Today 15.2

$\boxed{L20}$

Today 15.2

$\boxed{L20}$

General plane motion, velocity

Today 15.2

Wednesday 15.3

$\boxed{L20}$

Today 15.2

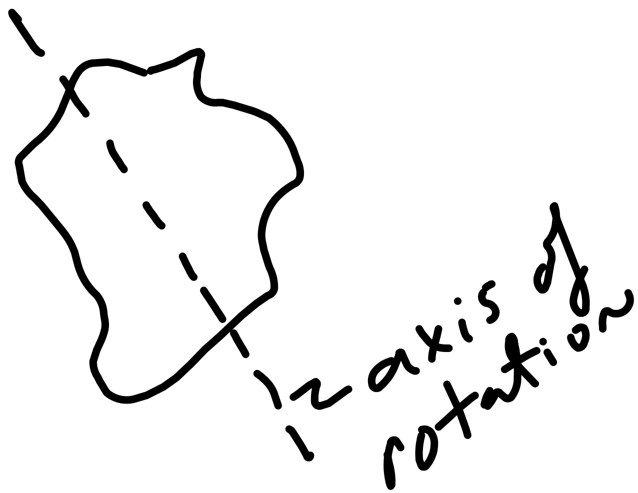
Wednesday 15.3

L20

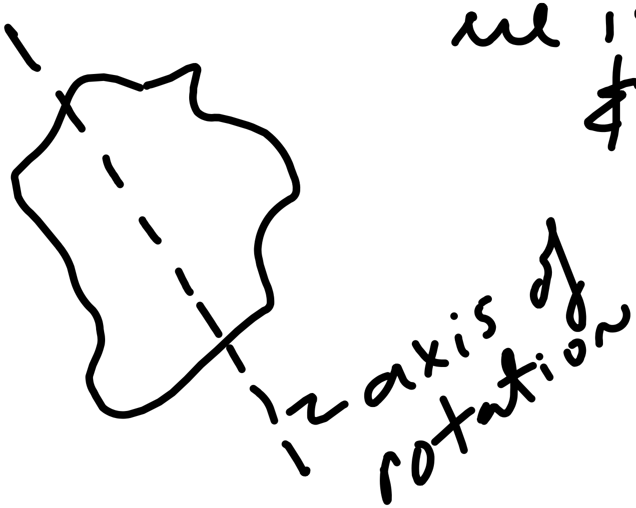
Instantaneous center of rotation

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$

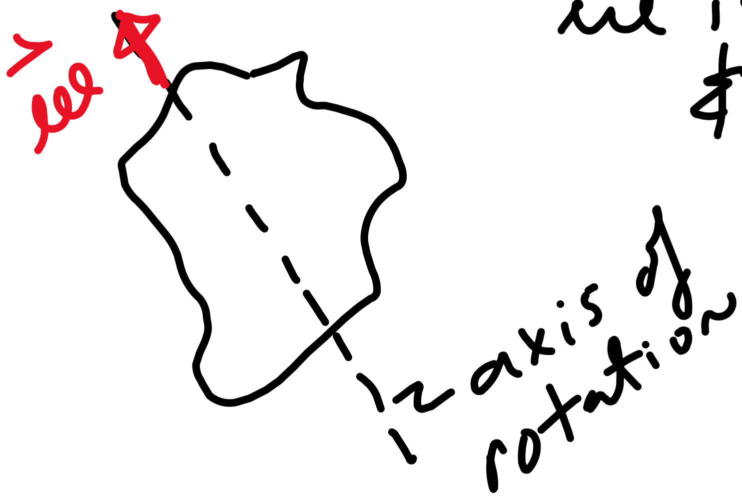
Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$



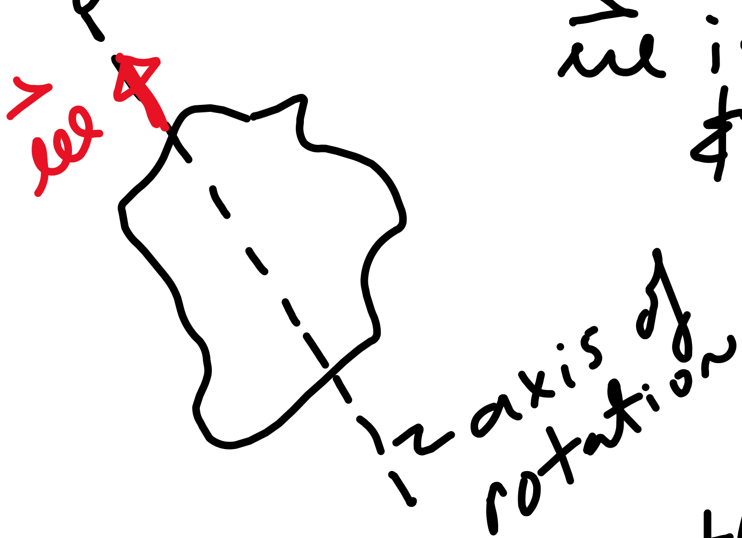
Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule



Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.



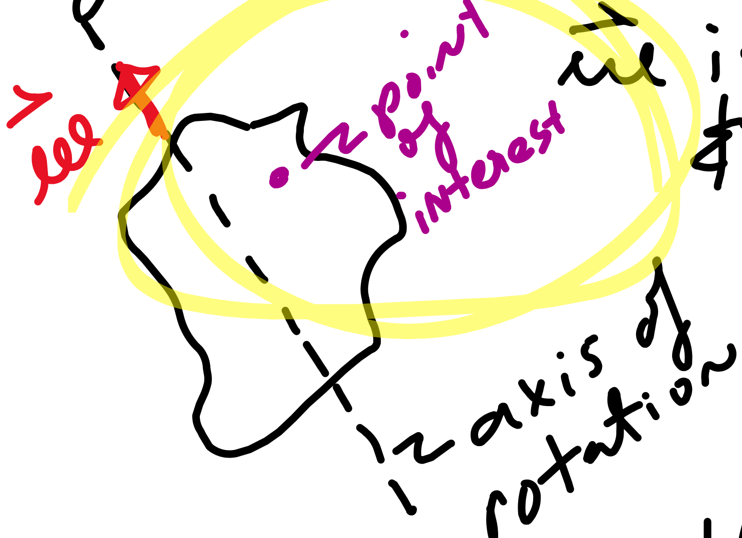
Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where



$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest.

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.



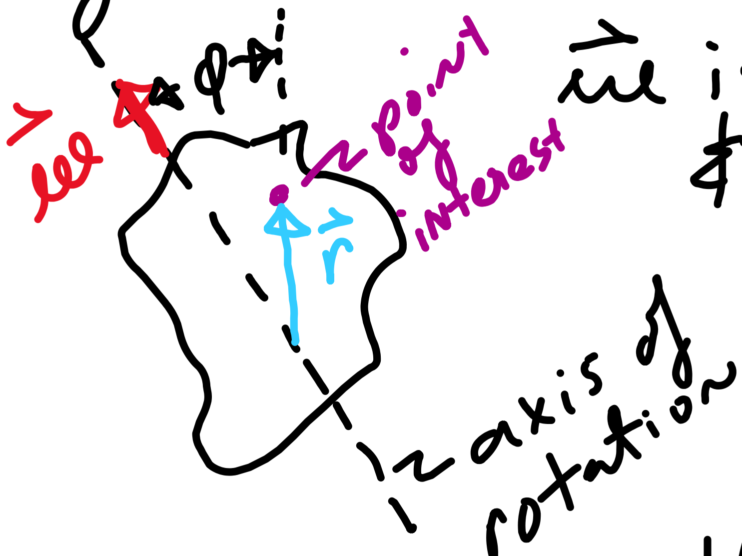
The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest.

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where

$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where



$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

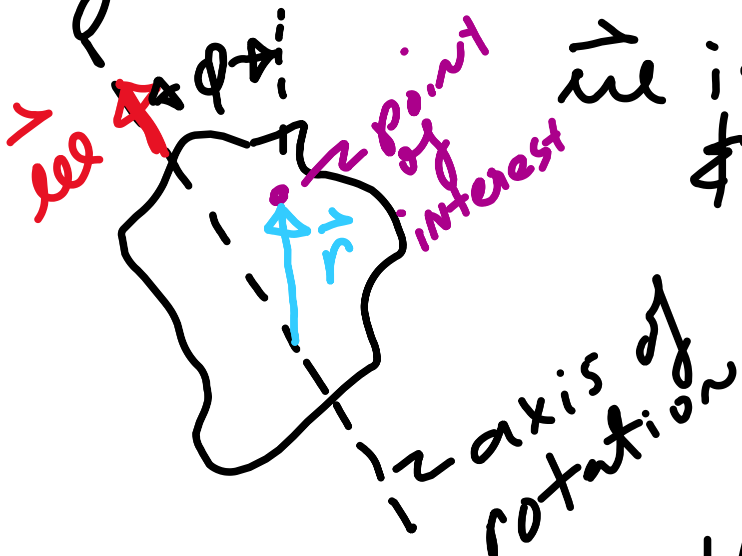
The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest. The angle between $\vec{\omega}$ & $\vec{r}$ is ϕ

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where

$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest. The angle between $\vec{\omega}$ & $\vec{r}$ is φ

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where



$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest. The angle between $\vec{\omega}$ & $\vec{r}$ is ϕ. <u>Special case</u>: Let us start $\vec{r}$ on axis of rotation such that $\phi = 90^\circ$

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where

$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest. The angle between $\vec{\omega}$ & $\vec{r}$ is φ. Special

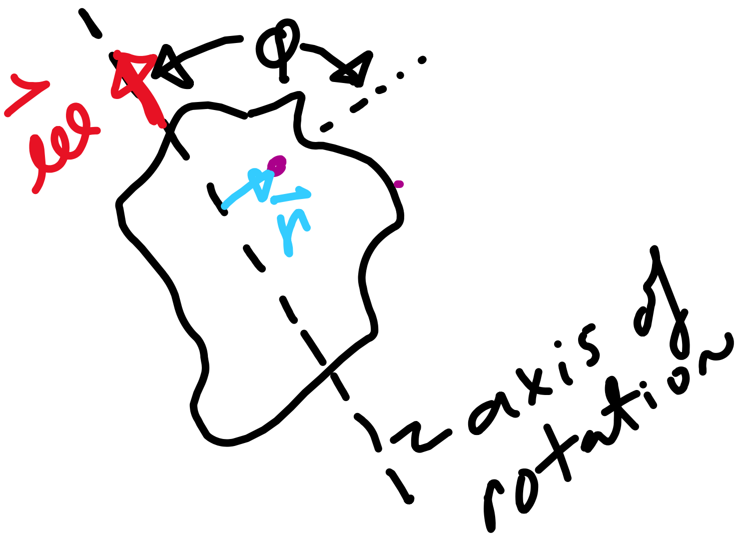
<u>case</u>: Let us start $\vec{r}$ on axis of rotation such that $\varphi = 90°$

Previously we looked at rotations about a fixed axis using $\vec{v} = \vec{\omega} \times \vec{r}$, where

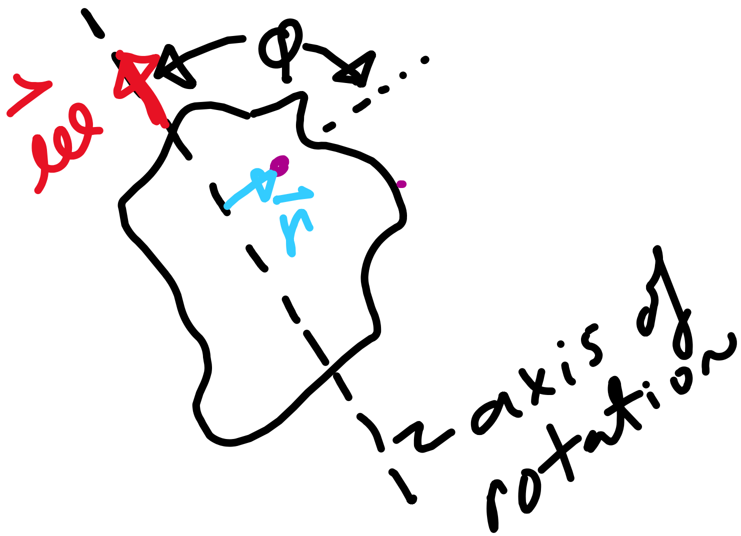
$\vec{\omega}$ is along axis of rotation & points in direction determined by the right-hand rule.

The vector $\vec{r}$ must start somewhere along the axis of rotation & end at the point of interest. The angle between $\vec{\omega}$ & $\vec{r}$ is ϕ. <u>Special case</u>: Let us start $\vec{r}$ on axis of rotation such that $\phi = 90°$

Special case $\phi = 90°$



Special case $\phi = 90°$

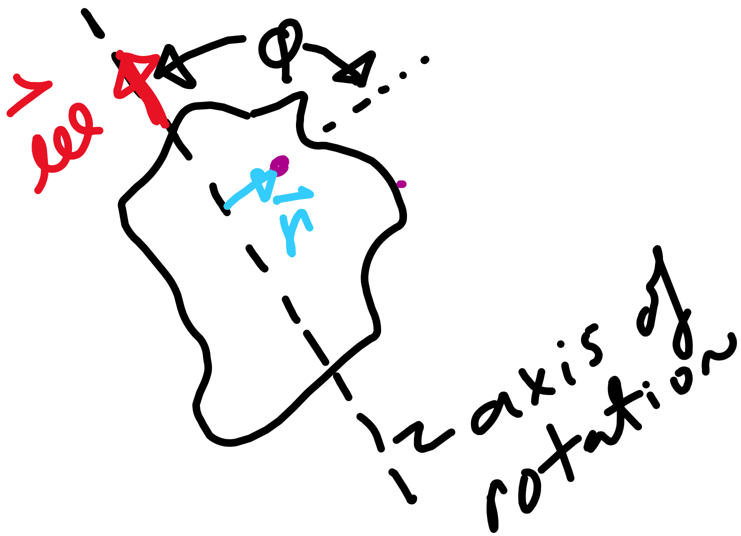


"Top" view

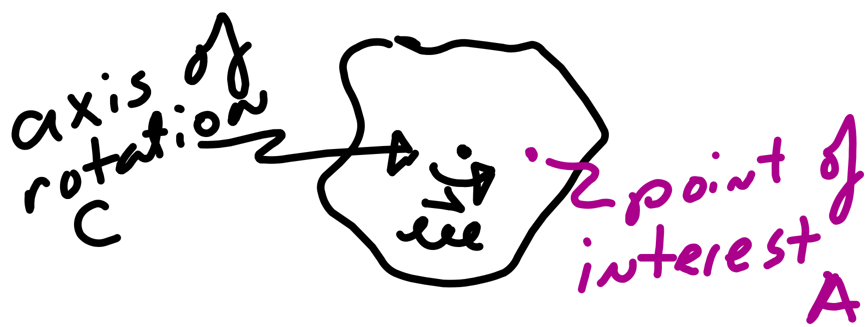
Special case $\varphi = 90°$

"Top" view

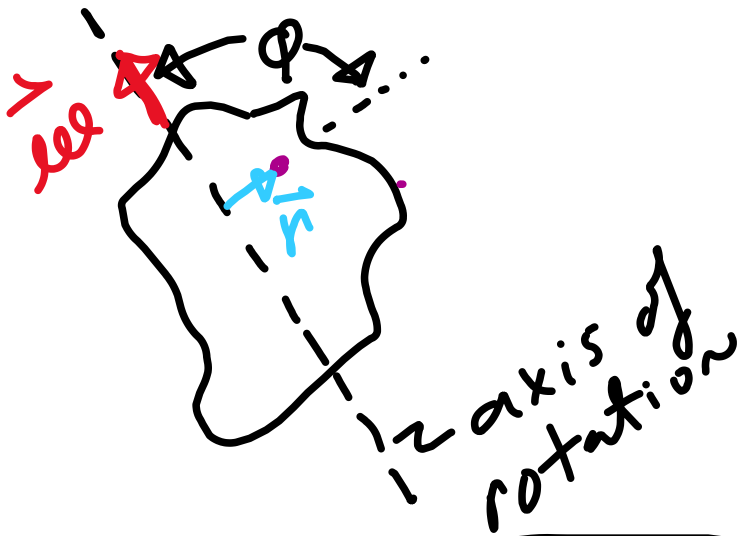
Special case $\phi = 90°$



"Top" view



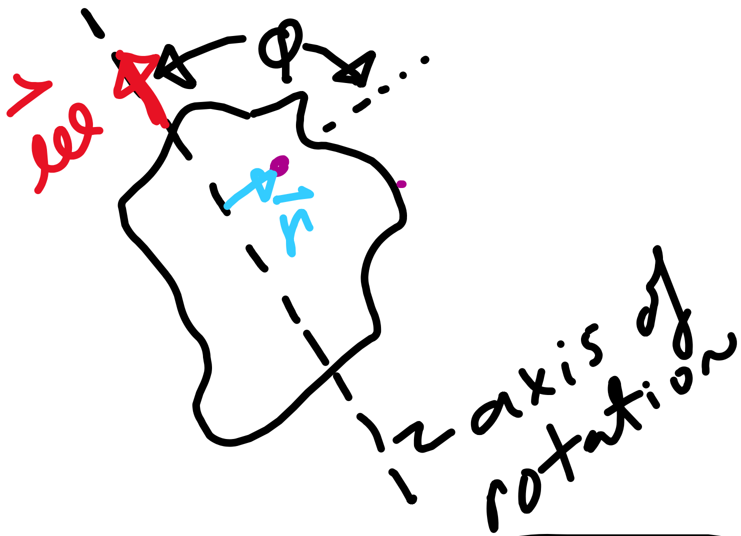
Special case $\phi = 90°$



"Top" view

I have labeled the point of interest as "A" & the axial point as "C"

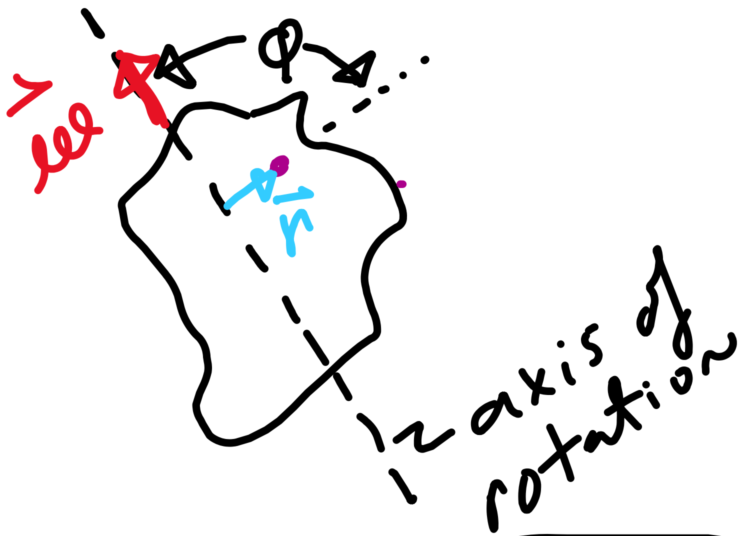
Special case $\phi = 90°$



"Top" view

I have labeled the point of interest as "A" & the axial point as "C" so that $\vec{r} = \vec{r}_{A/C}$.

Special case $\phi = 90°$



"Top" view

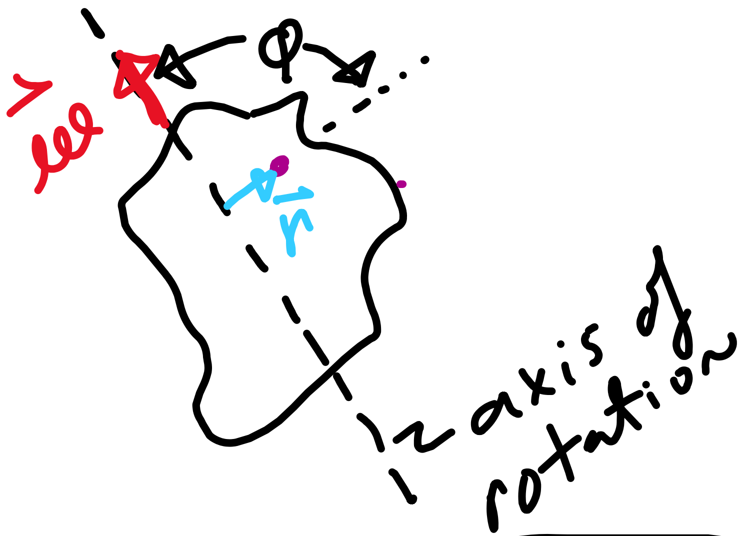
I have labeled the point of interest as "A" & the axial point as "C" so that $\vec{r} = \vec{r}_{A/C}$. For a fixed axis of rotation,

Special case $\phi = 90°$

"Top" view

I have labeled the point of interest as "A" & the axial point as "C" so that $\vec{r} = \vec{r}_{A/C}$. For a fixed axis of rotation, we can fully specify the orientation of the object by knowing how much it has rotated from some initial state

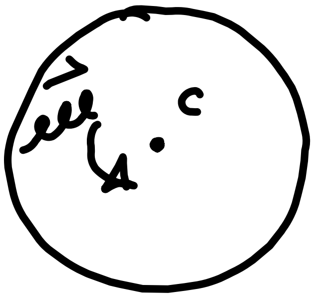
Special case $\phi = 90°$



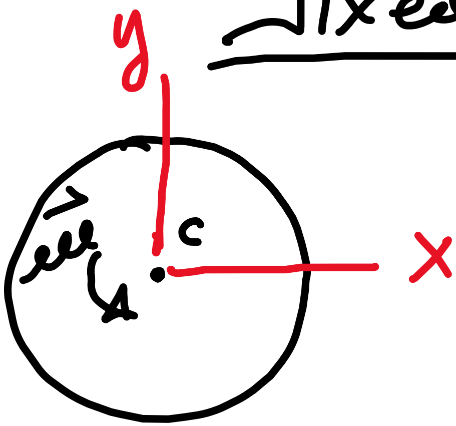
"Top" view

I have labeled the point of interest as "A" & the axial point as "C" so that $\vec{r} = \vec{r}_{A/C}$. For a fixed axis of rotation, we can fully specify the orientation of the object by knowing how much it has rotated from some initial state ☺

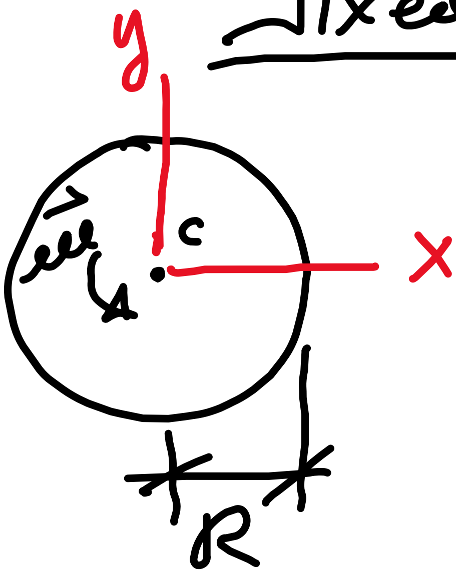
Fixed axis rotation of a wheel



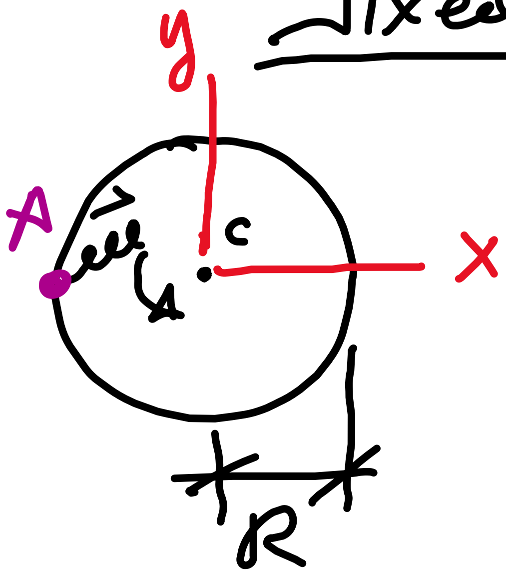
Fixed axis rotation of a wheel



Fixed axis rotation of a wheel

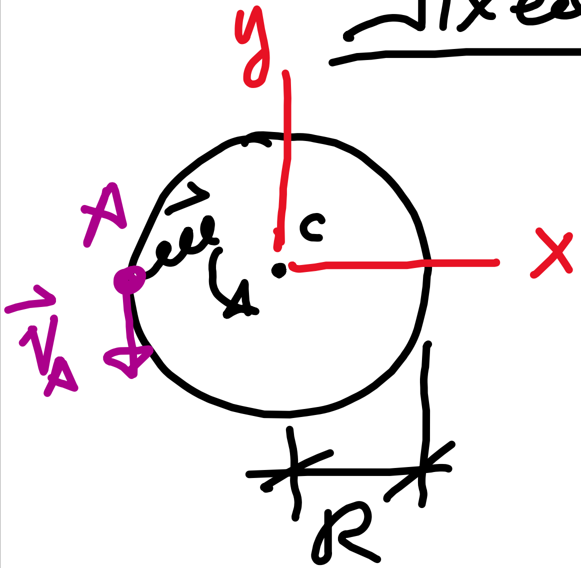


Fixed axis rotation of a wheel

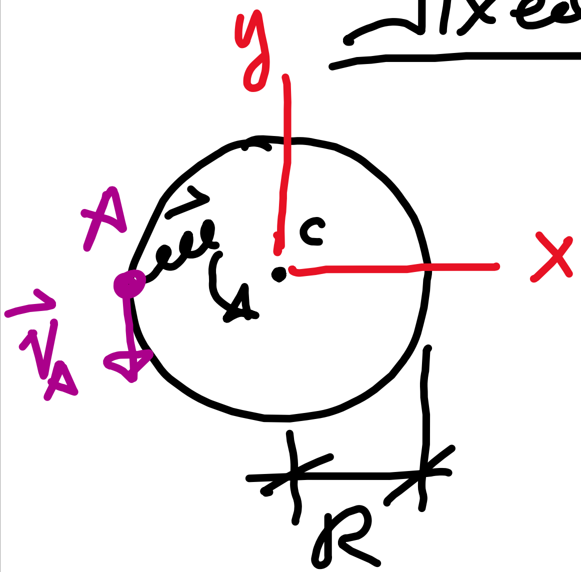


Fixed axis rotation of a wheel

Here $\vec{\omega} = \omega \hat{z}$



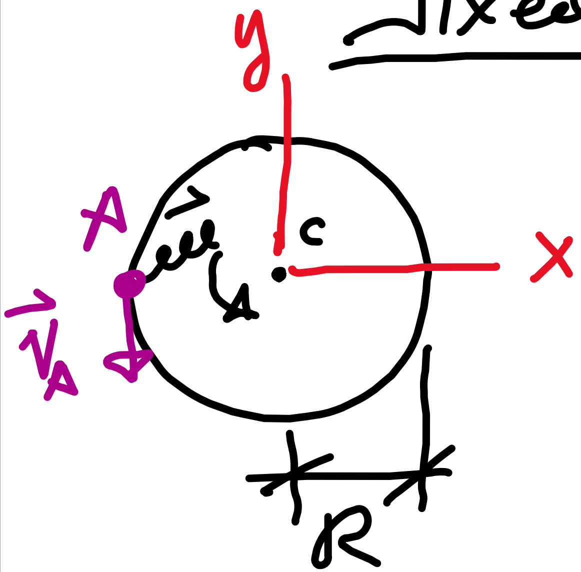
Fixed axis rotation of a wheel



Here $\vec{\omega} = \omega \hat{z}$

& $\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c}$

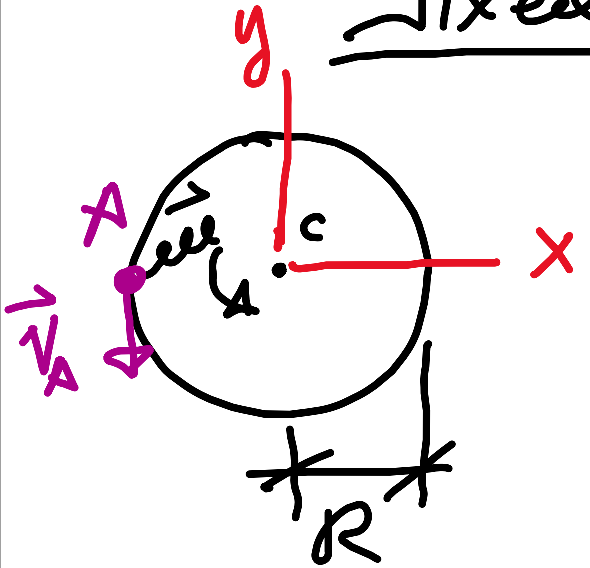
Fixed axis rotation of a wheel

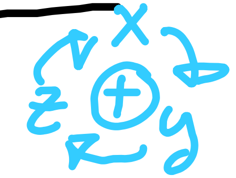


Here $\vec{\omega} = \omega\,\hat{z}$

& $\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\,\hat{z} \times (-\hat{x})$

