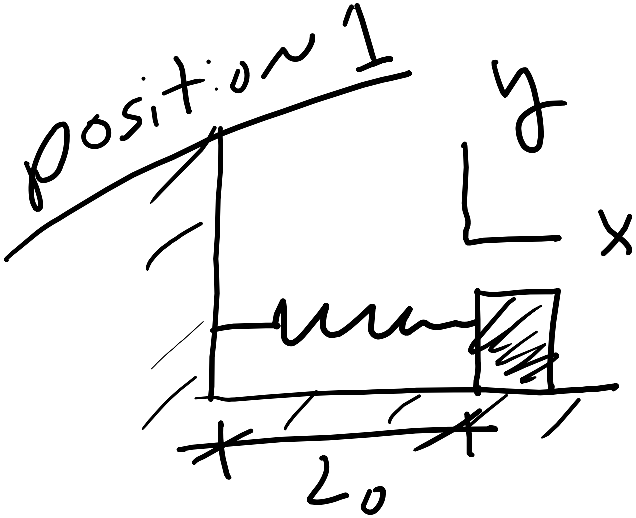
Spring

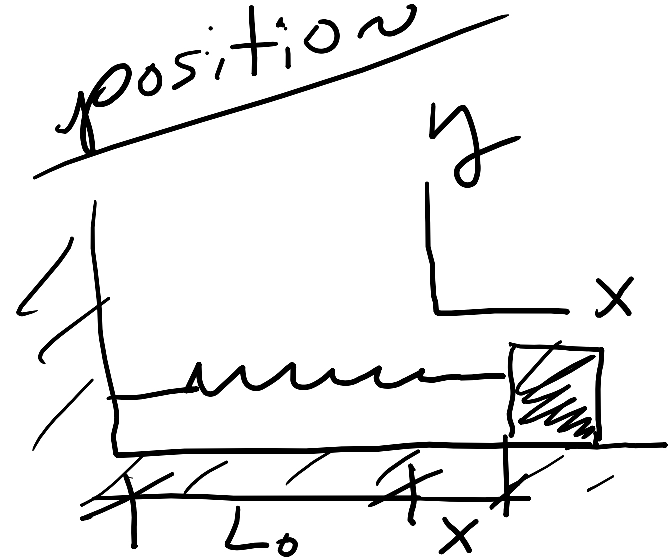




Here $\vec{F} = -kx\hat{i}$

Spring

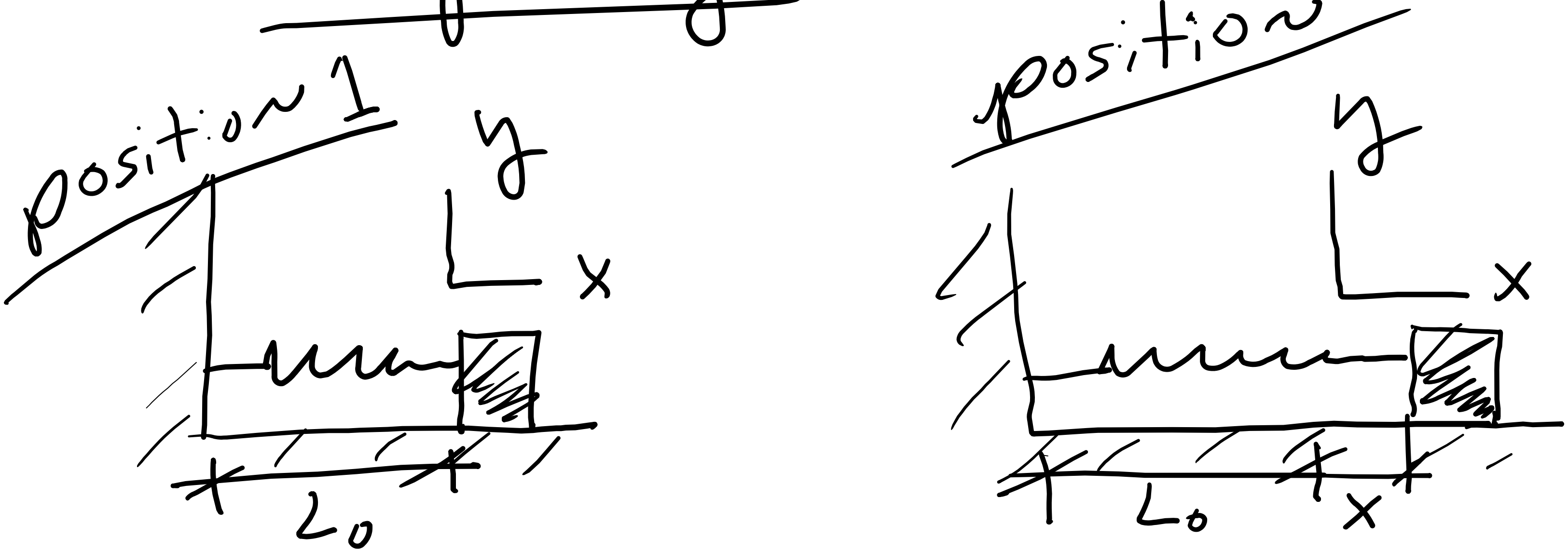




Here $\vec{F} = -kx\hat{i} \Rightarrow$

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

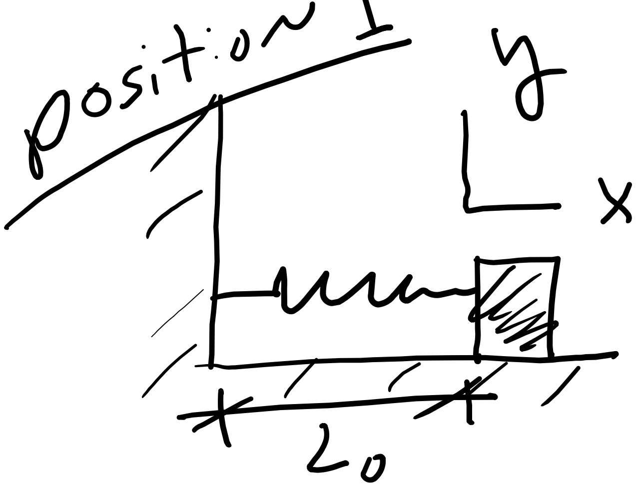
Spring

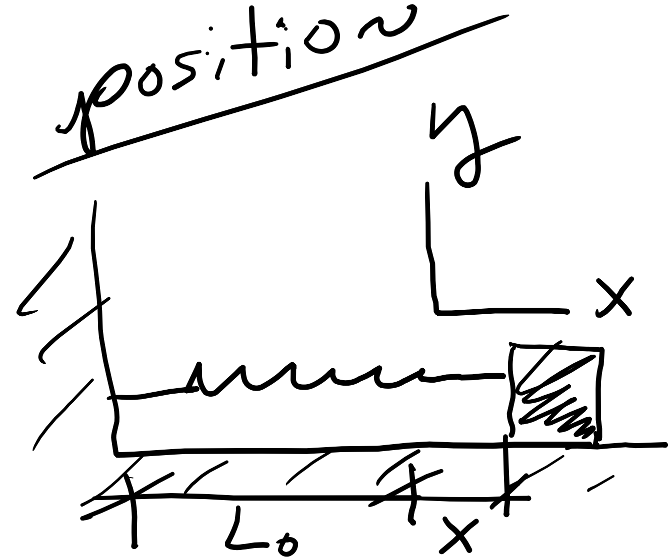


Here $\vec{F} = -kx\hat{i} \Rightarrow$

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\, dx$$

Spring

