Today 15.2 & 15.3

$\boxed{L21}$

Today 15.2 & 15.3

$\boxed{L21}$

velocity of rigid body

Today 15.2 & 15.3

L21

Instantaneous Center of rotation

Today 15.2 & 15.3

Friday 15.3 & 15.4

$\boxed{L21}$

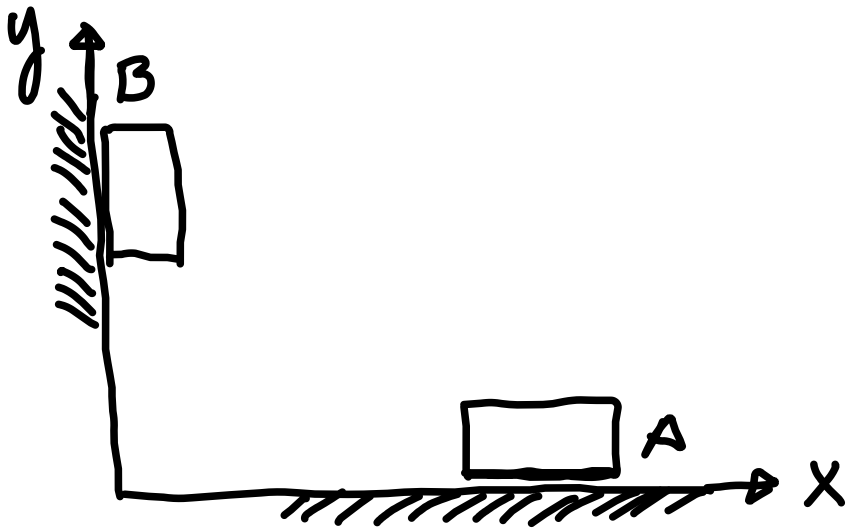
Today 15.2 & 15.3

Friday 15.3 & 15.4

L21

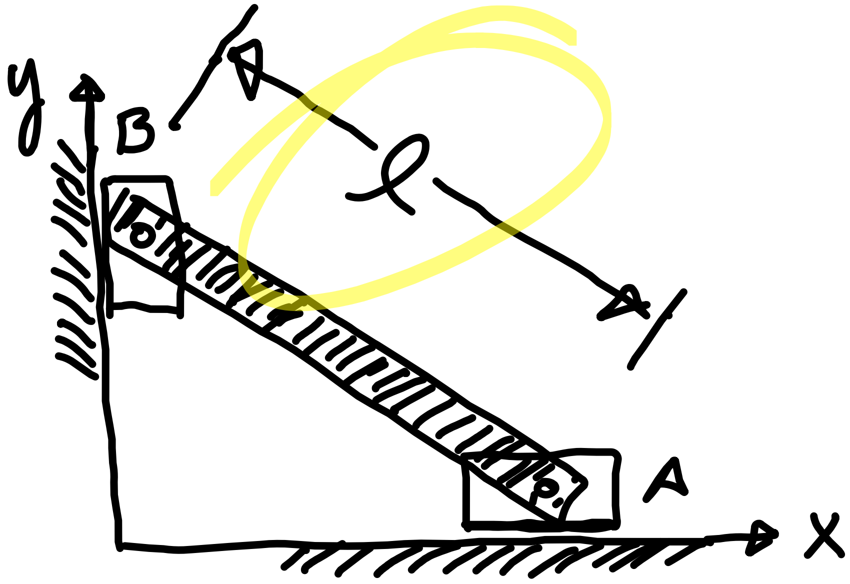
Rigid body acceleration

Example

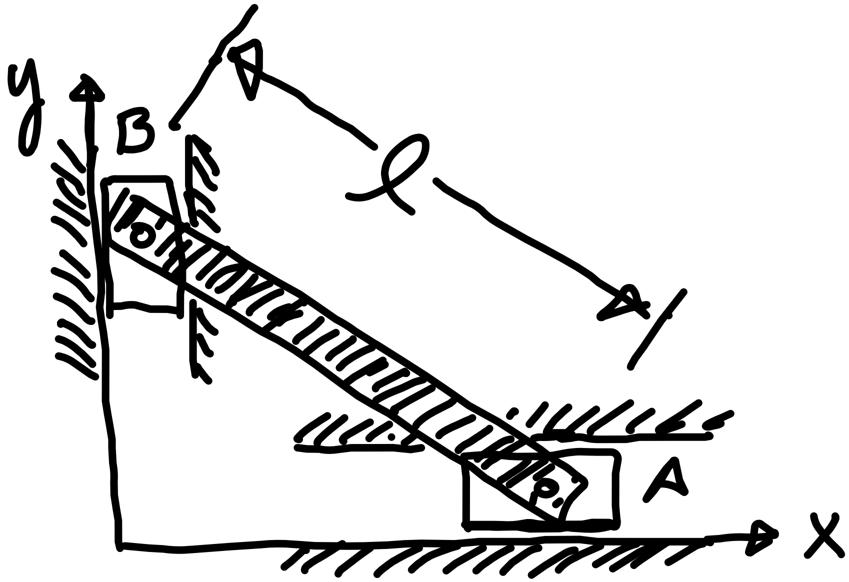


Example

Given l



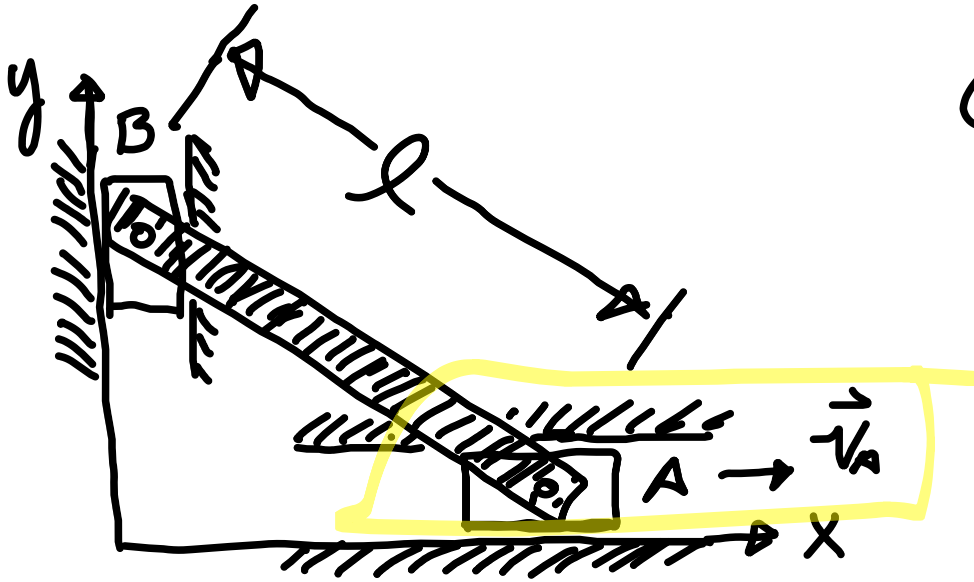
Example



Given l

Example

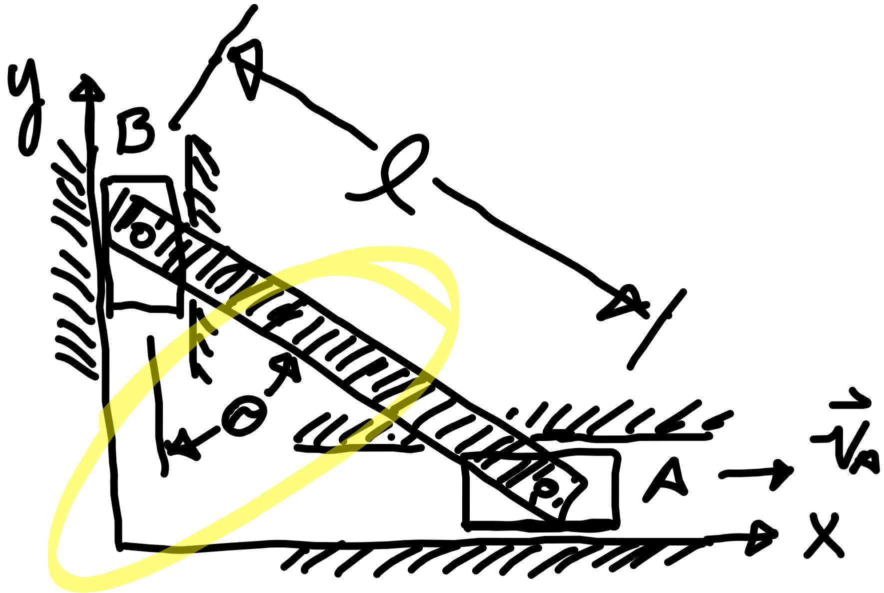
Given l, $\vec{V}_A = V_A \hat{x}$



Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

θ

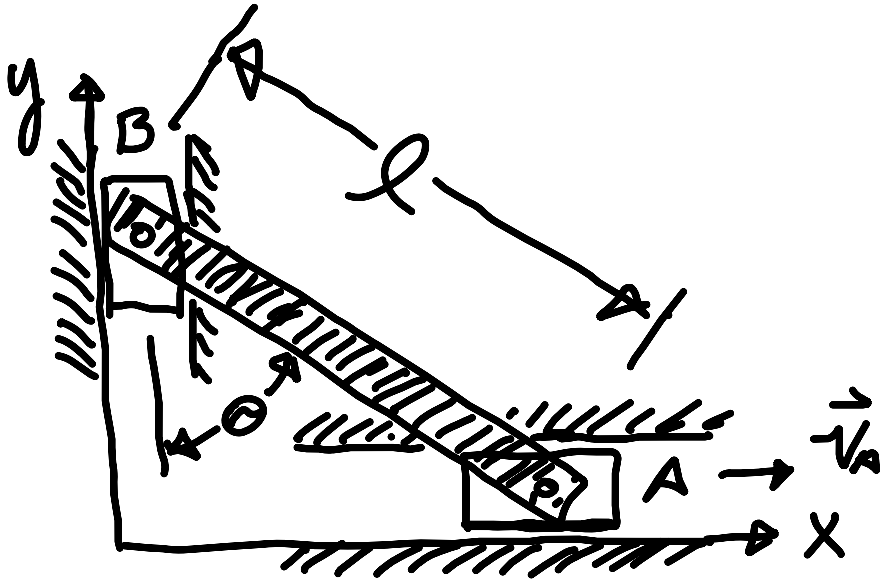


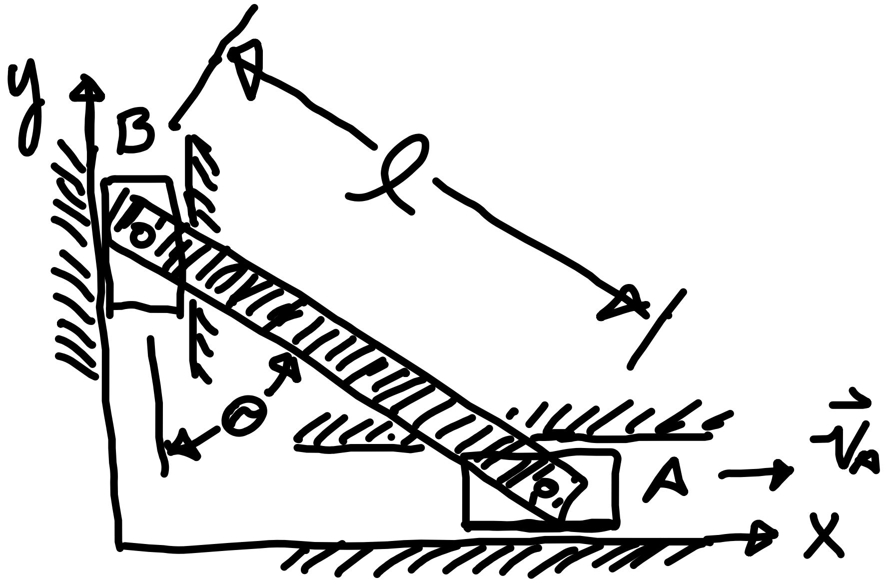
Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

θ

Find $\vec{V}_B$

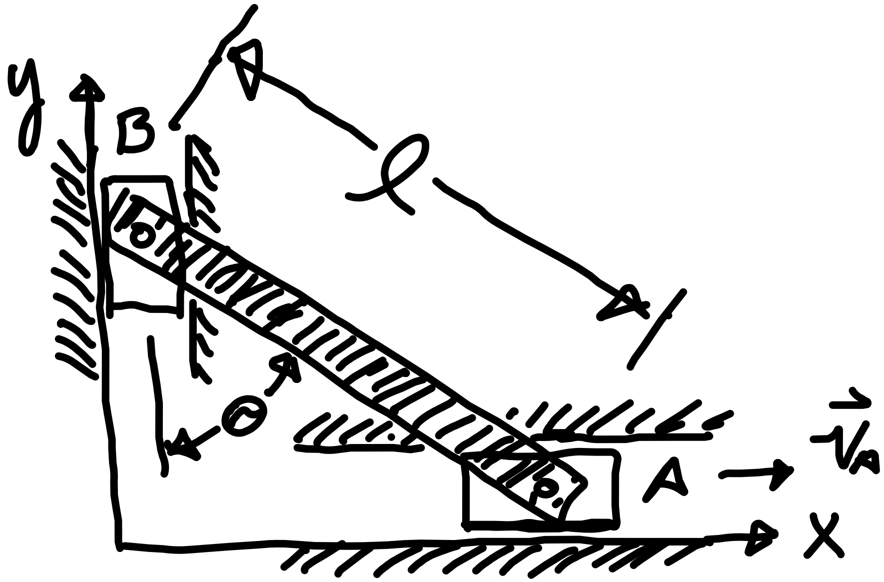




Example

Given ℓ, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

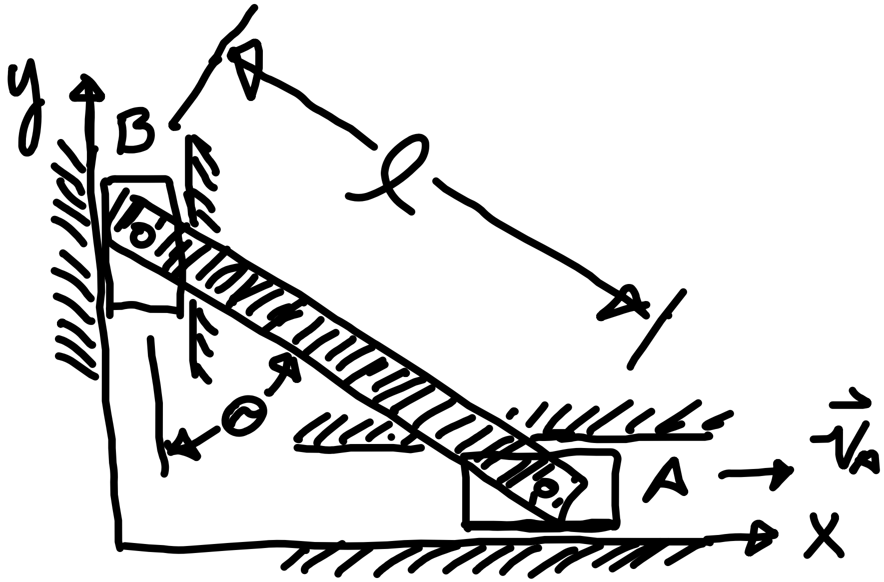


Example

Given l, $\vec{V}_A = V_A \hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$



Example

Given ℓ, $\vec{V}_A = V_A \hat{x}$ &
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Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
$$= \vec{\omega} \times \vec{r}_{B/A} + V_A \hat{x}$$

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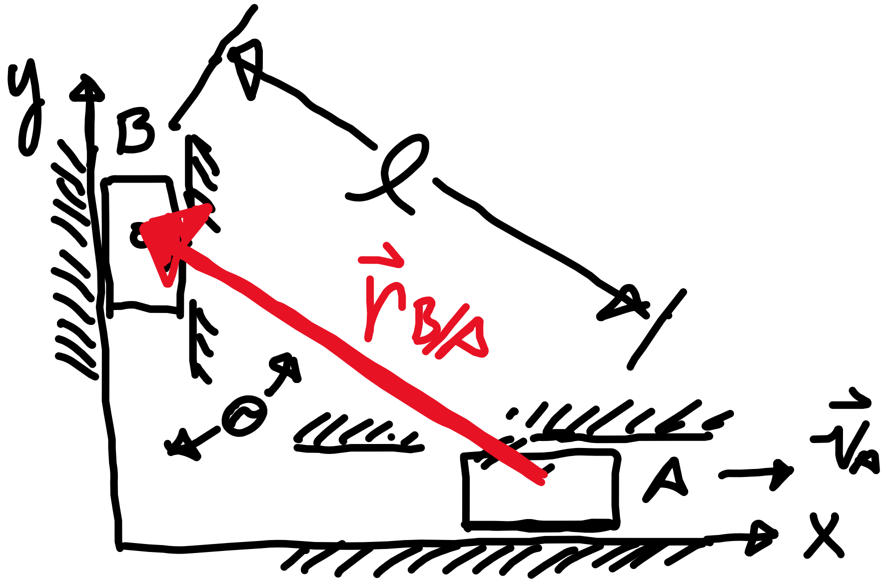
Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

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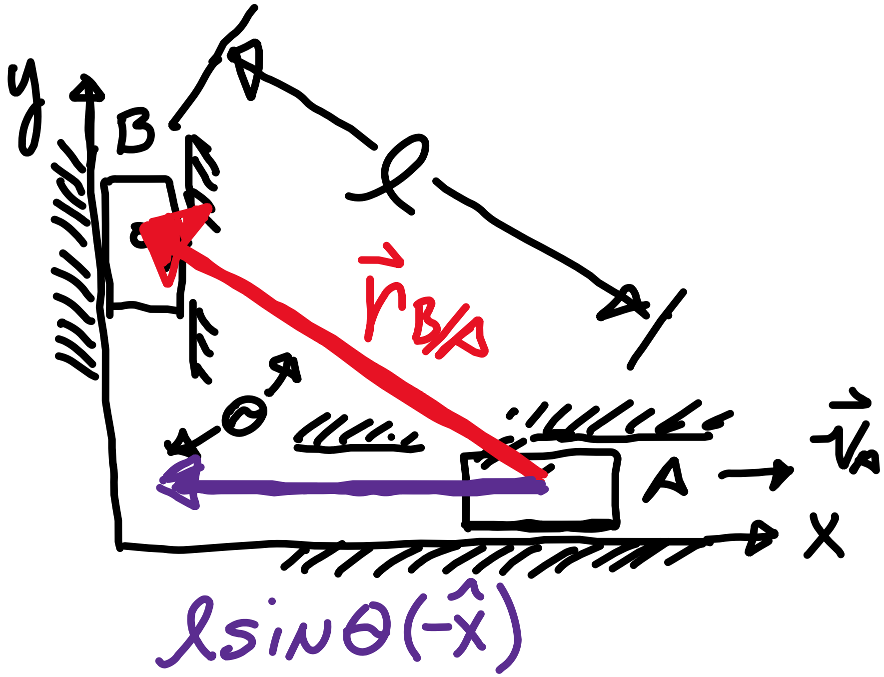