Fixed axis rotation of a wheel

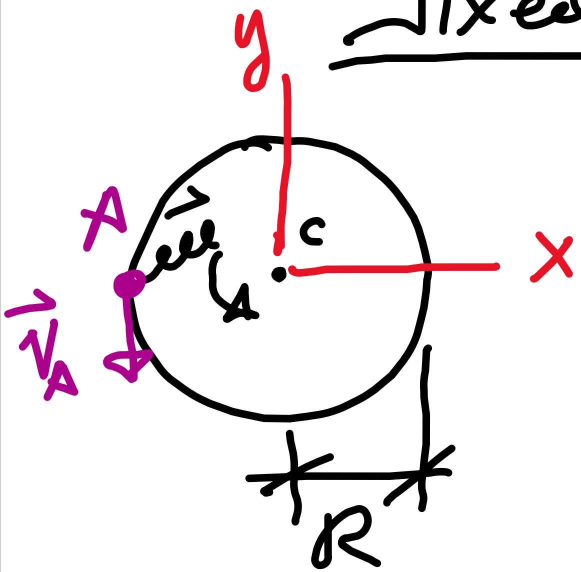




Here $\vec{\omega} = \omega \hat{z}$

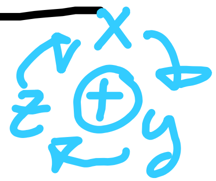
& $\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R \, \hat{z} \times (-\hat{x})$

Fixed axis rotation of a wheel

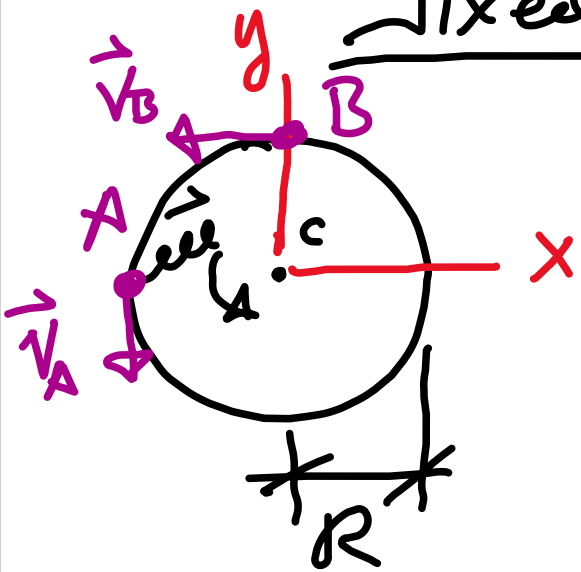


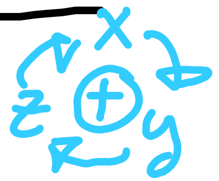
Here $\vec{\omega} = \omega \hat{z}$

$\therefore \vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R \, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$



Fixed axis rotation of a wheel



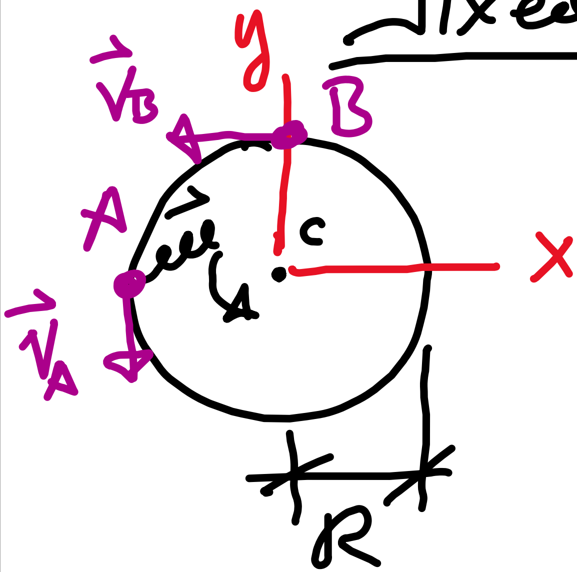


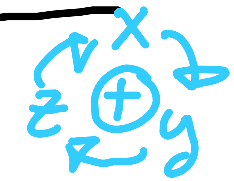
Here $\vec{\omega} = \omega \hat{z}$

$\therefore \vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

$\therefore \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$

Fixed axis rotation of a wheel



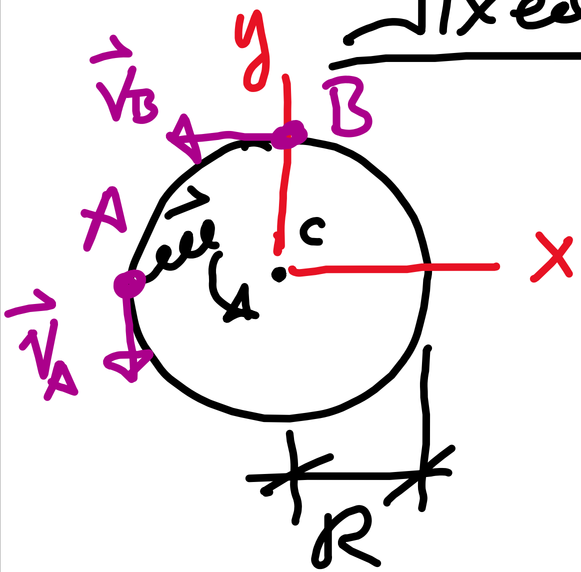


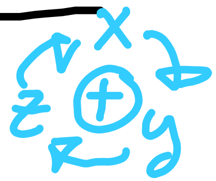
Here $\vec{\omega} = \omega \hat{z}$

& $\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R \, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

& $\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R \, \hat{z} \times \hat{y}$

Fixed axis rotation of a wheel



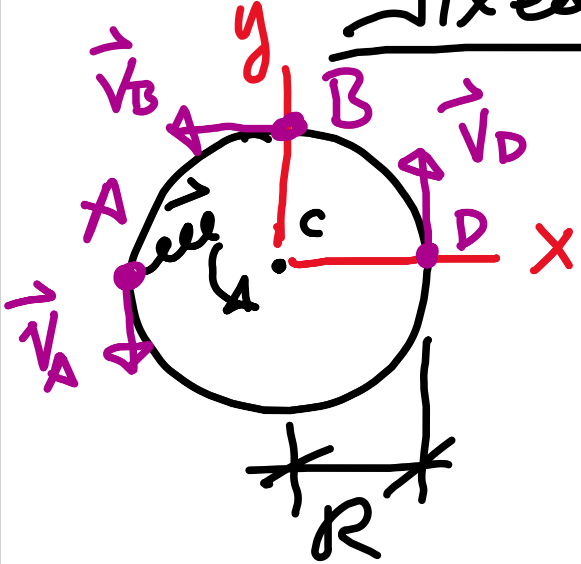


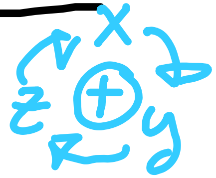
Here $\vec{\omega} = \omega \hat{z}$

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

Fixed axis rotation of a wheel





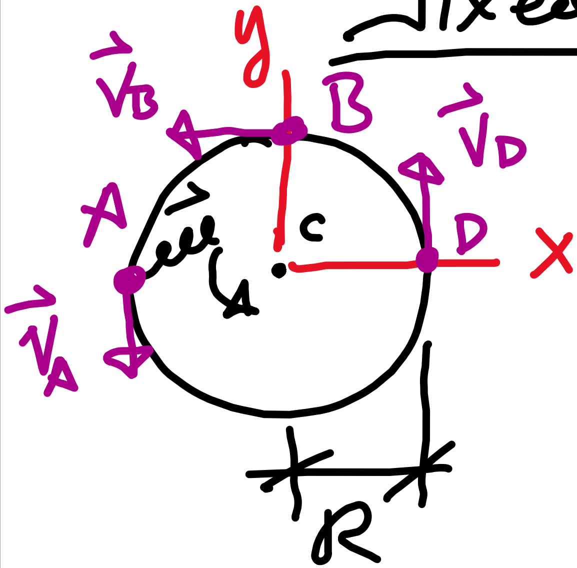
Here $\vec{\omega} = \omega \hat{z}$

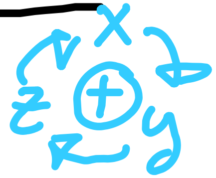
$\$\ \vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

$\$\ \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$

$\$\ \vec{V}_D = \vec{\omega} \times \vec{r}_{D/C}$

Fixed axis rotation of a wheel





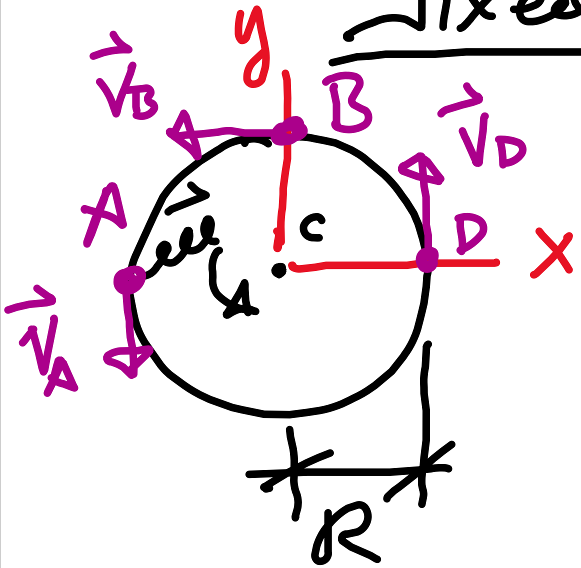
Here $\vec{\omega} = \omega\hat{z}$

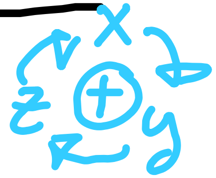
$\$ \; \vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R\,\hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

$\$ \; \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R\,\hat{z} \times \hat{y} = \omega R(-\hat{x})$

$\$ \; \vec{V}_D = \vec{\omega} \times \vec{r}_{D/C} = \omega R\,\hat{z} \times \hat{x}$

Fixed axis rotation of a wheel





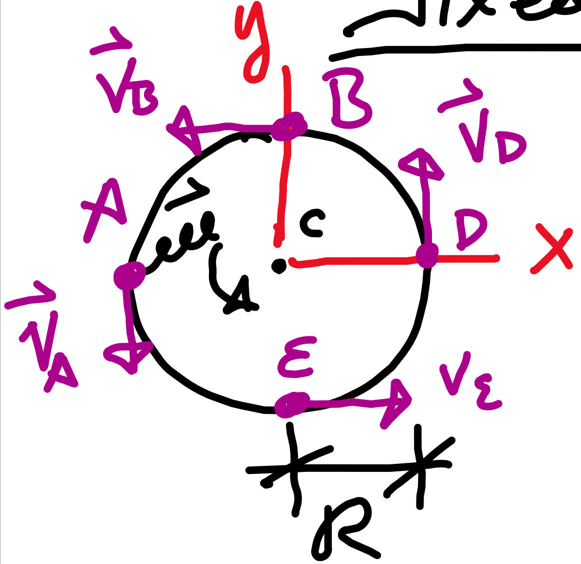
Here $\vec{\omega} = \omega \hat{z}$

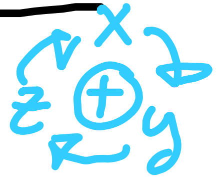
$\$ \quad \vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

$\$ \quad \vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$

$\$ \quad \vec{V}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$

Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

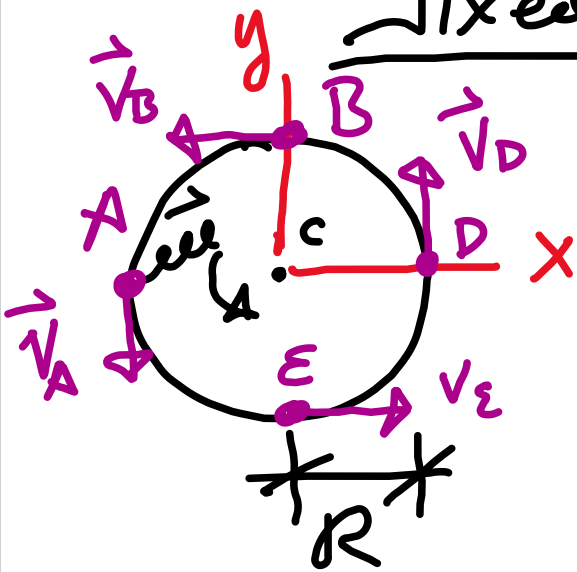
$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

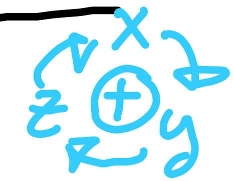
$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$

$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R \hat{z} \times \hat{x} = \omega R \hat{y}$

$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/c}$

Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

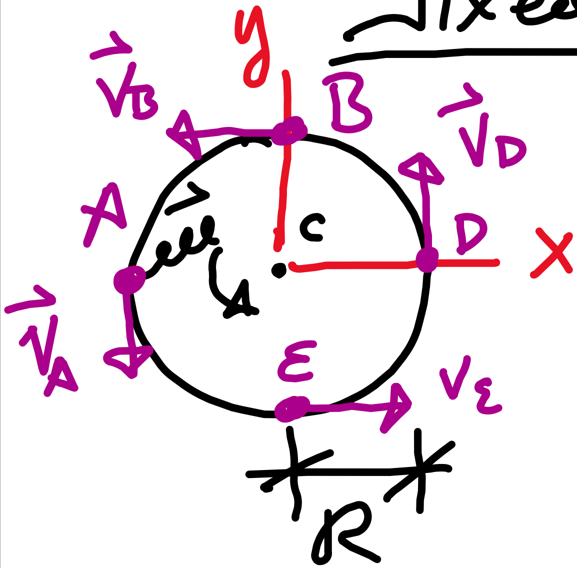
$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

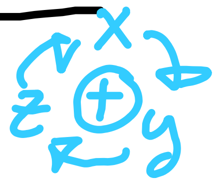
$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/C} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$$

$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/C} = \omega R\, \hat{z} \times (-\hat{y})$$

Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

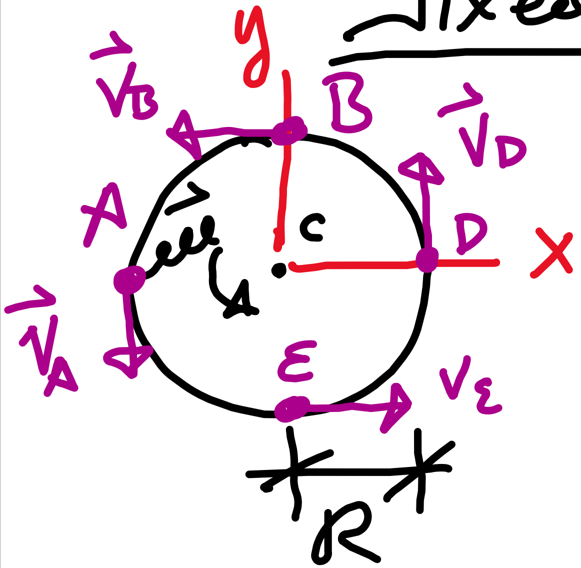
$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R \, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

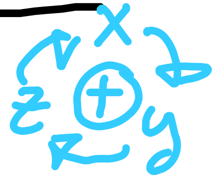
$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R \, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/C} = \omega R \, \hat{z} \times \hat{x} = \omega R \hat{y}$$

$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/C} = \omega R \, \hat{z} \times (-\hat{y}) = \omega R \hat{x}$$

Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

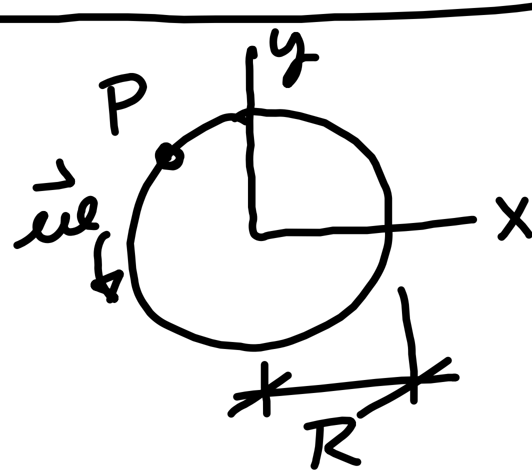
$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

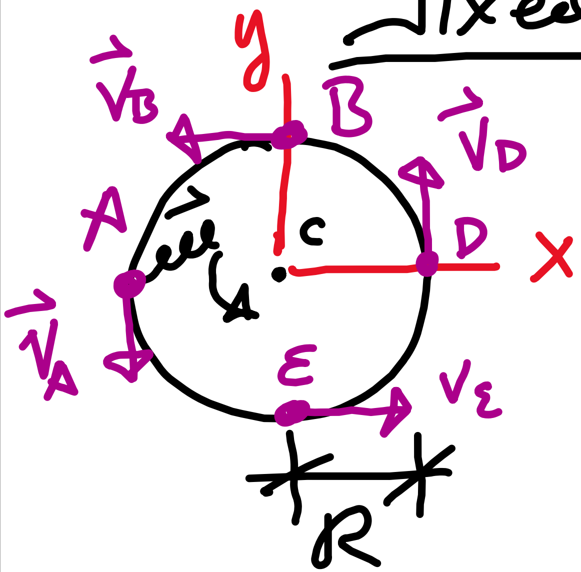
$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$$

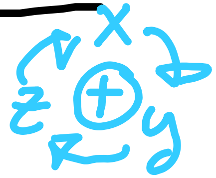
$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/c} = \omega R\, \hat{z} \times (-\hat{y}) = \omega R\, \hat{x}$$

For some arbitrary point P:



Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

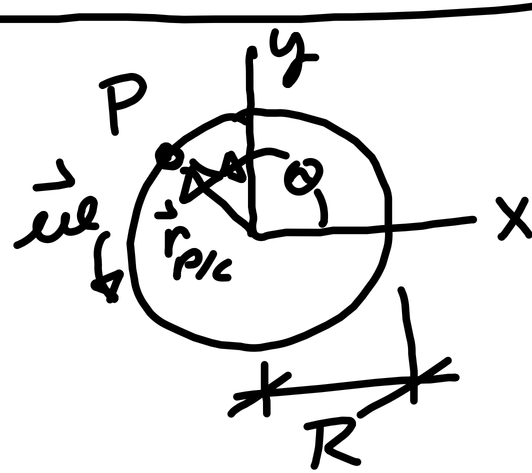
$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$

$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$

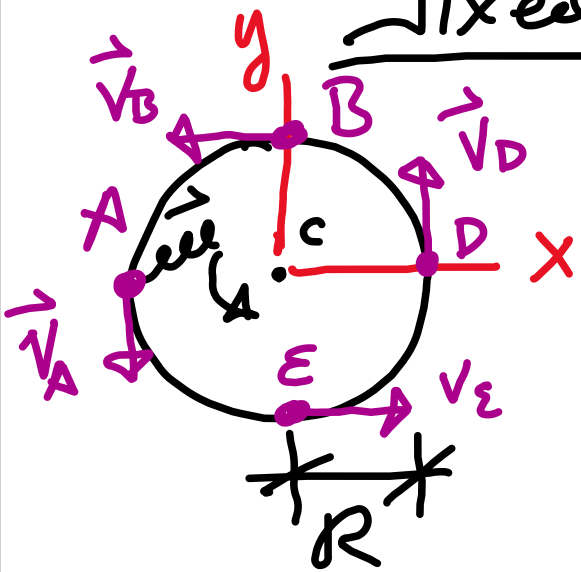
$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$

$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/c} = \omega R\, \hat{z} \times (-\hat{y}) = \omega R\, \hat{x}$

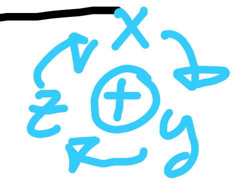
For some arbitrary point P:



Fixed axis rotation of a wheel



Here $\vec{\omega} = \omega \hat{z}$



$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

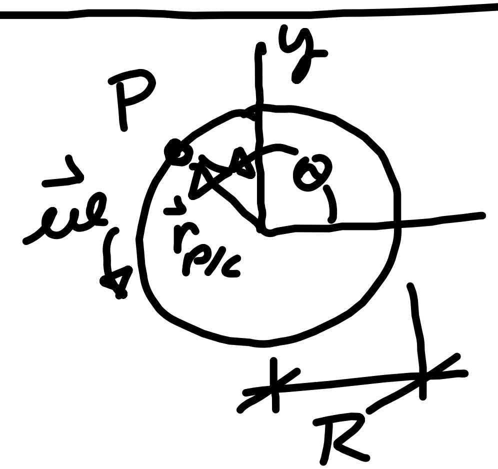
$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$$

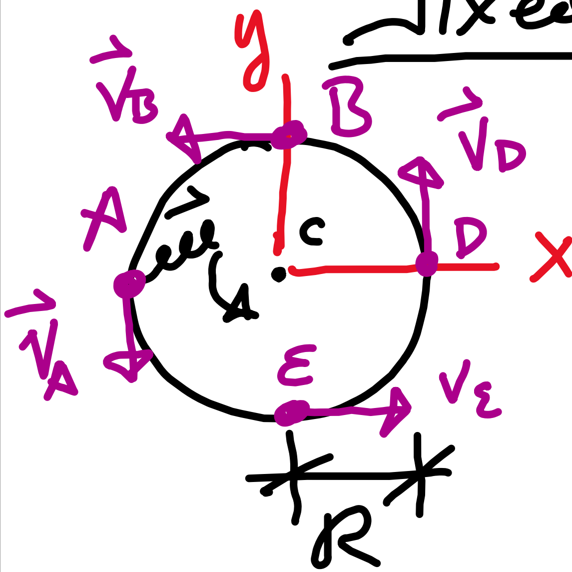
$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/c} = \omega R\, \hat{z} \times (-\hat{y}) = \omega R\, \hat{x}$$

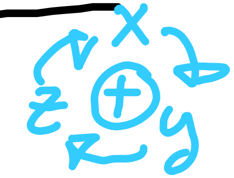
For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/c}$



Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/c} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

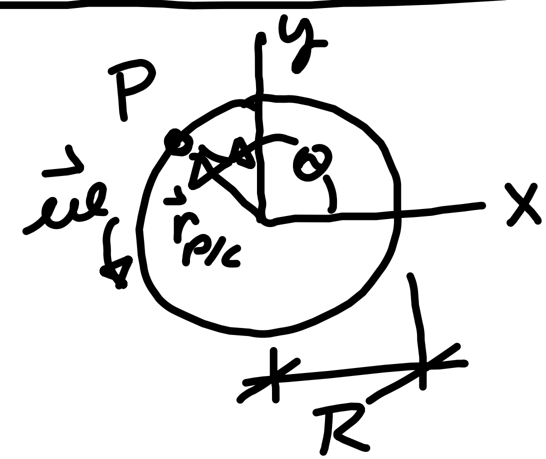
$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/c} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

$$\vec{v}_D = \vec{\omega} \times \vec{r}_{D/c} = \omega R\, \hat{z} \times \hat{x} = \omega R \hat{y}$$

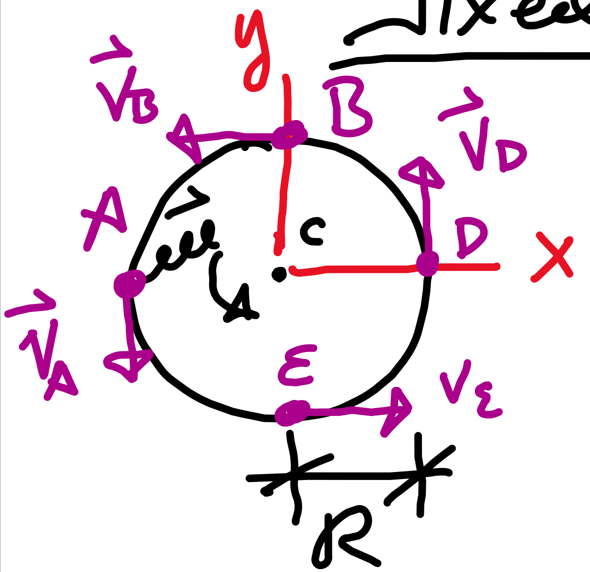
$$\vec{v}_E = \vec{\omega} \times \vec{r}_{E/c} = \omega R\, \hat{z} \times (-\hat{y}) = \omega R \hat{x}$$

