Today: 13.3

$\boxed{L12}$

Today: $\underbrace{13.3}$

$\boxed{L12}$

Impulse & momentum

L12

Today: 13.3

Wednesday: $\underbrace{13.4}_{\text{Impacts}}$, 14.1

L12

Today: 13.3

Wednesday: 13.4, $\underbrace{14.1}$

System of particles: Newton's 2nd law

L12

Today: 13.3

Wednesday: 13.4, 14.1

Reminder:

HW#4 Due Wednesday

Impulse & momentum

Impulse & momentum

$\vec{F} = m\vec{a}$

Impulse & momentum

$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \frac{d}{dt}\vec{L}$

Impulse & momentum

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \frac{d}{dt}\vec{L} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{L}_1}^{\vec{L}_2} d\vec{L}$$

Impulse & momentum

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \frac{d}{dt}\vec{L} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{L}_1}^{\vec{L}_2} d\vec{L} = \Delta\vec{L}$$

Impulse & momentum

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \frac{d}{dt}\vec{L} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{L}_1}^{\vec{L}_2} d\vec{L} = \Delta\vec{L}$$

$$\Rightarrow \quad \vec{I}_{\substack{mp \\ 1\to 2}} = \Delta\vec{L}, \text{ where } \vec{I}_{\substack{mp \\ 1\to 2}} \equiv \int_{t_1}^{t_2} \vec{F}\,dt$$

ASU

Impulse & momentum

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \frac{d}{dt}\vec{L} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{L}_1}^{\vec{L}_2} d\vec{L} = \Delta\vec{L}$$

$$\Rightarrow \vec{I}_{\substack{mp \\ 1\to 2}} = \Delta\vec{L}, \text{ where } \vec{I}_{\substack{mp \\ 1\to 2}} \equiv \int_{t_1}^{t_2} \vec{F}\,dt$$

For multiple impulses :

$$\sum \vec{I}_{mp} = \Delta\vec{L}$$

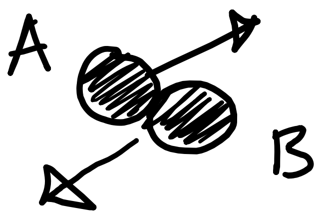
For multiple particles :

$$\sum \vec{I}_{mp} = \sum \Delta\vec{L}$$

Internal impulses

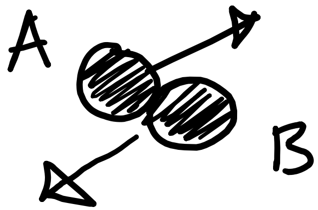
Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Internal impulses

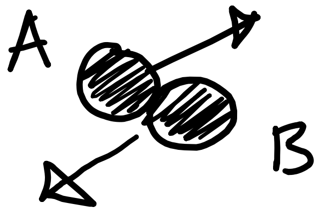
Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact,

Internal impulses

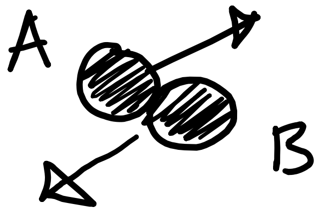
Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$

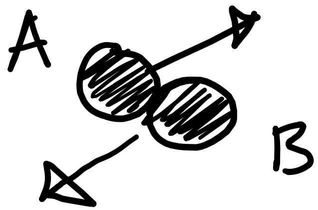
Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A\to B}$ & force of B on A $\equiv \vec{F}_{B\to A}$.

Internal impulses Example: Two particles collide and there is NO external impulses provided.



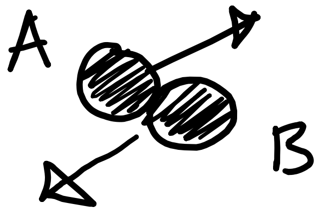
Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A\to B}$ & force of B on A $\equiv \vec{F}_{B\to A}$.

The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$ & force of B on A $\equiv \vec{F}_{B \to A}$.

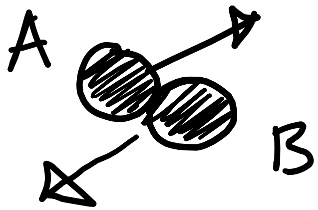
The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\sum \vec{Imp} = \sum \Delta \vec{L}$ we obtain

$$\int \vec{F}_{A \to B} dt + \int \vec{F}_{B \to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$$

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$ & force of B on A $\equiv \vec{F}_{B \to A}$.

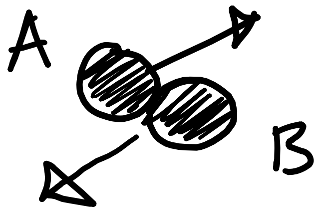
The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$ we obtain

$$\int \vec{F}_{A \to B} dt + \int \vec{F}_{B \to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$$

But $\vec{F}_{A \to B} = -\vec{F}_{B \to A}$

<u>Internal impulses</u> Example: Two particles collide and there is <u>NO</u> external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$ & force of B on A $\equiv \vec{F}_{B \to A}$.

The interaction time can be taken as $\Delta t = t_F - t_I$.

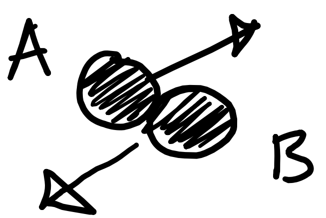
Using our new equation: $\sum \vec{Imp} = \sum \Delta \vec{L}$ we obtain

$$\int \vec{F}_{A \to B} dt + \int \vec{F}_{B \to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$$ <u>But</u> $\vec{F}_{A \to B} = -\vec{F}_{B \to A}$ so

$$0 = \Delta \vec{L}_A + \Delta \vec{L}_B$$

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$ & force of B on A $\equiv \vec{F}_{B \to A}$.

The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$ we obtain

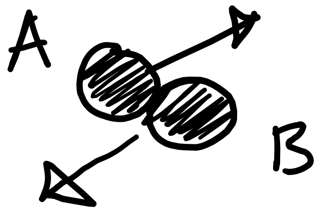
$$\int \vec{F}_{A \to B} dt + \int \vec{F}_{B \to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$$

But $\vec{F}_{A \to B} = -\vec{F}_{B \to A}$ so

$$0 = \Delta \vec{L}_A + \Delta \vec{L}_B \Rightarrow \vec{L}_{AI} + \vec{L}_{BI} = \vec{L}_{AF} + \vec{L}_{BF}$$

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A\to B}$ & force of B on A $\equiv \vec{F}_{B\to A}$.

