Today Review

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Monday Exam #2

Work & energy

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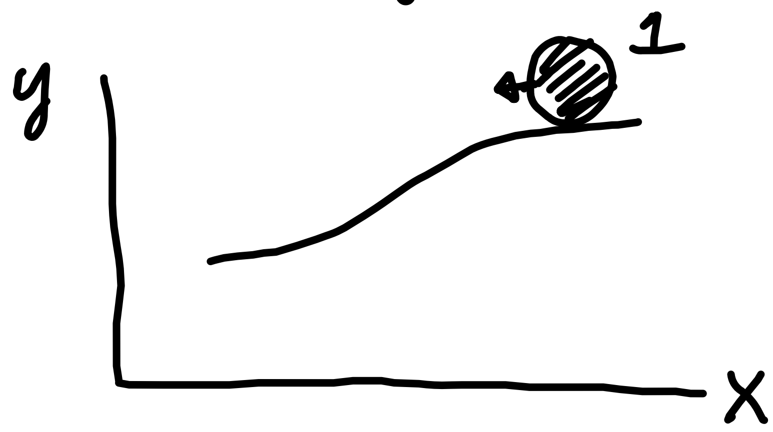

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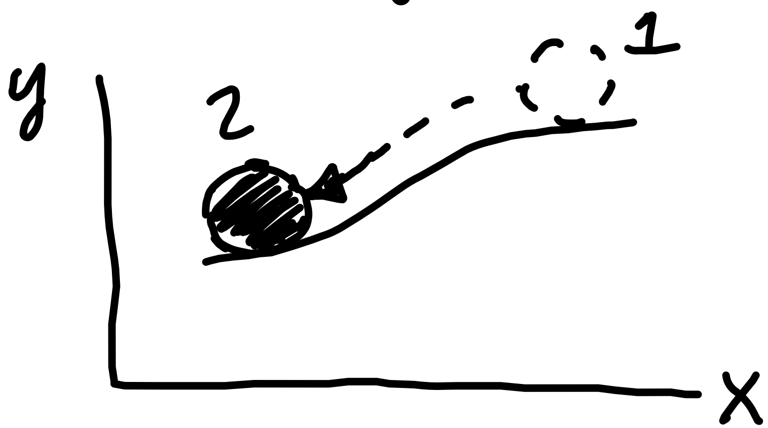

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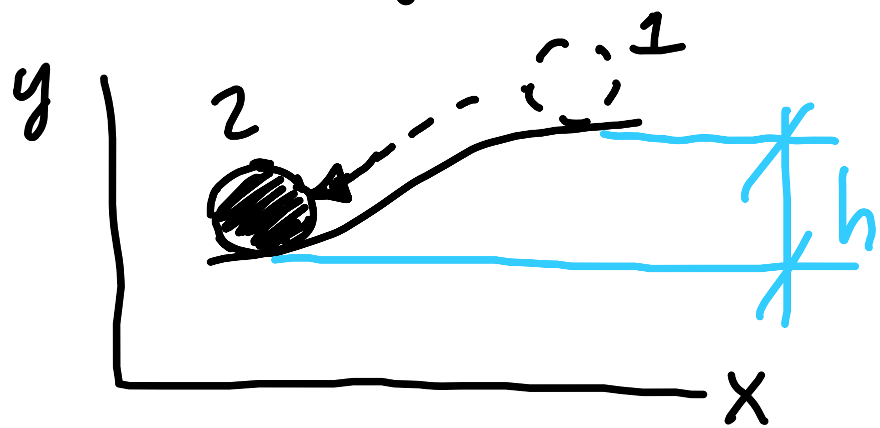

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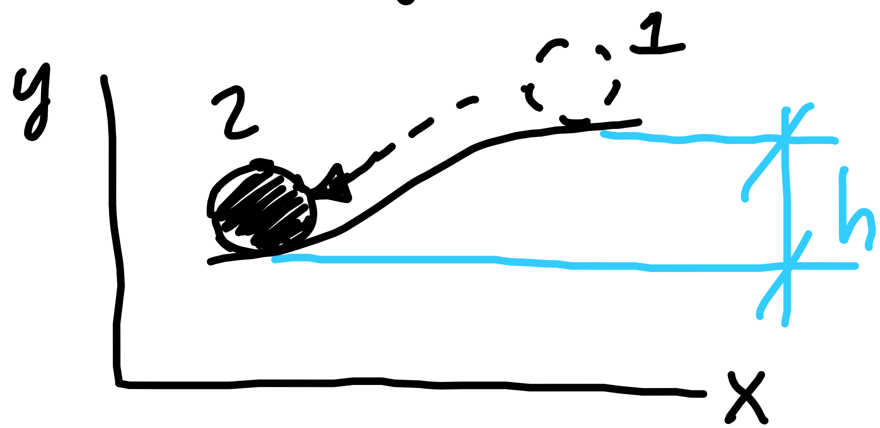

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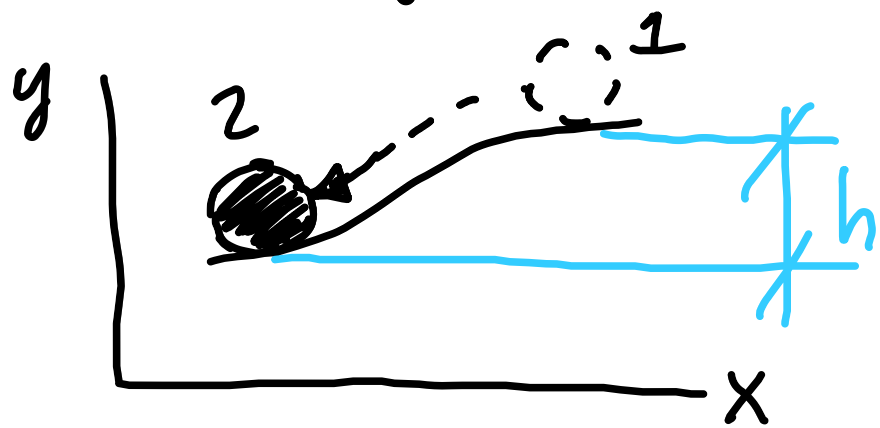

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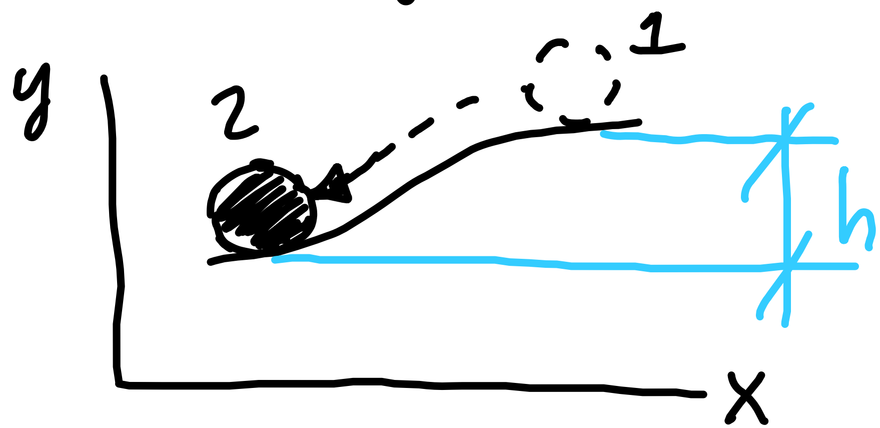

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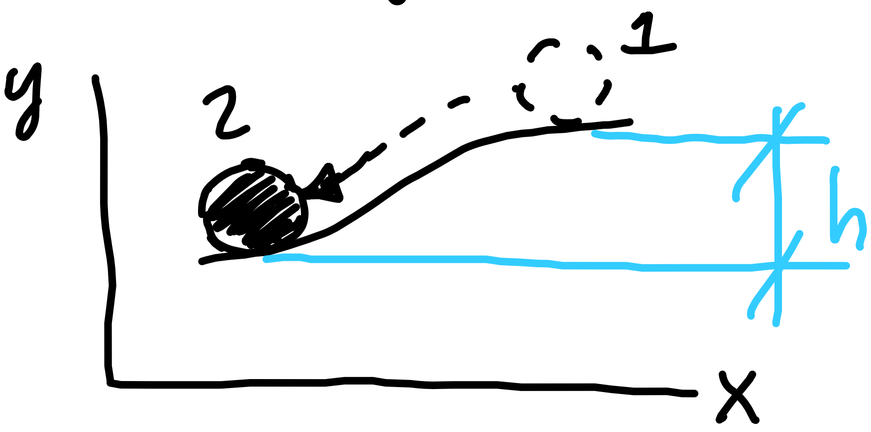

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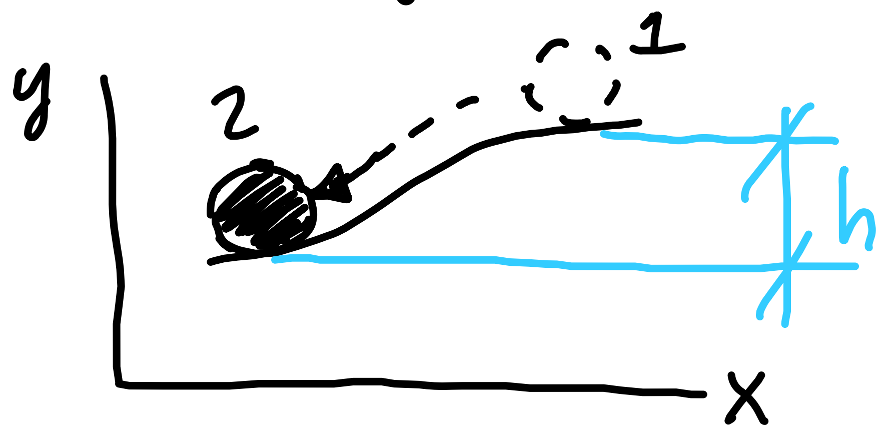

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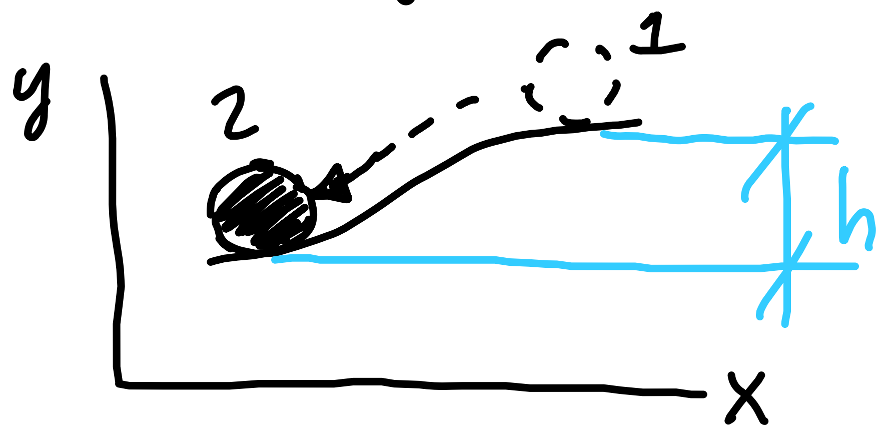

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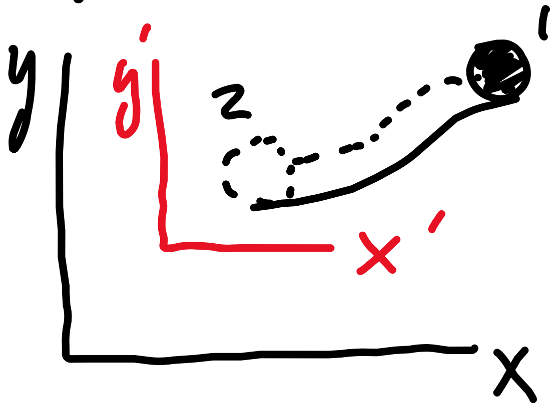
For non-conservative force, we must use work integral: $U_{1\to 2}^{NC} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}^{NC} \cdot d\vec{r}$

Note: Can always add a constant to V & still be valid since potential energy is defined as a difference



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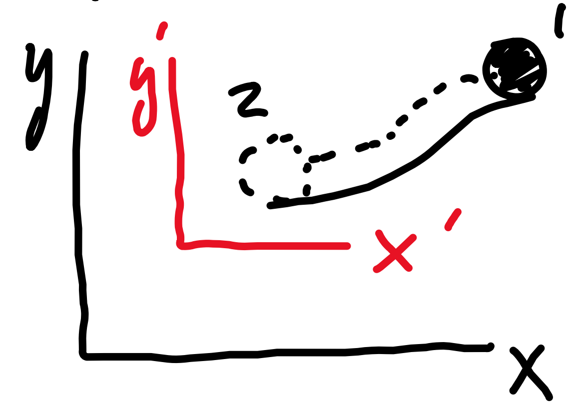

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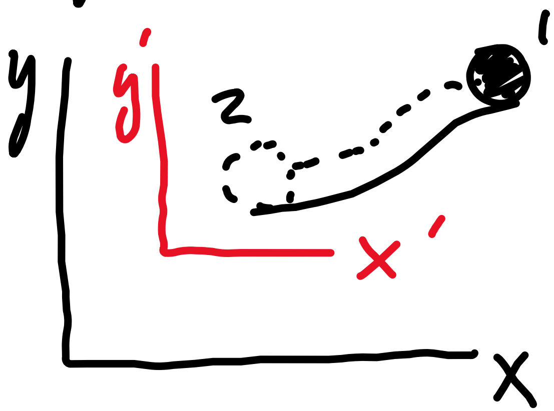

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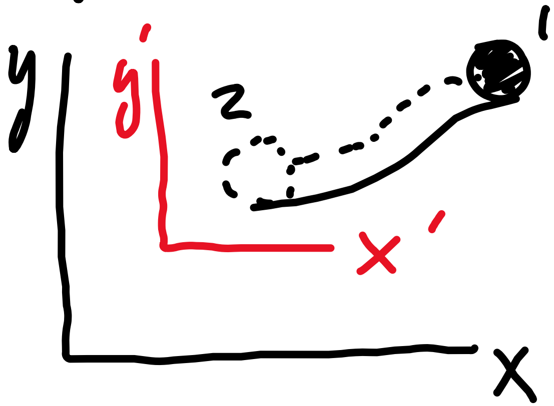

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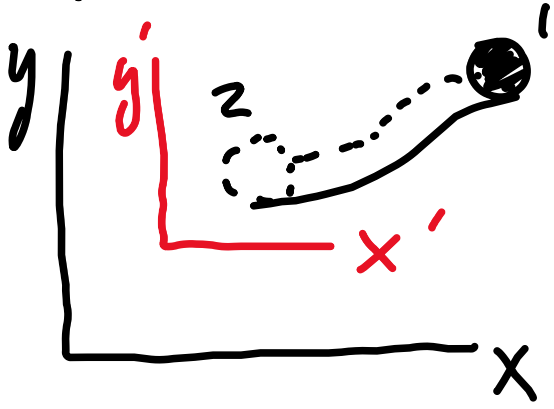

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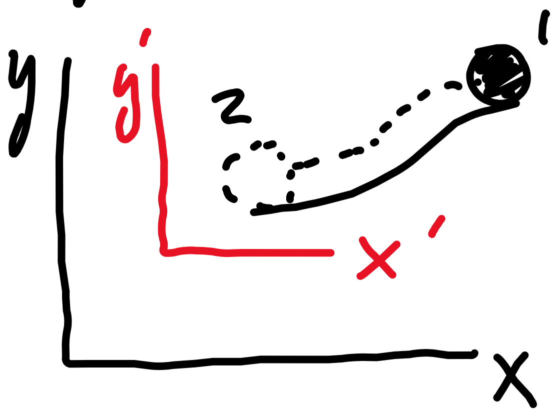

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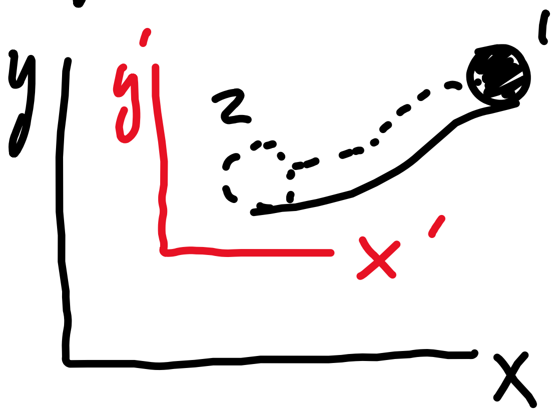