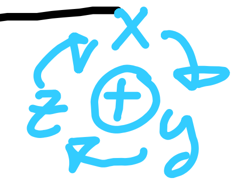
For some arbitrary point P:

$$\vec{v}_P = \vec{\omega} \times \vec{r}_{P/c} = \omega \hat{z} \times \vec{r}_{P/c}$$



Fixed axis rotation of a wheel





Here $\vec{\omega} = \omega \hat{z}$

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R\, \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R\, \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

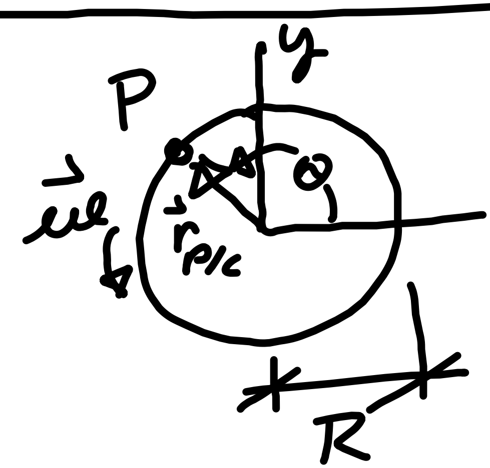
$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/C} = \omega R\, \hat{z} \times \hat{x} = \omega R\, \hat{y}$$

$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/C} = \omega R\, \hat{z} \times (-\hat{y}) = \omega R\, \hat{x}$$

For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

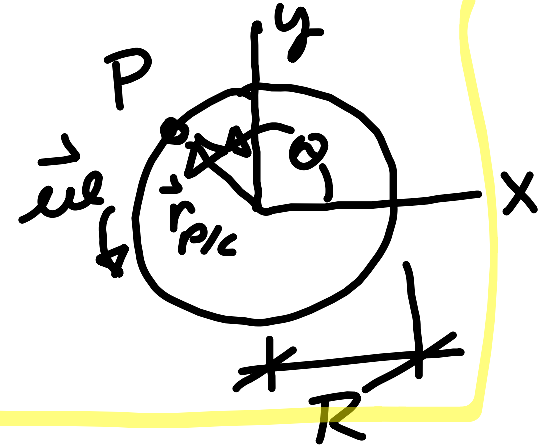
$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$



For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$

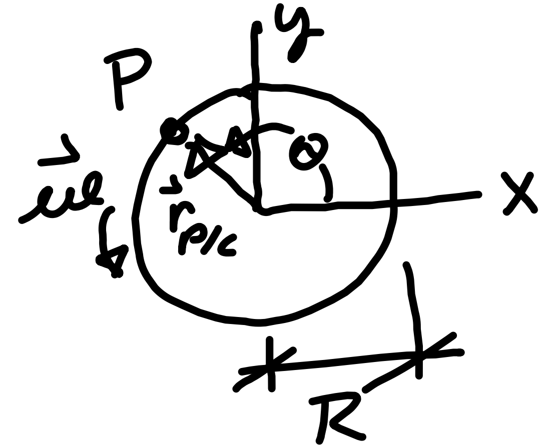


From previous slide

For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/c} = \omega \hat{z} \times \vec{r}_{P/c}$, where

$\vec{r}_{P/c} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$

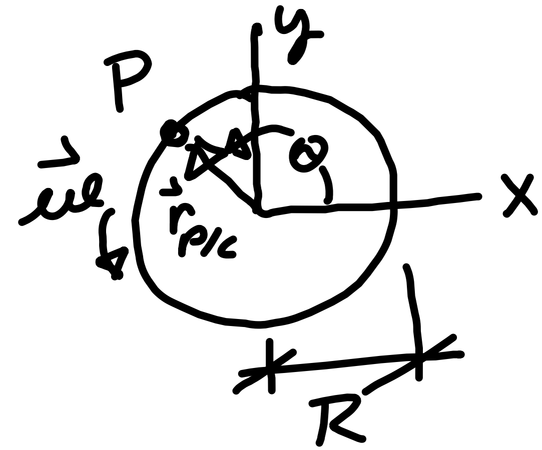


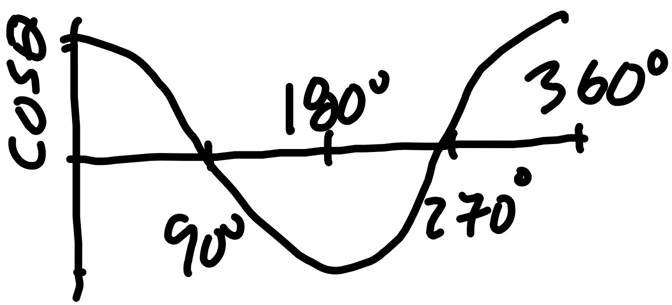
Lets check that the signs are correct for each Quadrant

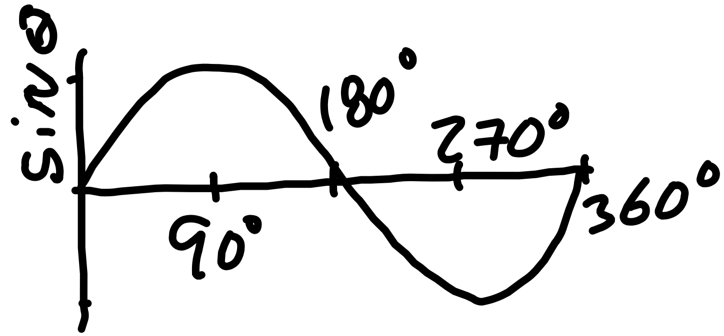
For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Note:



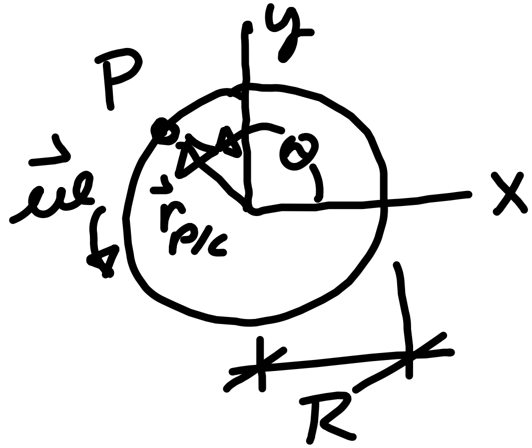


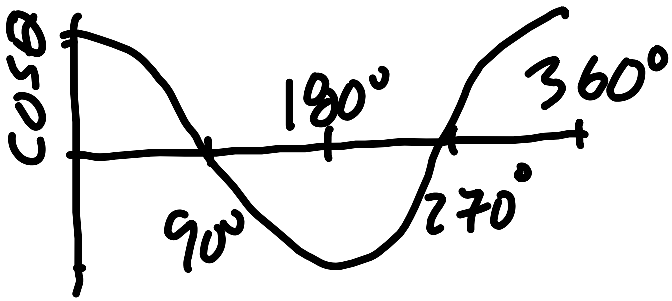


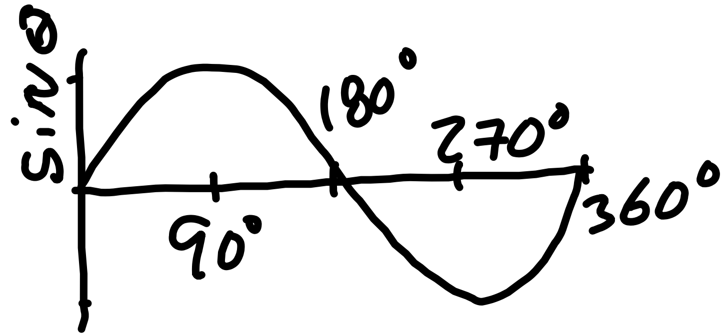
For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ <u>Note</u>:

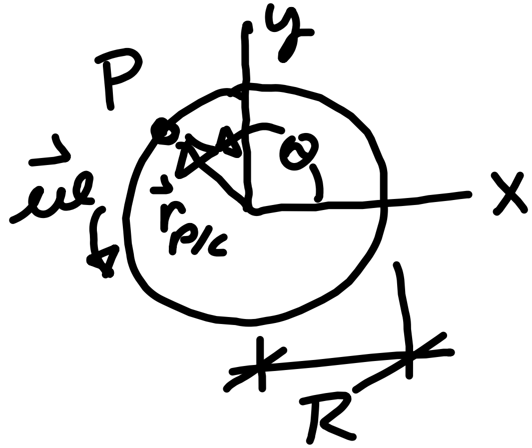






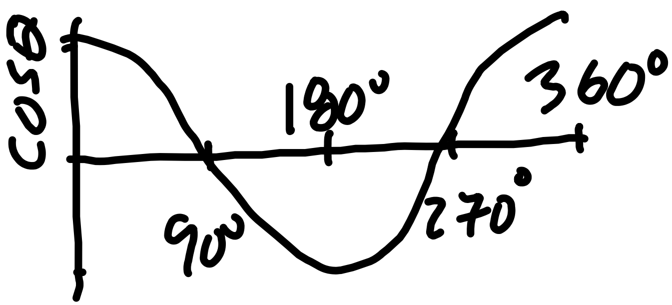
$\Rightarrow$ the signs will work out just fine.

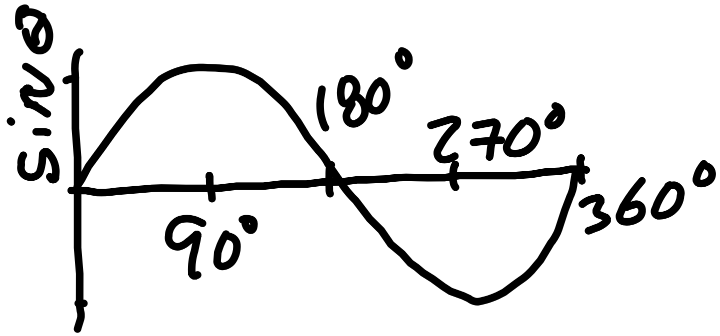
For some arbitrary point P:



$\vec{v}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ <u>Note</u>:





$\Rightarrow$ the signs will work out just fine.

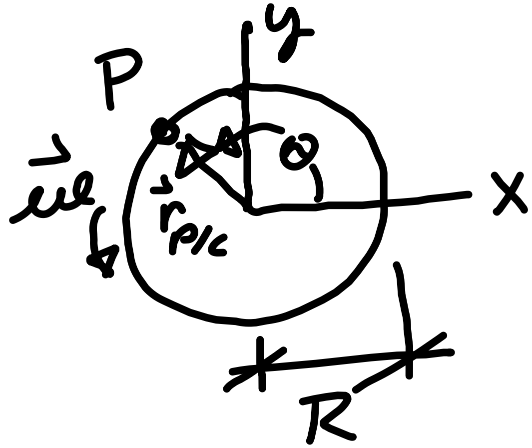
$\cos\theta$ is positive from 0°–90° & 270°–360°

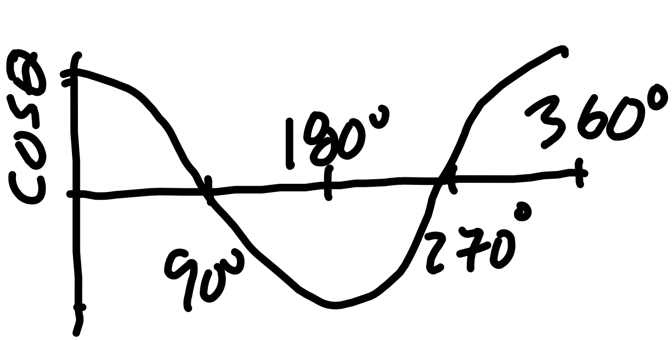
& negative from 90°–270°

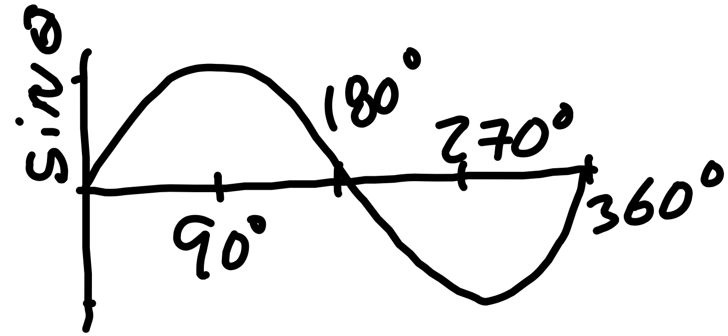
For some arbitrary point P:

$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/c} = \omega \hat{z} \times \vec{r}_{P/c}$, where

$\vec{r}_{P/c} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Note:







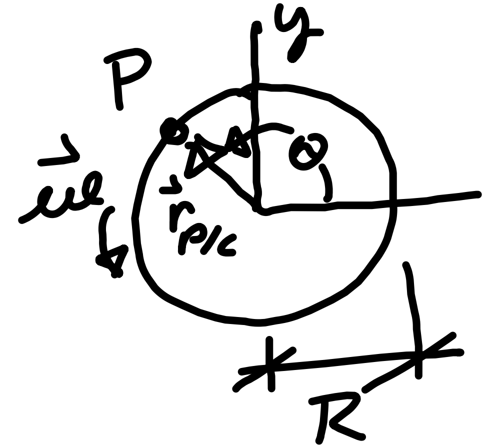
the signs will work out just fine.

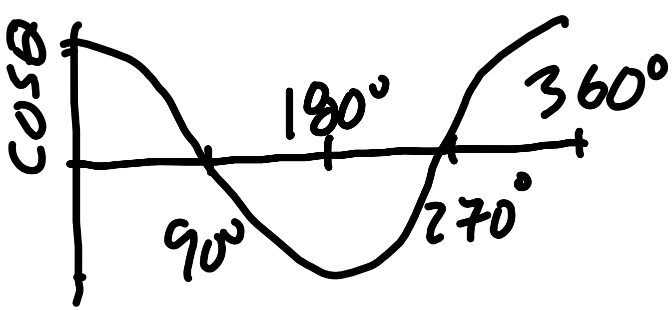
$\cos\theta$ is positive from 0°–90° & 270°–360° & negative from 90°–270° while $\sin\theta$ is positive from 0°–180°

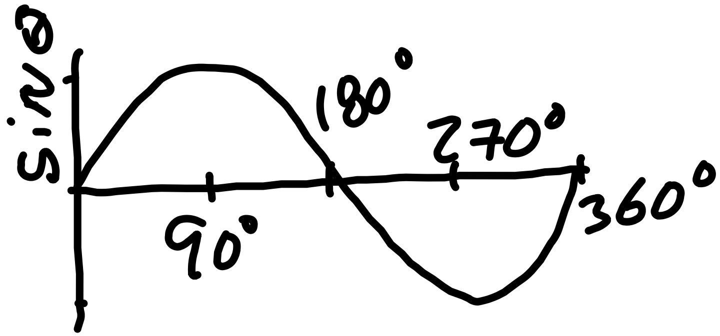
For some arbitrary point P:

$\vec{v}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Note:







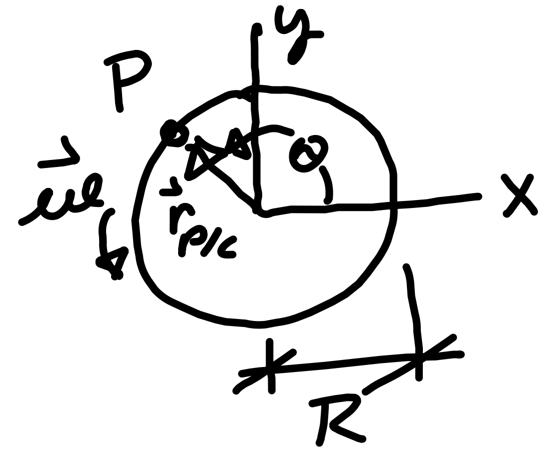
$\Rightarrow$ the signs will work out just fine.

$\cos\theta$ is positive from 0°–90° & 270°–360° & negative from 90°–270° while $\sin\theta$ is positive from 0°–180° & negative from 180°–360°

For some arbitrary point P:

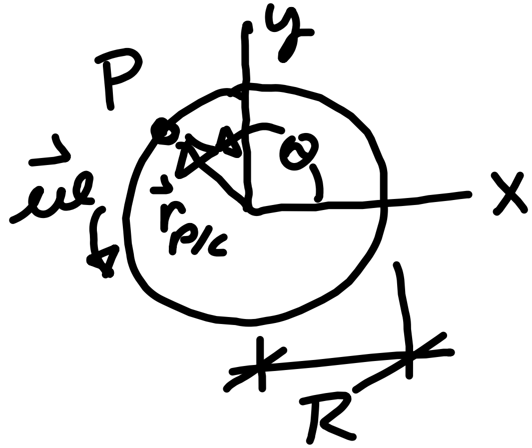
$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/c} = \omega \hat{z} \times \vec{r}_{P/c}$, where

$\vec{r}_{P/c} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Now



$\vec{V}_P = (\omega R)\hat{z} \times [\hat{x}\cos\theta + y\sin\theta]$

For some arbitrary point P:



$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C} = \omega \hat{z} \times \vec{r}_{P/C}$, where

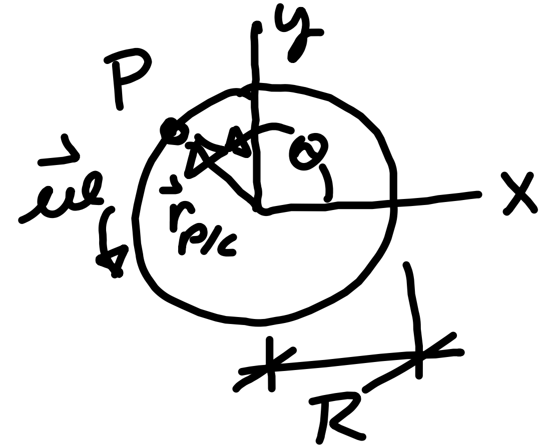
$\vec{r}_{P/C} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Now

$$\vec{V}_P = (\omega R)\hat{z} \times [\hat{x}\cos\theta + y\sin\theta]$$

$\Rightarrow$

$$\vec{V}_P = (\omega R)[\hat{y}\cos\theta - \hat{x}\sin\theta]$$

For some arbitrary point P:



$\vec{v}_P = \vec{\omega} \times \vec{r}_{P/c} = \omega \hat{z} \times \vec{r}_{P/c}$, where

$\vec{r}_{P/c} = R[\hat{x}\cos\theta + \hat{y}\sin\theta]$ Now

$$\vec{v}_P = (\omega R)\hat{z} \times [\hat{x}\cos\theta + y\sin\theta]$$

$\Rightarrow$

$\vec{v}_P = (\omega R)[\hat{y}\cos\theta - \hat{x}\sin\theta]$, Assuming that

θ is measured from the x-axis!

Rolling wheel (General plane motion):

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{v}_p = \vec{v}_{p/c} + \vec{v}_c$

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{V}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \Rightarrow$ Rotation

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{V}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \to$ Rotation & $\vec{V}_C \to$ Translation,

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{V}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \rightarrow$ Rotation & $\vec{V}_C \rightarrow$ Translation, then the motion of P is a rotation plus a translation.

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{v}_p = \vec{V}_{p/c} + \vec{V}_c$ & $\vec{V}_{p/c} \to$ Rotation & $\vec{V}_c \to$ Translation, then the motion of p is a rotation plus a translation.

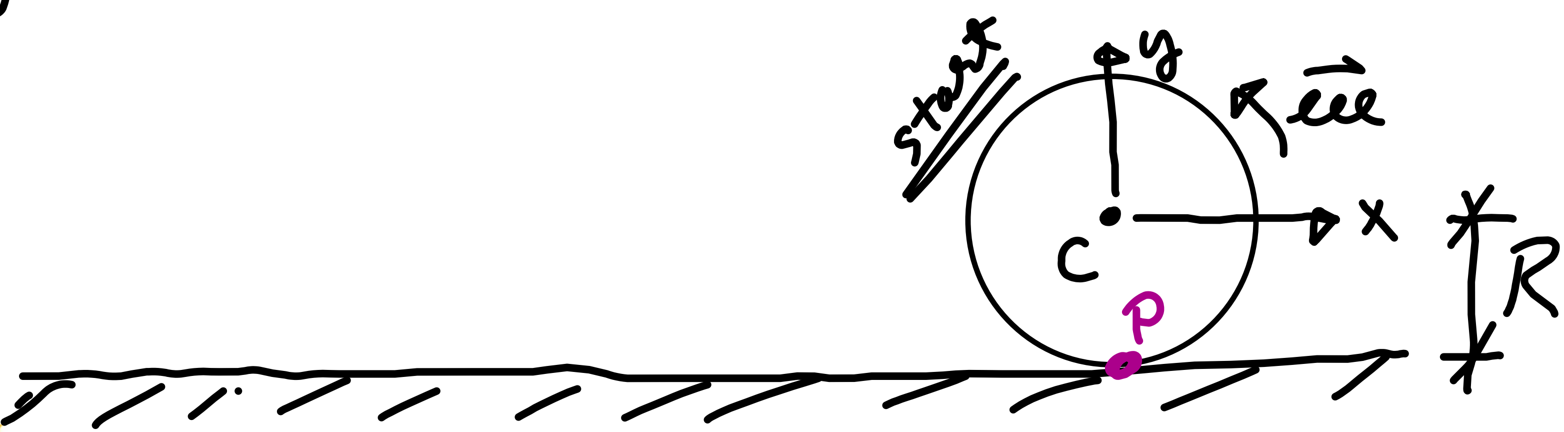
Let a wheel roll without slipping for one full rotation:

Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{v}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \rightarrow$ Rotation & $\vec{V}_C \rightarrow$ Translation, then the motion of P is a rotation plus a translation.

Let a wheel roll without slipping for one full rotation:

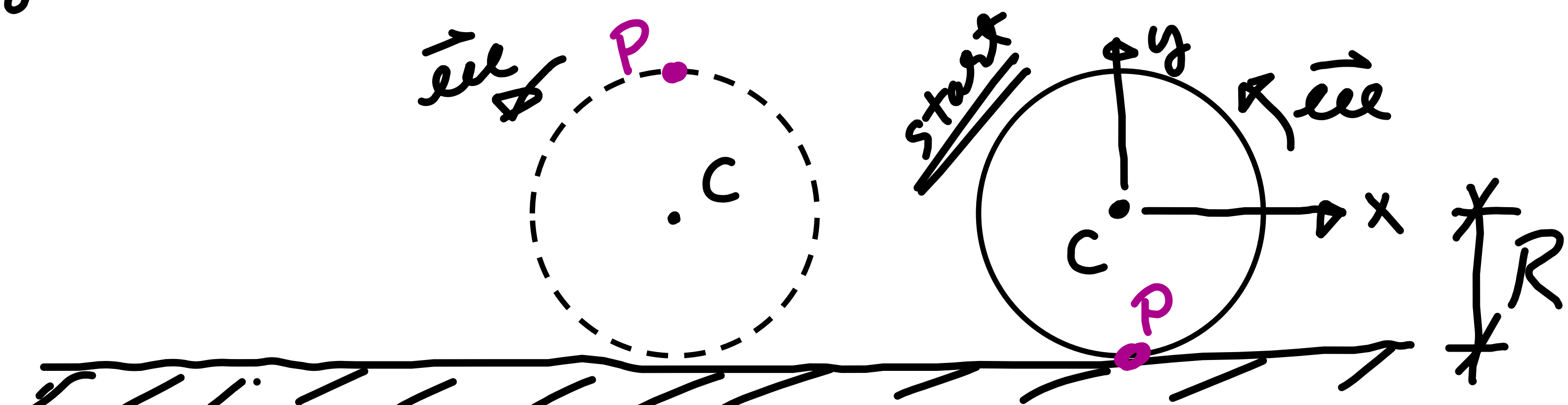


Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{V}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \to$ Rotation & $\vec{V}_C \to$ Translation, then the motion of P is a rotation plus a translation.

Let a wheel roll without slipping for one full rotation:

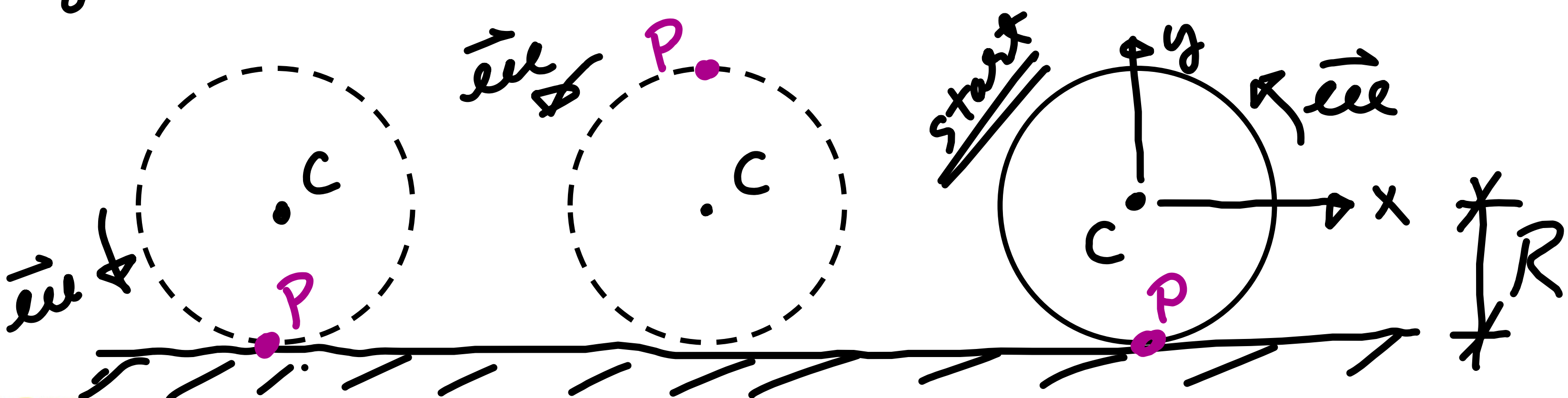


Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

Since $\vec{v}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \rightarrow$ Rotation & $\vec{V}_C \rightarrow$ Translation, then the motion of P is a rotation plus a translation.