The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$ we obtain

$$\int \vec{F}_{A\to B}\,dt + \int \vec{F}_{B\to A}\,dt = \Delta\vec{L}_A + \Delta\vec{L}_B$$

But $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$ so

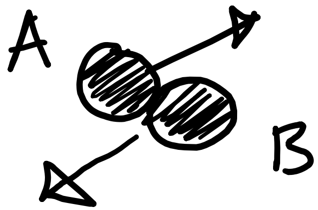
$$0 = \Delta\vec{L}_A + \Delta\vec{L}_B \Rightarrow \vec{L}_{AI} + \vec{L}_{BI} = \vec{L}_{AF} + \vec{L}_{BF}$$

The momentum of $\vec{L}_A$ & $\vec{L}_B$ changed ($\vec{L}_{AI} \neq \vec{L}_{AF}$ & $\vec{L}_{BI} \neq \vec{L}_{BF}$)

ASU

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A\to B}$ & force of B on A $\equiv \vec{F}_{B\to A}$.

The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$ we obtain

$$\int \vec{F}_{A\to B}\, dt + \int \vec{F}_{B\to A}\, dt = \Delta\vec{L}_A + \Delta\vec{L}_B$$

But $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$ so

$$0 = \Delta\vec{L}_A + \Delta\vec{L}_B \Rightarrow \vec{L}_{AI} + \vec{L}_{BI} = \vec{L}_{AF} + \vec{L}_{BF}$$

The momentum of $\vec{L}_A$ & $\vec{L}_B$ changed ($\vec{L}_{AI} \neq \vec{L}_{AF}$ & $\vec{L}_{BI} \neq \vec{L}_{BF}$)

But the sum: $\vec{L}_A + \vec{L}_B = \text{constant}$.

So, no external impulses $\Rightarrow$ conservation of linear momentum

We can also write $\sum \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

So, no external impulses $\Rightarrow$ conservation of linear momentum

We can also write $\sum \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

Examples: Rail car A at rest and rail car B moving with $v = 2.5 \frac{mi}{h}$.

So, no external impulses $\Rightarrow$ conservation of linear momentum

---

We can also write $\Sigma \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

---

<u>Examples</u>: Rail car A at rest and rail car B moving with $v = 2.5 \frac{mi}{h}$. When in contact A & B couple together.

So, no external impulses ⇒ conservation of linear momentum

We can also write $\sum \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

Examples: Rail car A at rest and rail car B moving with $v = 2.5 \frac{mi}{h}$. When in contact A & B couple together. Car A has locked wheels with $\mu_K = 0.30$.

So, no external impulses $\Rightarrow$ Conservation of linear momentum

---

We can also write $\Sigma \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

---

Examples: Rail car A at rest and rail car B moving with $v = 2.5 \frac{mi}{h}$. When in contact A & B couple together. Car A has locked wheels with $\mu_K = 0.30$. Find V_{AB} right after coupling:

So, no external impulses $\Rightarrow$ conservation of linear momentum

---

We can also write $\Sigma \vec{F}_{AVE}\,\Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

---

Examples: Rail car A at rest and rail car B moving with $v = 2.5\,\frac{mi}{h}$. When in contact A & B couple together. Car A has locked wheels with $\mu_K = 0.30$. Find V_{AB} right after coupling:

Here, we take Δt so small that friction has not changed motion of A+B

So, no external impulses $\Rightarrow$ conservation of linear momentum

---

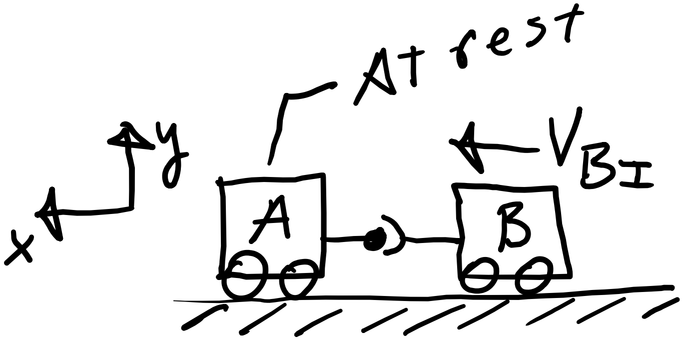
We can also write $\Sigma \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

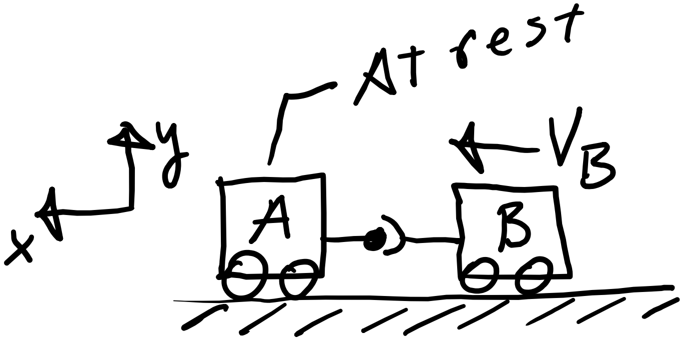
---

<u>Examples</u>: Rail car A at rest and rail car B moving with $v = 2.5 \frac{mi}{h}$. When in contact A & B couple together. Car A has locked wheels with $\mu_K = 0.30$. <u>Find V_{AB} right after coupling</u>:

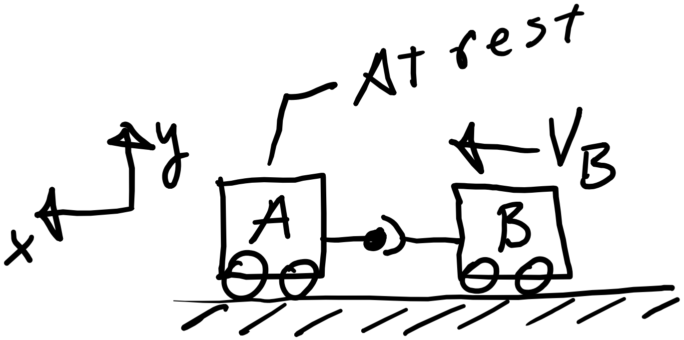
Here, we take Δt so small that friction has not changed motion of A+B $\rightarrow$

Conservation of linear momentum $\rightarrow$


At rest
V_{BI}
x
y
A
B

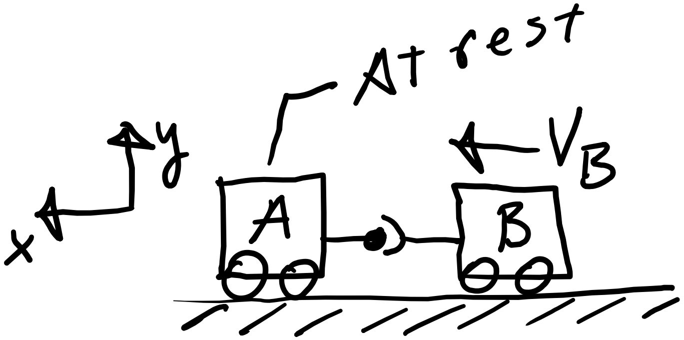


$$\vec{L}_{A_1} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$



$$\cancel{\vec{L}_{A_1}}^{\,0} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