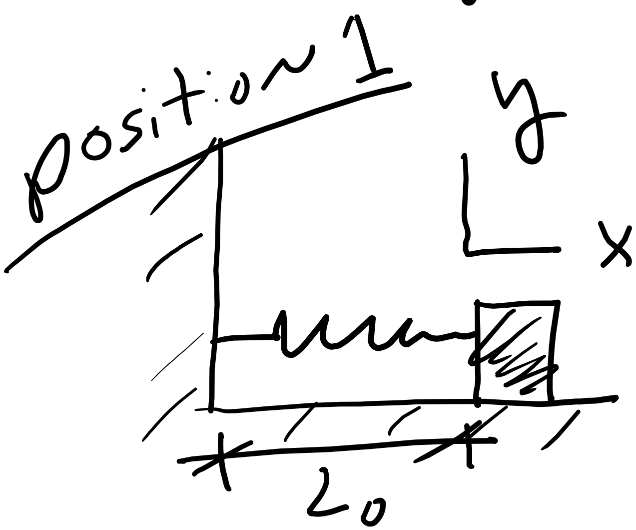


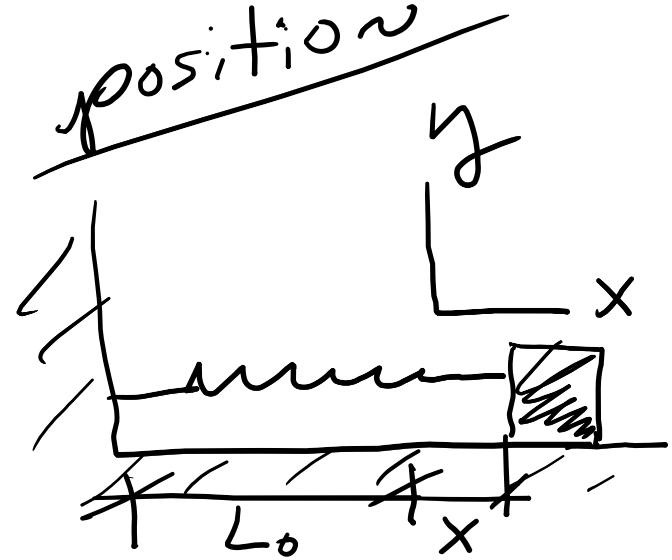


Here $\vec{F} = -kx\hat{i} \Rightarrow$

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\, dx = -k\int_{x_1}^{x_2} x\, dx$$

Spring





Here $\vec{F} = -kx\hat{i} \Rightarrow$

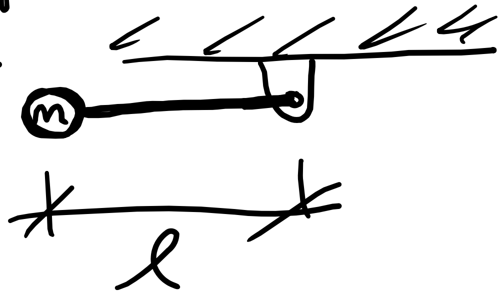
$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\, dx = -k\int_{x_1}^{x_2} x\, dx \Rightarrow$$

$$U_{1\to 2} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

Pendulum problem

