Today 14.1

L14

2nd law
for system
of particles

Today 14.1

Monday 14.2

L14

Energy & work

for system

of particles

Things to note: You could run into a problem with a system of particles where $\sum \vec{F}_i = \emptyset$ [no external forces] and $\sum \vec{M}_0 = \emptyset$ [no external torques].

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ASU

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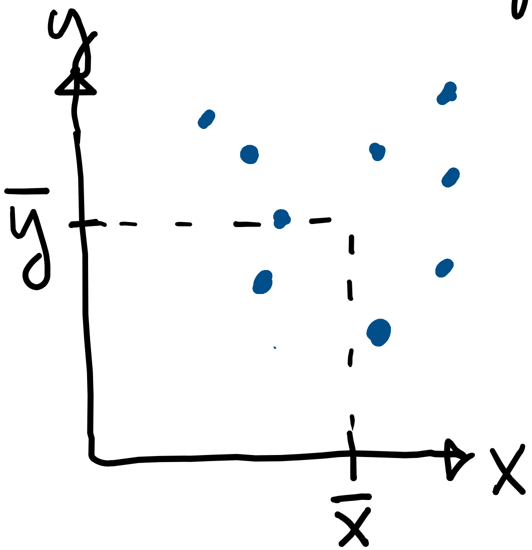
{A good example is the universe}

The center-of-mass system has some nice properties:



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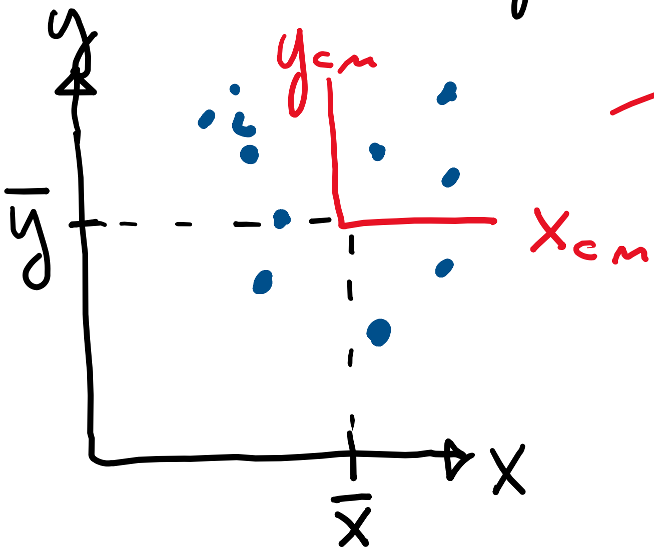

Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

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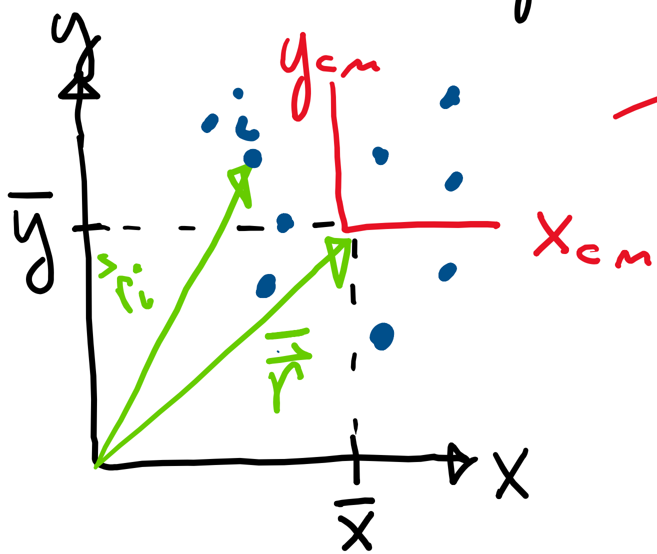

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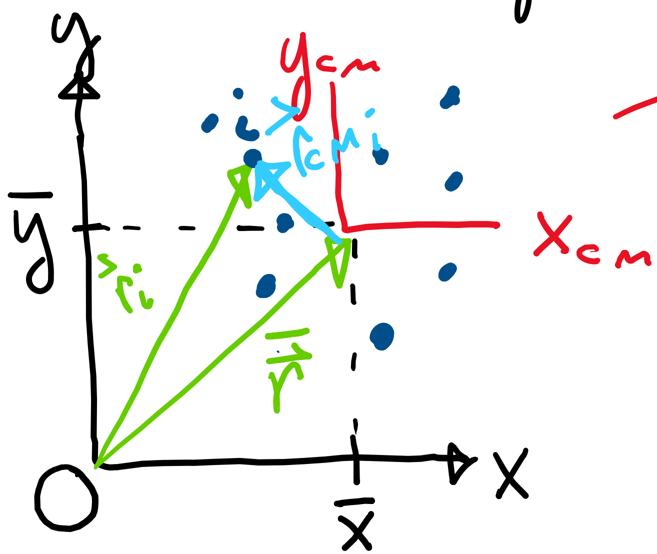

→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* Non-stop along $\vec{r}_i$
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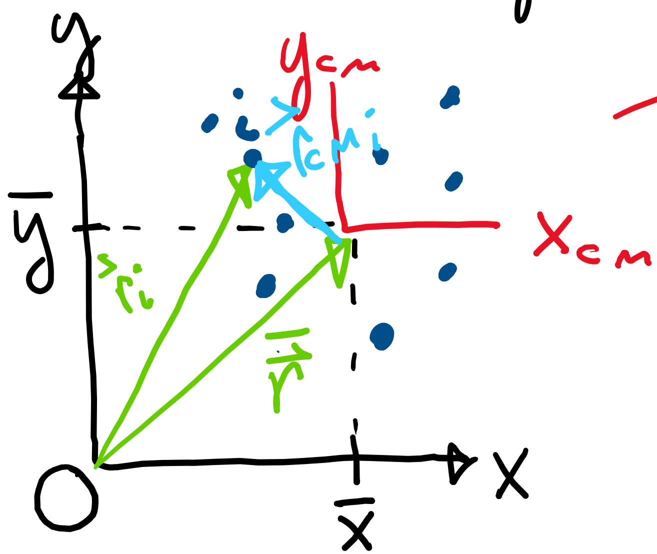

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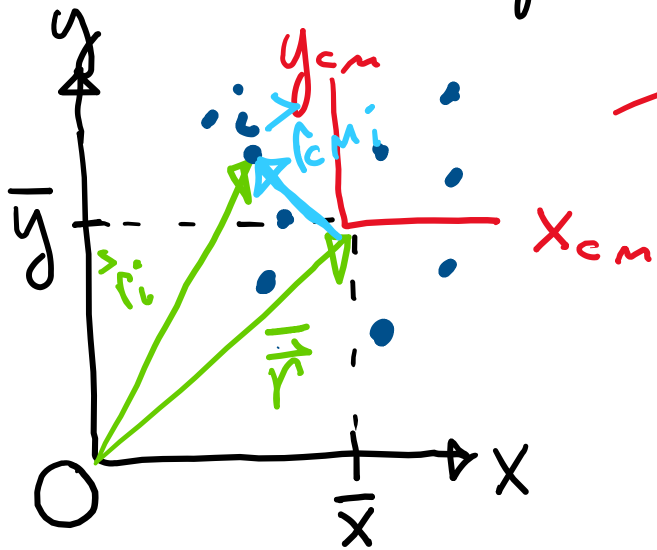

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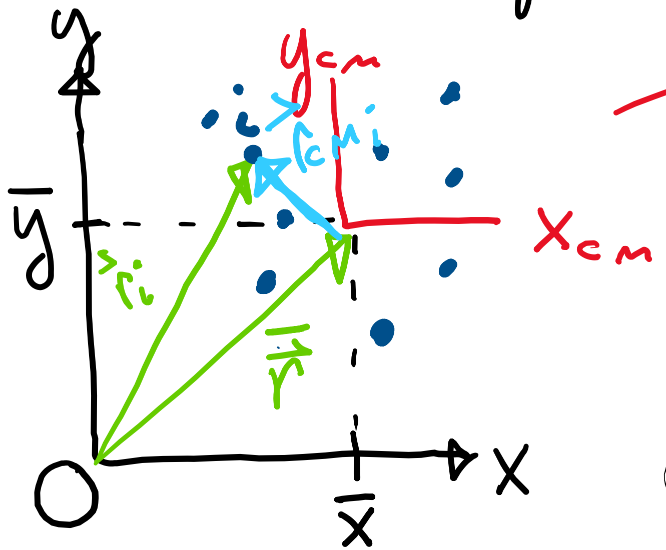