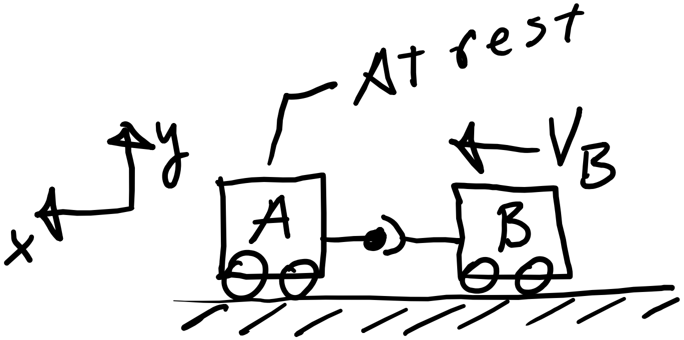
$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$



$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

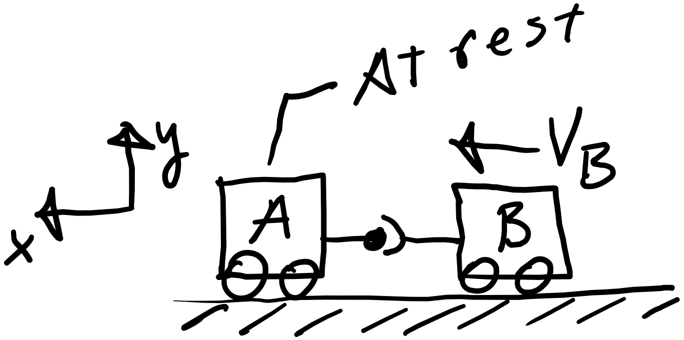


$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

<u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$



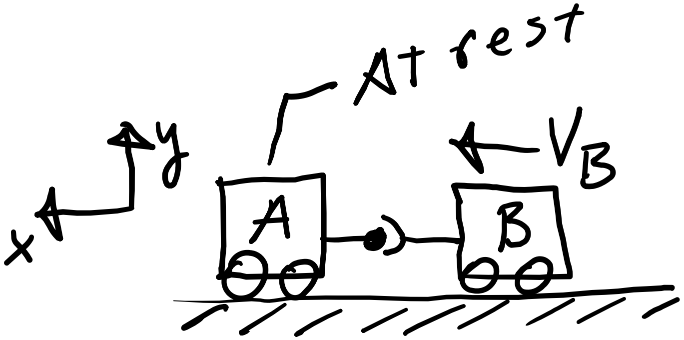
$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

<u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

So $\quad m_B \vec{V}_{B_1} = (m_A + m_B)\vec{V}_2$



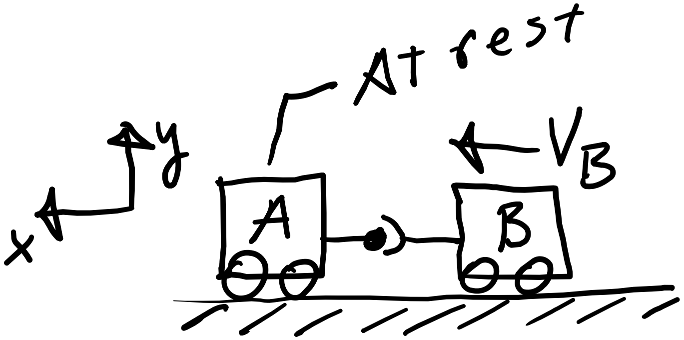
$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = M_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

<u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $\quad m_B \vec{V}_{B_1} = (M_A + M_B)\vec{V}_2 \quad$ so $\quad \boxed{\vec{V}_2 = \left[\dfrac{m_B}{(M_A + M_B)}\right]\vec{V}_{B_1}}$



$$\cancel{\vec{L}_{A_1}}^{\,0} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

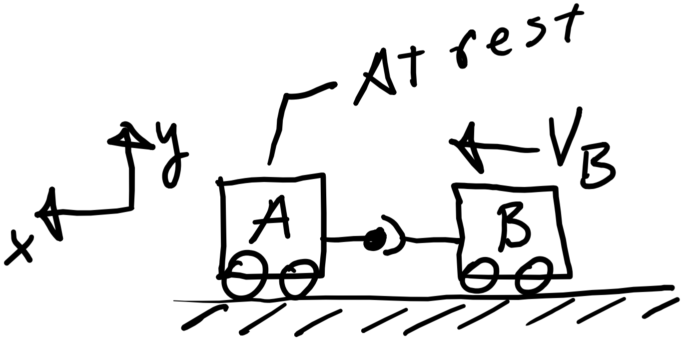
$$\Rightarrow \quad m_B \vec{V}_{B_1} = M_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

<u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $m_B \vec{V}_{B_1} = (M_A + M_B)\vec{V}_2$ so

$$\boxed{\vec{V}_2 = \left[\frac{m_B}{(M_A + M_B)}\right]\vec{V}_{B_1}}$$

<u>Find time to come to rest:</u>



$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = M_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

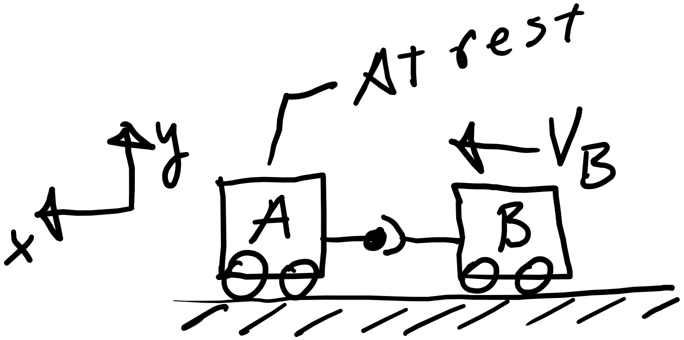
But $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $\quad m_B \vec{V}_{B_1} = (M_A + M_B)\vec{V}_2 \quad$ so

$$\boxed{\vec{V}_2 = \left[\frac{m_B}{(M_A + M_B)}\right]\vec{V}_{B_1}}$$

<u>Find time to come to rest:</u>

Friction only acting on car A



$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

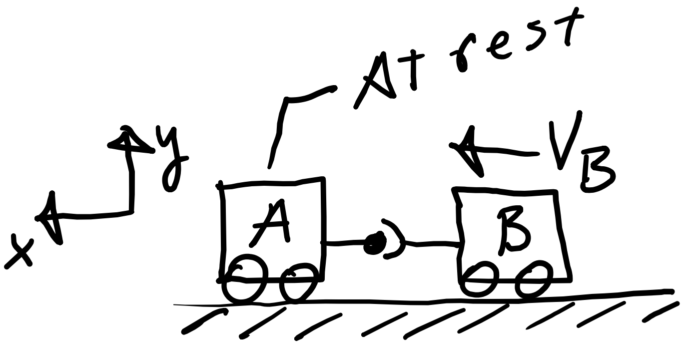
$$\Rightarrow \quad m_B \vec{V}_{B_1} = M_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

