Today 13.4, 14.1

$\boxed{L13}$

Today 13.4, 14.1

$\boxed{L13}$

Impacts

Today 13.4, 14.1

$\boxed{L13}$

System of particles: Newton's 2nd law

$\boxed{L13}$

Today 13.4, 14.1

Friday 14.1, 14.2

Today 13.4, 14.1

Friday 14.1, 14.2

$\boxed{L13}$

Energy & momentum methods for a system of particles

Today 13.4, 14.1 $\boxed{L13}$

Friday 14.1, 14.2

Reminder:

HW#4 Due by end of today

Section 13.4

Elastic impact

Section 13.4

Elastic impact $\Rightarrow$ momentum is conserved

Section 13.4

Elastic impact ⇒ momentum is conserved

AND kinetic energy is conserved

Section 13.4

Elastic impact ⇒ momentum is conserved **AND** kinetic energy is conserved

Non-elastic impact

Section 13.4

Elastic impact ⇒ Momentum is conserved **AND** Kinetic energy is conserved

Non-elastic impact ⇒ Momentum is conserved

Section 13.4

Elastic impact $\Rightarrow$ momentum is conserved AND kinetic energy is conserved

Non-elastic impact $\Rightarrow$ momentum is conserved But kinetic energy is NOT conserved

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Note: Non-elastic is also called "plastic".

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Elastic impact ⇒ Momentum is conserved **AND** Kinetic energy is conserved

Non-elastic impact ⇒ Momentum is conserved But Kinetic energy is NOT conserved

Note: Non-elastic is also called "plastic".

A completely non-elastic {completely plastic} impact is one where the two items stick together

Section 14.1 : System of particles

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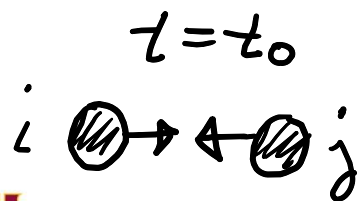

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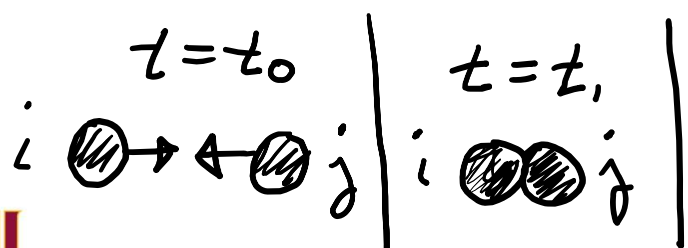

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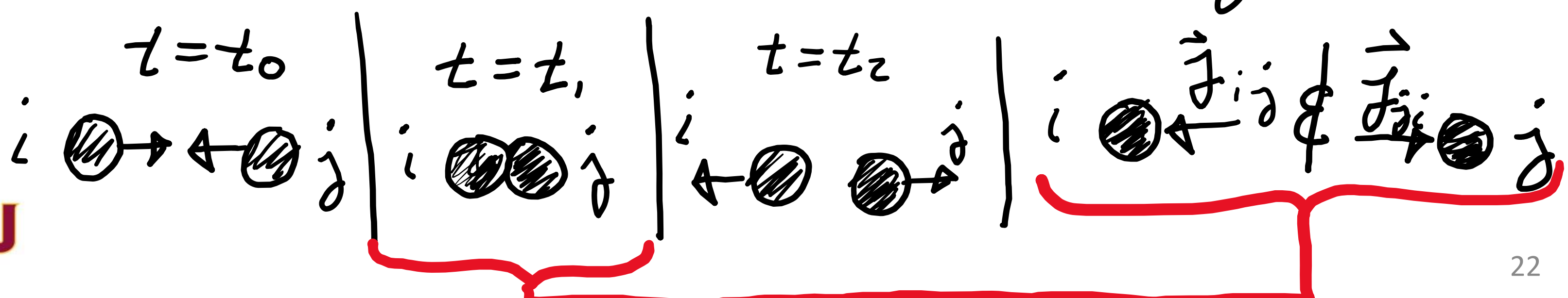

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Let us now take the cross product:
$\vec{r}_i \times \left[\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij}\right] = \vec{r}_i \times [m_i \vec{a}_i]$, where $\vec{r}_i$ is the position vector of particle i.

ASU

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$$\vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{f}_{i,j} = m_i \vec{r}_i \times \vec{a}_i$$

From the previous slide, we have

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$$\vec{F}_i + \sum_{j=1}^{N} \vec{J}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{J}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{J}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

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Next, we want to get rid of the term $\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij}$ from equation 2 →

First we look at the case where there are only two particles:

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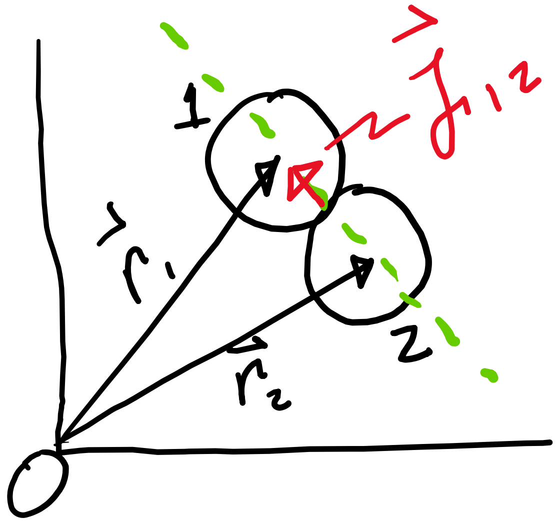

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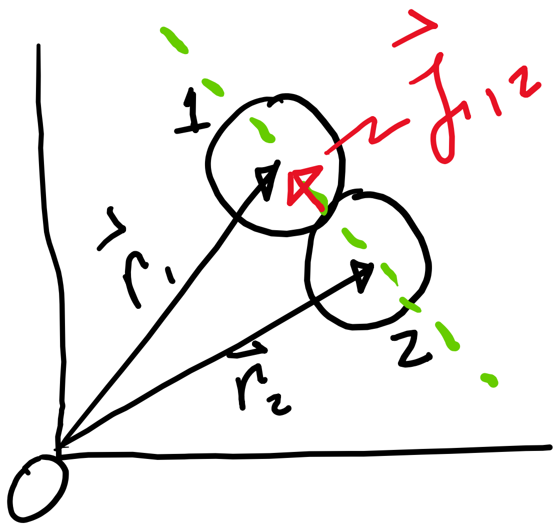

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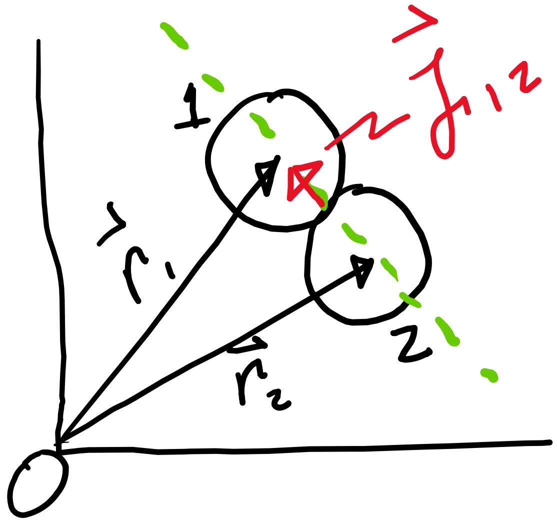

But $\vec{f}_{12} = -\vec{f}_{21}$ so

$$\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} - \vec{r}_2 \times \vec{f}_{12}$$

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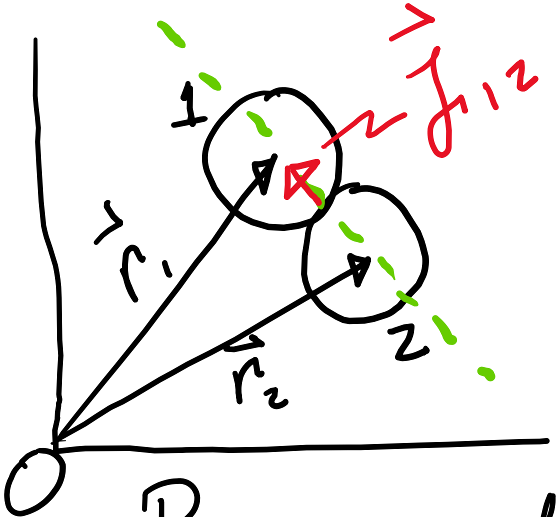

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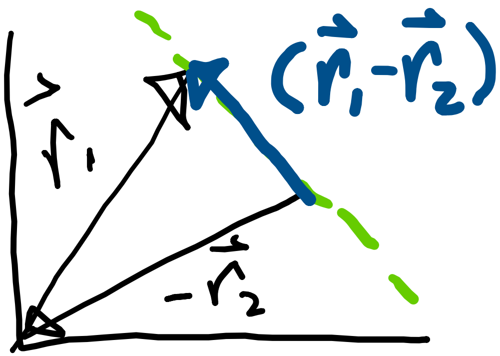
Remove objects & reset $\vec{r}_2$ to $-\vec{r}_2$



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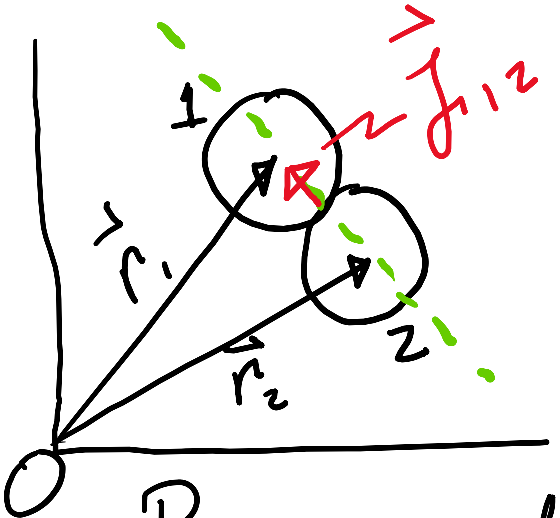