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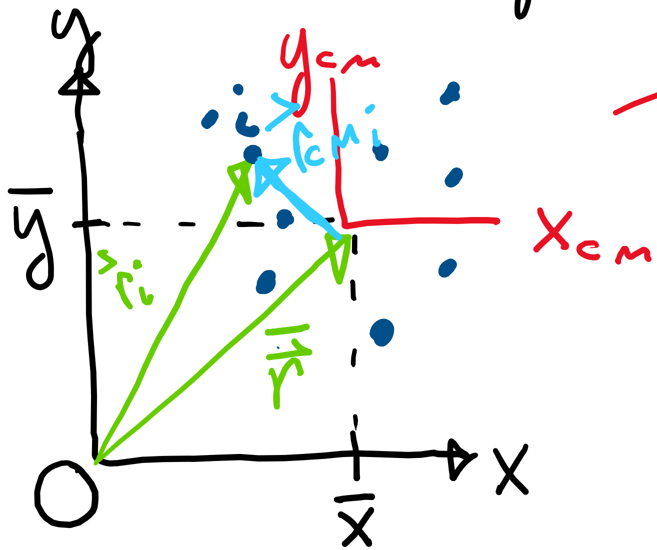

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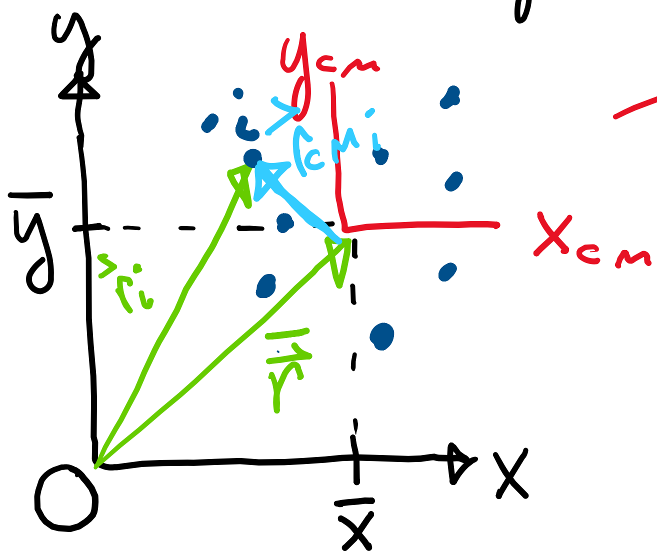

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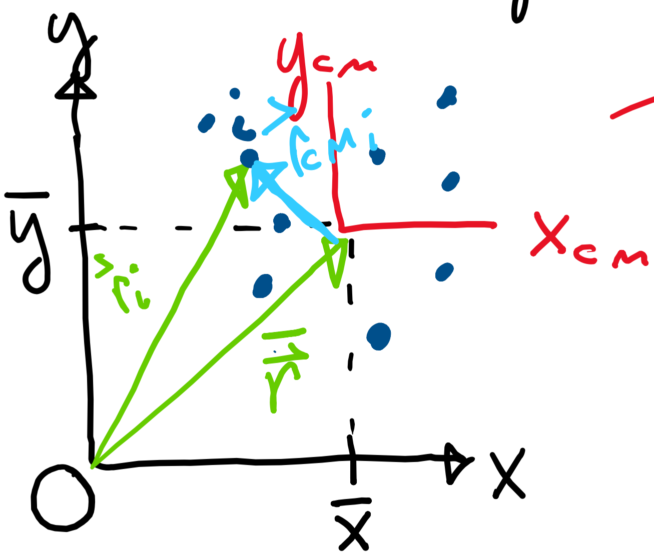

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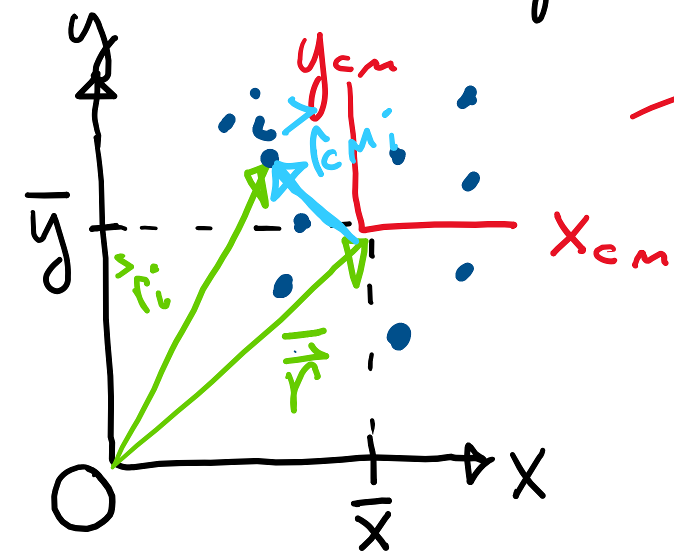

$\Rightarrow$ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

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This means that $M\vec{\bar{V}}_{cm} = 0$

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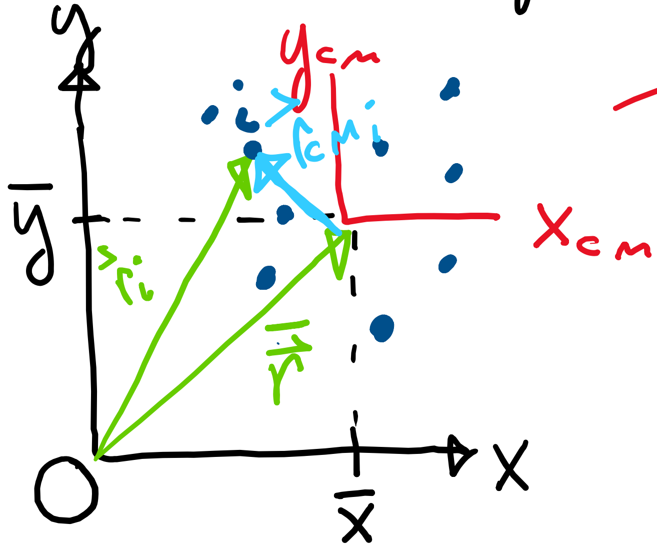

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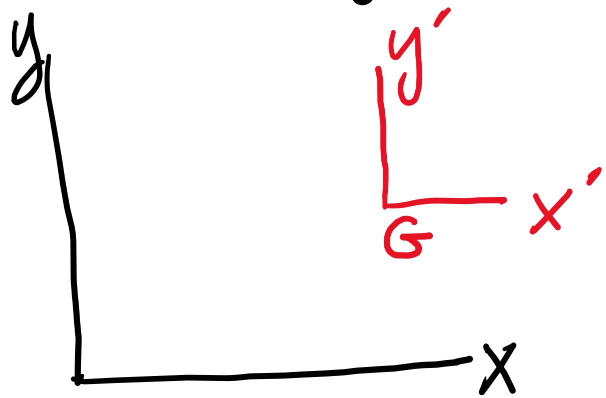
This means that $M\vec{\bar{v}}_{cm} = 0 \Rightarrow$ Center of mass system is also center of momentum system

We will define reference frame G as the center-of-mass frame



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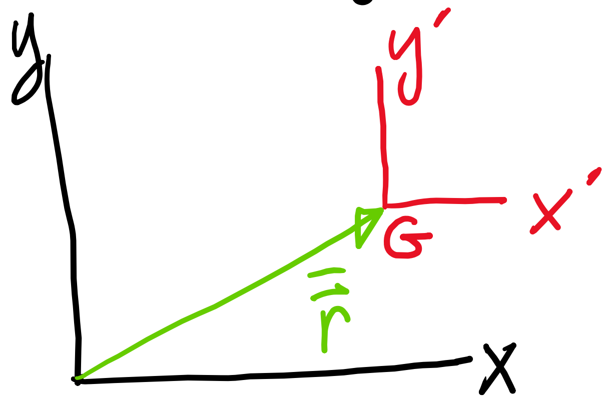

