Today 14.2

$\boxed{L15}$

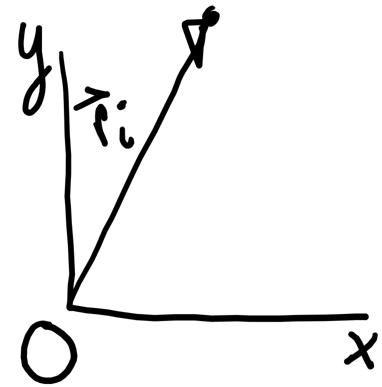
Today 14.2

Wednesday 14.2 } Energy & work

$\boxed{L15}$

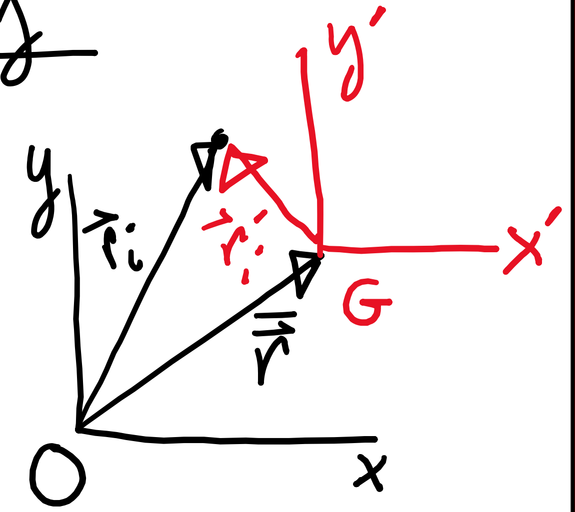
Kinetic energy

$$T = \frac{1}{2}\sum_i m_i v_i^2$$



Kinetic energy

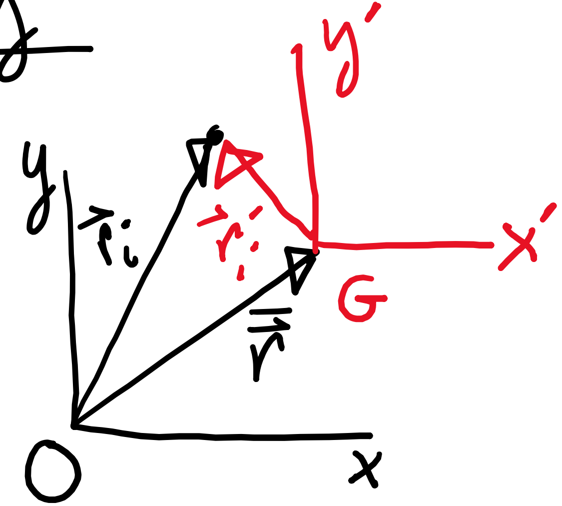
$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \bar{\vec{r}} + \vec{r}_i{}'$



Kinetic energy

$T = \frac{1}{2} \sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}'$

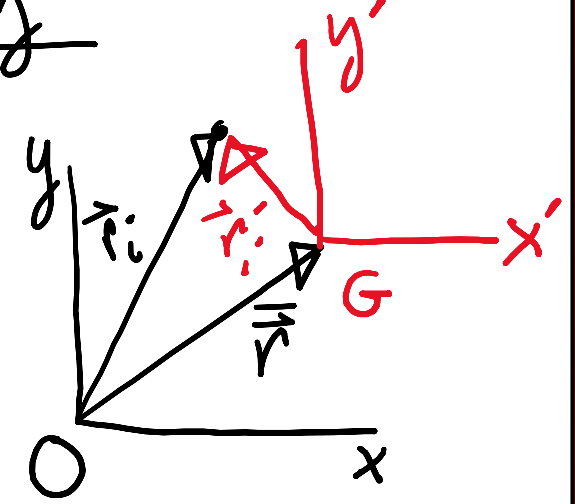
$\Rightarrow \vec{v}_i = \vec{\bar{V}} + \vec{v}_i{}'$



Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}'$

$\Rightarrow \vec{v}_i = \vec{\bar{V}} + \vec{v}_i{}'$ Also $v_i^2 = |\vec{v}_i|^2$

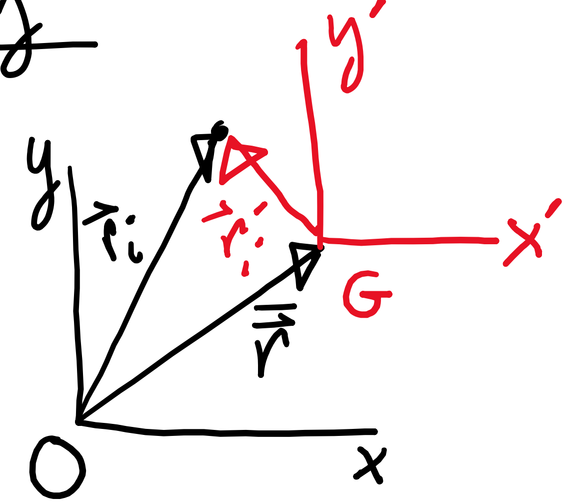


Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}'$

$\Rightarrow \vec{v}_i = \vec{\bar{V}} + \vec{v}_i{}'$ Also $v_i^2 = |\vec{v}_i|^2$

$= \vec{v}_i \cdot \vec{v}_i$

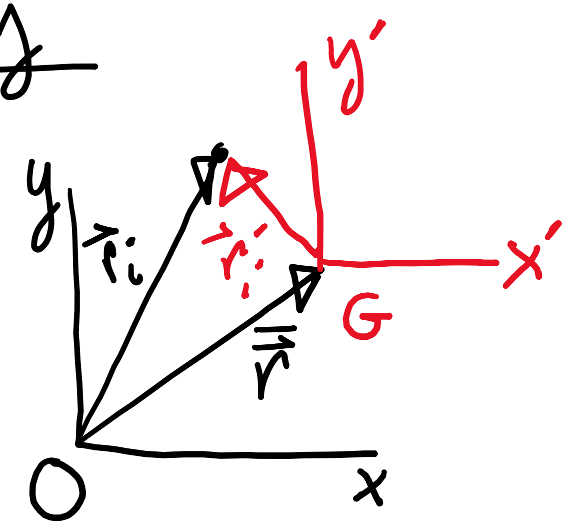


Kinetic energy

$T = \frac{1}{2} \sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i'$

$\Rightarrow \vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ Also $v_i^2 = |\vec{v}_i|^2$