position 1

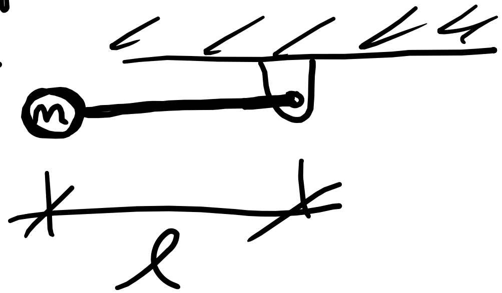
$V_1 = 0$



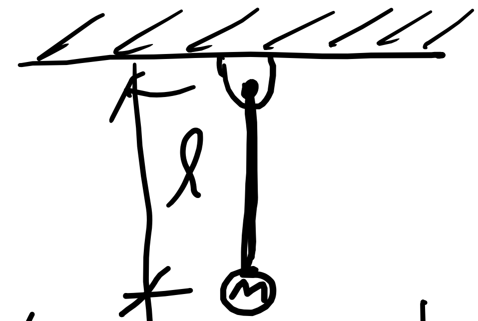
Pendulum problem

position 1

$V_1 = 0$



position 2

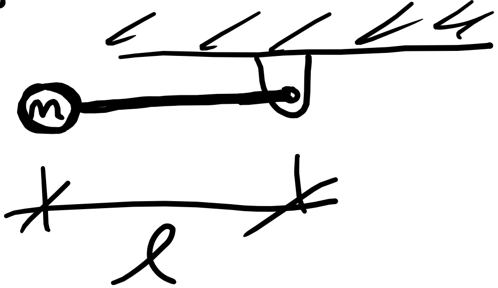


Assume rod has no mass & system has no friction

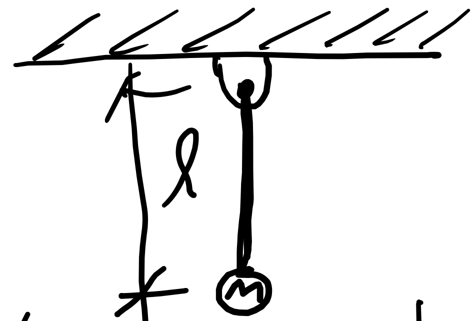
Pendulum problem

position 1

$V_1 = 0$



position 2

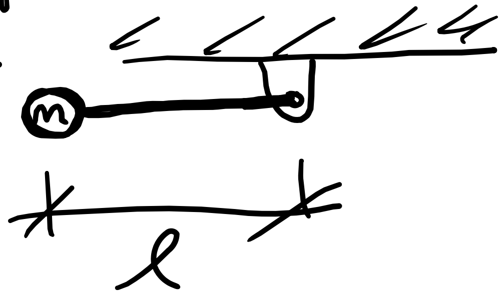


Assume rod has no mass & system has no friction. Find speed of mass at position 2:

