Today 14.2

$\boxed{L16}$

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Friday Review

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Monday Exam #2

Mass center

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& if $\Sigma \vec{F} = \vec{0}$ [No ext. forces]

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& if $\Sigma \vec{M}_O = \vec{0}$ [No ext. torques]

then $\vec{H}_O$ is conserved

ASU

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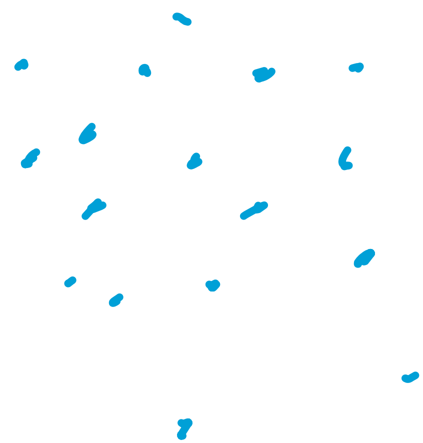

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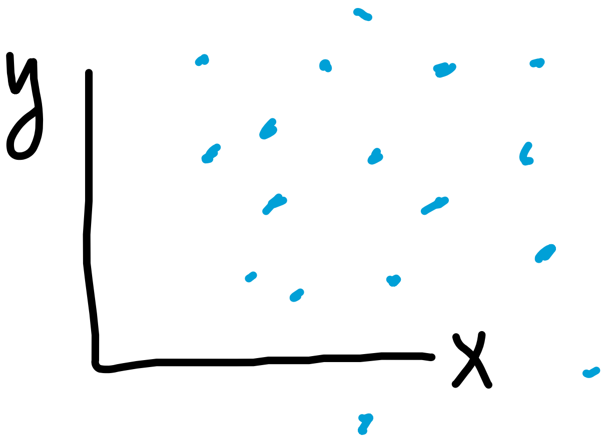

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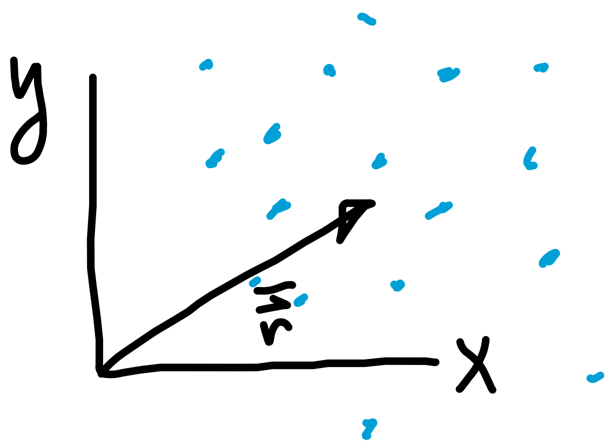

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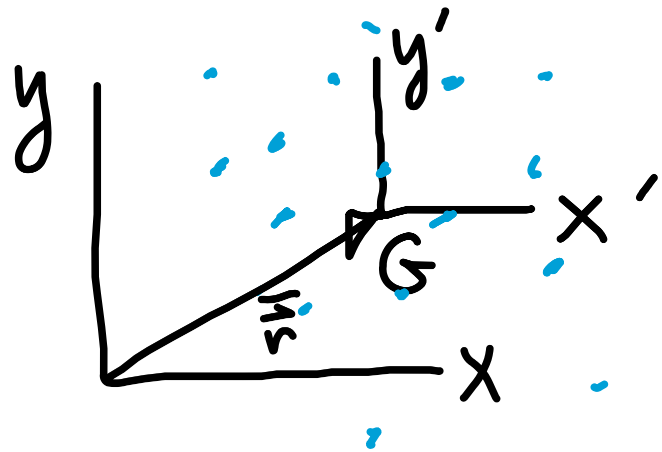

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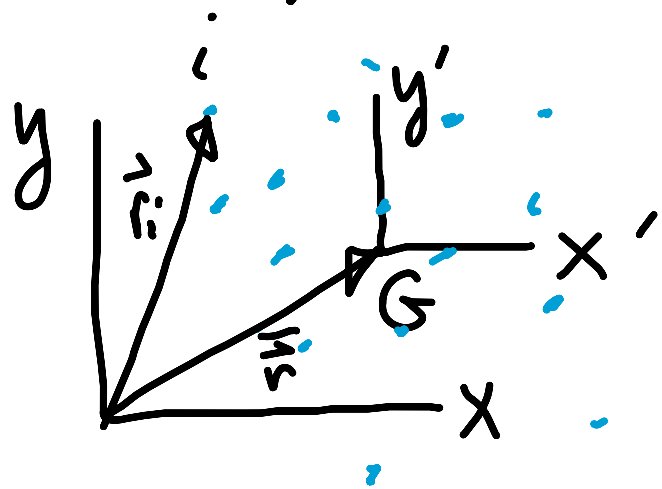

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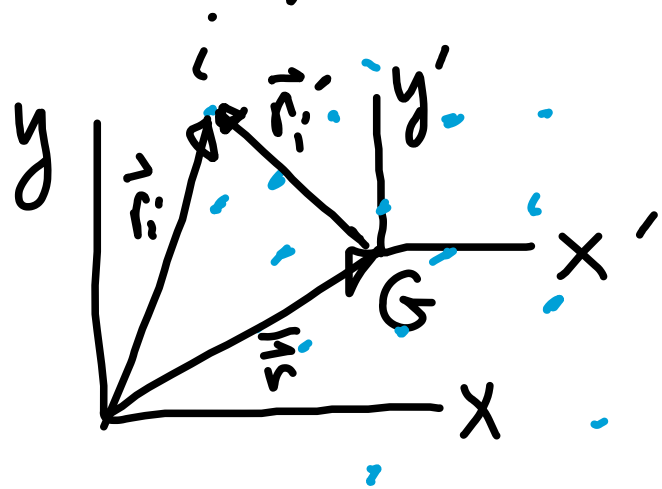