$= \vec{v}_i \cdot \vec{v}_i = (\vec{\bar{v}} + \vec{v}_i') \cdot (\vec{\bar{v}} + \vec{v}_i')$



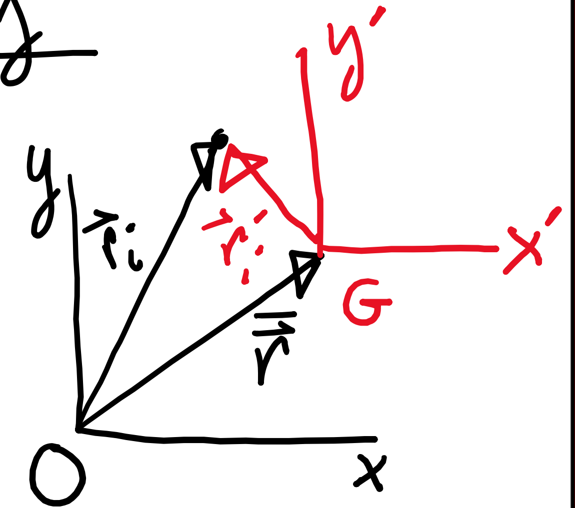
Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i'$

$\Rightarrow \vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ Also $v_i^2 = |\vec{v}_i|^2$

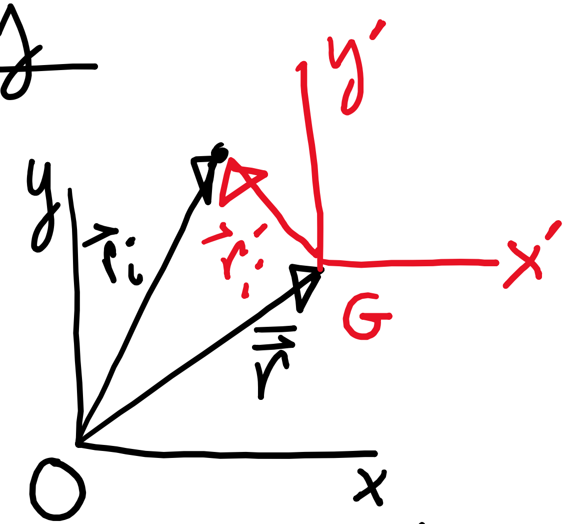
$= \vec{v}_i \cdot \vec{v}_i = (\vec{\bar{v}} + \vec{v}_i') \cdot (\vec{\bar{v}} + \vec{v}_i')$

$= |\vec{\bar{v}}|^2 + |\vec{v}_i'|^2 + 2\vec{\bar{v}} \cdot \vec{v}_i'$



Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i\,'$



$\Rightarrow \vec{v}_i = \vec{\bar{v}} + \vec{v}_i\,'$ Also $v_i^2 = |\vec{v}_i|^2$

$= \vec{v}_i \cdot \vec{v}_i = (\vec{\bar{v}} + \vec{v}_i\,') \cdot (\vec{\bar{v}} + \vec{v}_i\,')$

$= |\vec{\bar{v}}|^2 + |\vec{v}_i\,'|^2 + 2\vec{\bar{v}} \cdot \vec{v}_i\,'$ This means that

$$T = \frac{1}{2}\bar{v}^2 \sum_i m_i + \frac{1}{2}\sum_i m_i (v_i\,')^2 + \vec{\bar{v}} \cdot \left(\sum_i m_i \vec{v}_i\,'\right)$$

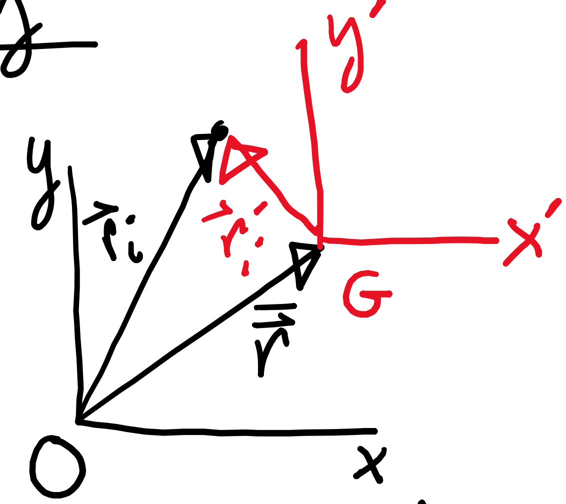
Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i\,'$

$\Rightarrow \vec{v}_i = \vec{\bar{V}} + \vec{v}_i\,'$ Also $v_i^2 = |\vec{v}_i|^2$

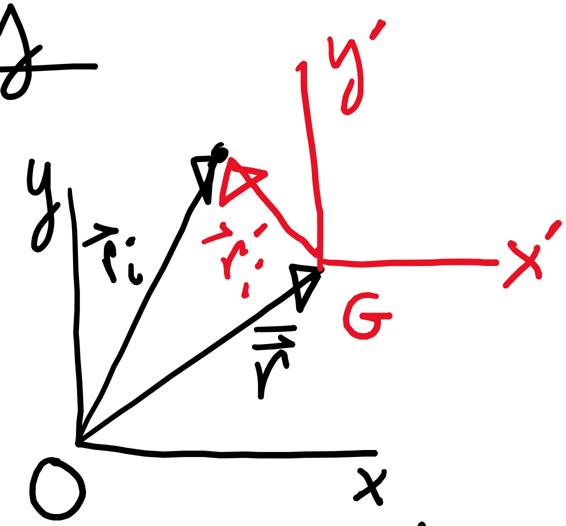
$= \vec{v}_i \cdot \vec{v}_i = (\vec{\bar{V}} + \vec{v}_i\,') \cdot (\vec{\bar{V}} + \vec{v}_i\,')$

