L19

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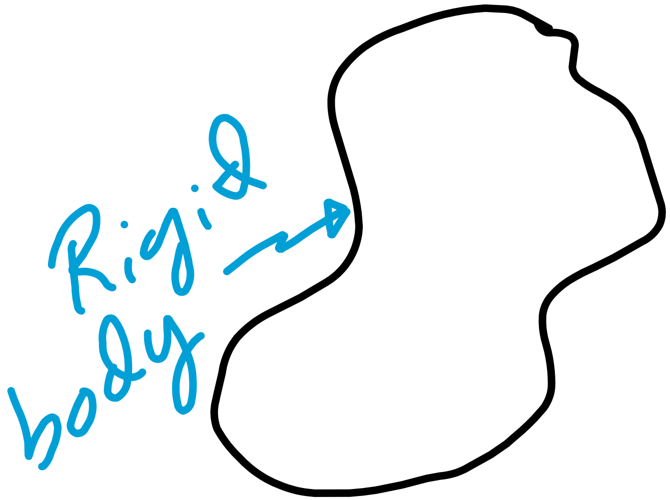
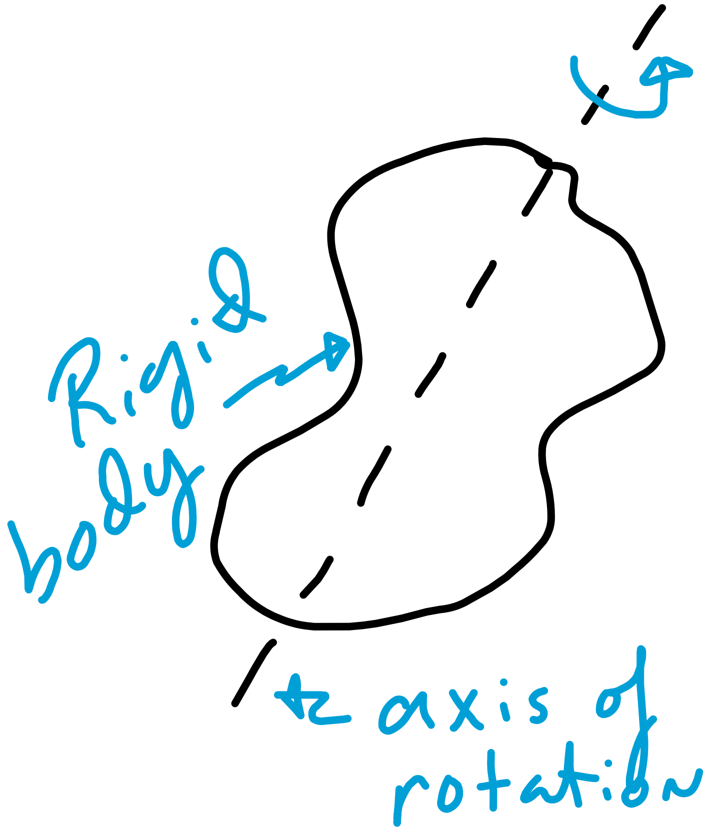
Notation:

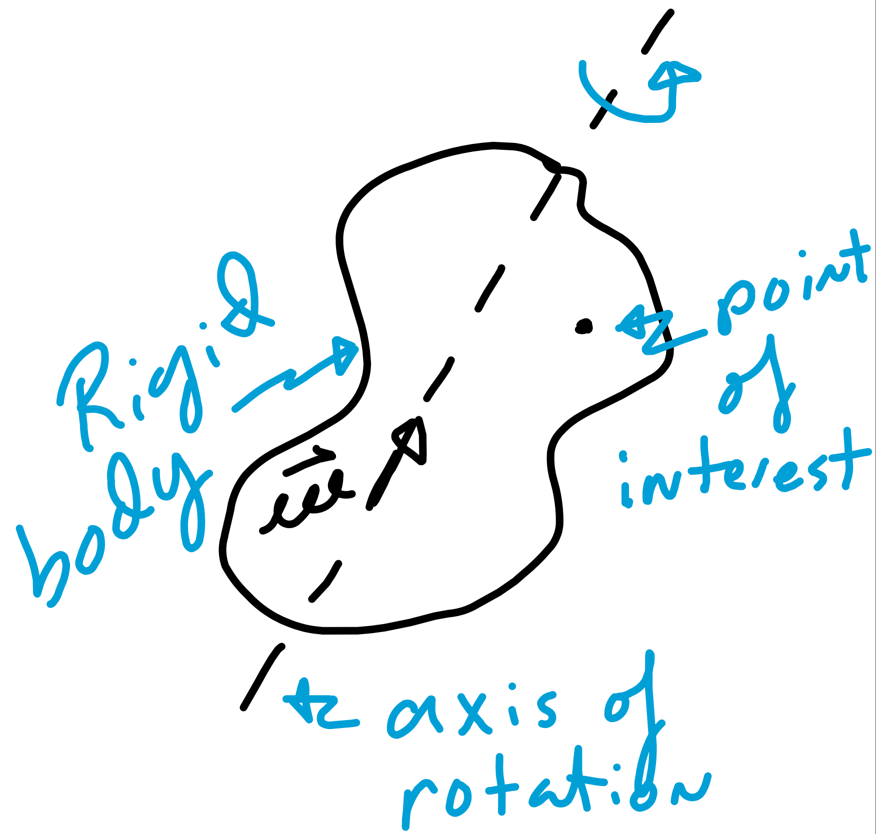
Let $\vec{a}$ be some vector. Define $\hat{a}$ as the unit vector $\hat{a} \equiv \frac{\vec{a}}{|\vec{a}|}$. So, with this notation $\hat{i} = \hat{x}$, $\hat{j} = \hat{y}$ & $\hat{k} = \hat{z}$.


Rigid
body

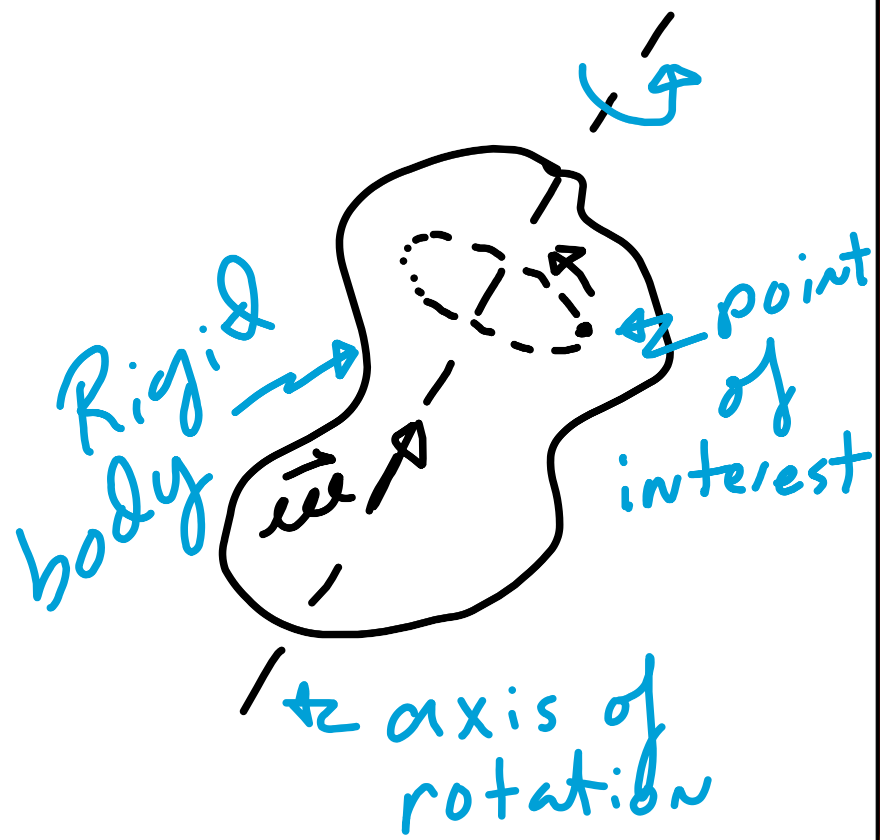

Rigid body
axis of rotation

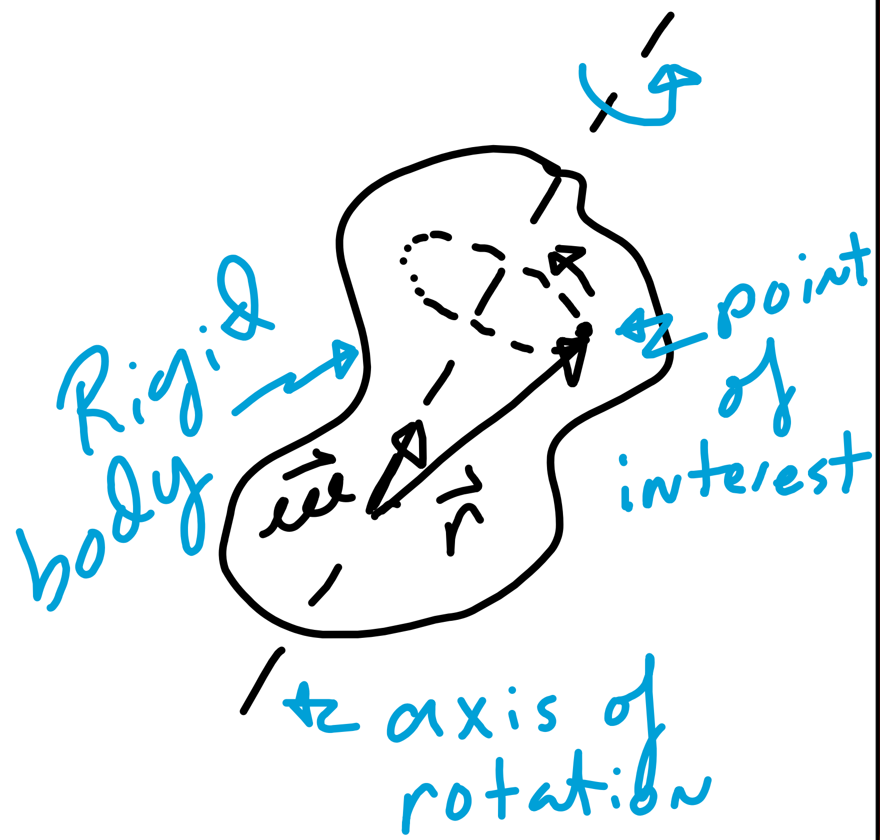

Rigid body
axis of rotation

Rigid body
$\vec{\omega}$
axis of rotation



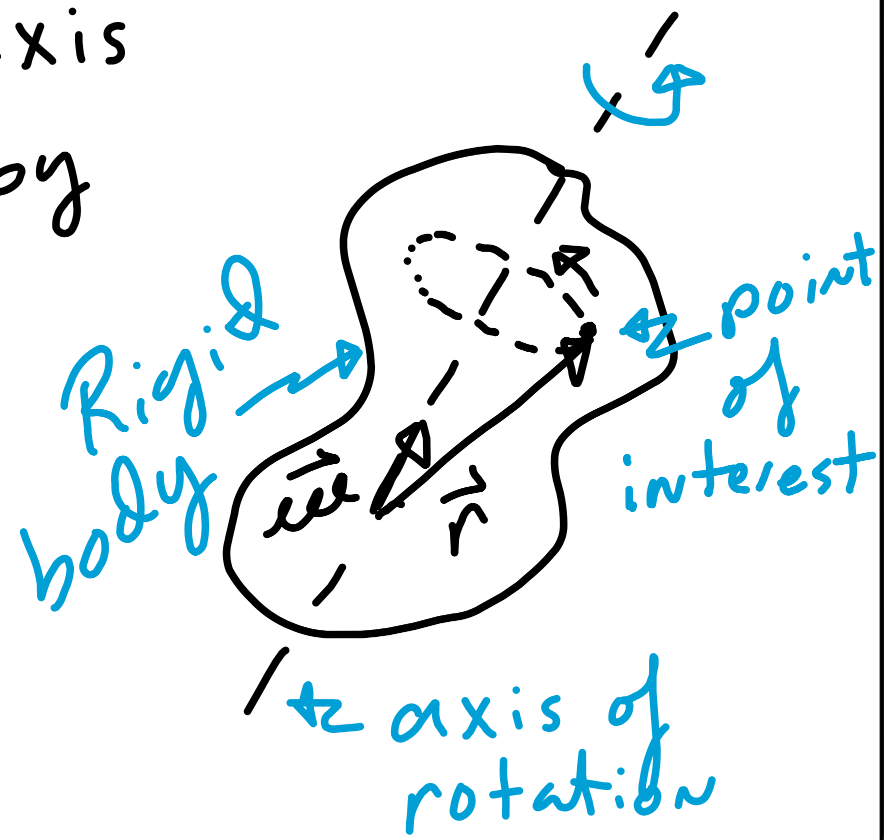
Rigid body
$\vec{\omega}$
point of interest
axis of rotation