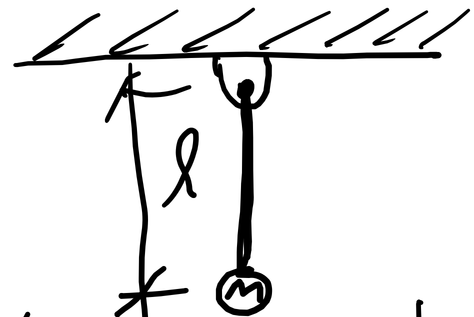
Pendulum problem

position 1

$V_1 = 0$



position 2

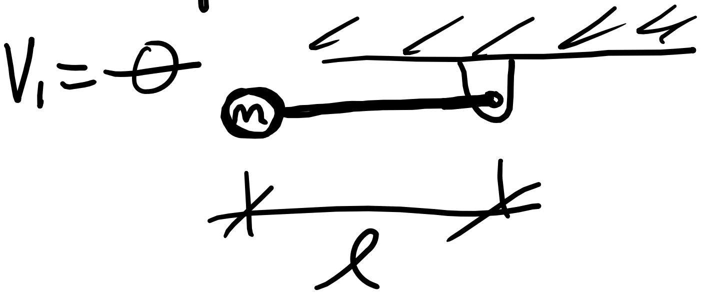


Assume rod has no mass & system has no friction. Find speed of mass at position 2:

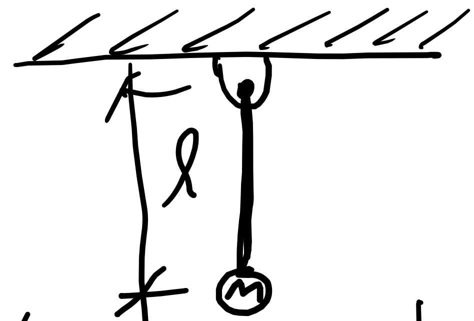
$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

Pendulum problem

position 1



position 2



Assume rod has no mass & system has no friction. Find speed of mass at position 2:

$$U_{1\to 2} = \int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dx$$

Pendulum problem

position 1

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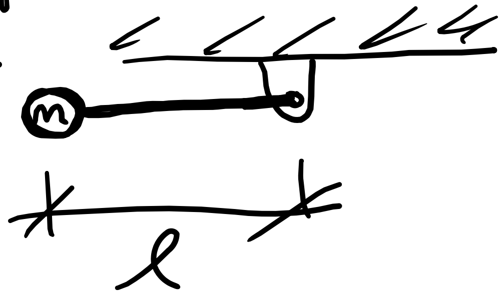
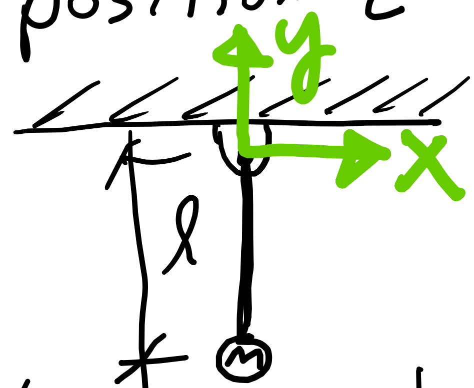


position 2



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Pendulum problem

position 1

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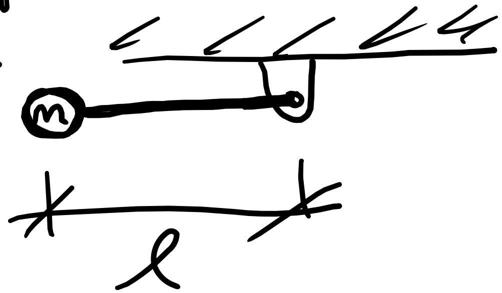
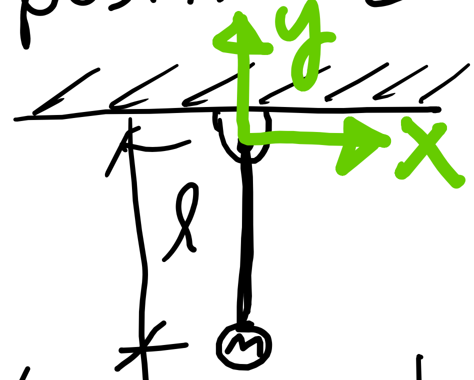