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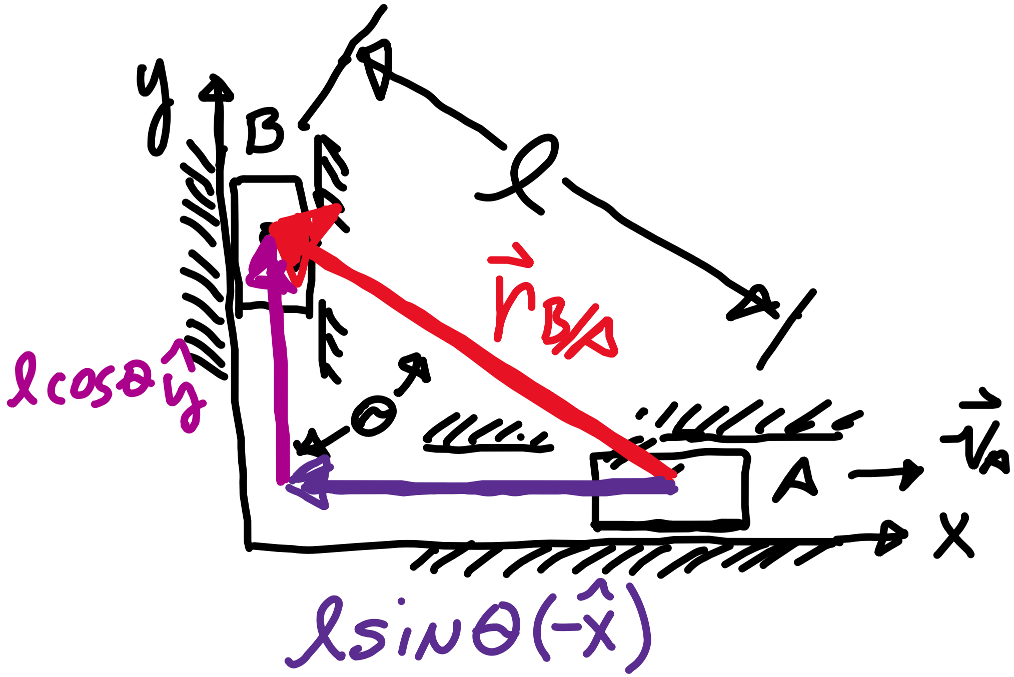

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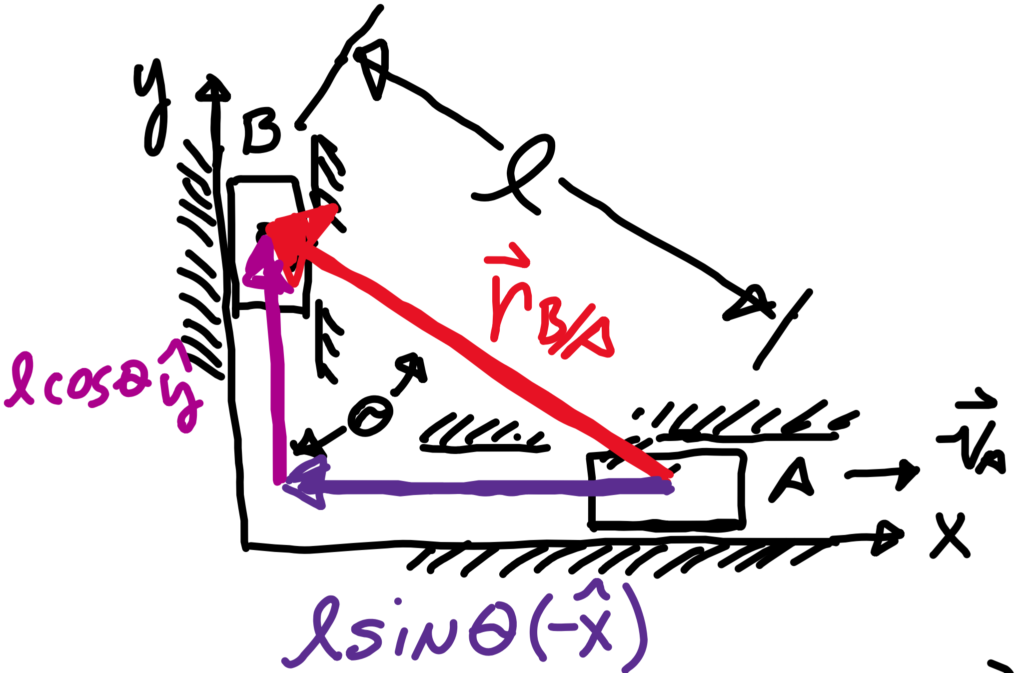

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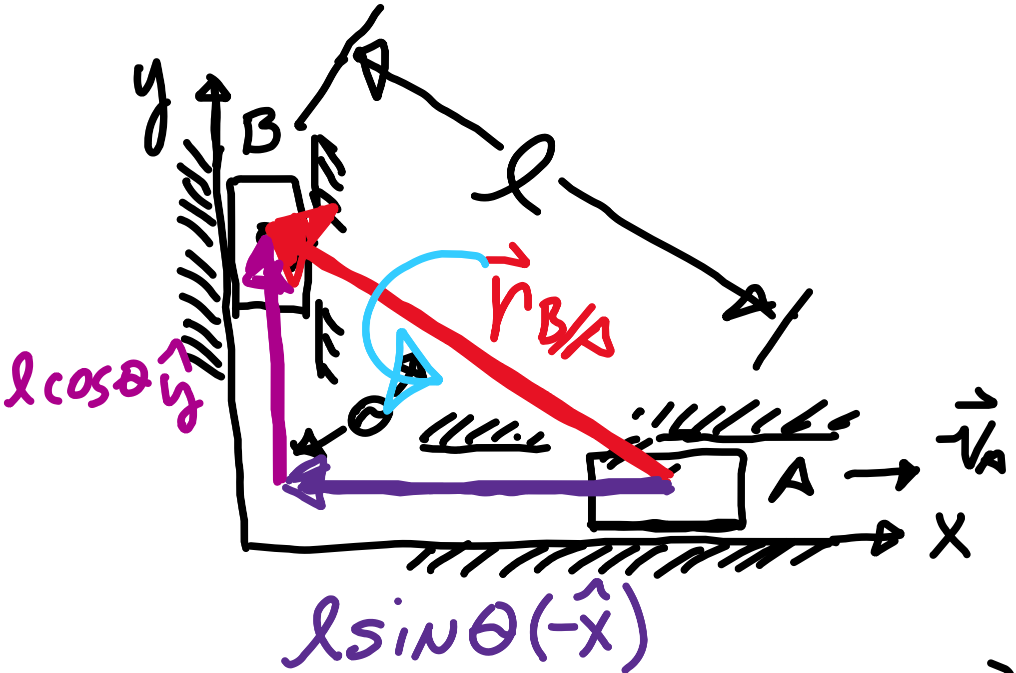
$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega} \times \vec{r}_{B/A} + V_A\hat{x}$$

$$\Rightarrow \vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$$



Example

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θ

Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

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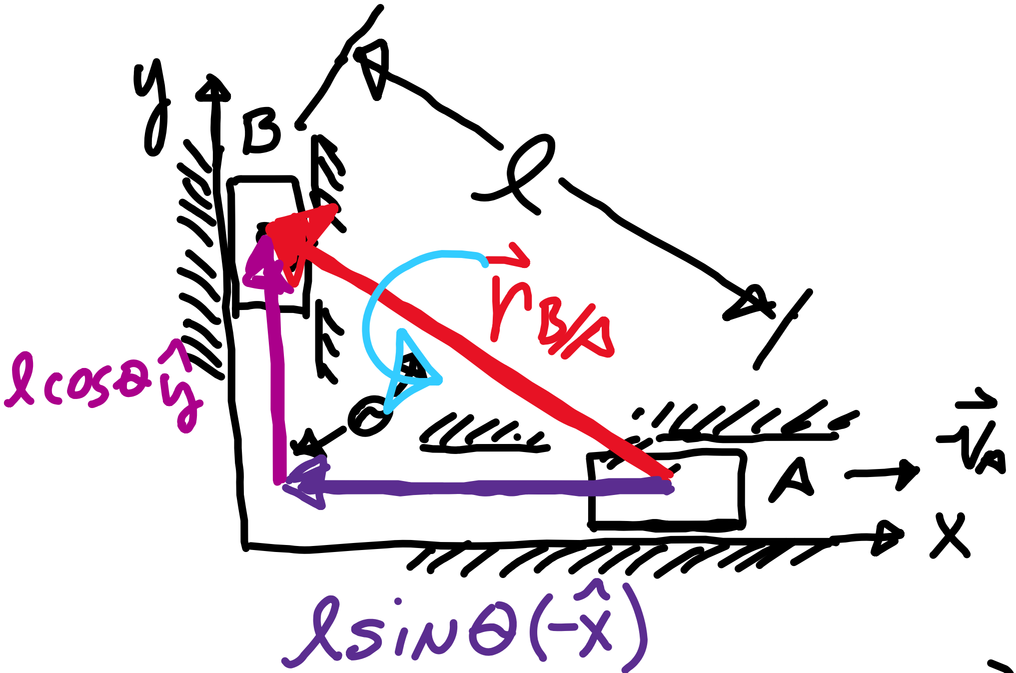
$\Rightarrow \vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$



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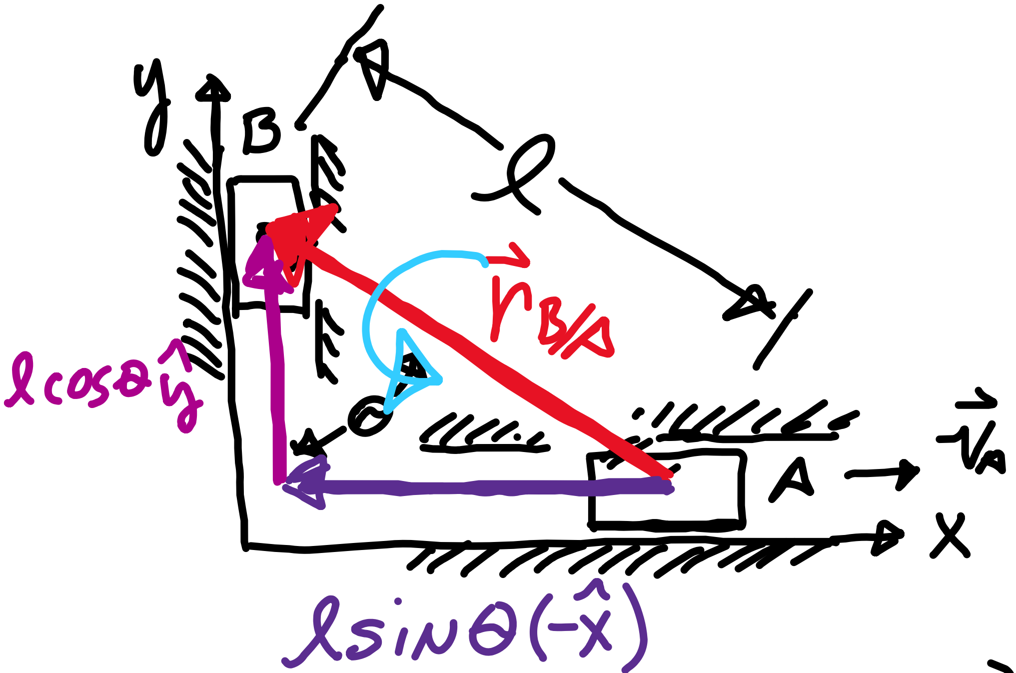
$\Rightarrow$ $\vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega\hat{z}\times\ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}] + V_A\hat{x}$



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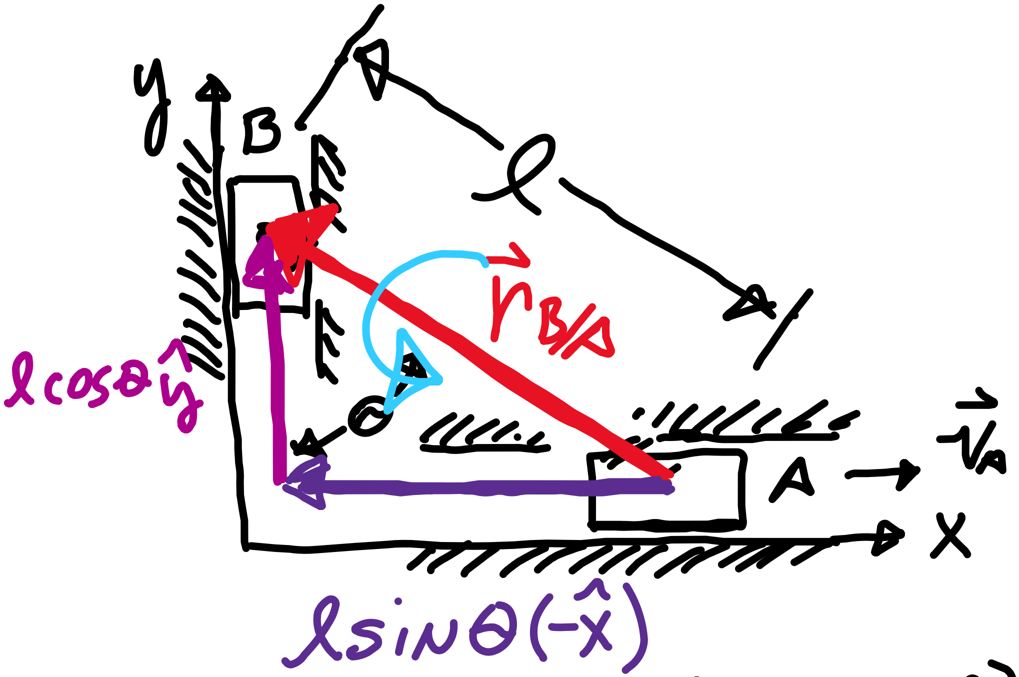
$\Rightarrow$ $\vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega\ell\,[-\sin\theta(\hat{z}\times\hat{x}) + \cos\theta(\hat{z}\times\hat{y})] + V_A\hat{x}$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
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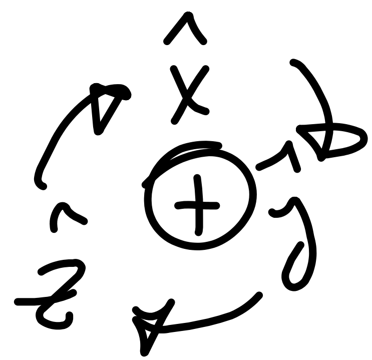
so $\vec{V}_B = \omega l\,[-\sin\theta(\hat{z}\times\hat{x}) + \cos\theta(\hat{z}\times\hat{y})] + V_A\hat{x}$

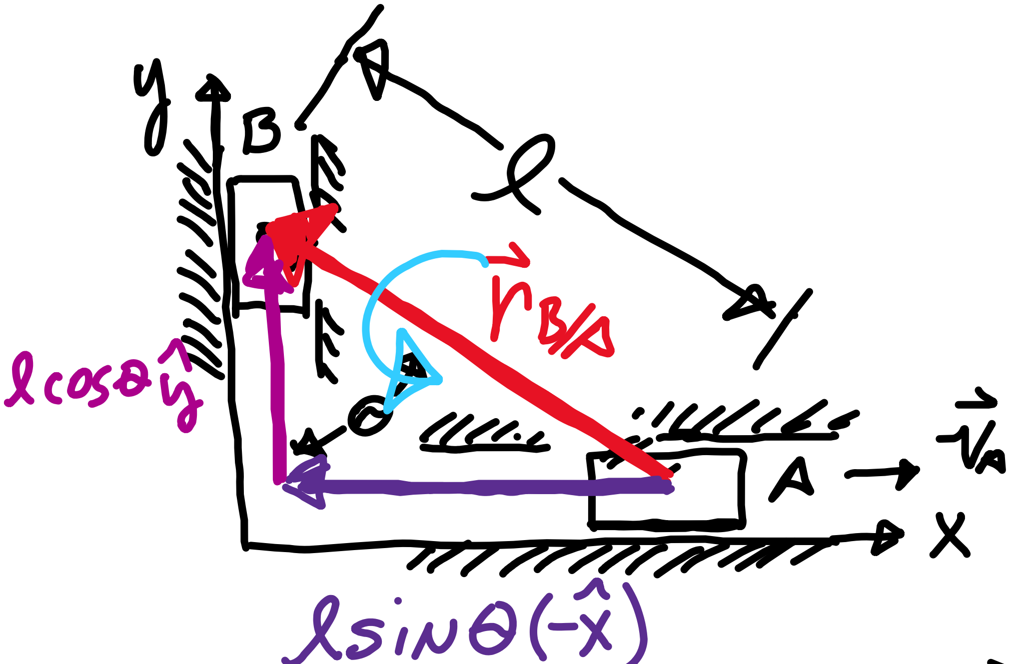


Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

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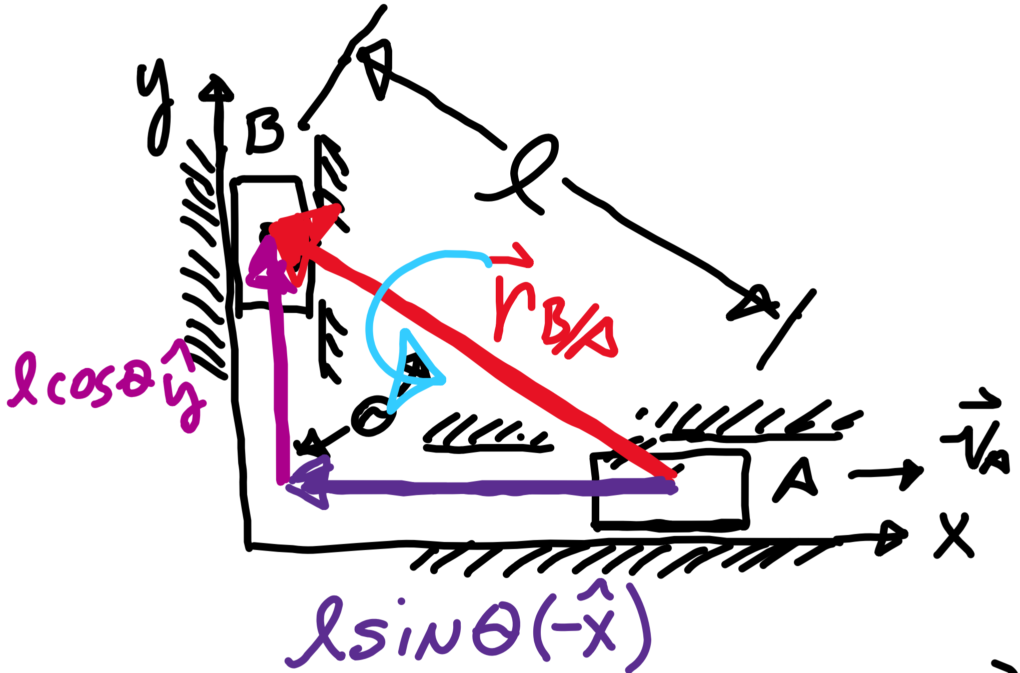
so $\vec{V}_B = \omega l[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + V_A \hat{x}$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