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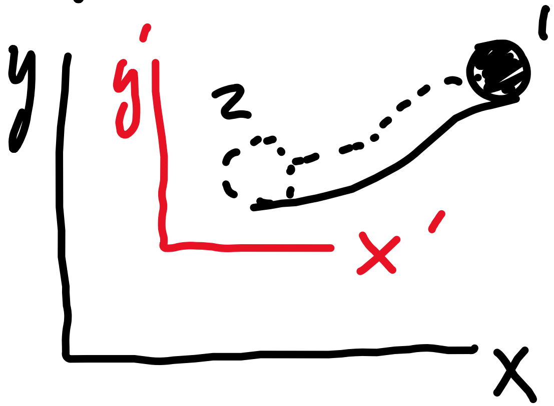

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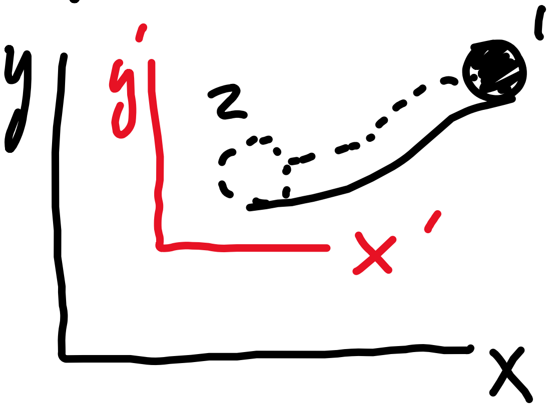

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Conservation of energy:

$$V_1 + T_1 + U_{1\to 2}^{nc} = V_2 + T_2$$

Spring

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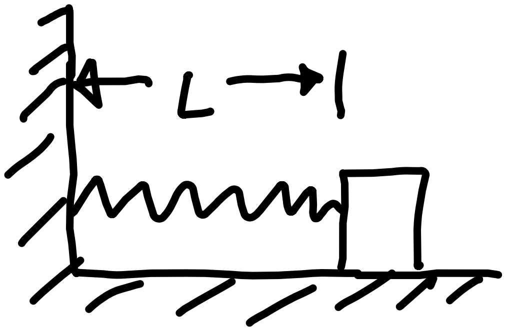

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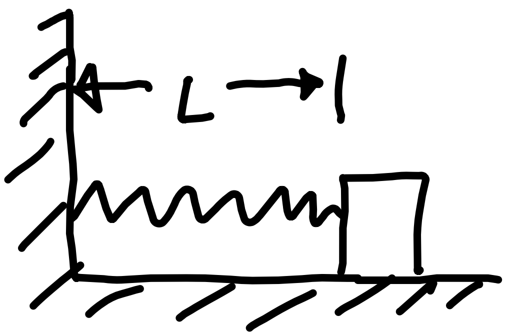

$L = L_0 + X \Rightarrow$

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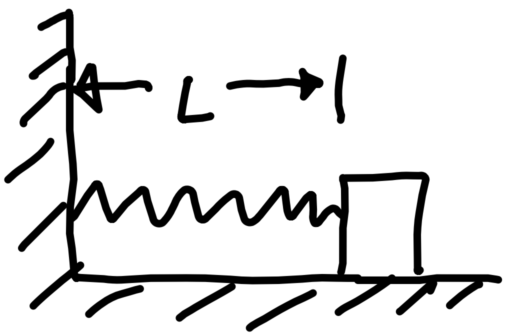

$L = L_0 + x \Rightarrow x = L - L_0$

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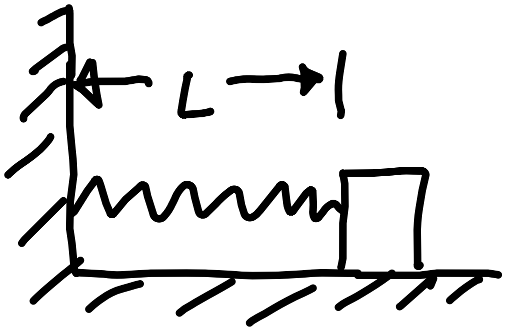

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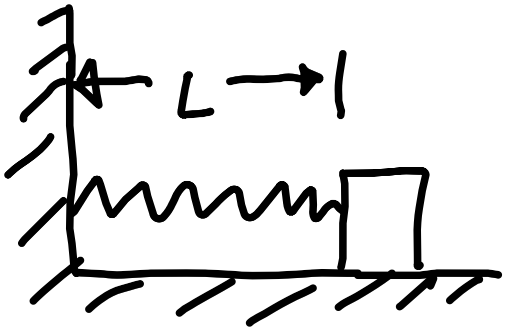

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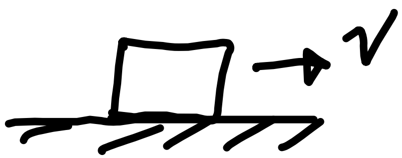
Most common non-conservative force is friction

__Example__: Box of mass m slides on surface with coefficient of kinetic friction μ_K for a distance L



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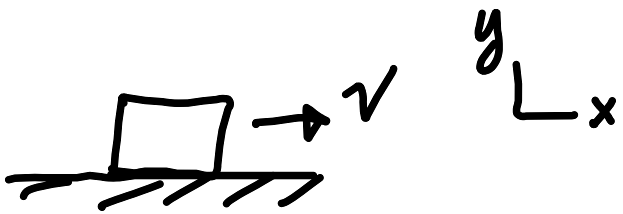

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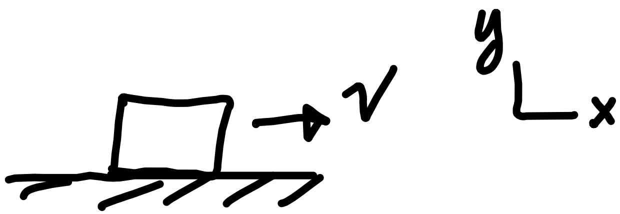

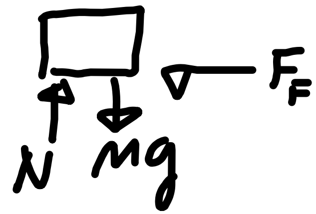

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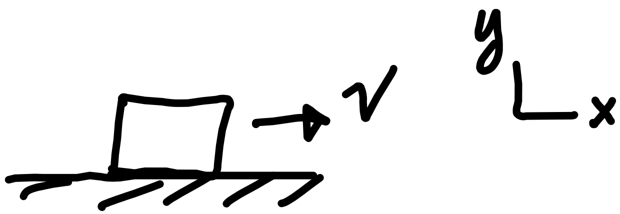

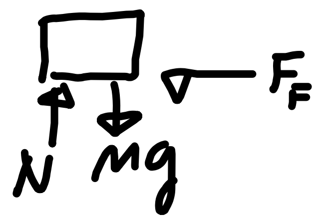

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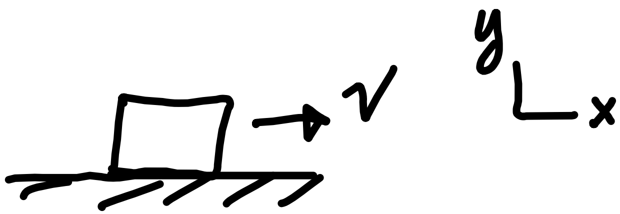

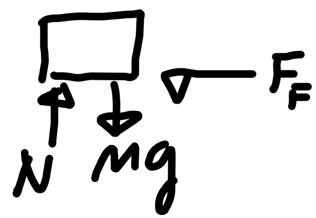

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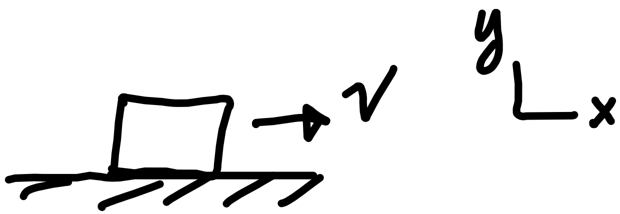

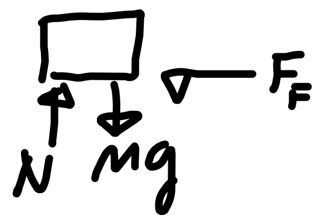

here $\vec{F}_F = N\mu_k(-\hat{i})$ & $N = mg$

so $U_{1\to2}^{NC} = \int_0^L \vec{F}\cdot d\vec{x} = (mg\mu_k)\left(-\int_0^L dx\right) = -mg\mu_k L$

Conservation of energy

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$$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_1$$

Conservation of energy

$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_1$, for our sliding box

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Conservation of energy

$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_1$, for our sliding box $V_1 = V_2 \Rightarrow U^{nc}_{1\to 2} = \Delta T \Rightarrow -mg\mu_k L = \Delta T$

$\Rightarrow$ Kinetic energy gets smaller

Conservation of energy

$T_1 + V_1 + U_{1\to 2}^{nc} = T_2 + V_1$, for our sliding box $V_1 = V_2 \Rightarrow U_{1\to 2}^{nc} = \Delta T \Rightarrow -mg\mu_k L = \Delta T$

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Box slows down

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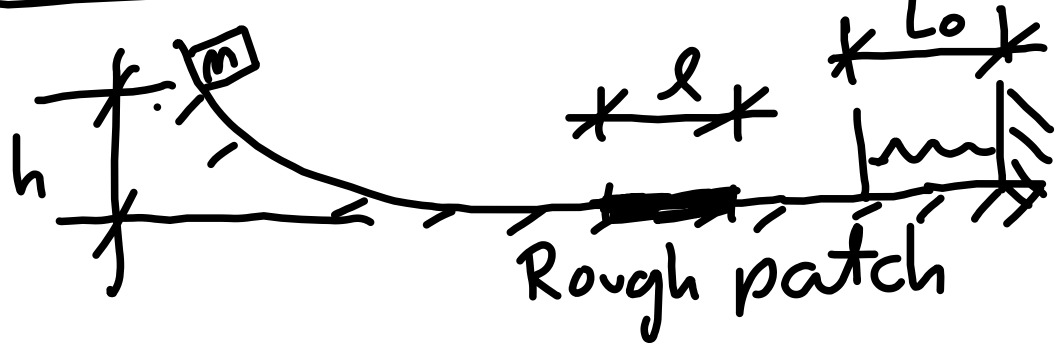

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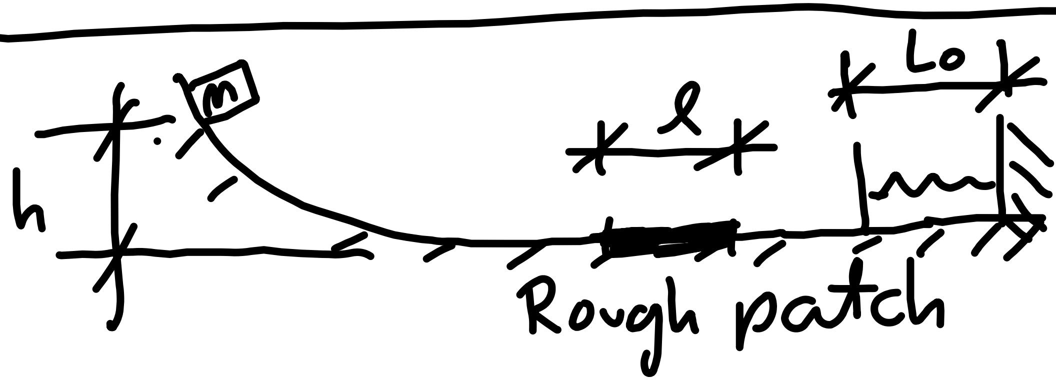

mass starts at rest.

Conservation of energy

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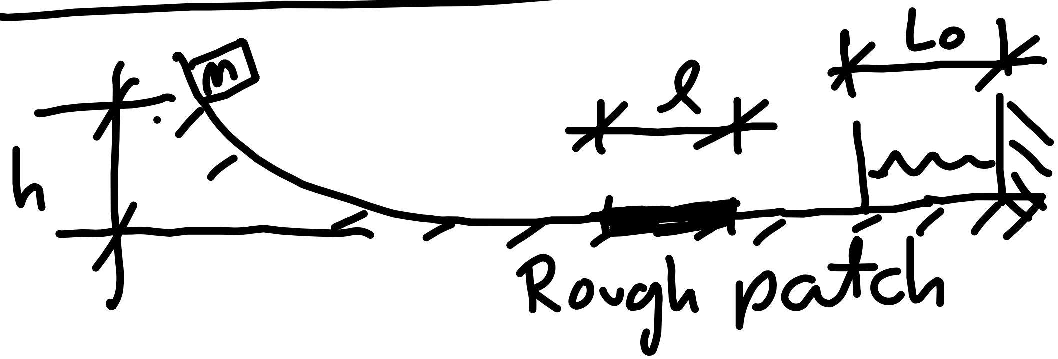

Mass starts at rest. How far is spring compressed? x

Conservation of energy

$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_1$, for our sliding box $V_1 = V_2 \Rightarrow U^{nc}_{1\to 2} = \Delta T \Rightarrow -mg\mu_k L = \Delta T$

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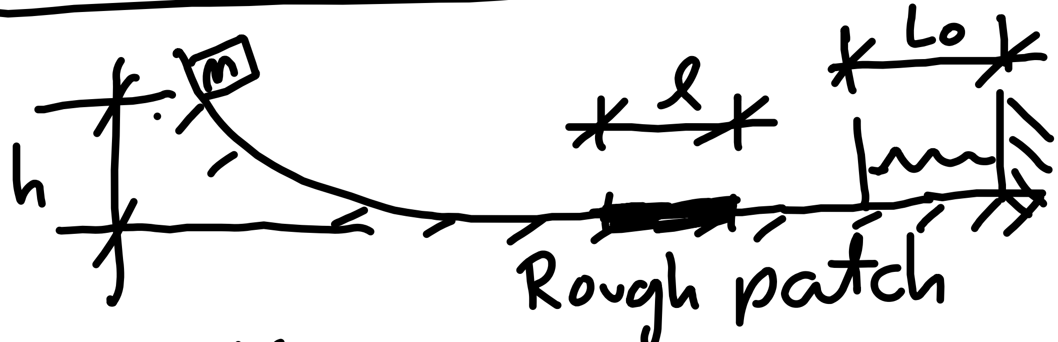

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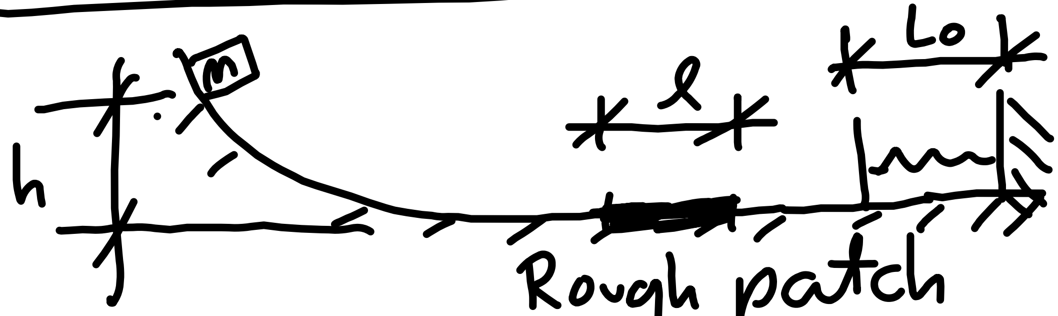

Mass starts at rest. How far is spring compressed? x

$V_1+T_1+U^{nc}_{1\to2} = V_2+T_2$ here $T_1=T_2=0$

Conservation of energy

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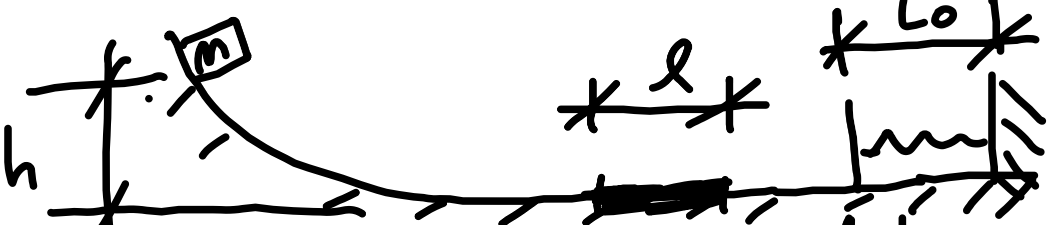

Rough patch

mass starts at rest. How far is spring compressed? x

$V_1 + T_1 + U^{nc}_{1\to 2} = V_2 + T_2$ here $T_1 = T_2 = 0$ & $V_1 = mgh$

Conservation of energy

$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_1$, for our sliding box $V_1 = V_2 \Rightarrow U^{nc}_{1\to 2} = \Delta T \Rightarrow -mg\mu_k L = \Delta T$

