Today 13.2

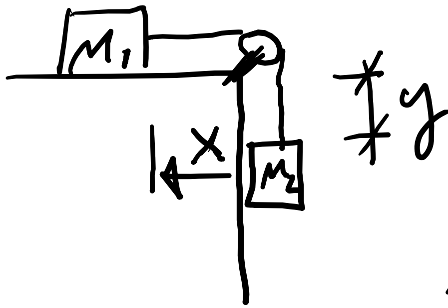
$\boxed{L11}$

$\boxed{L11}$

Today 13.2

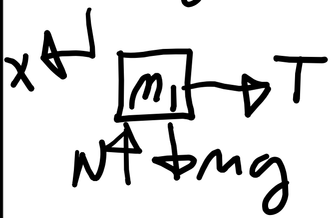
Monday 13.3

Pulley problem: system starts at rest. Find v_F after m_2 moves a distance L.

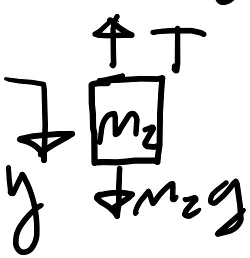


Assume no friction

$$x+y=\text{const} \Rightarrow \Delta y=-\Delta x = L$$



so $\sum F_x = m_1 a_x \Rightarrow -T = m_1 a_x$



so $\sum F_y = m_2 a_y \Rightarrow m_2 g - T = m_2 a_y$

Now $U_{1\to 2} = \int \sum \vec{F}\cdot d\vec{r} = \int F_x\,dx + \int F_y\,dy$

$$\Rightarrow U_{1\to 2} = -T\int_{x_1}^{x_2} dx + (m_2 g - T)\int_{y_1}^{y_2} dy$$

From previous slide $U_{1+2} = -T\int_{x_1}^{x_2} dx + (m_2 g - T)\int_{y_1}^{y_2} dy$

From previous slide $U_{1\to 2} = -T\int_{x_1}^{x_2} dx + (m_2 g - T)\int_{y_1}^{y_2} dy$

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ASU

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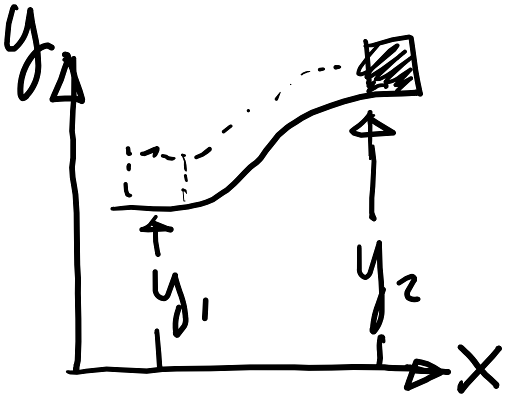
so $v_F = \sqrt{\dfrac{2m_2 gL}{m_1 + m_2}}$

Earlier we saw that the work **due to gravity** along some curved path is



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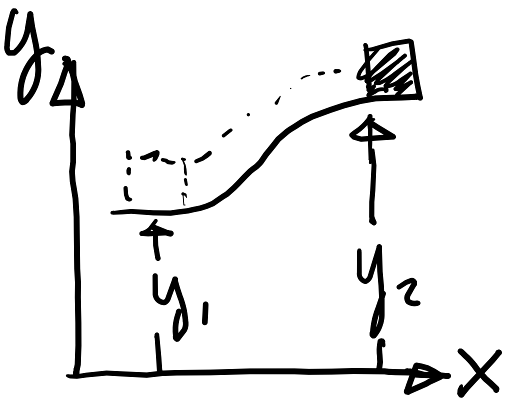

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{y_1}^{y_2} F_y\, dy = -mg\Delta y$$

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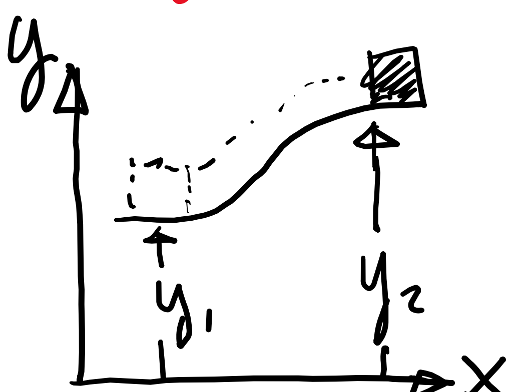

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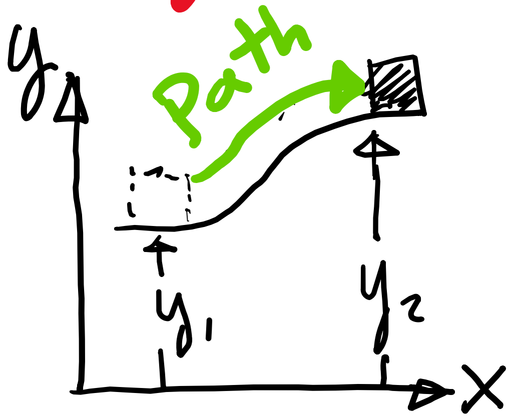

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For our problem, we need to integrate the sum of forces over the path shown.