<u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $m_B \vec{V}_{B_1} = (M_A + M_B)\vec{V}_2$ so $\boxed{\vec{V}_2 = \left[\dfrac{m_B}{(M_A + M_B)}\right]\vec{V}_{B_1}}$

<u>Find time to come to rest:</u>

Friction only acting on car A so $\vec{F}_F = M_A g \mu_k (-\hat{i})$



$$\cancel{\vec{L}_{A_1}}^{\,0} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

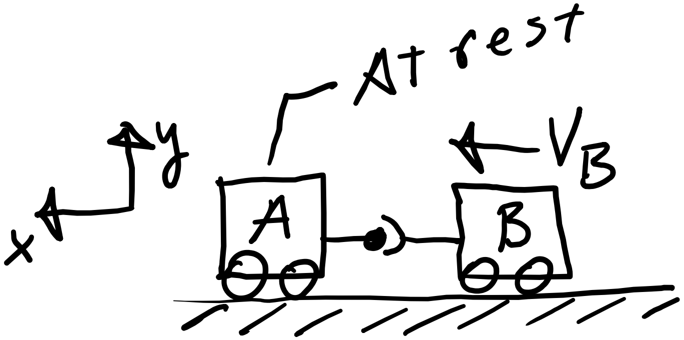
$$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$ <u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $m_B \vec{V}_{B_1} = (m_A + m_B)\vec{V}_2$ so $\boxed{\vec{V}_2 = \left[\dfrac{m_B}{(m_A+m_B)}\right]\vec{V}_{B_1}}$

<u>Find time to come to rest:</u>

Friction only acting on car A so $\vec{F}_F = m_A g \mu_k (-\hat{i})$

Now $\int_{t_2}^{t_3} \vec{F}\,dt = \vec{L}_3 - \vec{L}_2$



$$\cancelto{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

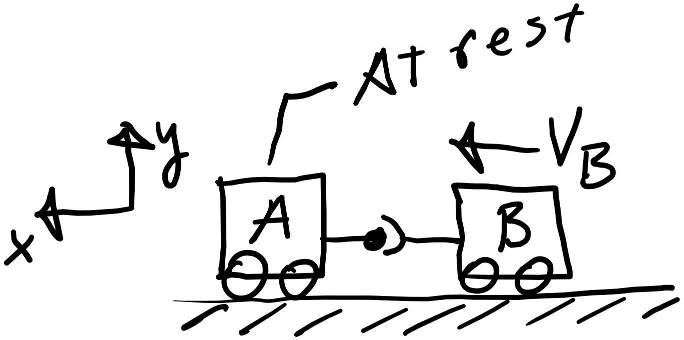
$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$ <u>But</u> $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $m_B \vec{V}_{B_1} = (m_A + m_B)\vec{V}_2$ so $\boxed{\vec{V}_2 = \left[\dfrac{m_B}{(m_A + m_B)}\right]\vec{V}_{B_1}}$

<u>Find time to come to rest:</u>

Friction only acting on car A so $\vec{F}_F = m_A g \mu_k (-\hat{i})$

Now $\int_{t_2}^{t_3} \vec{F}\,dt = \cancelto{0}{\vec{L}_3} - \vec{L}_2 \Rightarrow -m_A g \mu_k \Delta t = -(m_A + m_B)\vec{V}_2$



$$\cancel{\vec{L}_{A_1}}^{\,0} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$\Rightarrow \; m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$ But $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

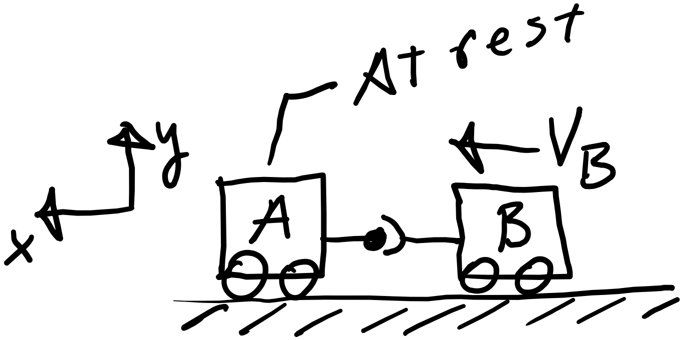
so $m_B \vec{V}_{B_1} = (m_A + m_B)\vec{V}_2$ so $\boxed{\vec{V}_2 = \left[\frac{m_B}{(m_A+m_B)}\right]\vec{V}_{B_1}}$

Find time to come to rest:

Friction only acting on car A so $\vec{F}_F = m_A g \mu_k(-\hat{i})$

Now $\int_{t_2}^{t_3} \vec{F}\,dt = \cancel{\vec{L}_3}^{\,0} - \vec{L}_2 \Rightarrow -m_A g \mu_k \Delta t = -(m_A+m_B)\vec{V}_2$

But $\vec{V}_2 = \left(\frac{m_B}{m_A+m_B}\right) V_1 \hat{i}$



$$\overset{0}{\vec{L}_{A_1}} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$

$$\Rightarrow \quad m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$$

But $\vec{V}_{A_2} = \vec{V}_{B_2} \equiv \vec{V}_2$

so $m_B \vec{V}_{B_1} = (m_A + m_B)\vec{V}_2$ so

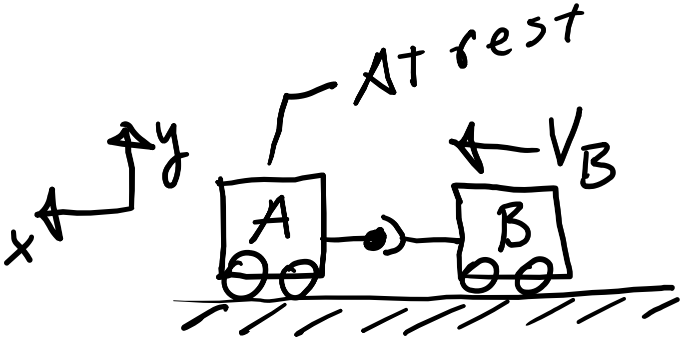
$$\boxed{\vec{V}_2 = \left[\frac{m_B}{(m_A + m_B)}\right]\vec{V}_{B_1}}$$