$= |\vec{\bar{V}}|^2 + |\vec{v}_i\,'|^2 + 2\vec{\bar{V}} \cdot \vec{v}_i\,'$ This means that



$T = \frac{1}{2}\bar{V}^2 \sum_i m_i + \frac{1}{2}\sum_i m_i (v_i\,')^2 + \vec{\bar{V}} \cdot \left(\sum_i m_i \vec{v}_i\,'\right)$ <u>Note</u>: $\sum_i m_i = M$

Kinetic energy



$T=\frac{1}{2}\sum_i m_i v_i^2$ But $\vec r_i=\vec{\bar r}+\vec r_i'$

$\Rightarrow \vec v_i=\vec{\bar v}+\vec v_i'$ Also $v_i^2=|\vec v_i|^2$

$=\vec v_i\cdot\vec v_i=(\vec{\bar v}+\vec v_i')\cdot(\vec{\bar v}+\vec v_i')$

$=|\vec{\bar v}|^2+|\vec v_i'|^2+2\vec{\bar v}\cdot\vec v_i'$ This means that

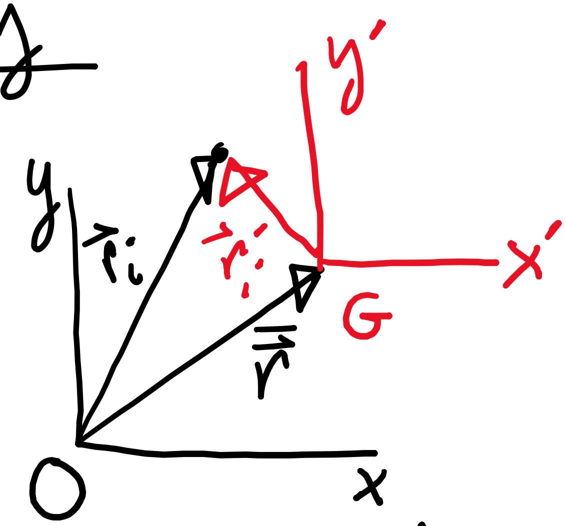
$T=\frac{1}{2}\bar v^2\sum_i m_i+\frac{1}{2}\sum_i m_i(v_i')^2+\vec{\bar v}\cdot\left(\sum_i m_i\vec v_i'\right)$ <u>Note</u>: $\sum_i m_i=M$

& $\sum_i m_i\vec v_i'=\vec L_G=0$ → Linear momentum in CM frame

So $T=\frac{1}{2}M\bar v^2+\frac{1}{2}\sum_i m_i(\vec v_i')^2$

Kinetic energy

$T = \frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i\,'$



$\Rightarrow \vec{v}_i = \vec{\bar{V}} + \vec{v}_i\,'$ Also $v_i^2 = |\vec{v}_i|^2$

$= \vec{v}_i \cdot \vec{v}_i = (\vec{\bar{V}} + \vec{v}_i\,') \cdot (\vec{\bar{V}} + \vec{v}_i\,')$

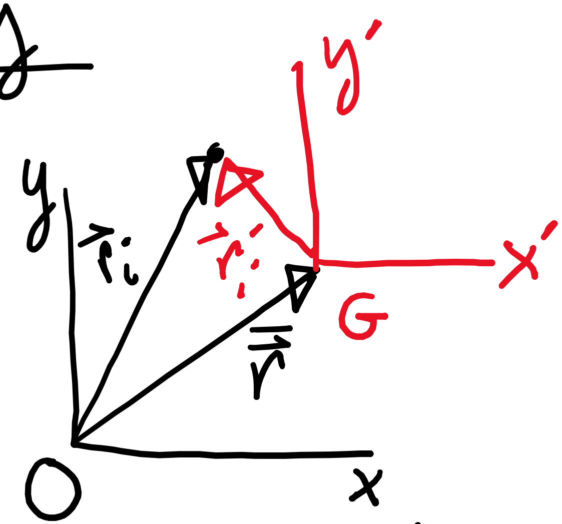
$= |\vec{\bar{V}}|^2 + |\vec{v}_i\,'|^2 + 2\vec{\bar{V}} \cdot \vec{v}_i\,'$ This means that

$T = \frac{1}{2}\bar{V}^2 \sum_i m_i + \frac{1}{2}\sum_i m_i (v_i')^2 + \vec{\bar{V}} \cdot \left(\sum_i m_i \vec{v}_i\,'\right)$ Note: $\sum_i m_i = M$

& $\sum_i m_i \vec{v}_i\,' = \vec{L}_G = 0$ → Linear momentum in CM frame

So $T = \frac{1}{2} M \bar{V}^2 + \frac{1}{2}\sum_i m_i (\vec{v}_i\,')^2$ → K.E. as seen in the CM frame

Kinetic energy



$T=\frac{1}{2}\sum_i m_i v_i^2$ But $\vec{r}_i=\vec{\bar{r}}+\vec{r}_i'$

$\Rightarrow \vec{v}_i=\vec{\bar{V}}+\vec{v}_i'$ Also $v_i^2=|\vec{v}_i|^2$

$=\vec{v}_i\cdot\vec{v}_i=(\vec{\bar{V}}+\vec{v}_i')\cdot(\vec{\bar{V}}+\vec{v}_i')$

$=|\vec{\bar{V}}|^2+|\vec{v}_i'|^2+2\vec{\bar{V}}\cdot\vec{v}_i'$ This means that

$T=\frac{1}{2}\bar{V}^2\sum_i m_i+\frac{1}{2}\sum_i m_i(v_i')^2+\vec{\bar{V}}\cdot\left(\sum_i m_i\vec{v}_i'\right)$ <u>Note</u>: $\sum_i m_i=M$

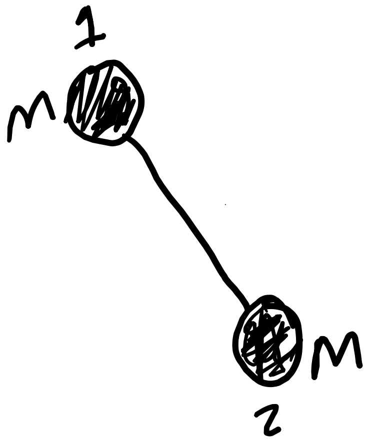
& $\underbrace{\sum_i m_i\vec{v}_i'=\vec{L}_G=0}$ → Linear momentum in CM frame