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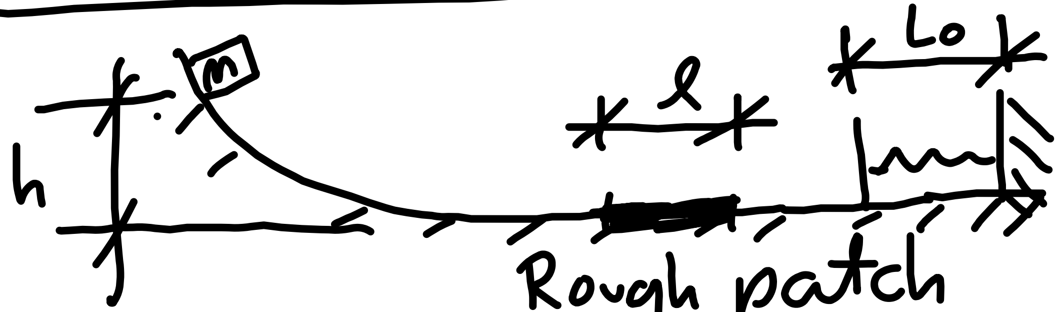

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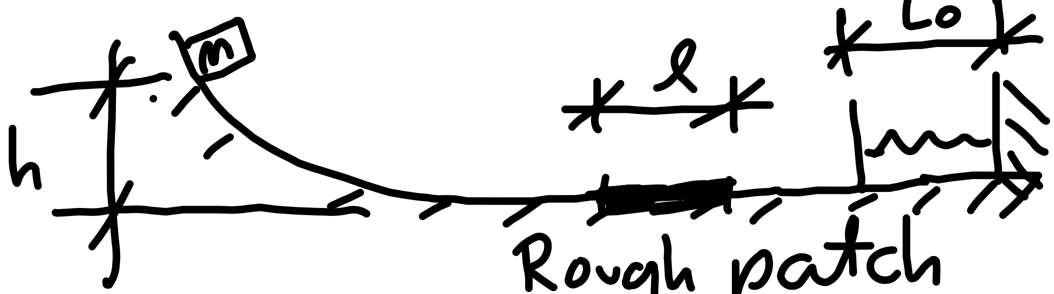

Mass starts at rest. How far is spring compressed? x

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Conservation of energy

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$mg[h - \mu_K l] = \frac{1}{2}Kx^2$

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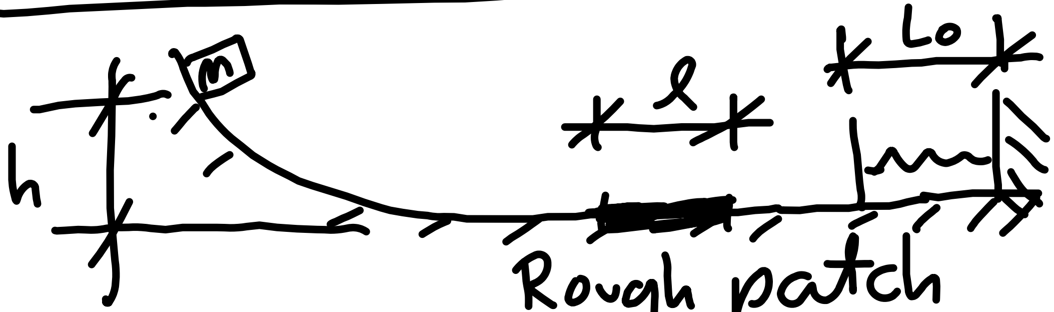

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$mg[h - \mu_K l] = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\left(\frac{2mg}{K}\right)(h - \mu_K l)}$

Impulse and momentum

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$$\int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{L}$$

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$$\int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{L} \quad \underline{or} \quad \vec{I}_{\underset{1\to2}{mp}} = \Delta\vec{L}$$

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<u>Example</u>: Ball on horizontal surface bounces off of wall.

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Impulse and momentum

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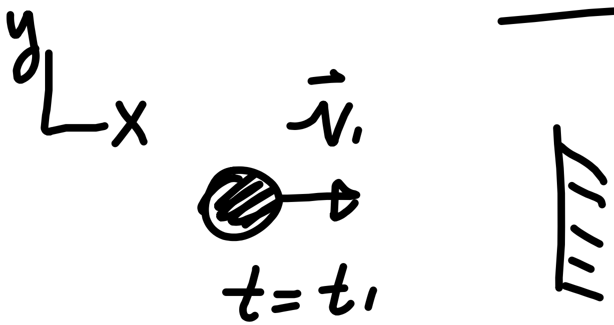

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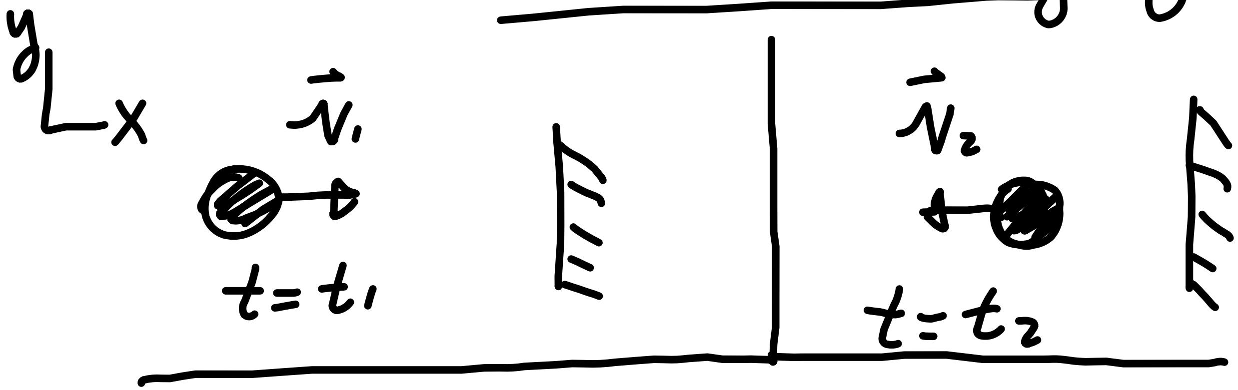

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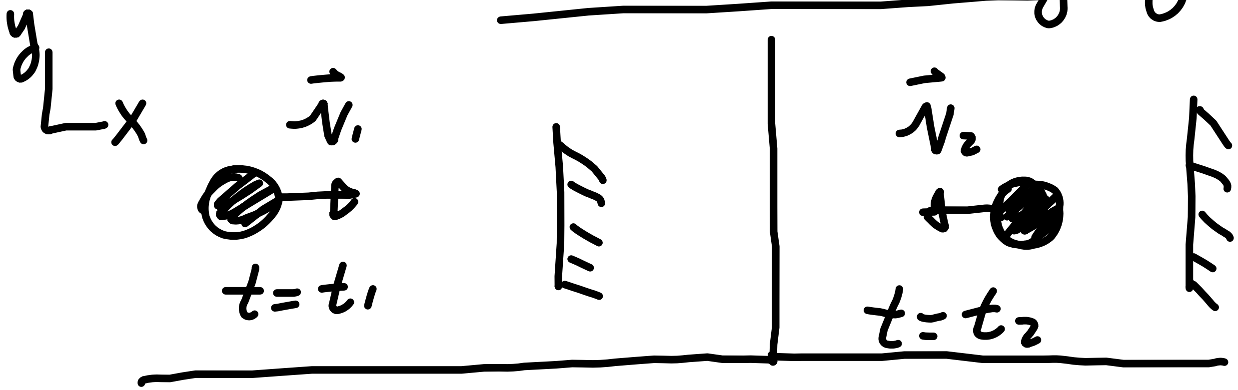
Here $\vec{L}_1 = mv_1\hat{i}$



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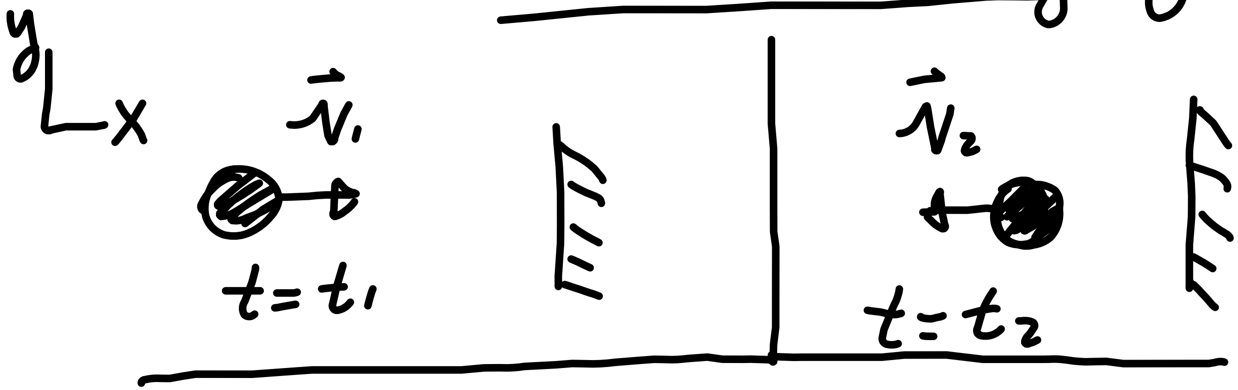

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Impulse and momentum

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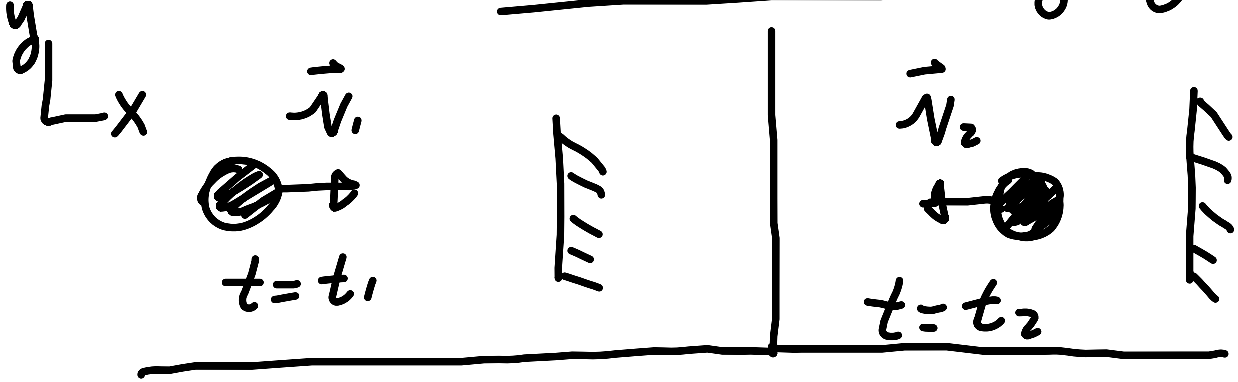

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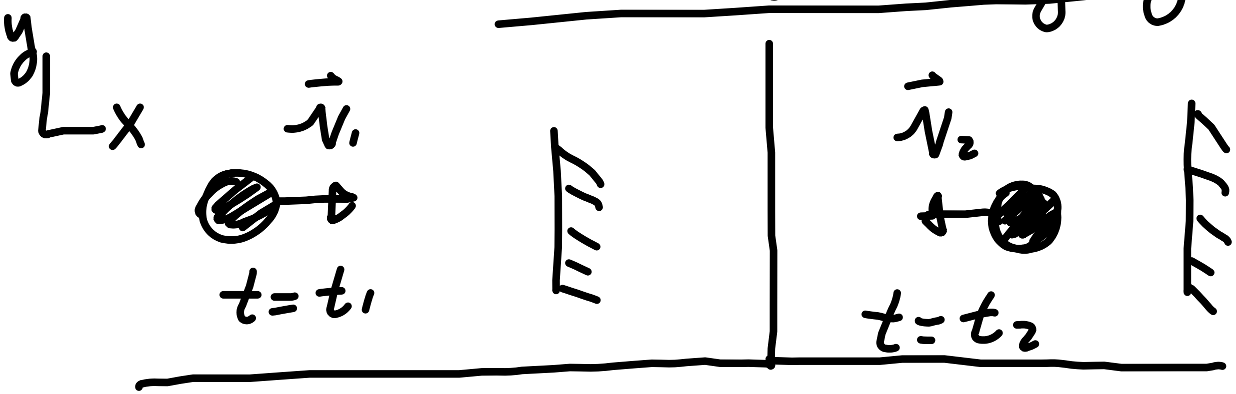

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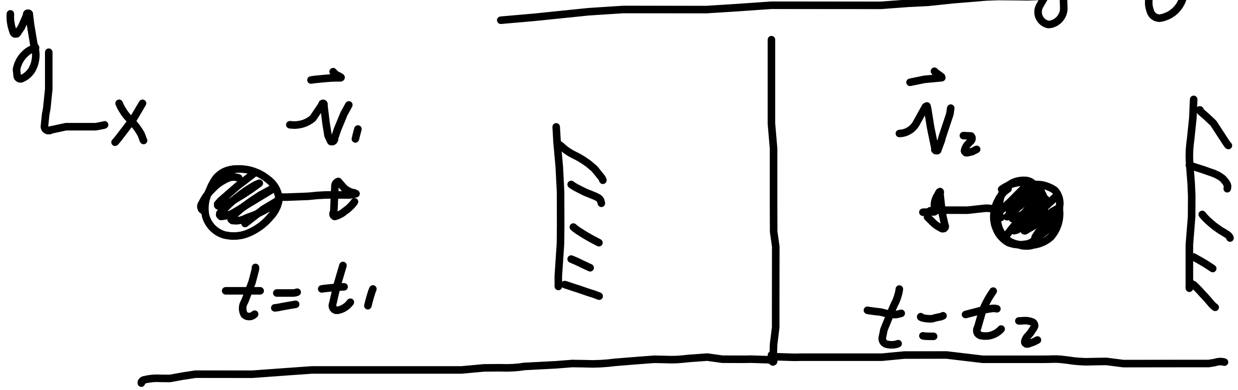

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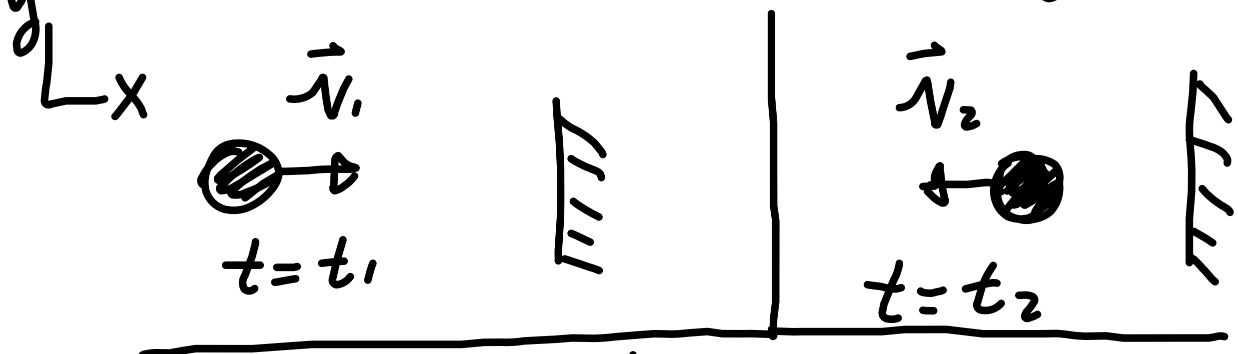

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Impulse and momentum

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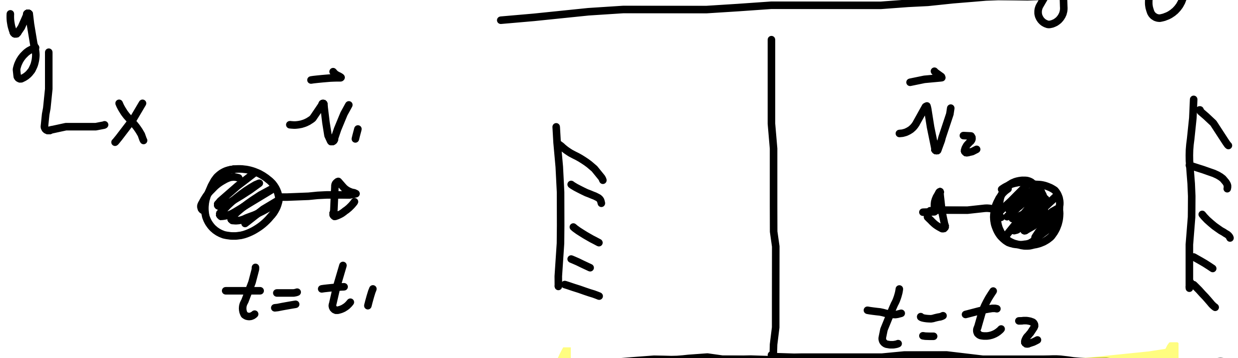