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$$= \vec{\omega}\times\vec{r}_{B/A} + V_A\hat{x}$$

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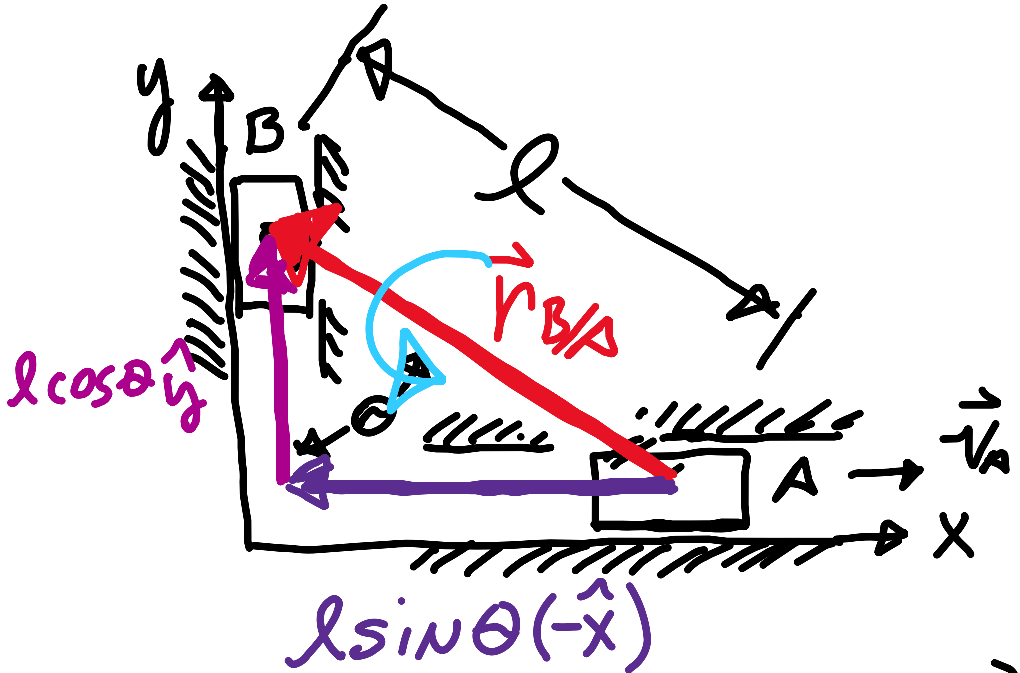
so $\vec{V}_B = \omega l[-\sin\theta\,\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

2 unknowns but only single eqn.

ASU



Example

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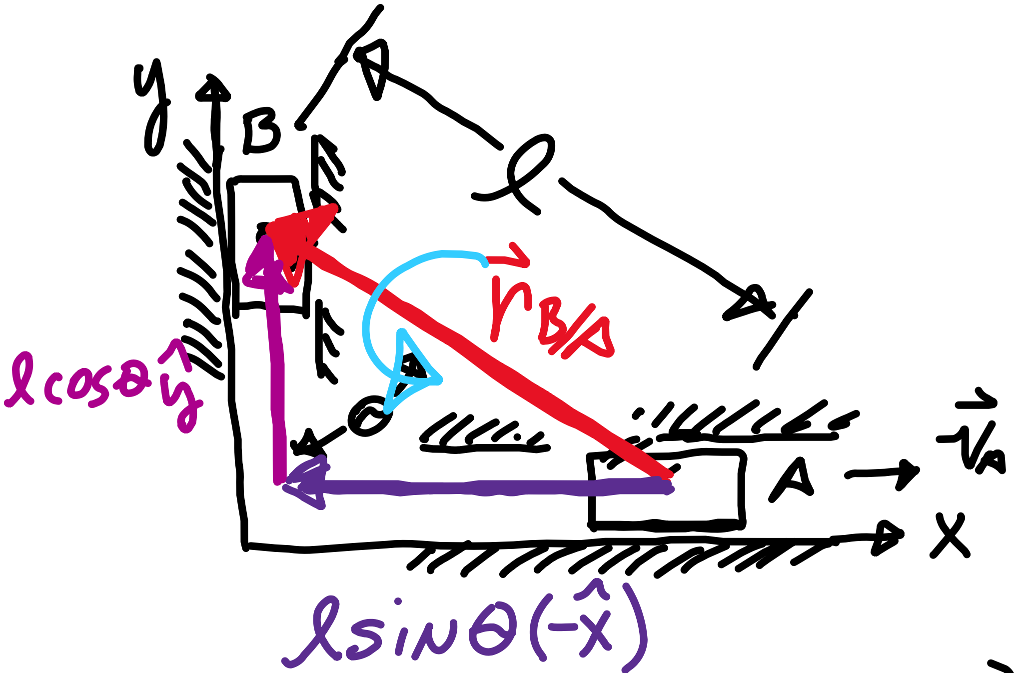
2 unknowns but only single eqn.

Need another equation!

Example

Given ℓ, $\vec{V}_A = V_A \hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
$$= \vec{\omega} \times \vec{r}_{B/A} + V_A\hat{x}$$

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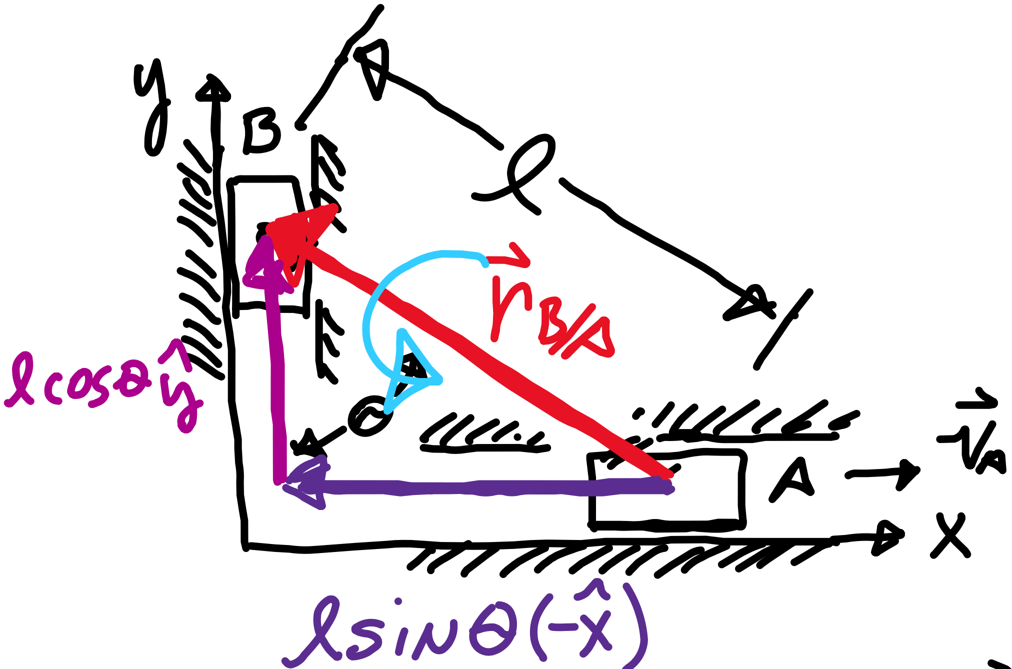
so $\vec{V}_B = \omega\ell[-\sin\theta\,\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

2 unknowns but only single eqn.
Need another equation!

Constraint: B can only move in y-direction



Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

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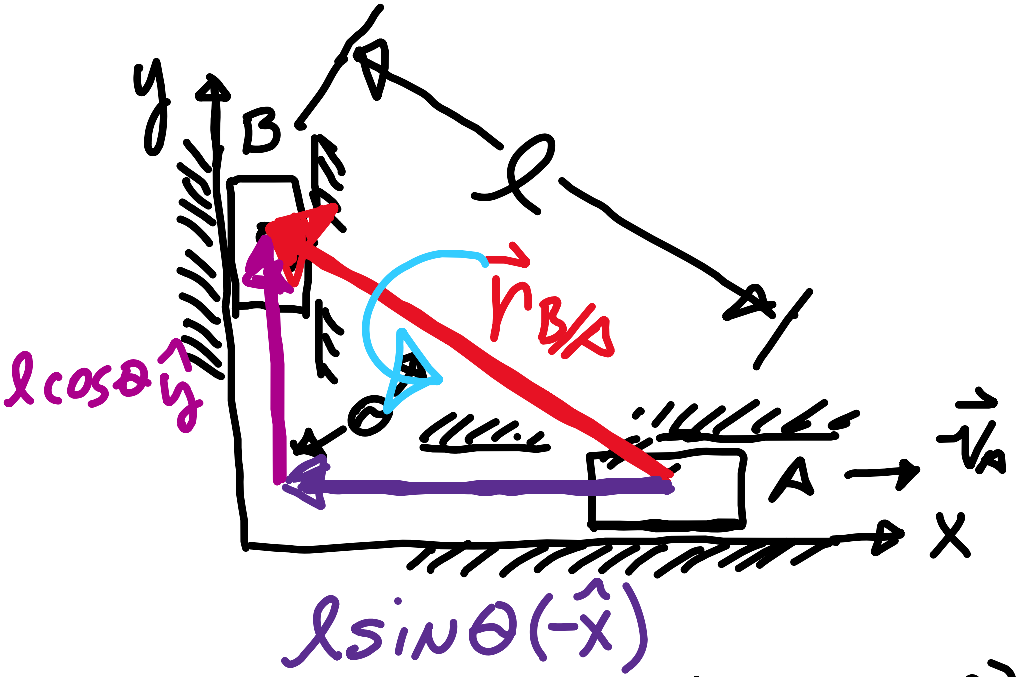
so $\vec{V}_B = \omega l[-\sin\theta\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

MUST EQUAL ZERO



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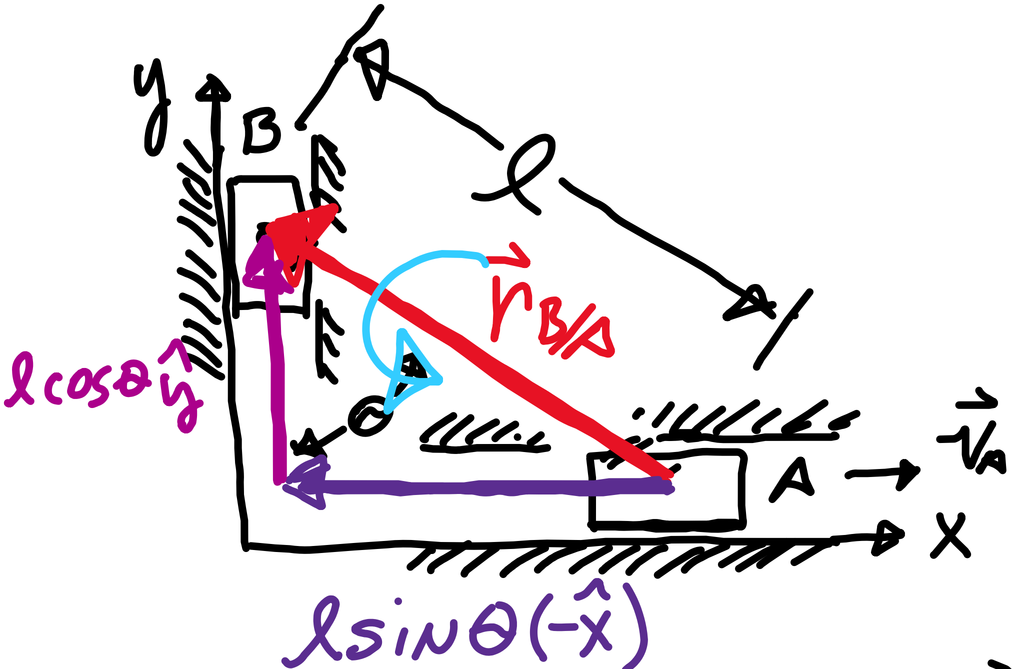
MUST EQUAL ZERO

so $\vec{V}_B = \omega l \sin\theta(-\hat{y})$



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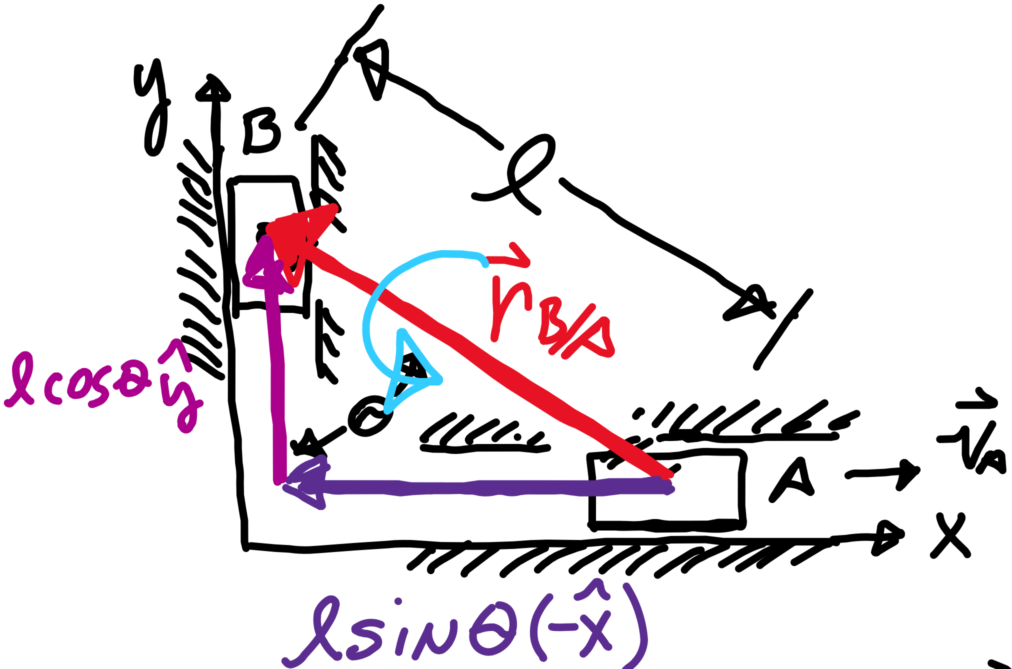
so $\vec{V}_B = \omega l\sin\theta(-\hat{y})$ & $V_A = \omega l\cos\theta$



Example

Given ℓ, $\vec{V}_A = V_A\hat{x}$ & θ

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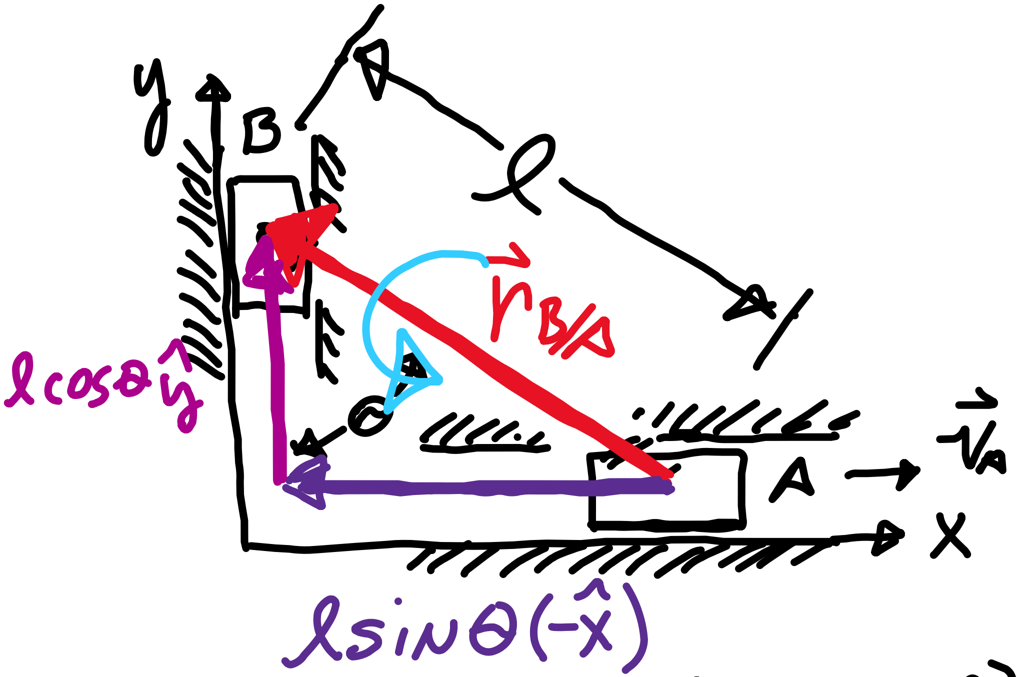
Now we have 2 equations & 2 unknowns



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Given l, $\vec{V}_A = V_A \hat{x}$ & θ