Find time to come to rest:

Friction only acting on car A so $\vec{F}_F = m_A g \mu_k (-\hat{i})$

Now $\int_{t_2}^{t_3} \vec{F}\,dt = \overset{0}{\vec{L}_3} - \vec{L}_2 \Rightarrow -m_A g \mu_k \Delta t = -(m_A + m_B)\vec{V}_2$

But $\vec{V}_2 = \left(\frac{m_B}{m_A + m_B}\right) V_1 \hat{i}$ so $m_A g \mu_k \Delta t = m_B V_1$



$$\cancelto{0}{\vec L_{A_1}} + \vec L_{B_1} = \vec L_{A_2} + \vec L_{B_2}$$

$$\Rightarrow \vec L_{B_1} = \vec L_{A_2} + \vec L_{B_2}$$

$$\Rightarrow m_B \vec V_{B_1} = m_A \vec V_{A_2} + m_B \vec V_{B_2}$$

But $\vec V_{A_2} = \vec V_{B_2} \equiv \vec V_2$

so $m_B \vec V_{B_1} = (m_A + m_B)\vec V_2$ so $\boxed{\vec V_2 = \left[\frac{m_B}{(m_A+m_B)}\right]\vec V_{B_1}}$

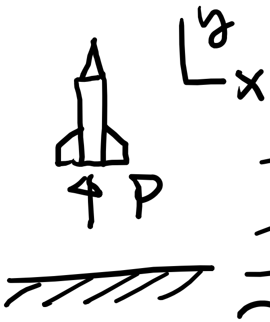
Find time to come to rest:

Friction only acting on car A so $\vec F_F = m_A g \mu_k(-\hat i)$

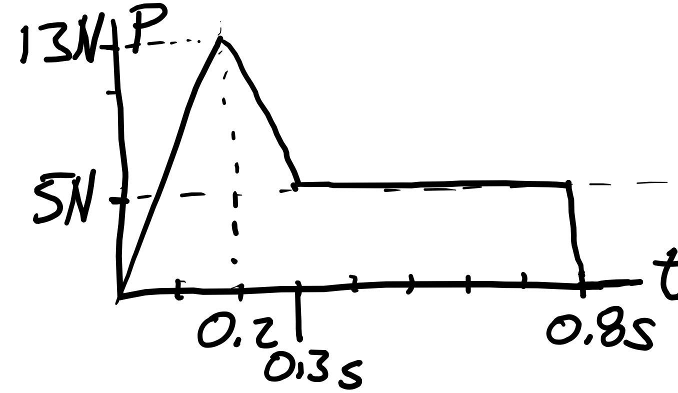
Now $\int_{t_2}^{t_3} \vec F\, dt = \cancelto{0}{\vec L_3} - \vec L_2 \Rightarrow -m_A g \mu_k \Delta t = -(m_A+m_B)\vec V_2$

But $\vec V_2 = \left(\frac{m_B}{m_A+m_B}\right)V_1 \hat i$ so $m_A g \mu_k \Delta t = m_B V_1$

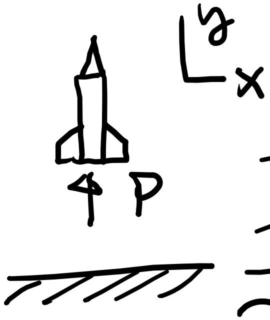
$$\Rightarrow \boxed{\Delta t = \frac{m_B V_1}{m_A g \mu_k}}$$



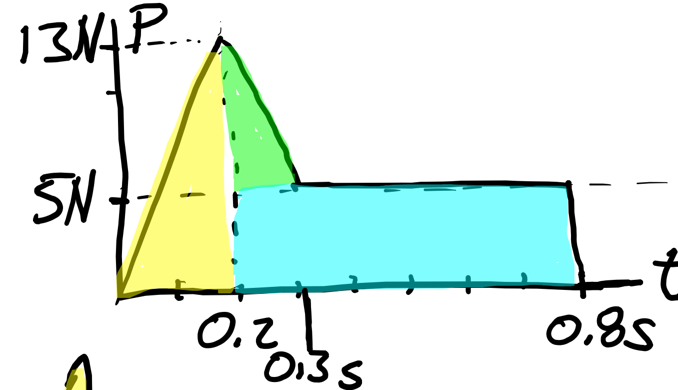
A 60g rocket has a thrust as given in plot.



Find speed at end of thrust:

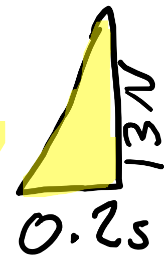


A 60g rocket has a thrust as given in plot.

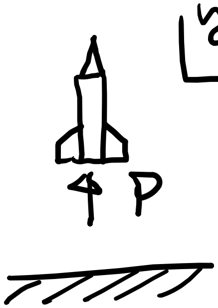


Find speed at end of thrust:

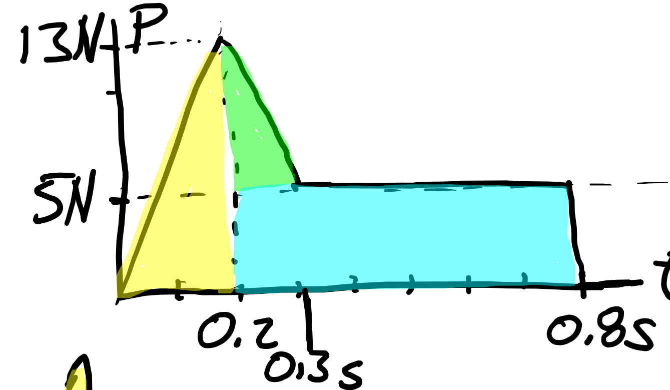
Area of 1st triangle



$$= \left(\frac{0.2s}{2}\right) 13N = 1.3 Ns$$

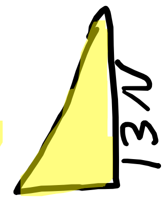


A 60g rocket has a thrust as given in plot.