So $T=\underbrace{\frac{1}{2}M\bar{V}^2}_{\text{K.E. for single particle of mass } M=\sum_i m_i \text{ with velocity of cm frame } \vec{\bar{V}}}+\underbrace{\frac{1}{2}\sum_i m_i(\vec{v}_i')^2}_{\text{K.E. as seen in the cm frame}}$

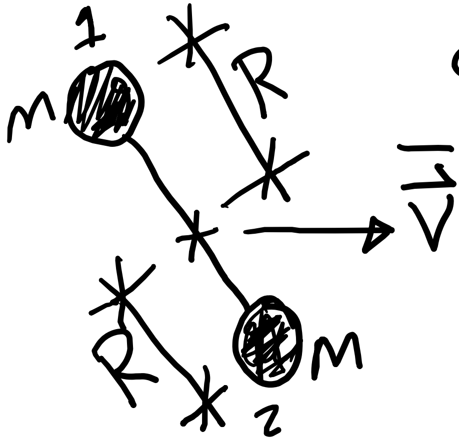
→ K.E. as seen in the cm frame

→ K.E. for single particle of mass $M=\sum_i m_i$ with velocity of cm frame $\vec{\bar{V}}$

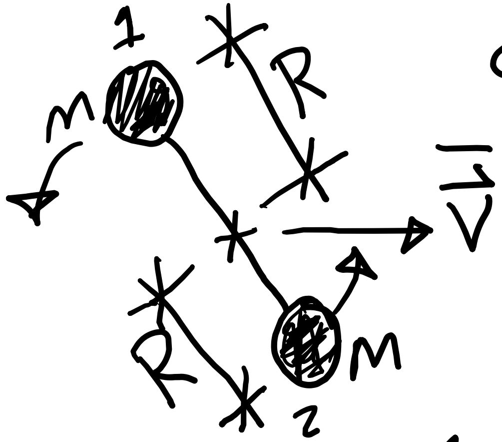
Example: Two equal masses held together by a massless string.



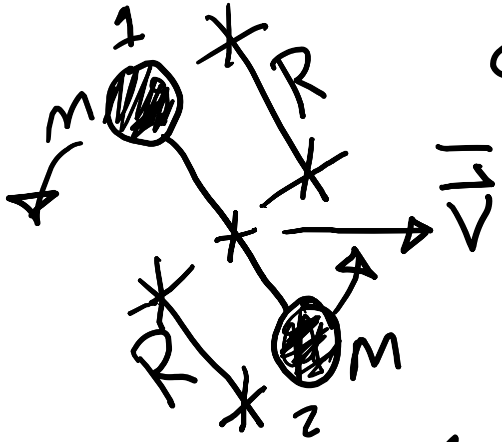
Example: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\vec{V}$



Example: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\vec{V}$. Given that the masses revolve about center of mass with angular speed $\dot{\theta}$,

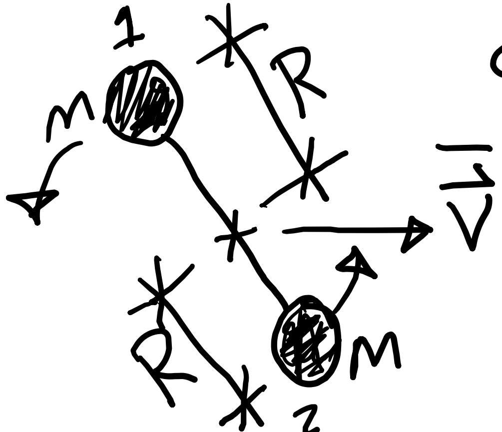


Example: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\vec{V}$. Given that the masses revolve about center of mass with angular speed $\dot{\theta}$, find an expression for the kinetic energy T:



$V_1' = V_2' = R\dot{\theta}$

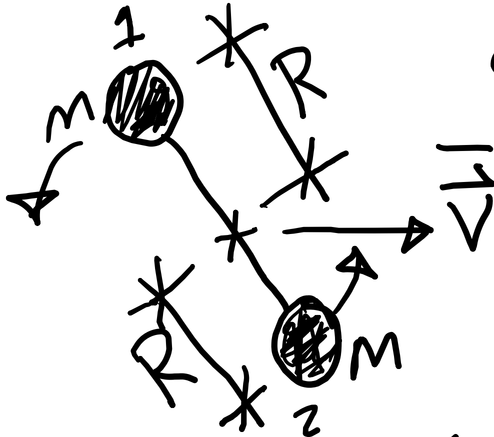
Example: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\vec{\bar{V}}$. Given that the masses revolve about center of mass with angular speed $\dot{\Theta}$, find an expression for the kinetic energy T:



$$V_1' = V_2' = R\dot{\Theta} \Rightarrow T = \tfrac{1}{2}M\bar{V}^2 + \tfrac{1}{2}M_1R^2\dot{\Theta}^2 + \tfrac{1}{2}M_2R^2\dot{\Theta}^2$$

But $M = \sum_i M_i = M_1 + M_2$ & $M_1 = M_2 = M$

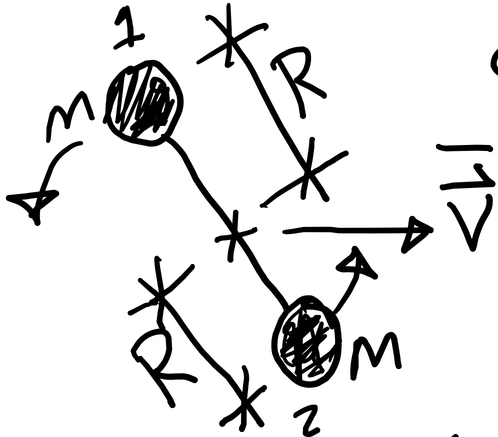
Example: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\bar{\vec{V}}$. Given that the masses revolve about center of mass with angular speed $\dot{\Theta}$, find an expression for the kinetic energy T:



$$V_1' = V_2' = R\dot{\Theta} \Rightarrow T = \frac{1}{2}M\bar{V}^2 + \frac{1}{2}M_1R^2\dot{\Theta}^2 + \frac{1}{2}M_2R^2\dot{\Theta}^2$$