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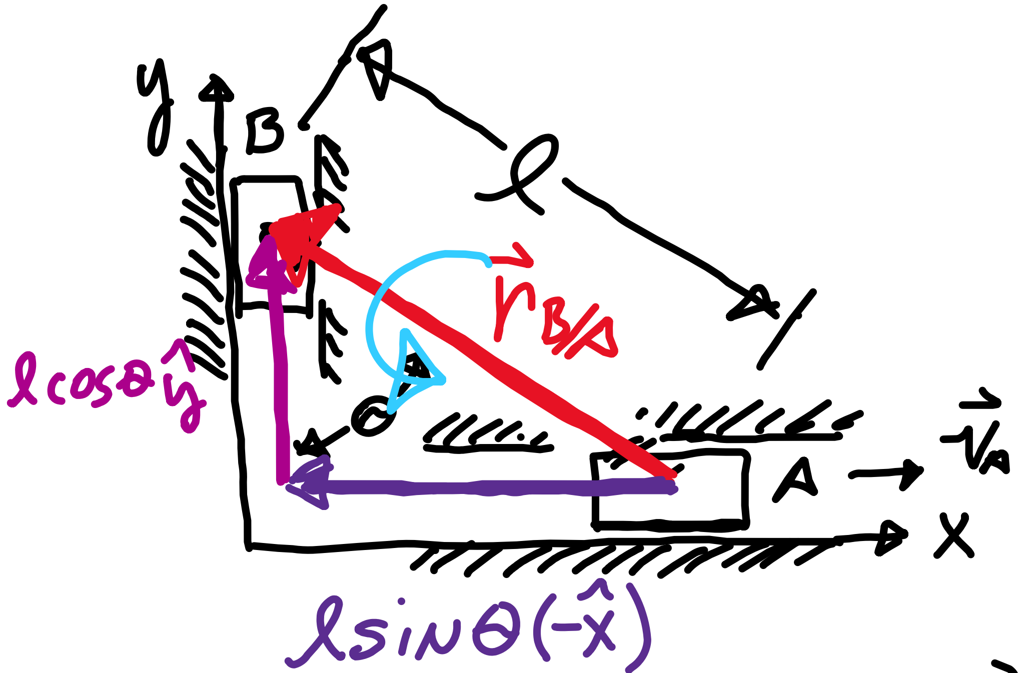

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
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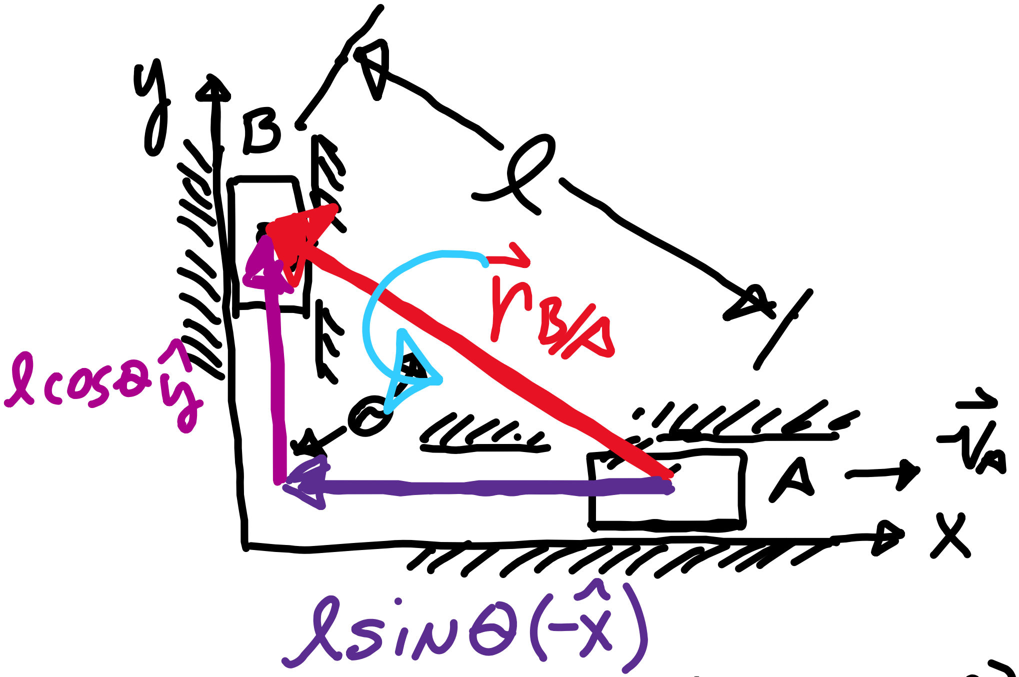
so $\quad \vec{V}_B = \omega l\,[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + V_A \hat{x}$

so $\quad \vec{V}_B = \omega l \sin\theta(-\hat{y})$ (1) & $\quad V_A = \omega l \cos\theta$ (2)



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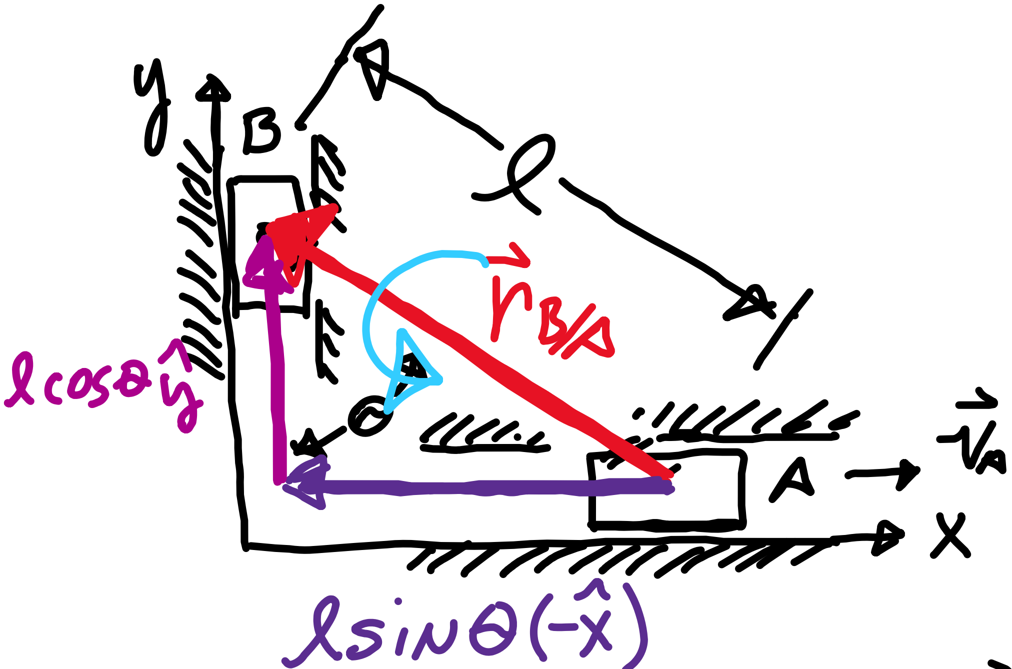
so $\vec{V}_B = \omega\ell\sin\theta(-\hat{y})$ (1) & $V_A = \omega\ell\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(\ell\cos\theta)}$



Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

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$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\hat{z}$

so $\vec{V}_B = \omega l[-\sin\theta\,\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

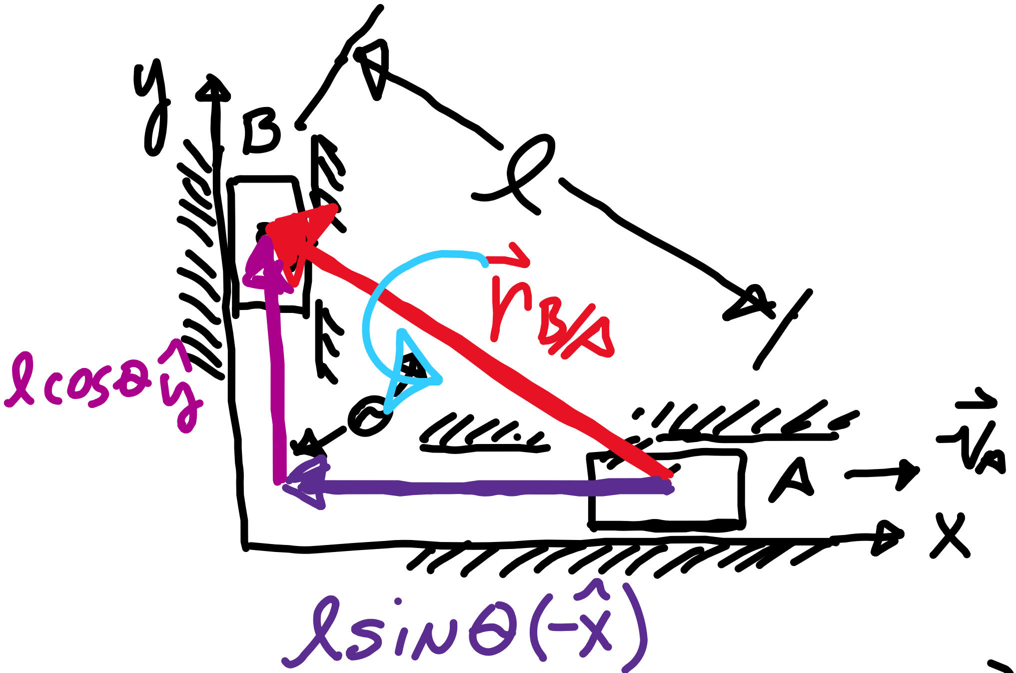
so $\vec{V}_B = \omega l\sin\theta(-\hat{y})$ (1) & $V_A = \omega l\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(l\cos\theta)}$ (3)

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



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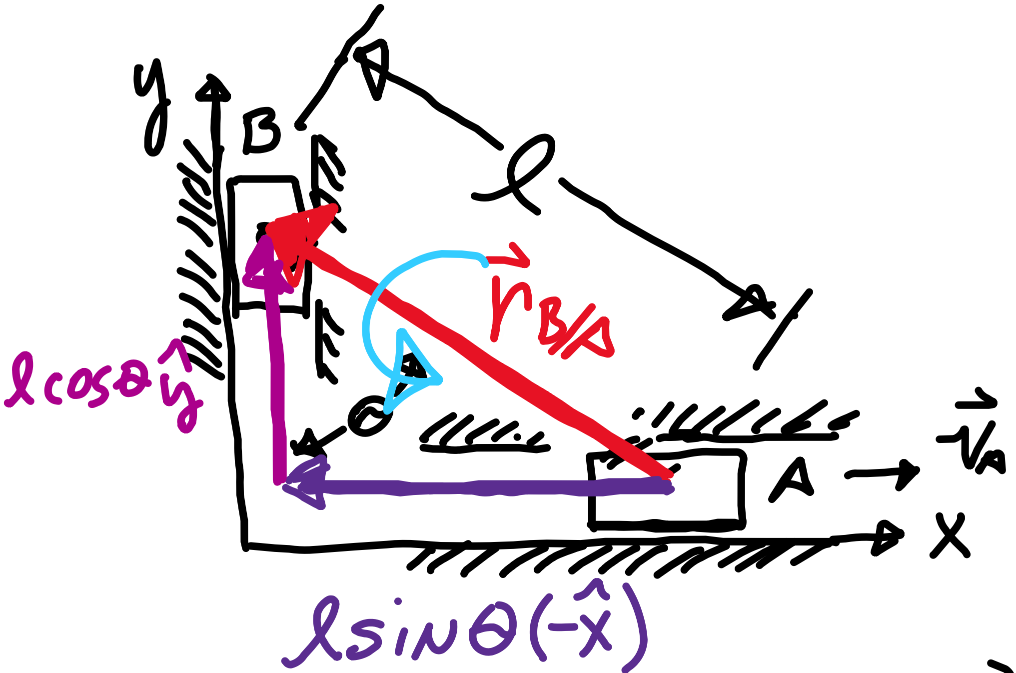
so $\quad \vec{V}_B = \omega l[-\sin\theta\,\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

so $\quad \vec{V}_B = \omega l\sin\theta(-\hat{y})$ (1) & $V_A = \omega l\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(l\cos\theta)}$ (3) EQN 1&3 $\Rightarrow$

$$\vec{V}_B = \left(\frac{lV_A}{l\cos\theta}\right)\sin\theta(-\hat{y})$$

Example



Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = \vec{\omega} \times \vec{r}_{B/A} + V_A\hat{x}$$

$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

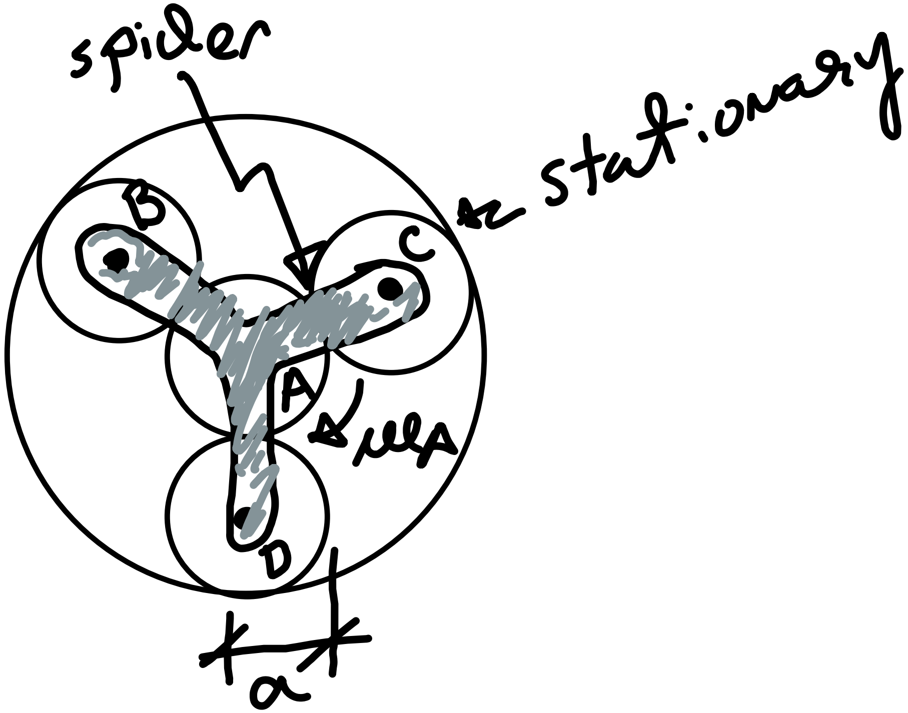
so $\vec{V}_B = \omega l\,[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + V_A\hat{x}$

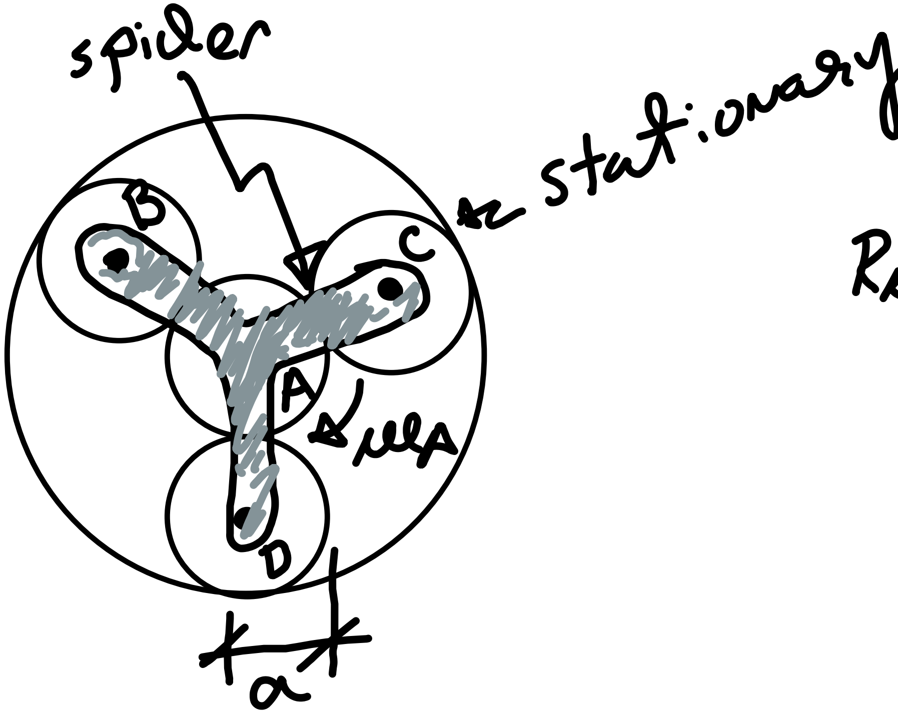
so $\vec{V}_B = \omega l \sin\theta(-\hat{y})$ (1) & $V_A = \omega l\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(l\cos\theta)}$ (3) EQN 1&3 $\Rightarrow$

$$\vec{V}_B = \left(\frac{lV_A}{l\cos\theta}\right)\sin\theta(-\hat{y}) \Rightarrow \boxed{\vec{V}_B = (V_A\tan\theta)(-\hat{y})}$$