Find speed at end of thrust:

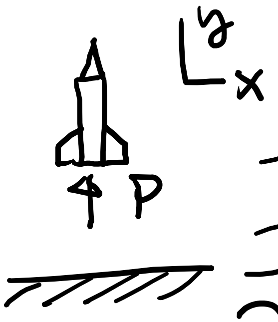
Area of 1st triangle



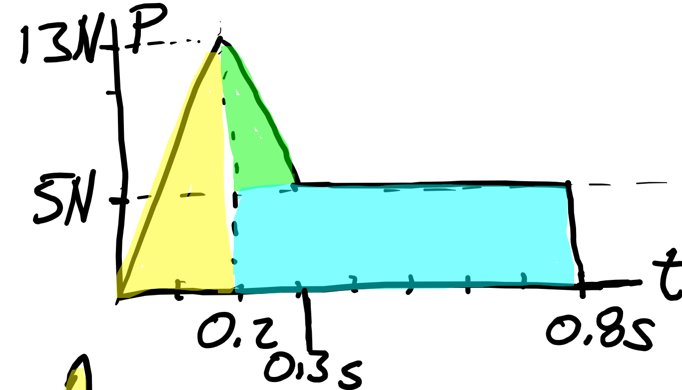
$$= \left(\frac{0.2s}{2}\right) 13N = 1.3Ns$$

Area of 2nd triangle

$$= \left(\frac{0.1s}{2}\right) 8N = 0.4Ns$$

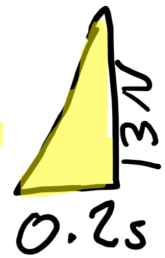


A 60g rocket has a thrust as given in plot.



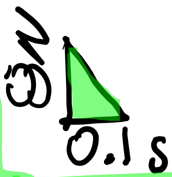
Find speed at end of thrust:

Area of 1ST triangle



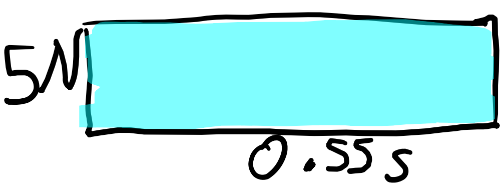
$= \left(\frac{0.2s}{2}\right) 13N = 1.3Ns$

Area of 2ND triangle

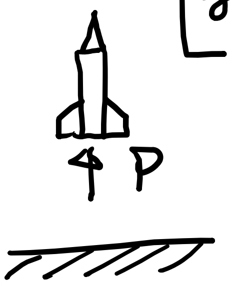


$= \left(\frac{0.1s}{2}\right) 8N = 0.4Ns$

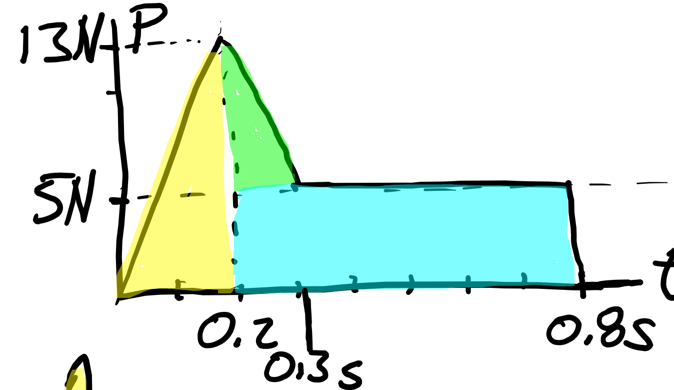
Area of rectangle



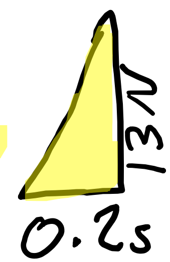
$= (0.6\ s)\, 5N = 3.0Ns$



A 60g rocket has a thrust as given in plot. Find speed at end of thrust:



Area of 1^{ST} triangle

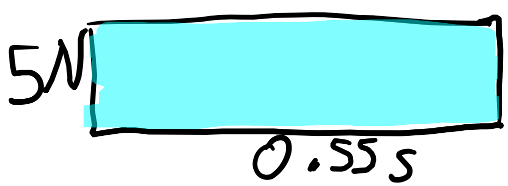


$$=\left(\frac{0.2s}{2}\right)13N = 1.3Ns$$

Area of 2^{nd} triangle

$$=\left(\frac{0.1s}{2}\right)8N = 0.4Ns$$

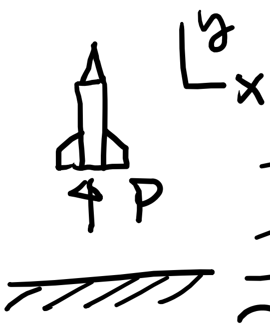
Area of rectangle



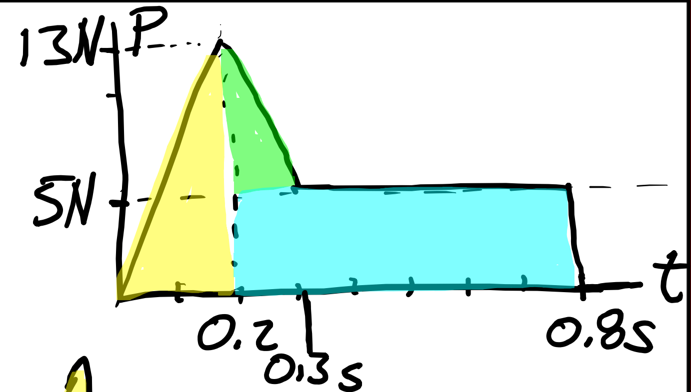
$$=(0.6\ s)5N = 3.0Ns$$

$$\int_0^{0.8s} \vec{P}dt = \hat{j}(1.3+0.4+3.0)Ns = 4.7\hat{j}Ns$$

ASU

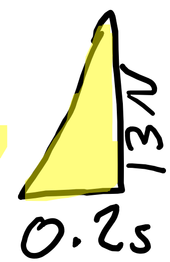


A 60g rocket has a thrust as given in plot.



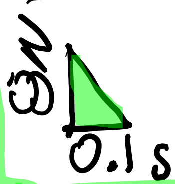
Find speed at end of thrust:

Area of 1st triangle



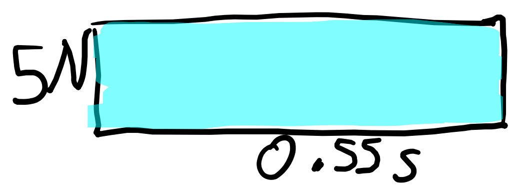
$= \left(\frac{0.2s}{2}\right) 13N = 1.3Ns$

Area of 2nd triangle



$= \left(\frac{0.1s}{2}\right) 8N = 0.4Ns$

Area of rectangle



$= (0.6\ s) 5N = 3.0Ns$

$$\int_0^{0.8s} \vec{P} dt = \hat{j}(1.3 + 0.4 + 3.0)Ns = 4.7\hat{j}Ns$$

Just need to add in gravity ⇒

Gravity

$$\int_0^{0.8s} \vec{F}_g \, dt$$

Gravity

$$\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$$

$$0.06kg * 9.81 m/s^2 * 0.8s (-\hat{j}) = 0.471 Ns (-\hat{j})$$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$0.06kg * 9.81 m/s^2 * 0.8s (-\hat{j}) = 0.471 Ns (-\hat{j})$