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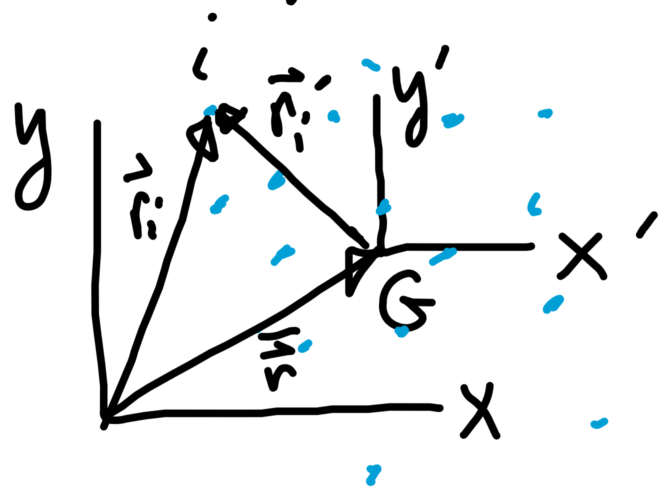

And $\vec{H}_G' = \vec{H}_G$

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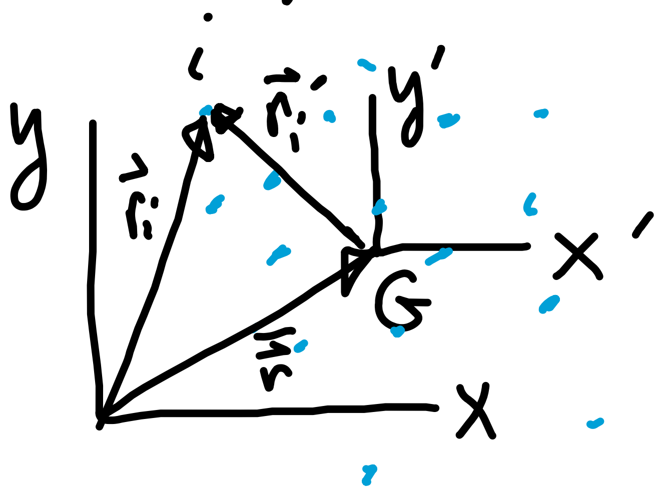

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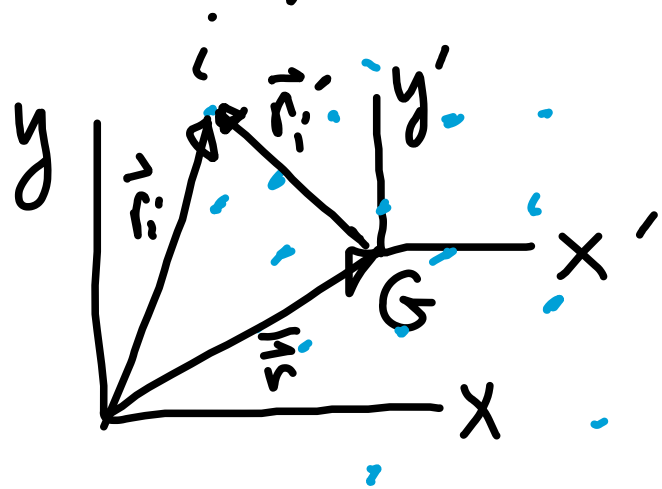

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{Does not matter if using $\vec{v}_i'$ or $\vec{v}_i$}

Special case:

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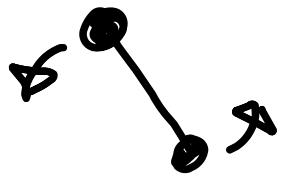

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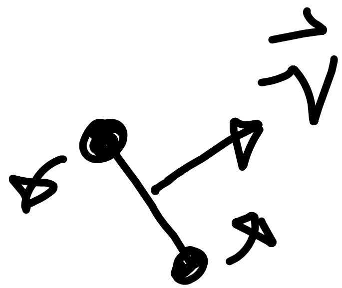

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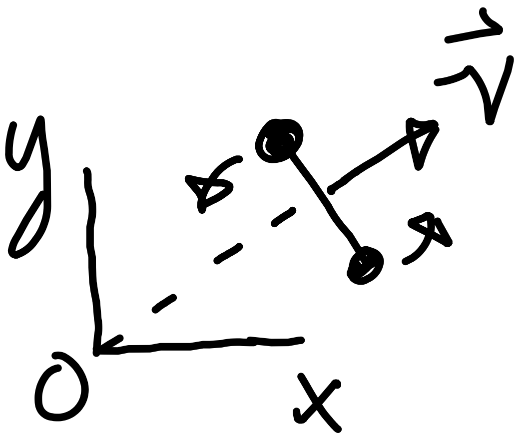

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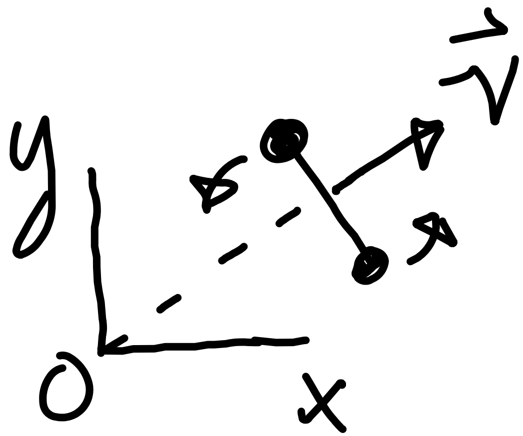

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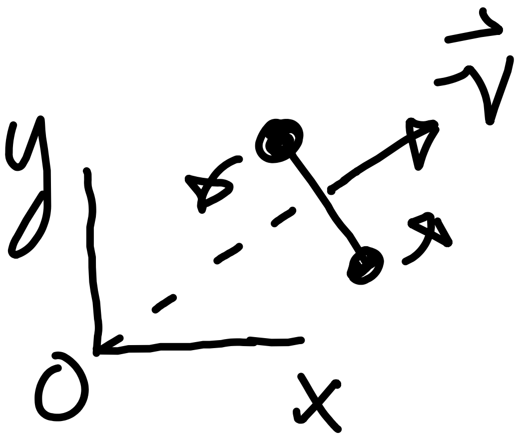