Let a wheel roll without slipping for one full rotation:

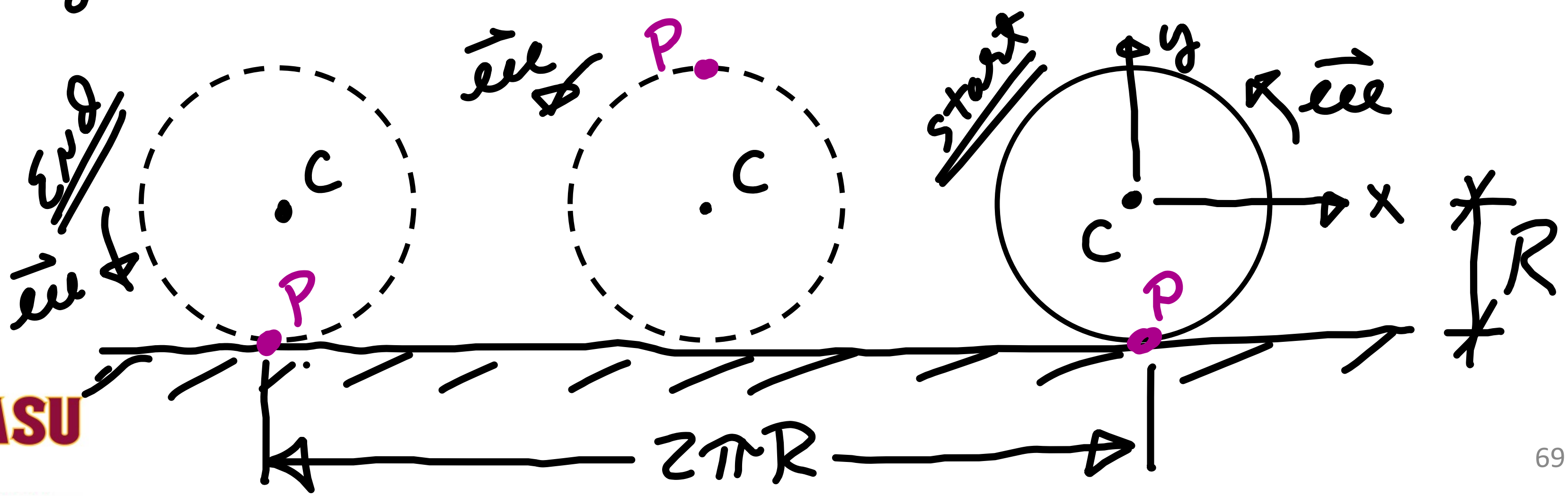


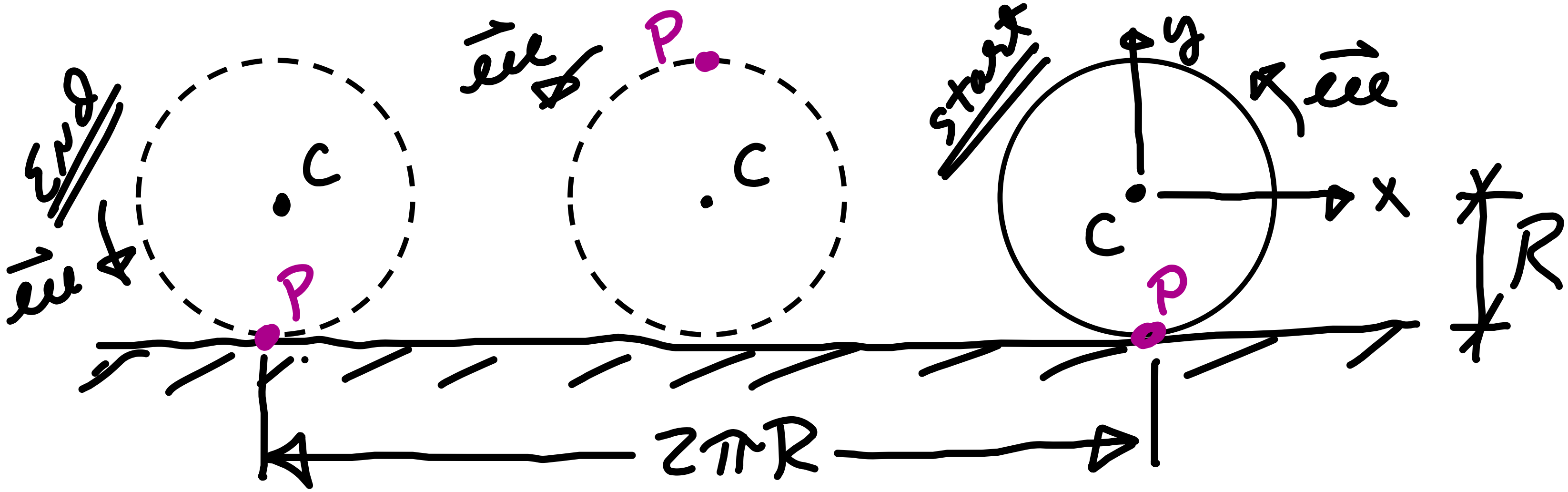
Rolling wheel (General plane motion):

If the axis of rotation is translating, we have to include that motion

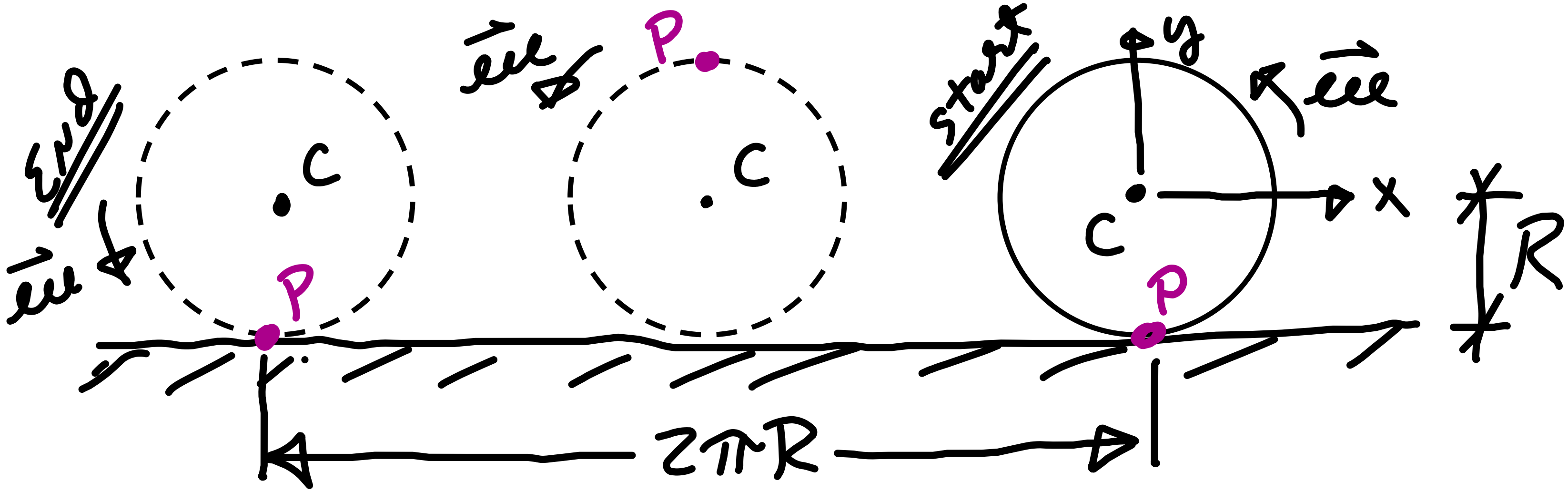
Since $\vec{v}_p = \vec{V}_{p/c} + \vec{V}_c$ & $\vec{V}_{p/c} \rightarrow$ Rotation & $\vec{V}_c \rightarrow$ Translation, then the motion of p is a rotation plus a translation.

Let a wheel roll without slipping for one full rotation:

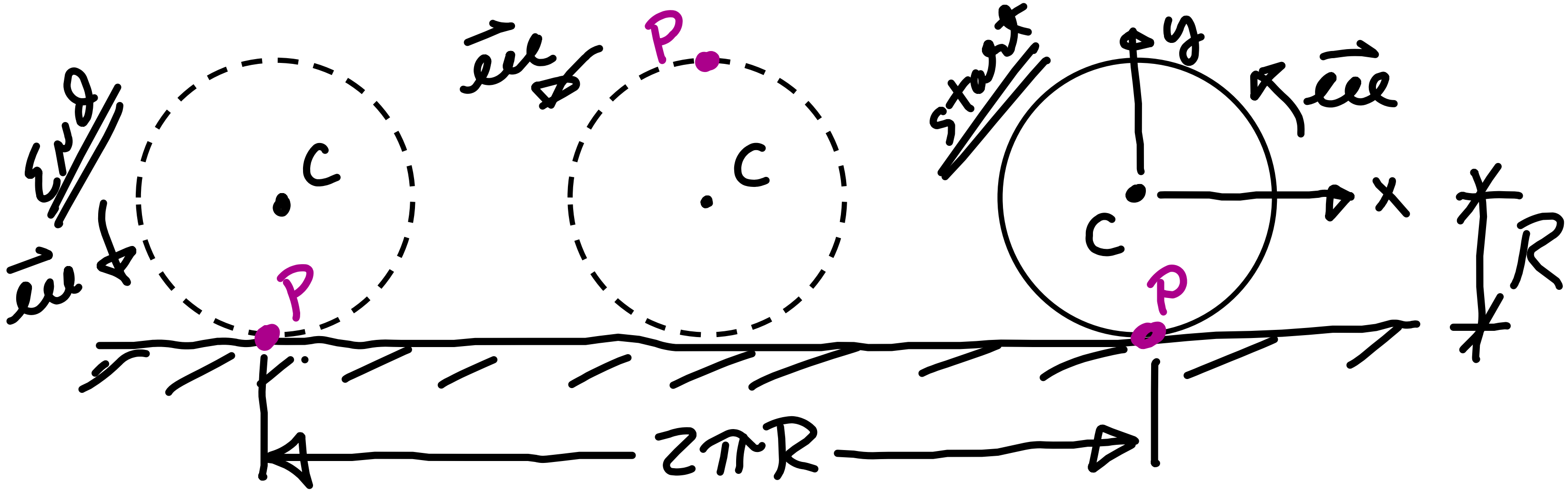




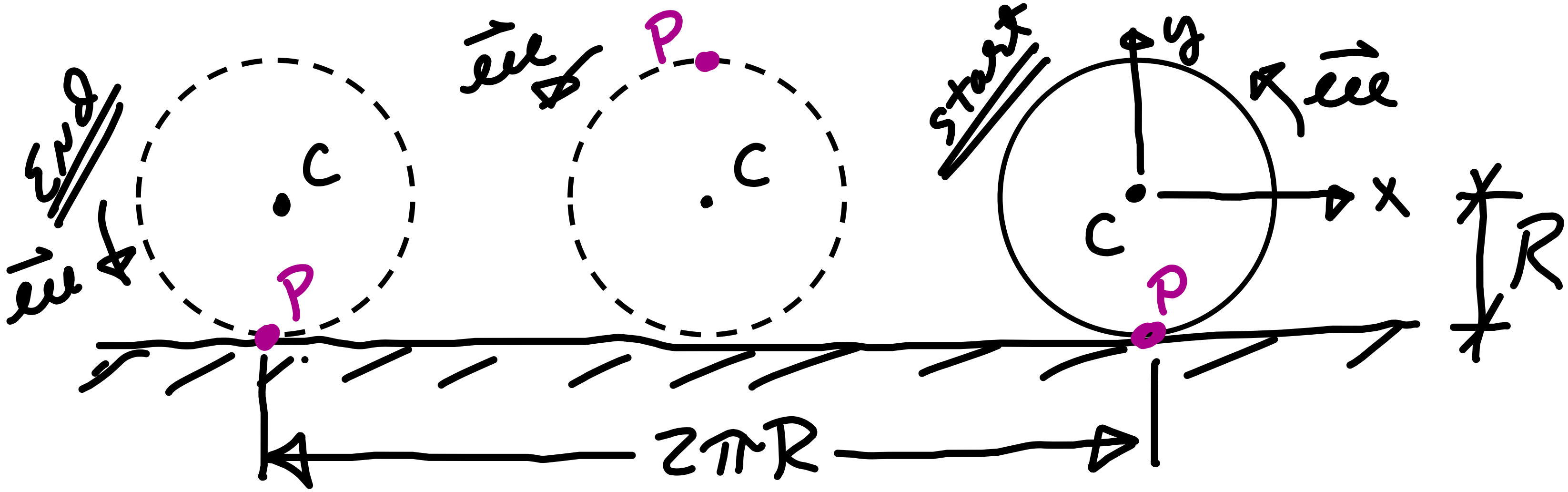
End
Start
P
P
P
C
C
C
y
x
R
2πR



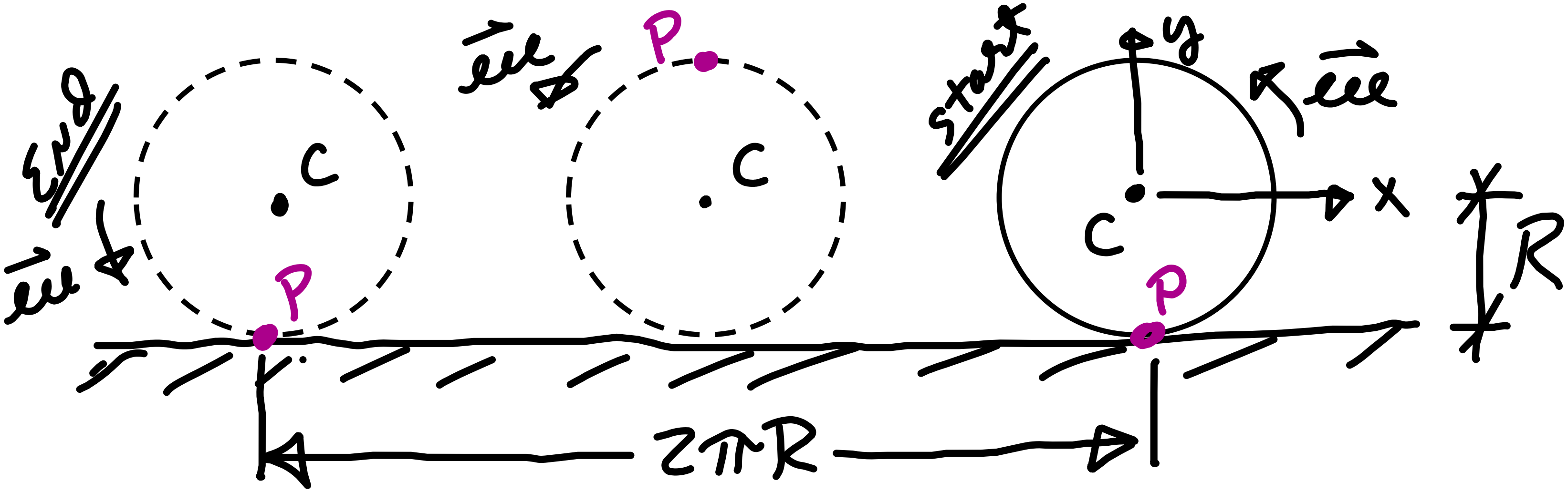
$$V_c = \frac{2\pi R}{\Delta t}$$



$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x})$$



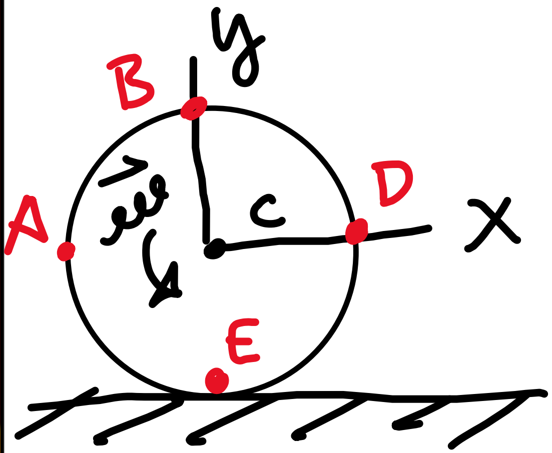
$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x}) \quad \& \quad \omega = \frac{2\pi}{\Delta t}$$



$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x}) \quad \& \quad \omega = \frac{2\pi}{\Delta t}$$

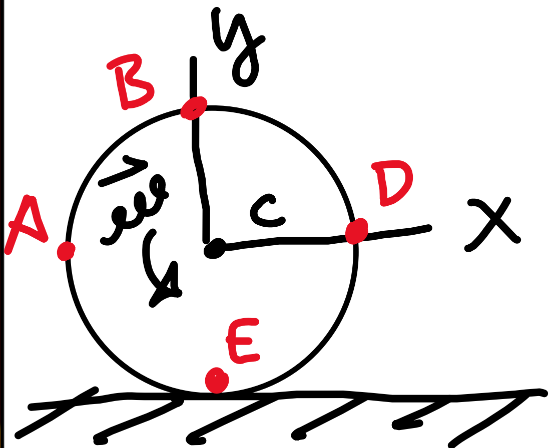
$$\Rightarrow \qquad \vec{V}_c = \omega R(-\hat{x})$$

For a wheel rotating <u>without slipping</u>

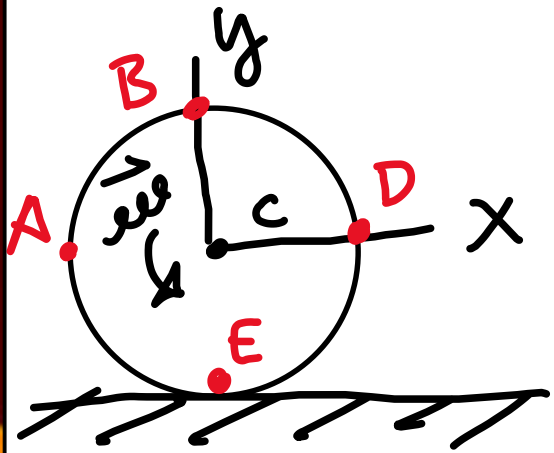


For a wheel rotating <u>without slipping</u>

$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C$$

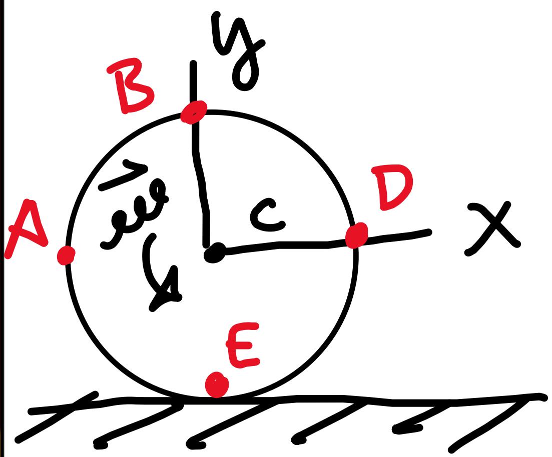


For a wheel rotating <u>without slipping</u>



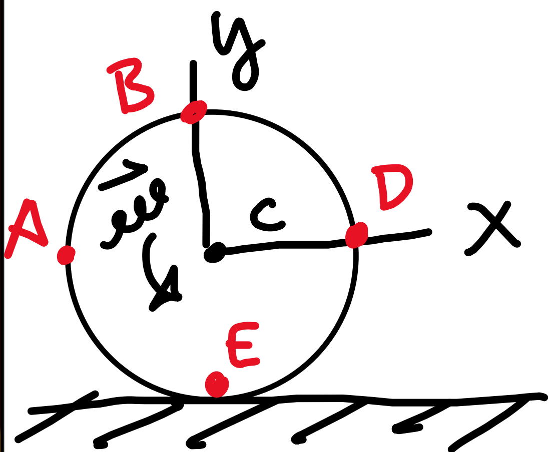
$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x}$$

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$

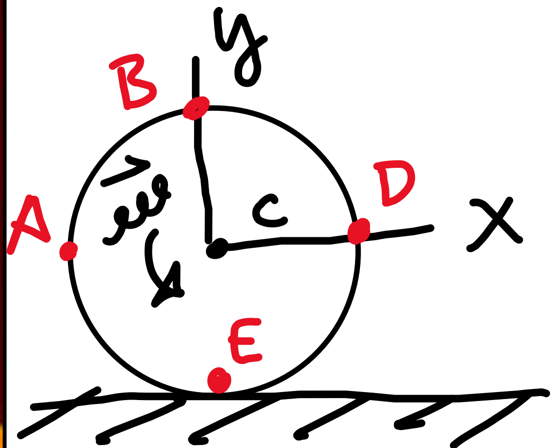
For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C =$$

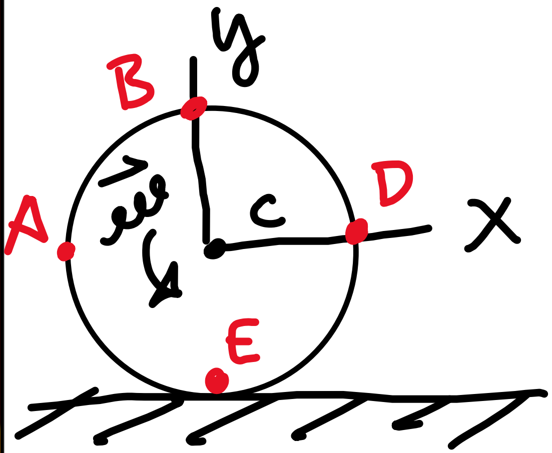
For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x}$$

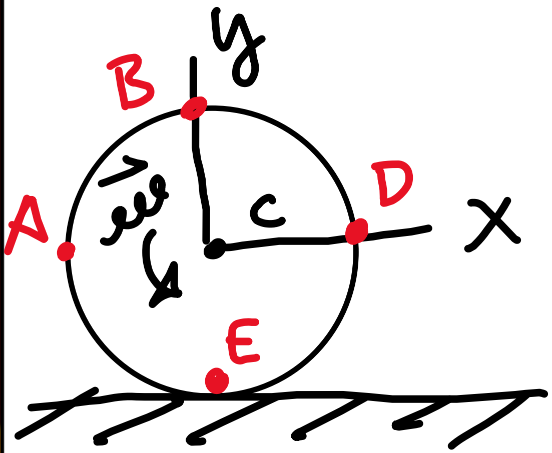
For a wheel rotating **without slipping**



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

For a wheel rotating <u>without slipping</u>

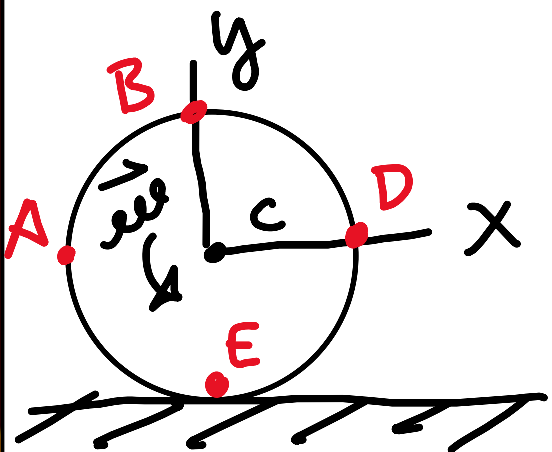


$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C$$

For a wheel rotating <u>without slipping</u>

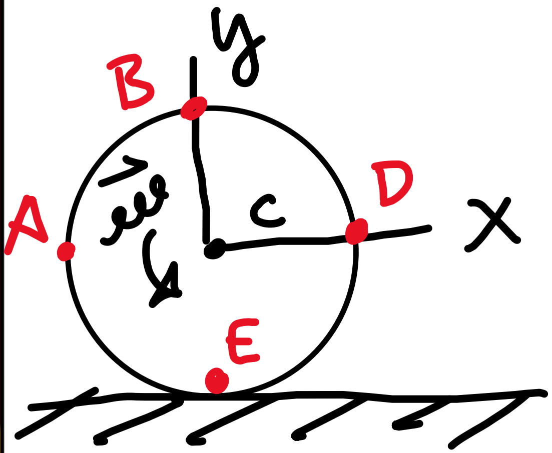


$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x}$$

For a wheel rotating without slipping

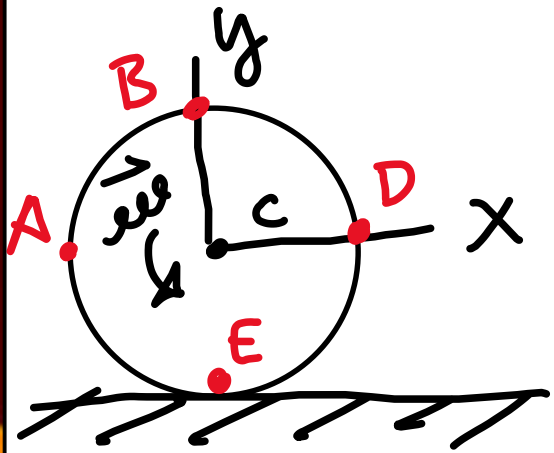


$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/c} + \vec{V}_c = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

For a wheel rotating <u>without slipping</u>



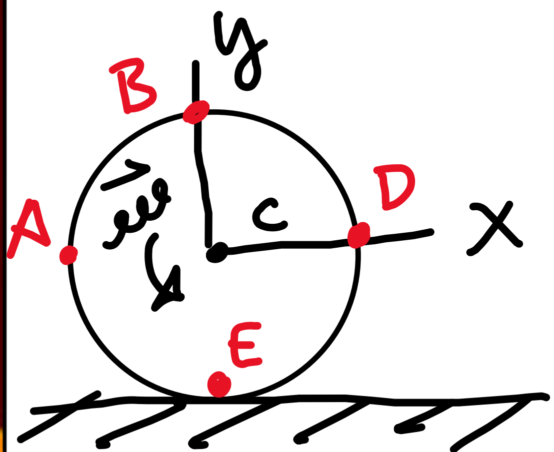
$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/c} + \vec{V}_c = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/c} + \vec{V}_c$$