But $M = \sum_i M_i = M_1 + M_2$ & $M_1 = M_2 = M$ so

Example: Two equal masses held together by a massless string. For this case, the



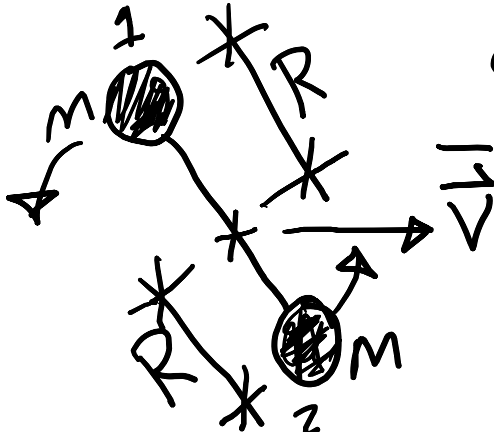
center of mass is at the mid-point along the string & is moving with velocity $\bar{V}$. Given that the masses revolve about center of mass with angular speed $\dot{\theta}$, find an expression for the kinetic energy T:

$v_1' = v_2' = R\dot{\theta} \Rightarrow T = \frac{1}{2}M\bar{V}^2 + \frac{1}{2}m_1R^2\dot{\theta}^2 + \frac{1}{2}m_2R^2\dot{\theta}^2$

But $M = \sum_i m_i = m_1 + m_2$ & $m_1 = m_2 = m$ so

$$T = \frac{1}{2}M\bar{V}^2 + \frac{1}{2}MR^2\dot{\theta}^2.$$

<u>Example</u>: Two equal masses held together by a massless string. For this case, the center of mass is at the mid-point along the string & is moving with velocity $\bar{\vec{V}}$. Given that the masses revolve about center of mass with angular speed $\dot{\theta}$, find an expression for the kinetic energy T:



$$V_1' = V_2' = R\dot{\theta} \Rightarrow T = \tfrac{1}{2}M\bar{V}^2 + \tfrac{1}{2}M_1R^2\dot{\theta}^2 + \tfrac{1}{2}M_2R^2\dot{\theta}^2$$

But $M = \sum_i M_i = M_1 + M_2$ & $M_1 = M_2 = M$ so

$$T = \tfrac{1}{2}M\bar{V}^2 + \tfrac{1}{2}MR^2\dot{\theta}^2.$$ Now find $\vec{L}$ & $\vec{H}_G'$

The evaluation of $\vec{L}$ & $\vec{H}'_G$ are straight-forward

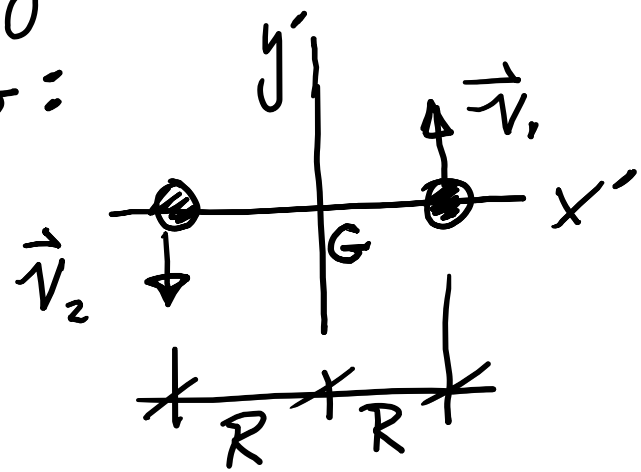
The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$\vec{L} = m\vec{v}$ & $\vec{H}_G' = [m_1 R v_1' + m_2 R v_2'] = m R^2 \dot{\theta}$

The evaluation of $\vec{L}$ & $\vec{H}'_G$ are straight-forward

$$\vec{L} = M\vec{V} \quad \& \quad \vec{H}'_G = \left[m_1 R V_1' + m_2 R V_2'\right](\circlearrowleft) = mR^2\dot{\theta}\,(\circlearrowleft)$$

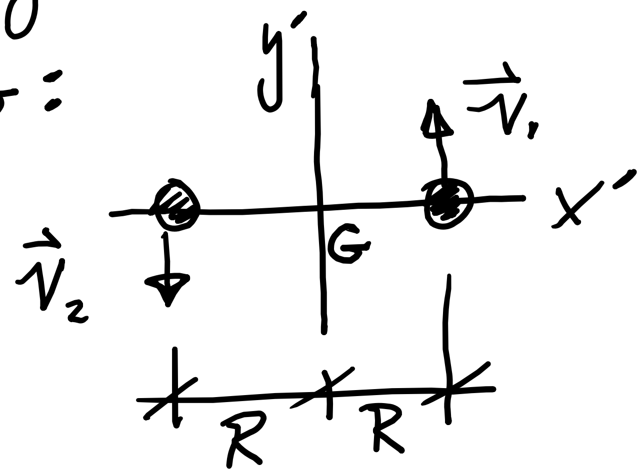
<u>Now</u> string breaks such that in C.M. Frame G:



The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$\vec{L} = m\vec{v}$ & $\vec{H}_G' = \left[m_1 R v_1' + m_2 R v_2'\right] \circlearrowleft = mR^2\dot{\theta} \circlearrowleft$

<u>Now</u> string breaks such that in C.M. Frame G:

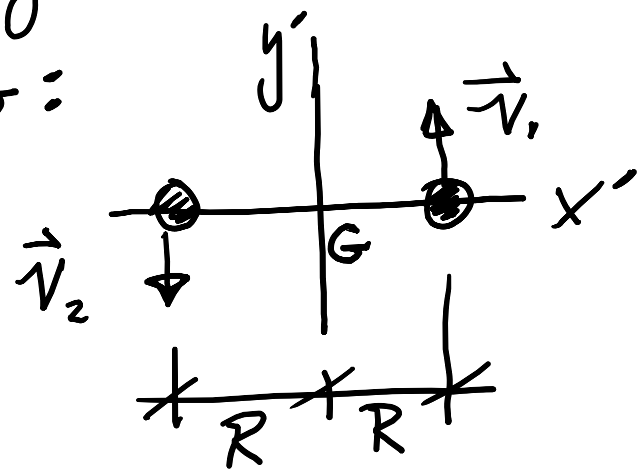


Find $\vec{H}_G'$:

The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$\vec{L} = M\vec{\bar{V}}$ & $\vec{H}_G' = [m_1 R V_1' + m_2 R V_2']\circlearrowleft = mR^2\dot{\theta}\circlearrowleft$

<u>Now</u> string breaks such that in C.M. Frame G:



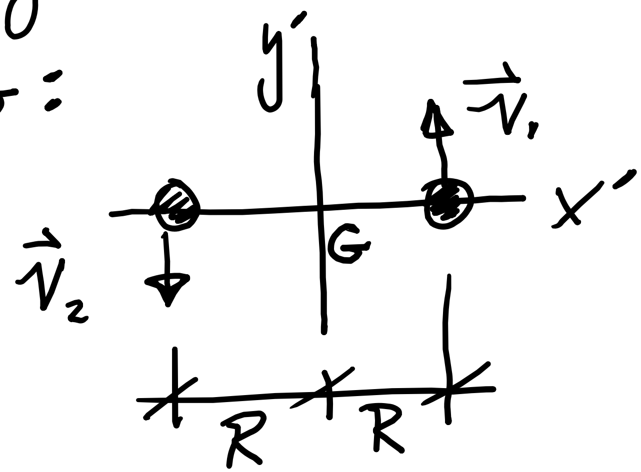
Find $\vec{H}_G'$:

No more spinning but $\vec{H}_G' = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$

The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$$\vec{L} = m\vec{\bar{v}} \quad \& \quad \vec{H}_G' = [m_1 R v_1' + m_2 R v_2'](\circlearrowleft) = mR^2\dot{\theta}(\circlearrowleft)$$

<u>Now</u> string breaks such that in C.M. Frame G:



Find $\vec{H}_G'$:

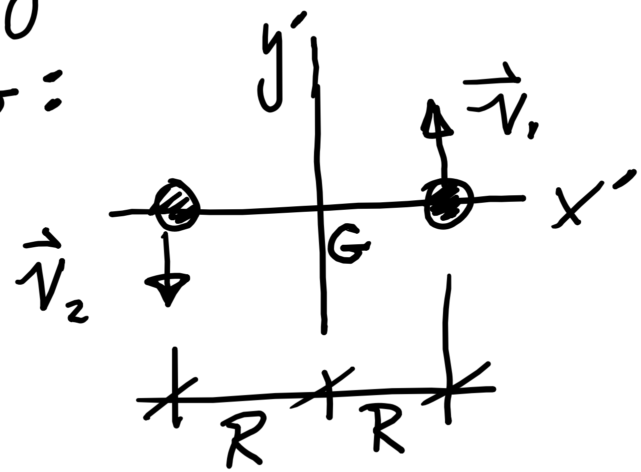
No more spinning but $\vec{H}_G' = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$

$$= (R\hat{i}) \times (m_1 v_1' \hat{j}) + (-R\hat{i}) \times (-m_2 v_2 \hat{j})$$

The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$$\vec{L} = m\vec{v} \quad \& \quad \vec{H}_G' = \left[m_1 R v_1' + m_2 R v_2'\right] \circlearrowleft = mR^2\dot{\theta} \circlearrowleft$$

__Now__ string breaks such that in C.M. Frame G:



Find $\vec{H}_G'$:

No more spinning but $\vec{H}_G' = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$

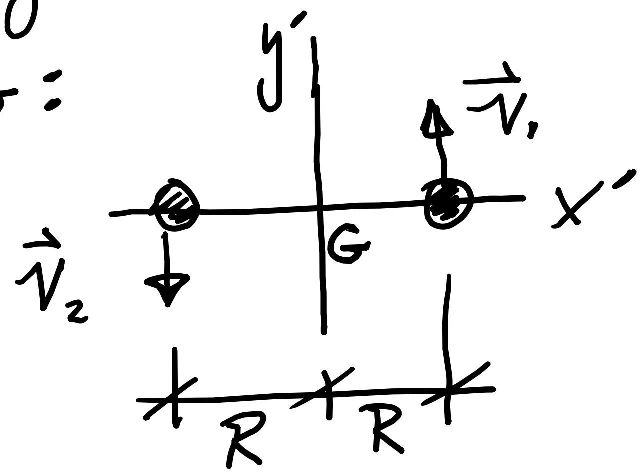
$$= (R\hat{i}) \times (m_1 v_1' \hat{j}) + (-R\hat{i}) \times (-m_2 v_2 \hat{j})$$

$$= m_1 v_1 R\hat{k} + m_2 v_2 R\hat{k}$$

The evaluation of $\vec{L}$ & $\vec{H}_G'$ are straight-forward

$\vec{L} = M\vec{V}$ & $\vec{H}_G' = [m_1 R V_1' + m_2 R V_2'](\circlearrowleft) = mR^2\dot{\theta}(\circlearrowleft)$

<u>Now</u> string breaks such that in C.M. Frame G:



Find $\vec{H}_G'$:

No more spinning but $\vec{H}_G' = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$

$= (R\hat{i}) \times (m_1 V_1' \hat{j}) + (-R\hat{i}) \times (-m_2 V_2 \hat{j})$

$= m_1 V_1 R \hat{k} + m_2 V_2 R \hat{k} = mR\dot{\theta}$ [as it should]

What if we measured v_i from the Stationary frame?

What if we measured v_i from the stationary frame?

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i$$

What if we measured v_i from the Stationary frame?*

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\vec{\bar{v}} + \vec{v}_i')$$

What if we measured v_i from the stationary frame?