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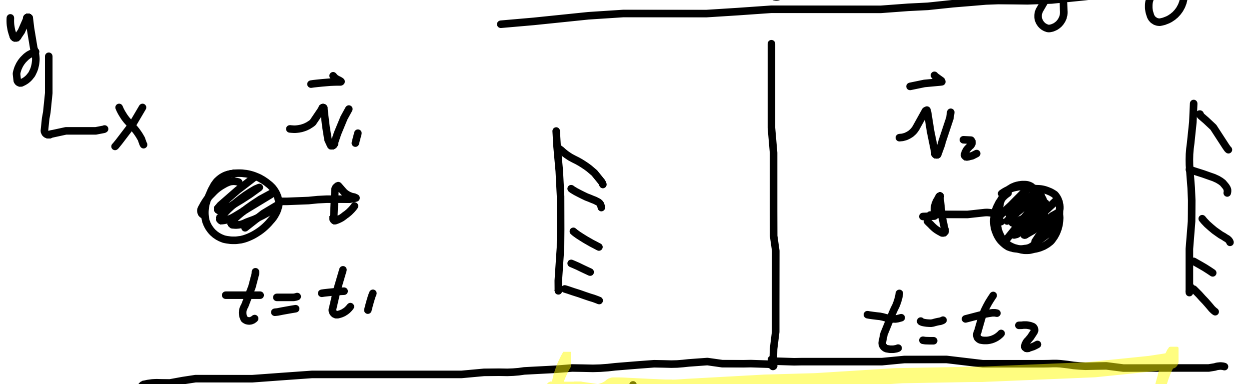

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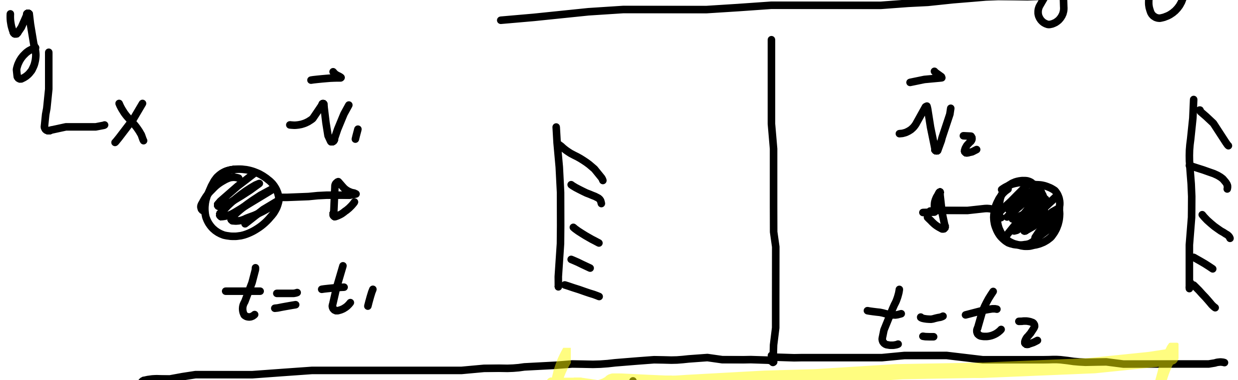

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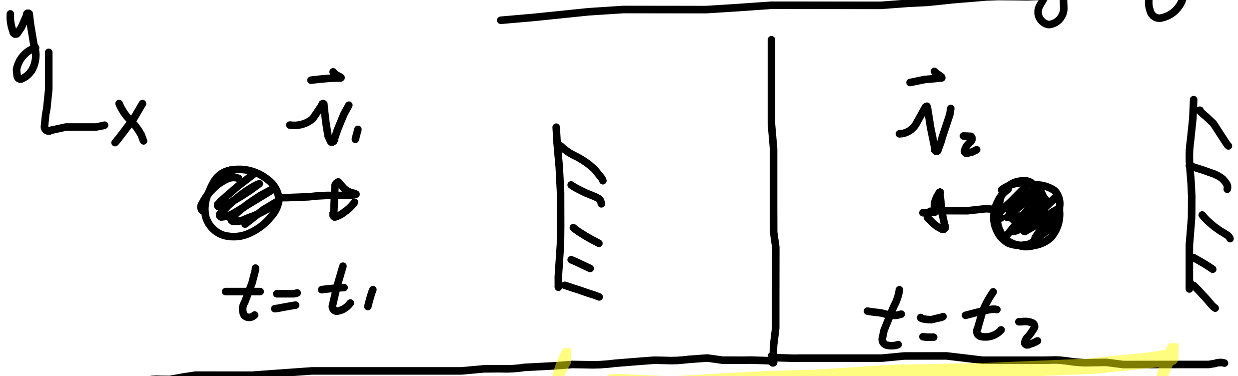

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For a system of particles

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$$\Sigma \vec{F}_i = \Sigma m_i \vec{a}_i$$

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No external forces

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No external torques

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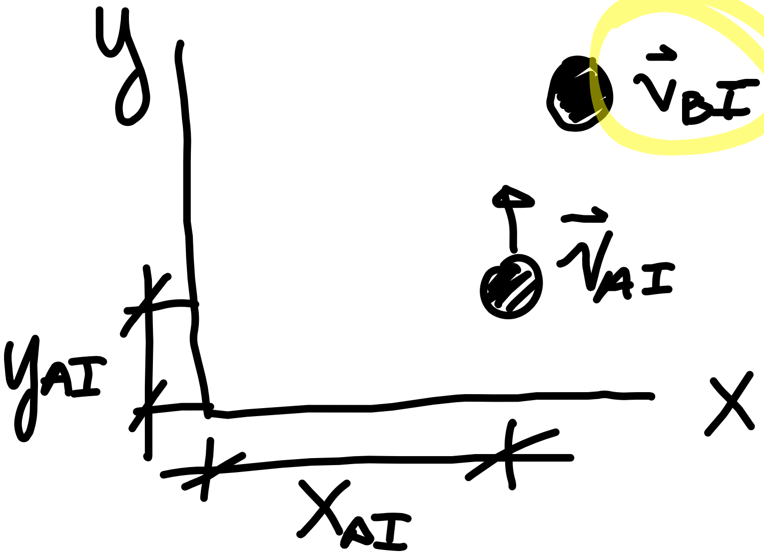
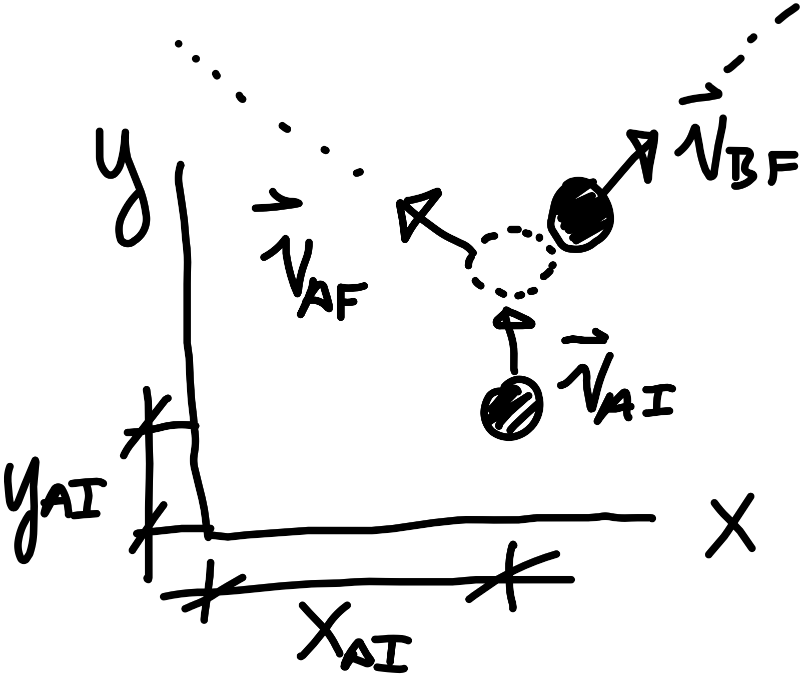
$\sum \vec{F} = \dot{\vec{L}}$ so, if $\sum \vec{F} = 0$ then $\vec{L} = \text{const.}$

& if $\sum \vec{M}_o = 0$ then $\vec{H}_o = \text{const.}$



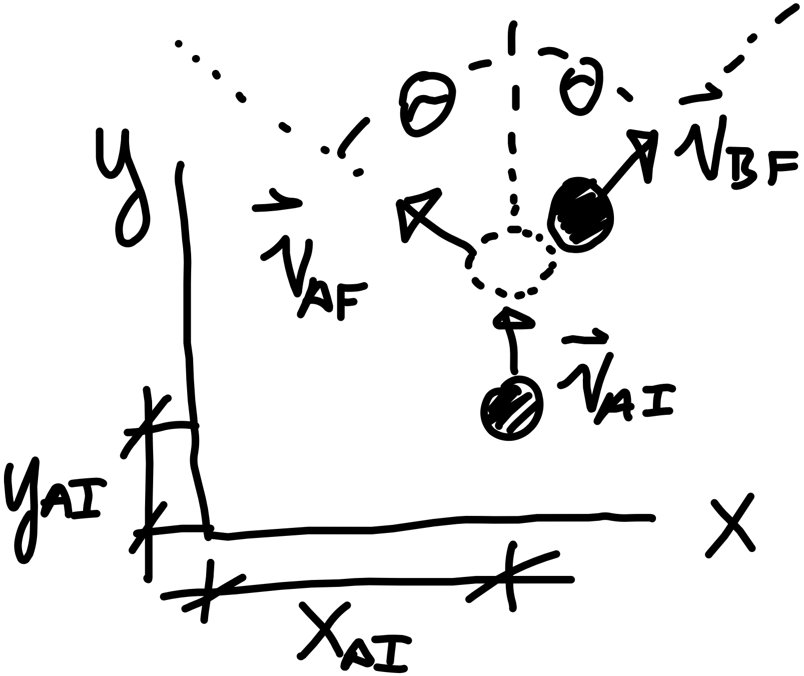
Given $V_{BI} = \emptyset$!

$M_A = M_B$



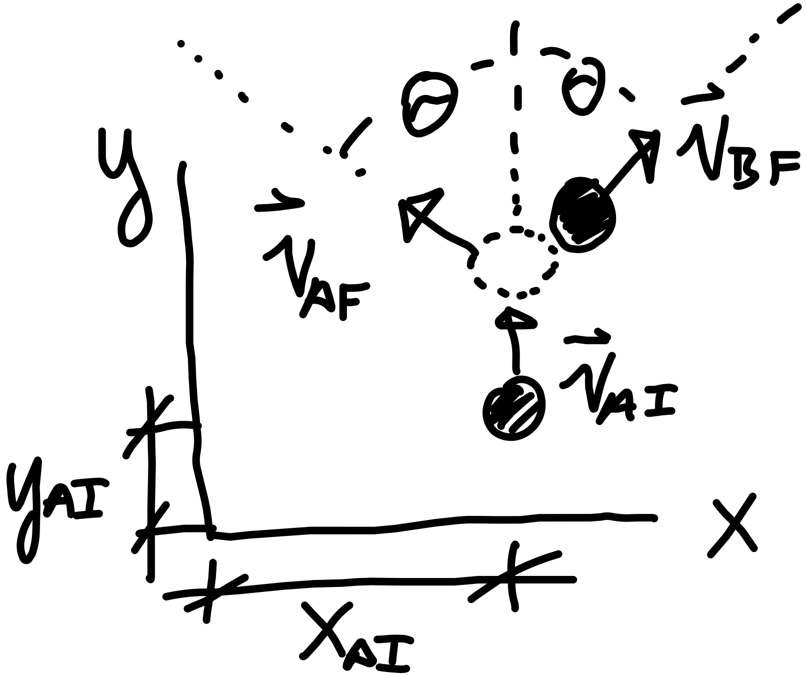
<u>Given</u> $V_{BI} = 0$

$M_A = M_B$



<u>Given</u> $V_{BI} = \cancel{0}$ ♥

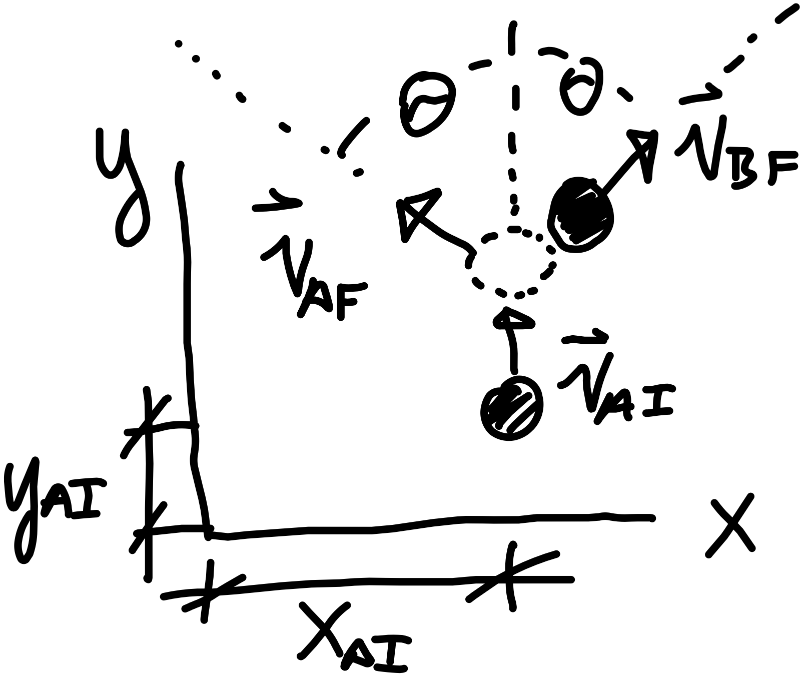
$M_A = M_B$



<u>Given</u> $V_{BI} = 0$

$M_A = M_B$ so

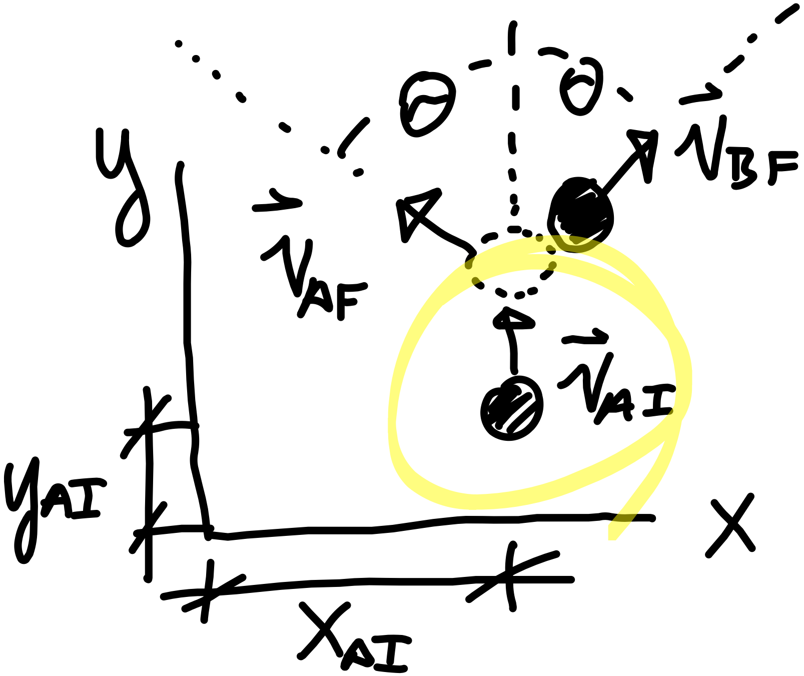
$\vec{L}_I = \vec{L}_{AI} + \vec{L}_{BI}$