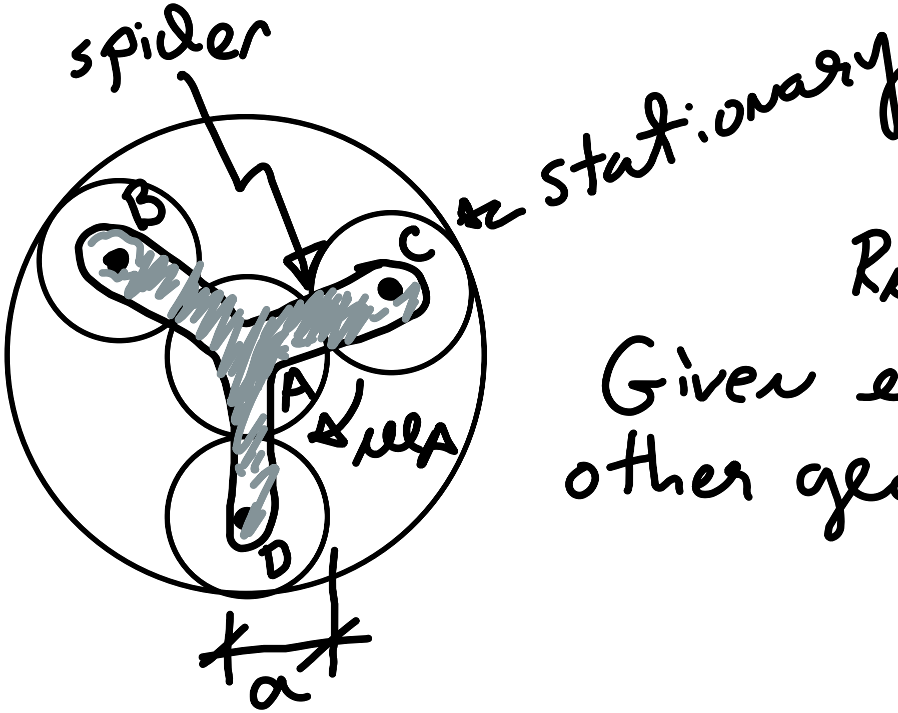
Example: Planetry gear system





Example: Planetry gear system

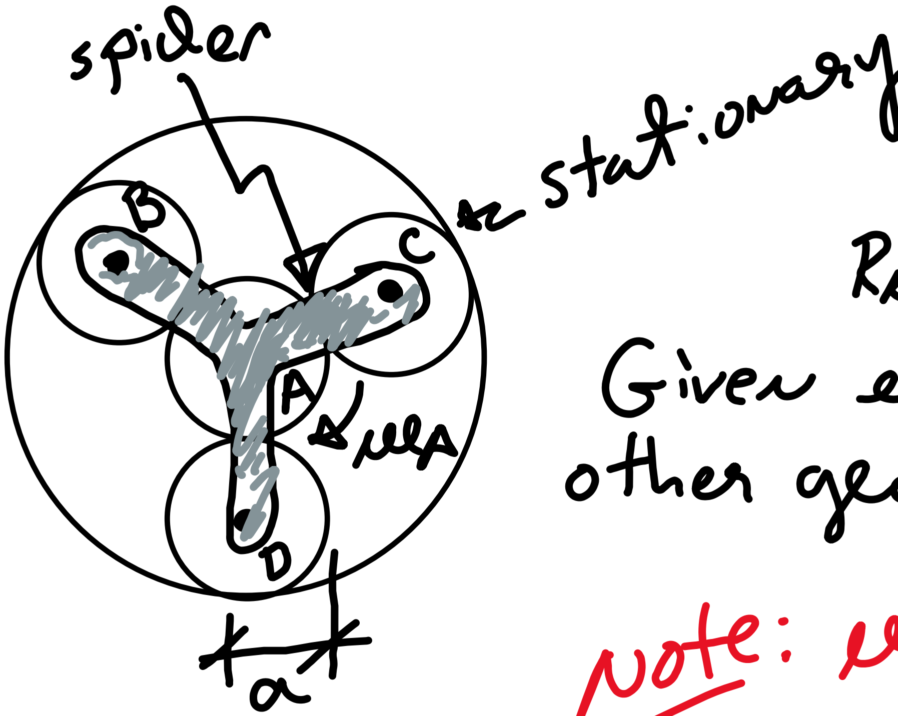
$R_A = R_B = R_C = R_D = a$



<u>Example</u>: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

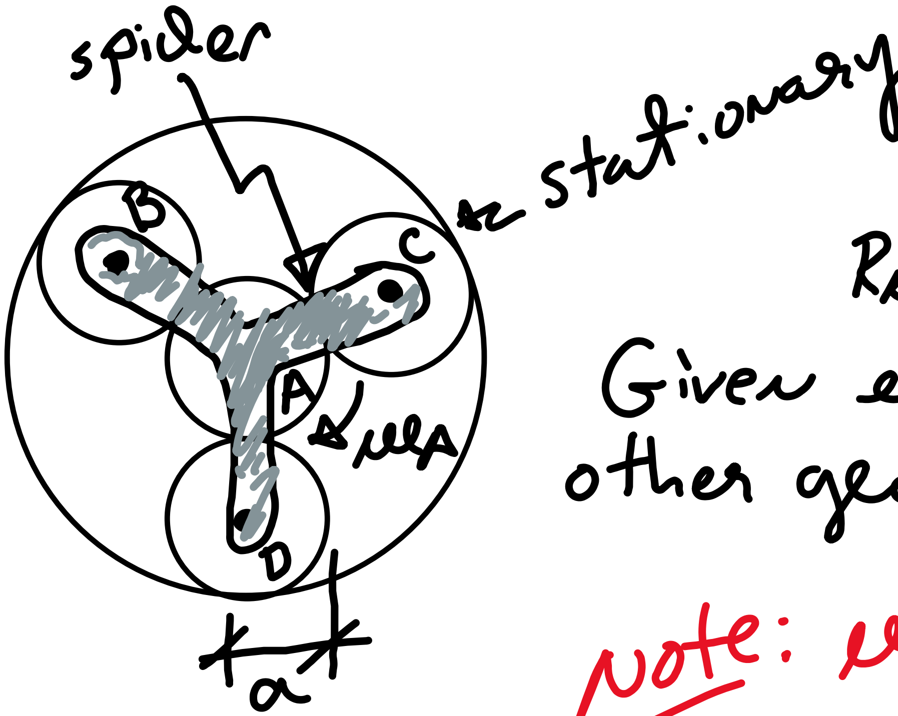


Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

Note: $\omega_B = \omega_C = \omega_D$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

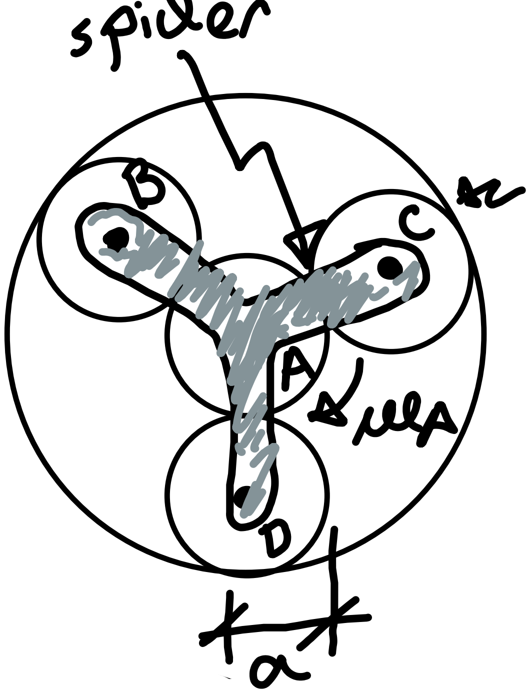
Note: $\omega_B = \omega_C = \omega_D$

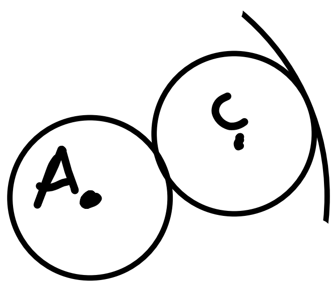
Just need to look at one of these

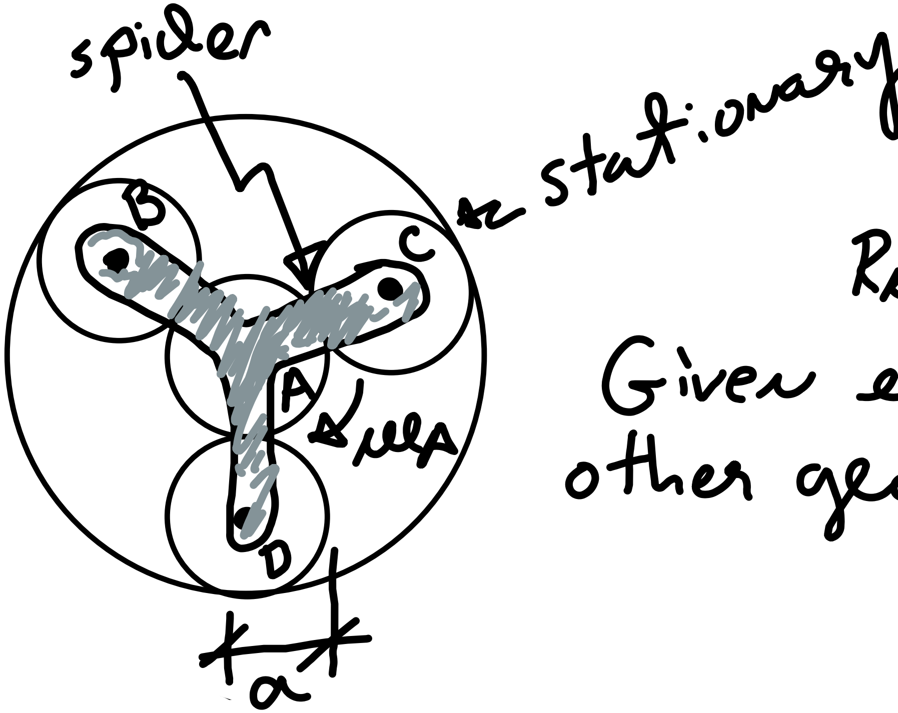
Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.



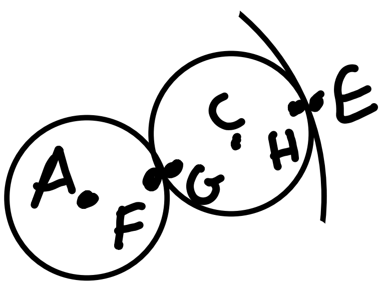




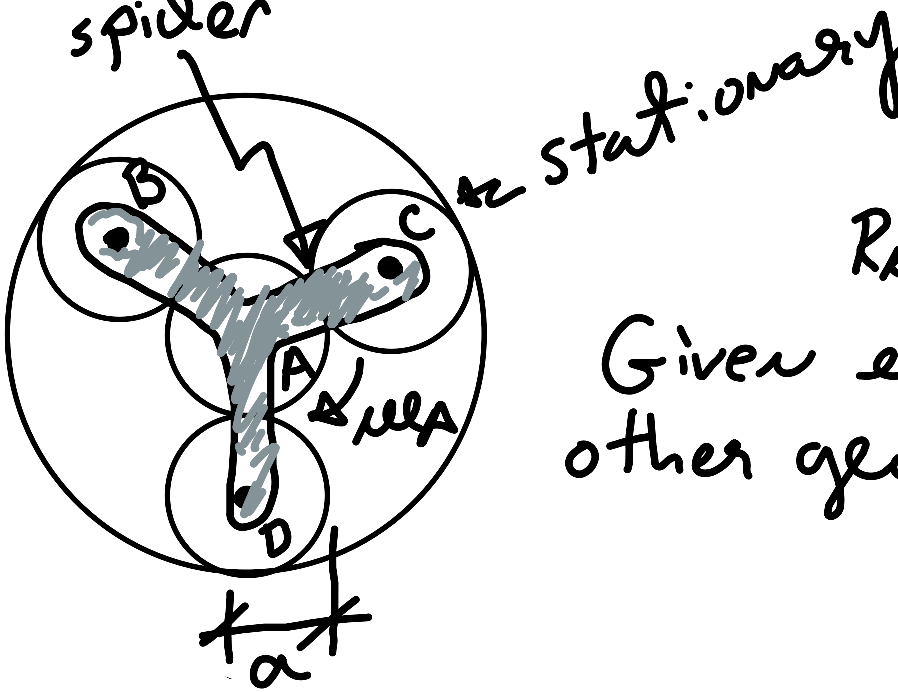
Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

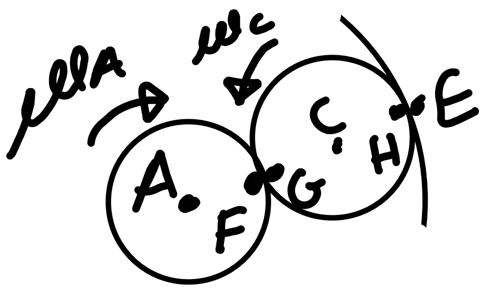


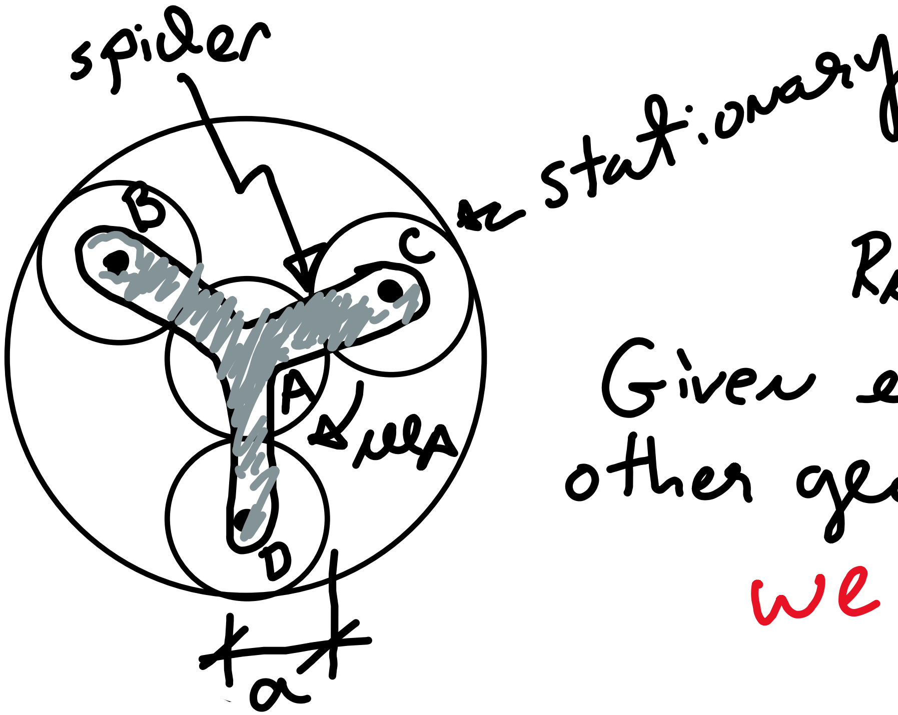
Example: Planetry gear system



$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



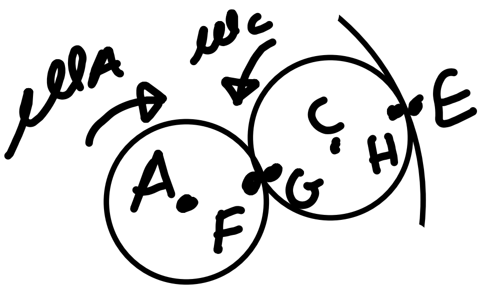


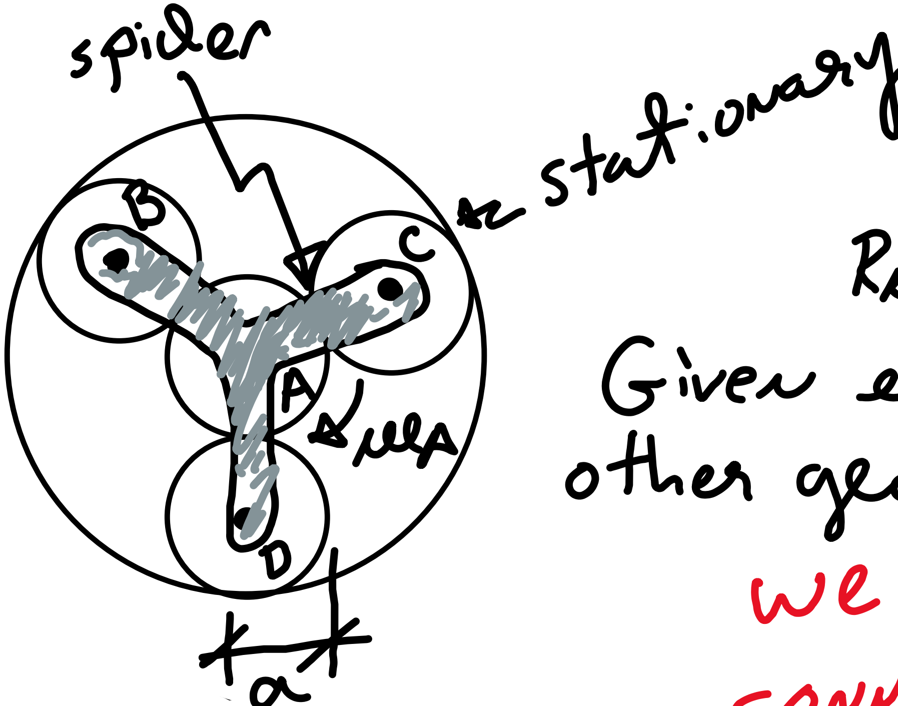
Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

we know V_E & V_F

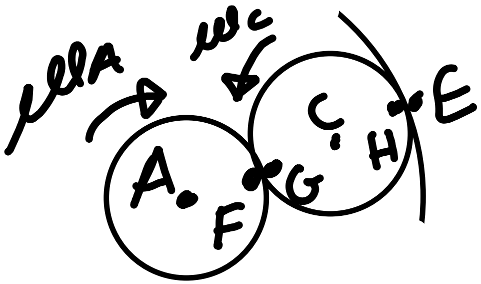




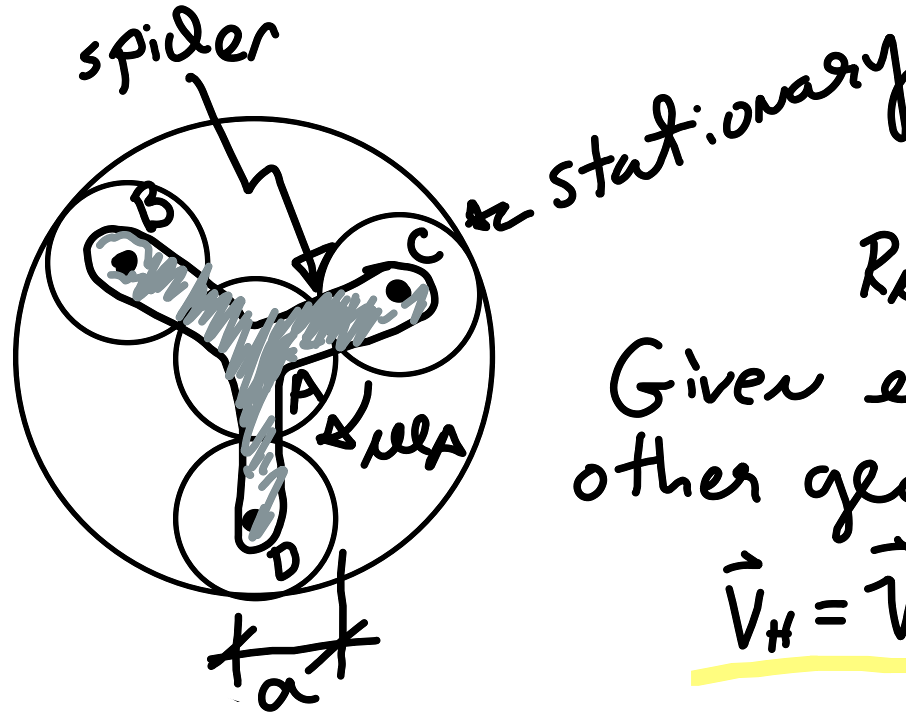
Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



we know V_E & V_F so connect the points!