For a wheel rotating <u>without slipping</u>



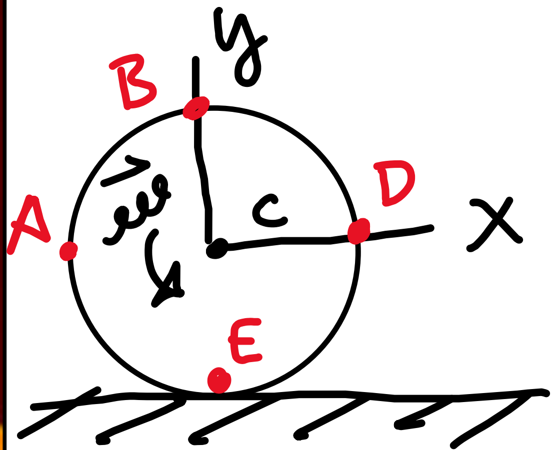
$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/c} + \vec{V}_c = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/c} + \vec{V}_c = \omega R\hat{x} - \omega R\hat{x}$$

For a wheel rotating without slipping



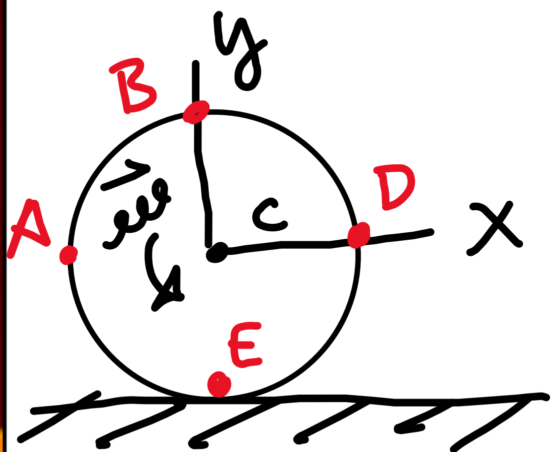
$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y} - \hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0 \;!!$$

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

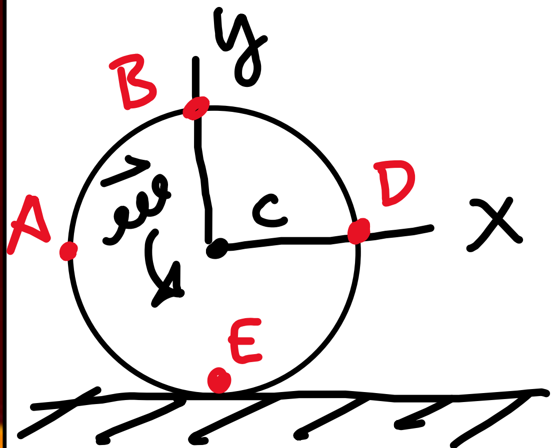
$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0$$

<u>In General</u>:

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

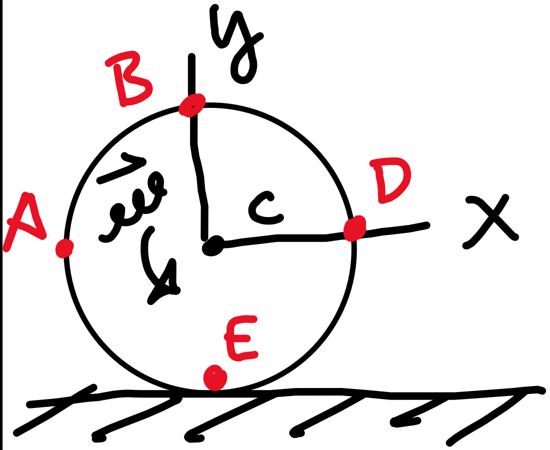
$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0$$

<u>In General</u>: If a body rotates <u>without</u> slipping against some object,

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

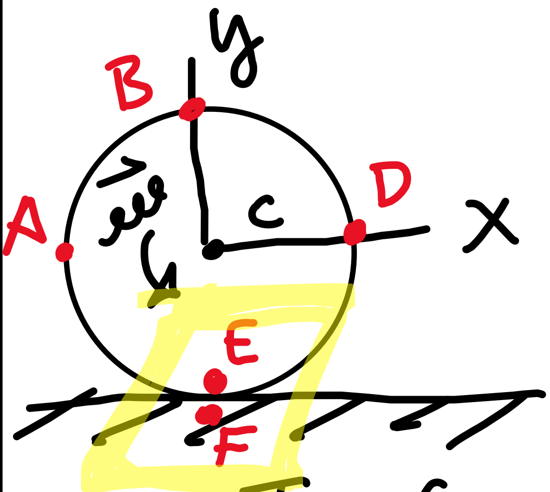
$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0$$

<u>In General</u>: If a body rotates <u>without</u> slipping against some object, the relative velocity between those two objects is zero.

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

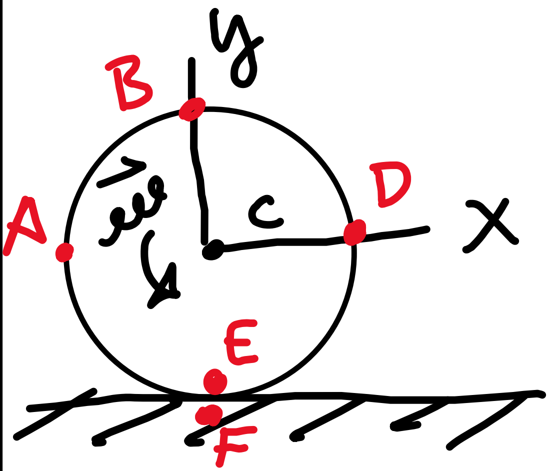
$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y}-\hat{x}]$$

$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0$$

<u>In General</u>: If a body rotates <u>without</u> slipping against some object, the relative velocity between those two objects is zero. In the above case, where the wheel touches the road (point E) has the same velocity as the point on the road touching the wheel (point F)

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C = -\omega R\hat{x} - \omega R\hat{x} = -2\omega R\hat{x}$$

$$\vec{V}_D = \vec{V}_{D/C} + \vec{V}_C = \omega R\hat{y} - \omega R\hat{x} = \omega R[\hat{y} - \hat{x}]$$

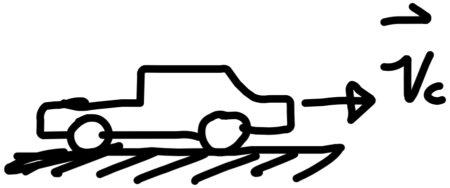
$$\vec{V}_E = \vec{V}_{E/C} + \vec{V}_C = \omega R\hat{x} - \omega R\hat{x} = 0$$

<u>In General</u>: If a body rotates <u>without</u> slipping against some object, the relative velocity between those two objects is zero. In the above case, where the wheel touches the road (point E) has the same velocity as the point on the road touching the wheel (point F). That is $\vec{V}_{E/F} = 0$

The intuition problem:

The intuition problem: If I see a car moving down the road,

The intuition problem: If I see a car moving down the road,



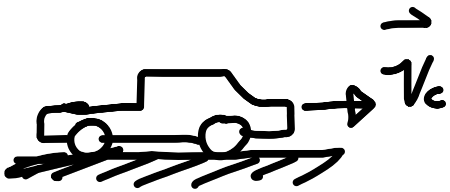
The intuition problem: If I see a car moving down the road, my mind wants to say that **all** of the car is moving down the road at the same speed.

$\vec{v}_c$

The intuition problem: If I see a car moving down the road, my mind wants to say that all of the car is moving down the road at the same speed. My mind will allow for additional motion of the wheels

$\vec{v}_c$

The intuition problem: If I see a car moving down the road, my mind wants to say that **all** of the car is moving down the road at the same speed. My mind will allow for additional motion of the wheels



But my intuition is not compatible with the idea that there is a point on each wheel that has zero velocity.

The intuition problem: If I see a car moving down the road, my mind wants to say that **all** of the car is moving down the road at the same speed. My mind will allow for additional motion of the wheels.

$\vec{v}_c$

But my intuition is not compatible with the idea that there is a point on each wheel that has zero velocity.

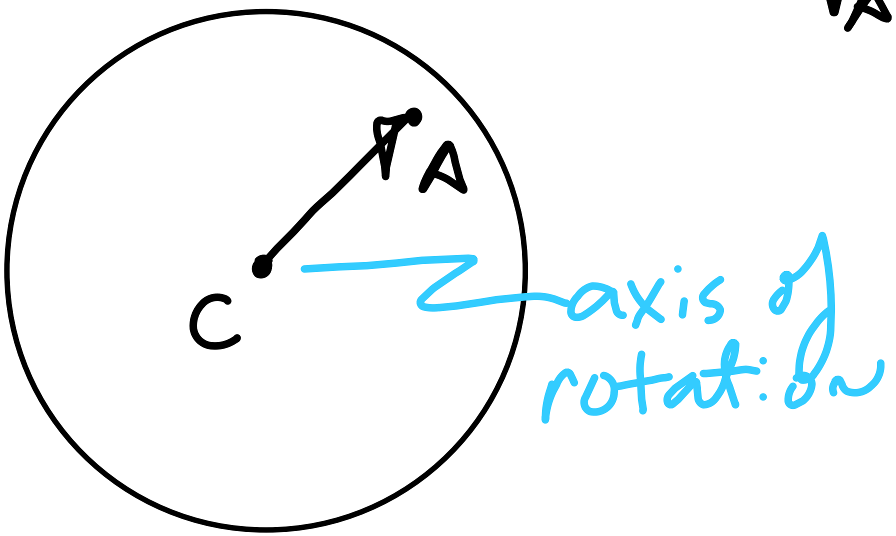
You may not share this intuition problem with me, but many in this class will

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation.

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

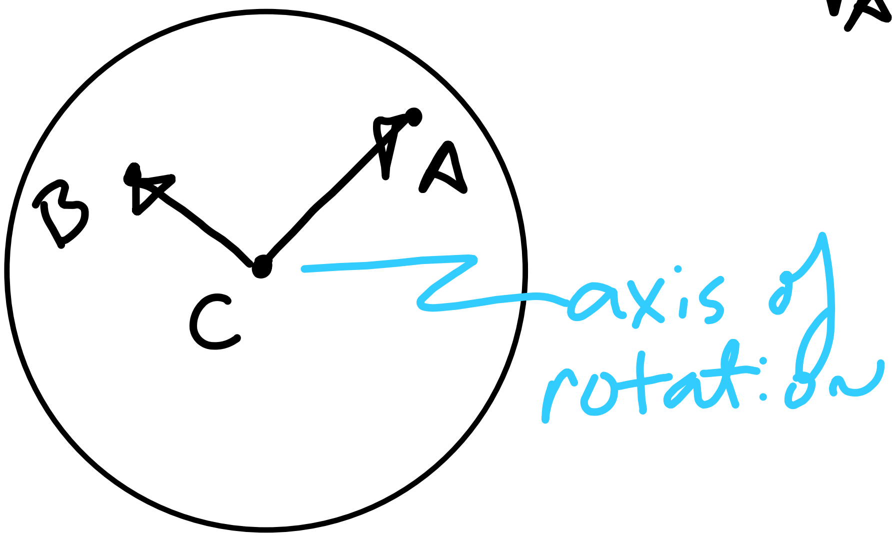
So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/C}$$



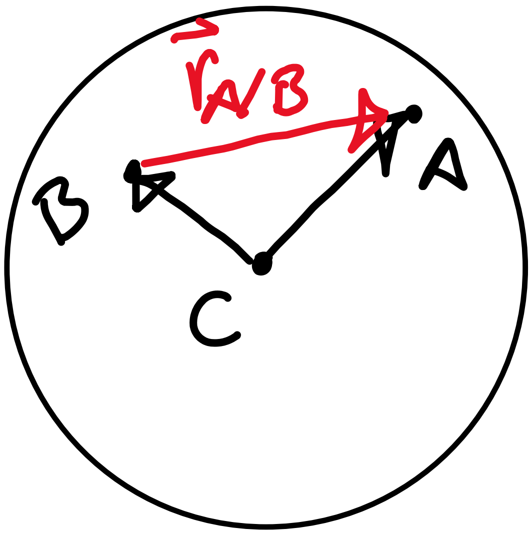
So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C}$ & $\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$



So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

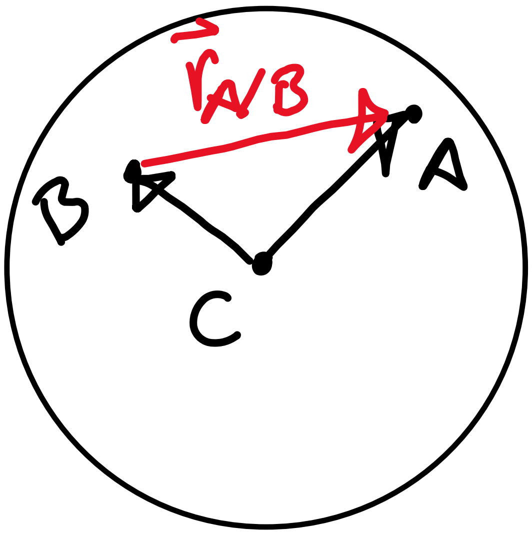
$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/C} \quad \& \quad \vec{v}_B = \vec{\omega} \times \vec{r}_{B/C}$$



Now, since $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

So far, our derivation of $\vec{v}=\vec{\omega}\times\vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$$\vec{V}_A=\vec{\omega}\times\vec{r}_{A/C} \;\&\; \vec{V}_B=\vec{\omega}\times\vec{r}_{B/C}$$

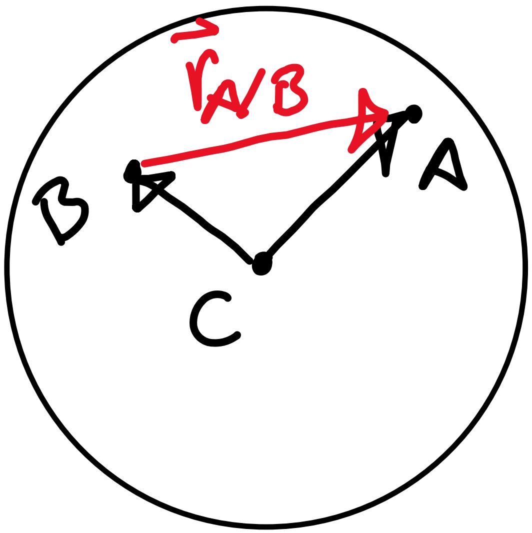


Now, since $\vec{V}_{A/B}=\vec{V}_A-\vec{V}_B$

then $\vec{V}_{A/B}=\vec{\omega}\times\vec{r}_{A/C}-\vec{\omega}\times\vec{r}_{B/C}$

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} \;\&\; \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$$



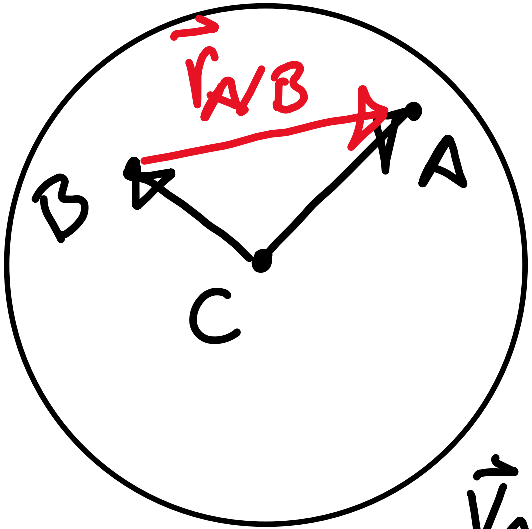
Now, since $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

then $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/C} - \vec{\omega} \times \vec{r}_{B/C}$

$$\Rightarrow V_{A/B} = \vec{\omega} \times \left[\vec{r}_{A/C} - \vec{r}_{B/C}\right]$$

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} \ \& \ \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$$



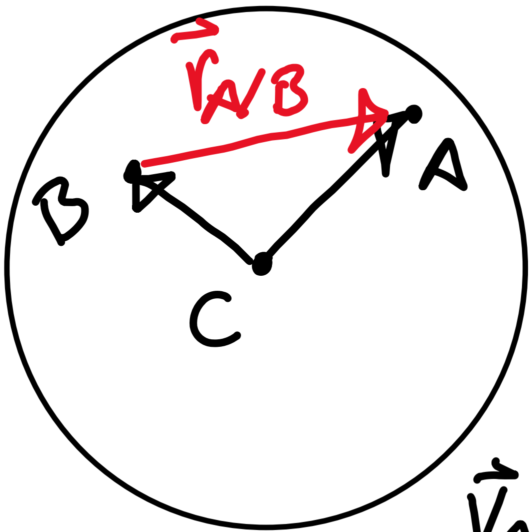
Now, since $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

then $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/C} - \vec{\omega} \times \vec{r}_{B/C}$

$$\Rightarrow V_{A/B} = \vec{\omega} \times [\vec{r}_{A/C} - \vec{r}_{B/C}] \Rightarrow$$

$$\vec{V}_{A/B} = \vec{\omega} \times [(\vec{r}_A - \vec{r}_C) - (\vec{r}_B - \vec{r}_C)]$$

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} \ \& \ \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$$

Now, since $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

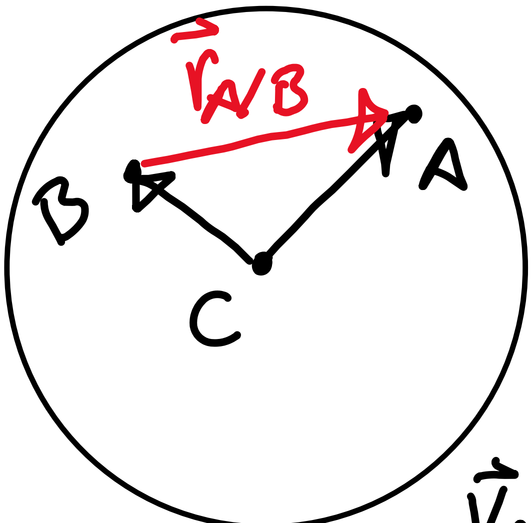
then $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/C} - \vec{\omega} \times \vec{r}_{B/C}$

$$\Rightarrow V_{A/B} = \vec{\omega} \times [\vec{r}_{A/C} - \vec{r}_{B/C}] \Rightarrow$$

$$\vec{V}_{A/B} = \vec{\omega} \times [(\vec{r}_A - \vec{r}_C) - (\vec{r}_B - \vec{r}_C)] = \vec{\omega} \times (\vec{r}_A - \vec{r}_B)$$

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} \;\&\; \vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$$



Now, since $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

then $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/C} - \vec{\omega} \times \vec{r}_{B/C}$

$$\Rightarrow V_{A/B} = \vec{\omega} \times [\vec{r}_{A/C} - \vec{r}_{B/C}] \Rightarrow$$

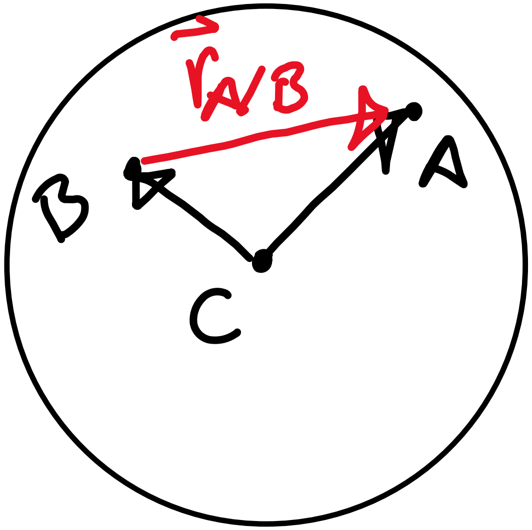
$$\vec{V}_{A/B} = \vec{\omega} \times [(\vec{r}_A - \vec{r}_C) - (\vec{r}_B - \vec{r}_C)] = \vec{\omega} \times (\vec{r}_A - \vec{r}_B)$$

$$\Rightarrow \boxed{\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}}$$

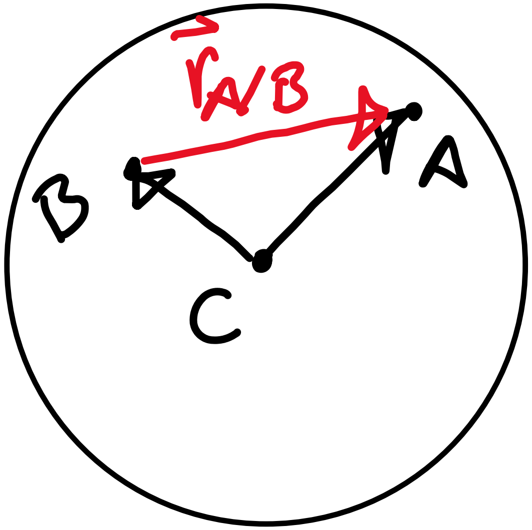
So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

We have $\vec{v}_A = \vec{\omega} \times \vec{r}_A$

Motion of point A as seen standing at point C



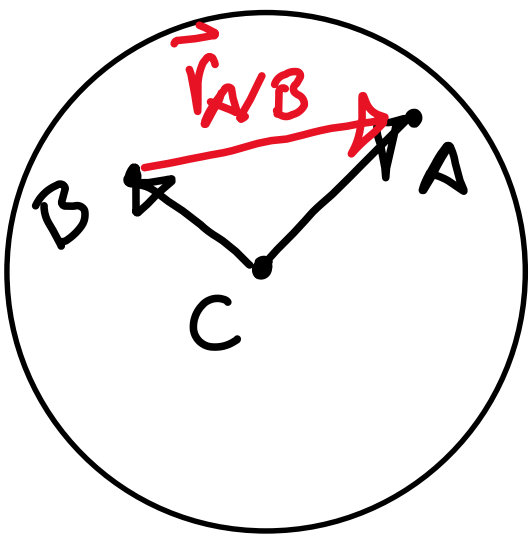
So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

& $\underbrace{\vec{V}_B = \vec{\omega} \times \vec{r}_B}_{\text{Motion of point B as seen standing at point C}}$

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



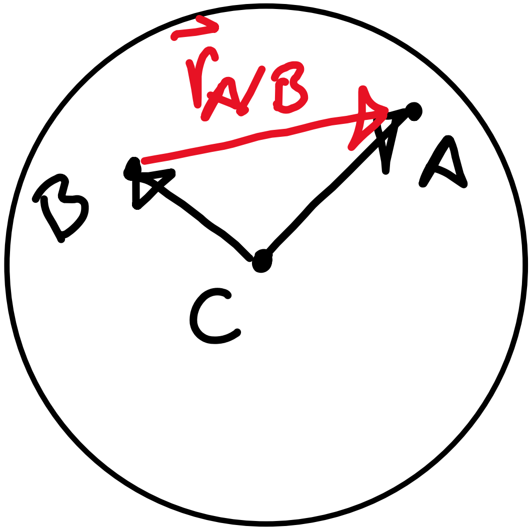
We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

& $\vec{V}_B = \vec{\omega} \times \vec{r}_B$

& $\underbrace{\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}}$

Motion of point A as seen from point B

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



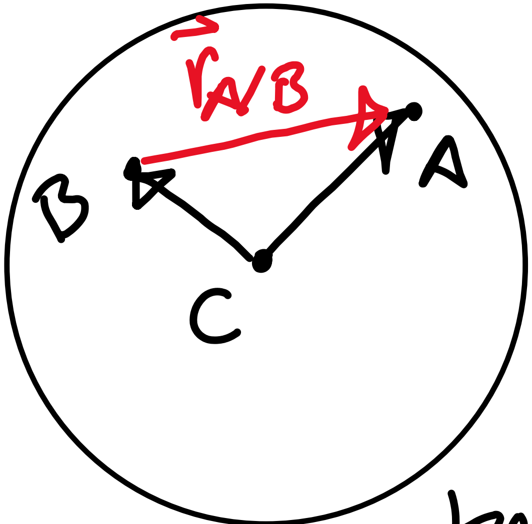
We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

& $\vec{V}_B = \vec{\omega} \times \vec{r}_B$

& $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$

The $\vec{\omega}$ is identical, regardless of the reference point

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



We have $\vec{v}_A = \vec{\omega} \times \vec{r}_A$

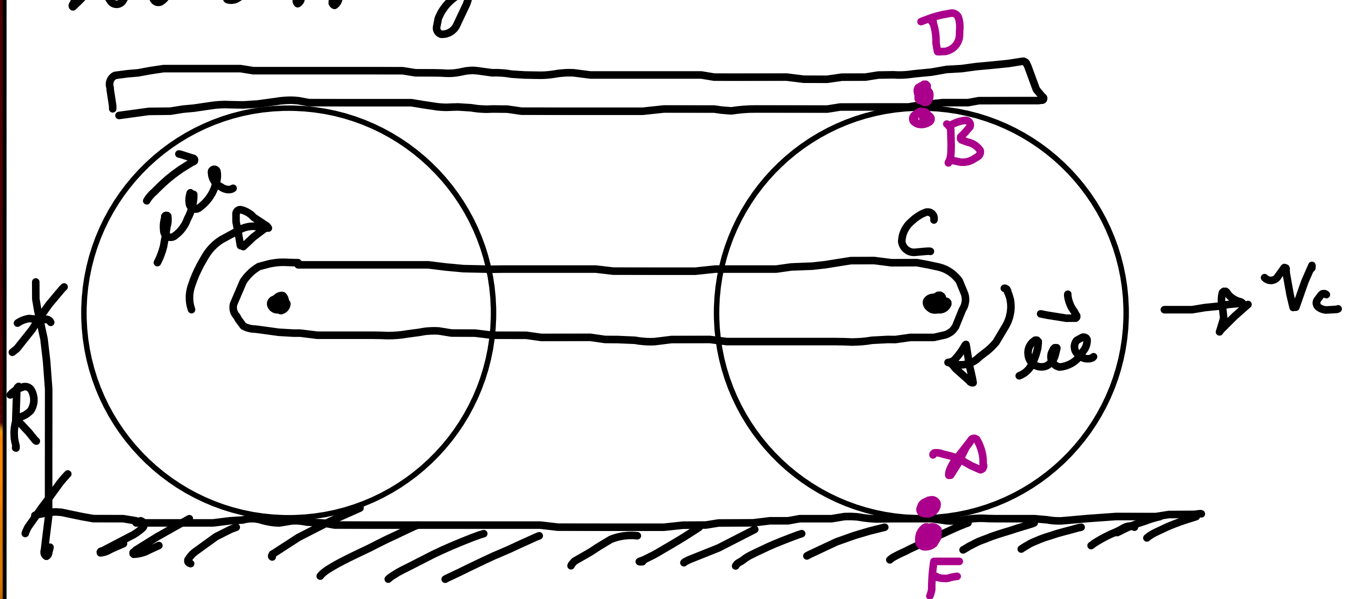
& $\vec{v}_B = \vec{\omega} \times \vec{r}_B$

& $\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$

<u>Angular velocity of a rigid body in plane motion is independent of the reference point</u>

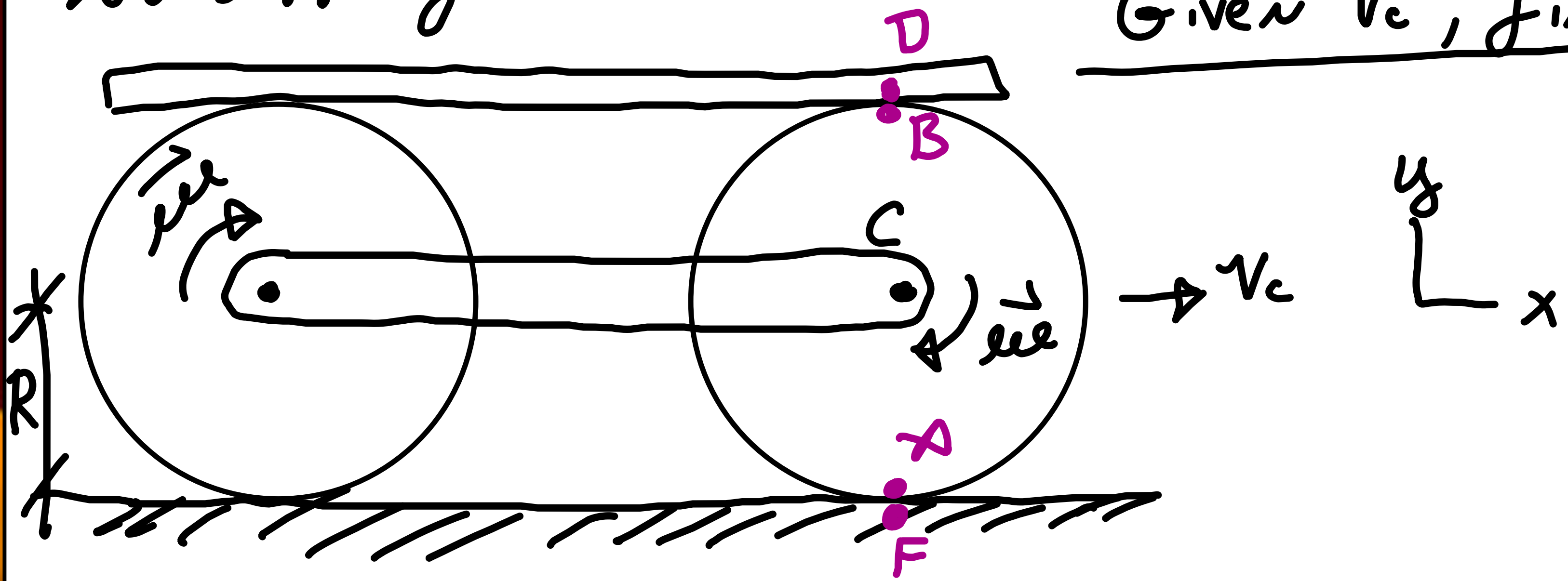
→ <u>Not</u> intuitive

Example: Board sitting on wheels. Rolling no slipping.