position 2



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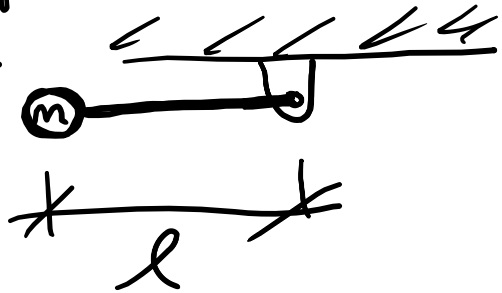
Here $F_x = 0$ & $F_y = -mg$ & $y_1 = 0$ & $y_2 = -l$

so $U_{1\to 2} = -mg\int_0^{-l} dy$

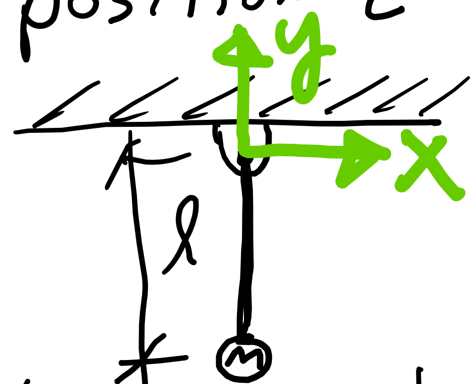
Pendulum problem

position 1

$V_1 = 0$



position 2



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So $U_{1\to 2} = -mg\int_0^{-\ell} dy = (-mg)(-\ell) = mg\ell$

Pendulum problem

position 1

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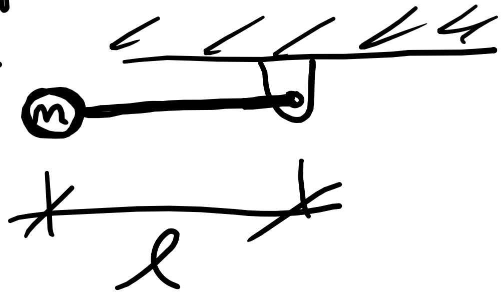
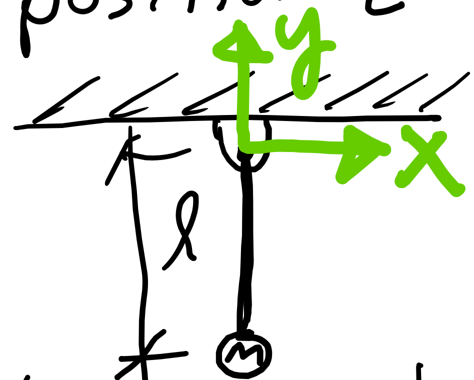


position 2



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Here $F_x = 0$ & $F_y = -mg$ & $y_1 = 0$ & $y_2 = -l$

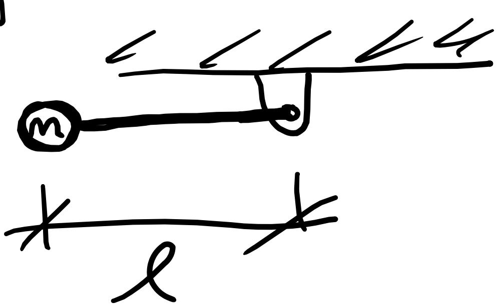
so $U_{1\to2} = -mg \int_0^{-l} dy = (-mg)(-l) = mgl$ &