Given $V_{BI} = 0$

$M_A = M_B$ so θ

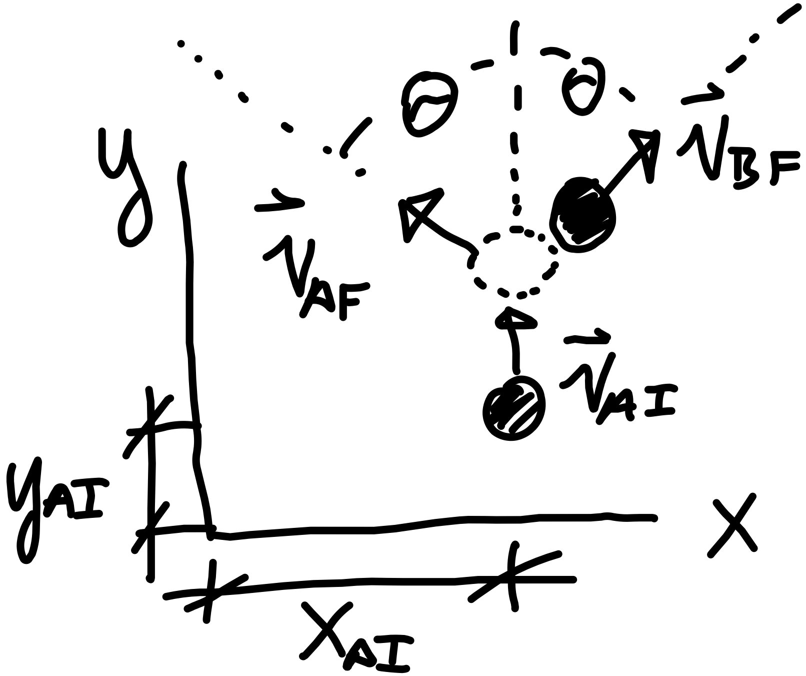
$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}}$



Given $V_{BI} = 0\,\hat{j}$

$M_A = M_B$ SO 0 $\cancel{0}$

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\,\hat{j}$

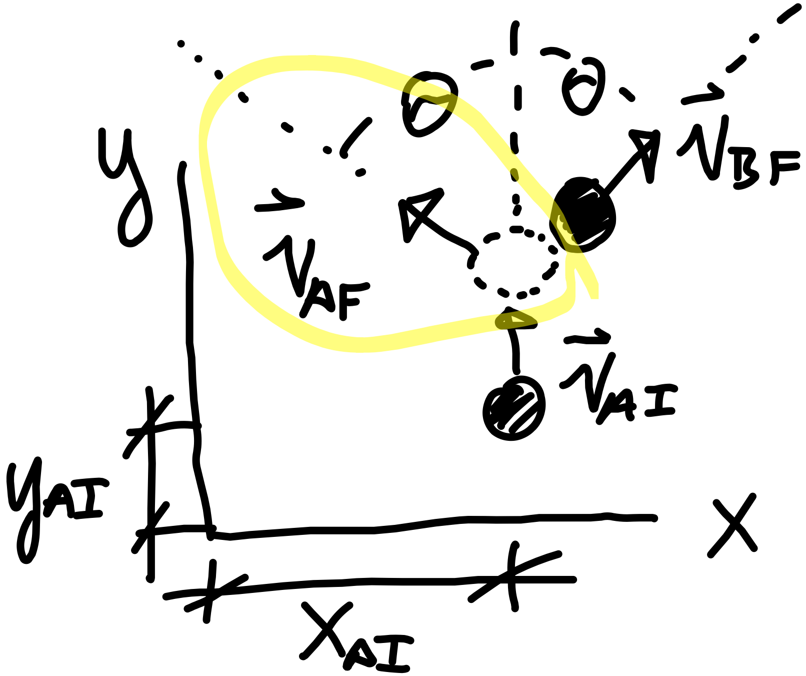


Given $V_{BI} = 0$

$M_A = M_B$ so 0

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\hat{j}$

$\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$



Given $V_{BI} = 0$

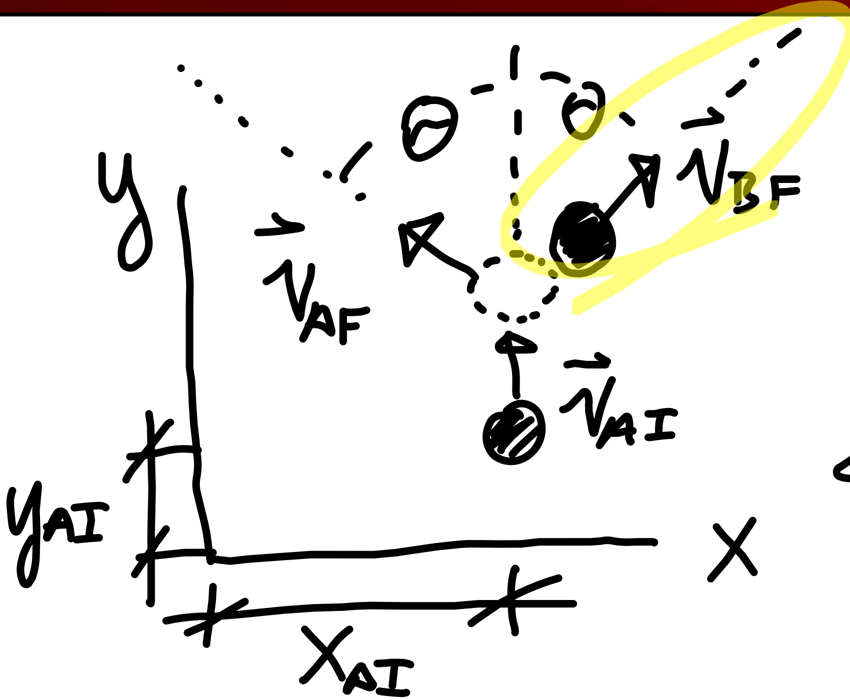
$M_A = M_B$ so 0

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\hat{j}$

$\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF}(-\sin\theta\hat{i} + \cos\theta\hat{j})$

$+$



Given $V_{BI} = 0\,\hat{j}$

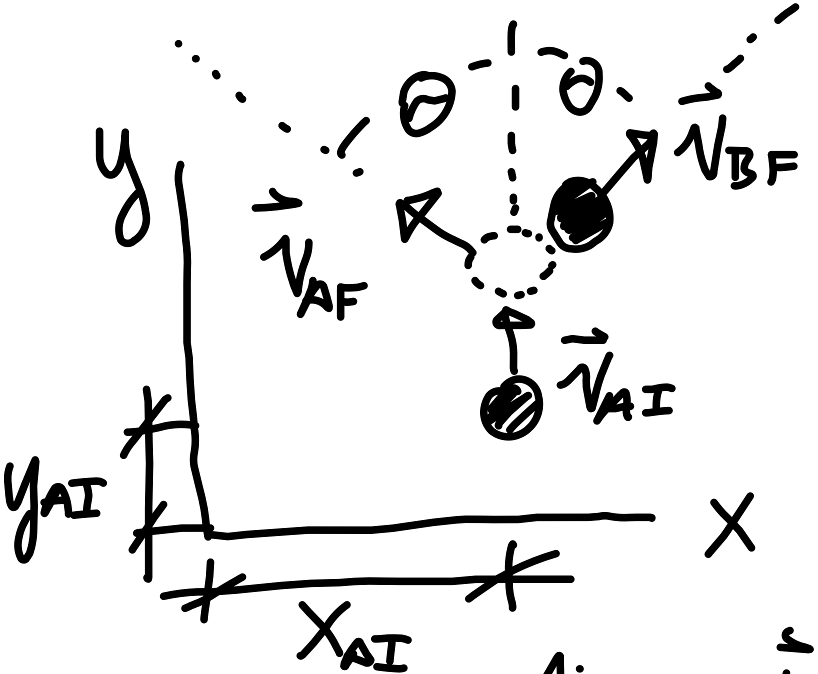
$M_A = M_B$ $\Delta\theta$

$$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\,\hat{j}$$

$$\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$$

$$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

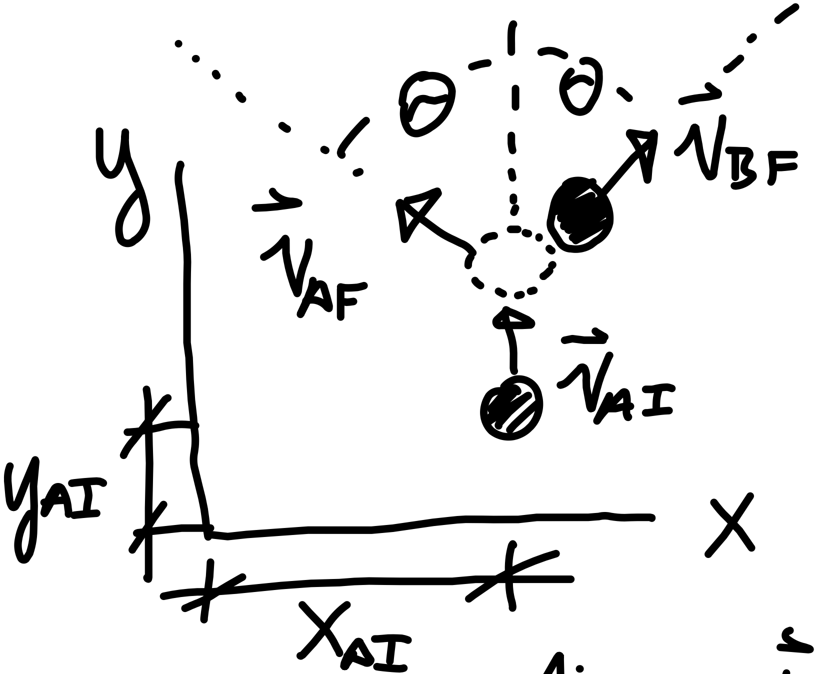
$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Since $\vec{L}_I = \vec{L}_F$



Given $V_{BI} = \cancel{\theta}$ &

$M_A = M_B$ & θ

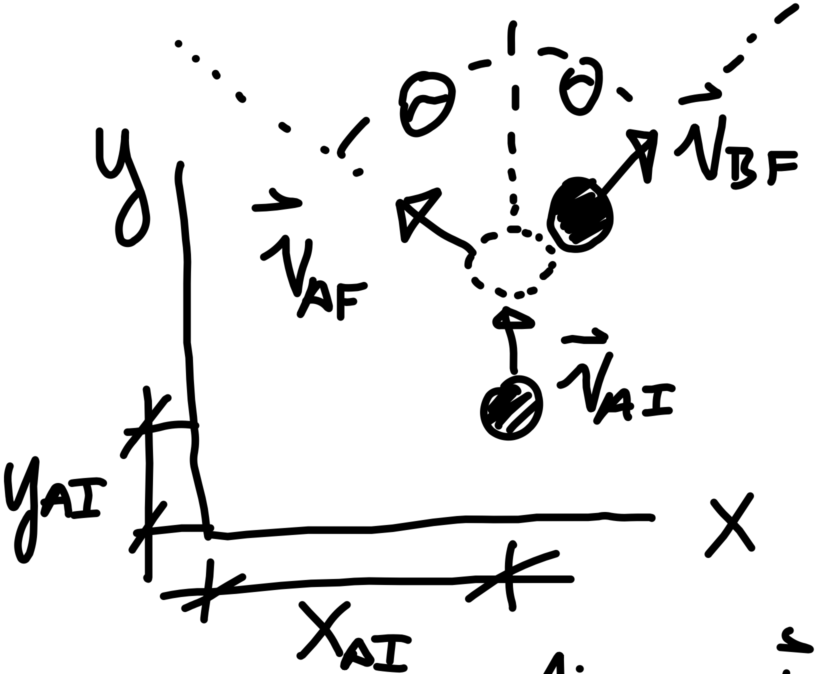
$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI} \hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF} (-\sin\theta \hat{i} + \cos\theta \hat{j})$

$+ M_B V_{BF} (\sin\theta \hat{i} + \cos\theta \hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx}$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

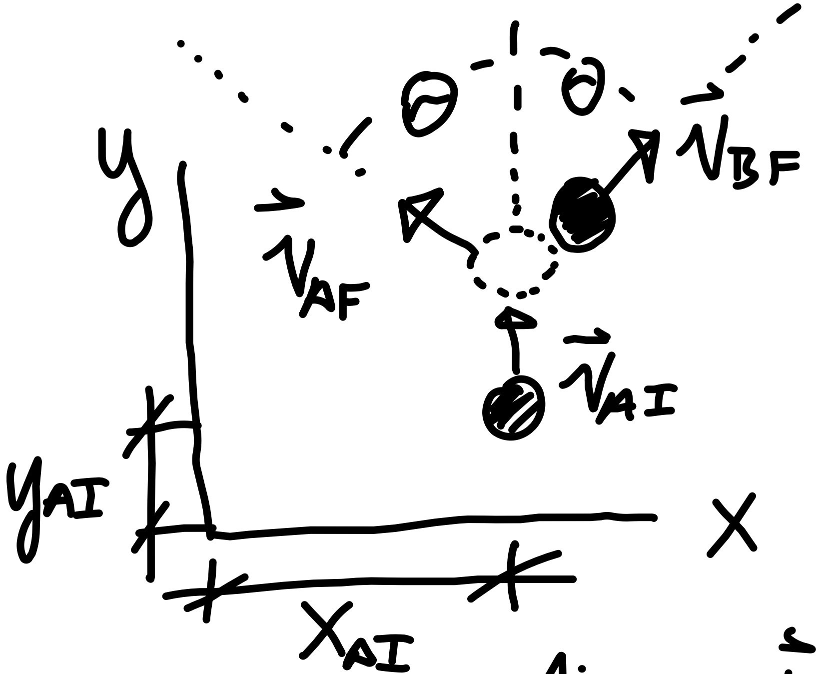
$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\,\hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$



Given $V_{BI} = 0$ ∴

$M_A = M_B$ so 0 $\cancel{0}$

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\hat{j}$

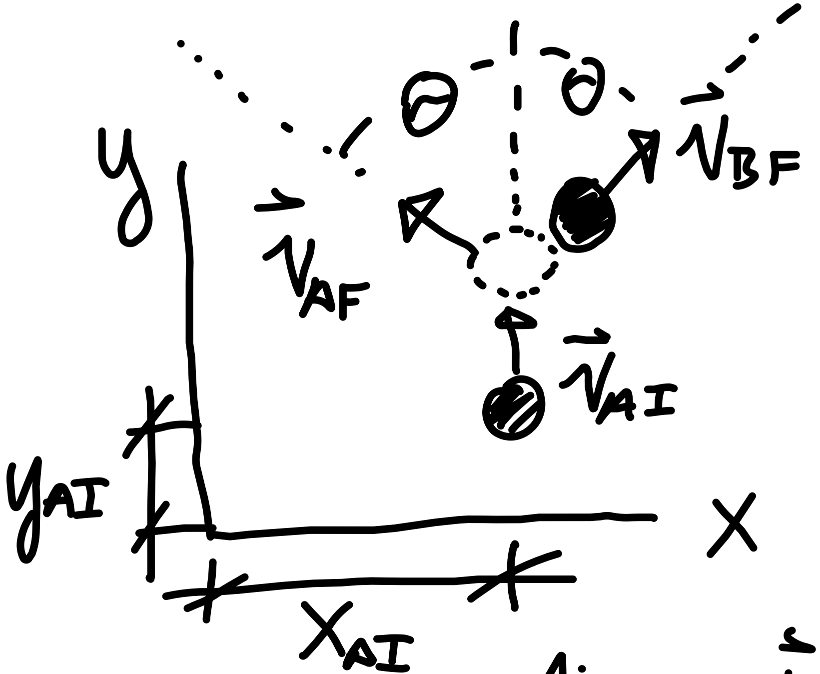
∴ $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

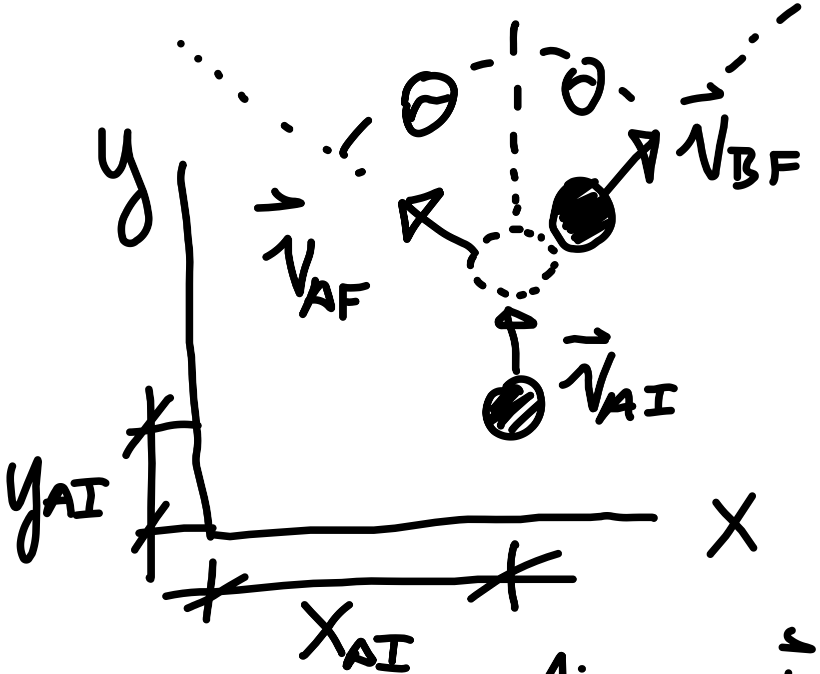
$\vec{L}_I = \vec{L}_{AI} + \vec{L}_{BI} = M_A V_{AI}\hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$$= \boxed{M_A V_{AF}(-\sin\theta\,\hat{i}} + \cos\theta\,\hat{j}) + \boxed{M_B V_{BF}(\sin\theta\,\hat{i}} + \cos\theta\,\hat{j})$$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$