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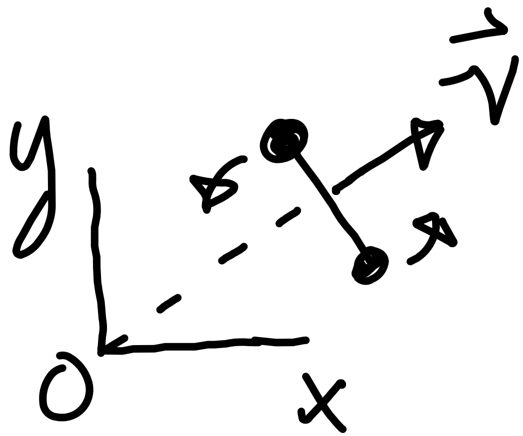

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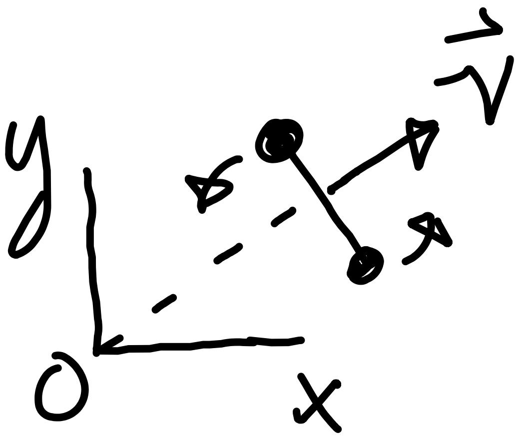

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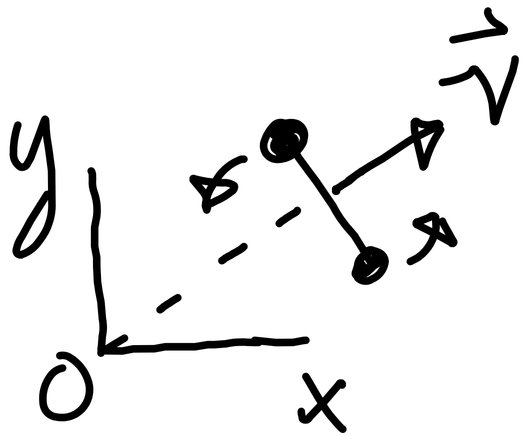

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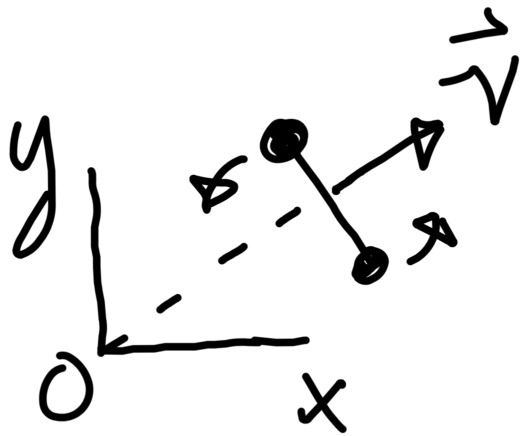

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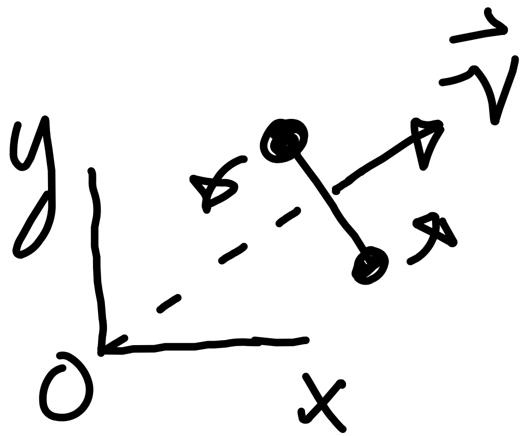

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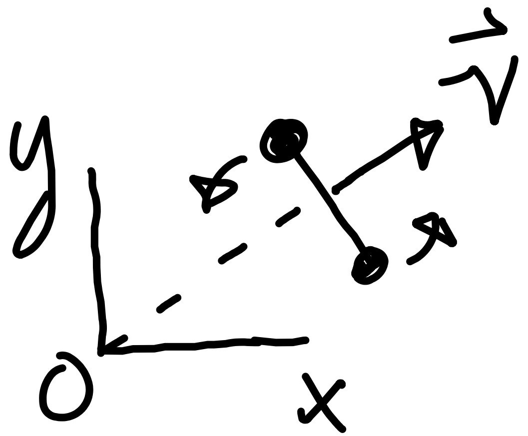

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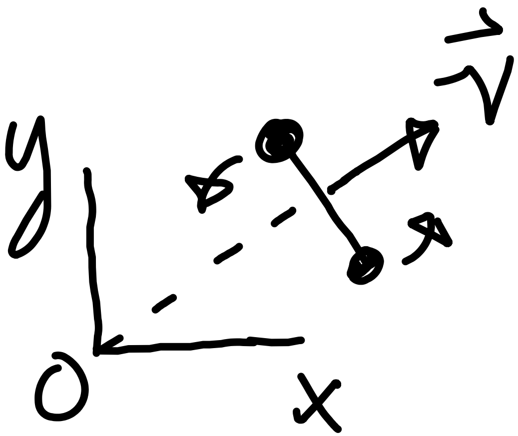

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we set $\bar{\vec{v}}$ in line with origin