Rigid body
$\vec{\omega}$
point of interest
axis of rotation

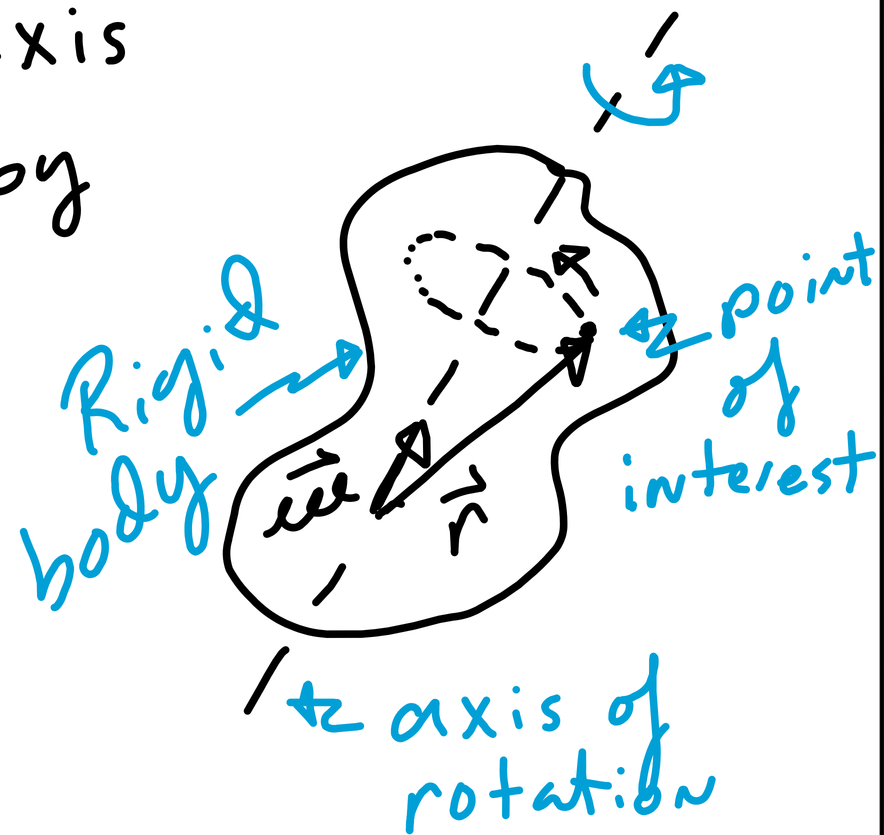


Rigid body
$\vec{\omega}$
$\vec{r}$
point of interest
axis of rotation

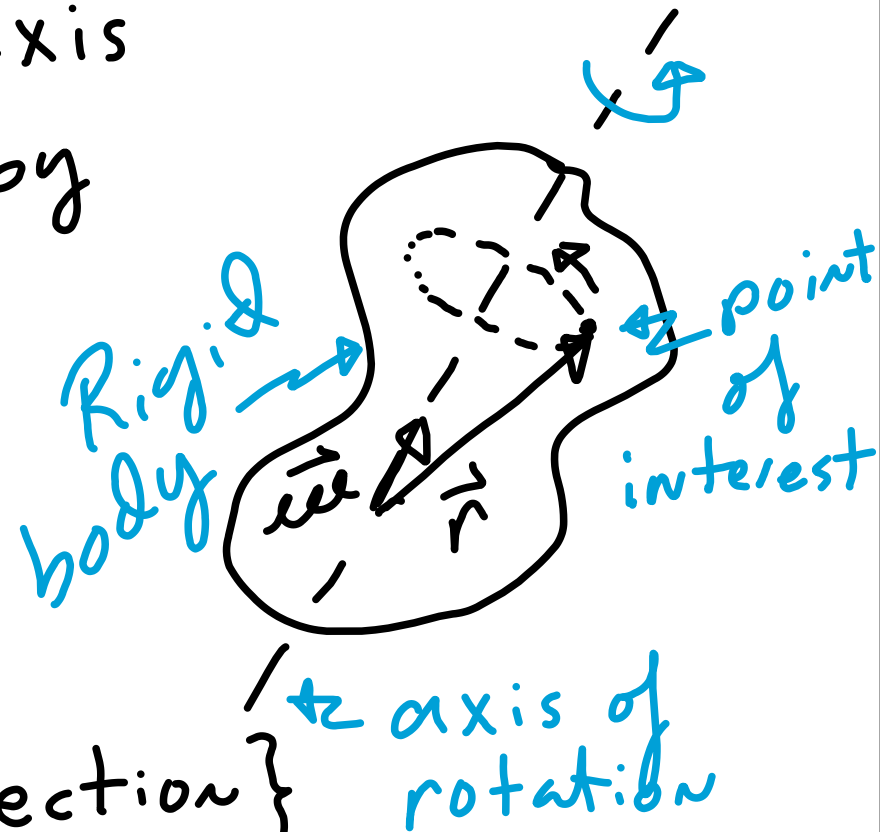
$\vec{\omega}$ is along axis of rotation defined by the right hand rule



$\vec{\omega}$ is along axis of rotation defined by the right hand rule {grab axis of rotation such that fingers wrap in direction of rotation}

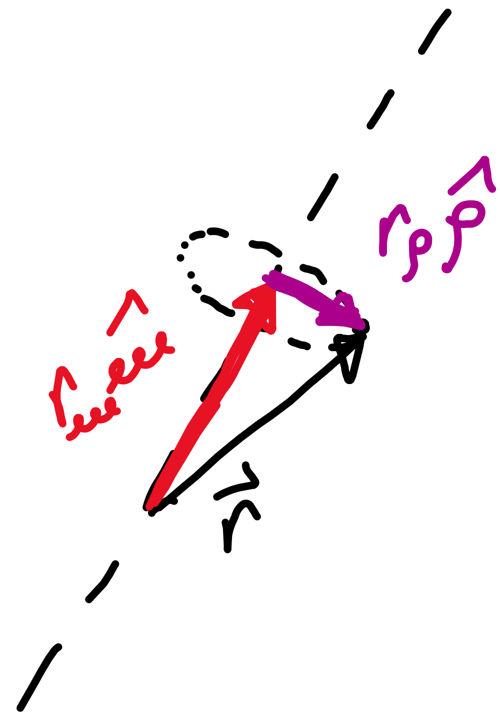


$\vec{\omega}$ is along axis of rotation defined by the right hand rule {grab axis of rotation such that fingers wrap in direction of rotation & thumb points in $\vec{\omega}$ direction}

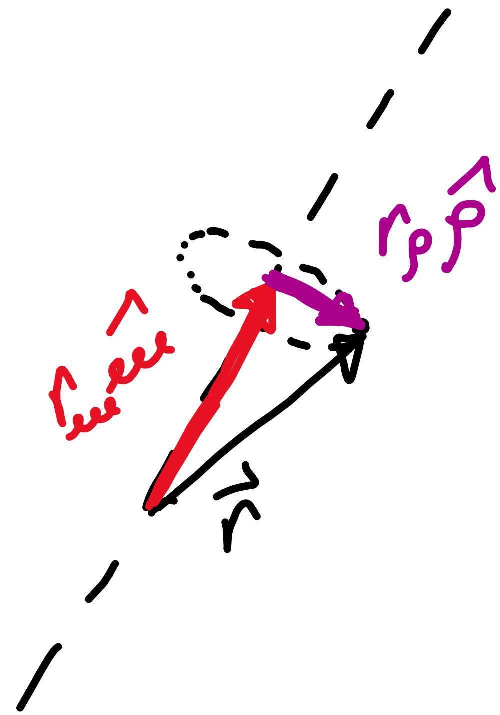


We can break up $\vec{r}$ into orthogonal directions of $\hat{p}$ & $\hat{u}$:

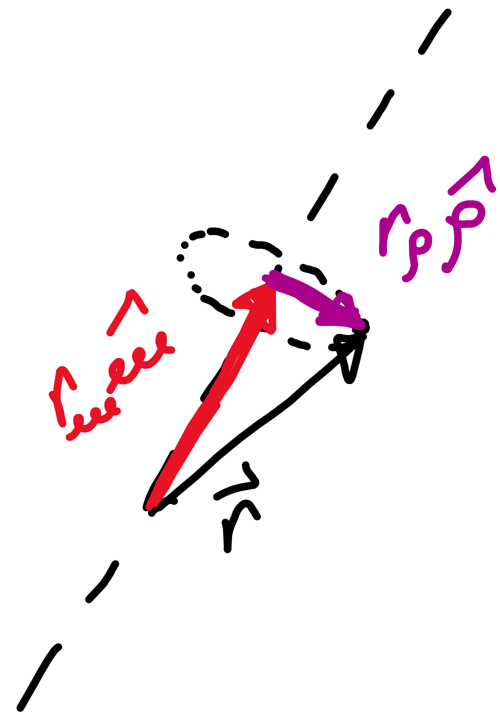
We can break up $\vec{r}$ into orthogonal directions of $\hat{\rho}$ & $\hat{\mu}$:



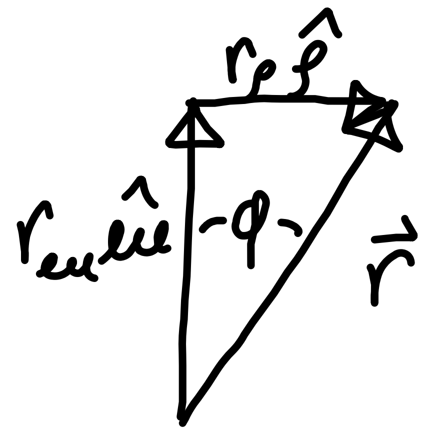
We can break up $\vec{r}$ into orthogonal directions of $\hat{\rho}$ & $\hat{\ell\ell}$: $\vec{r} = r_{\ell\ell\ell}\,\hat{\ell\ell\ell} + r_\rho\,\hat{\rho}$



We can break up $\vec{r}$ into orthogonal directions of $\hat{\rho}$ & $\hat{ee}$: $\vec{r} = r_{ee}\,\hat{ee} + r_\rho\,\hat{\rho}$

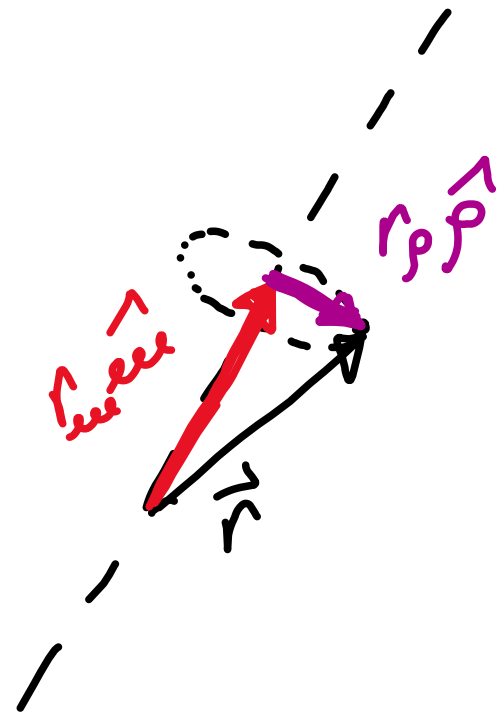


"side" view:



We can break up $\vec{r}$ into orthogonal directions of $\hat{\rho}$ & $\hat{\ell\ell}$: $\vec{r} = r_{\ell\ell\ell}\,\hat{\ell\ell\ell} + r_\rho \hat{\rho}$

Now $\vec{r} = r\cos\varphi\,\hat{\ell\ell} + r\sin\varphi\,\hat{\rho}$



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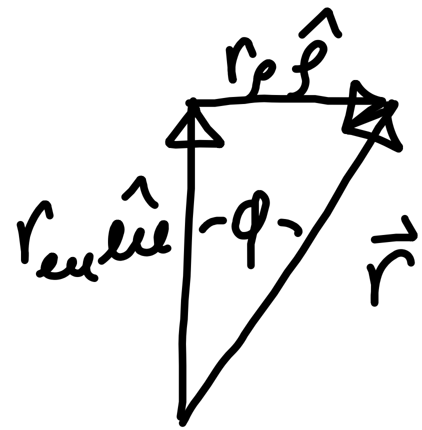

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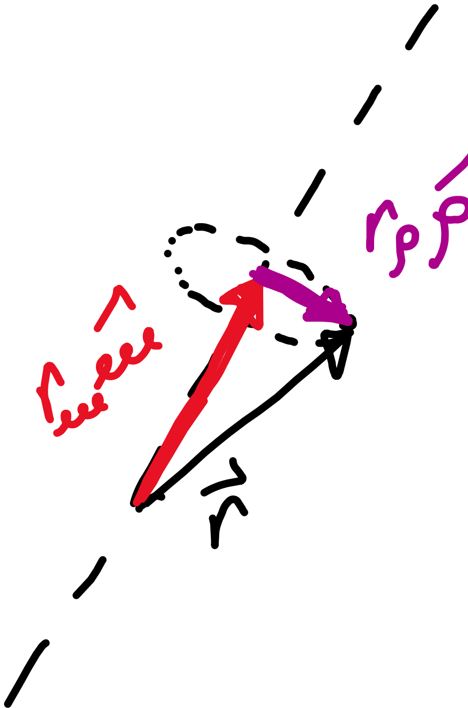
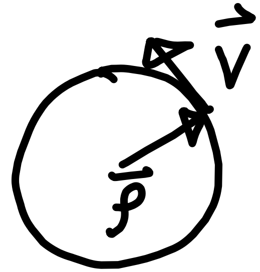
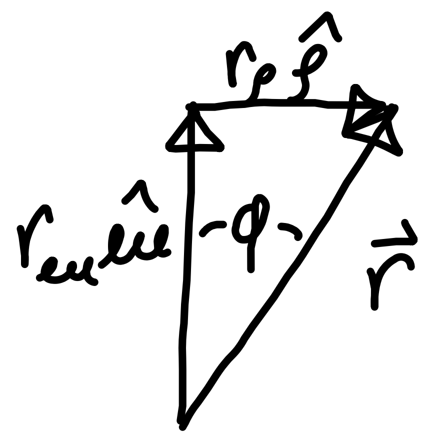


"Top" view:



"Side" view:



We can break up $\vec{r}$ into orthogonal directions of $\hat{\rho}$ & $\hat{\omega}$: $\vec{r} = r_{\omega}\hat{\omega} + r_{\rho}\hat{\rho}$

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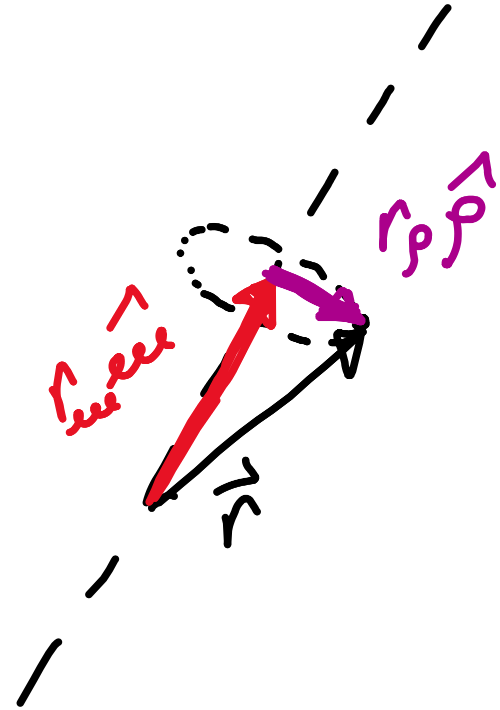

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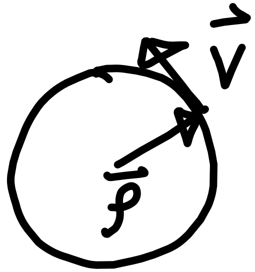
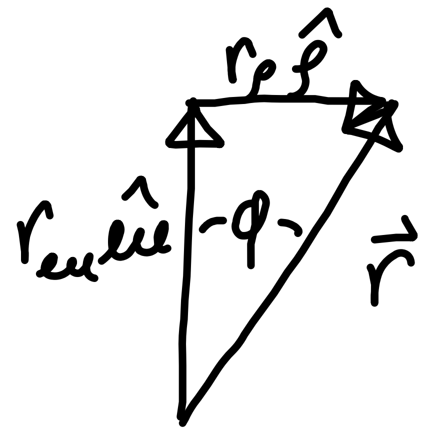


we know that

$V = \rho\dot{\theta} = \rho\omega$

"Side" view:



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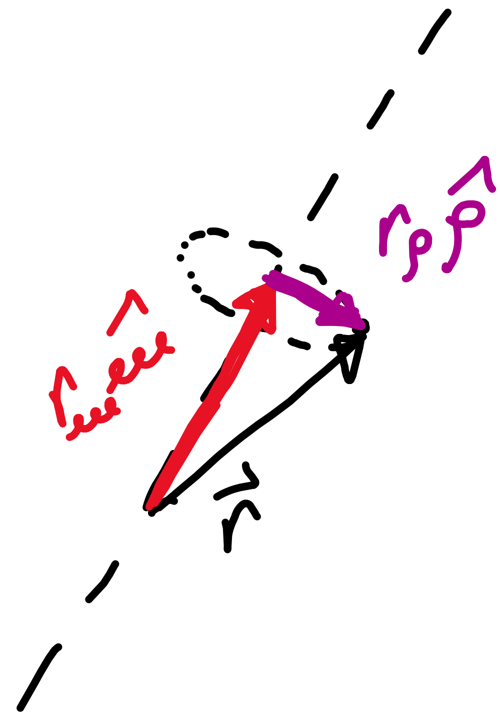

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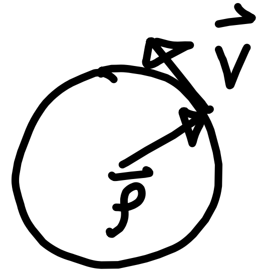
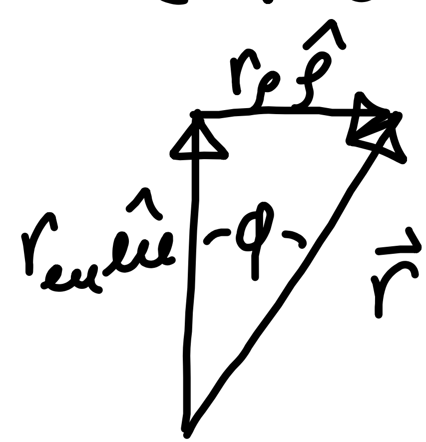


we know that

$v = \rho\dot{\theta} = \rho\omega$, but $\rho = r\sin\varphi$

"Side" view:



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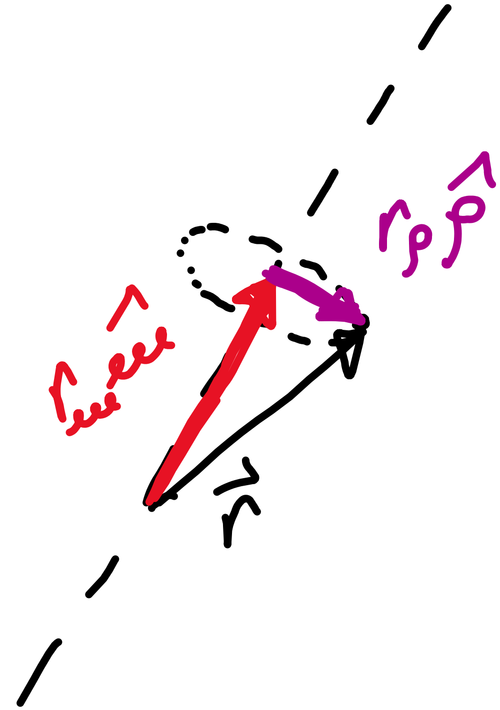

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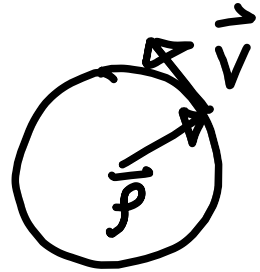
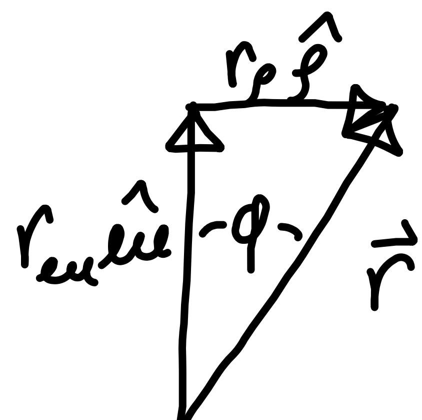


we know that

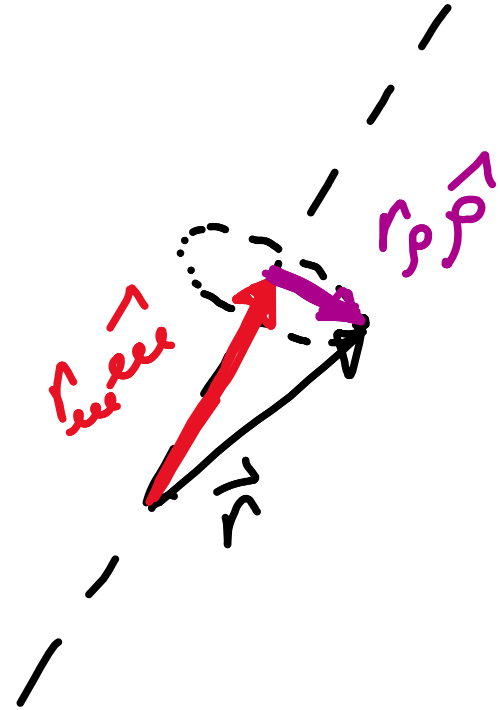
$V = \rho\dot{\theta} = \rho\omega$, but $\rho = r\sin\varphi$

$\Rightarrow V = (r\sin\varphi)\omega$

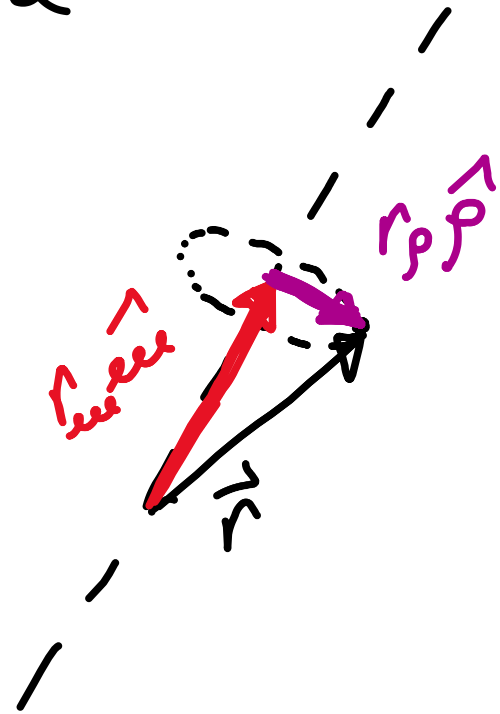
"Side" view:



We have $v = (r \sin\varphi)$eee

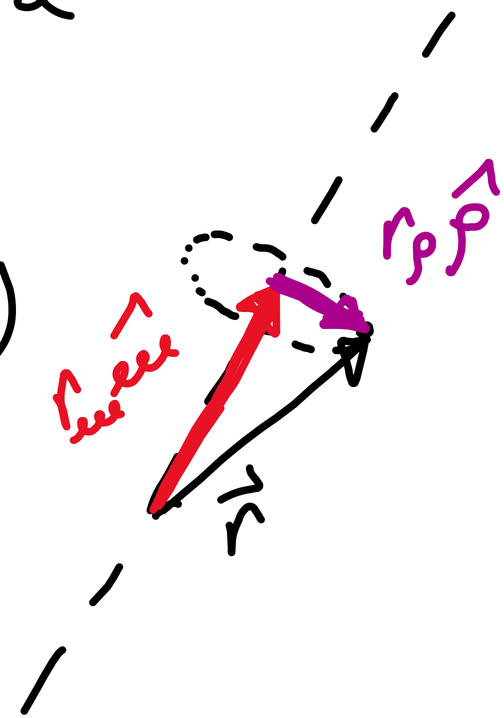


We have $v = (r \sin\varphi)\omega$ and only need to obtain the direction:



We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:

<u>Note</u>: $\vec{\omega} \times \vec{r} = \omega\hat{\omega} \times (r_\omega\hat{\omega} + r_\rho\hat{\rho})$



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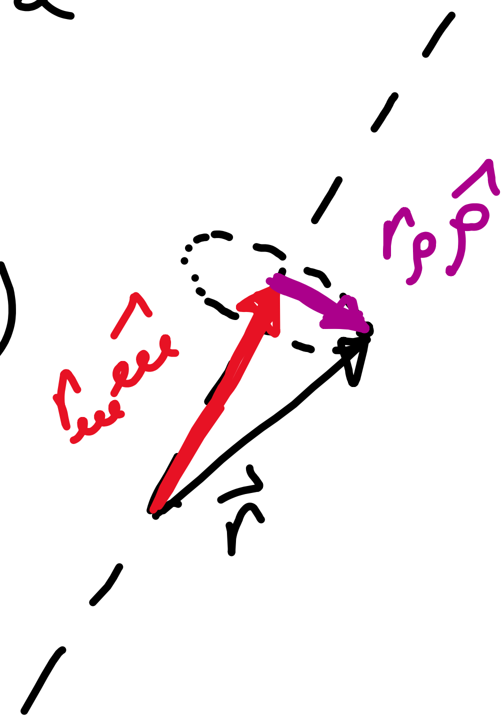
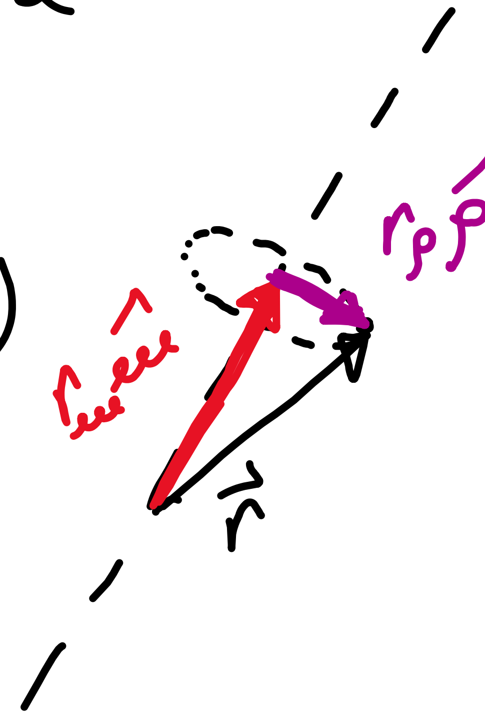


<u>Note</u>: $\vec{\omega} \times \vec{r} = \omega\hat{\omega} \times (r_\omega\hat{\omega} + r_\rho\hat{\rho})$

But $\rho = r_\rho = r\sin\varphi$

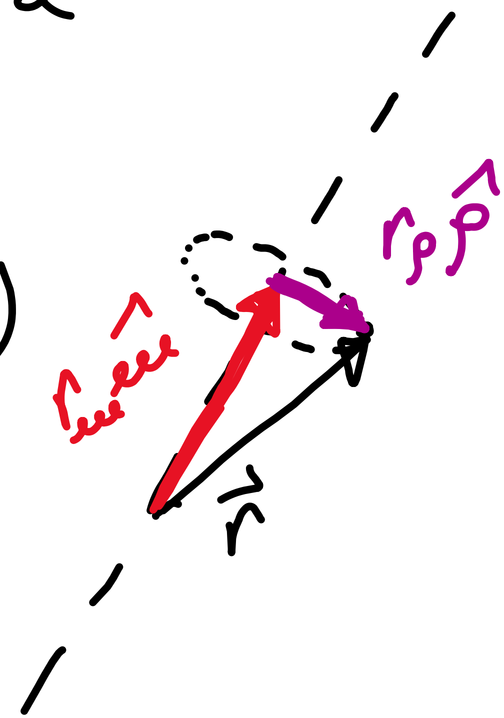
We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:



Note: $\vec{\omega} \times \vec{r} = \omega\hat{\omega} \times (r_\omega \hat{\omega} + r_\rho \hat{\rho})$

But $\hat{\omega} \times \hat{\omega} = 0$

We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:



Note: $\vec{\omega} \times \vec{r} = \omega \hat{\omega} \times (r_\omega \hat{\omega} + r_\rho \hat{\rho})$

But $\hat{\omega} \times \hat{\omega} = 0$ so

$\vec{\omega} \times \vec{r} = \omega r_\rho \hat{\omega} \times \hat{\rho}$

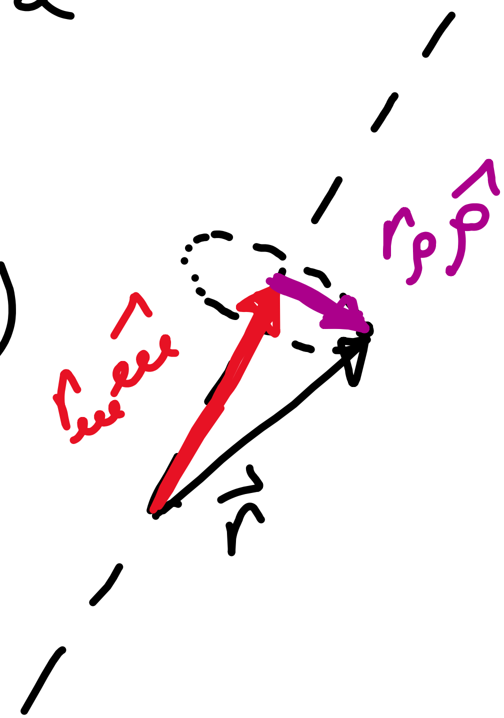
We have $v = (r\sin\varphi)\,\omega$ and only need to obtain the direction:

<u>Note</u>: $\vec{\omega}\times\vec{r} = \omega\,\hat{\omega}\times(r_\omega\hat{\omega} + r_\rho\hat{\rho})$

But $\hat{\omega}\times\hat{\omega} = 0$ so

$\vec{\omega}\times\vec{r} = \omega\, r_\rho\, \hat{\omega}\times\hat{\rho}$ But

$|\hat{\omega}\times\hat{\rho}| = 1$



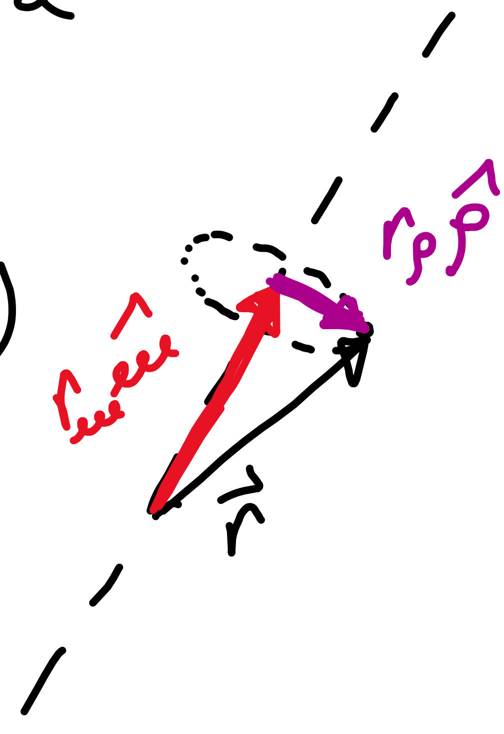
We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:

Note: $\vec{\omega}\times\vec{r} = \omega\,\hat{\omega}\times(r_\omega\hat{\omega} + r_\rho\hat{\rho})$

But $\hat{\omega}\times\hat{\omega} = \theta$ so

$\vec{\omega}\times\vec{r} = \omega\, r_\rho\, \hat{\omega}\times\hat{\rho}$ But

$|\hat{\omega}\times\hat{\rho}| = 1$ [$\hat{\omega}$ orthogonal to $\hat{\rho}$]



We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:

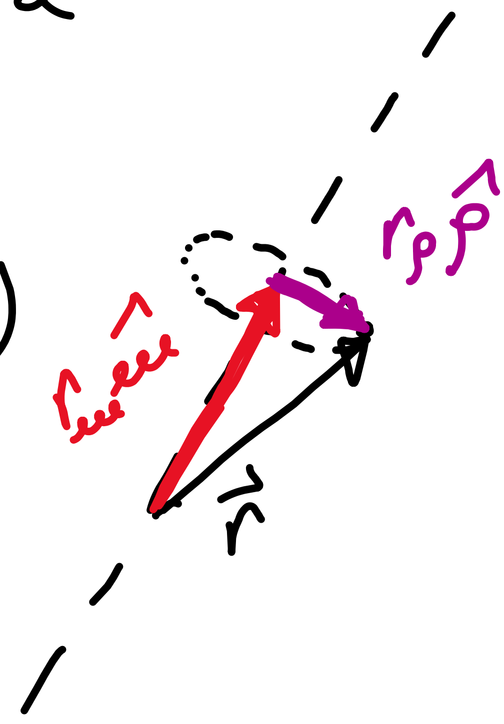
<u>Note</u>: $\vec{\omega} \times \vec{r} = \omega\,\hat{\omega} \times (r_\omega \hat{\omega} + r_\rho \hat{\rho})$

But $\hat{\omega} \times \hat{\omega} = \cancel{0}$ so

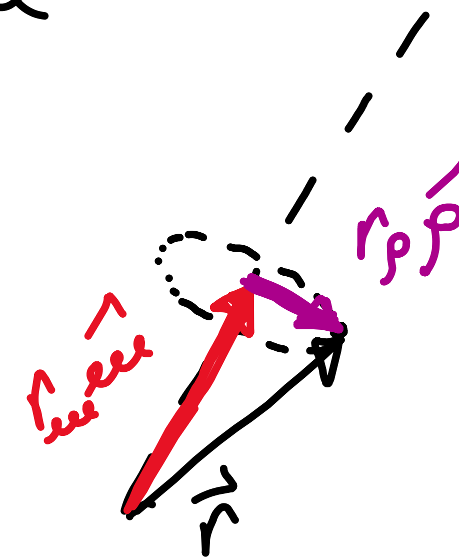
$\vec{\omega} \times \vec{r} = \omega\, r_\rho\, \hat{\omega} \times \hat{\rho}$ But

$|\hat{\omega} \times \hat{\rho}| = 1$ [$\hat{\omega}$ orthogonal to $\hat{\rho}$]

Even better, $\hat{\omega} \times \hat{\rho} = \hat{v}$



We have $v=(r\sin\varphi)\omega$ and only need to obtain the direction:



Note: $\vec{\omega}\times\vec{r}=\omega\hat{\omega}\times(r_\omega\hat{\omega}+r_\rho\hat{\rho})$