Take an arbitrary position along the path.

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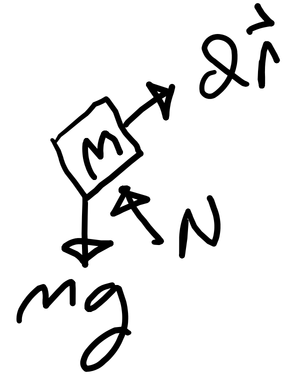

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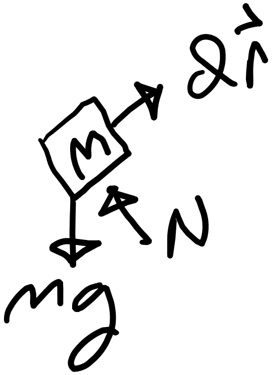

, where I have included the $d\vec{r}$ element in the diagram.

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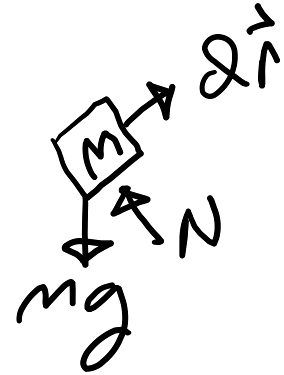

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$$\sum \vec{F}\cdot d\vec{r} = mg(-\hat{j})\cdot d\vec{r} + \vec{N}\cdot d\vec{r}$$

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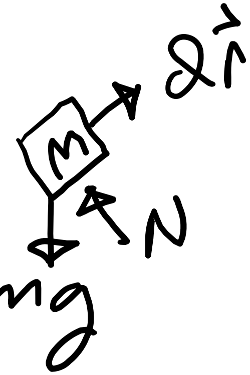

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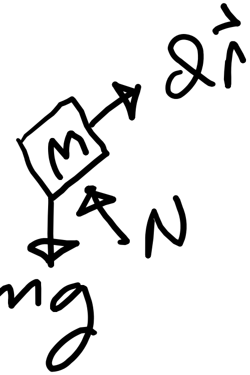

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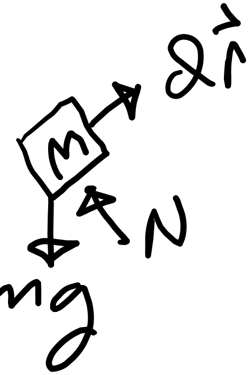

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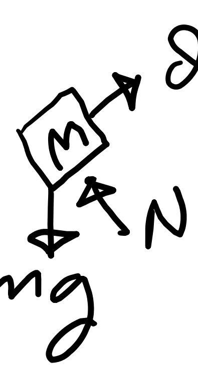

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Now we have $\sum \vec{F} \cdot d\vec{r} = mg(-\hat{j}) \cdot d\vec{r}$ & since $d\vec{r} = \hat{\imath}\,dx + \hat{j}\,dy$, we get

$$\sum \vec{F} \cdot d\vec{r} = -mg\,dy$$

Since $\sum \vec{F} \cdot d\vec{r} = -mg\,dy$, then

$$U_{1\to 2} = -mg\Delta y.$$

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The force perpindicular to the motion did not matter & the actual path taken was not important.

We call such a force, where the work done by that force is independent of the path, a

Conservative Force!!

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* Mostly friction

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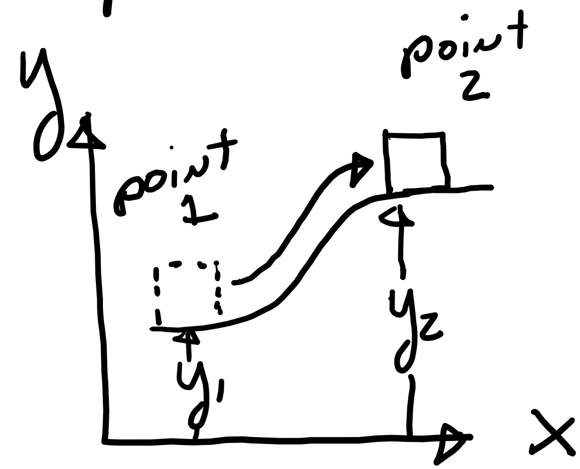
Non-conservative forces?

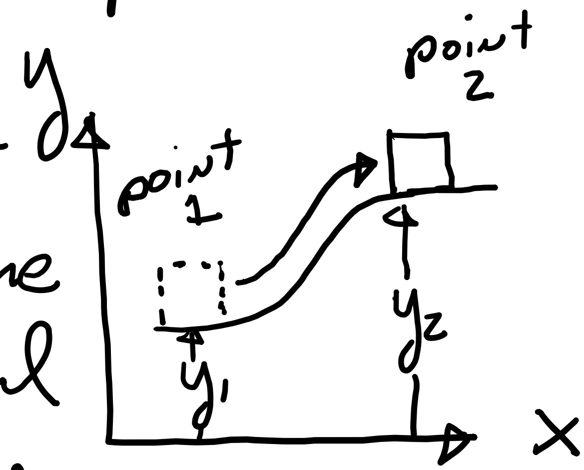
* Mostly friction
* Anything that generates heat

Going back to our box problem:



Going back to our box problem:

This time Bob pushes the box up the hill such that the initial kinetic energy = final kinetic energy = zero {at rest at points 1 & 2}.