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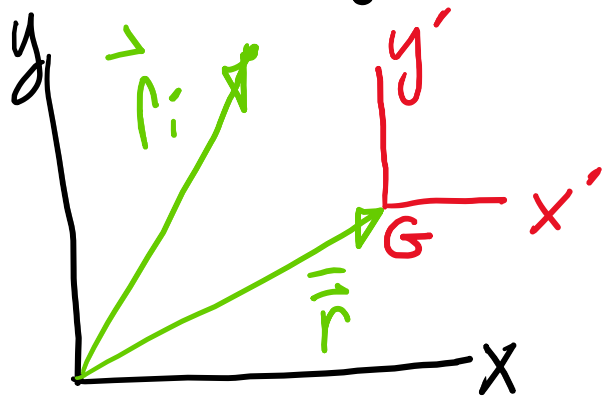

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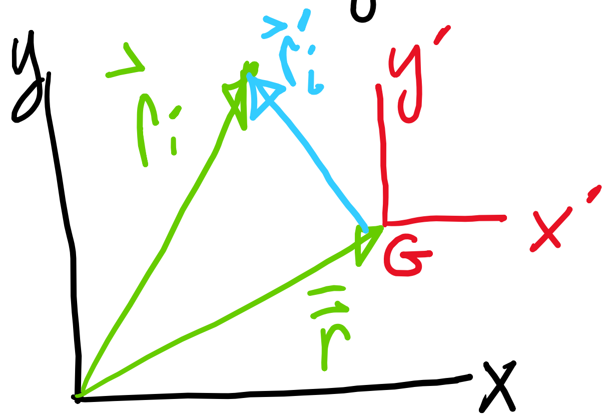

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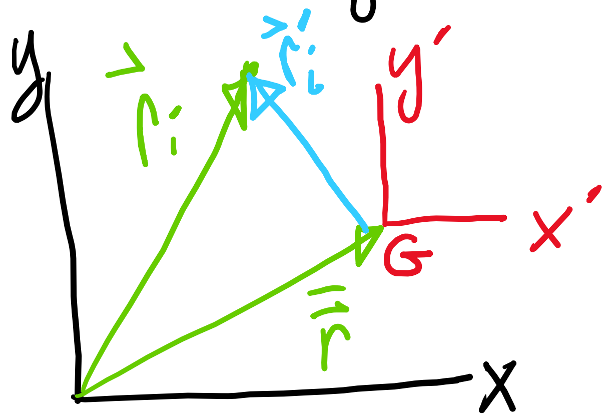

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$$

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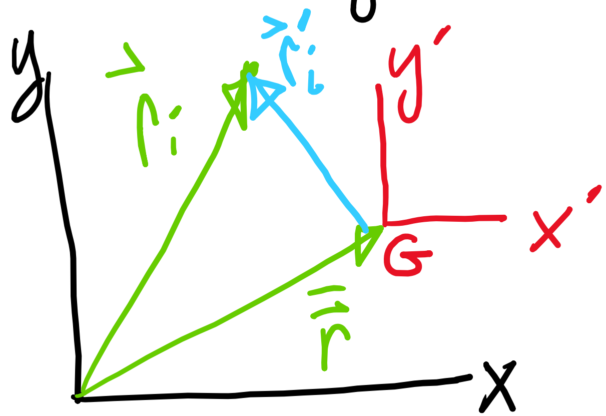

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

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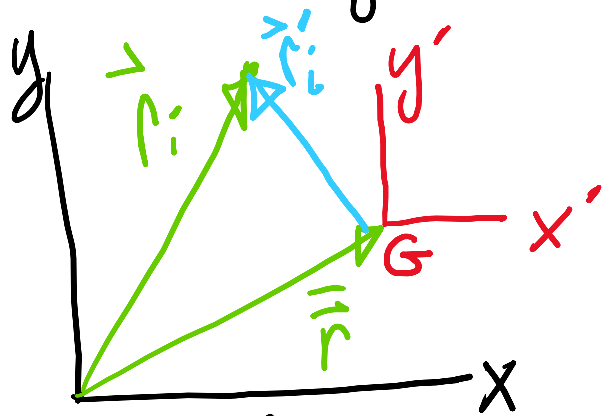

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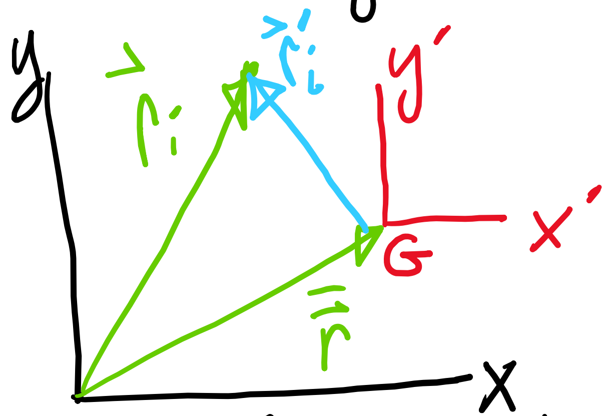

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But $\dot{\vec{r}}_i{}' = \vec{v}_i{}'$ & $\vec{v}_i{}' \times \vec{v}_i{}' = 0$

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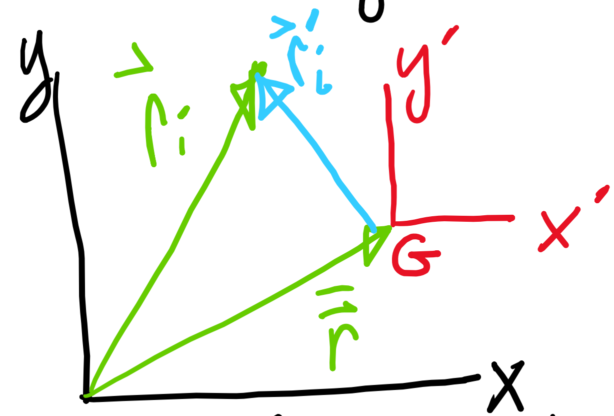

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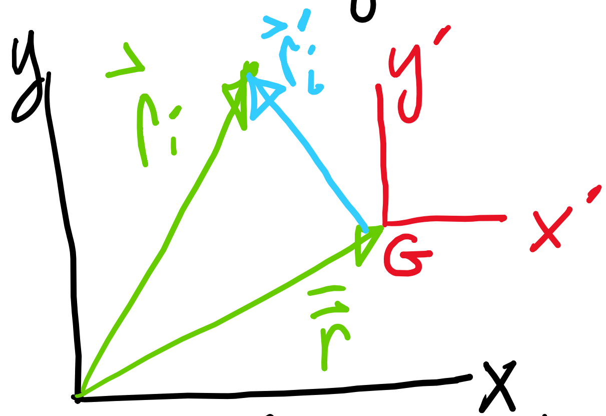

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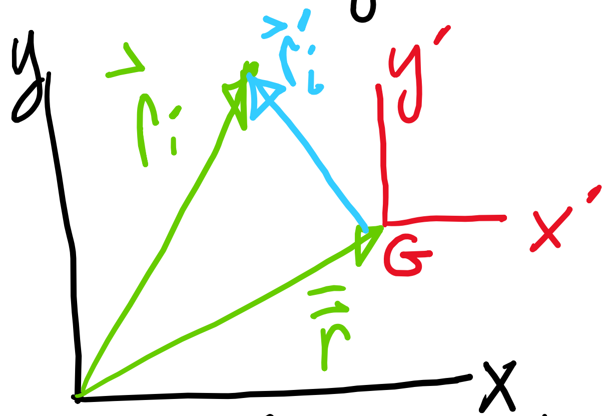