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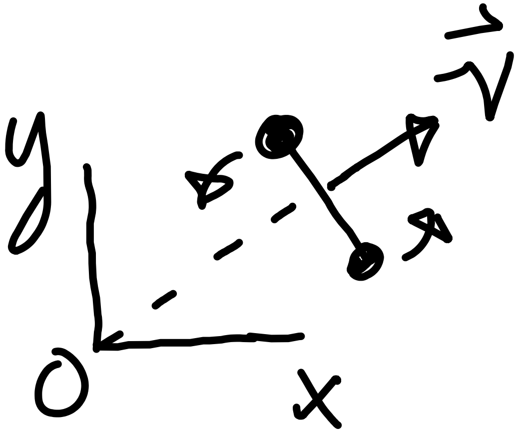

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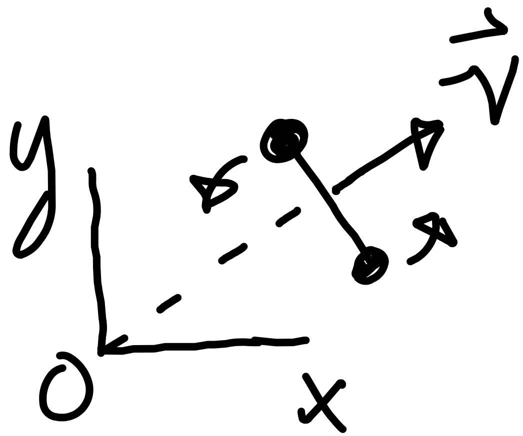

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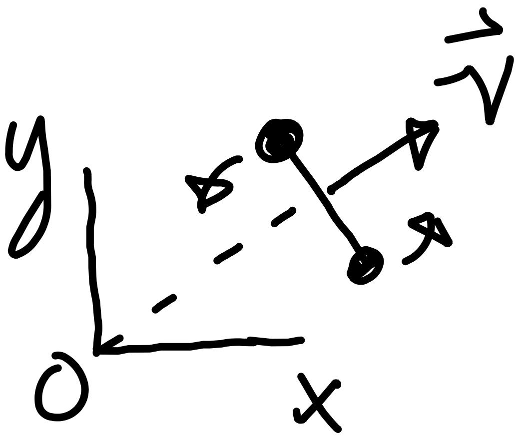
then, for this special case $\vec{H}_0 = \vec{H}_G$

If, however, $\bar{\vec{r}} \times \bar{\vec{v}}$ is non-zero

Special case: What if we measure $\vec{H}_O$, where origin O is directly in line with $\vec{\bar{v}}$?



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& since $\sum m_i \vec{v}_i = m\vec{\bar{v}}_i$ & $\vec{\bar{r}} \times \vec{\bar{v}} = \cancel{0}$

then, for this special case $\vec{H}_O = \vec{H}_G$

If, however, $\vec{\bar{r}} \times \vec{\bar{v}}$ is non-zero we would have $\vec{H}_O = \vec{H}_G + m\vec{\bar{r}} \times \vec{\bar{v}}$

We also saw that

Work: $T_1 + U_{1\to 2} = T_2$

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Work & Energy: $T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$

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Work & Energy: $T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

Impulse-Momentum: $\Sigma \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$

We also saw that

Work: $T_1 + U_{1\to 2} = T_2$

Work & Energy: $T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$

Impulse-momentum: $\sum \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$

Torque-impulse: $\sum \int_{t_1}^{t_2} \vec{M}_o\,dt = \Delta \vec{H}_o$

Notes on problem 14.38

Notes on problem 14.38: Bound energy

$= 120\,J$

Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$

Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$ & $\theta = 30^\circ$

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This is in center of mass frame

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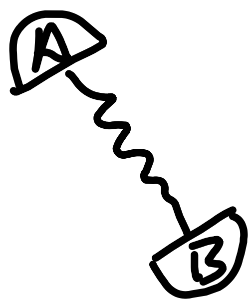

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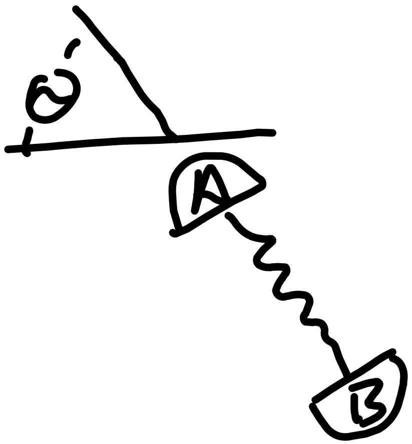

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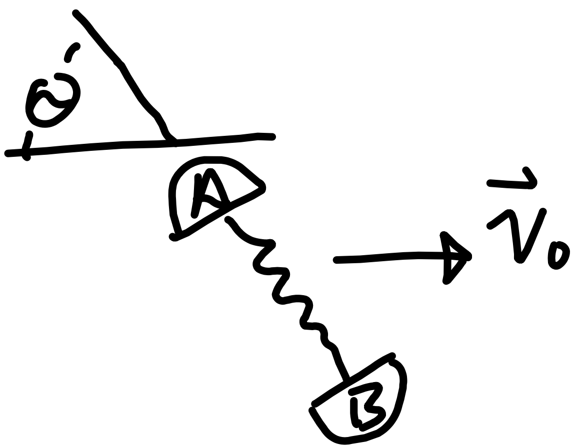
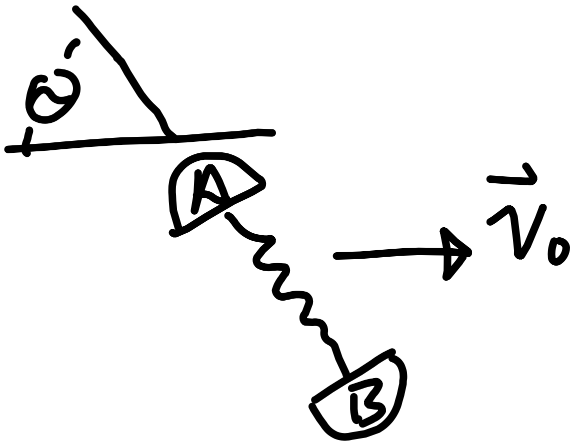


Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$ & $\theta = 30°$

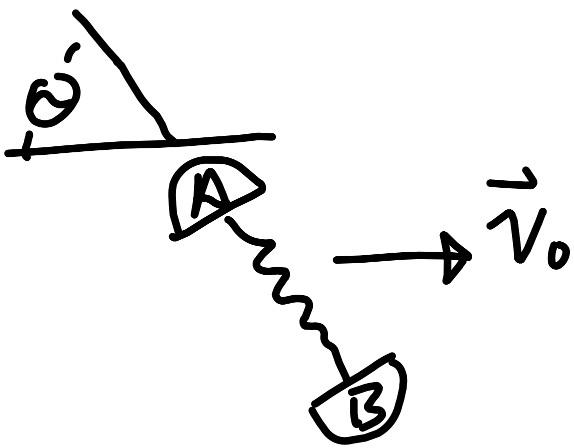
⌊This is in center of mass frame



* Go into the c-m frame

Notes on problem 14.38: Bound energy $=120\,J$, $v_0 = 8\,m/s$ & $\theta = 30°$

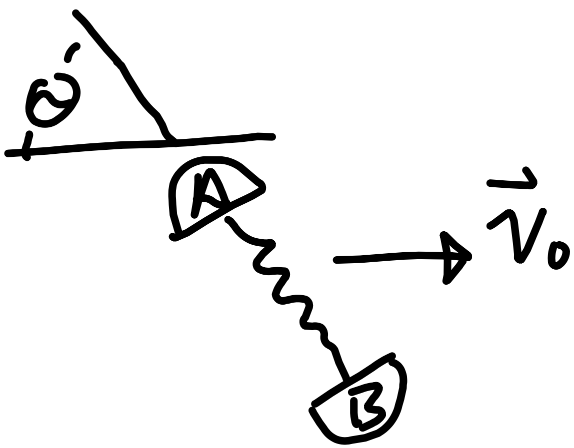
LThis is in center of mass frame



* Go into the c-m frame
* Release the bound energy in c-m frame

Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$ & $\theta = 30^\circ$

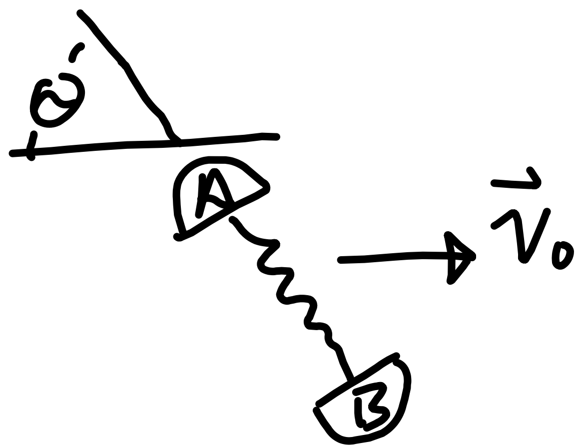
↳ This is in center of mass frame



* Go into the c-m frame
* Release the bound energy in c-m frame
* Conserve momentum in c-m frame

Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$ & $\theta = 30°$

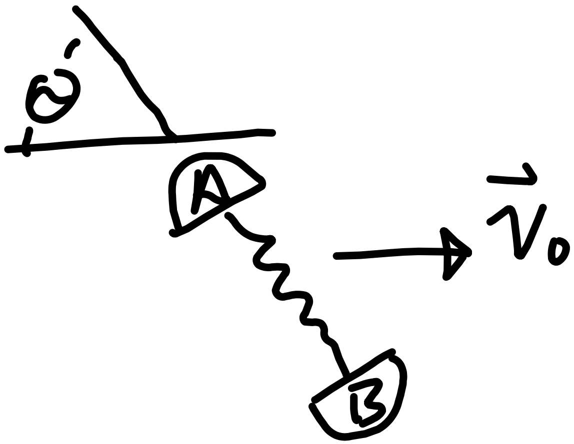
This is in center of mass frame



* Go into the c-m frame
* Release the bound energy in c-m frame
* Conserve momentum in c-m frame $\vec{L}_{cm_I} = \vec{L}_{cm_F}$

Notes on problem 14.38: Bound energy $= 120\,\mathrm{J}$, $v_0 = 8\,\mathrm{m/s}$ & $\theta = 30^\circ$

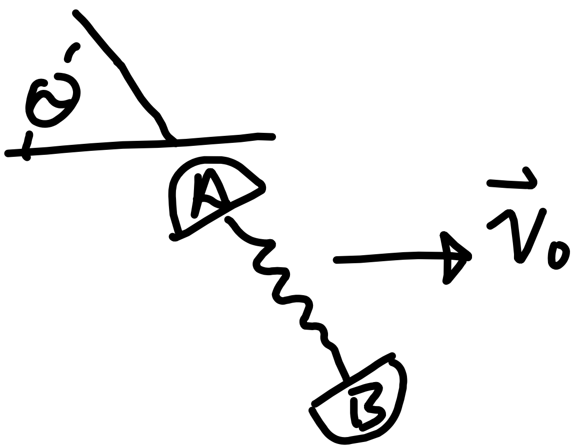
↳ This is in center of mass frame



* Go into the c-m frame
* Release the bound energy in c-m frame
* Conserve momentum in c-m frame $\vec{L}_{cm_I} = \vec{L}_{cm_F}$
* Boost back into the lab frame

Notes on problem 14.38: Bound energy $= 120\,J$, $v_0 = 8\,m/s$ & $\underline{\theta = 30^\circ}$

This is in center of mass frame



* Go into the c-m frame
* Release the bound energy in c-m frame
* Conserve momentum in c-m frame $\vec{L}_{cm_I} = \vec{L}_{cm_F}$
* Boost back into the lab frame [add $\vec{v}_0$ to each velocity]

Notes on problem 14.52:

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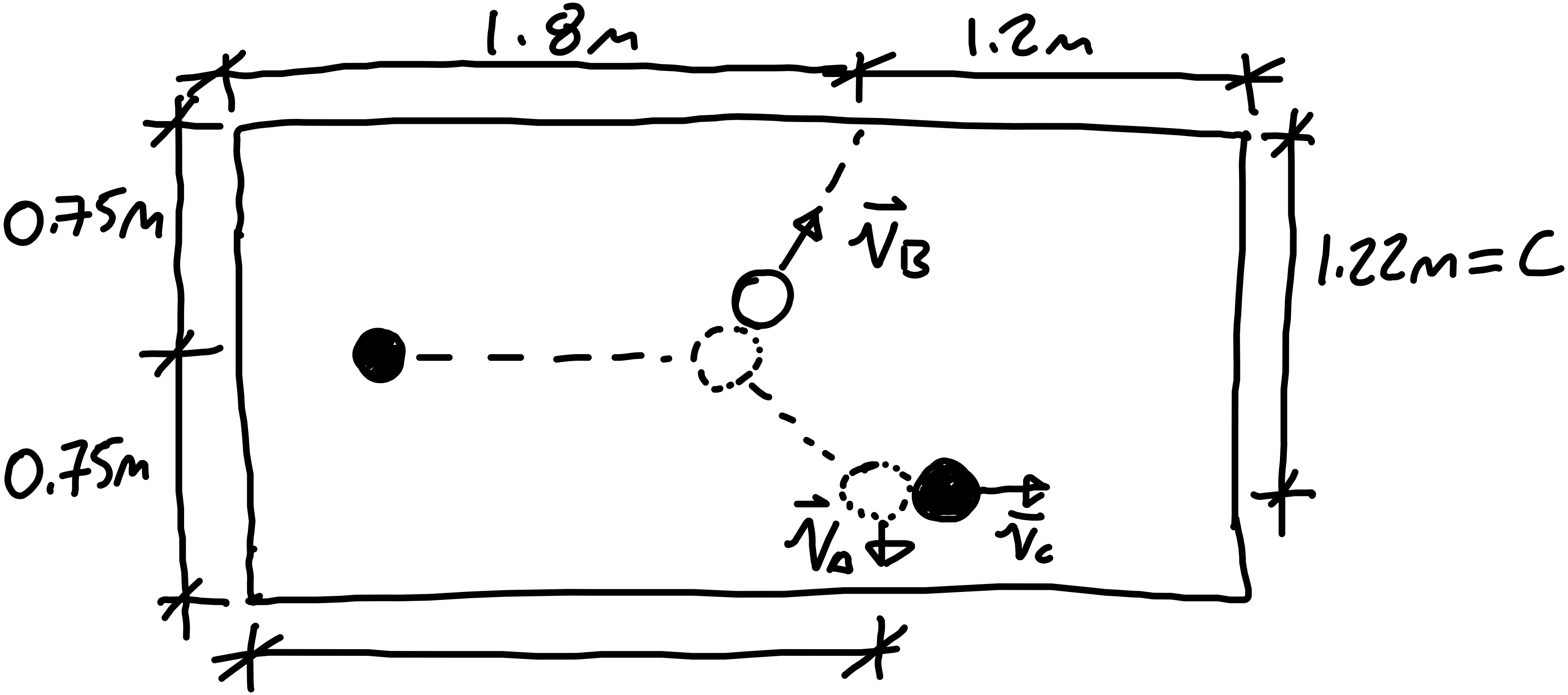