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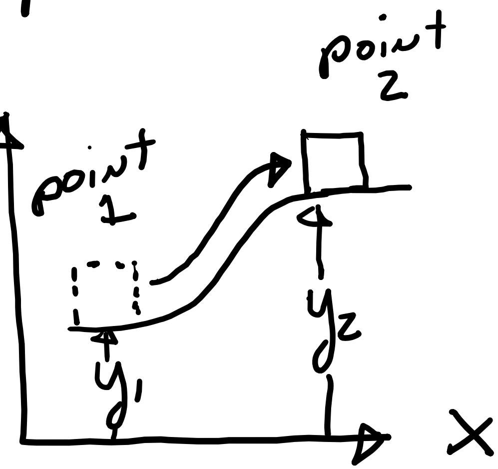

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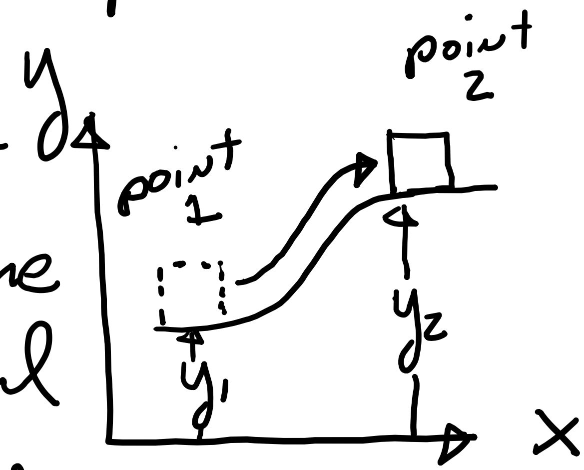

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<u>A new point of view:</u>

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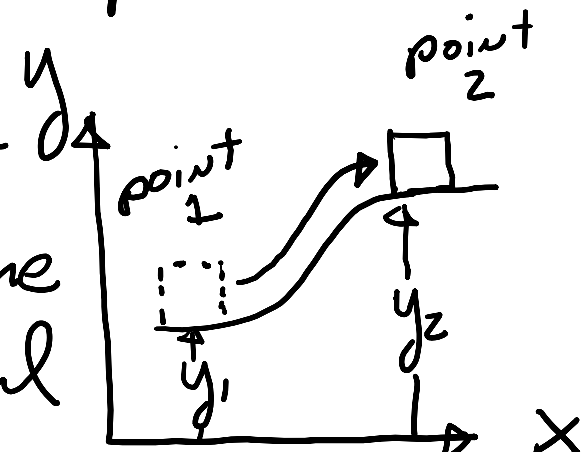

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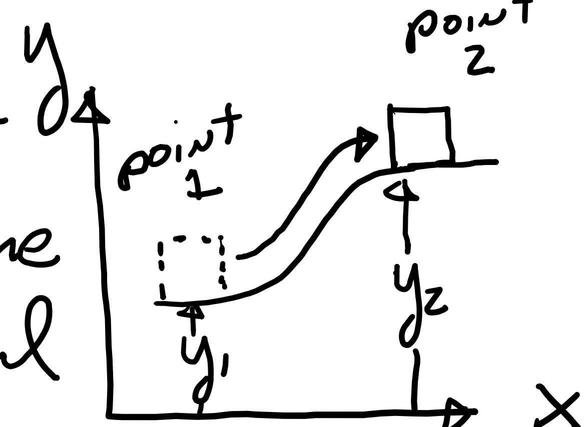

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<u>A new point of view</u>: Let's say that Bob put energy into the box. At point 2, the box has gained "Potential Energy"

Let $\begin{pmatrix}\text{potential energy} \\ \text{at point } i\end{pmatrix} \equiv V_i$

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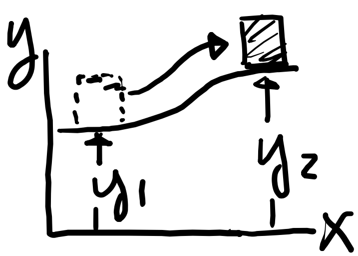
__Please notice__: The __difference__ in potential energy is relatable to work put in.