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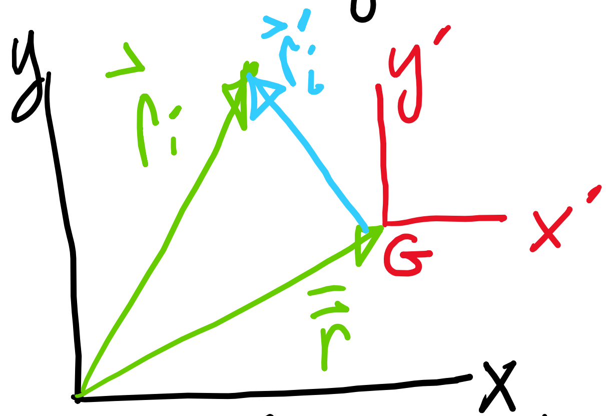

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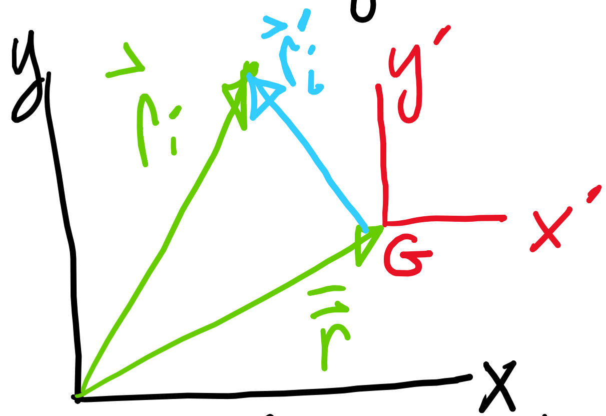

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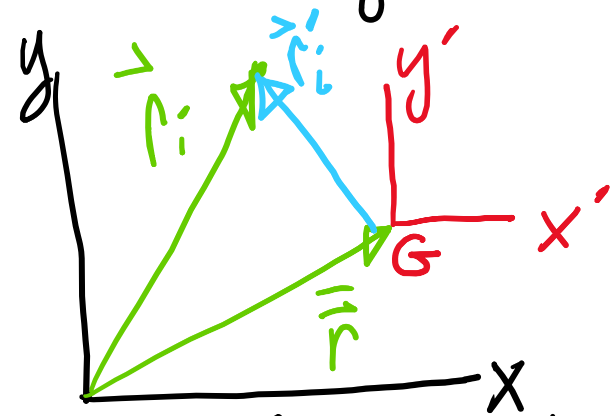

$$\vec H_G = \sum_i \vec r_i' \times m_i \vec v_i' \Rightarrow$$

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But $\dot{\vec r}_i' = \vec v_i'$ & $\vec v_i' \times \vec v_i' = 0$ so $\dot{\vec H}_G = \sum_i \vec r_i' \times m_i \vec a_i'$

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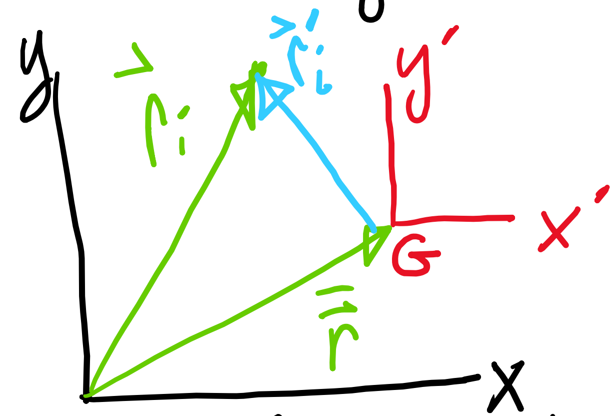

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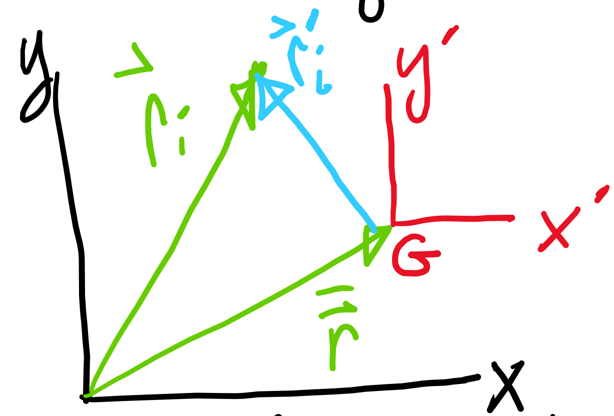
$$\Rightarrow \dot{\vec H}_G = \sum \vec r_i' \times m_i \vec a_i = \sum \vec r_i' \times \vec F_i \Rightarrow$$

$$\sum \vec M_G = \dot{\vec H}_G$$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

But $\dot{\vec{r}}_i' = \vec{v}_i'$ & $\vec{v}_i' \times \vec{v}_i' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i' \times m_i \vec{a}_i'$

But $\vec{r}_i = \bar{\vec{r}} + \vec{r}_i' \Rightarrow \vec{r}_i' = \vec{r}_i - \bar{\vec{r}} \Rightarrow \vec{a}_i' = \vec{a}_i + \bar{\vec{a}}$

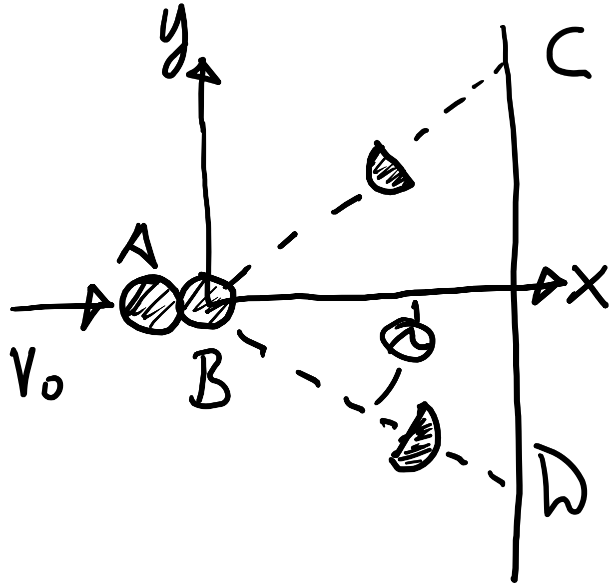
So $\dot{\vec{H}}_G = \sum_i \vec{r}_i' \times m_i (\vec{a}_i + \bar{\vec{a}}) = \sum_i m_i \vec{r}_i' \times \vec{a}_i + \overbrace{\left(\sum m_i \vec{r}_i'\right)}^{0} \bar{\vec{a}}$

$$\Rightarrow \dot{\vec{H}}_G = \sum \vec{r}_i' \times m_i \vec{a}_i = \sum \vec{r}_i' \times \vec{F}_i \Rightarrow$$

$$\sum \vec{M}_G = \dot{\vec{H}}_G$$

ASU

Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$.



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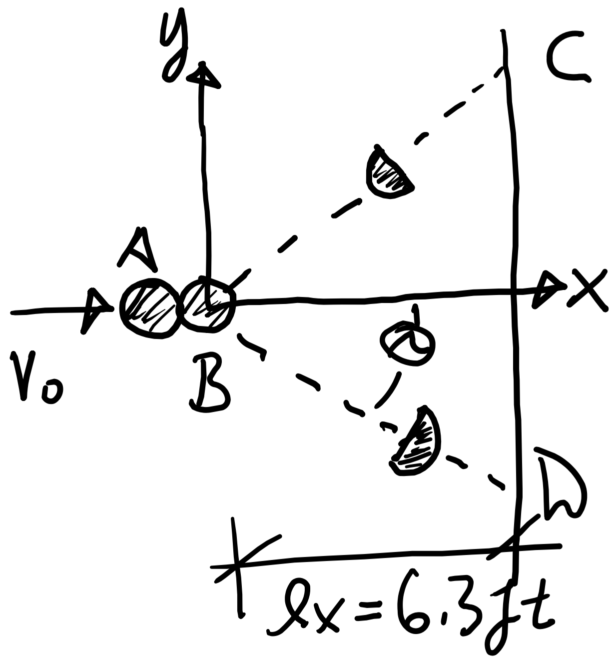

$V_0 = 16\,ft/s$, hits C at $\Delta t_C = 0.7\,s$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9\,s$.