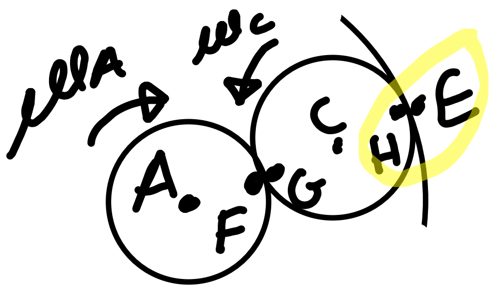


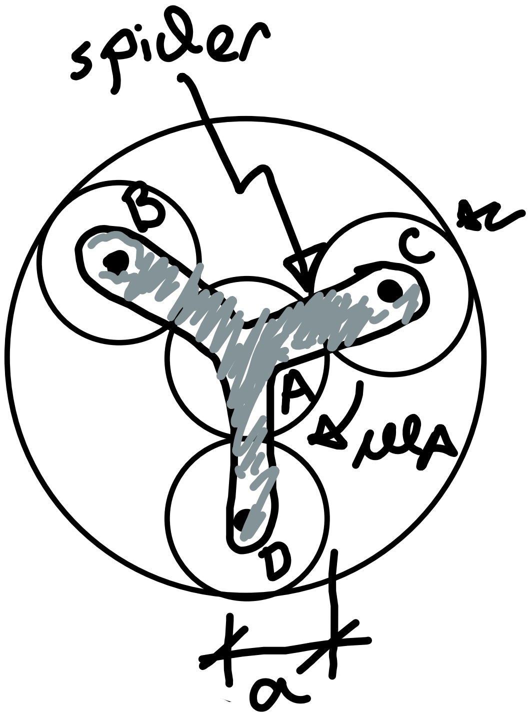
Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$





Example: Planetry gear system

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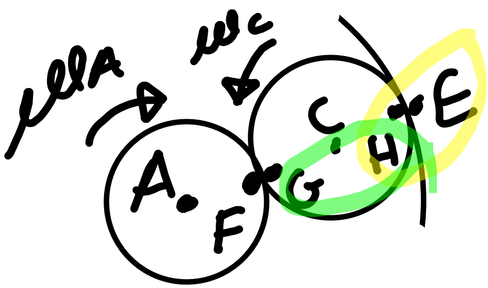
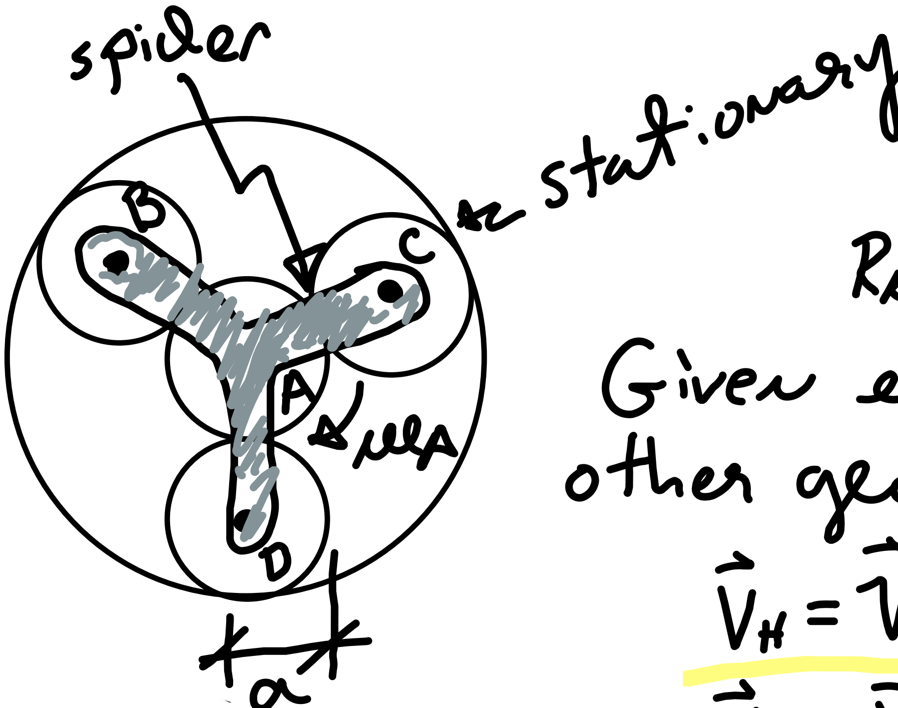
Given ω_A, find ω of the other gears.



$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

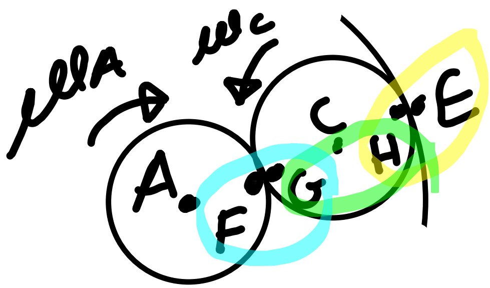
$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

Example: Planetry gear system



$$R_A = R_B = R_C = R_D = a$$

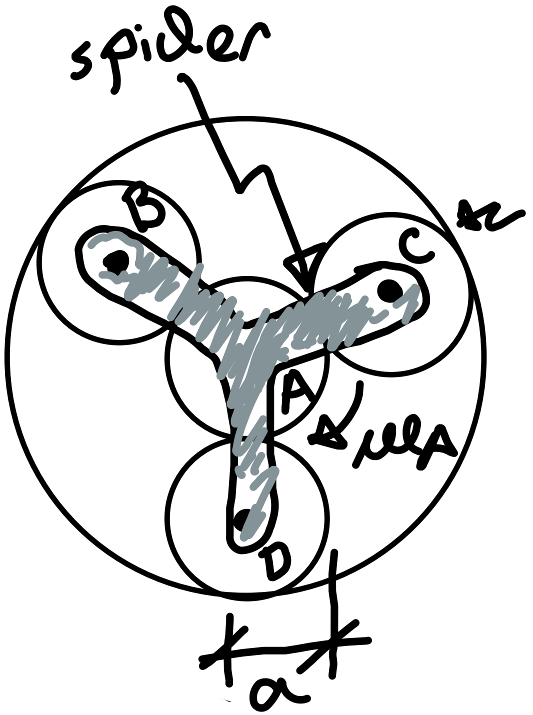
Given ω_A, find ω of the other gears.



$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$



Example: Planetry gear system

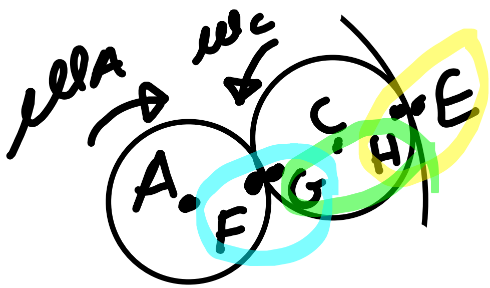
$R_A = R_B = R_C = R_D = a$

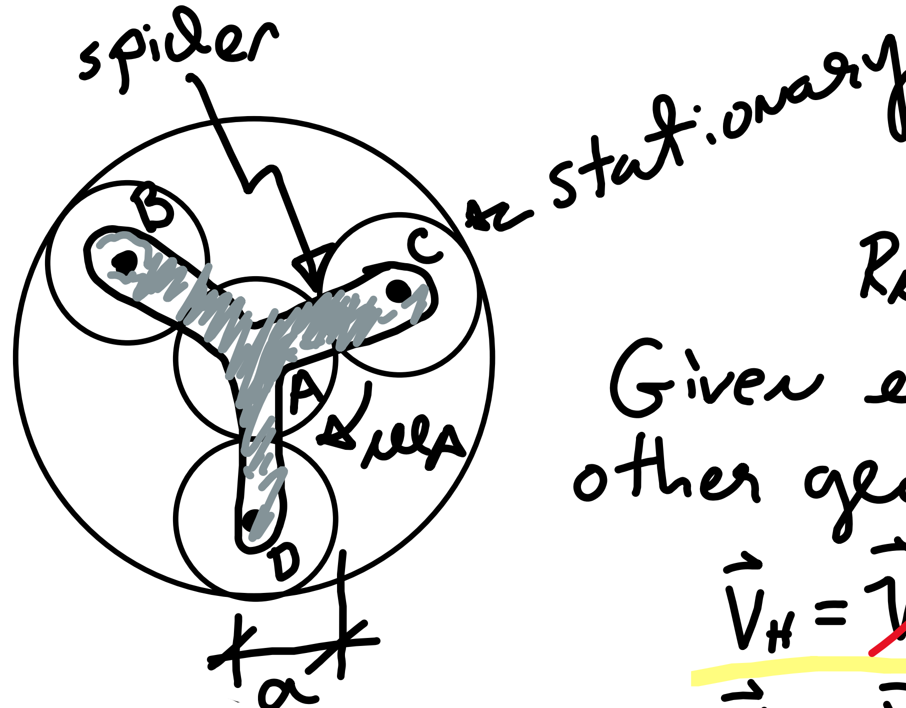
Given ω_A, find ω of the other gears.

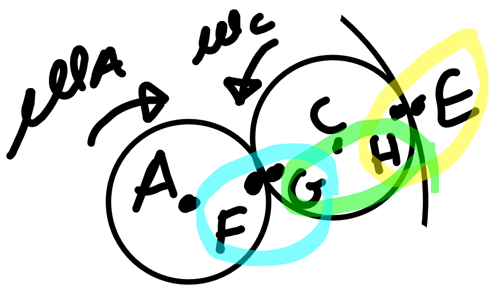
$$\vec{V}_H = \vec{V}_{H/E} + \cancel{\vec{V}_E}^{\,0}$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$







Example: Planetry gear system

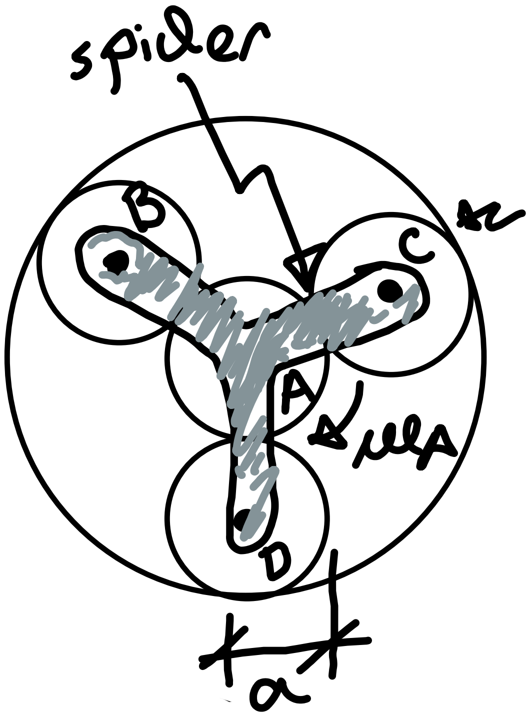
$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\vec{V}_H = \cancelto{0}{\vec{V}_{H/E}} + \cancelto{0}{\vec{V}_E}$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$



Example: Planetry gear system

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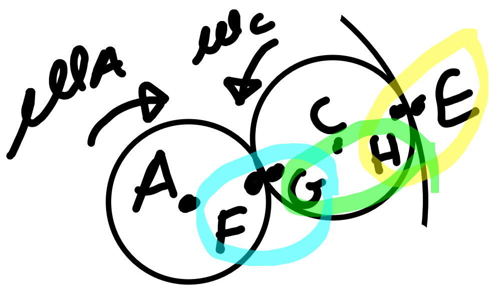
Given ω_A, find ω of the other gears.



$$\overset{0}{\cancel{\vec{V}_H}} = \overset{0}{\cancel{\vec{V}_{H/E}}} + \overset{0}{\cancel{\vec{V}_E}}$$

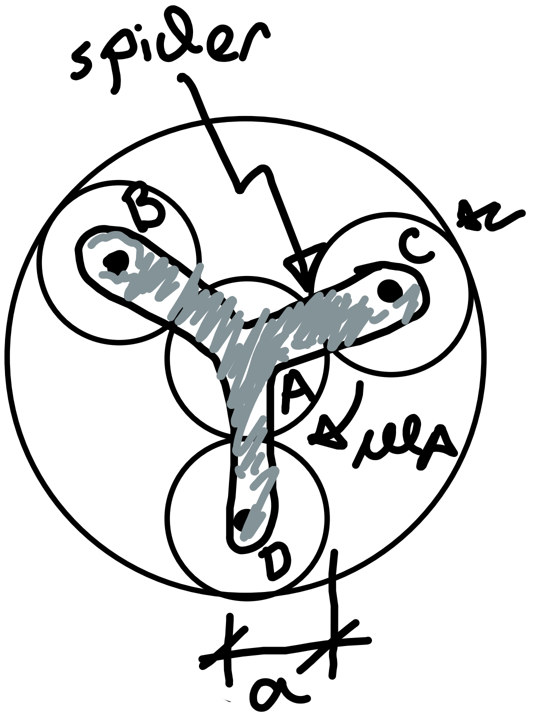
$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

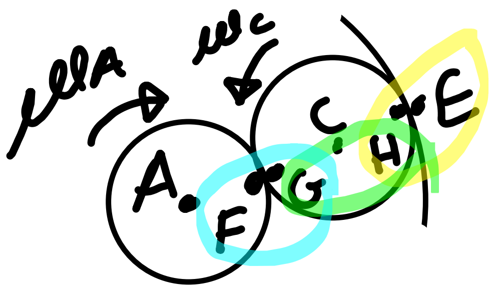
$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$

Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

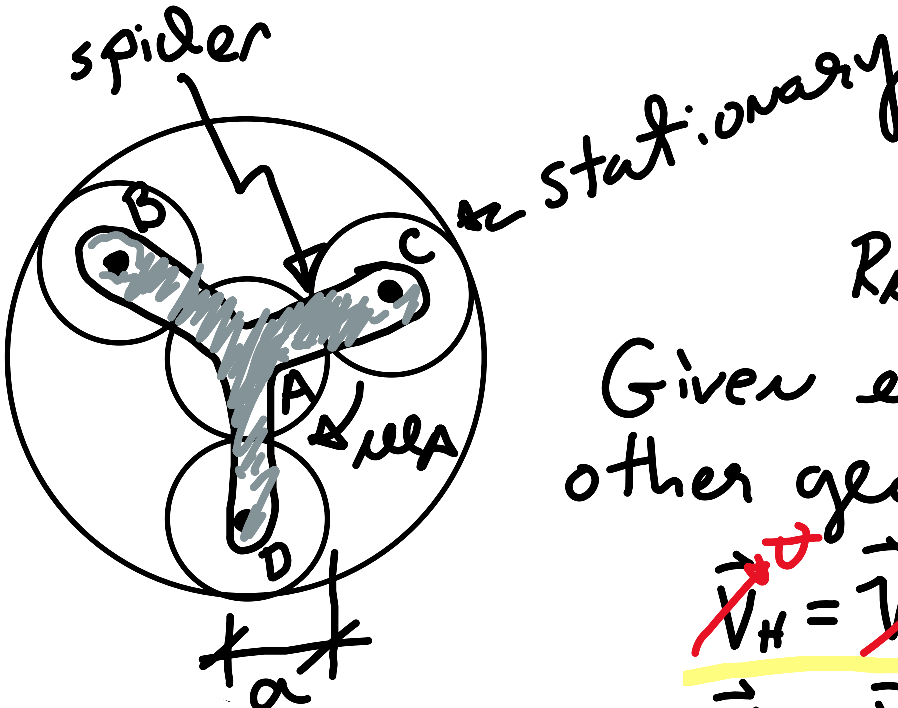




$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

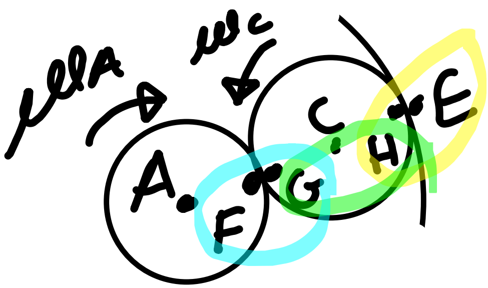
$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

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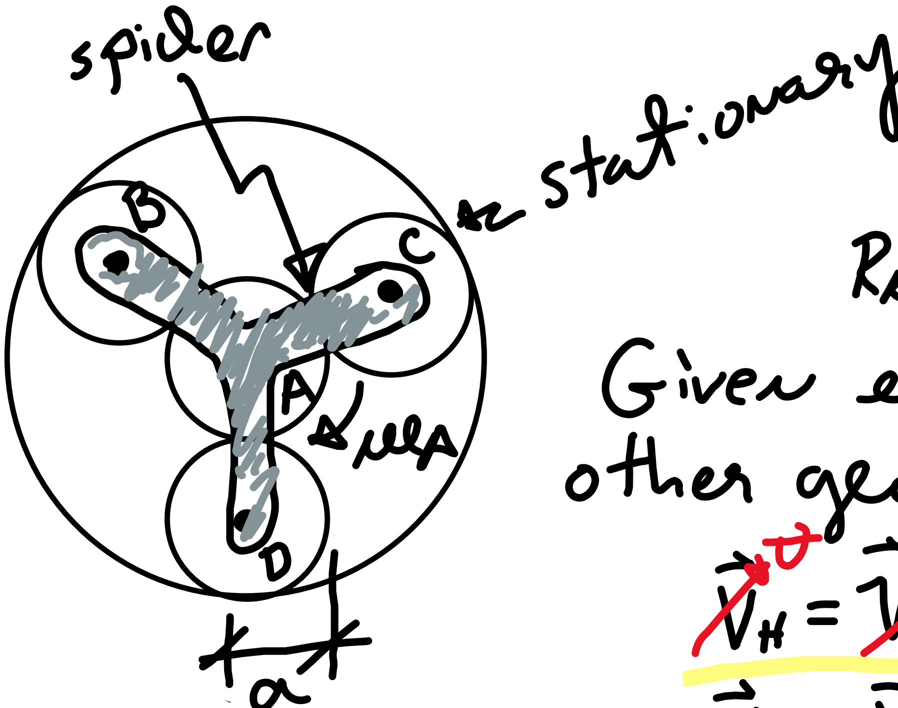
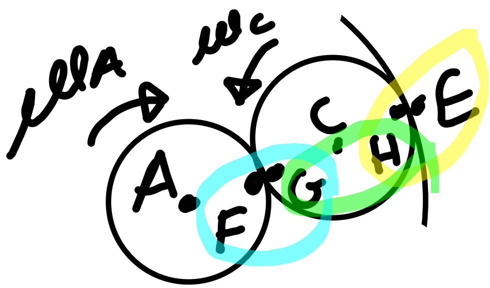




Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

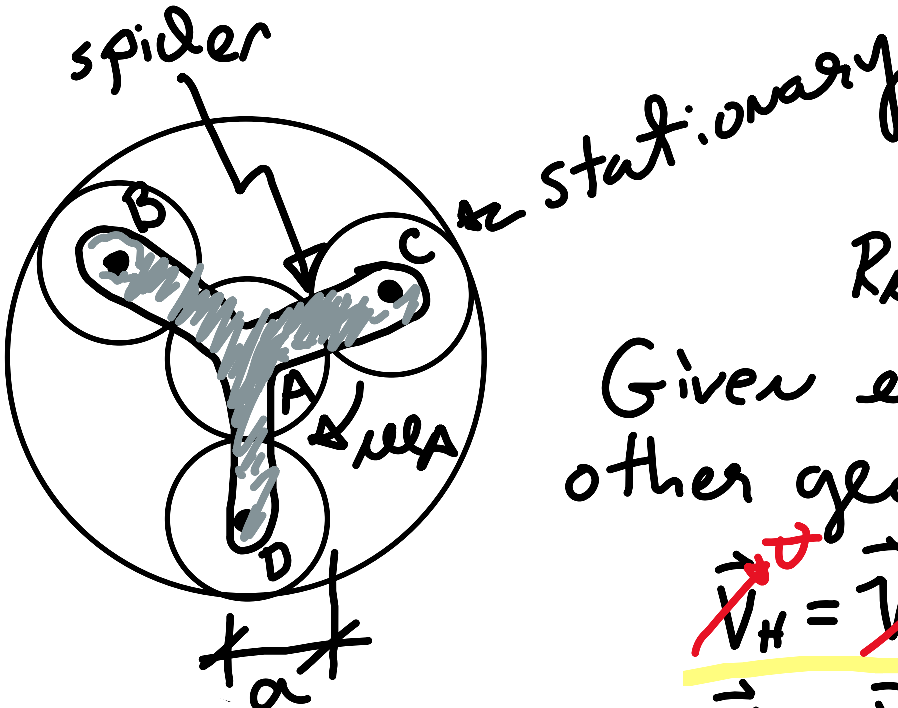
Given ω_A, find ω of the other gears.