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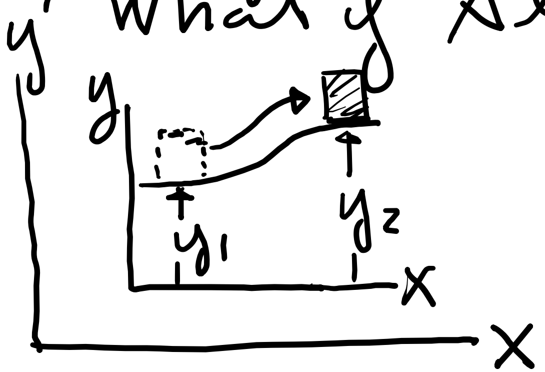
What if Alice had a coordinate system $x'y'$



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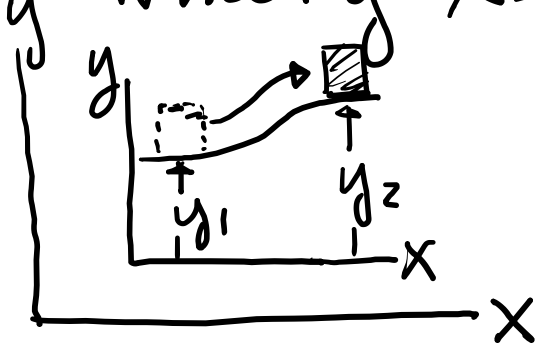

We, with coordinate system x, y could say $V_1 = mgy_1$ & $V_2 = mgy_2$

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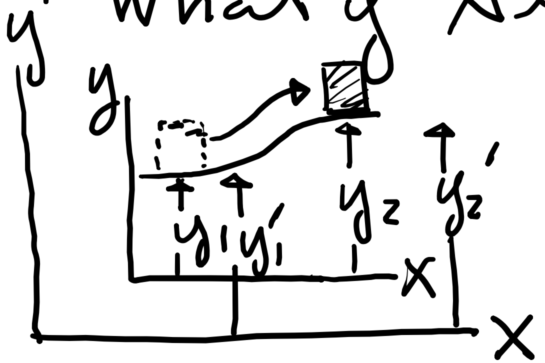

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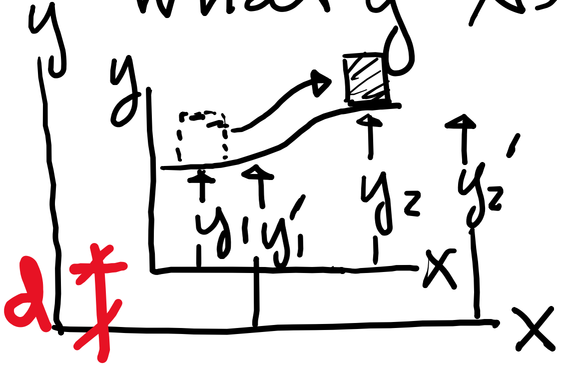

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<u>But</u> $y_1' = y_1 + d$ & $y_2' = y_2 + d$

So $U_{1\to 2}^{Bob} = mg(y_2' - y_1') = mg(y_2 - y_1)$

So $U_{1\to 2 \atop \text{Bob}} = mg(y_2' - y_1') = mg(y_2 - y_1)$

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& we agree on the <u>difference</u> in potential energy but <u>NOT</u> on the value at a given point.

So $\underset{\text{Bob}}{U_{1\to 2}} = mg(y_2' - y_1') = mg(y_2 - y_1)$

& we agree on the difference in potential energy but NOT on the value at a given point.

You can always add a constant to potential energy as long as it is the same constant for all points.

Potential energy:

Gravity : $V_g = mgy$, near earth

Potential energy:

Gravity : $\begin{cases} V_g = mgy \text{, Near earth} \\ V_g = -\dfrac{GmM_E}{r} \text{, far from earth} \end{cases}$

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Spring [elastic] : $V_e = \frac{1}{2}kx^2$, where x is displacement from equilibrium

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For any conservative forces present,

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Conservation of energy

For any conservative forces present, then

$$V_1 + T_1 + U_{1\to 2}^{NC} = V_2 + T_2$$

Potential energy:

Gravity : $\begin{cases} V_g = mgy & \text{, near earth} \\ V_g = -\dfrac{GmM_E}{r} & \text{, far from earth} \end{cases}$

Spring [elastic] : $V_e = \frac{1}{2}kx^2$, where x is displacement from equilibrium

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Conservation of energy

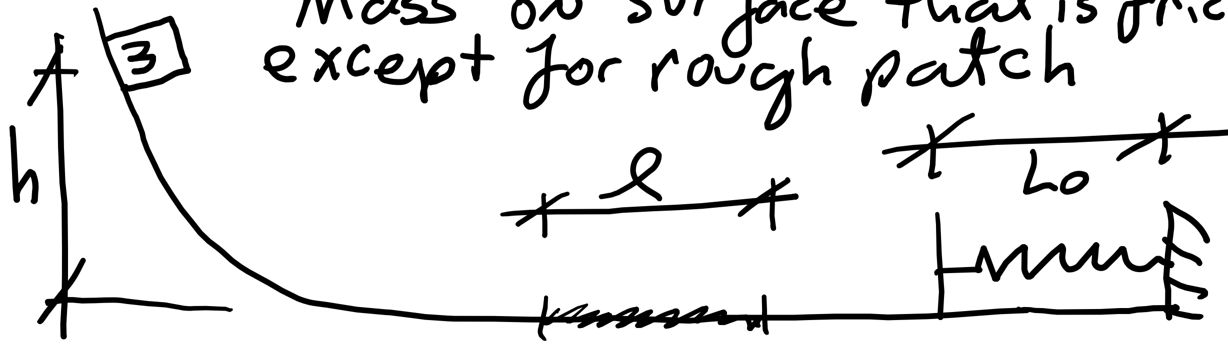
If any conservative forces present, then