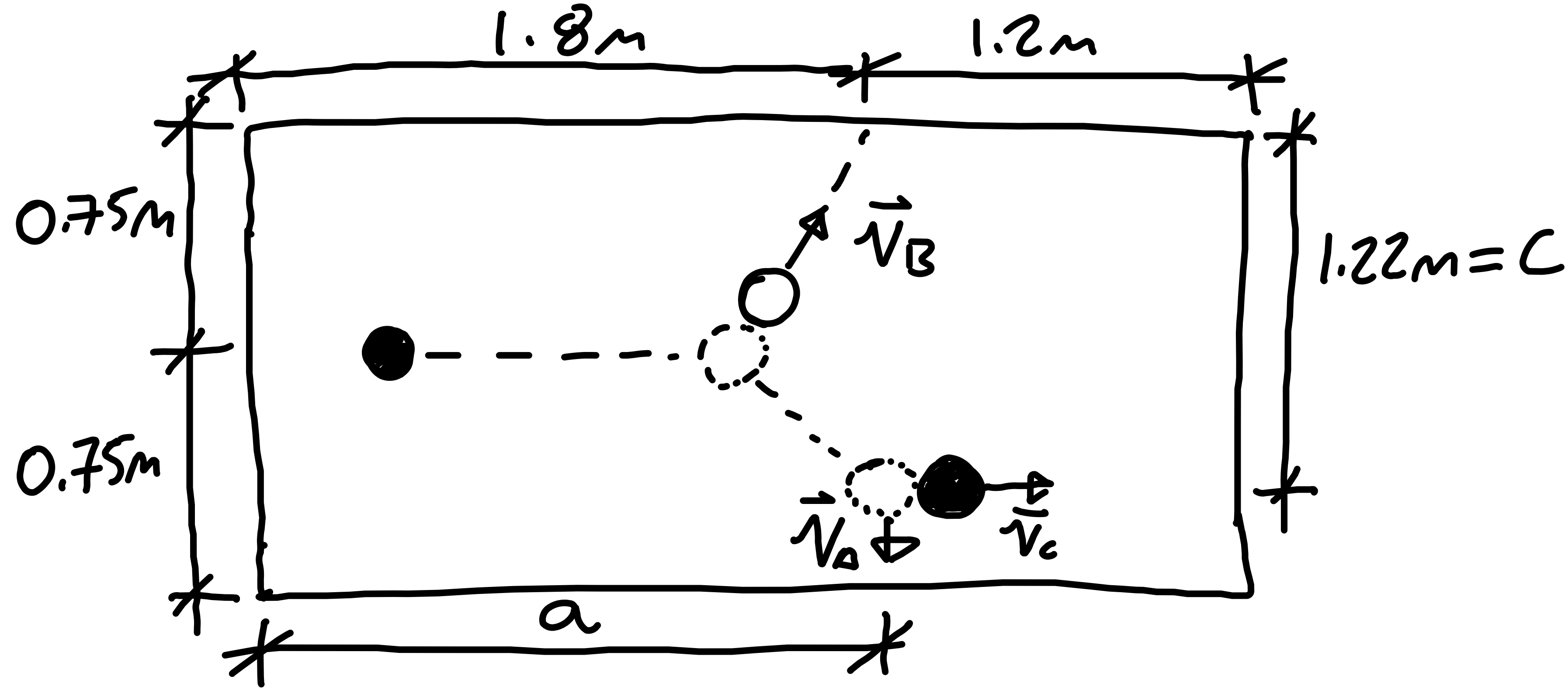
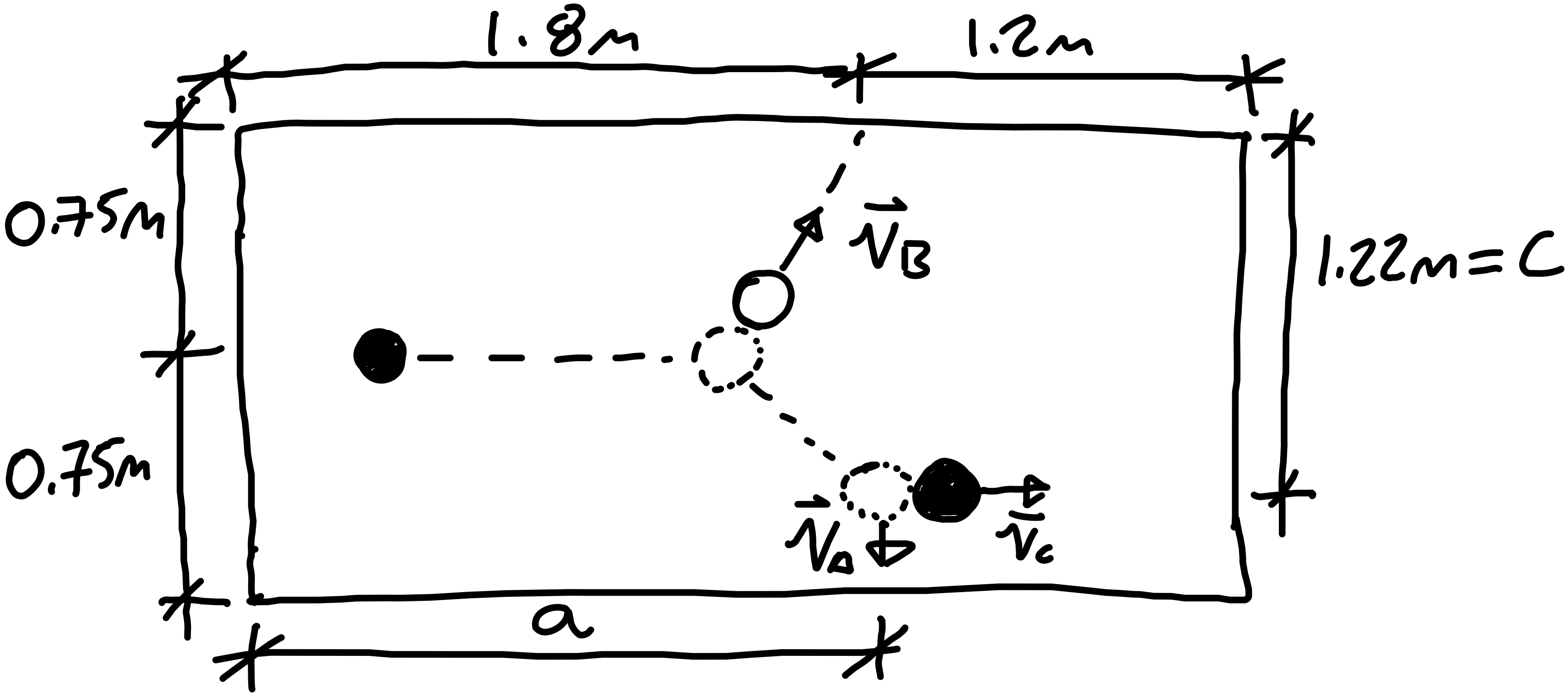