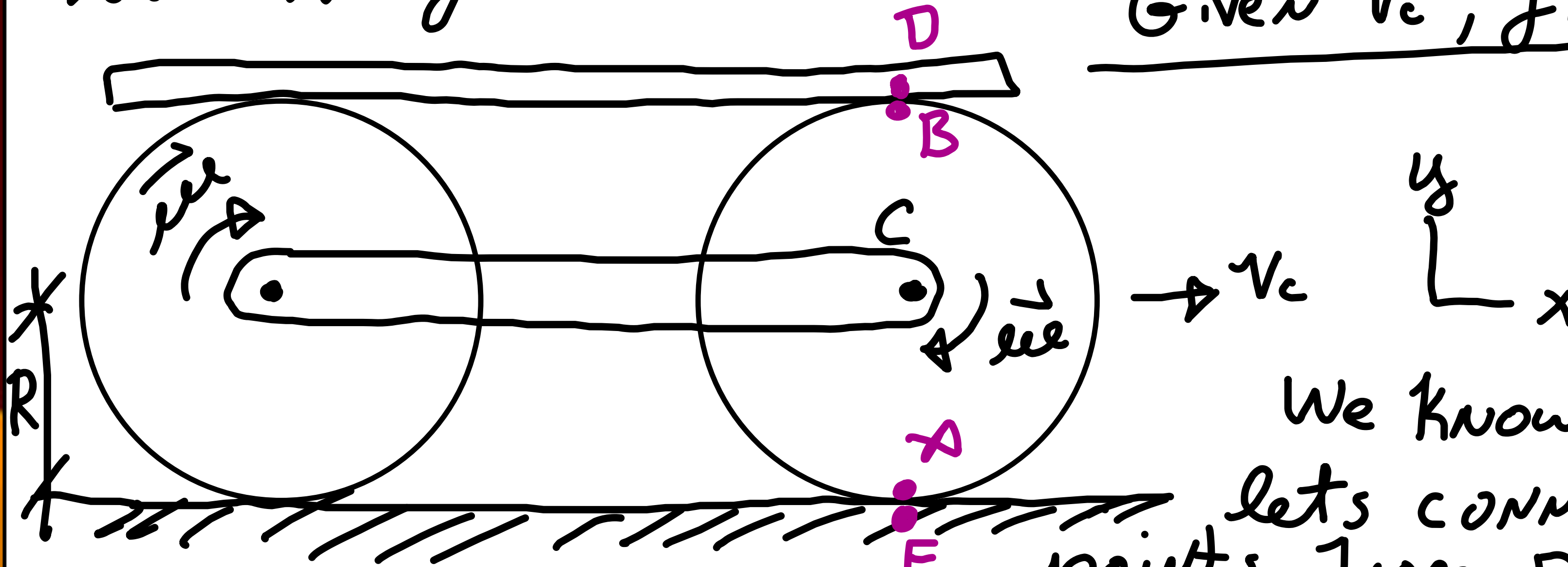
Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



Example: Board sitting on wheels. Rolling no slipping.

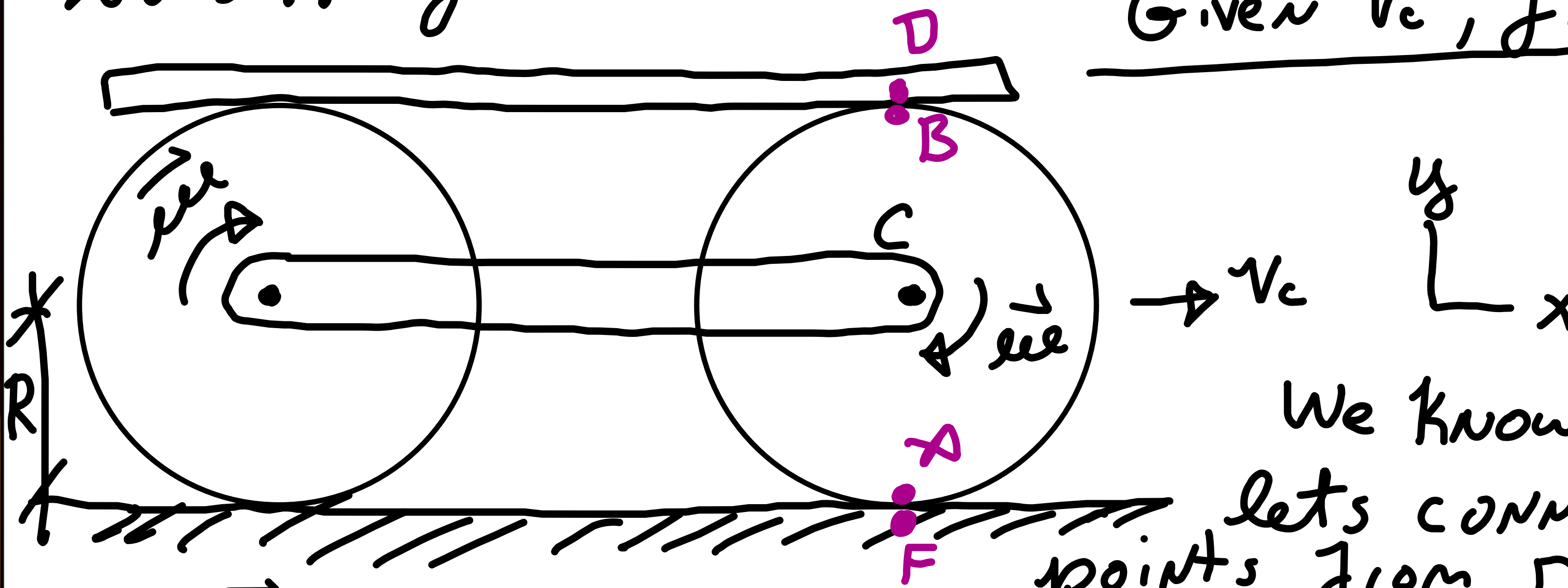
Given v_C, find v_D



We know $\vec{v}_C$ so lets connect points from D to C

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

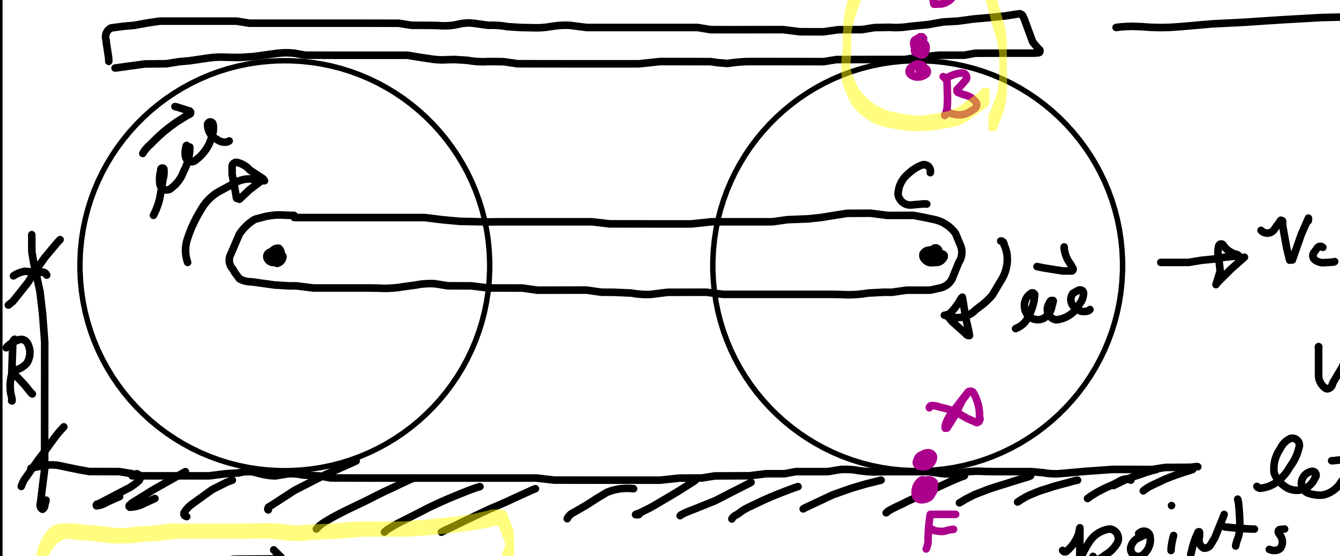


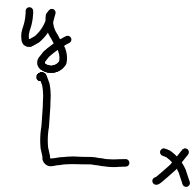
We know $\vec{V}_c$ so lets connect points from D to C

$$\vec{V}_D = \vec{V}_{D/B} + V_B$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



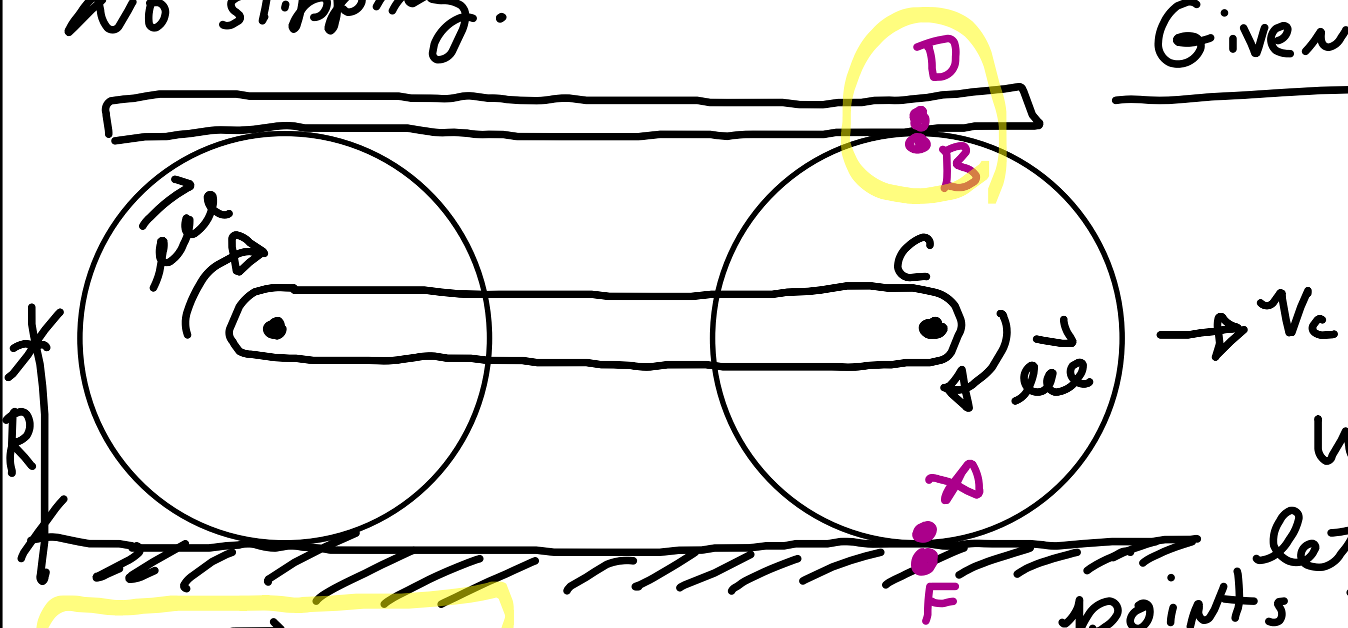


We know $\vec{v}_c$ so lets connect points from D to C

$$\boxed{\vec{v}_D = \vec{v}_{D/B} + v_B}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



y x

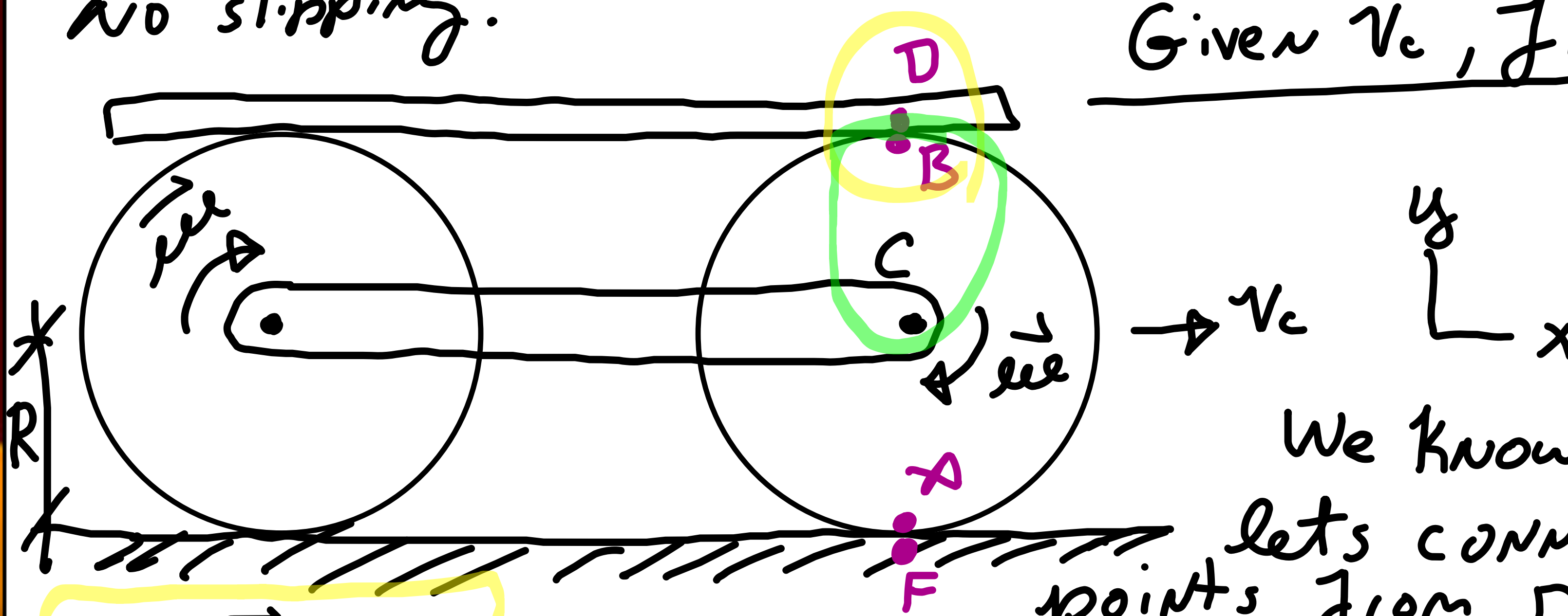
We know $\vec{v}_c$ so lets connect points from D to C

$$\boxed{\vec{v}_D = \vec{v}_{D/B} + \vec{v}_B}$$

$$\vec{v}_B = \vec{v}_{B/C} + \vec{v}_C$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



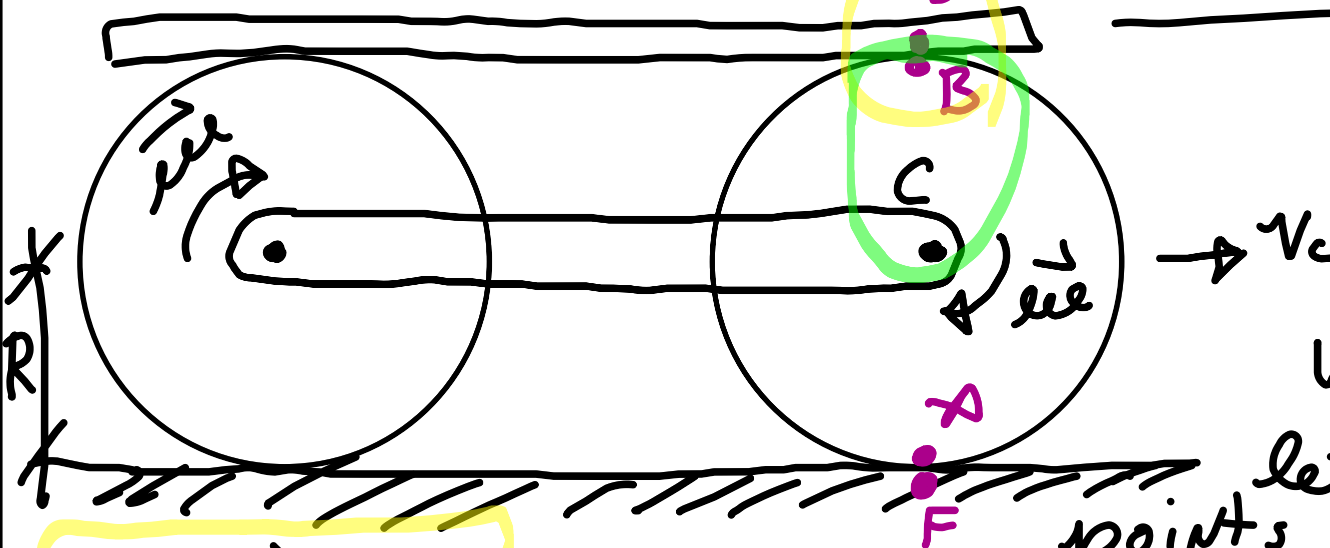
We know $\vec{v}_c$ so lets connect points from D to C

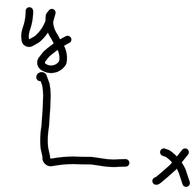
$$\boxed{\vec{v}_D = \vec{v}_{D/B} + \vec{v}_B}$$

$$\boxed{\vec{v}_B = \vec{v}_{B/c} + \vec{v}_c}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



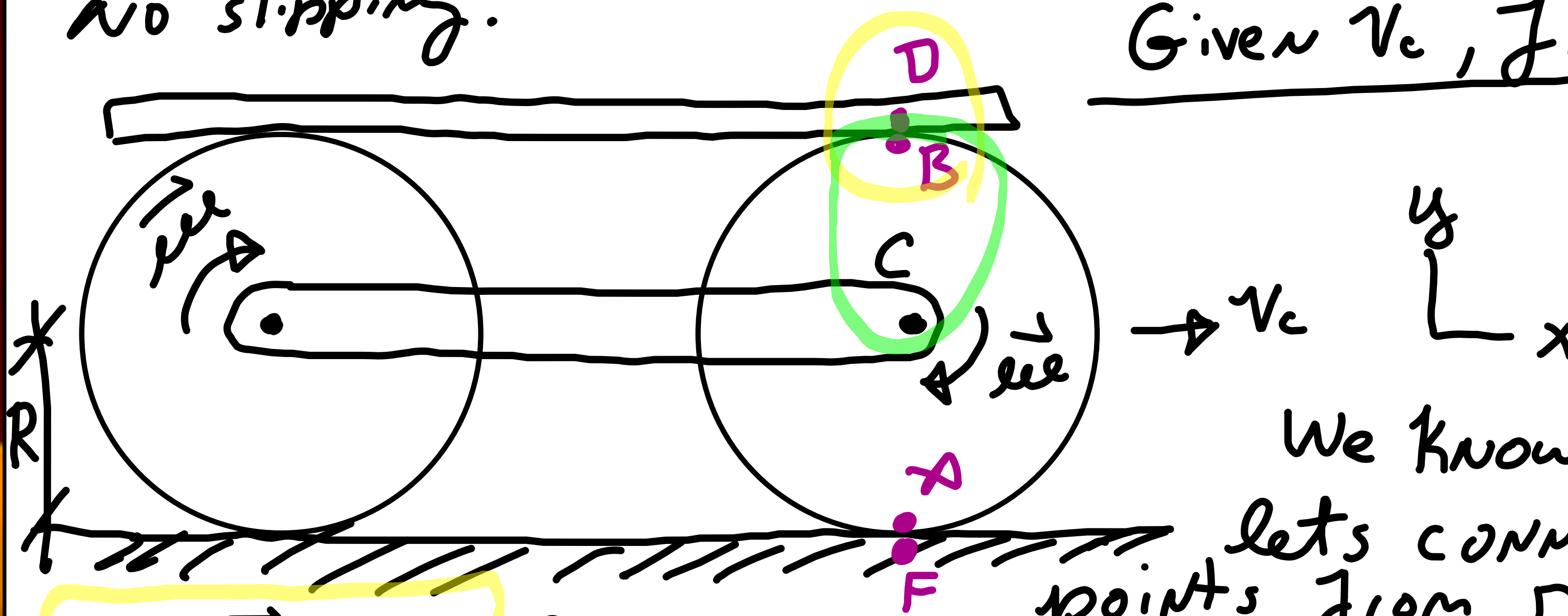


We know $\vec{v}_c$ so lets connect points from D to C

$$\left.\begin{aligned} \vec{V}_D &= \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B &= \vec{V}_{B/C} + \vec{V}_C \end{aligned}\right\} \text{But roll no slip}$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

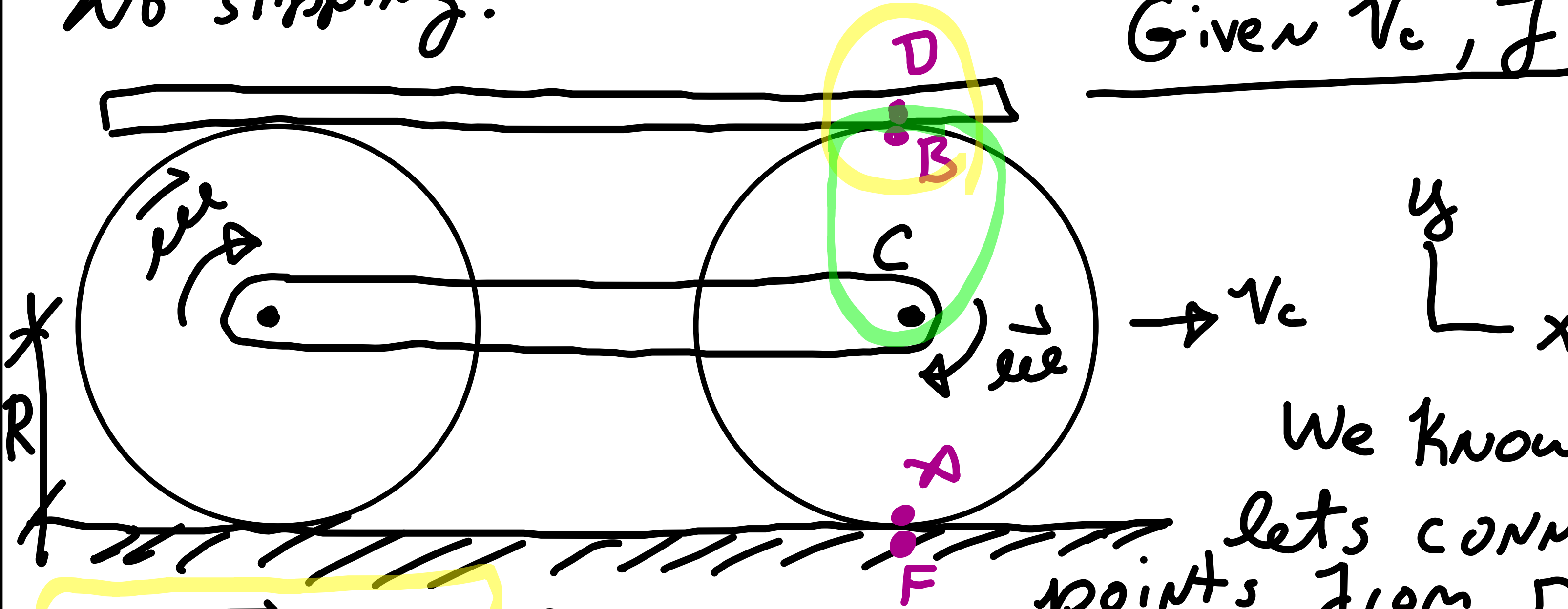
$$\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c$$