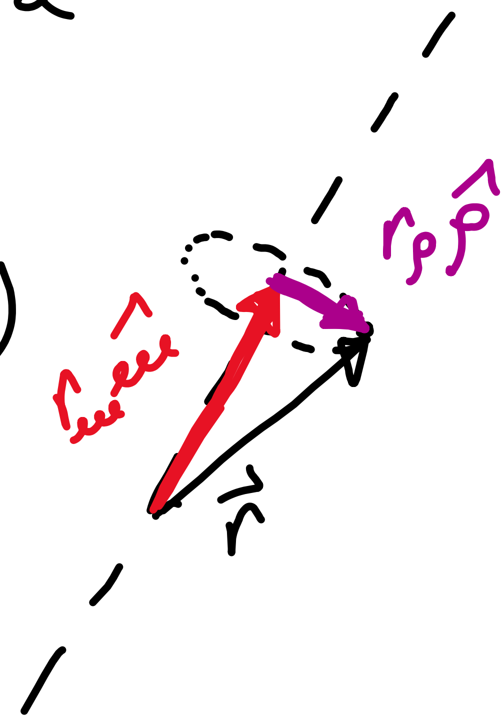
But $\hat{\omega}\times\hat{\omega}=0$ so

$\vec{\omega}\times\vec{r}=\omega r_\rho\hat{\omega}\times\hat{\rho}$ But

$|\hat{\omega}\times\hat{\rho}|=1$ [$\hat{\omega}$ orthogonal to $\hat{\rho}$]

Even better, $\hat{\omega}\times\hat{\rho}=\hat{v}$ [$\vec{v}$ orthogonal to $\hat{\omega}$ & $\hat{\rho}$ & points the correct way]

We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:



Note: $\vec{\omega} \times \vec{r} = \omega\hat{\omega} \times (r_\omega \hat{\omega} + r_\rho \hat{\rho})$

But $\hat{\omega} \times \hat{\omega} = 0$ so

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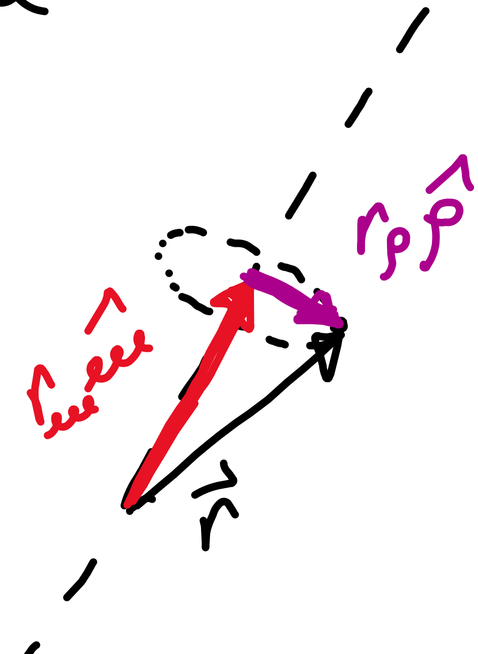
Even better, $\hat{\omega} \times \hat{\rho} = \hat{v}$ [$\vec{v}$ orthogonal to $\hat{\omega}$ & $\hat{\rho}$ & points the correct way] so $\vec{\omega} \times \vec{r} = v\hat{v}$

We have $v = (r\sin\varphi)\omega$ and only need to obtain the direction:



Note: $\vec{\omega} \times \vec{r} = \omega\hat{\omega} \times (r_\omega\hat{\omega} + r_\rho\hat{\rho})$

But $\hat{\omega} \times \hat{\omega} = 0$ so

$\vec{\omega} \times \vec{r} = \omega r_\rho \hat{\omega} \times \hat{\rho}$ But

$|\hat{\omega} \times \hat{\rho}| = 1$ [$\hat{\omega}$ orthogonal to $\hat{\rho}$]

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$\Rightarrow \vec{v} = \vec{\omega} \times \vec{r}$

ASU

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

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What about $\vec{a} = \frac{d\vec{v}}{dt}$?

x

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What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right]$$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right] = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}$$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

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$\dot{\vec{r}} = \vec{v}$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right] = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}$, but

$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

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but $\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right] = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}, \text{ but}$$

$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$ & since $\vec{v} = \vec{\omega} \times \vec{r}$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right] = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}},$$ but

$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$ & since $\vec{v} = \vec{\omega} \times \vec{r}$

then $$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

We have seen that $\vec{v} = \vec{\omega} \times \vec{r}$

What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need to work out the derivitive:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\vec{\omega} \times \vec{r}\right] = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}},$$

but $\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

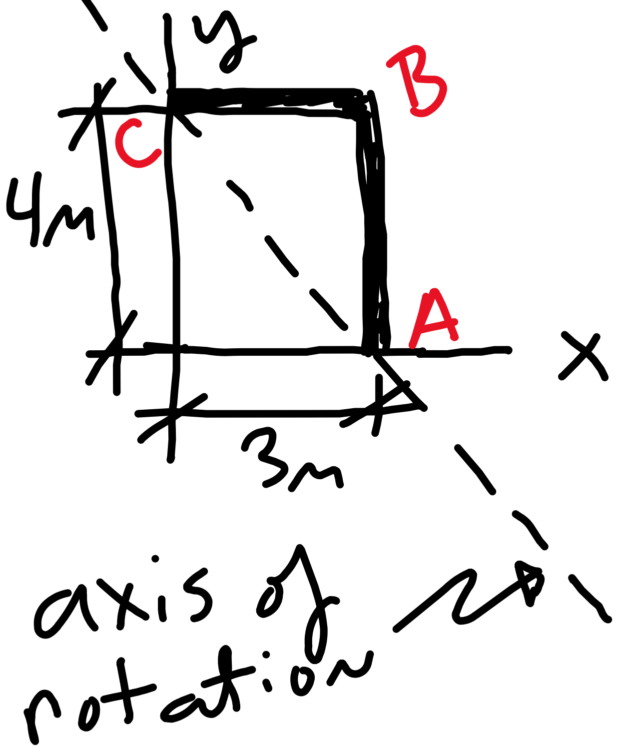
& since $\vec{v} = \vec{\omega} \times \vec{r}$

then $\boxed{\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})}$

ASU

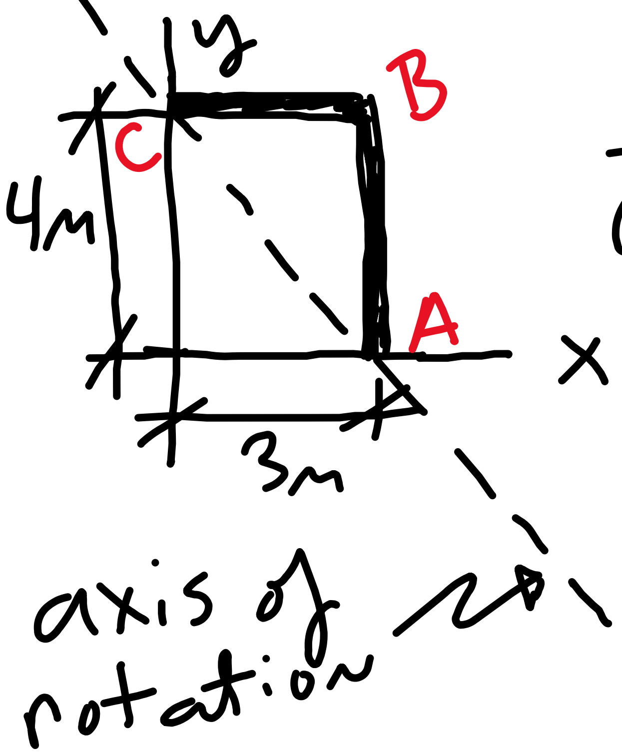
Example similar to problem 15.10

Example similar to problem 15.10

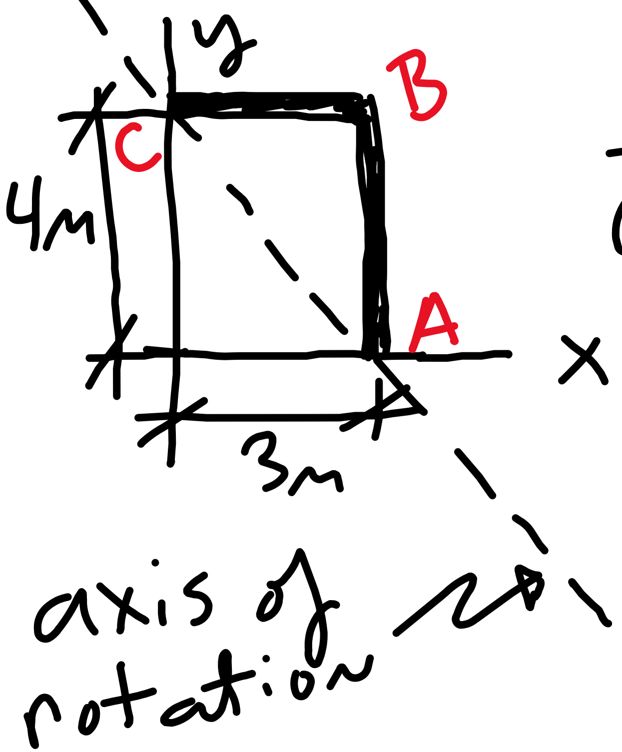


Example similar to problem 15.10

Rotation clockwise as viewed from point A.



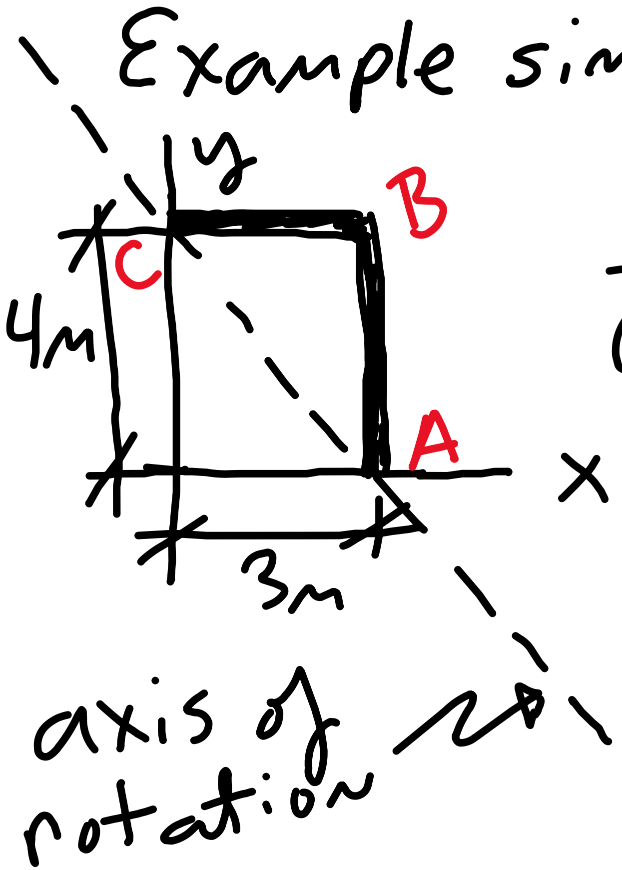
Example similar to problem 15.10



Rotation clockwise as viewed from point A. **Find $\vec{v}_B$:**

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

Example similar to problem 15.10

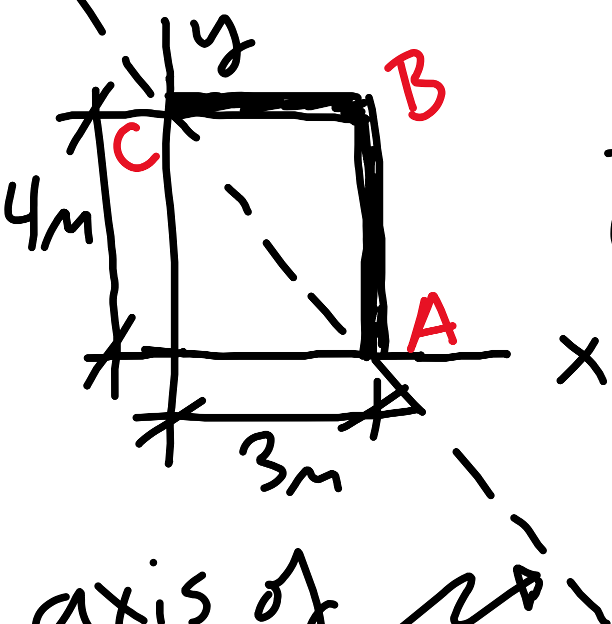


Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

Need to determine $\vec{r}$ & $\vec{\omega}$.