$$V_1 + T_1 + U_{1\to2}^{NC} = V_2 + T_2$$

, where $U_{1\to2}^{NC}$ is the work due to non-conservative forces

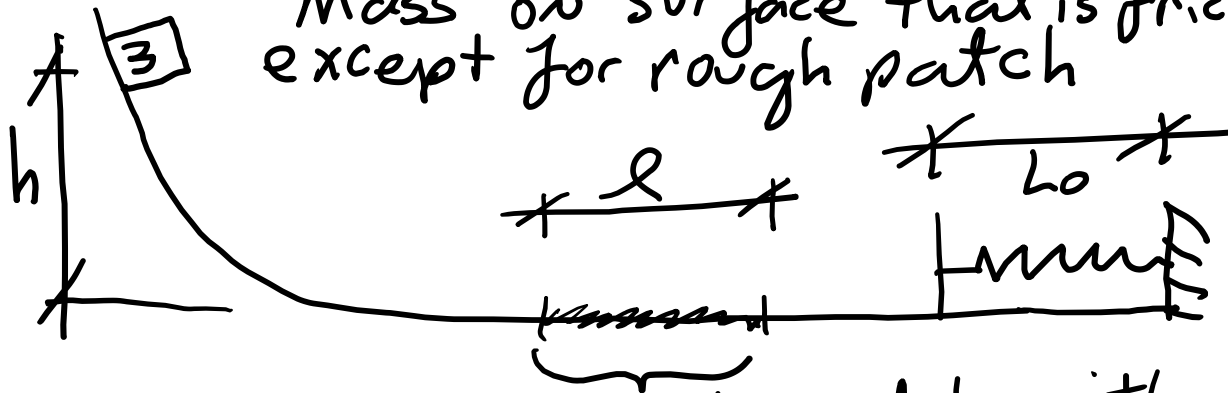
Example:

Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

Example: Mass on surface that is frictionless except for rough patch

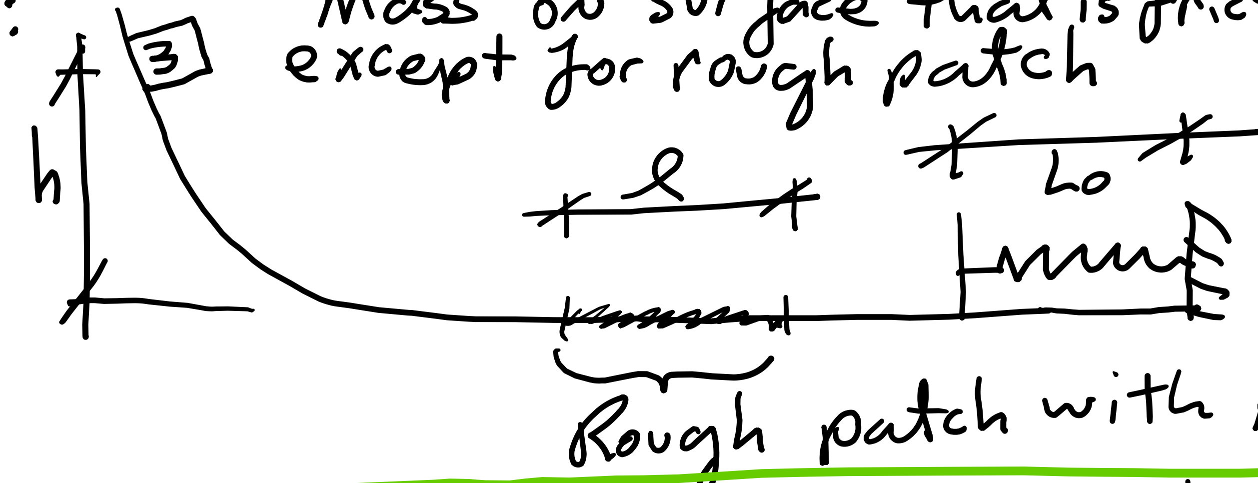


Rough patch with $\mu_k \neq 0$

Mass initially at rest, then let go. How far can spring be compressed? x

Example:

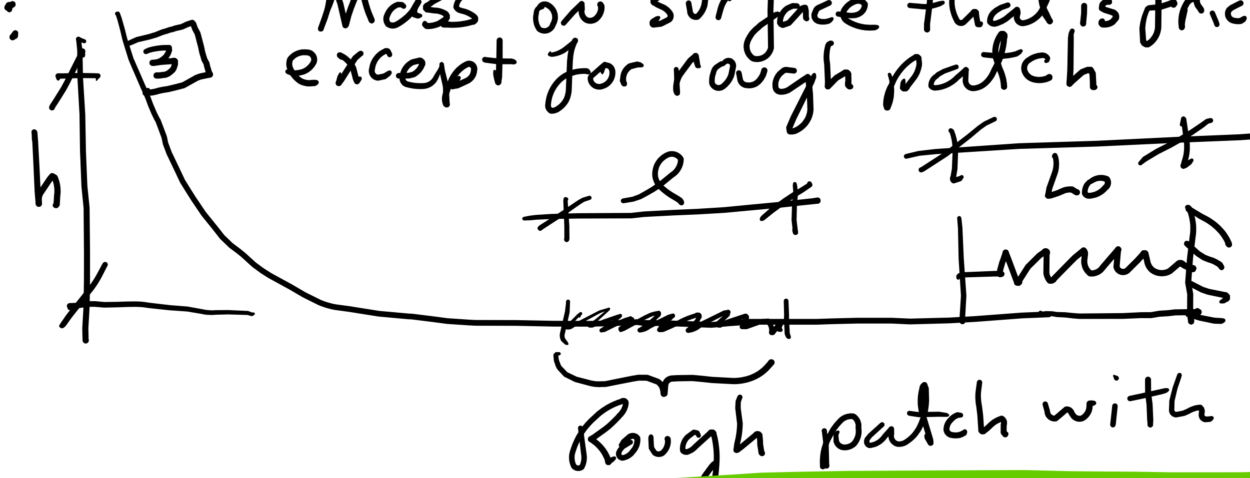
Mass on surface that is frictionless except for rough patch



Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U^{NC}_{1\to 2} = V_2 + T_2$$

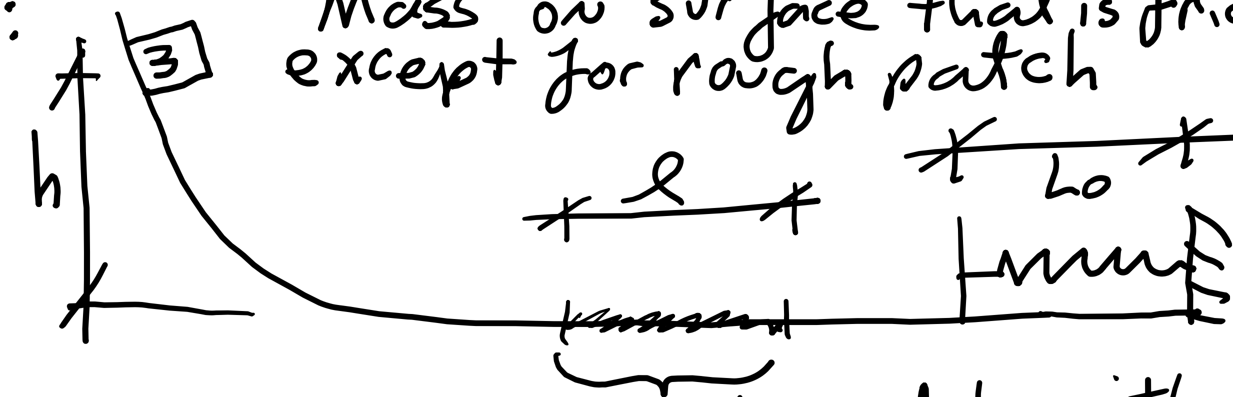
Example: Mass on surface that is frictionless except for rough patch



Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U_{1\to 2}^{NC} = V_2 + T_2 \quad , \text{Here } T_1 = T_2 = 0$$

Example: Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

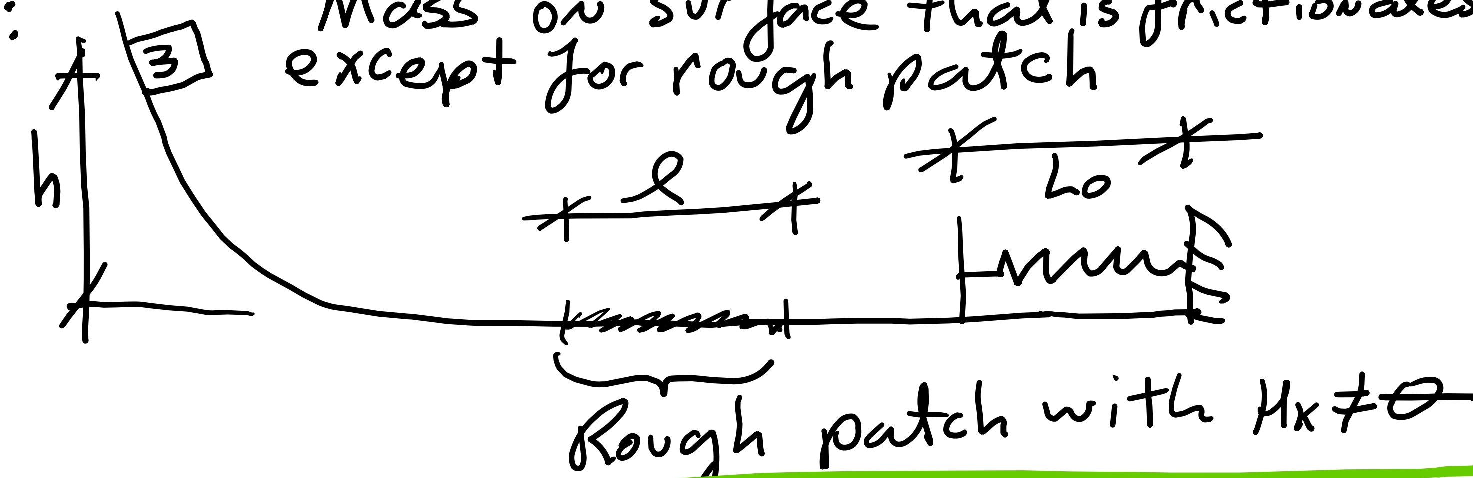
Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U^{NC}_{1\to2} = V_2 + T_2$$, Here $T_1 = T_2 = 0$