since $U_{1\to2} = \Delta T$

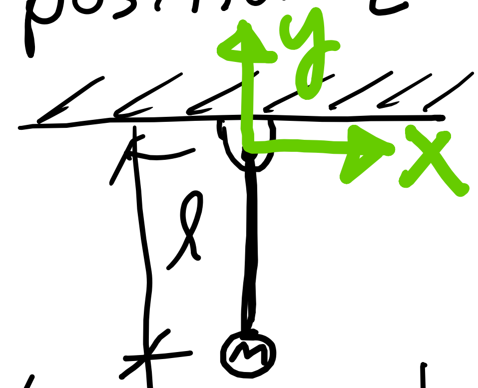
Pendulum problem

position 1

$V_1 = 0$



position 2



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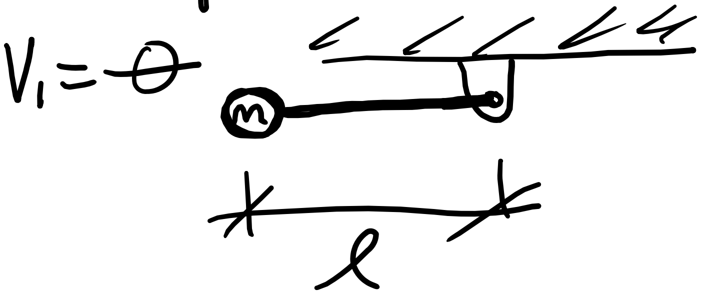
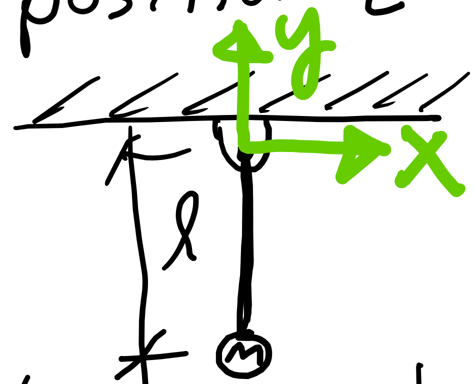
since $U_{1\to 2} = \Delta T$ then $mg\ell = T_2$

Pendulum problem

position 1



position 2



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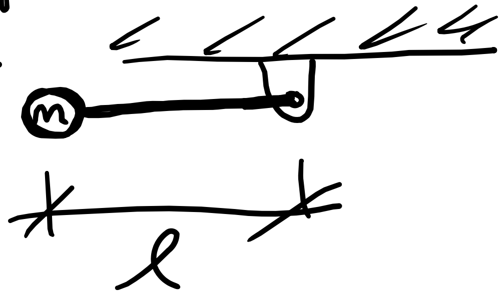
so $U_{1\to2} = -mg\int_0^{-\ell} dy = (-mg)(-\ell) = mg\ell$ &

since $U_{1\to2} = \Delta T$ then $mg\ell = T_2$ But $T_2 = \frac{1}{2} m V_2^2$

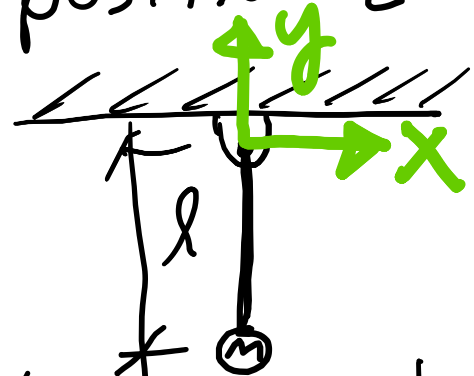
Pendulum problem

position 1

$V_1 = 0$



position 2



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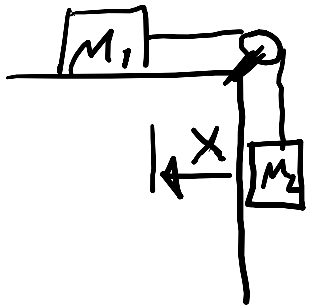
so $U_{1\to 2} = -mg \int_0^{-l} dy = (-mg)(-l) = mgl$ &

since $U_{1\to 2} = \Delta T$ then $mgl = T_2$ But $T_2 = \frac{1}{2} m V_2^2$

so $\frac{1}{2} m V_2^2 = mgl \Rightarrow V_2 = \sqrt{2gl}$

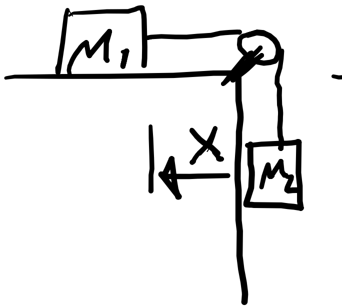
<u>Pulley problem</u>: system starts at rest. Find v_F after



if m_2 moves a distance L.