Now $\sum \overrightarrow{I_{mp}} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$0.06kg * 9.81 m/s^2 * 0.8s (-\hat{j}) = 0.471 Ns (-\hat{j})$

Now $\sum \overrightarrow{I_{mp}} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$= (-0.471 + 4.7) \hat{j} Ns$

$= 4.229 \hat{j} N \cdot s$

Gravity $\int_0^{0.8s} \vec{F}_g\,dt = (-\hat{j})mg * 0.8s =$

$0.06kg * 9.81 m/s^2 * 0.8s(-\hat{j}) = 0.471 Ns(-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g\,dt + \int_0^{0.8s} P\,dt$

$= (-0.471 + 4.7)\hat{j}\,Ns$

$= 4.229\,\hat{j}\,N\cdot s$

So $\sum \vec{I}_{mp} = \Delta \vec{L}$

<u>Gravity</u> $\int_0^{0.8s} \vec{F}_g\,dt = (-\hat{j})mg * 0.8s =$

$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g\,dt + \int_0^{0.8s} P\,dt$

$= (-0.471 + 4.7)\hat{j}\,Ns$

$= 4.229\,\hat{j}\,N{\cdot}s$

So $\sum \vec{I}_{mp} = \Delta\vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \vec{L}_I$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$0.06 kg * 9.81 m/s^2 * 0.8s (-\hat{j}) = 0.471 Ns (-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$= (-0.471 + 4.7) \hat{j} Ns$

$= 4.229 \hat{j} N \cdot s$

So $\sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancel{\vec{L}_I}^{\,0}$

$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m \vec{V}_F$

Gravity $\int_0^{0.8s} \vec{F}_g dt = (-\hat{j}) mg * 0.8s =$

$0.06kg * 9.81 m/s^2 * 0.8s(-\hat{j}) = 0.471 Ns(-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g dt + \int_0^{0.8s} P dt$

$= (-0.471 + 4.7)\hat{j} Ns$

$= 4.229 \hat{j} N \cdot s$

So $\sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancel{\vec{L}_I}^{0}$

$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m\vec{V}_F \Rightarrow \vec{V}_F = \frac{\sum \vec{I}_{mp}}{m} = \frac{4.23 Ns \hat{j}}{0.06 kg}$

Gravity $\displaystyle\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$$

Now $\displaystyle\sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$$= (-0.471 + 4.7)\hat{j}\,Ns$$

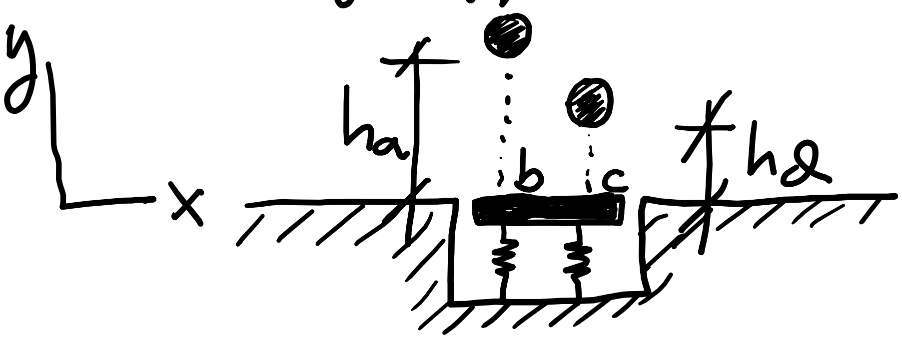
$$= 4.229\,\hat{j}\,N \cdot s$$

So $\sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancelto{0}{\vec{L}_I}$

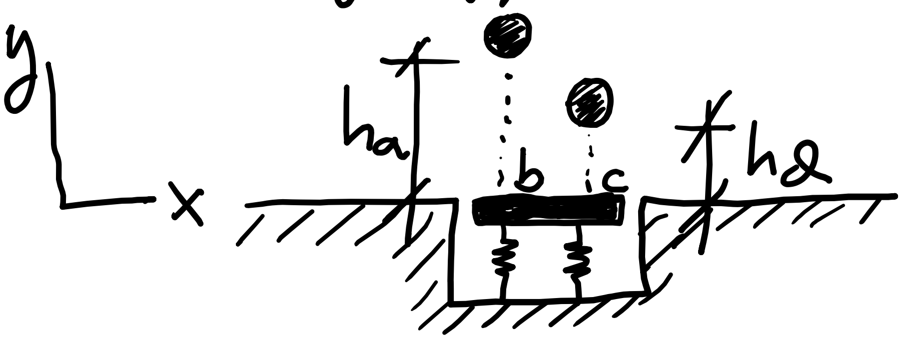
$$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m\vec{V}_F \Rightarrow \vec{V}_F = \frac{\sum \vec{I}_{mp}}{m} = \frac{4.23\,Ns\,\hat{j}}{0.06\,kg}$$

$$\Rightarrow \sum \vec{I}_{mp} = (70.5\,\tfrac{m}{s})\hat{j}$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.

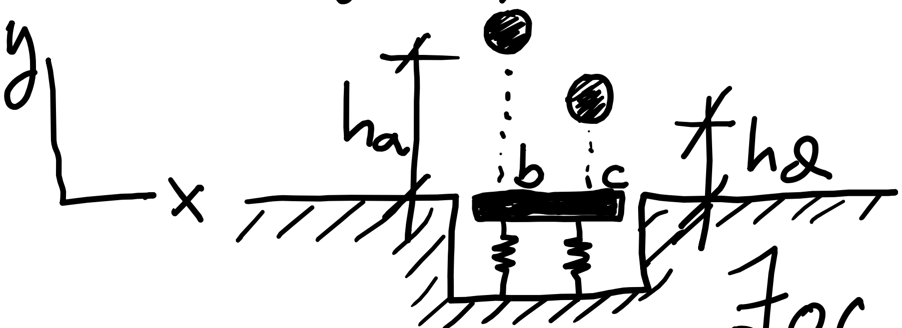


Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

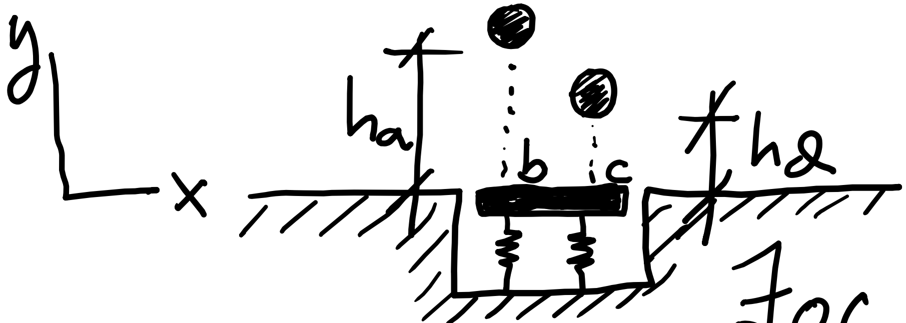
Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