& $V_1 = mgh$

Example:

Mass on surface that is frictionless except for rough patch

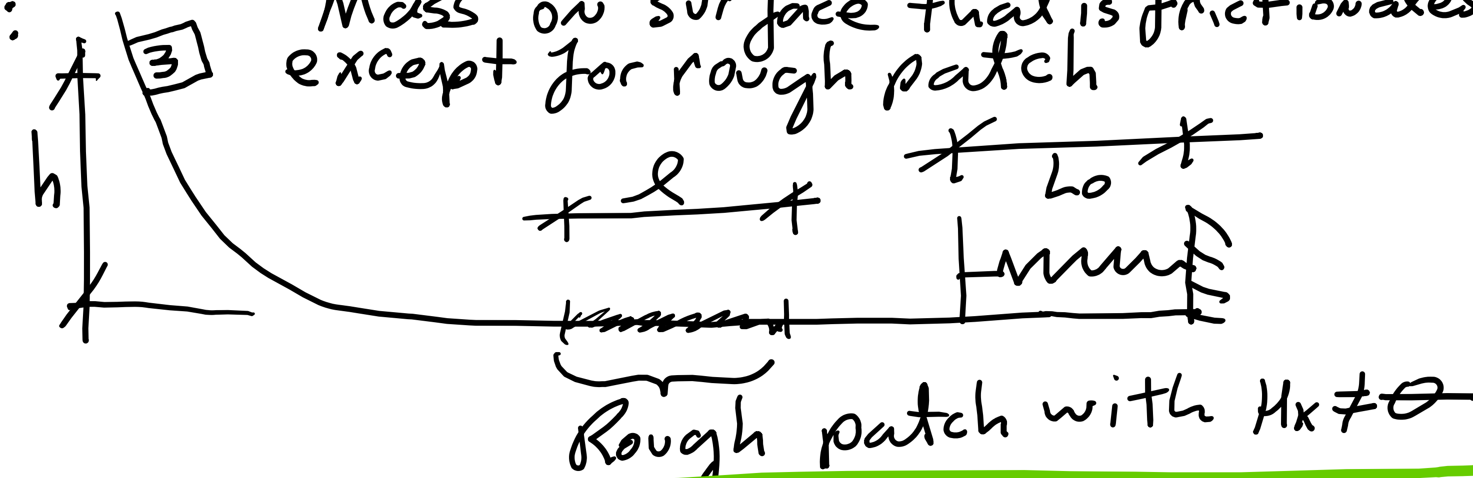


Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U_{1\to2}^{NC} = V_2 + T_2 \quad , \text{Here } T_1 = T_2 = 0$$

$$\& \; V_1 = mgh \;\; \& \; V_2 = \tfrac{1}{2}kx^2$$

Example: Mass on surface that is frictionless except for rough patch



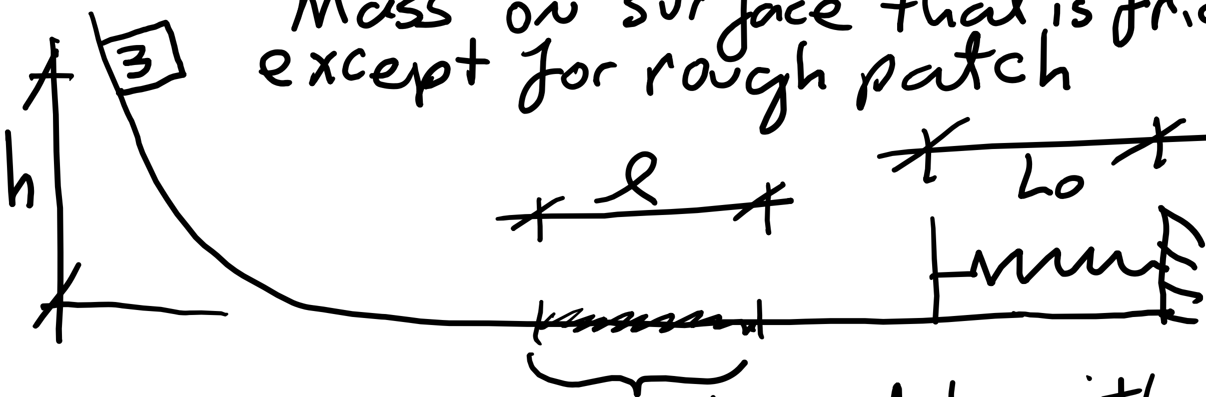
Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U^{NC}_{1\to 2} = V_2 + T_2 \quad , \text{ Here } T_1 = T_2 = 0$$

$$\& \ V_1 = mgh \ \& \ V_2 = \tfrac{1}{2}kx^2 \ \& \ U^{NC}_{1\to 2} = [-mg\mu_k]$$