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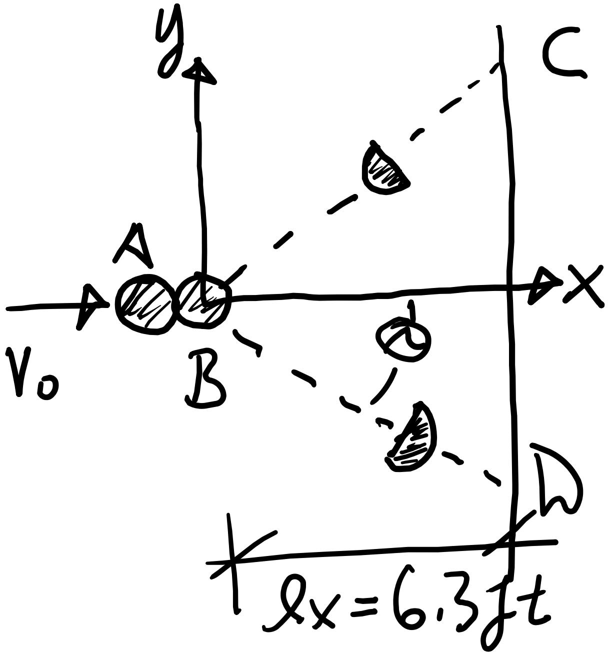

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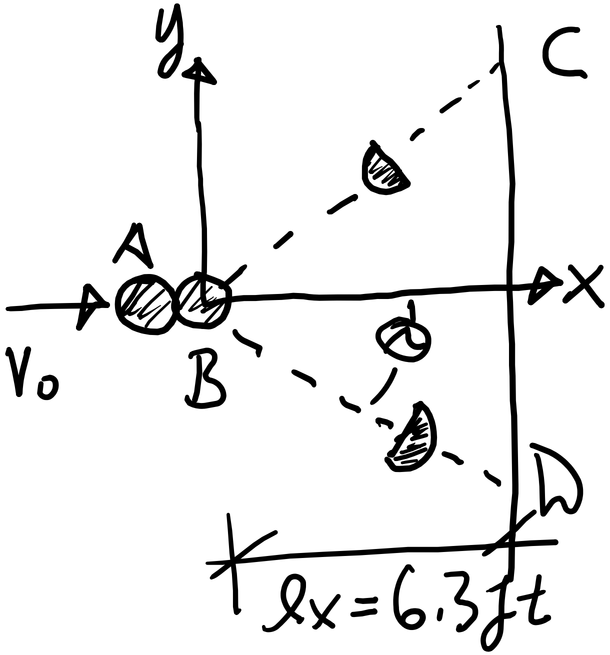

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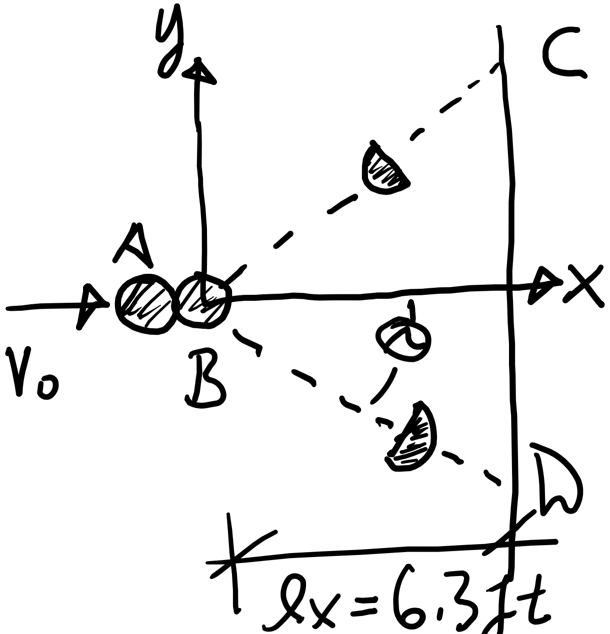

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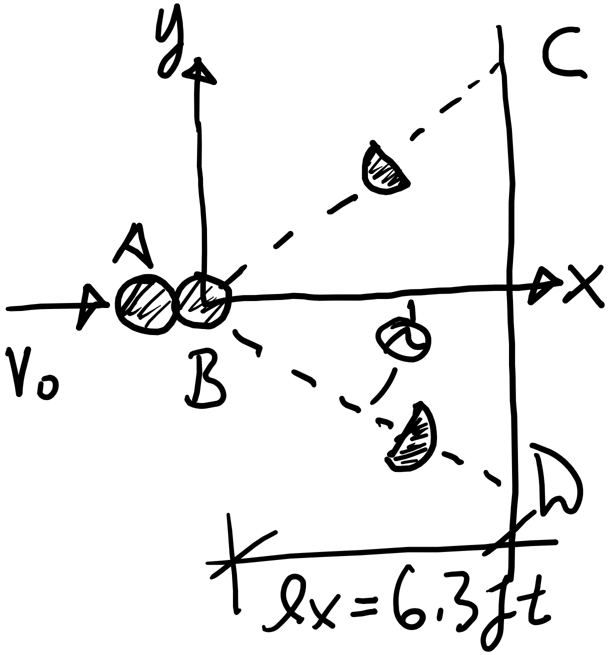

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& $\vec{L}_F = \frac{1}{2}M\vec{V}_C + \frac{1}{2}M\vec{V}_D + M\vec{V}_A$

$\Sigma O \quad mV_0 = \frac{1}{2}m\left(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}\right) + mV_{AF}\hat{i}$

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ASU

$\mathcal{L}$ O $\quad M V_0 = \frac{1}{2} M \left(V_{cx} \hat{i} + V_{cy} \hat{j} + V_{Dx} \hat{i} + V_{Dy} \hat{j} \right) + M V_{AF} \hat{i}$

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$V_A = V_0 - \frac{1}{2}l_x\left(\frac{1}{\Delta t_c} + \frac{1}{\Delta t_D}\right)$

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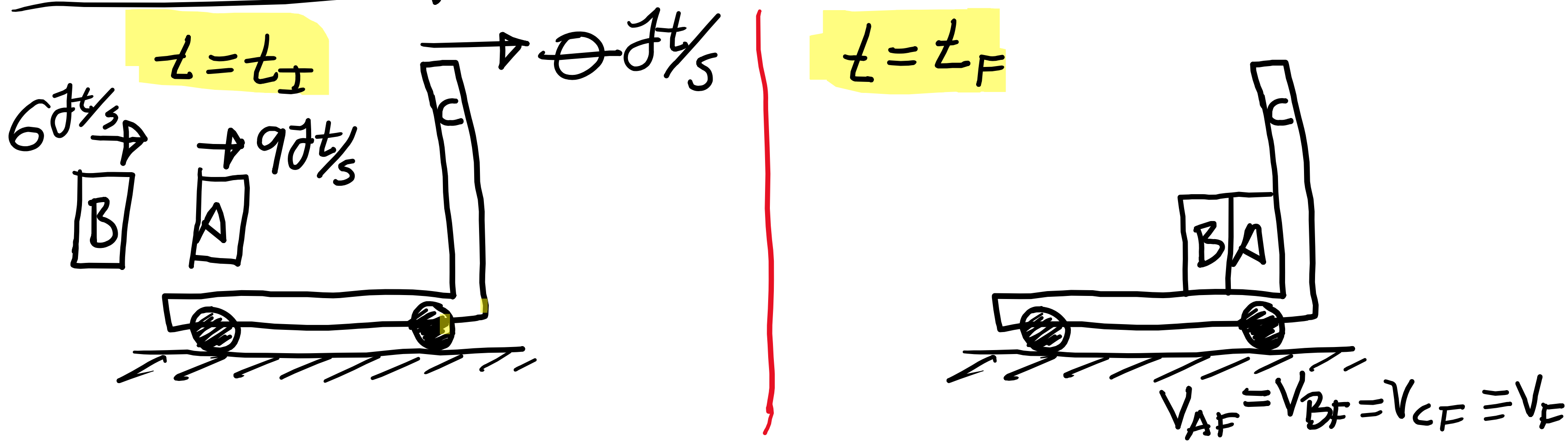
$$V_A = V_0 - \frac{1}{2}l_x\left(\frac{1}{\Delta t_c} + \frac{1}{\Delta t_D}\right) = \left[16 - \left(\frac{6.3}{2}\right)\left(\frac{1}{0.7} + \frac{1}{0.9}\right)\right] ft/s$$

$$= \left[16 - \left(\frac{6.3}{2}\right)\left(\frac{0.9+0.7}{0.63}\right)\right] ft/s = \left[16 - (5)(1.6)\right] ft/s = 8\ ft/s$$