But roll no slip $\Rightarrow \vec{V}_{D/B} = \Theta$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



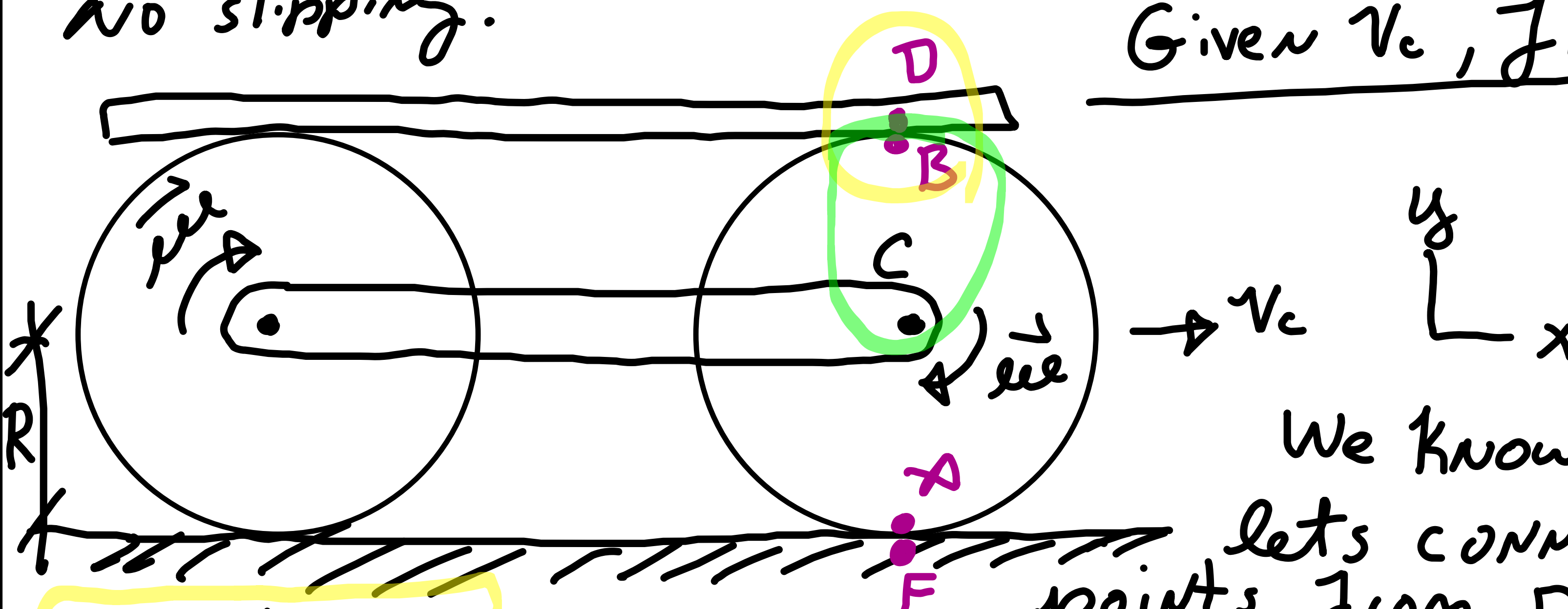
We know $\vec{v}_c$ so lets connect points from D to C

$$\left.\begin{array}{l} \boxed{\vec{v}_D = \vec{v}_{D/B} + \vec{v}_B} \\ \boxed{\vec{v}_B = \vec{v}_{B/c} + \vec{v}_c} \end{array}\right\}$$

But roll no slip $\Rightarrow \vec{v}_{D/B} = \theta$ so $\begin{cases} \vec{v}_D = \vec{v}_B \\ \vec{v}_B = \vec{v}_{B/c} + \vec{v}_c \end{cases}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

$\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B$

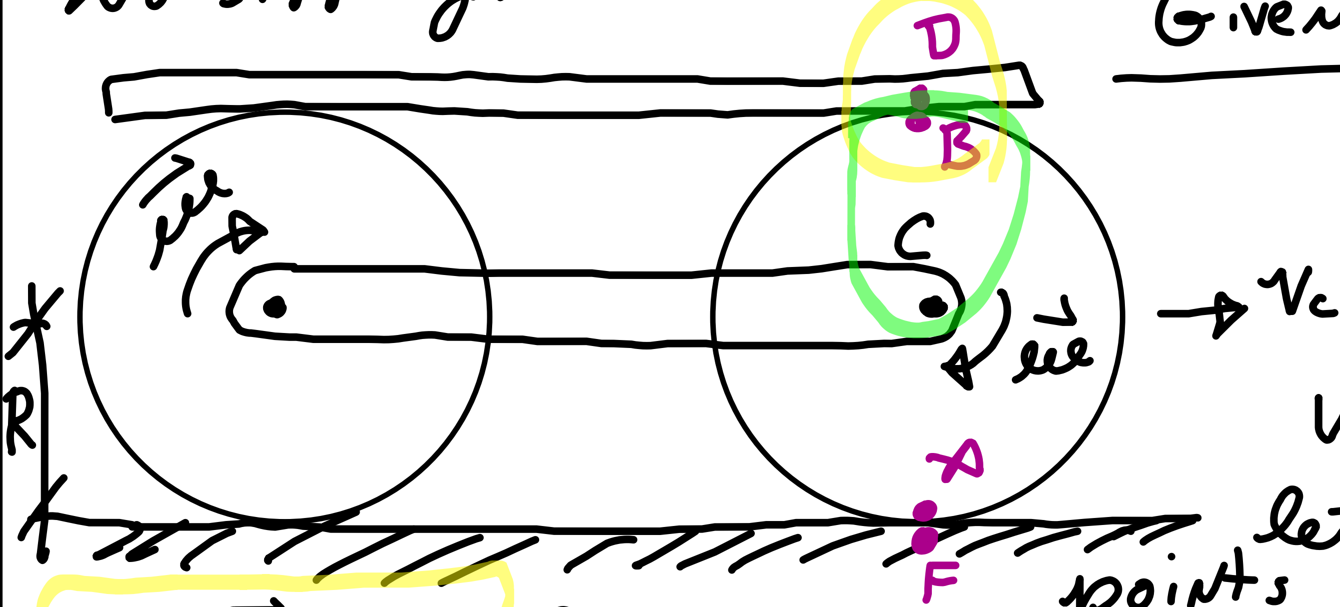
$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c$

But roll no slip $\Rightarrow \vec{V}_{D/B} = \theta$ so $\begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c \end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



y
x

We know $\vec{v}_c$ so lets connect points from D to C

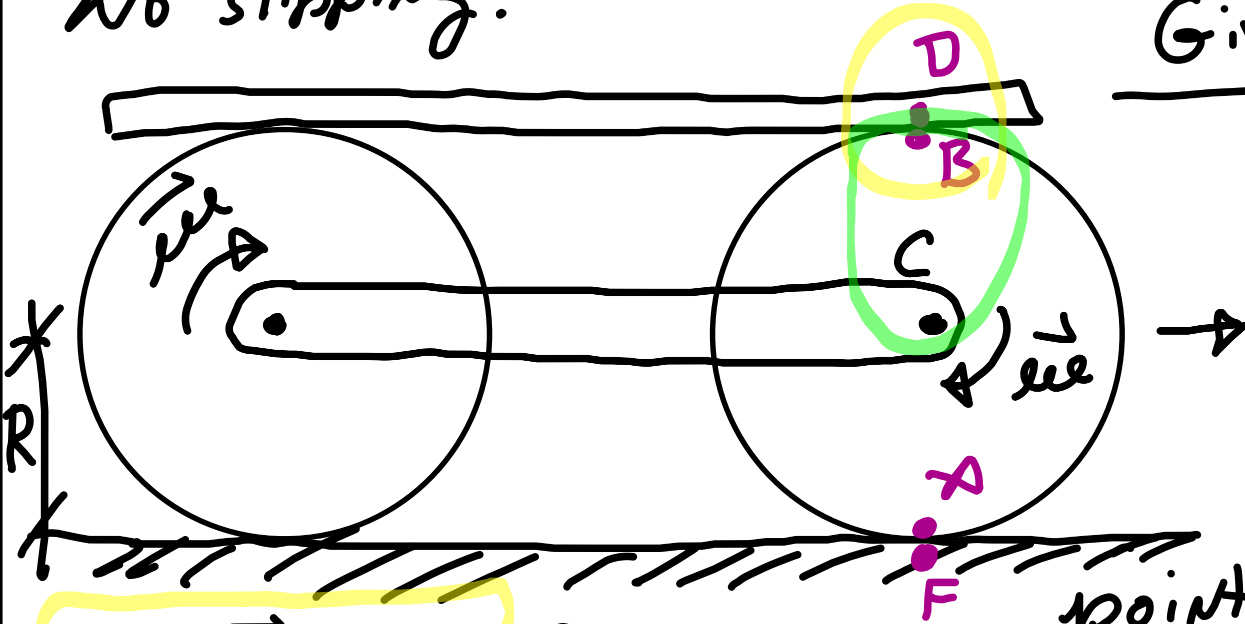
$$\left.\begin{array}{l} \boxed{\vec{v}_D = \vec{v}_{D/B} + \vec{v}_B} \\ \boxed{\vec{v}_B = \vec{v}_{B/c} + \vec{v}_c} \end{array}\right\}$$

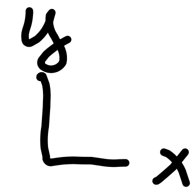
But roll no slip $\Rightarrow \vec{v}_{D/B} = 0$ so $\begin{cases} \vec{v}_D = \vec{v}_B \\ \vec{v}_B = \vec{v}_{B/c} + \vec{v}_c \end{cases}$

$\Rightarrow \vec{v}_D = \vec{v}_{B/c} + \vec{v}_c$ But $\vec{v}_c = R\omega\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_C, Find V_D





We know $\vec{V}_C$ so lets connect points from D to C

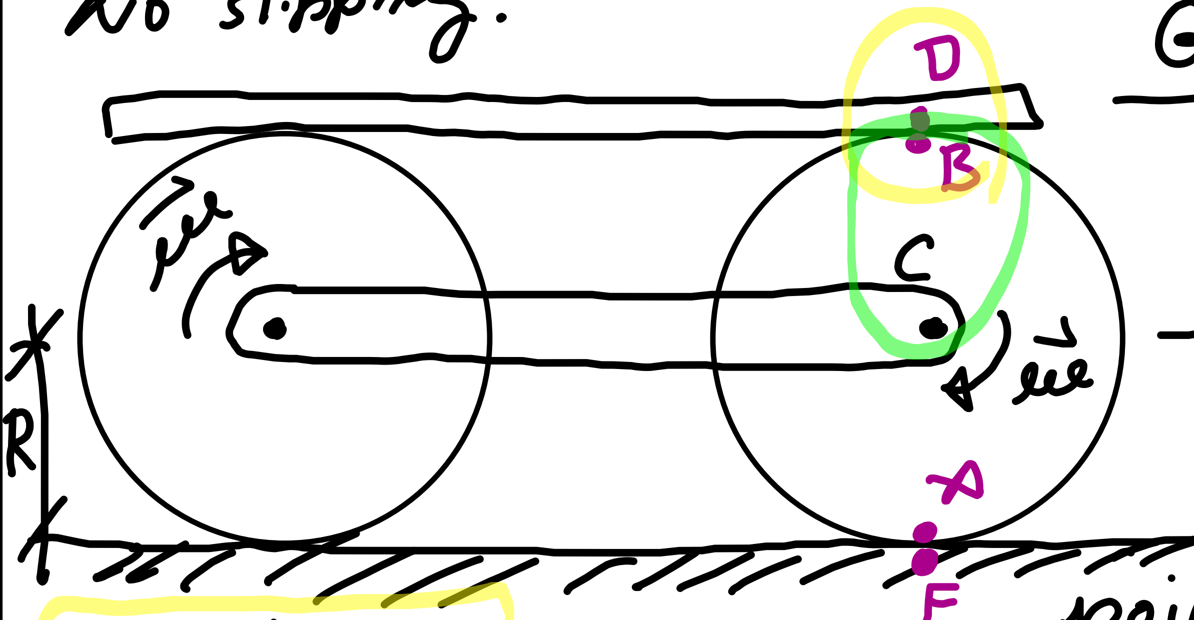
$$\left.\begin{aligned} &\boxed{\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B} \\ &\boxed{\vec{V}_B = \vec{V}_{B/C} + \vec{V}_C} \end{aligned}\right\}$$

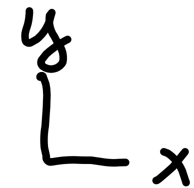
But roll no slip $\Rightarrow \vec{V}_{D/B} = 0$ so $\begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/C} + \vec{V}_C \end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/C} + \vec{V}_C$ But $\vec{V}_C = R\omega\,\hat{x}$ & $\vec{V}_{B/C} = R\omega\,\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D





We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/B} + V_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right\}$$

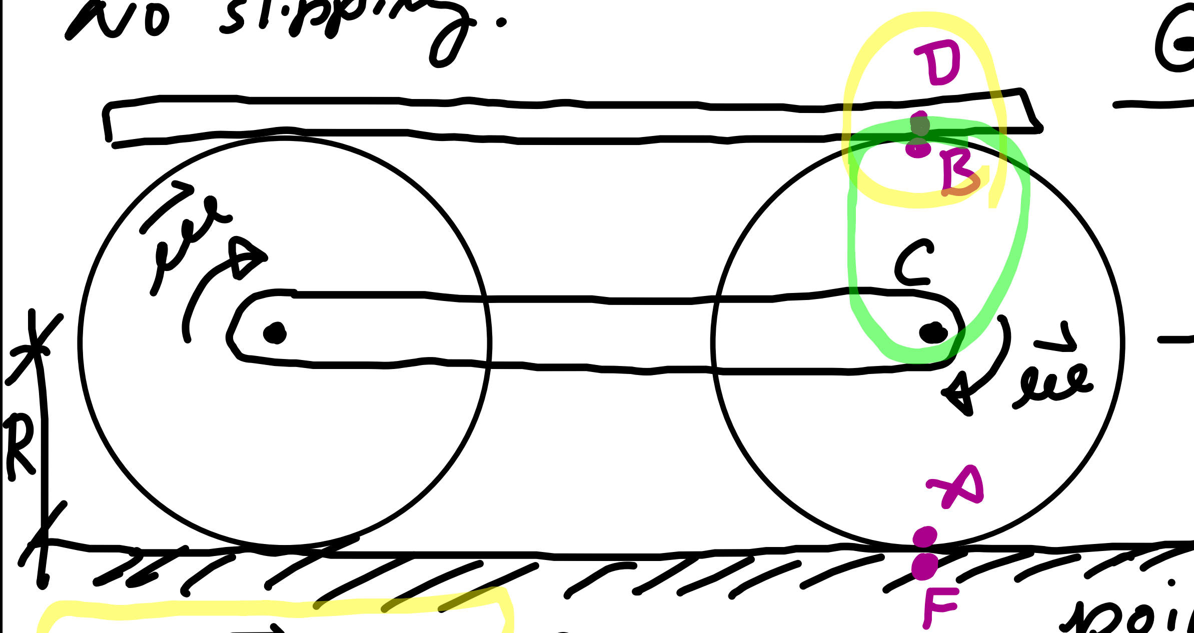
But roll no slip $\Rightarrow \vec{V}_{D/B} = \theta$ so $\begin{cases}\vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, Find V_D



We know $\vec{V}_c$ so lets connect points from D to C

$\vec{V}_D = \vec{V}_{D/B} + V_B$

$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c$

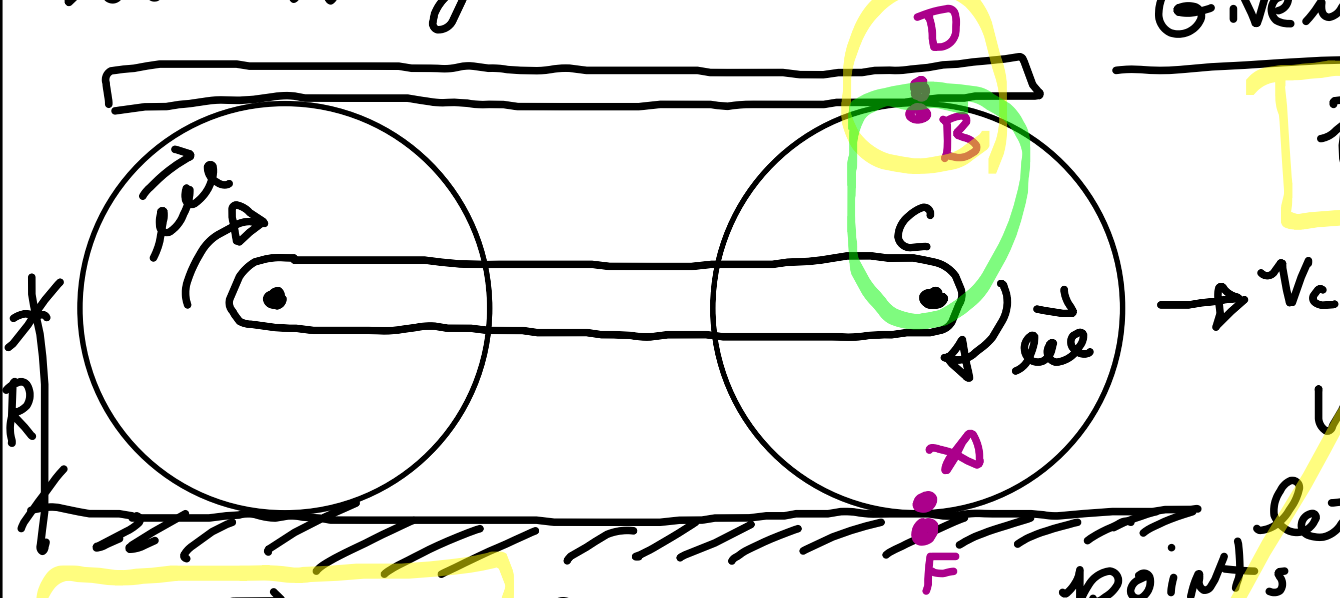
But roll no slip $\Rightarrow \vec{V}_{D/B} = \theta$ so $\begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c \end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$ or $\vec{V}_D = 2\vec{V}_c$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

We know $\vec{V}_c$ so lets connect points from D to C

$$\boxed{\vec{V}_D = \vec{V}_{D/B} + V_B}$$

$$\boxed{\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c}$$

But roll no slip $\Rightarrow \vec{V}_{D/B} = \theta$ so $\begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c \end{cases}$

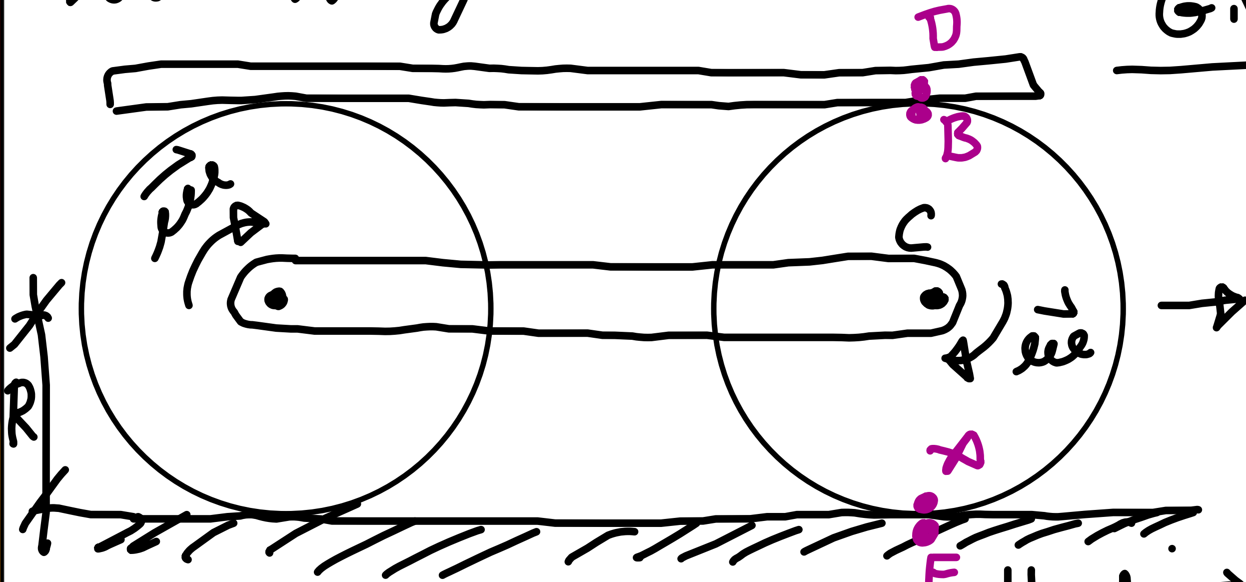
$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$ or $\boxed{\vec{V}_D = 2\vec{V}_c}$

ASU

Example: Board sitting on wheels. Rolling no slipping.

Given v_C, find v_D



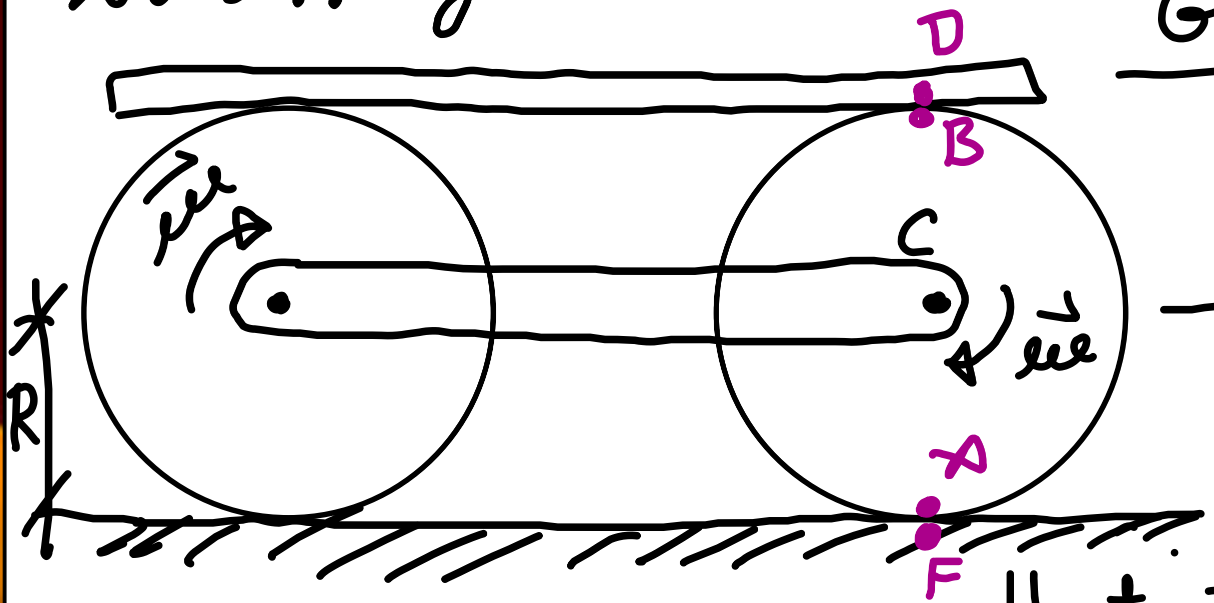
$$\boxed{\vec{V}_D = 2\vec{V}_C}$$

Another way is to notice that $\vec{\omega}$ is independent of the reference point

Example: Board sitting on wheels. Rolling no slipping.