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$

Example: Planetry gear system



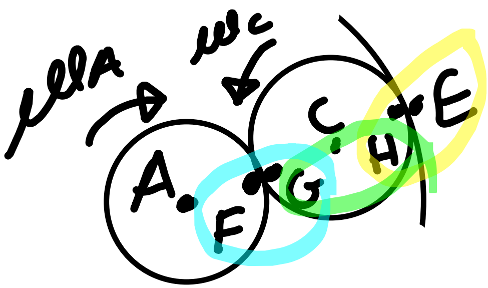
$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

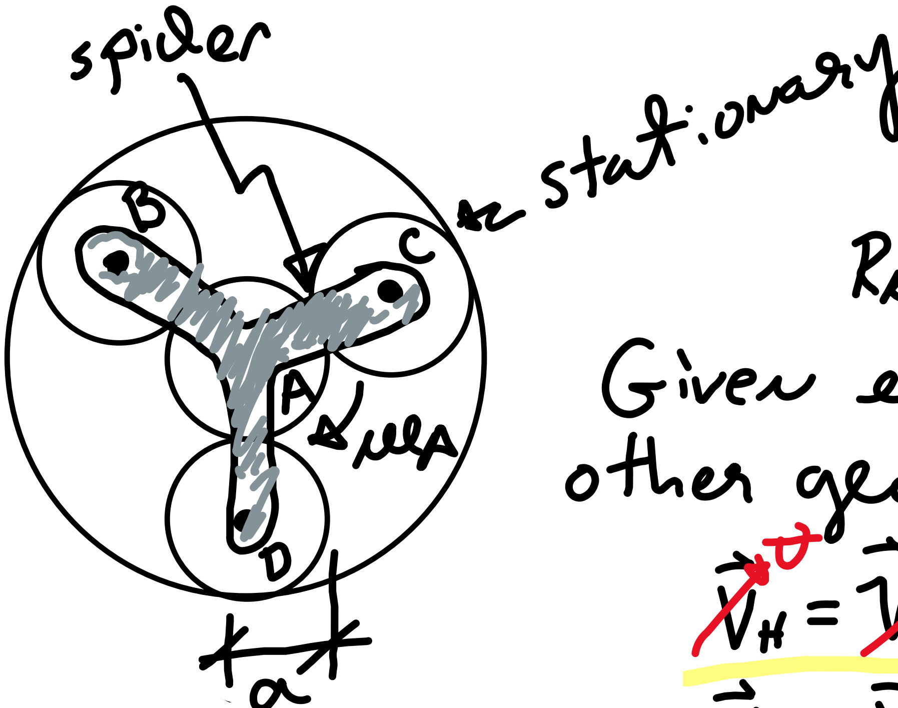
$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$



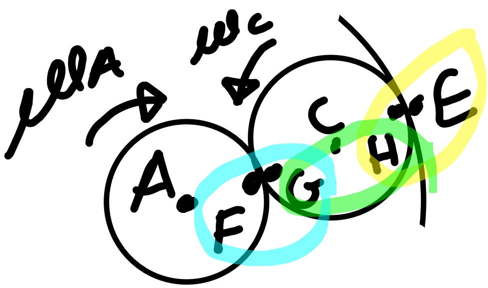
So $2a\omega_C = a\omega_A$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

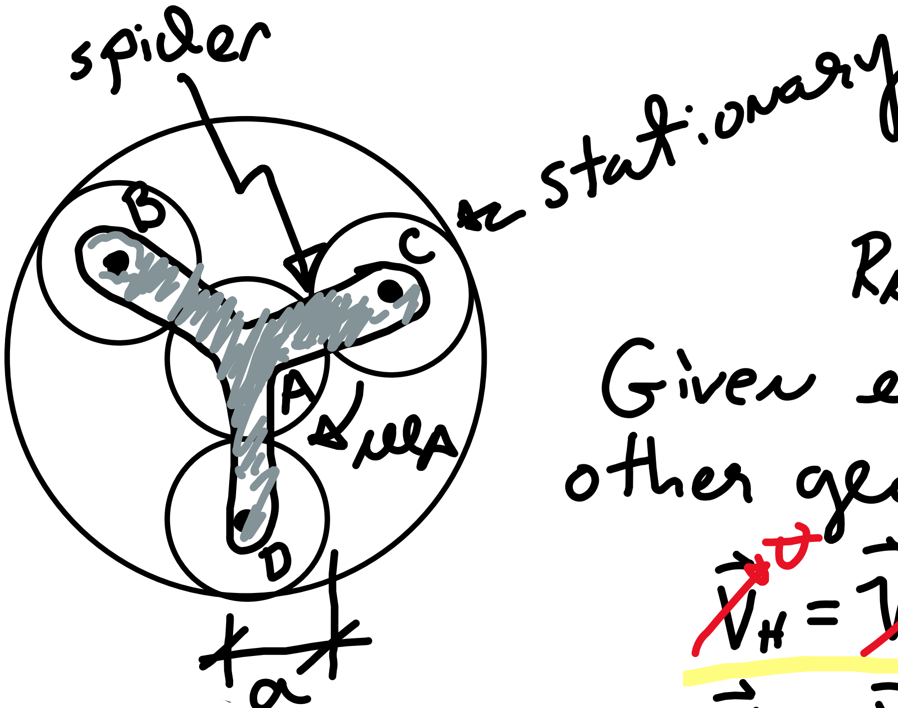


$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$

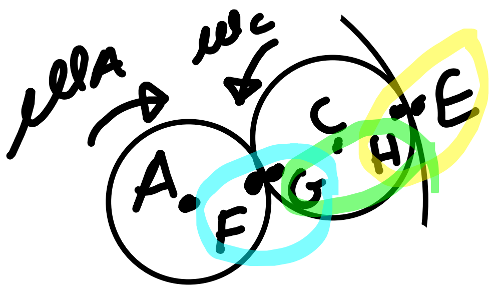
So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



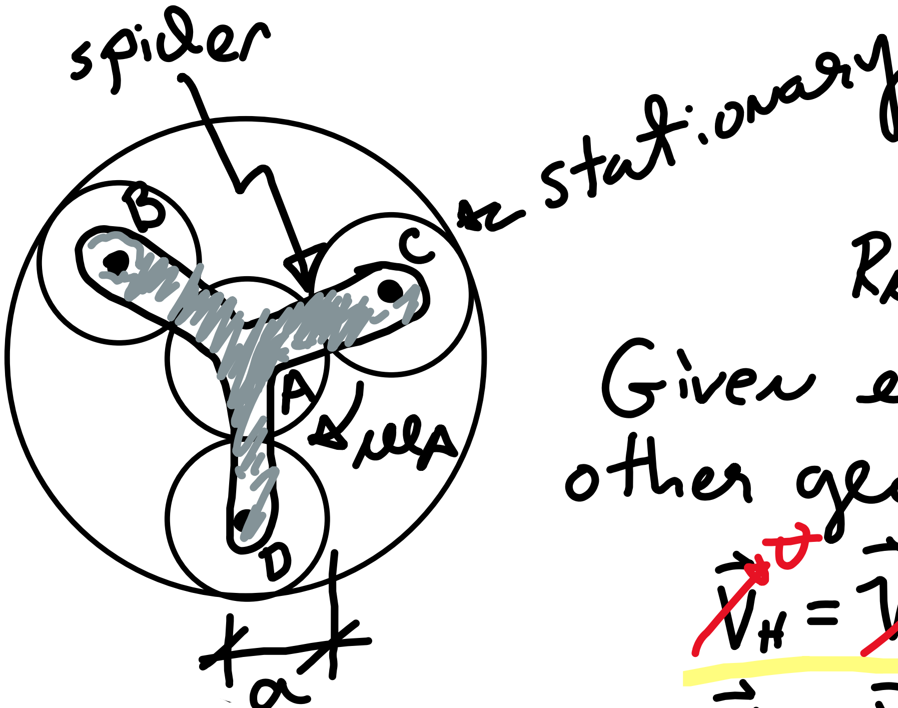
$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$

So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

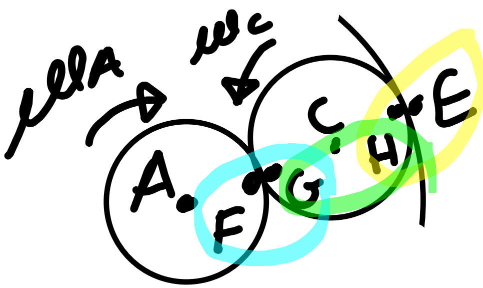
Find ω_{spider}:



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



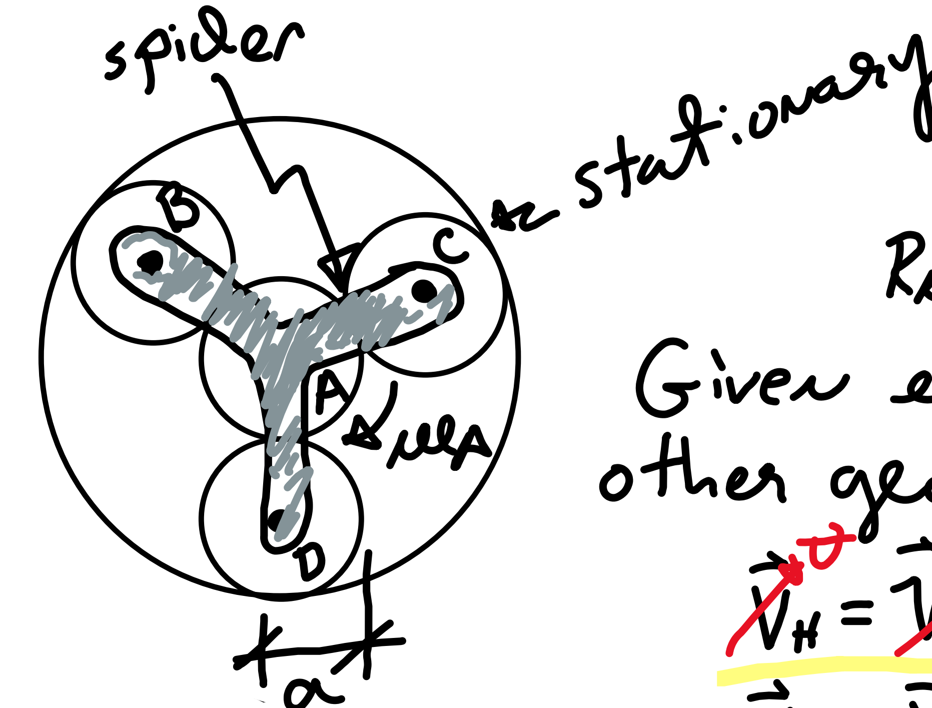
$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$

So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A$



Example: Planetry gear system

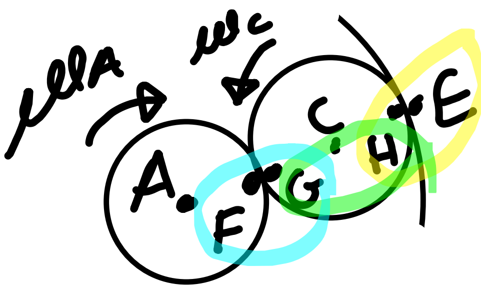
$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\cancelto{0}{\vec{V}_H} = \cancelto{0}{\vec{V}_{H/E}} + \cancelto{0}{\vec{V}_E}$$

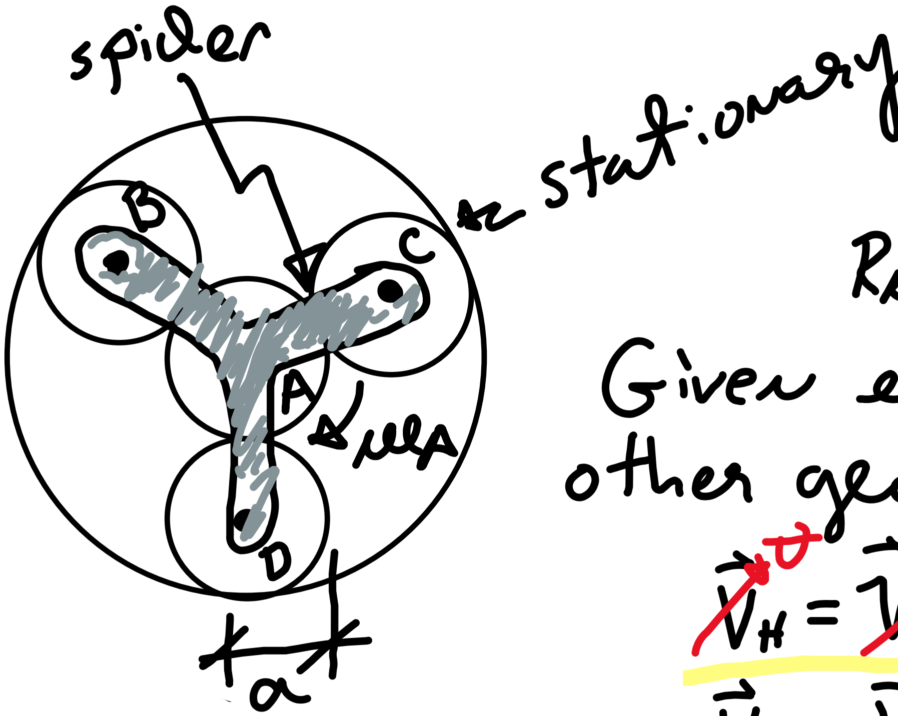
$$\vec{V}_G = \vec{V}_{G/H} + \cancelto{0}{\vec{V}_H} = 2a\omega_C$$

$$\vec{V}_G = \cancelto{0}{\vec{V}_{G/F}} + \vec{V}_F = a\omega_A$$



So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \cancelto{0}{\vec{V}_A}$



Example: Planetry gear system

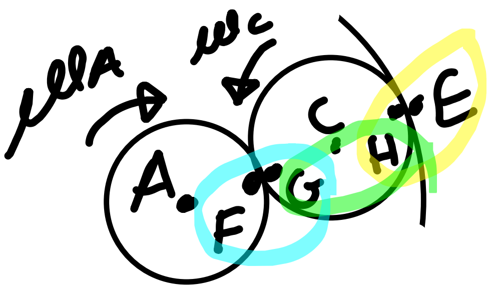
$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

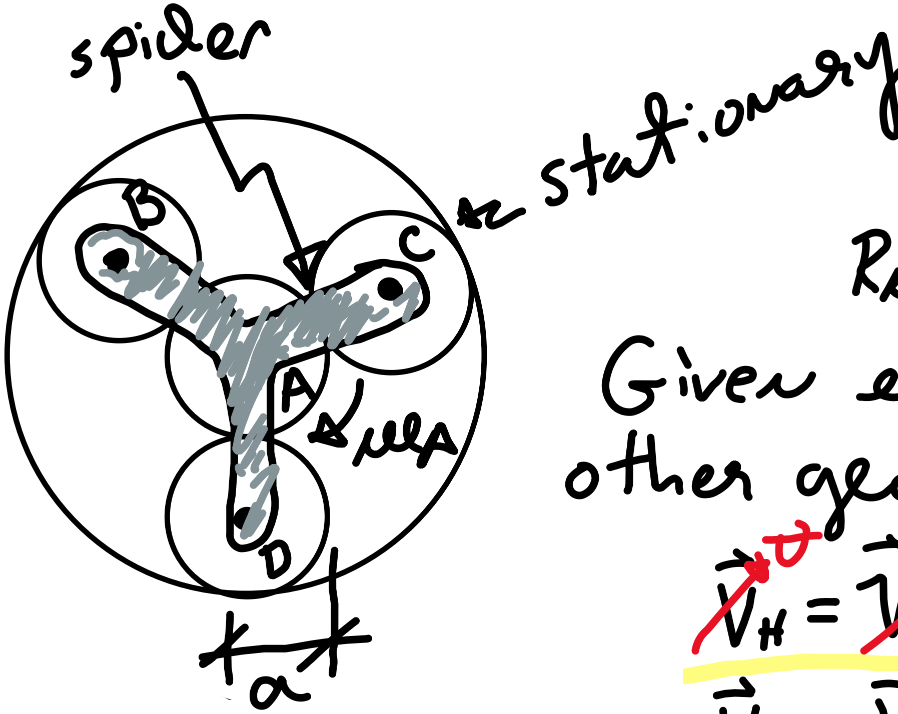
$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$