Another example: Two masses thrown onto carrier



$$W_A = 30\,lb,\quad W_B = 40\,lb,\quad W_C = 50\,lb$$

Find V_F:

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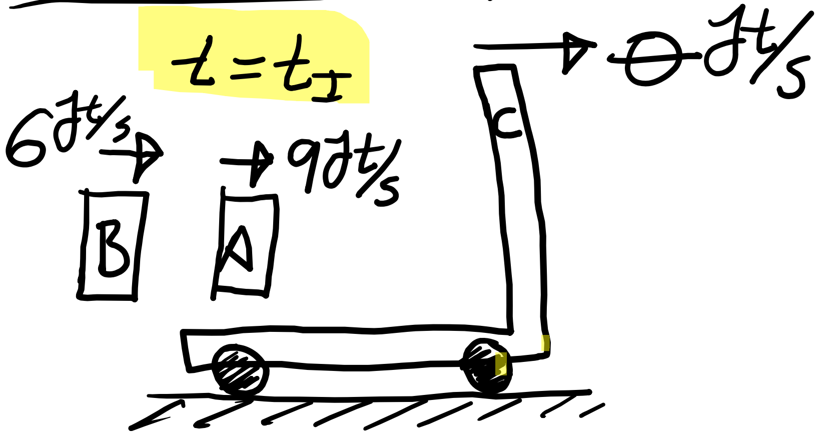

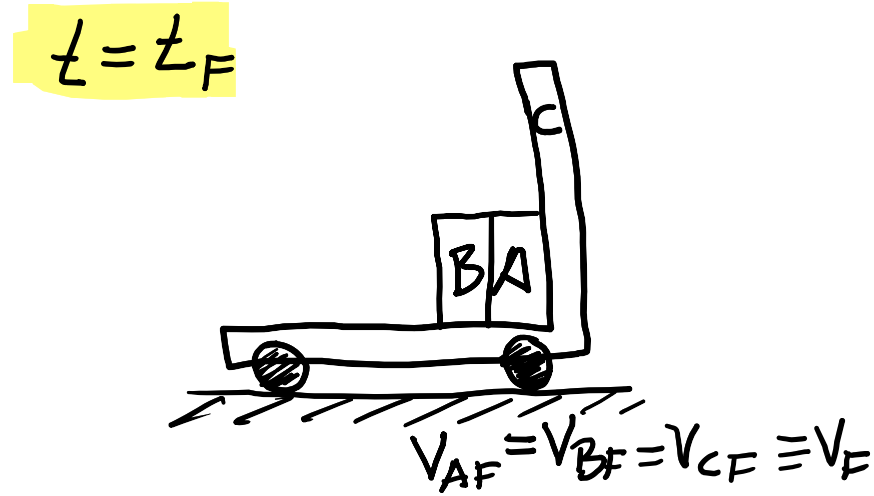

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Find v_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

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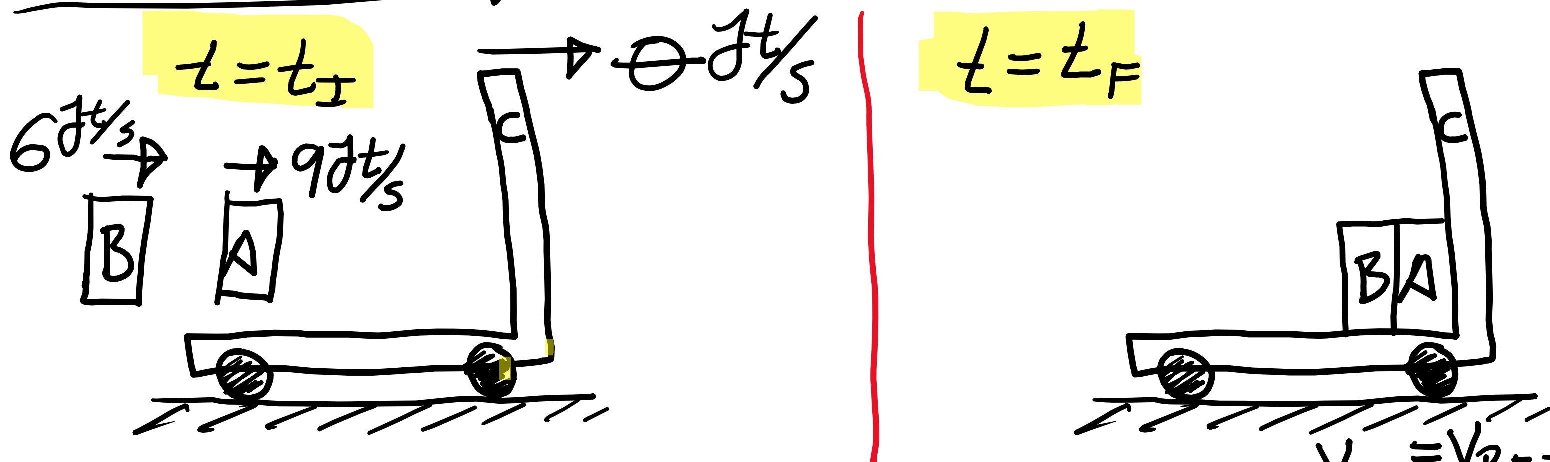

$$V_{AF} = V_{BF} = V_{CF} = V_F$$

$$W_A = 30\,lb,\ W_B = 40\,lb,\ W_c = 50\,lb$$

Find V_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} + M_c V_{CI} = M_A V_{AF} + M_B V_{BF} + M_c V_{CF}$$

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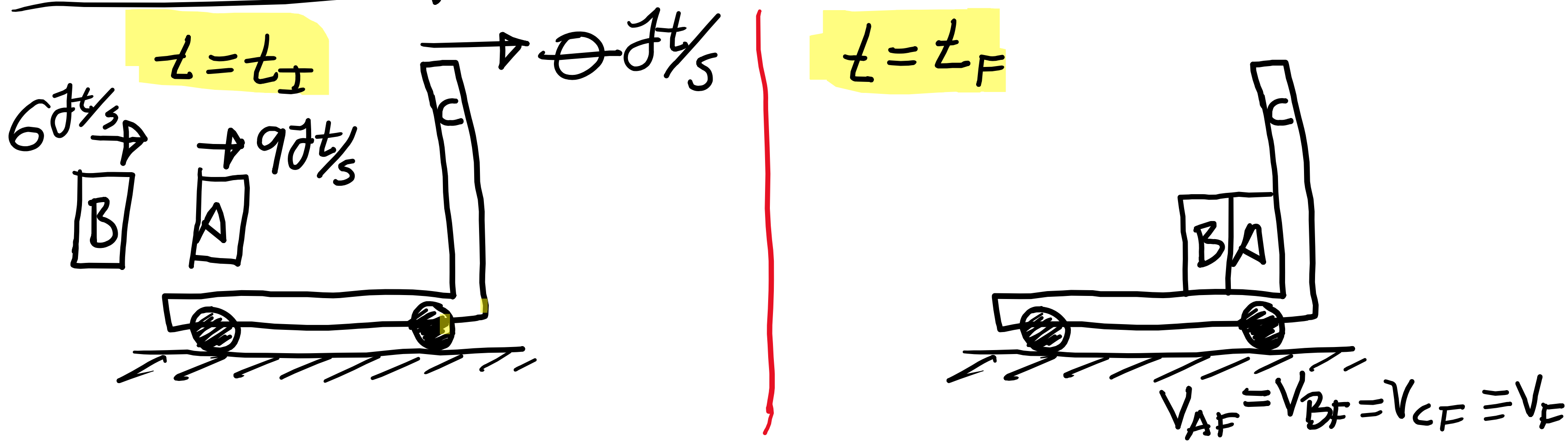