Given v_C, find v_D

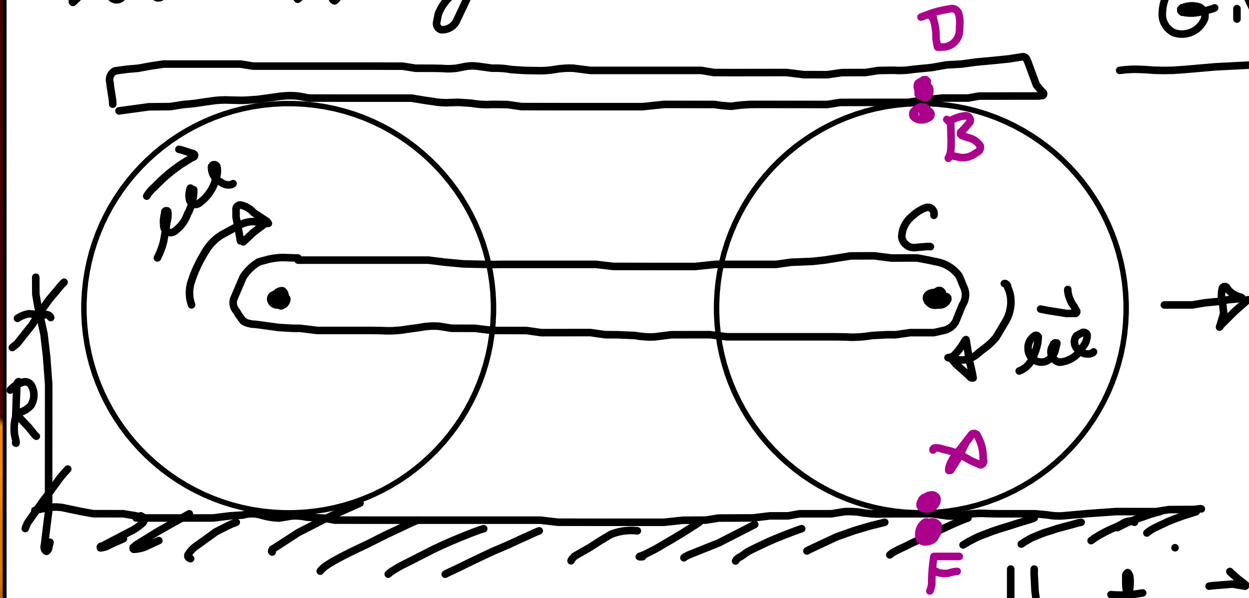
$$\boxed{\vec{V}_D = 2\vec{V}_C}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

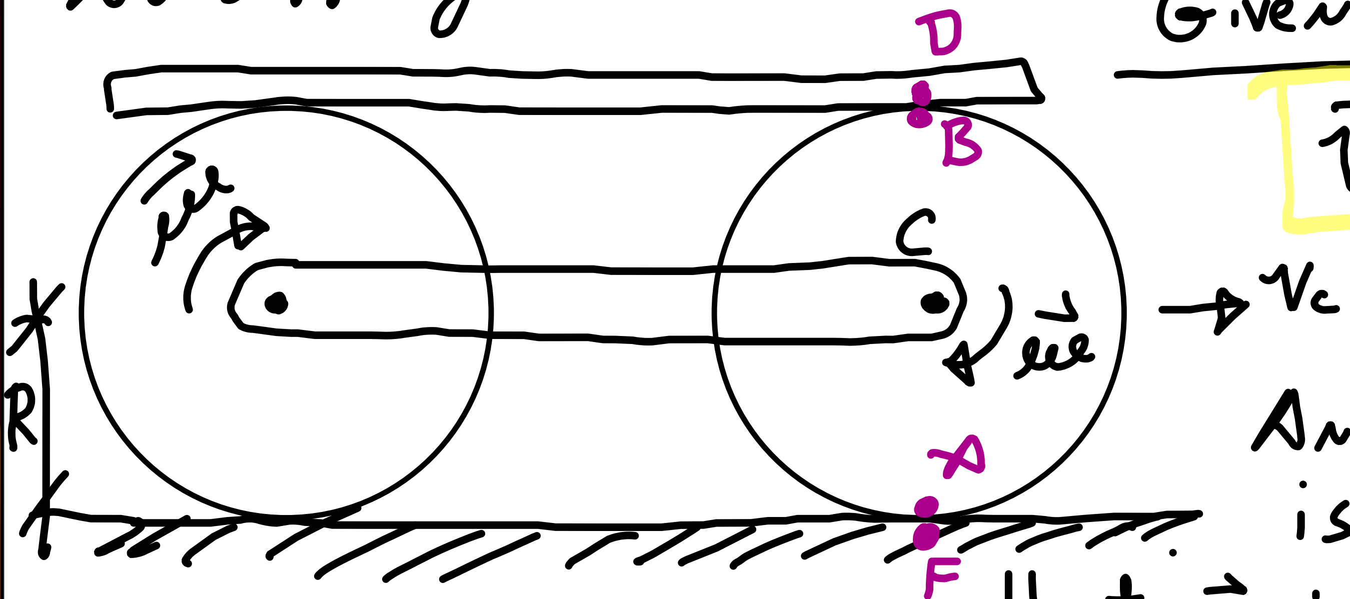
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



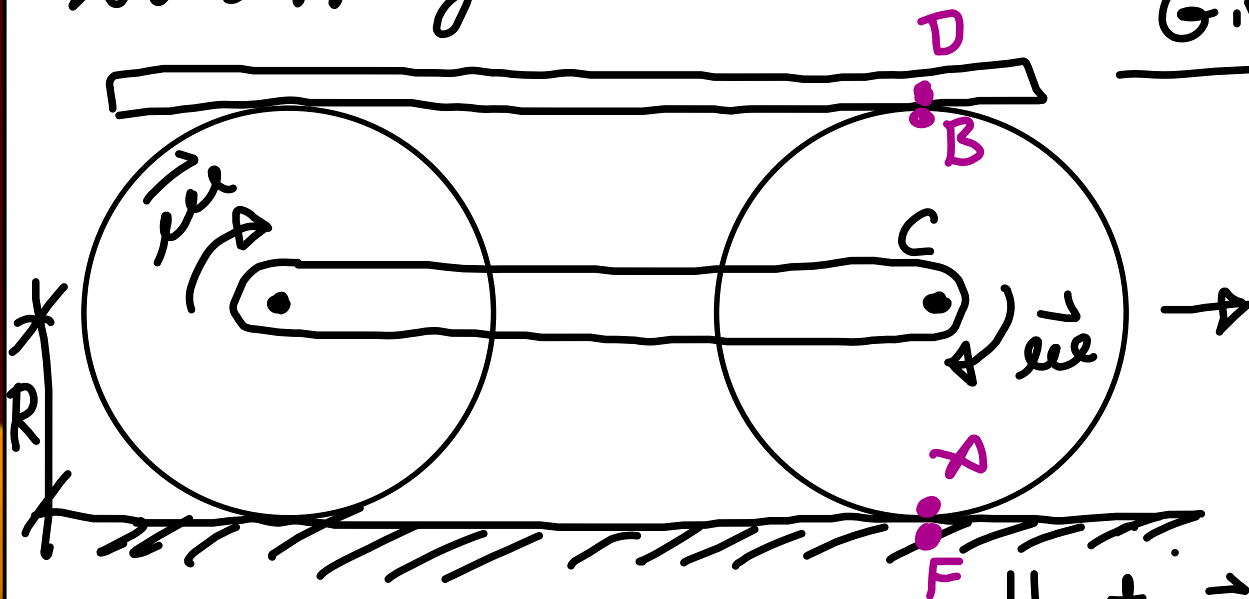
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

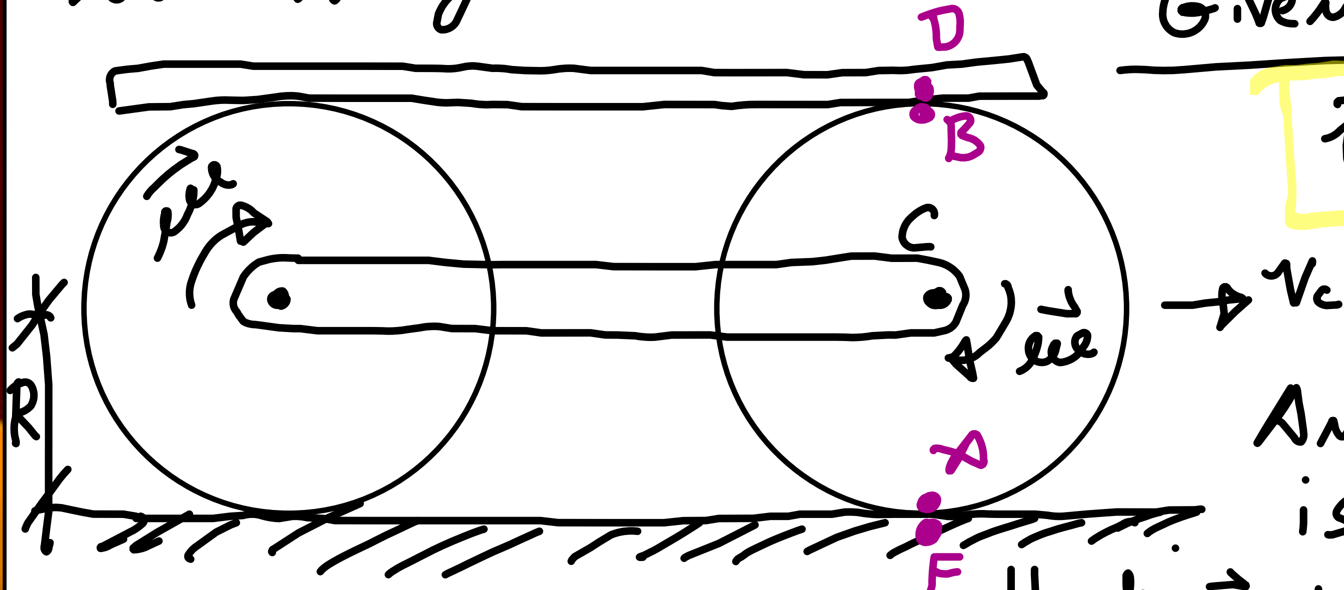


Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



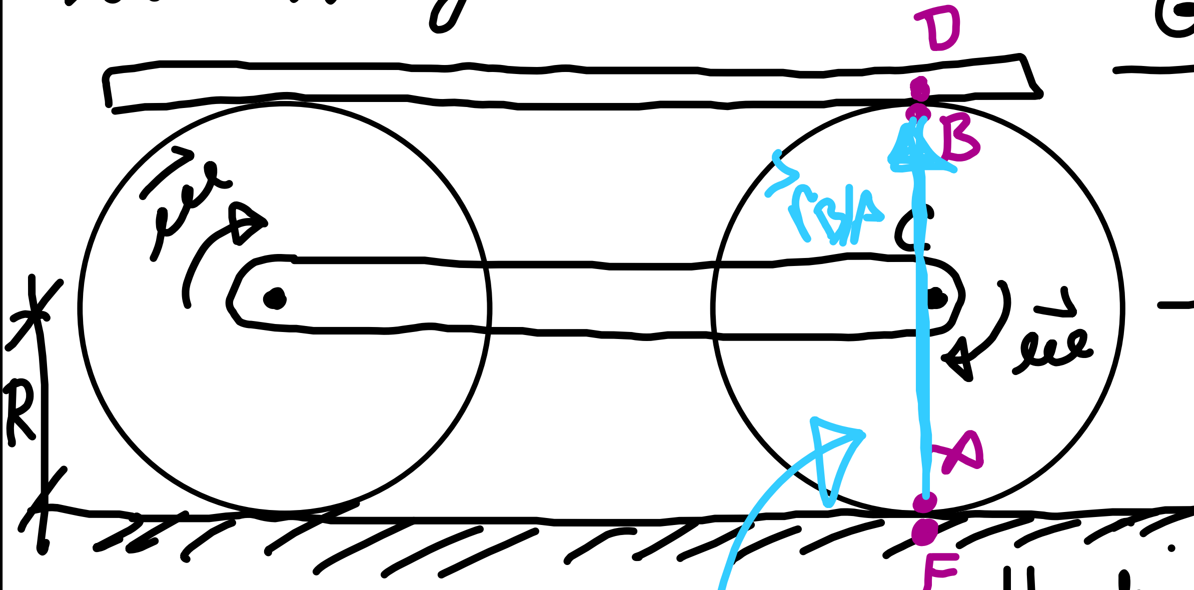
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

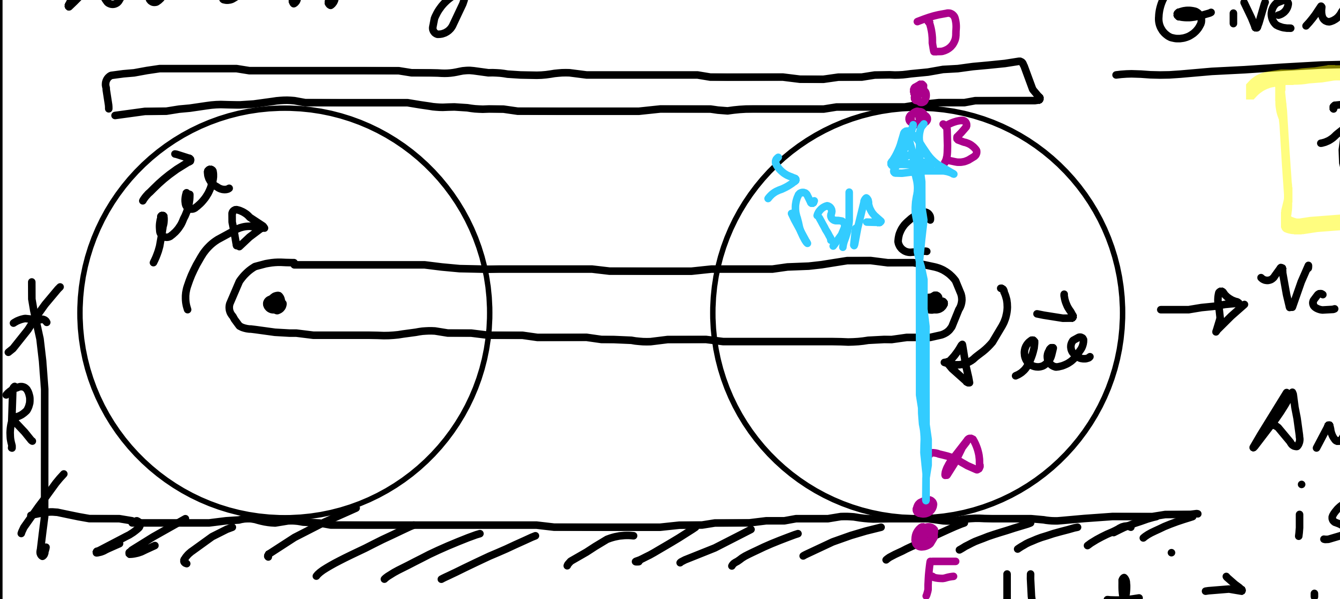
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = \vec{0}$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



$$\boxed{\vec V_D = 2\vec V_c}$$

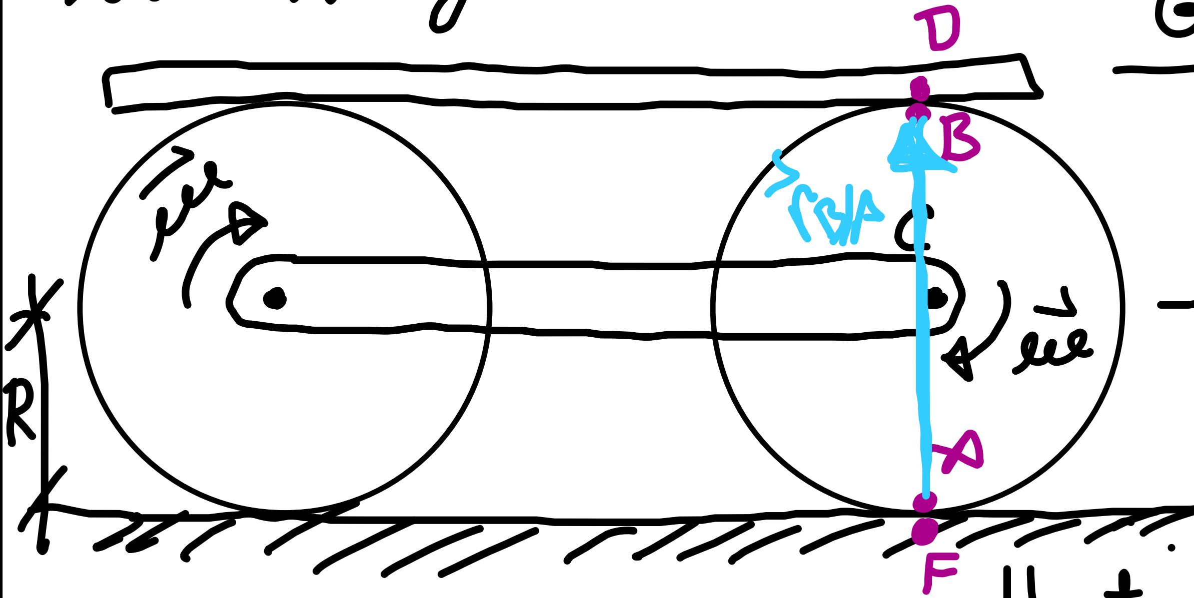
Another way is to notice that $\vec\omega$ is independent of the reference point & take $\vec V_B = \vec V_{B/A} + \vec V_A$

& since roll no slip $\vec v_A = \vec V_F = 0$ so $\vec V_B = \vec V_{B/A}$

$$\Rightarrow \vec V_B = \vec V_{B/A} = \vec\omega \times \vec r_{B/A} = \vec\omega \times 2R\hat y$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

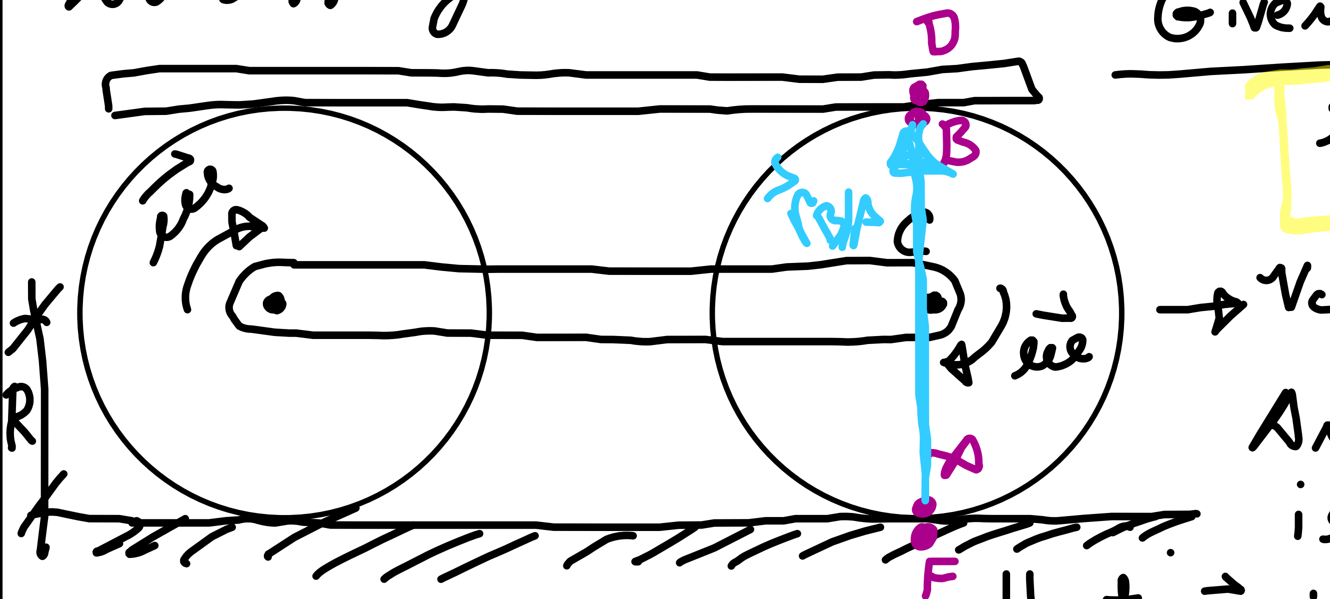
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



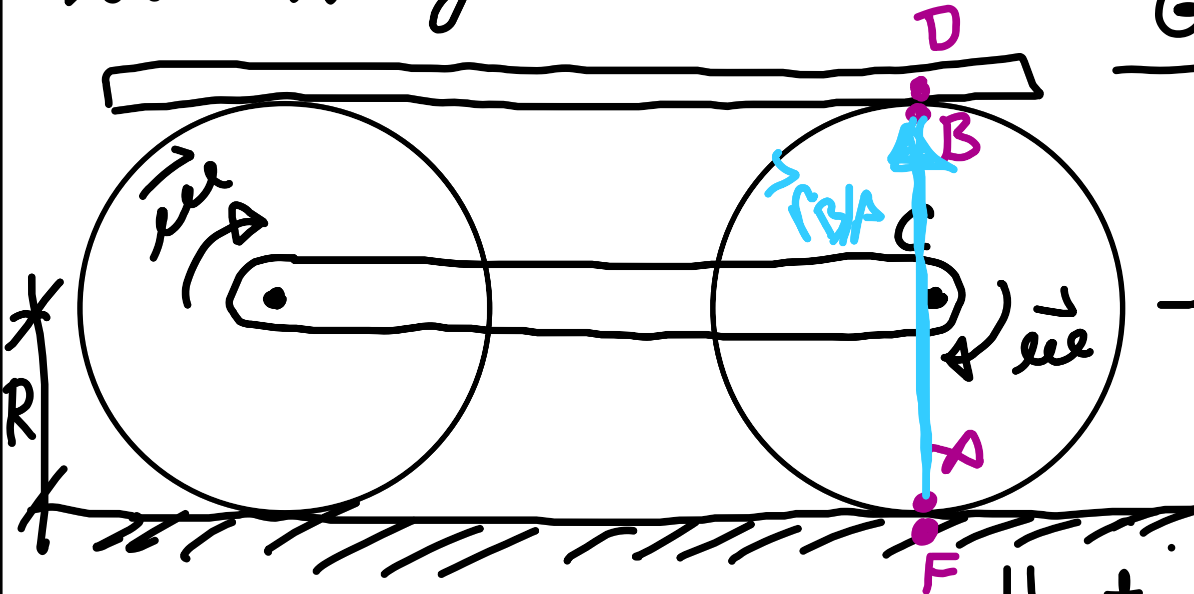
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ So $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$

Example: Board sitting on wheels. Rolling no slipping.



Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

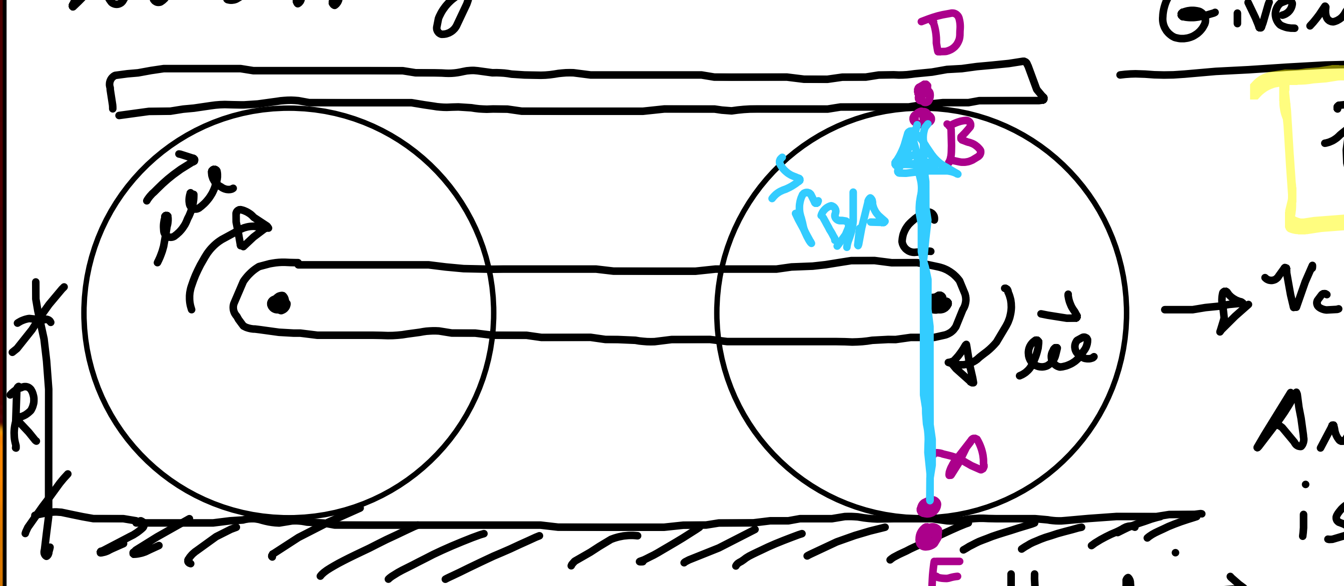
$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$ & since roll no slip

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

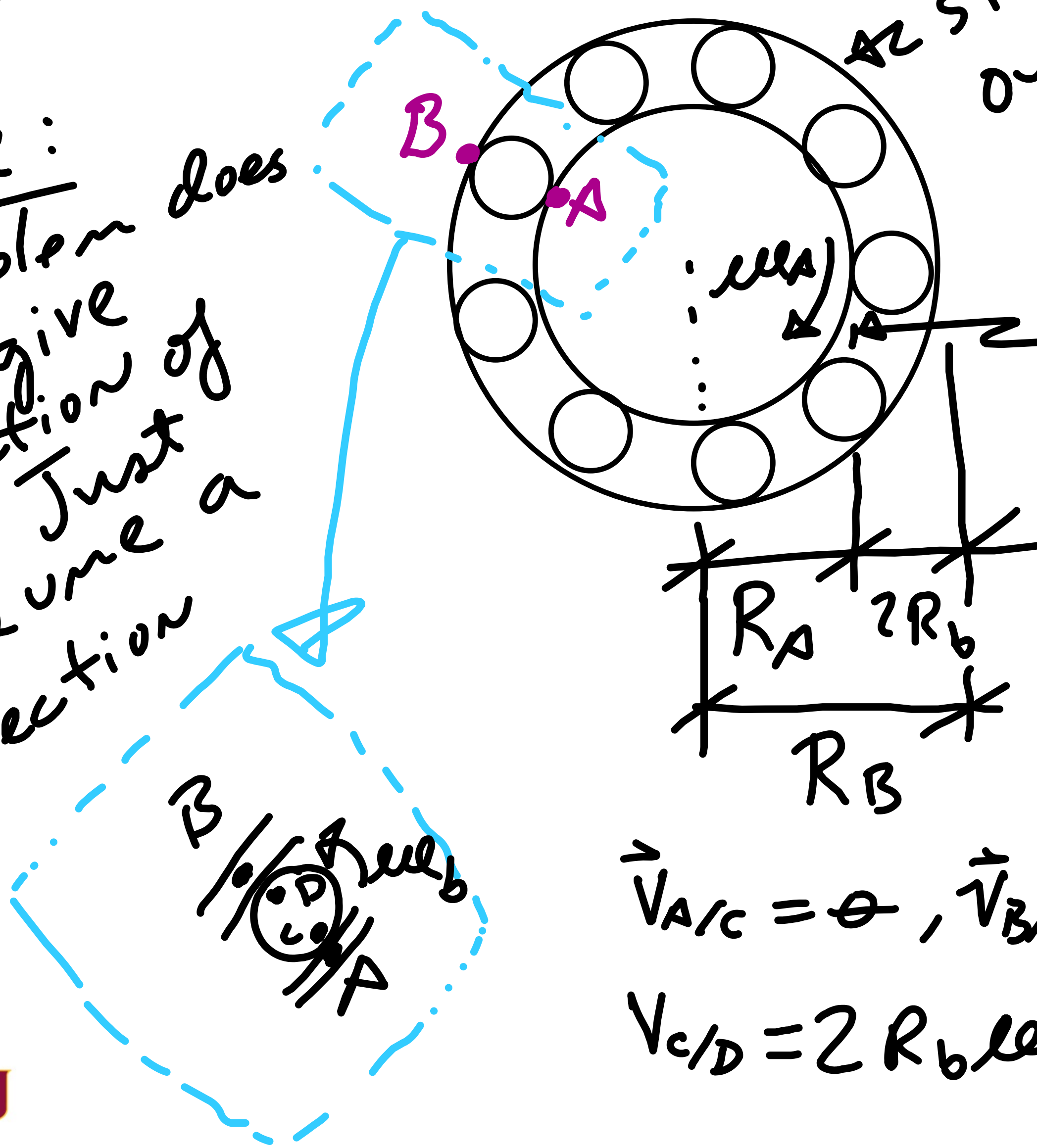
$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$ & since roll no slip

$$\vec{V}_D = \vec{V}_B = 2\vec{V}_c$$

$\boxed{15.51}$

Note: problem does not give direction of ω_A. Just assume a direction



$\vec{V}_{A/C} = 0$, $\vec{V}_{B/D} = 0$ → Assuming roll no slip

$V_{C/D} = 2R_b\omega_b$