Since $\vec{v}_i = \bar{\vec{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\bar{\vec{v}} + \vec{v}_i') = \sum_i m_i \vec{r}_i' \times (\bar{\vec{v}} + \vec{v}_i)$$

What if we measured v_i from the Stationary frame?

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\vec{\bar{v}} + \vec{v}_i') = \sum_i m_i \vec{r}_i' \times (\vec{\bar{v}} + \vec{v}_i')$$

$$= \left(\sum_i m_i \vec{r}_i'\right) \times \vec{\bar{v}} + \sum_i m_i \vec{r}_i' \times \vec{v}_i'$$

What if we measured v_i from the Stationary frame?

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\vec{\bar{v}} + \vec{v}_i') = \sum_i m_i \vec{r}_i' \times (\vec{\bar{v}} + \vec{v}_i')$$

$$= \left(\sum_i m_i \vec{r}_i'\right) \times \vec{\bar{v}} + \sum_i m_i \vec{r}_i' \times \vec{v}_i'$$ But $\sum m_i \vec{r}_i' = 0$

& $\vec{H}_G' = \sum_i m_i \vec{r}_i' \times \vec{v}_i'$

What if we measured v_i from the Stationary frame?

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\vec{\bar{v}} + \vec{v}_i') = \sum_i m_i \vec{r}_i' \times (\vec{\bar{v}} + \vec{v}_i')$$

$$= \left(\sum_i m_i \vec{r}_i'\right) \times \vec{\bar{v}} + \sum_i m_i \vec{r}_i' \times \vec{v}_i'$$

But $\sum m_i \vec{r}_i' = 0$

& $\vec{H}_G' = \sum_i m_i \vec{r}_i' \times \vec{v}_i'$

So $\boxed{\vec{H}_G = \vec{H}_G'}$

What if we measured v_i from the Stationary frame?

Since $\vec{v}_i = \vec{\bar{v}} + \vec{v}_i'$ then

$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i = \sum_i \vec{r}_i' \times m_i (\vec{\bar{v}} + \vec{v}_i') = \sum_i m_i \vec{r}_i' \times (\vec{\bar{v}} + \vec{v}_i')$$

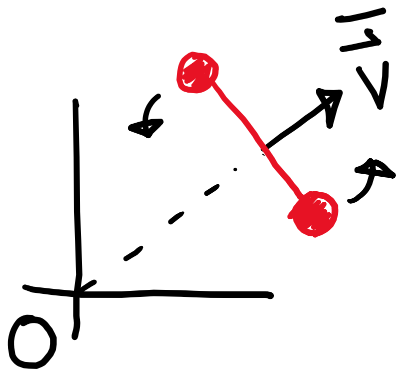
$$= \left(\sum_i m_i \vec{r}_i'\right) \times \vec{\bar{v}} + \sum_i m_i \vec{r}_i' \times \vec{v}_i' \quad \text{But} \quad \sum m_i \vec{r}_i' = 0$$

$\&$ $\vec{H}_G' = \sum_i m_i \vec{r}_i' \times \vec{v}_i'$ So $\boxed{\vec{H}_G = \vec{H}_G'} \Rightarrow$

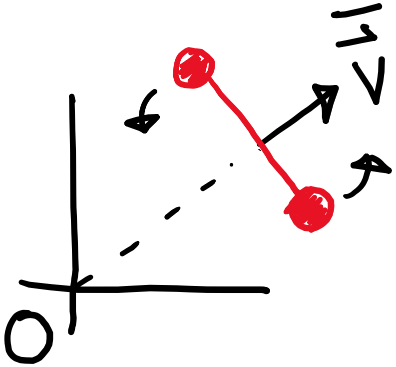
For $\vec{H}_G$ we can use $\vec{v}_i$ or $\vec{v}_i'$ and obtain the same result ☺

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{v}$?

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{v}$?

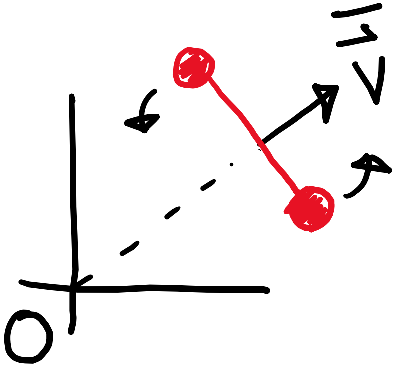


What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{v}$?



$$\vec{H}_o = \sum_i \vec{r}_i \times m_i \vec{v}_i$$

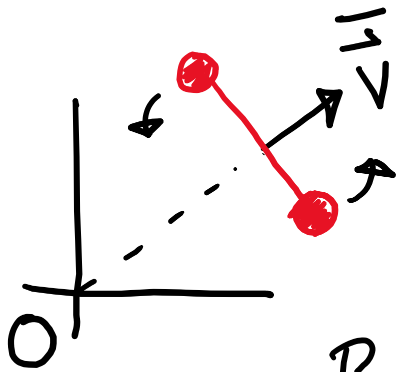
What if we measure $\vec{H}_O$, where the origin is directly in line with $\vec{v}$?



$$\vec{H}_O = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{r} + \vec{r}_i\,') \times m_i \vec{v}_i$$

$$= \vec{r} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i\,' \times m_i \vec{v}_i$$

What if we measure $\vec{H}_O$, where the origin is directly in line with $\vec{\bar{V}}$?

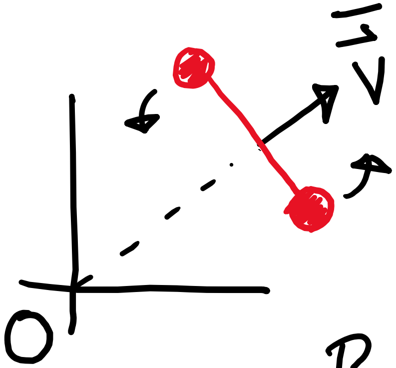


$$\vec{H}_O = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i{}') \times m_i \vec{v}_i$$

$$= \vec{\bar{r}} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i{}' \times m_i \vec{v}_i$$

But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = m \vec{\bar{V}}$

What if we measure $\vec{H}_O$, where the origin is directly in line with $\vec{\bar{v}}$?