Notes on problem 14.52:



* Conserve momentum & energy to find $\vec{v}_A$ & $\vec{v}_B$

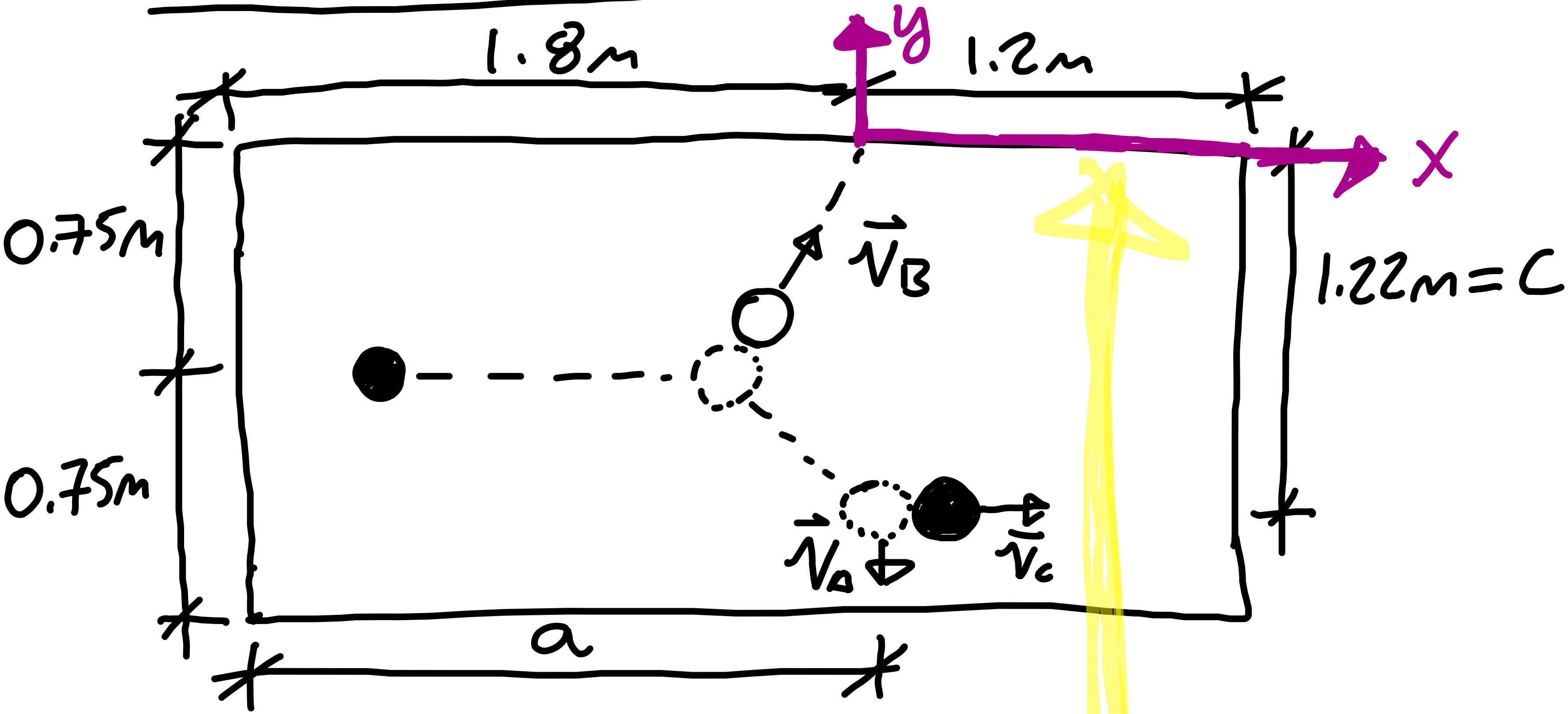
Notes on problem 14.52:



* Conserve momentum & energy to find $\vec{V}_A$ & $\vec{V}_B$

* Several ways to find a.

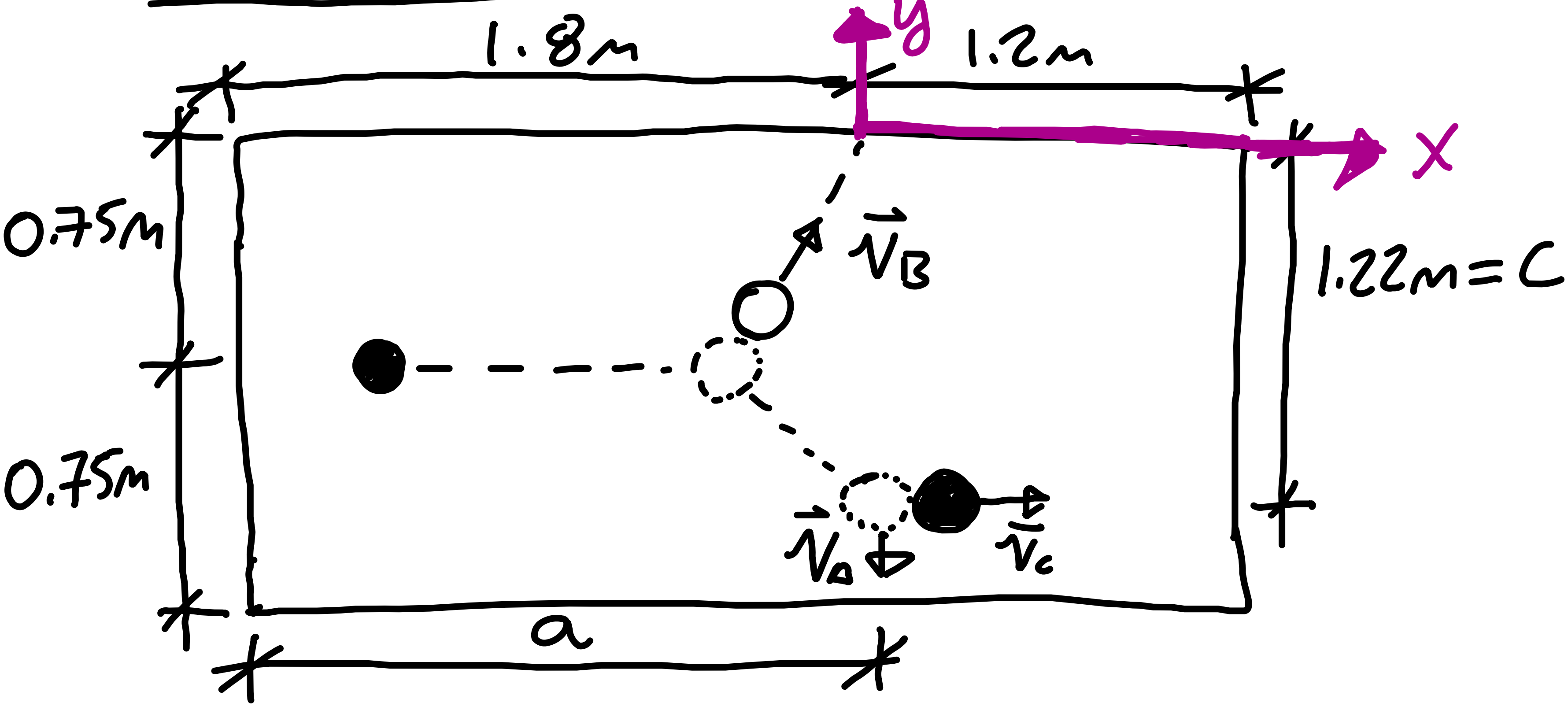
Notes on problem 14.52:



* Conserve momentum & energy to find $\vec{v}_A$ & $\vec{v}_B$

* Several ways to find a. One way is to put coordinate system as shown

Notes on problem 14.52:



* Conserve momentum & energy to find $\vec{v}_A$ & $\vec{v}_B$
* Several ways to find a. One way is to put coordinate system as shown & conserve $\vec{H}_O$