Given $V_{BI} = 0$ &

$m_A = m_B$ so 0

$$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = m_A v_{AI}\hat{j}$$

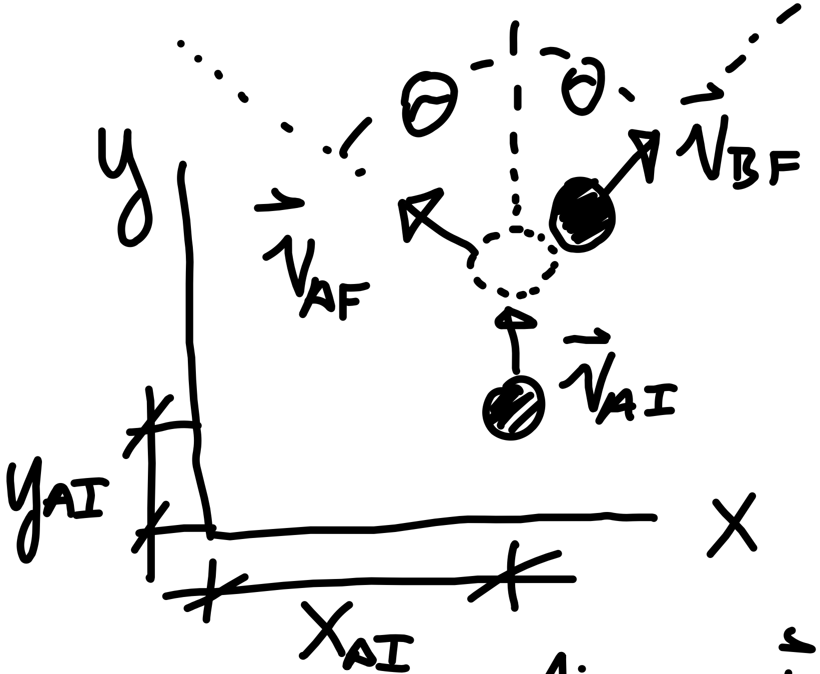
& $$\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$$

$$= m_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) + m_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(m_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$

Now $L_{Iy} = L_{Fy}$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

$\vec{L}_I = \vec{L}_{AI} + \vec{L}_{BI} = M_A V_{AI}\hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

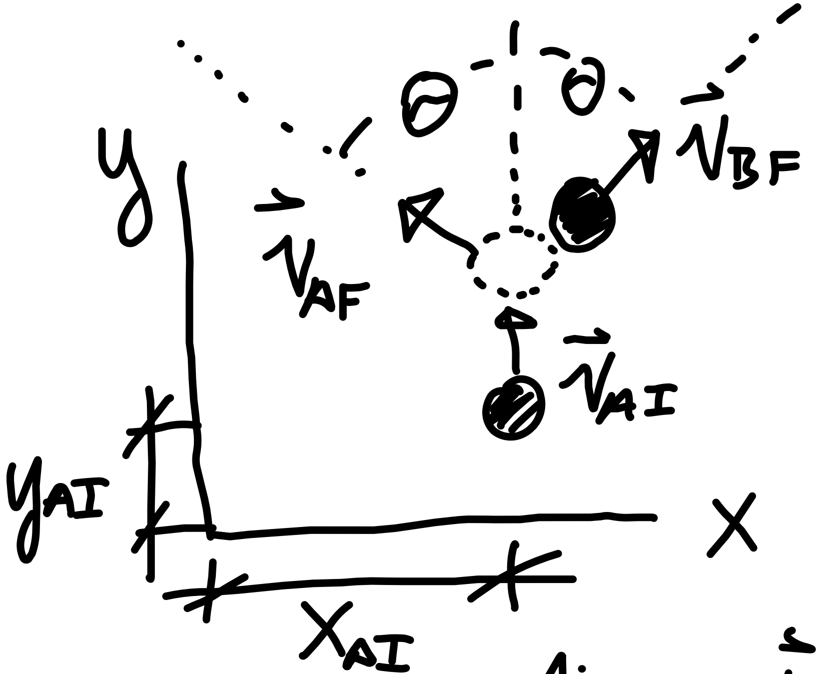
$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$

Now $L_{Iy} = L_{Fy} \Rightarrow M_A V_{AI} =$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

$$\vec{L}_I = \vec{L}_{AI} + \vec{L}_{BI} = M_A V_{AI}\hat{j}$$

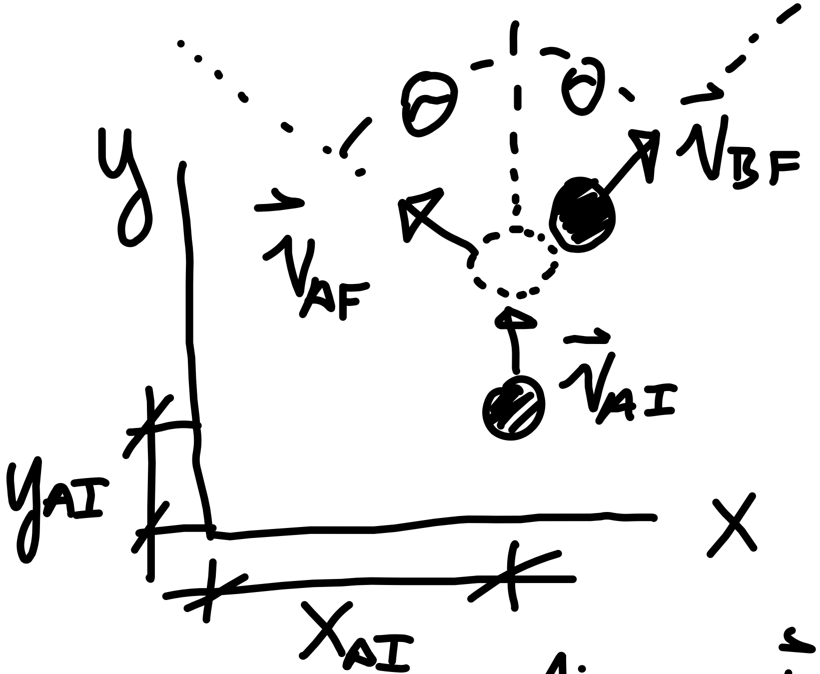
& $$\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$$

$$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) + M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$

Now $L_{Iy} = L_{Fy} \Rightarrow M_A V_{AI} = (M_A \cos\theta)(V_{AF} + V_{BF})$



Given $V_{BI} = 0$ ∴

$M_A = M_B$ so 0

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI}\hat{j}$

∴ $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

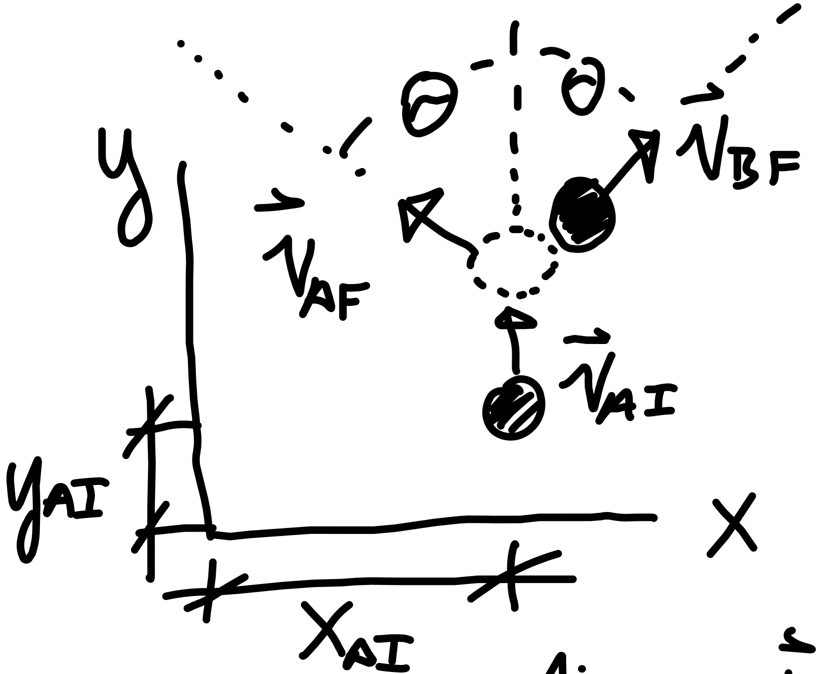
$+ M_B V_{BF}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$

Now $L_{Iy} = L_{Fy} \Rightarrow M_A V_{AI} = (M_A \cos\theta)(V_{AF} + V_{BF})$

$\Rightarrow V_{AI} = 2V_{AF}\cos\theta$



Given $V_{BI} = 0$ &

$M_A = M_B$ so 0

$\vec{L}_I = \vec{L}_{AI} + \cancel{\vec{L}_{BI}} = M_A V_{AI} \hat{j}$

& $\vec{L}_F = \vec{L}_{AF} + \vec{L}_{BF}$

$= M_A V_{AF} (-\sin\theta \hat{i} + \cos\theta \hat{j})$

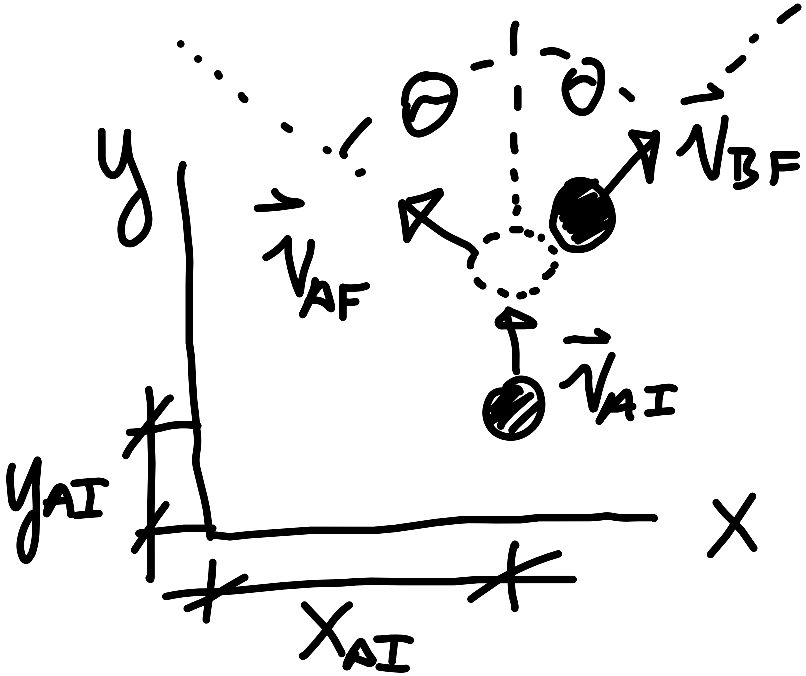
$+ M_B V_{BF} (\sin\theta \hat{i} + \cos\theta \hat{j})$

Since $\vec{L}_I = \vec{L}_F$, then $L_{Ix} = L_{Fx} = 0$

so $(M_A \sin\theta)(-V_{AF} + V_{BF}) = 0 \Rightarrow V_{AF} = V_{BF}$

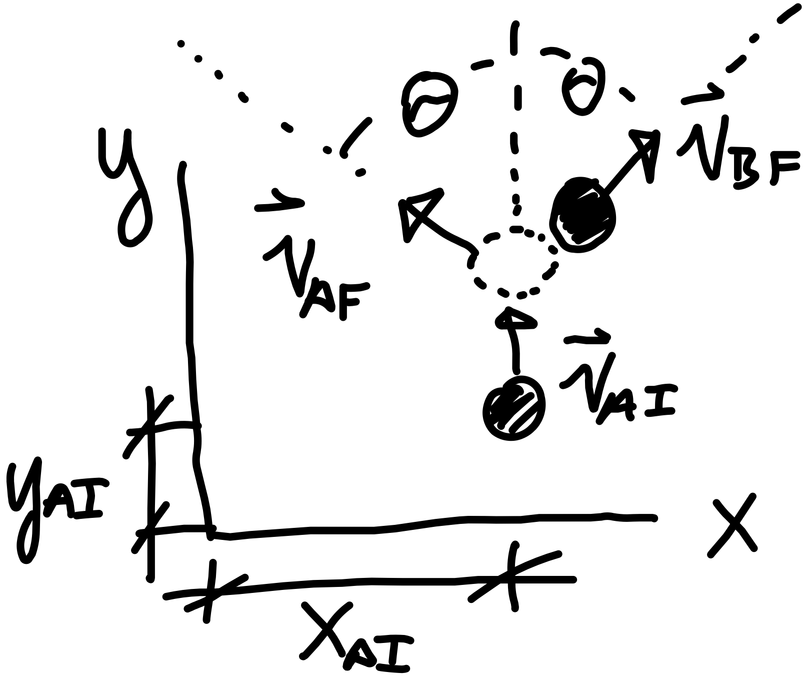
Now $L_{Iy} = L_{Fy} \Rightarrow M_A V_{AI} = (M_A \cos\theta)(V_{AF} + V_{BF})$

$\Rightarrow V_{AI} = 2 V_{AF} \cos\theta \Rightarrow V_{AF} = V_{BF} = \dfrac{V_{AI}}{2\cos\theta}$



Given $v_{BI} = 0$ &

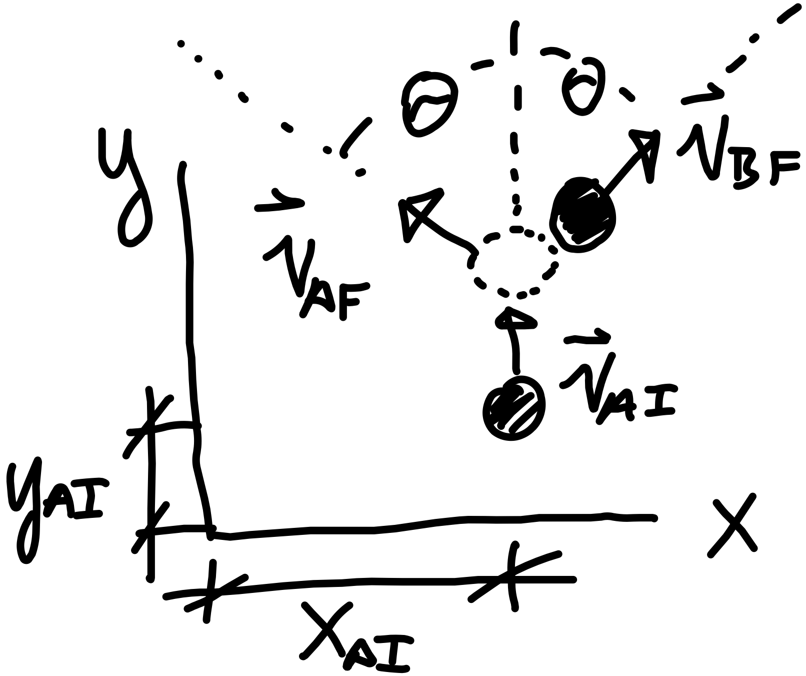
Sample calculation of $\vec{H}_0$:



Given $V_{BI} = 0$ &

Sample calculation of $\vec{H}_O$:

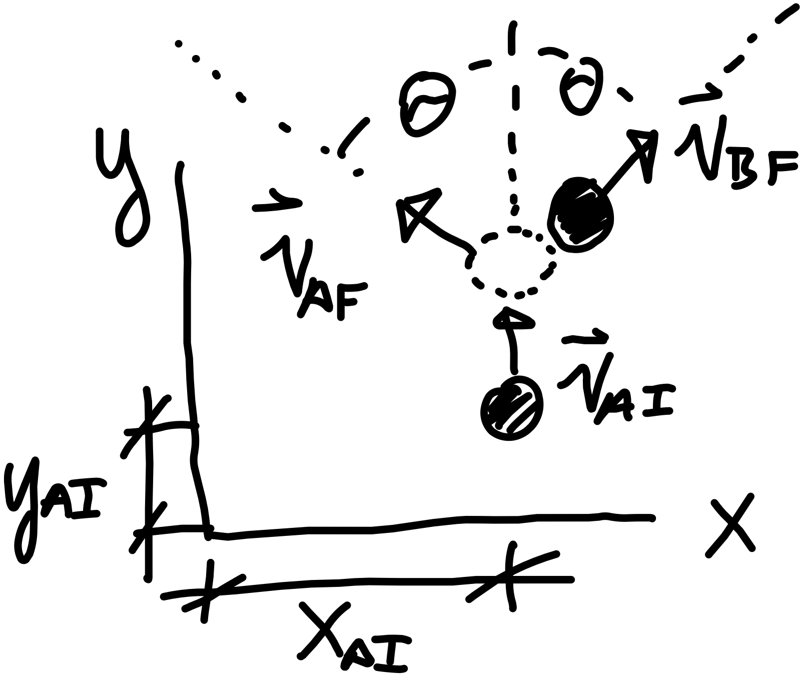
$$\vec{H}_{OI} = m_A \vec{r}_{AI} \times \vec{V}_{AI} +$$



Given $v_{BI} = \theta$ &

Sample calculation of $\vec{H}_0$:

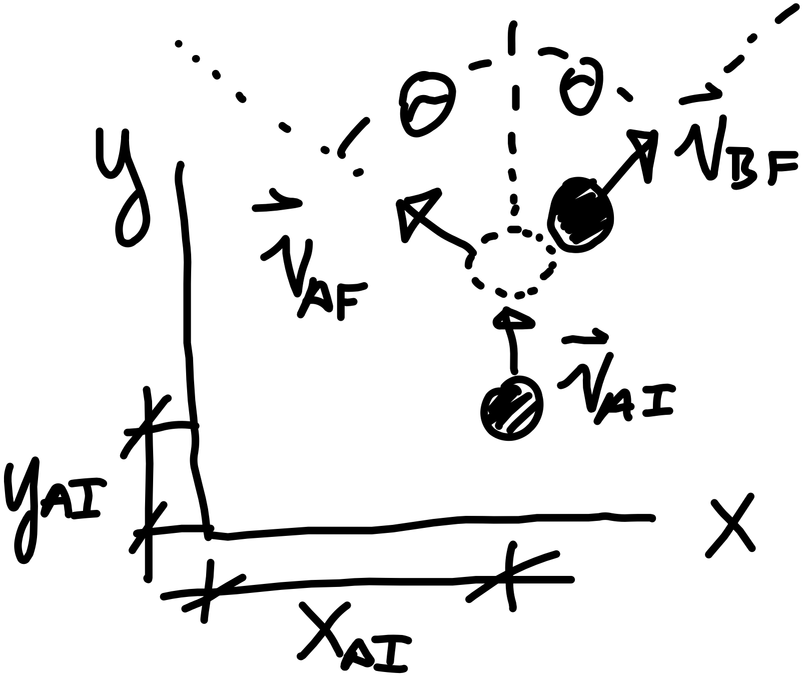
$$\vec{H}_{0I} = m_A \vec{r}_{AI} \times \vec{v}_{AI} + m_B \vec{r}_{BI} \times \vec{v}_{BI}$$



Given $V_{BI} = 0$ &

Sample calculation of $\vec{H}_O$:

$$\vec{H}_{OI} = m_A \vec{r}_{AI} \times \vec{V}_{AI} + m_B \vec{r}_{BI} \times \cancel{\vec{V}_{BI}}^{\,0}$$

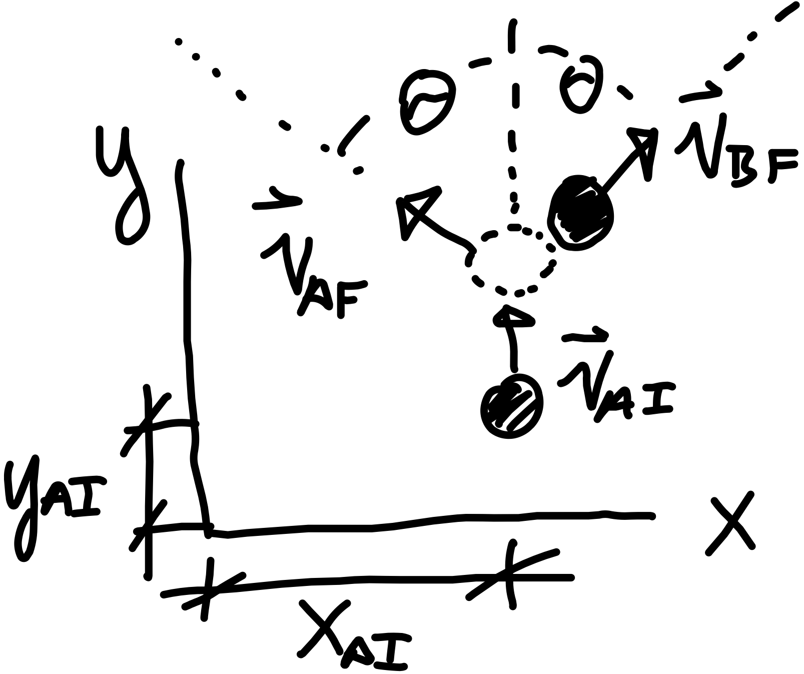


Given $V_{BI} = 0$;

Sample calculation of $\vec{H}_O$:

$$\vec{H}_{OI} = m_A \vec{r}_{AI} \times \vec{V}_{AI} + \overbrace{m_B \vec{r}_{BI} \times \vec{V}_{BI}}^{0}$$

$$= m_A (x_{AI}\hat{i} + y_{AI}\hat{j}) \times V_{AI}\hat{j}$$



Given $V_{BI} = 0$ ∴

Sample calculation of $\vec{H}_O$:

$$\vec{H}_{OI} = m_A \vec{r}_{AI} \times \vec{V}_{AI} + m_B \vec{r}_{BI} \times \cancelto{0}{\vec{V}_{BI}}$$

$$= m_A (x_{AI}\hat{i} + y_{AI}\hat{j}) \times V_{AI}\,\hat{j} = m_A x_{AI} V_{AI}\,\hat{k}$$

Center of mass:

Center of mass: Let

$\vec{r} \equiv$ center-of-mass position

Center of mass: Let

$\vec{r} \equiv$ center-of-mass position & $M = \sum m_i$

Center of mass: Let

$\vec{\bar{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\vec{\bar{r}} \equiv \sum m_i \vec{r}_i$

Center of mass: Let

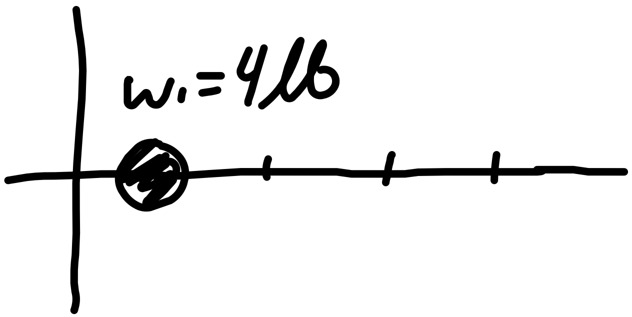
$\vec{r} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\vec{r} \equiv \sum m_i \vec{r}_i$ Example problem:

Center of mass: Let

$\vec{r} \equiv$ center-of-mass position & $m = \sum m_i$

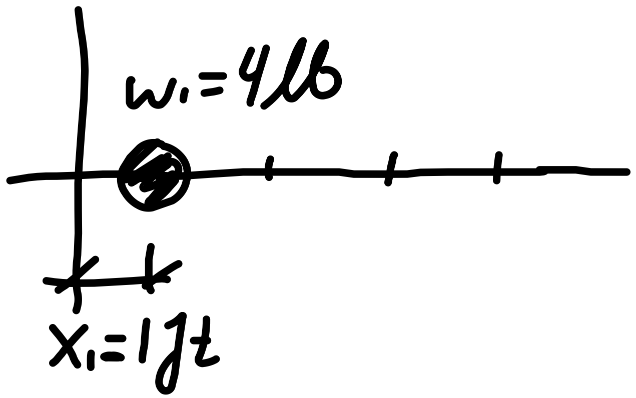
Now $m\vec{r} \equiv \sum m_i \vec{r}_i$ Example problem:



Center of mass: Let

$\vec{r} \equiv$ center-of-mass position & $m = \sum m_i$

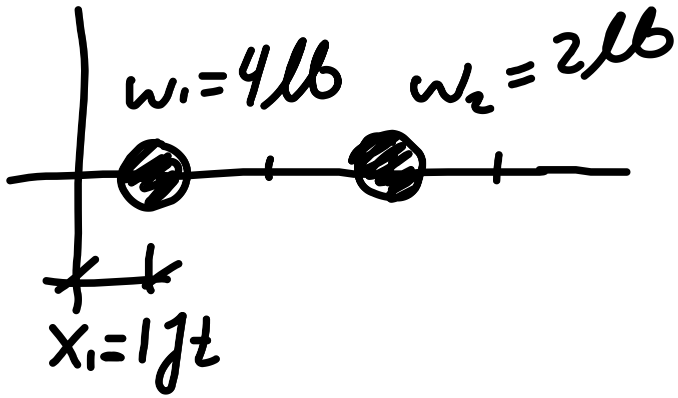
Now $m\vec{r} \equiv \sum m_i \vec{r}_i$ **Example problem:**



Center of mass: Let

$\vec{r} \equiv$ center-of-mass position & $m = \sum m_i$

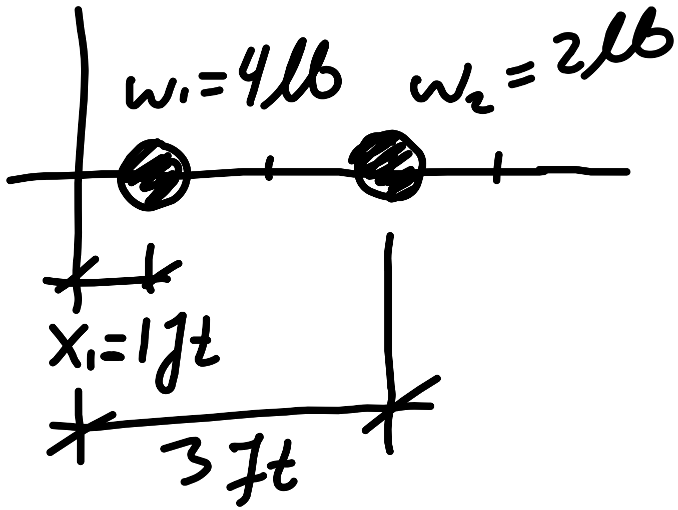
Now $m\vec{r} \equiv \sum m_i \vec{r}_i$ Example problem:



Center of mass: Let

$\vec{r} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\vec{r} \equiv \sum m_i \vec{r}_i$ **Example problem:**



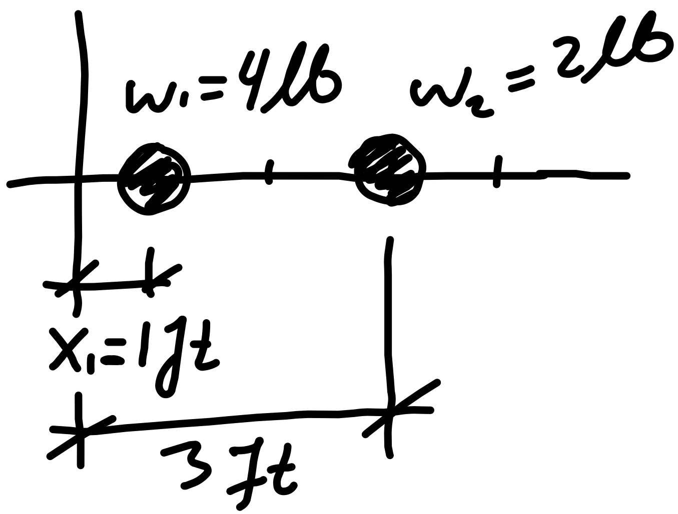
Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$

Example problem:

Find $\bar{\vec{r}}$



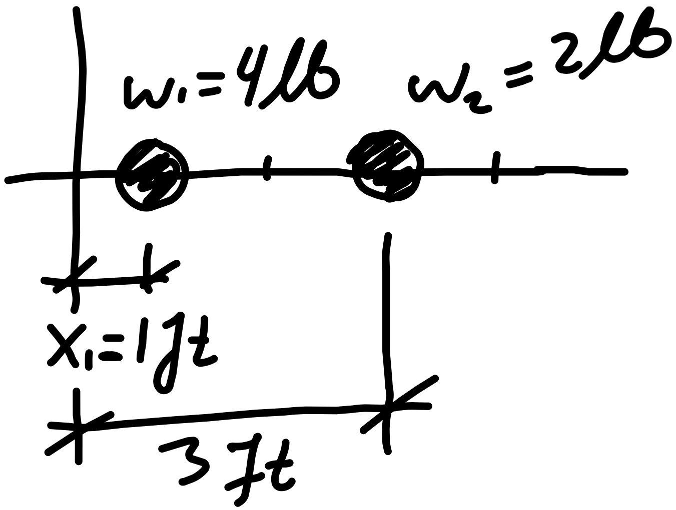
Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$

Example problem:

Find $\bar{\vec{r}}$ $m = \sum m$



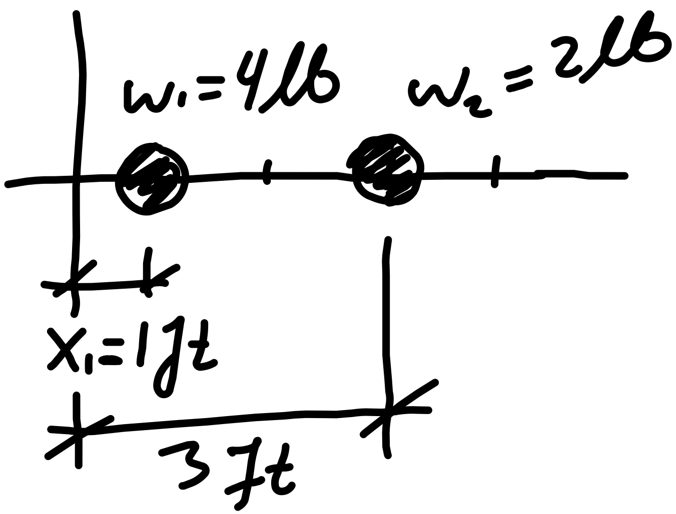
Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$

Example problem:

Find $\bar{\vec{r}}$ $\quad m = \sum m = \frac{4\,lb}{g} + \frac{2\,lb}{g}$



Center of mass: Let

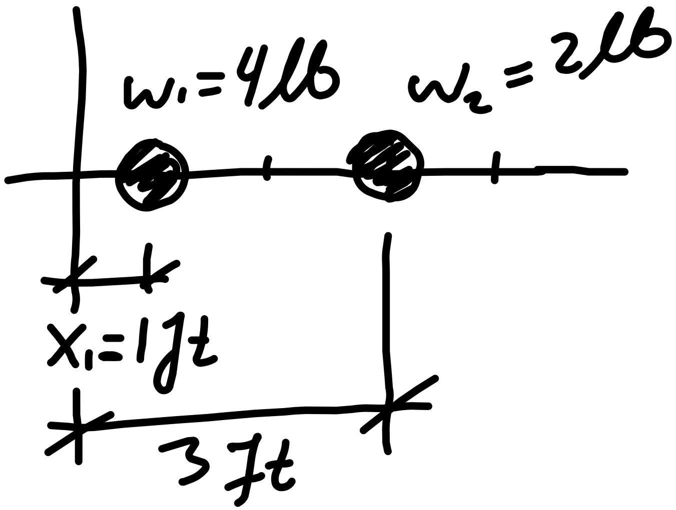
$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$

Example problem:

Find $\bar{\vec{r}}$ $\quad m = \sum m = \frac{4lb}{g} + \frac{2lb}{g}$

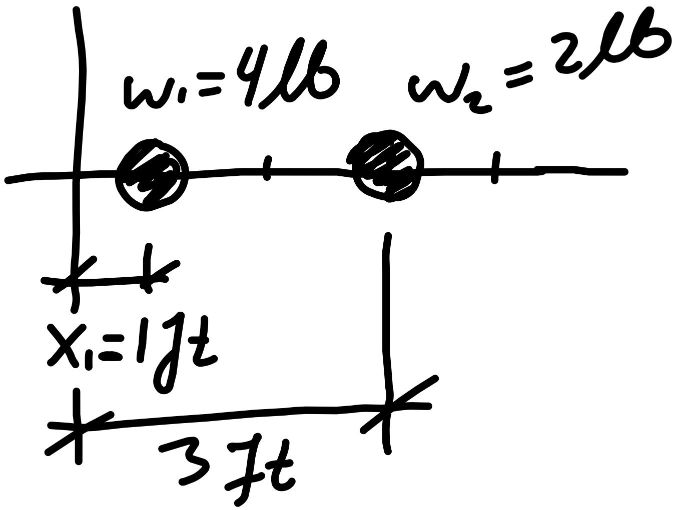
$\Rightarrow m = 6lb/g$



Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$



Example problem:

Find $\bar{\vec{r}}$

$m = \sum m = \frac{4lb}{g} + \frac{2lb}{g}$

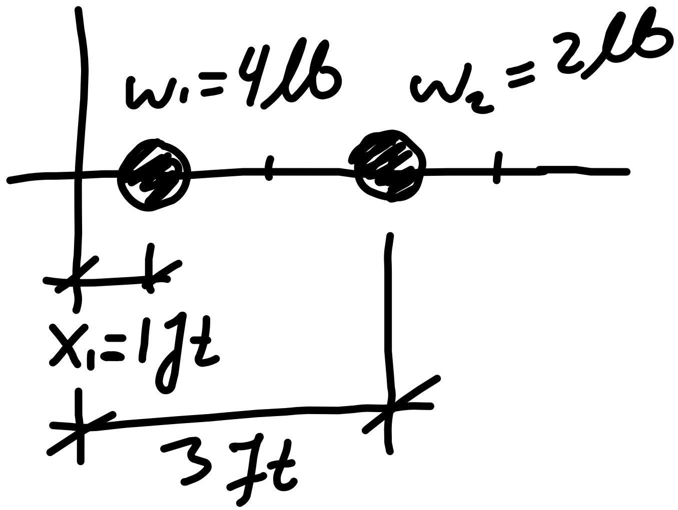
$\Rightarrow m = 6lb/g$ &

$$\sum m_i \vec{r}_i = \left(\frac{4lb}{g}\right)(1ft\,\hat{i}) + \left(\frac{2lb}{g}\right)(3ft\,\hat{i})$$

Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$



Example problem:

Find $\bar{\vec{r}}$

$$m = \sum m = \frac{4\,lb}{g} + \frac{2\,lb}{g}$$

$$\Rightarrow m = 6\,lb/g \quad \&$$

$$\sum m_i \vec{r}_i = \left(\frac{4\,lb}{g}\right)(1\,ft\,\hat{i}) + \left(\frac{2\,lb}{g}\right)(3\,ft\,\hat{i})$$

$$= (10\,lb\cdot ft/g)\,\hat{i}$$

Center of mass: Let

$\bar{\vec{r}} \equiv$ center-of-mass position & $m = \sum m_i$

Now $m\bar{\vec{r}} \equiv \sum m_i \vec{r}_i$

Example problem:

Find $\bar{\vec{r}}$ $\quad m = \sum m = \frac{4\,lb}{g} + \frac{2\,lb}{g}$

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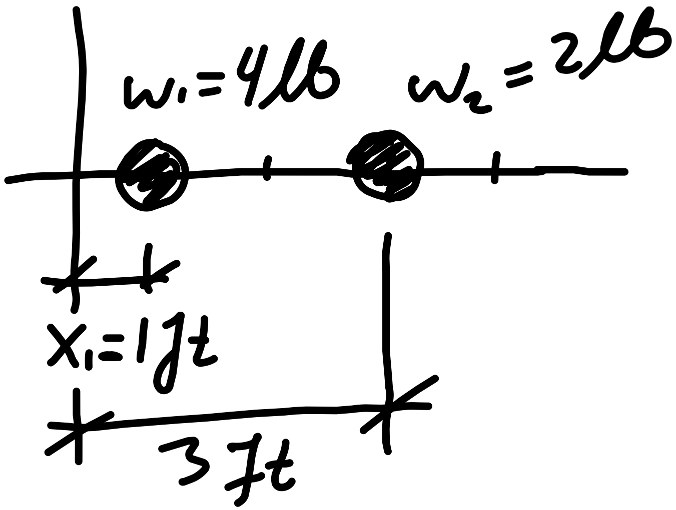

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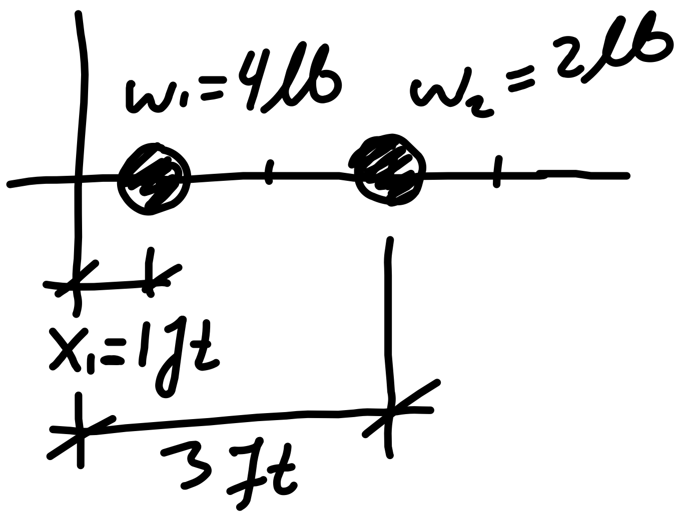

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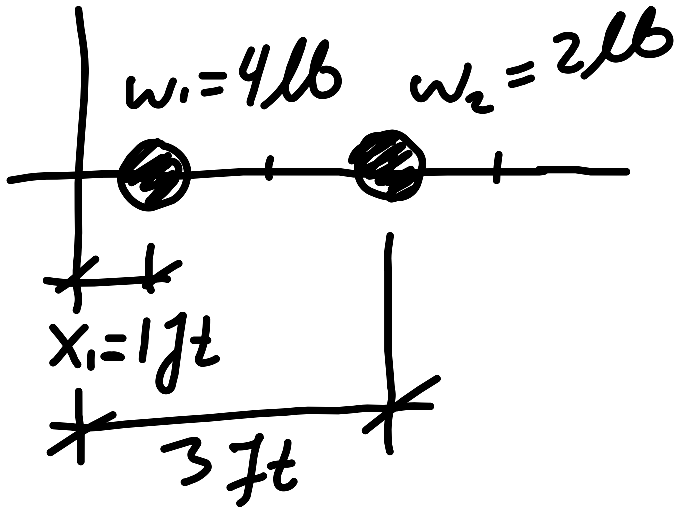

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$$\Rightarrow \boxed{\bar{\vec{r}} = \frac{5}{3}\,\text{ft}\,\hat{i}}$$

We can define the c-m frame as
coordinate system G

We can define the c-m frame as Coordinate system G with axis x', y', z':

We can define the c-m frame as coordinate system G with axis x'y'z':

$$\sum \vec{M}_G = \dot{\vec{H}}_G'$$

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$\sum \vec{M}_G = \dot{\vec{H}}_G'$ with $\vec{H}_G' = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$

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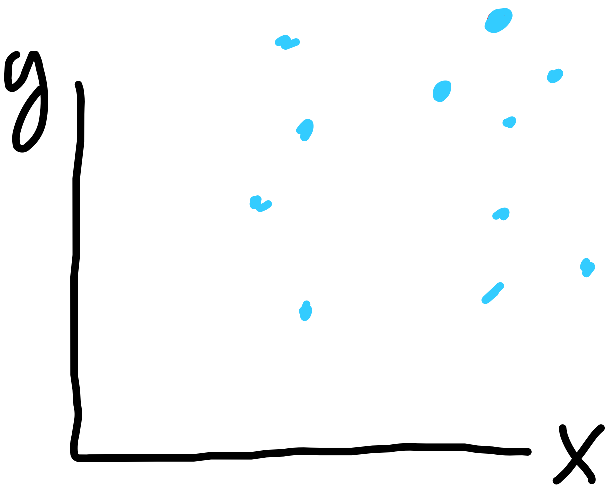

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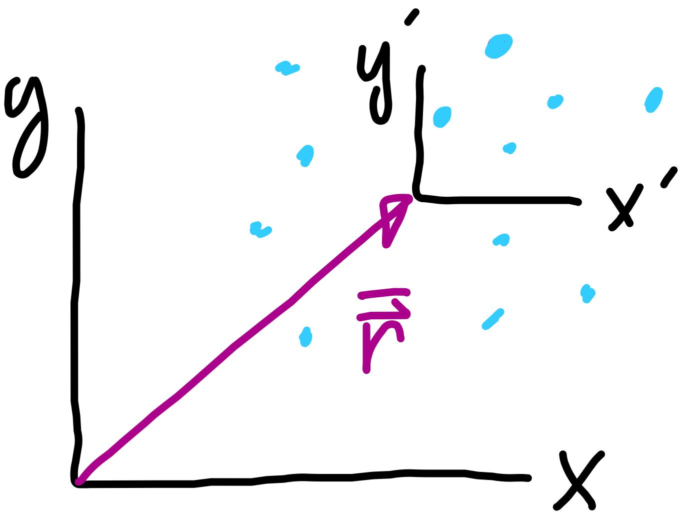

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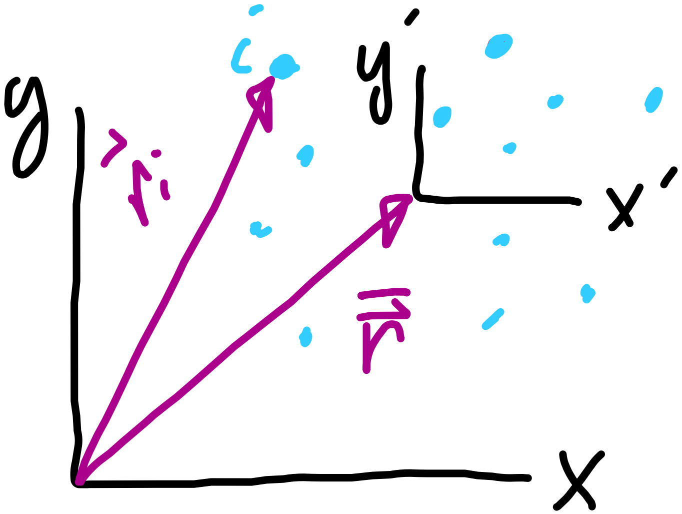

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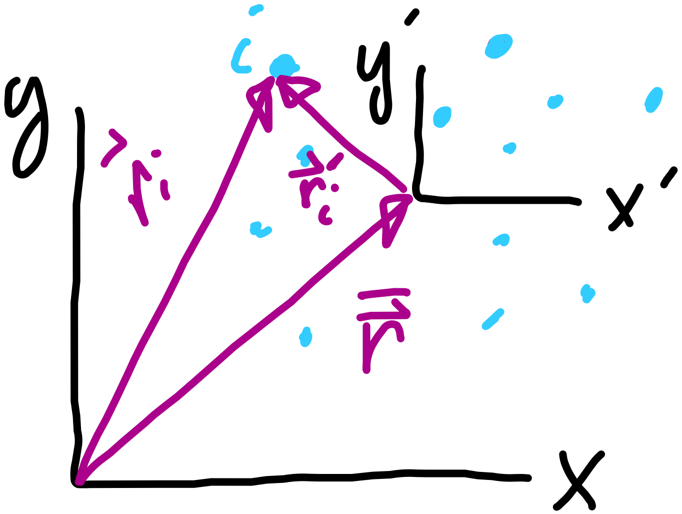

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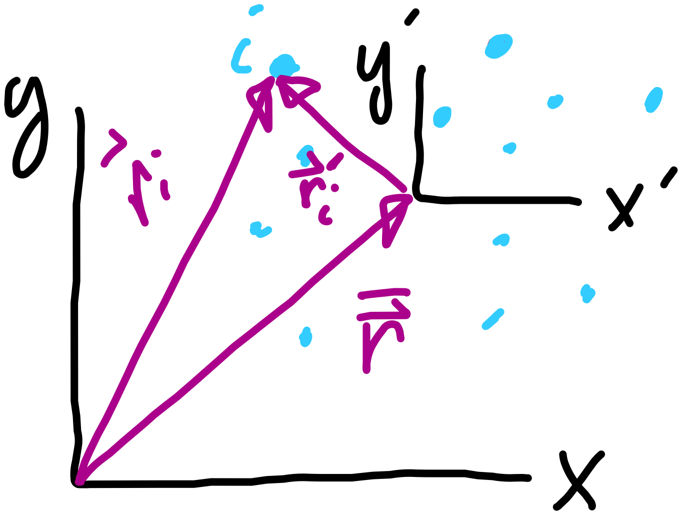

Kinetic energy $T = \frac{1}{2}\sum m_i v_i^2$

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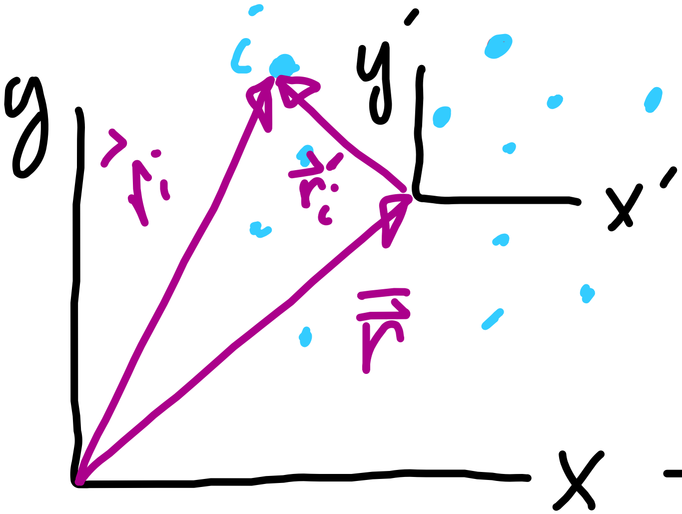

Kinetic energy $T = \frac{1}{2} \sum m_i v_i^2$

in terms of c-m frame

$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \sum m_i (v_i')^2$

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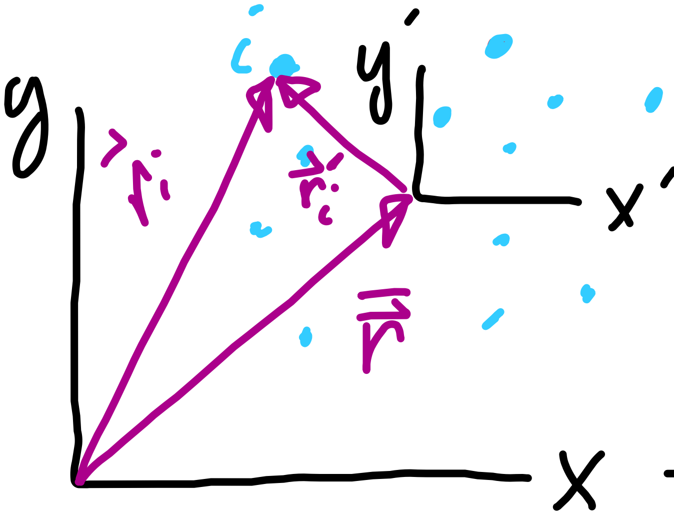

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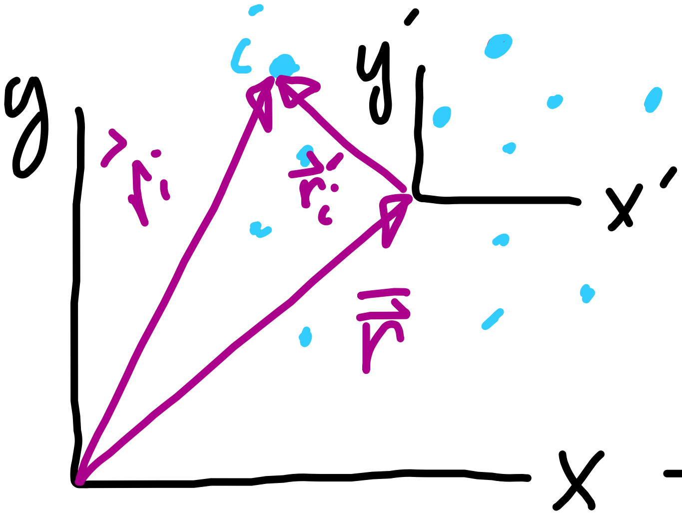

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$$\sum \int_{t_1}^{t_2} \vec{F}\,dt = \Delta \vec{L}$$