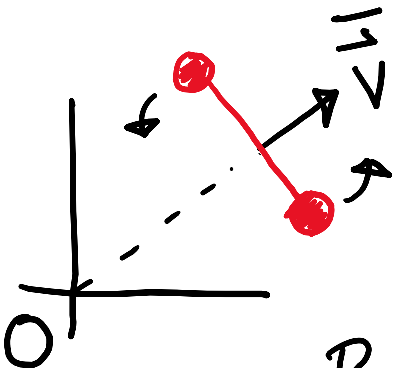
$$\vec{H}_O = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i') \times m_i \vec{v}_i$$

$$= \vec{\bar{r}} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i' \times m_i \vec{v}_i$$

But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = m \vec{\bar{v}}$

and $\sum \vec{r}_i' \times m_i \vec{v}_i = \vec{H}_G$

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{\bar{v}}$?



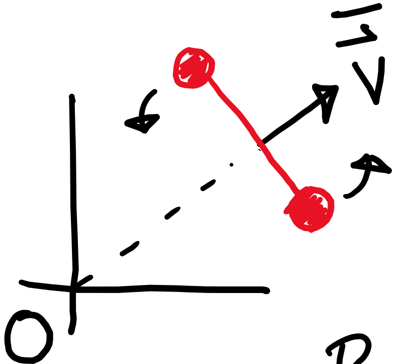
$$\vec{H}_o = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i{}') \times m_i \vec{v}_i$$

$$= \vec{\bar{r}} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i{}' \times m_i \vec{v}_i$$

But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = m \vec{\bar{v}}$

and $\sum \vec{r}_i{}' \times m_i \vec{v}_i = \vec{H}_G$ So $\vec{H}_o = \vec{\bar{r}} \times m \vec{\bar{v}} + \vec{H}_G$

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{\bar{v}}$?



$$\vec{H}_o = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i\,') \times m_i \vec{v}_i$$

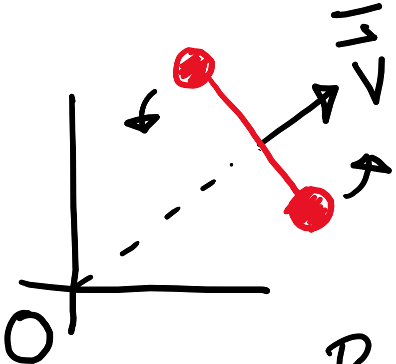
$$= \vec{\bar{r}} \times \underline{\sum_i m_i \vec{v}_i} + \underline{\sum_i \vec{r}_i\,' \times m_i \vec{v}_i}$$

But $\underline{\sum_i m_i \vec{v}_i} = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = \underline{m \vec{\bar{v}}}$

and $\underline{\sum \vec{r}_i\,' \times m_i \vec{v}_i} = \underline{\vec{H}_G}$ So $\vec{H}_o = \vec{\bar{r}} \times m \vec{\bar{v}} + \vec{H}_G$

But $\vec{\bar{r}} \times \vec{\bar{v}} = 0$ since they are in line

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{\bar{v}}$?



$$\vec{H}_o = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i\,') \times m_i \vec{v}_i$$

$$= \vec{\bar{r}} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i\,' \times m_i \vec{v}_i$$

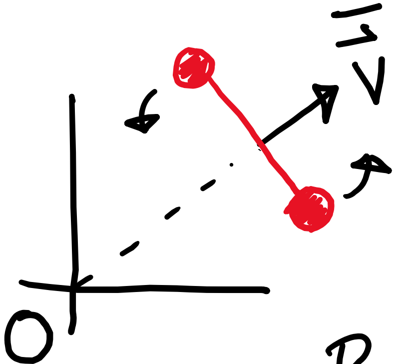
But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = m \vec{\bar{v}}$

and $\sum \vec{r}_i\,' \times m_i \vec{v}_i = \vec{H}_G$ so $\vec{H}_o = \vec{\bar{r}} \times m \vec{\bar{v}} + \vec{H}_G$

But $\vec{\bar{r}} \times \vec{\bar{v}} = 0$ since they are in line

$\Rightarrow$ $\vec{H}_o = \vec{H}_G$ (for this special case)

What if we measure $\vec{H}_o$, where the origin is directly in line with $\vec{\bar{v}}$?



$$\vec{H}_o = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum_i (\vec{\bar{r}} + \vec{r}_i{}') \times m_i \vec{v}_i$$

$$= \vec{\bar{r}} \times \sum_i m_i \vec{v}_i + \sum_i \vec{r}_i{}' \times m_i \vec{v}_i$$

But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m \vec{\bar{r}} = m \vec{\bar{v}}$

and $\sum \vec{r}_i{}' \times m_i \vec{v}_i = \vec{H}_G$ So $\vec{H}_o = \vec{\bar{r}} \times m \vec{\bar{v}} + \vec{H}_G$

But $\vec{\bar{r}} \times \vec{\bar{v}} = 0$ since they are in line

$\Rightarrow$ $\vec{H}_o = \vec{H}_G$ (for this special case). If, however, $\vec{\bar{r}} \times \vec{\bar{v}}$ is non-zero, we would have

$$\vec{H}_o = \vec{\bar{r}} \times m \vec{\bar{v}} + \vec{H}_G$$

Work $: T_1 + U_{1\to 2} = T_2$

Work : $T_1 + U_{1\to 2} = T_2$

Work & Energy : $T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

Work : $T_1 + U_{1\to 2} = T_2$

Work & Energy : $T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

Impulse-momentum : $\sum \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$

Work : $T_1 + U_{1\to 2} = T_2$

Work & Energy : $T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$

Impulse-momentum : $\sum \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$

Torque-impulse : $\sum \int_{t_1}^{t_2} \vec{M}_o\,dt = \Delta \vec{H}_o$

Work : $T_1 + U_{1\to 2} = T_2$

Work & Energy : $T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$

Impulse-momentum : $\sum \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$

Torque-impulse : $\sum \int_{t_1}^{t_2} \vec{M}_O\,dt = \Delta \vec{H}_O$

Just as one would Naively expect ☺