Example similar to problem 15.10



axis of rotation

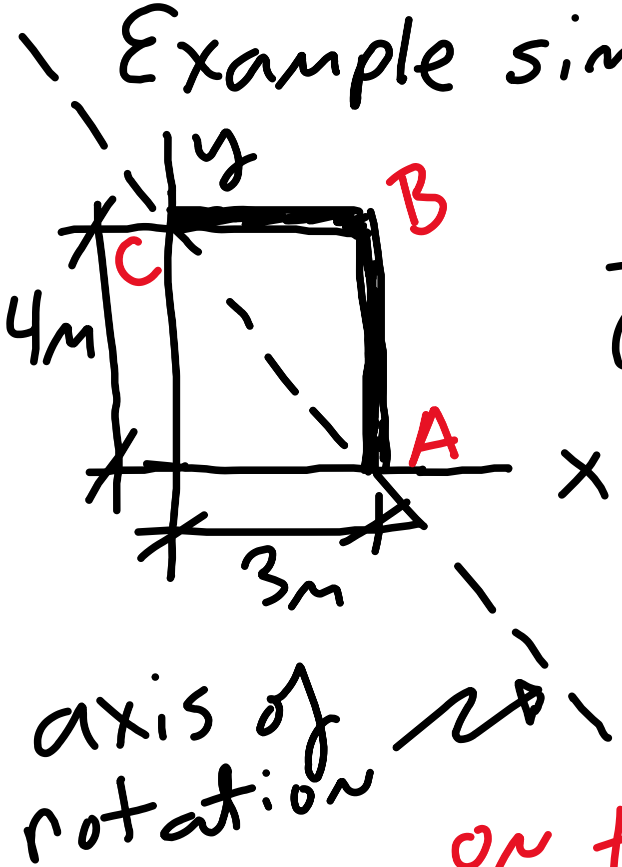
Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

Need to determine $\vec{r}$ & $\vec{\omega}$.

$\vec{r}$ must start somewhere on the axis of rotation

Example similar to problem 15.10



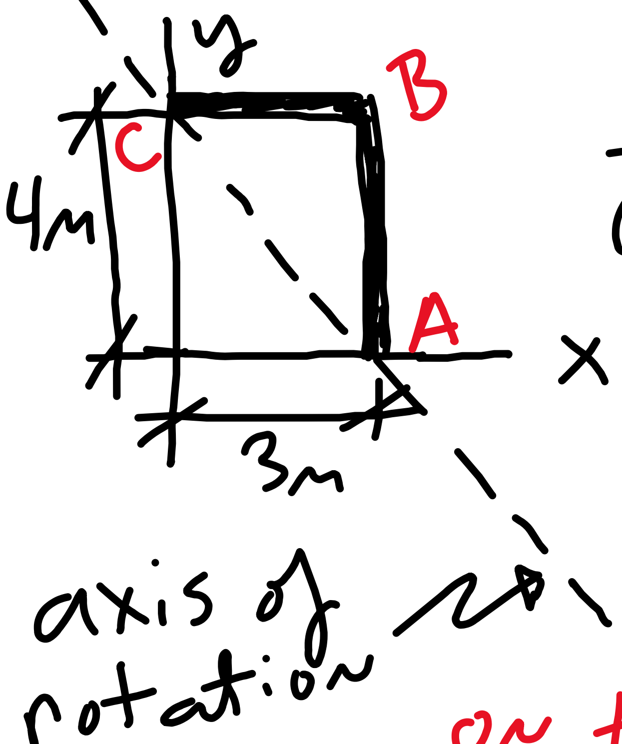
Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

Need to determine $\vec{r}$ & $\vec{\omega}$.

$\vec{r}$ must start somewhere on the axis of rotation & end at point B.

Example similar to problem 15.10



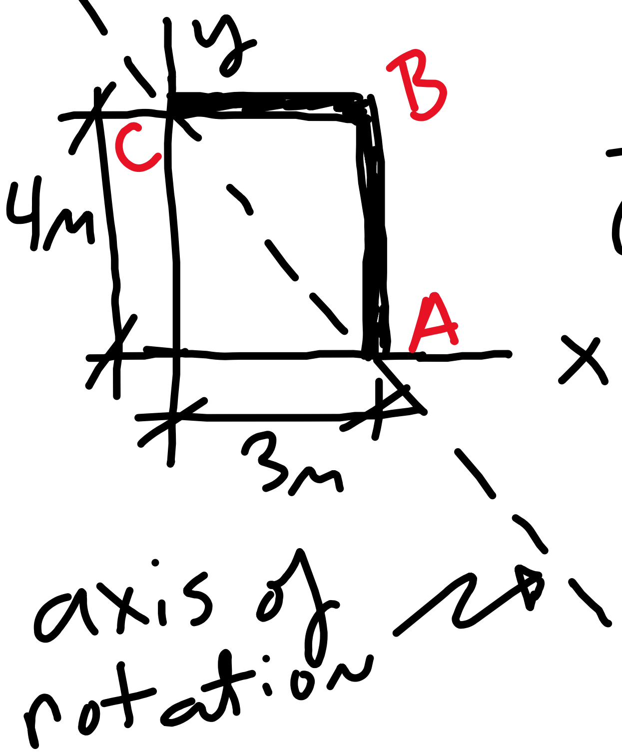
Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

Need to determine $\vec{r}$ & $\vec{\omega}$.

$\vec{r}$ must start somewhere on the axis of rotation & end at point B. $\vec{\omega}$ must point along direction from point A to C.

Example similar to problem 15.10

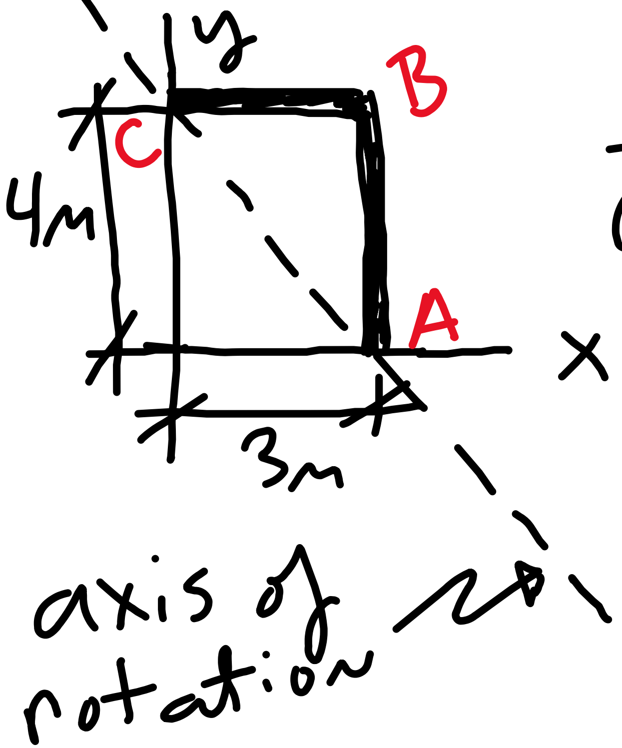


Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$:</u>

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction:

Example similar to problem 15.10

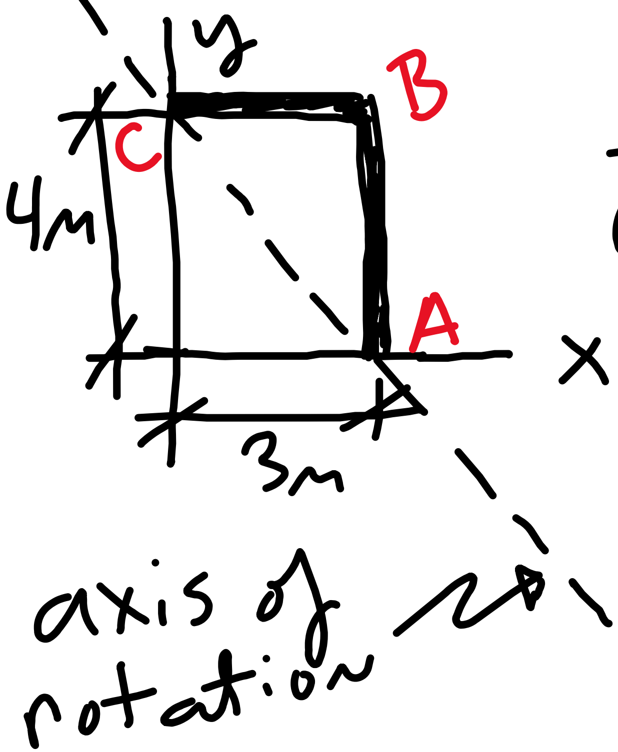


Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \dfrac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$

Example similar to problem 15.10



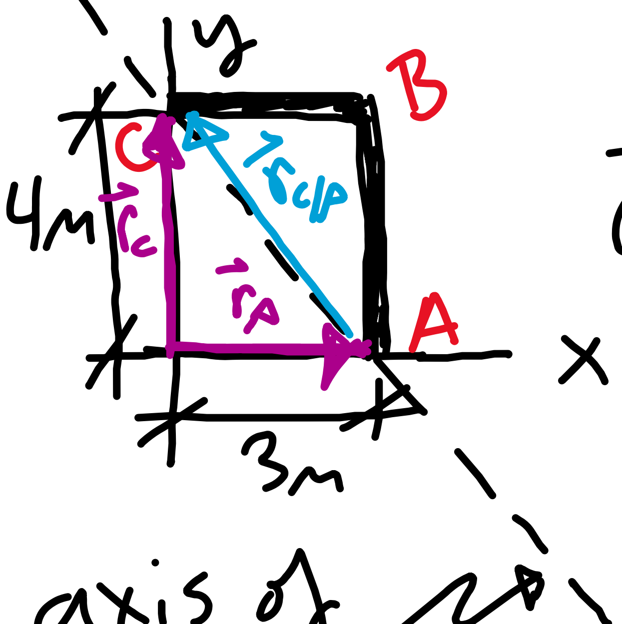
Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$</u>:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

ω-Direction: $\hat{\omega} = \dfrac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$</u>:

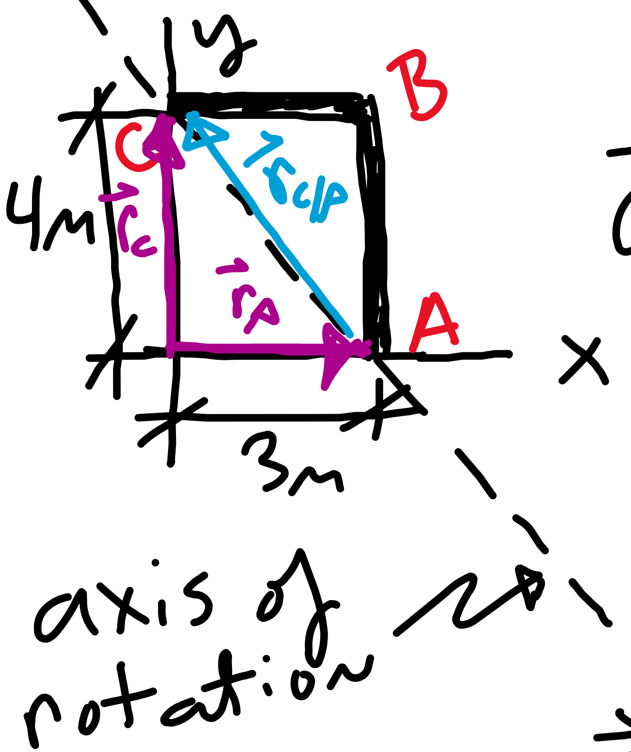
$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$</u>:

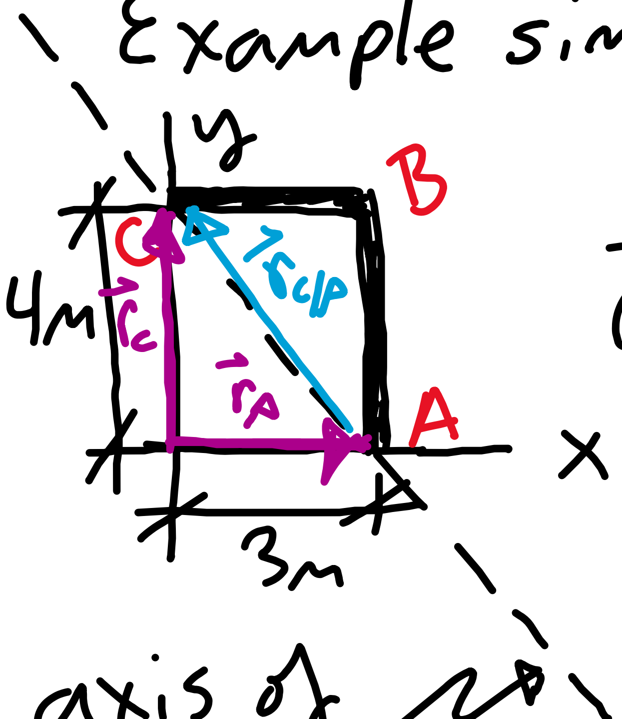
$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$

Example similar to problem 15.10



axis of rotation

Rotation clockwise as viewed from point A. **Find $\vec{v}_B$:**

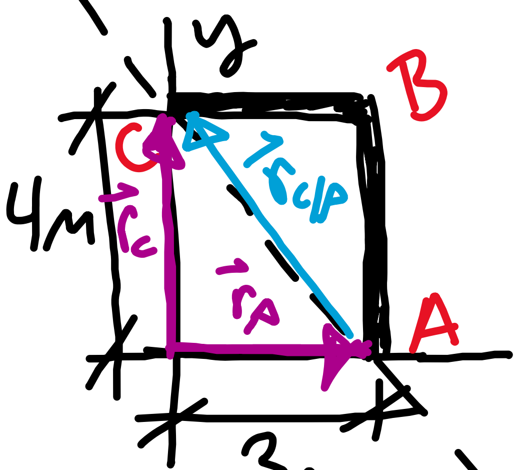
$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$</u>:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

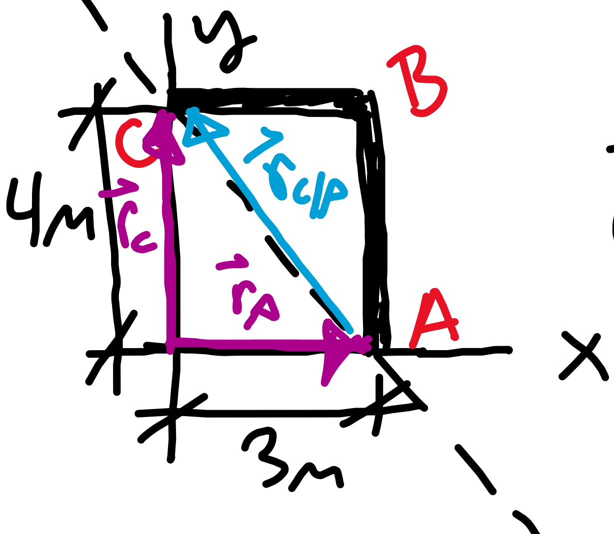
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

axis of rotation

$\vec{r}$ vector:

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{V}_B$:</u>

$\vec{V}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \dfrac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

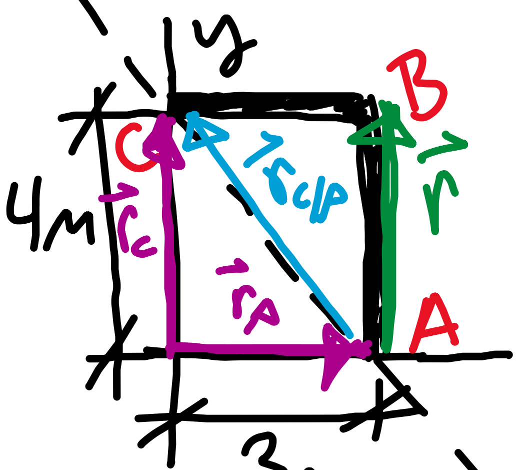
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

ASU

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$</u>:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

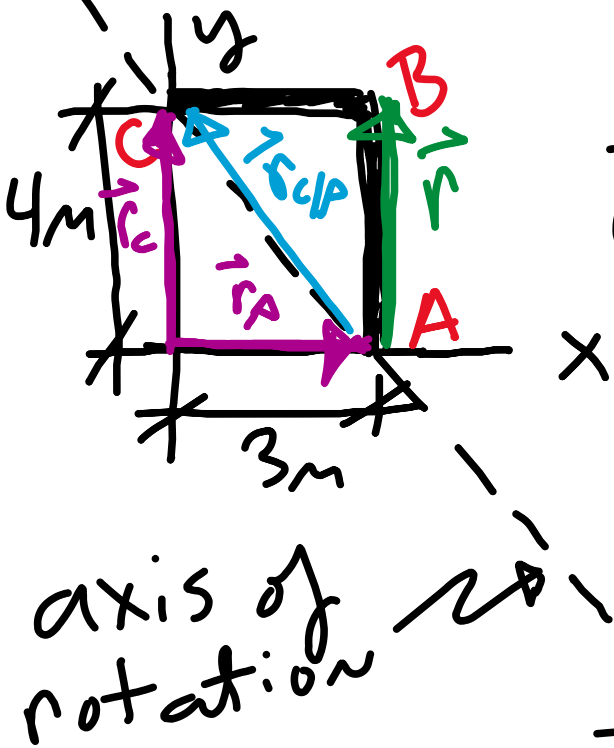
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

$\vec{r} = \vec{r}_{B/A}$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find $\vec{v}_B$:

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

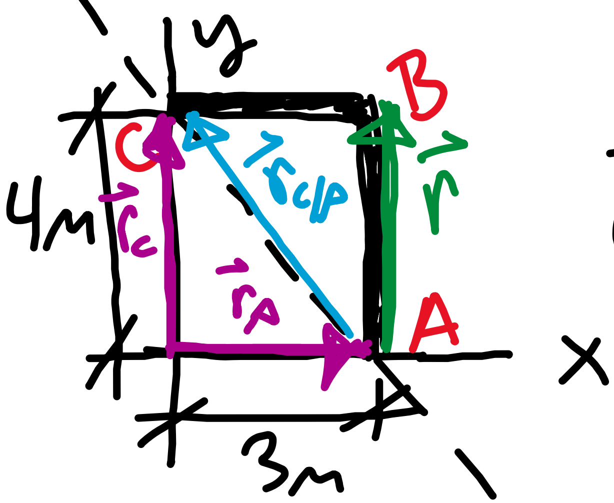
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

Example similar to problem 15.10



axis of rotation

Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$:</u>

$\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

ω-direction: $\hat{\omega} = \dfrac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

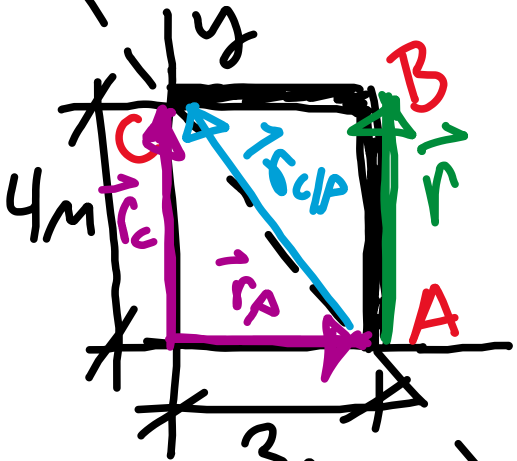
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\,\hat{x} + 4m\,\hat{y}) - 3m\,\hat{x}$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. <u>Find $\vec{v}_B$:</u>

× $\vec{v}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1 \frac{rad}{s}$

ω-direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

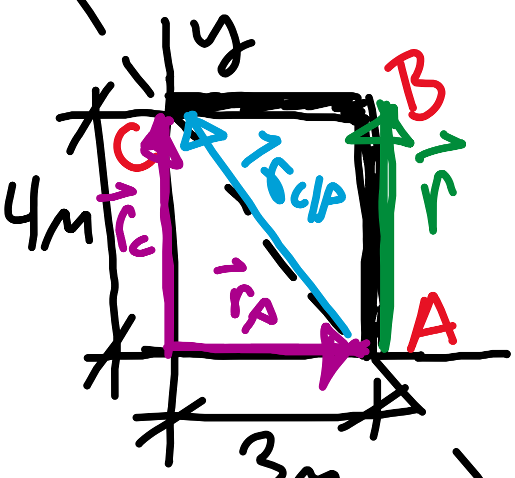
$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\,\hat{x} + 4m\,\hat{y}) - 3m\,\hat{x} = 4m\,\hat{y}$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find $\vec{V}_B$:

$\vec{V}_B = \vec{\omega} \times \vec{r}$, with $\omega = 1\,\frac{rad}{s}$

ω-Direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\,\hat{y} - 3m\,\hat{x}$

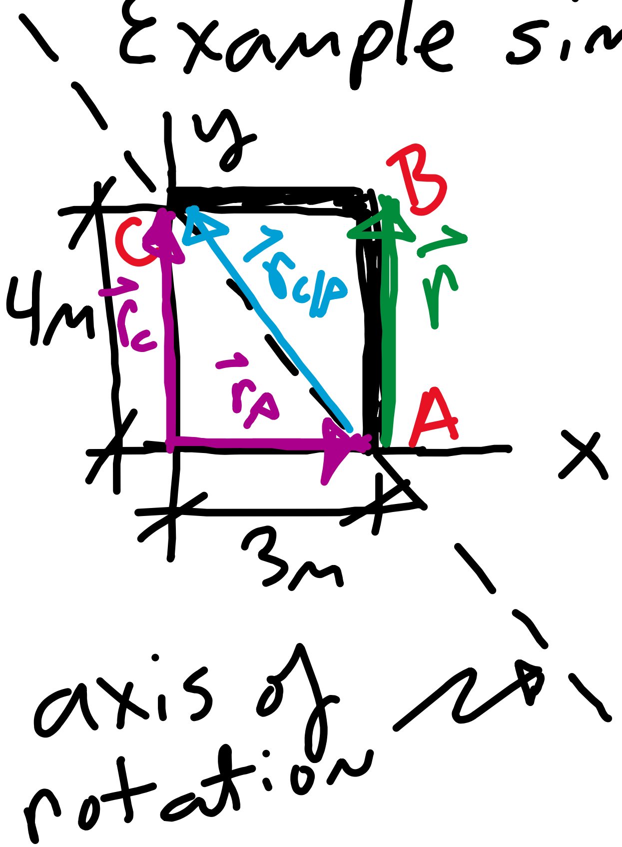
$\Rightarrow |\vec{r}_{C/A}| = 5m$ Now $\boxed{\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}}$

$\vec{r}$ vector: might as well start at point A

$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\,\hat{x} + 4m\,\hat{y}) - 3m\,\hat{x} = 4m\,\hat{y}$

Could have just read off of diagram

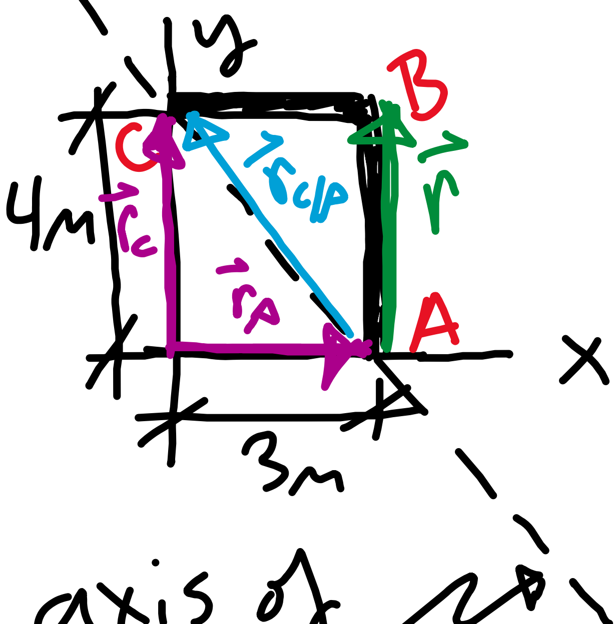
Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