Example:

Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

Mass initially at rest, then let go. How far can spring be compressed? x

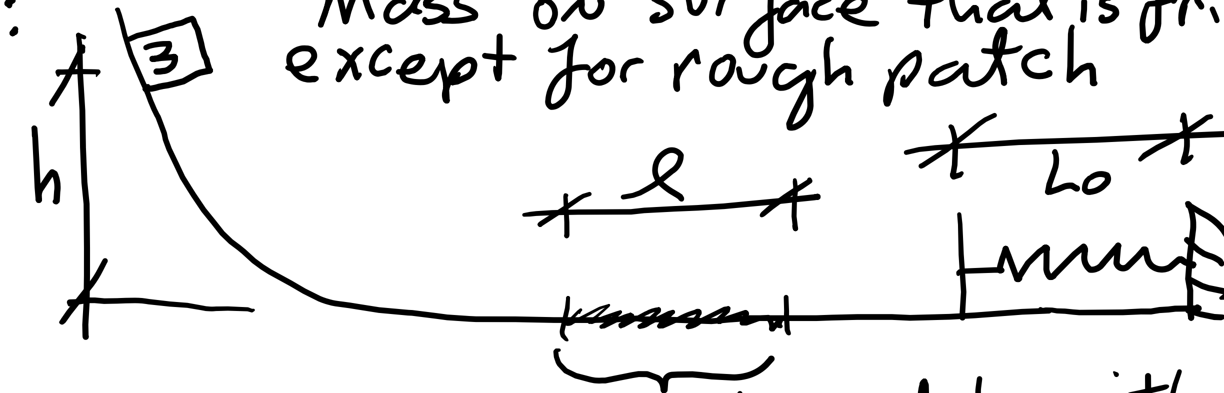
$$V_1 + T_1 + U^{NC}_{1\to2} = V_2 + T_2$$, Here $T_1 = T_2 = 0$

& $V_1 = mgh$ & $V_2 = \frac{1}{2}kx^2$ & $U^{NC}_{1\to2} = [-mg\mu_k]$ *

Notice the sign!!

Example:

Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

Mass initially at rest, then let go. How far can spring be compressed? x

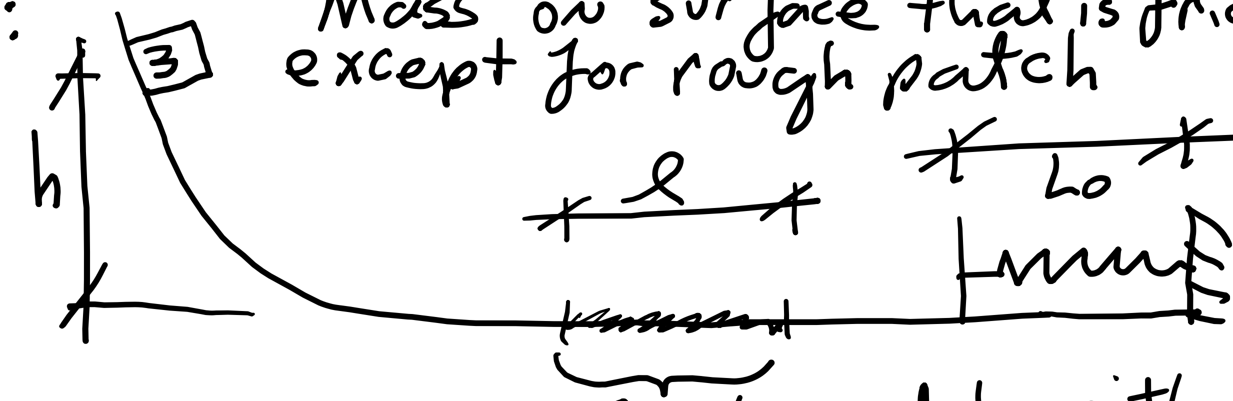
$V_1+T_1+U^{NC}_{1\to2} = V_2+T_2$, Here $T_1=T_2=0$

& $V_1 = mgh$ & $V_2 = \frac{1}{2}kx^2$ & $U^{NC}_{1\to2} = [-mg\mu_k]\ell$

So $mgh - mg\mu_k\ell = \frac{1}{2}kx^2$

Notice the sign!

Example: Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

Mass initially at rest, then let go. How far can spring be compressed? x

$$V_1 + T_1 + U^{nc}_{1\to 2} = V_2 + T_2 \quad , \text{ Here } T_1 = T_2 = 0$$

$$\& \; V_1 = mgh \; \& \; V_2 = \tfrac{1}{2}kx^2 \; \& \; U^{nc}_{1\to 2} = [-mg\mu_k]\ell$$

Notice the sign!

$$\text{So} \quad mgh - mg\mu_k\ell = \tfrac{1}{2}kx^2$$

$$\Rightarrow \quad x = \left[\left(\frac{2mg}{k}\right)(h - \mu_k \ell)\right]^{1/2}$$