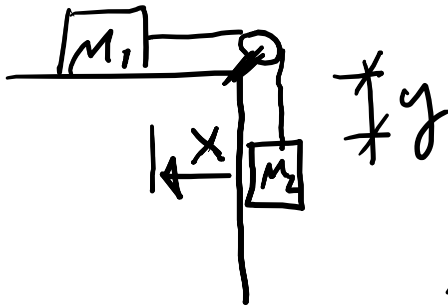
Assume no friction

Pulley problem: system starts at rest. Find v_F after m_2 moves a distance L. Assume no friction



$x + y = \text{const} \Rightarrow \Delta y = -\Delta x = L$

Pulley problem: system starts at rest. Find v_F after m_2 moves a distance L.



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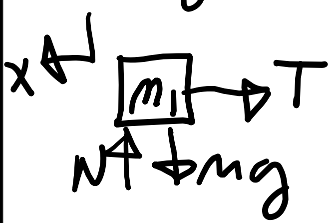

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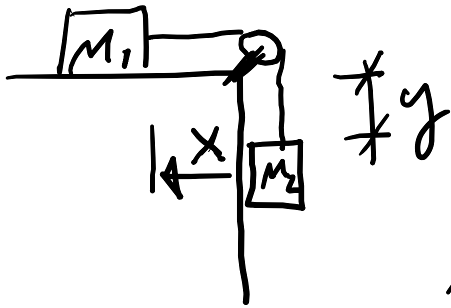

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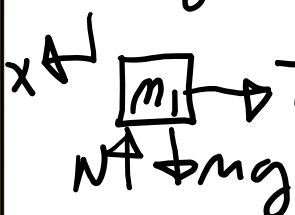

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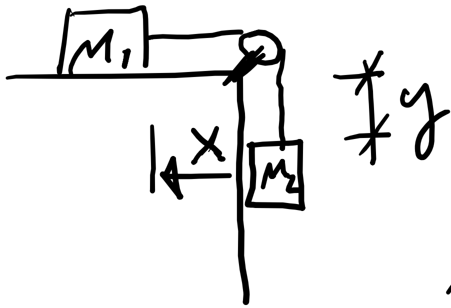

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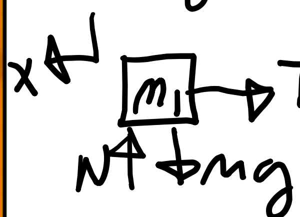

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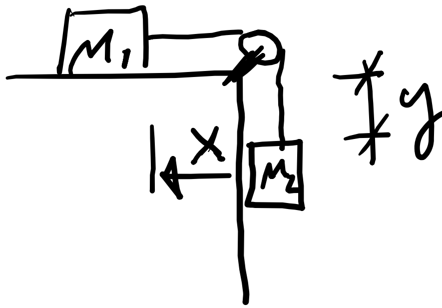

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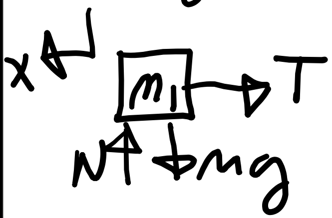

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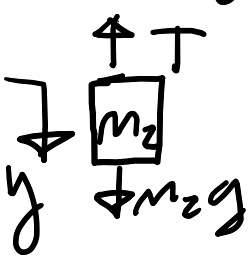

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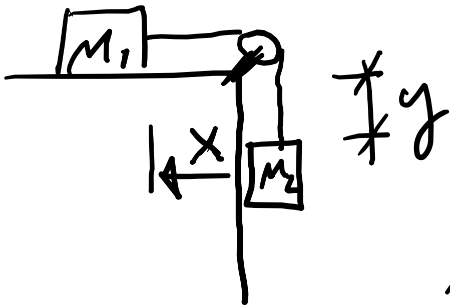

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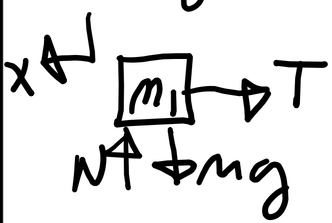

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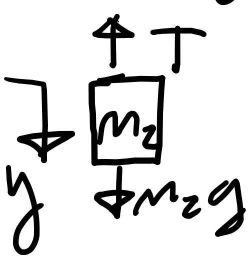