Example similar to problem 15.10

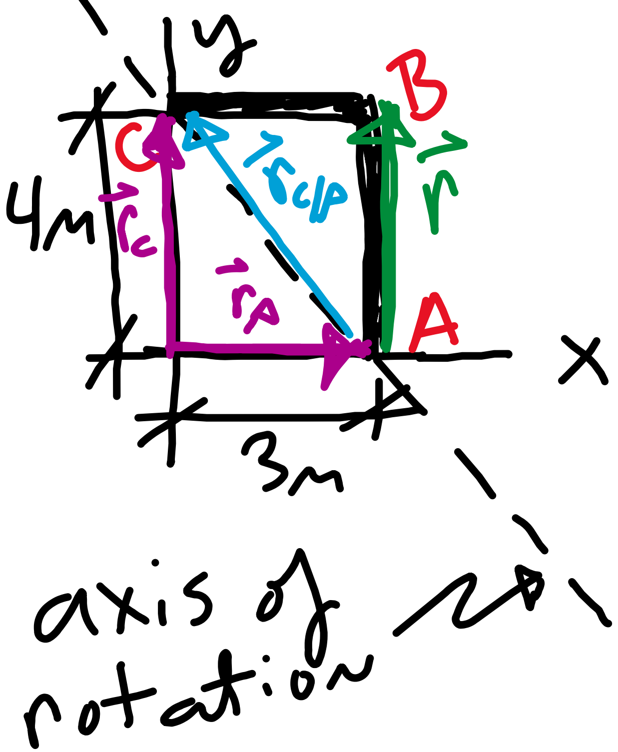


axis of rotation

Now we have

$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\,\hat{y}$

Example similar to problem 15.10



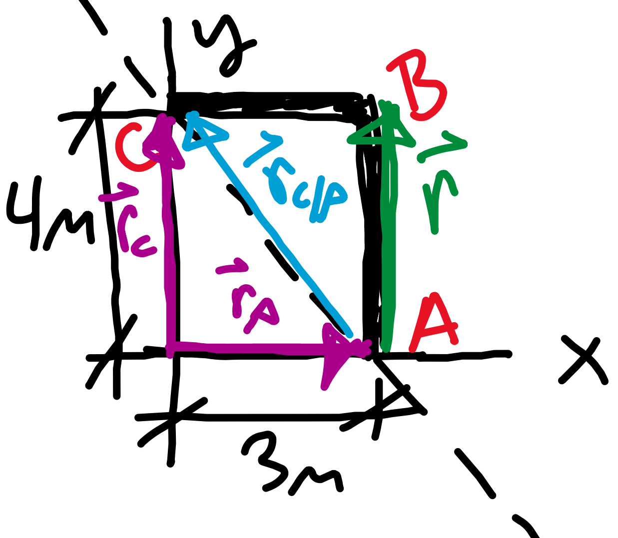
Now we have

$$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\,\hat{y}$$

So

$$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\,\hat{y}$$

Example similar to problem 15.10



axis of rotation

Now we have

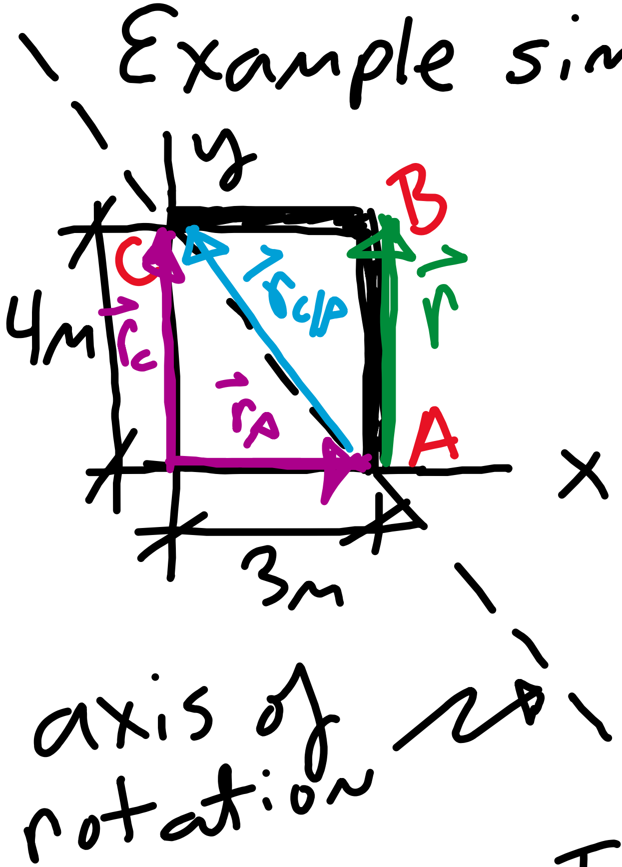
$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

So

$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}}$$

Example similar to problem 15.10



Now we have

$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

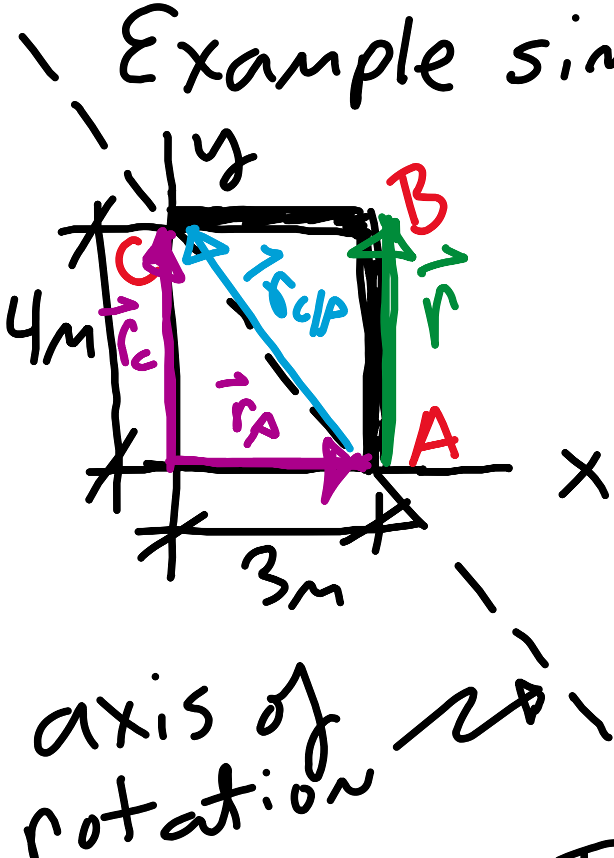
So

$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}}$$

To find $\vec{a}_B$

Example similar to problem 15.10



Now we have

$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

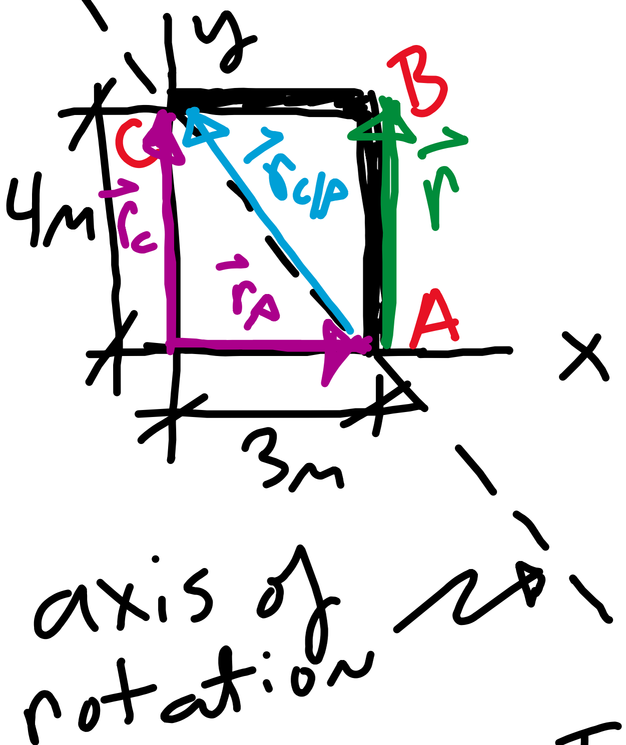
So

$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Example similar to problem 15.10



Now we have

$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

So

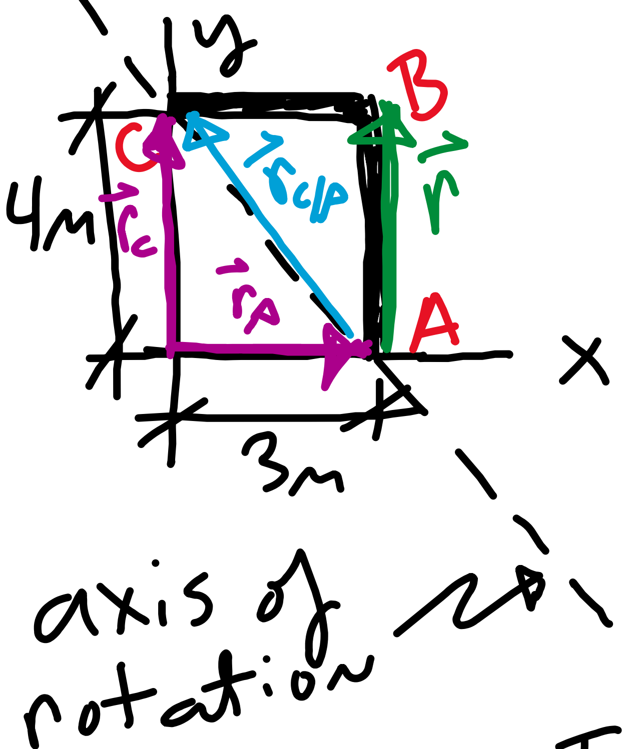
$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\,\hat{z}}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$

Example similar to problem 15.10



Now we have

$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

So

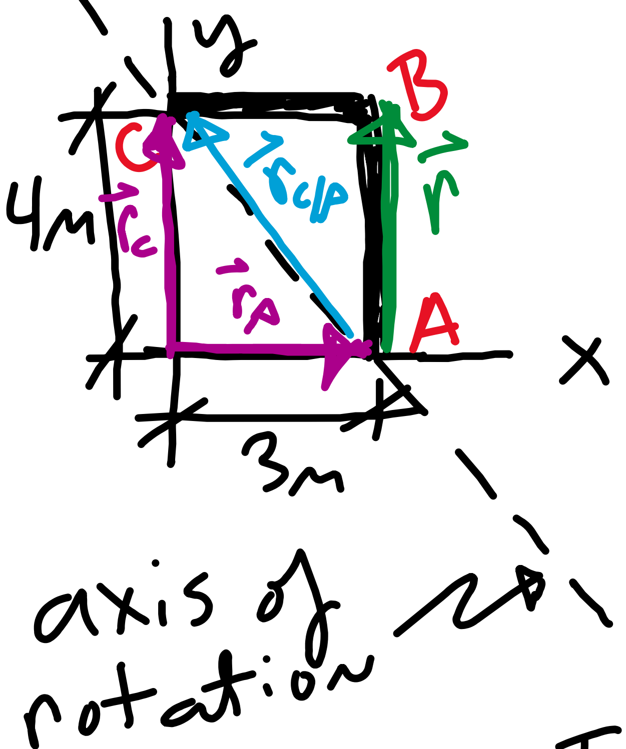
$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\,\hat{z}}$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$ & $\vec{\omega} \times \vec{r} = \vec{V}_B$

Example similar to problem 15.10



Now we have

$\hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$ & $\vec{r} = 4m\hat{y}$

So

$\vec{V}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}\right)\omega\right] \times 4m\hat{y}$

$$\boxed{\vec{V}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\,\hat{z}}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$ & $\vec{\omega} \times \vec{r} = \vec{V}_B$

$\Rightarrow \vec{a}_B = \vec{\omega} \times \vec{V}_B$

ASU

Angular Kinematics

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just replace $x \to \theta$

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Note: 1 rev $= 360°$

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so $\frac{\text{rev}}{2\pi} = 1$ & $\frac{\text{rev}}{360°} = 1$

& $\frac{2\pi\ \text{rad}}{360°} = 1$