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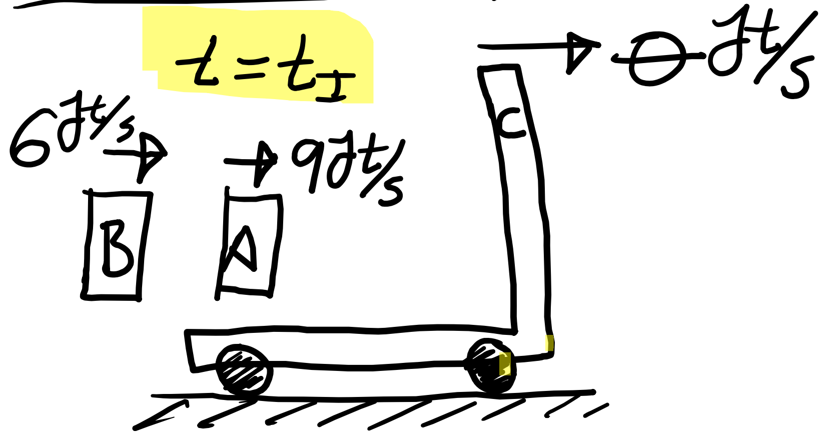

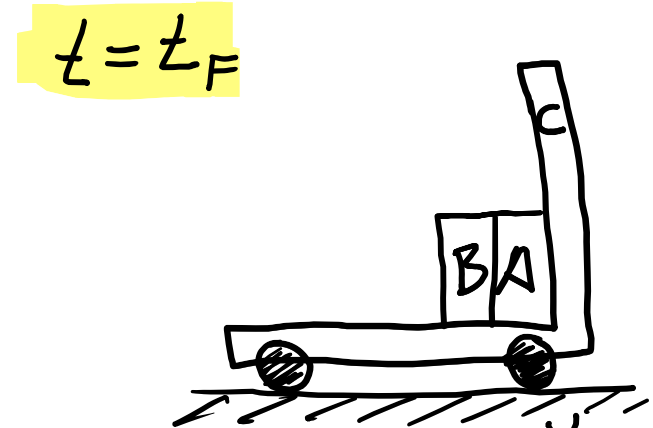

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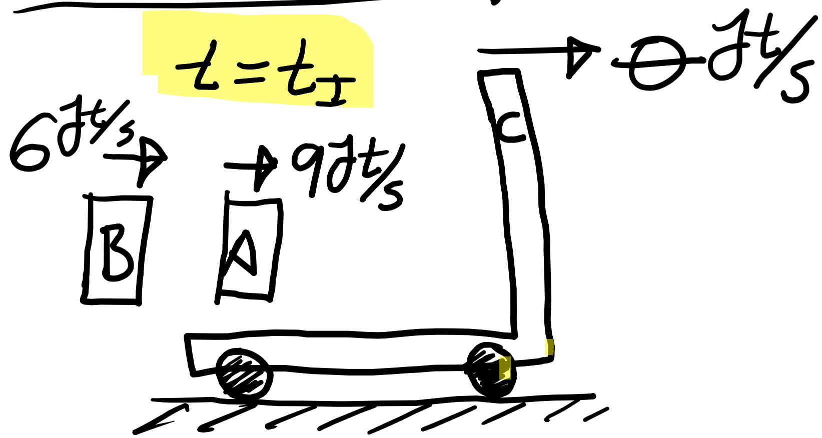

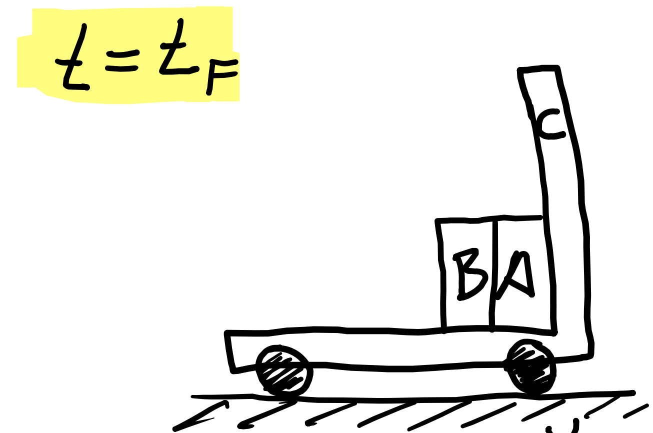

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$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} + M_c V_{CI} = M_A V_{AF} + M_B V_{BF} + M_c V_{CF}$$

$$V_{CI} = 0$$

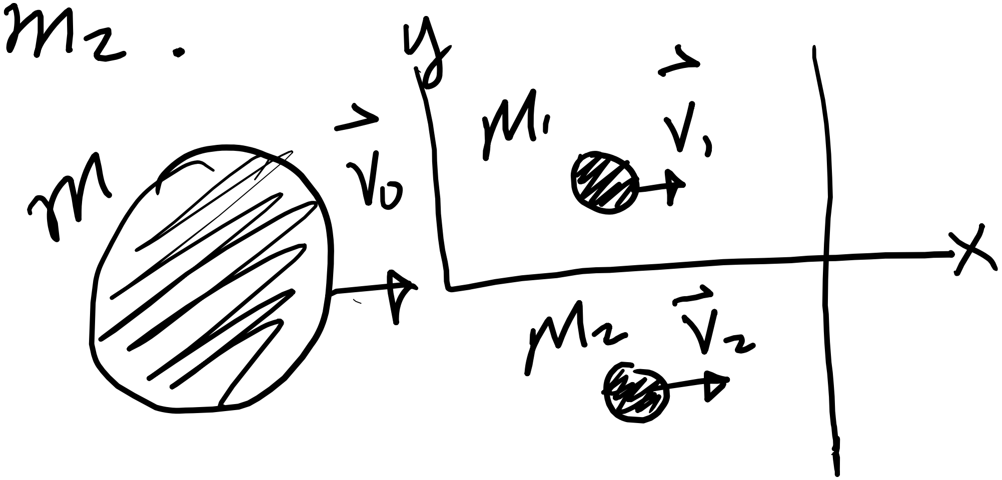
$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} = (M_A + M_B + M_c) V_F \Rightarrow$$

$$V_F = \left(\frac{M_A V_{AI} + M_B V_{BI}}{M_A + M_B + M_c}\right)\left(\frac{g}{g}\right) = \frac{W_A V_{AI} + W_B V_{BF}}{W_A + W_B + W_c} = \left(\frac{30*9 + 40*6}{30+40+50}\right) ft/s$$

$$\Rightarrow \quad V_F = 4.25\, ft/s$$

ASU

Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.



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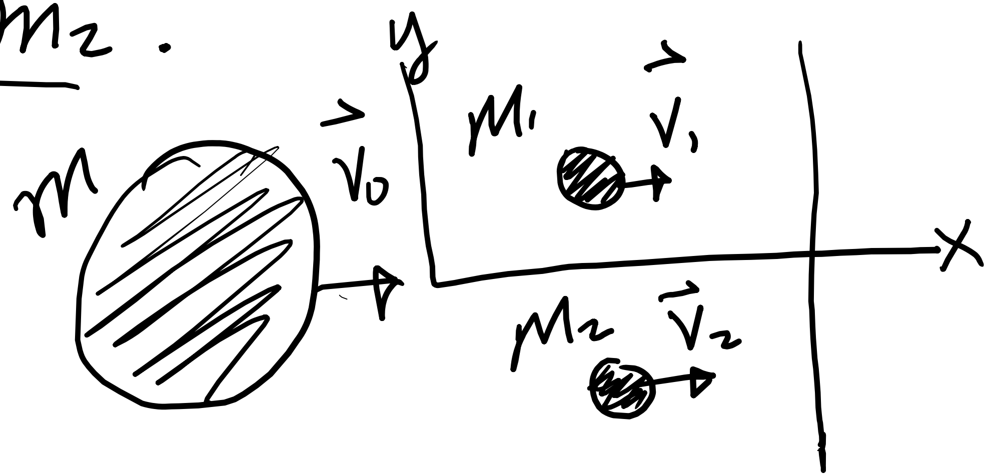

Given: $m = m_1 + m_2$ &

$m_1 = 6\,kg$, $m_2 = 4\,kg$ &

$\vec{v}_0 = 10\frac{m}{s}\hat{i}$, $\vec{v}_1 = 6\frac{m}{s}\hat{i}$,

m_1 hits wall at $y_1 = \frac{1}{4}m$ & m_2 hits wall at $y_2 = -\frac{1}{2}m$.