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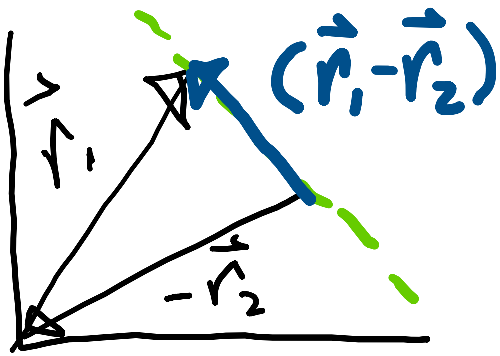

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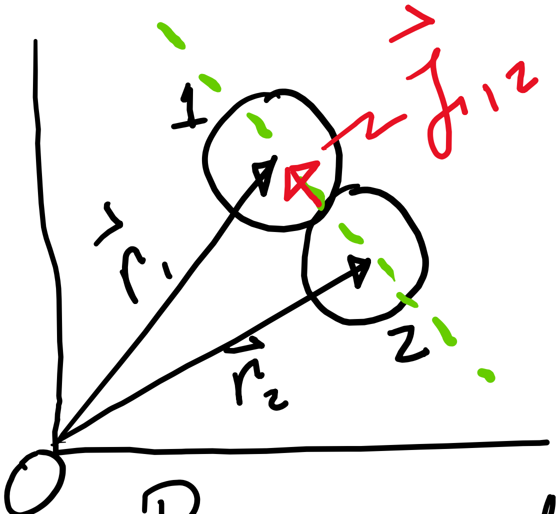

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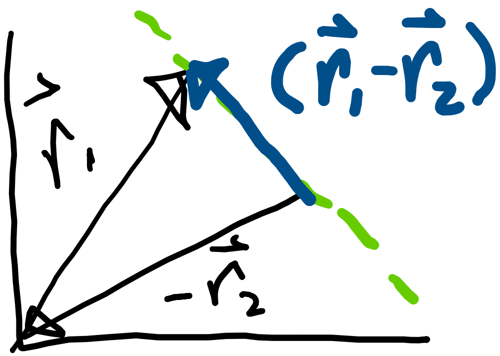

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$\Rightarrow$

$$\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{12} = 0$$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \quad \& \quad \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

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& $\vec{H}_{oi} = \vec{r}_i \times m \vec{v}_i \Rightarrow \dot{\vec{H}}_{oi} = (\dot{\vec{r}}_i \times m \vec{v}_i) + (\vec{r}_i \times m \vec{a}_i)$

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$\& \quad \vec{H}_{O_i} = \vec{r}_i \times m \vec{v}_i \Rightarrow \dot{\vec{H}}_{O_i} = \underbrace{(\dot{\vec{r}}_i \times m \vec{v}_i)}_{} + (\vec{r}_i \times m \vec{a}_i)$

↳ Equals zero since

$\dot{\vec{r}}_i = \vec{v}_i \;\; \& \;\; \vec{v}_i \times \vec{v}_i = \vec{0}$

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But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

$$\& \quad \vec{H}_{o_i} = \vec{r}_i \times m\vec{v}_i \Rightarrow \vec{H}_{o_i} = \underbrace{(\dot{\vec{r}}_i \times m\vec{v}_i)} + (\vec{r}_i \times m\vec{a}_i)$$

Equals zero since $\dot{\vec{r}}_i = \vec{v}_i$ & $\vec{v}_i \times \vec{v}_i = 0$

Now let $\sum \vec{L}_i$ & $\sum \vec{H}_{o_i}$

for some system be simply written as

$\vec{L}$ & $\vec{H}$ respectively.

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \quad \& \quad \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m\vec{a}_i$$

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Equals zero since $\dot{\vec{r}}_i = \vec{v}_i$ $\&$ $\vec{v}_i \times \vec{v}_i = 0$

Now let $\sum \vec{L}_i$ $\&$ $\sum \vec{H}_{o_i}$ for some system be simply written as $\vec{L}$ $\&$ $\vec{H}$ respectively. Now we get

$$\boxed{\sum \vec{F} = \dot{\vec{L}} \quad \& \quad \sum \vec{M}_o = \dot{\vec{H}}_o}$$

Mass center

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$$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i \quad ,$$

Mass center

$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i$, where $\bar{\vec{r}} \equiv$ mass center

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$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\bar{\vec{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\bar{\vec{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\bar{\vec{v}}$ & $\dot{\vec{L}} = m\bar{\vec{a}}$

& $\sum \vec{F} = m\bar{\vec{a}}$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\vec{\bar{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\vec{\bar{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\vec{\bar{v}}$ & $\dot{\vec{L}} = m\vec{\bar{a}}$

& $\sum \vec{F} = m\vec{\bar{a}}$ $\Rightarrow$ We can now treat the entire system of particles as a single particle located at $\vec{\bar{r}}$ with mass $m \equiv \sum_i m_i$ ☺