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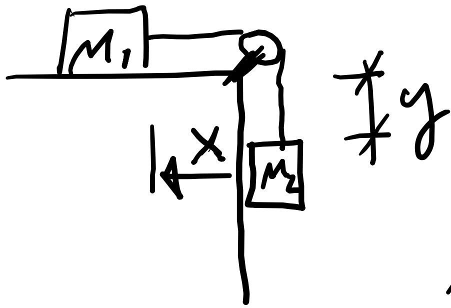

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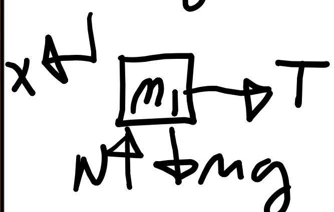

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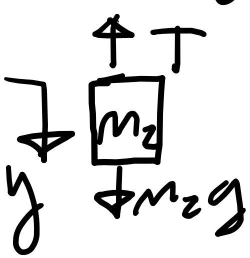

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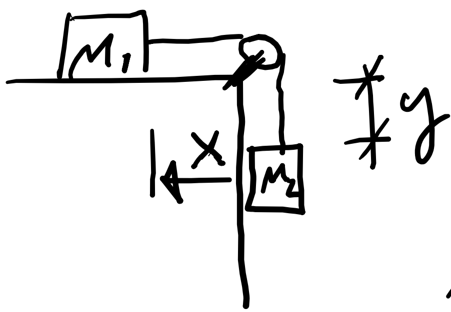

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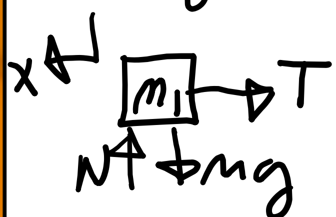

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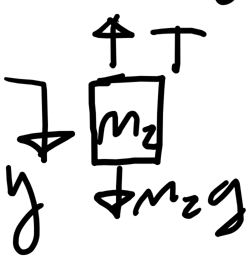

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Now $U_{1\to 2} = \int \sum \vec{F} \cdot d\vec{r} = \int F_x \, dx + \int F_y \, dy$

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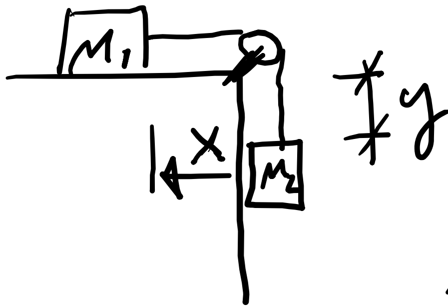

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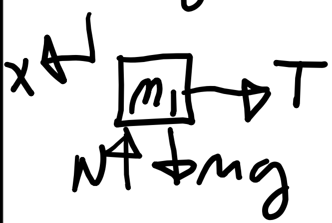

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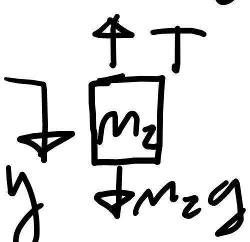

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Now $U_{1\to 2} = \int \sum \vec{F}\cdot d\vec{r} = \int F_x\,dx + \int F_y\,dy$

$$\Rightarrow U_{1\to 2} = -T\int_{x_1}^{x_2} dx + (m_2 g - T)\int_{y_1}^{y_2} dy$$

3d case

$\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{F}\cdot d\vec{r} = m\vec{a}\cdot d\vec{r}$, But

$d\vec{r} = \frac{d\vec{r}}{dt}dt = \vec{v}\,dt$ & $\vec{a} = \frac{d\vec{v}}{dt}$

so $\sum \vec{F}\cdot d\vec{r} = m\vec{v}\cdot\left(\frac{d\vec{v}}{dt}\right)dt$ & since

$d\vec{v} = \left(\frac{d\vec{v}}{dt}\right)dt$, then $\int \sum \vec{F}\cdot d\vec{r} = \int m\vec{v}\cdot d\vec{v}$

$= m\left\{\int v_x dv_x + \int v_y dv_y + \int v_z dv_z\right\} \Rightarrow \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \frac{m}{2}\left(v_x^2 + v_y^2 + v_z^2\right)\Big|_{v_I}^{v_F}$

$= \left(\frac{m}{2}\right)\left\{v_F^2 - v_I^2\right\} \Rightarrow \sum \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = T_2 - T_1$ or

$U_{1\to 2} = \Delta T$ with $U_{1\to 2} \equiv \sum \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$