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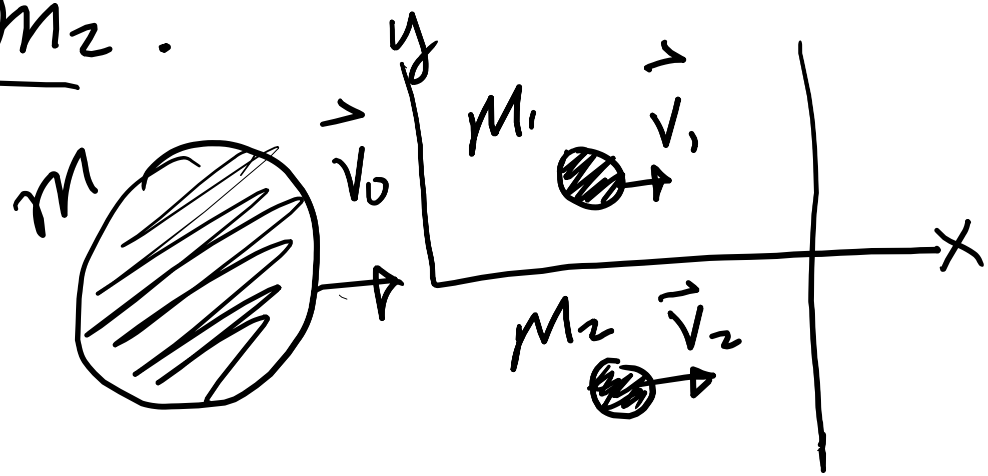

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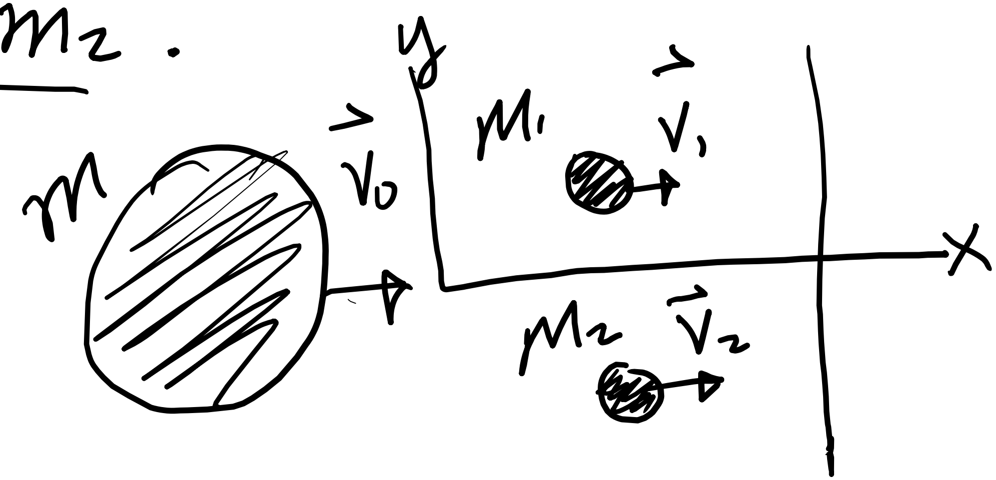

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We will use conservation of angular momentum & conservation of linear momentum.

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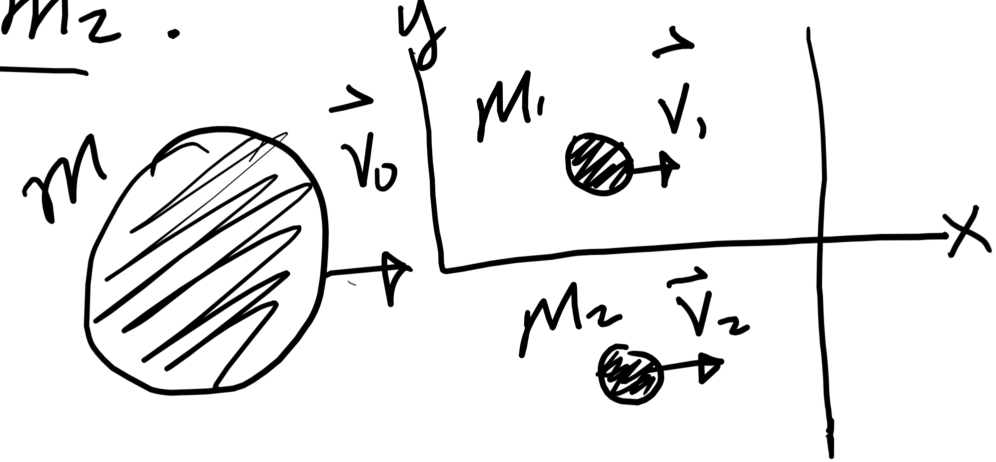

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$$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$$

$$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 \text{, where } \vec{L}_1 = m_1\vec{v}_1$$

& $\vec{L}_2 = m_2\vec{v}_2$.

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

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$$L_I = L_F \Rightarrow (m_1 + m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \quad \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \quad \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,kg)10\tfrac{m}{s}\hat{i} - (6\,kg)6\tfrac{m}{s}\hat{i}$$

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$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

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$$= [100-36]\frac{kg\cdot m}{s}\,\hat{i} = 64\frac{kg\cdot m}{s}\,\hat{i}$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

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$$= [100-36]\tfrac{kg\cdot m}{s}\hat{i} = 64\tfrac{kg\cdot m}{s}\hat{i}$$

$$\text{\&} \quad \vec{L}_1 = (6\,kg)(6\tfrac{m}{s})\hat{i} = 36\tfrac{kg\cdot m}{s}\hat{i}$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

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$$= [100-36]\tfrac{kg\cdot m}{s}\hat{i} = 64\tfrac{kg\cdot m}{s}\hat{i}$$

$$\text{\&}\ \vec{L}_1 = (6\,kg)(6\tfrac{m}{s})\hat{i} = 36\tfrac{kg\cdot m}{s}\hat{i}$$

Now $\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 = (r_{1x}\hat{i} + r_{1y}\hat{j}) \times (36\tfrac{kg\cdot m}{s})\hat{i}$

$+ (r_{2x}\hat{i} + r_{2y}\hat{j}) \times (64\tfrac{kg\cdot m}{s})\hat{i}$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

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$$= [100-36]\frac{kg\cdot m}{s}\hat{i} = 64\frac{kg\cdot m}{s}\hat{i}$$

$$\text{\& } \vec{L}_1 = (6\,kg)(6\tfrac{m}{s})\hat{i} = 36\frac{kg\cdot m}{s}\hat{i}$$

Now $\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 = (r_{1x}\hat{i} + r_{1y}\hat{j}) \times (36\frac{kg\cdot m}{s})\hat{i}$

$+ (r_{2x}\hat{i} + r_{2y}\hat{j}) \times (64\frac{kg\cdot m}{s})\hat{i}$ But $\hat{i}\times\hat{i} = 0$

& $\hat{j}\times\hat{i} = -\hat{k}$ So $\longrightarrow$

$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}),$$ where

$$r_{1y} = y_1 = \frac{1}{4}\,m \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,m$$

$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}), \text{ where}$$

$$r_{1y} = y_1 = \frac{1}{4}\,m \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,m$$

$$\Rightarrow \vec{H}_G = \left(\frac{36}{4} - \frac{64}{4}\right)(-\hat{k})\,\frac{kg\cdot m^2}{s}$$

$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}), \text{ where}$$

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$$\Rightarrow \vec{H}_G = \left(\frac{36}{4} - \frac{64}{4}\right)(-\hat{k})\,\frac{kg\cdot m^2}{s} = (9-16)(-\hat{k})\,\frac{kg\cdot m^2}{s}$$

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$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}), \text{ where}$$

$$r_{1y} = y_1 = \frac{1}{4}\,m \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,m$$

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$$\Rightarrow \quad \vec{H}_G = 7\hat{k}\left(\frac{kg\cdot m^2}{s}\right)$$

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