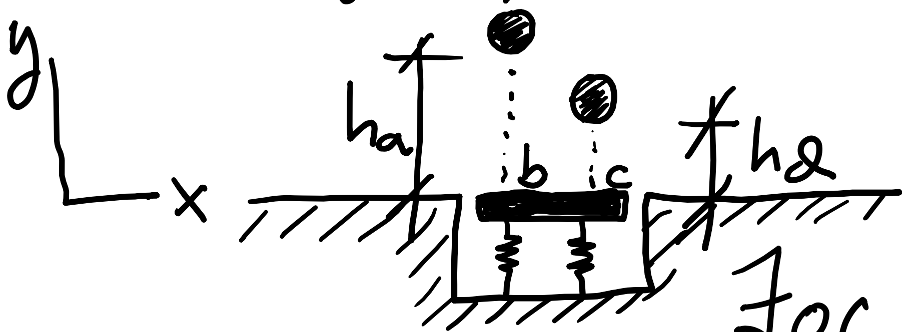
Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta \vec{L}_{Ball} + \Delta \vec{L}_{plate} = \cancel{0}$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



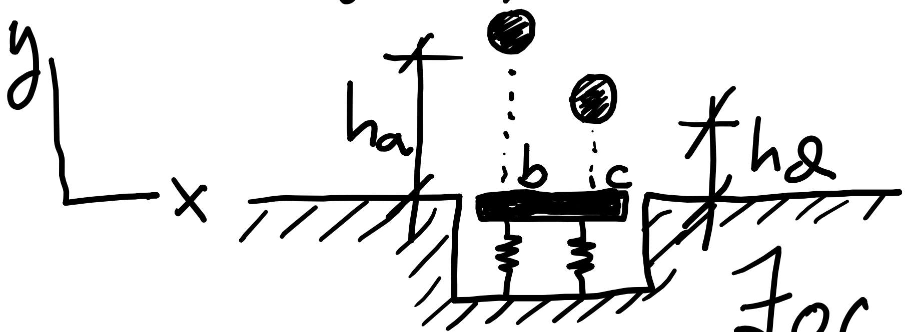
Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta\vec{L}_{Ball} + \Delta\vec{L}_{plate} = 0 \Rightarrow \Delta\vec{L}_p = -\Delta\vec{L}_B$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

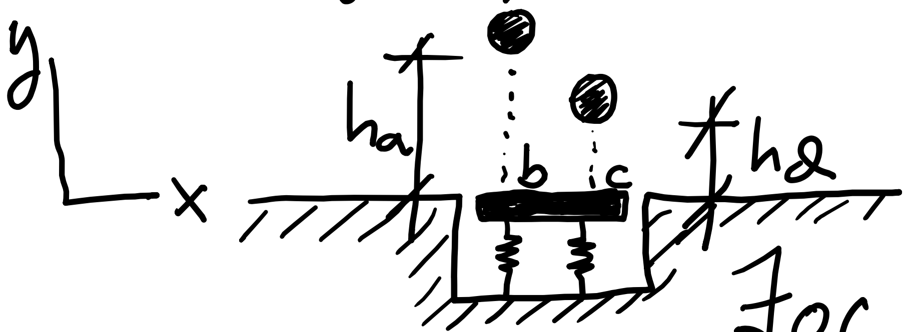
For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta \vec{L}_{Ball} + \Delta \vec{L}_{plate} = 0 \Rightarrow \Delta \vec{L}_p = -\Delta \vec{L}_B \Rightarrow$$

$$m_p v_{pF} - m_p v_{pI} = -\Delta \vec{L}_B$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta\vec{L}_{Ball} + \Delta\vec{L}_{plate} = 0 \Rightarrow \Delta\vec{L}_p = -\Delta\vec{L}_B \Rightarrow$$

$$m_p v_{pF} - \cancel{m_p v_{pI}}^{\,0} = -\Delta\vec{L}_B \Rightarrow v_{pF} = \frac{-\Delta\vec{L}_B}{m_p}$$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B / m_p$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + T_a = V_b + T_b$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \overset{0}{\cancel{T_a}} = \overset{0}{\cancel{V_b}} + T_b$

So far we have $\vec{V}_{PF} = -\Delta\vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $\cancel{V_a}^{\,0} + T_a = \cancel{V_b}^{\,0} + T_b$

<u>Note</u>: $T = \frac{1}{2} m v^2 = \frac{L^2}{2m}$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a}^{\,0} = \cancel{V_b}^{\,0} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.
need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$; $0 \quad 0$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{Bb}^2}{2m}$

$\Rightarrow \quad L_B^2 = 2m_B^2 g h_a$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a}^{\,0} = \cancel{V_b}^{\,0} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$V_c + T_c = V_d + T_d$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B / m_p$.

need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a}^{\,0} = \cancel{V_b}^{\,0} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancel{V_c}^{\,0} + T_c = V_e + \cancel{T_e}^{\,0} \Rightarrow \frac{L_c^2}{2m} = mgh_e \Rightarrow L_c^2 = 2m_B^2 g h_e$

So $\vec{L}_b = m\sqrt{2gh_a}(-\hat{j})$ & $L_c = m\sqrt{2gh_e}(+\hat{j})$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B / m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancel{V_c} + T_c = V_d + \cancel{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2gh_a}\,(-\hat{j})$ & $L_c = m\sqrt{2gh_d}\,(+\hat{j})$

Now $\Delta\vec{L}_B = \vec{L}_c - \vec{L}_b$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.

need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancel{V_c} + T_c = V_d + \cancel{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2gh_a}\,(-\hat{j})$ & $L_c = m\sqrt{2gh_d}\,(+\hat{j})$

Now $\Delta\vec{L}_B = \vec{L}_c - \vec{L}_b = (m\sqrt{2g})(h_d + h_a)\hat{j}$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancel{V_c} + T_c = V_d + \cancel{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2gh_a}\,(-\hat{j})$ & $L_c = m\sqrt{2gh_d}\,(+\hat{j})$

Now $\Delta\vec{L}_B = \vec{L}_c - \vec{L}_b = (m_B\sqrt{2g})(h_d + h_a)\hat{j}$ So

$$\boxed{\vec{V}_{pF} = \frac{-\Delta L_B}{m_p} = \left(\frac{M_B}{M_p}\right)\sqrt{2g}\,(h_d + h_a)(-\hat{j})}$$