So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A = 2a\omega_{spider}$



Example: Planetry gear system

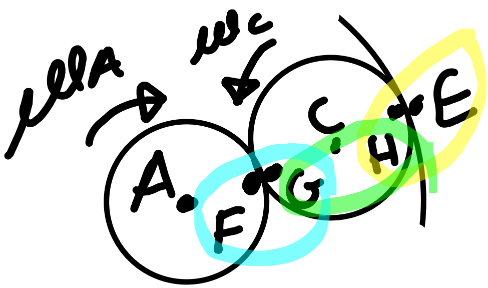
$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$

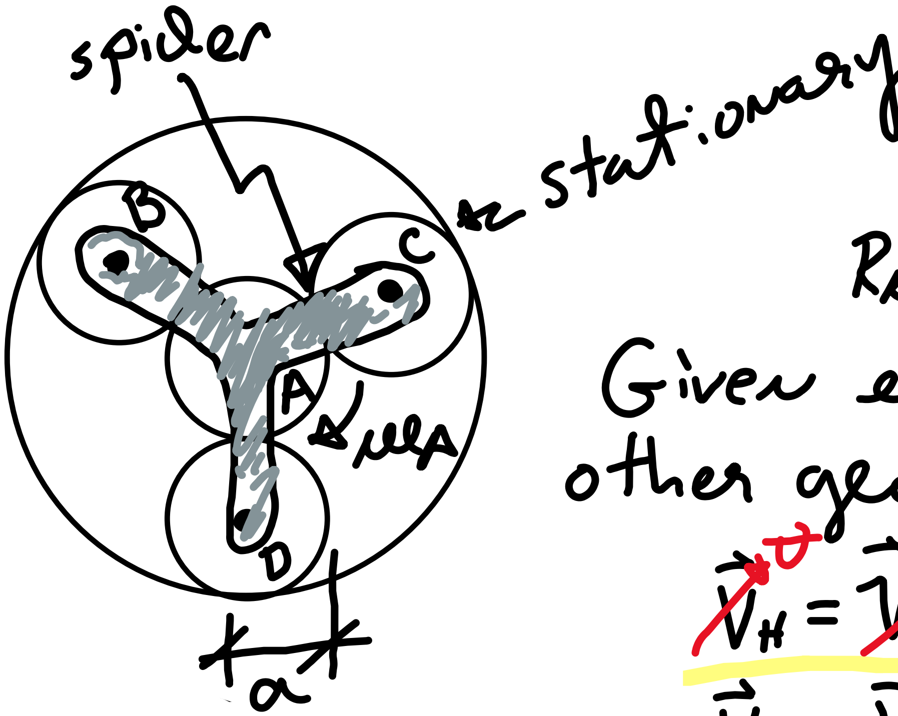


So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A = 2a\omega_{spider}$

Also $\vec{V}_C = \vec{V}_{C/E} + \vec{V}_E$

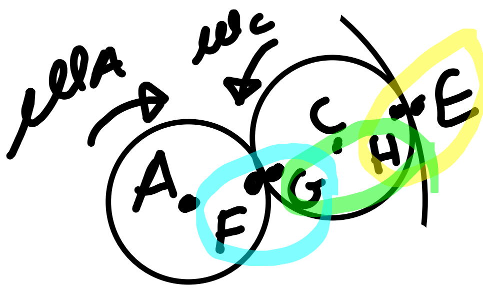
ASU



Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.



$$\cancelto{0}{\vec{V}_H} = \cancelto{0}{\vec{V}_{H/E}} + \cancelto{0}{\vec{V}_E}$$

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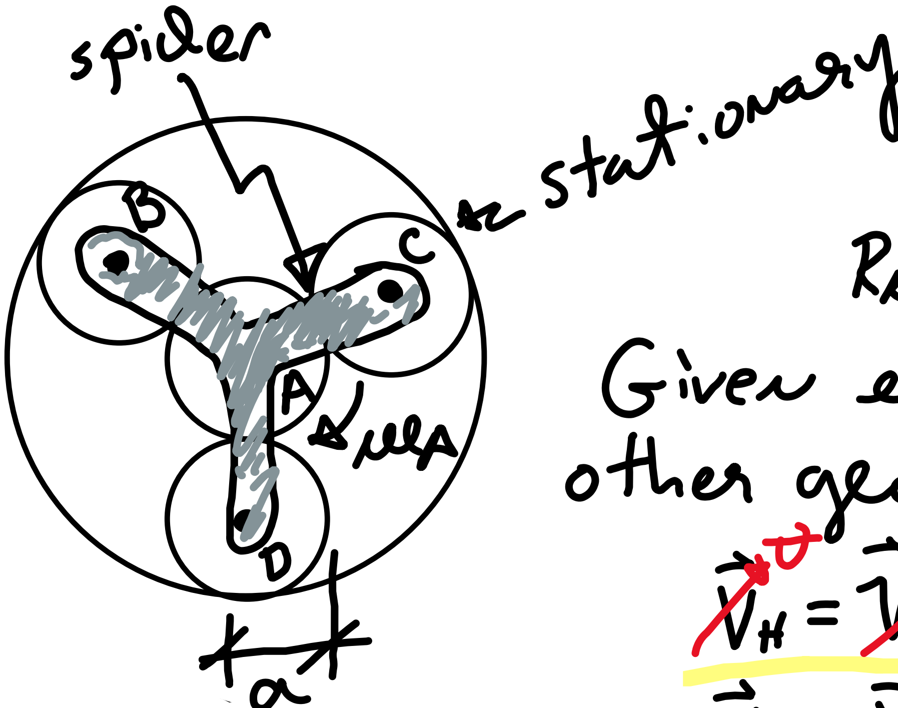

Example: Planetry gear system

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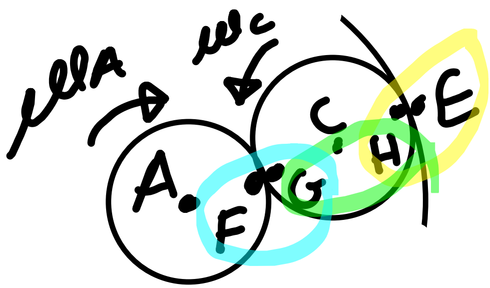

$$\cancel{\vec{V}_H}^{\,0} = \cancel{\vec{V}_{H/E}}^{\,0} + \cancel{\vec{V}_E}^{\,0}$$

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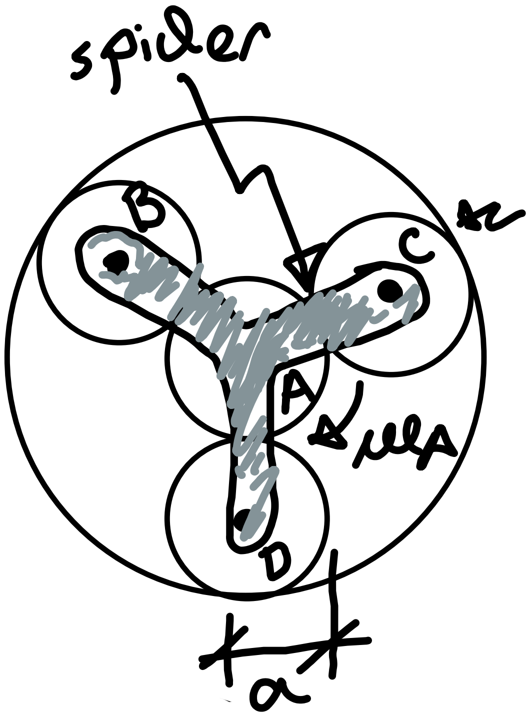

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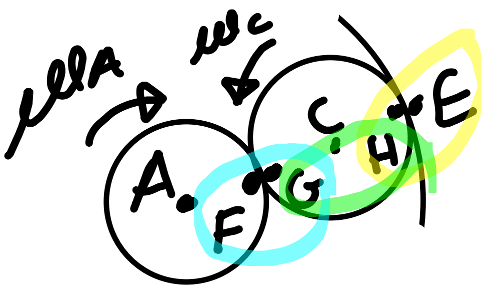

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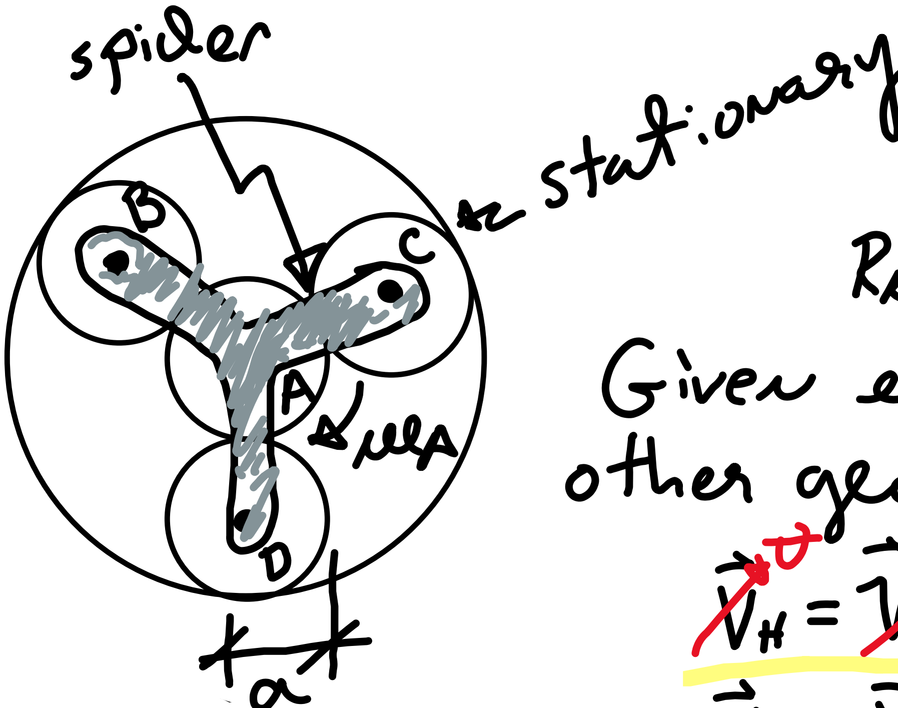

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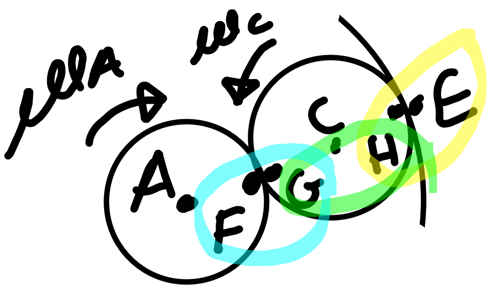

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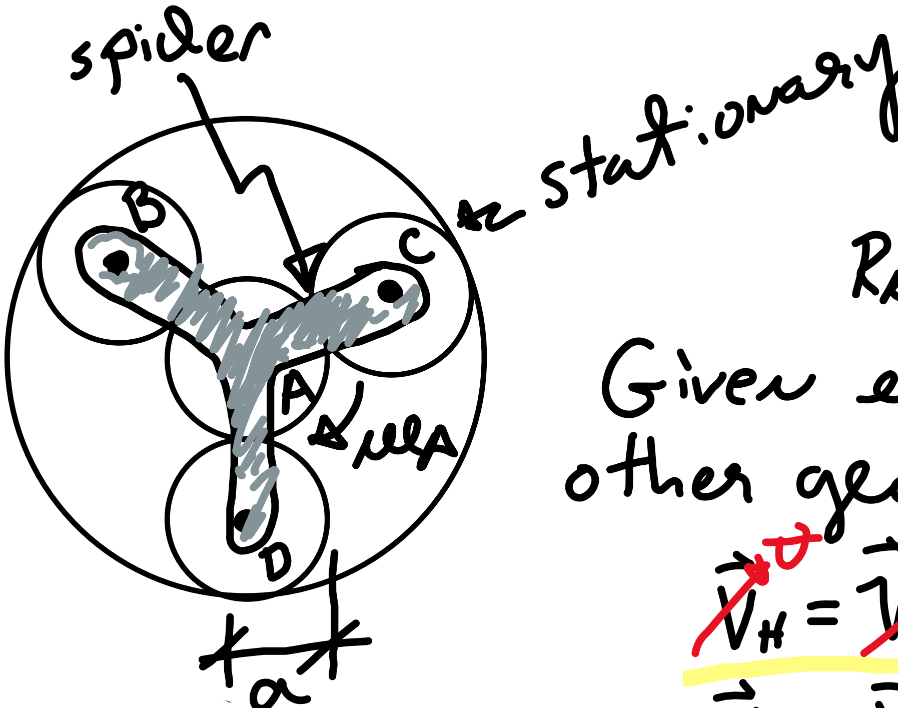

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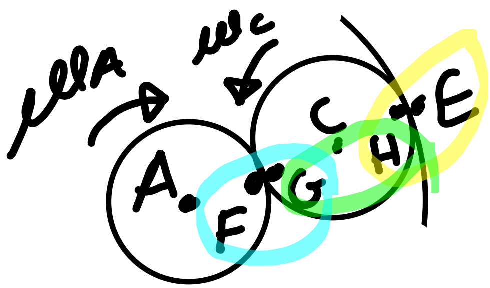

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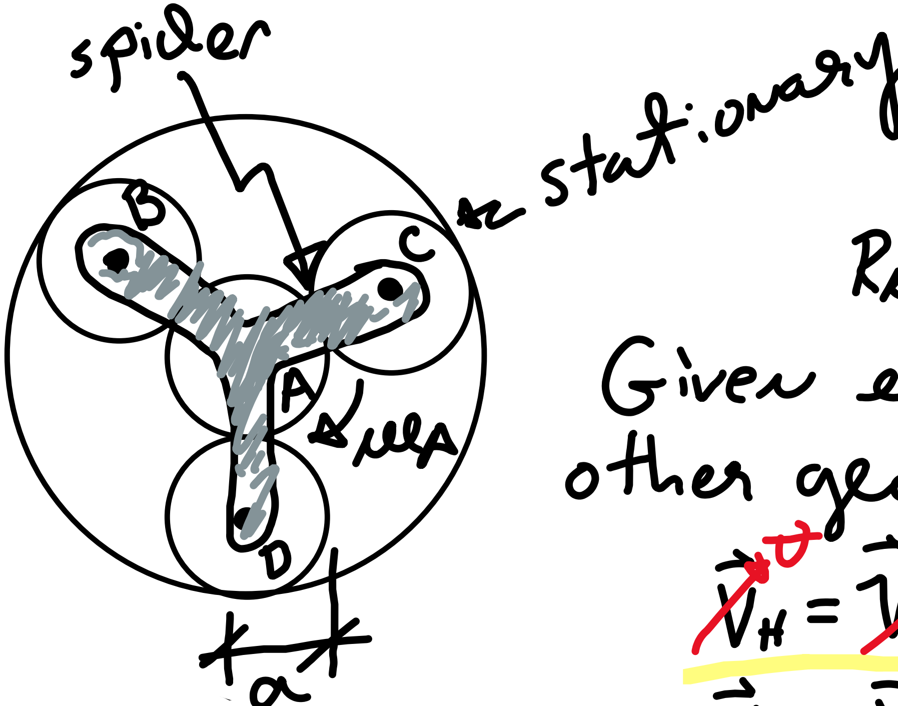
$$a\omega_A/2 = 2a\omega_{spider} \Rightarrow \omega_{spider} = \frac{\omega_A}{4}$$



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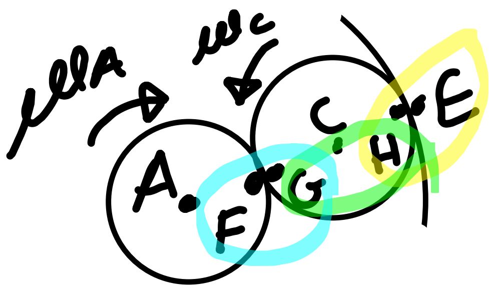

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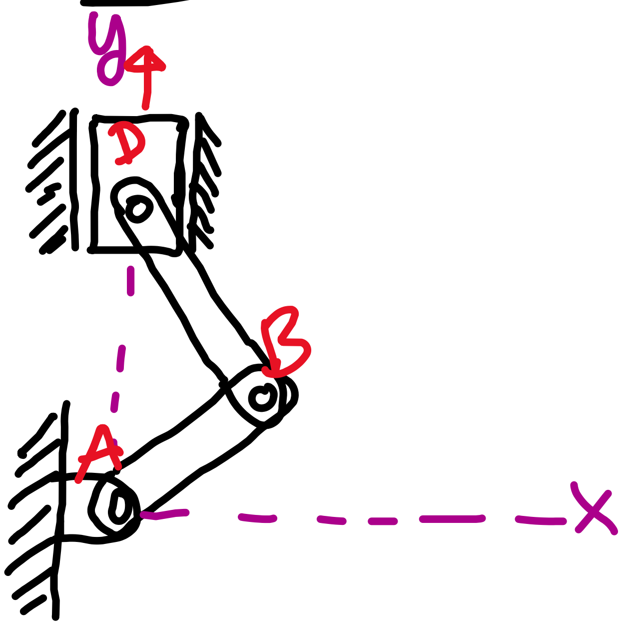
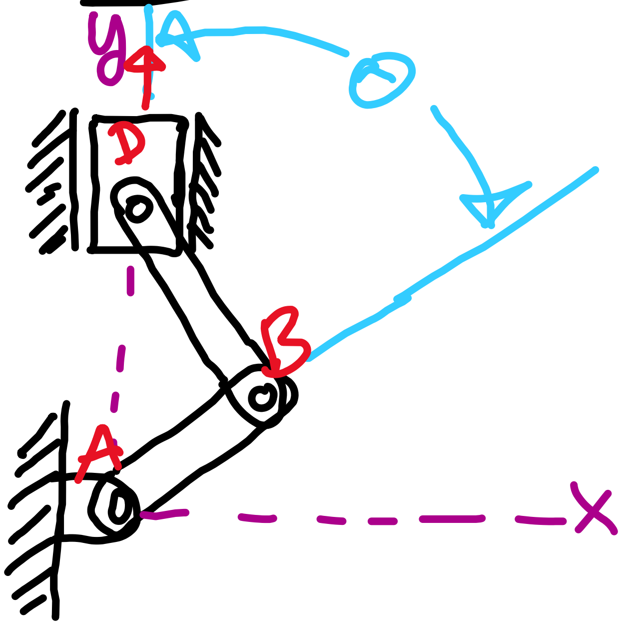
Notes on 15.62 :

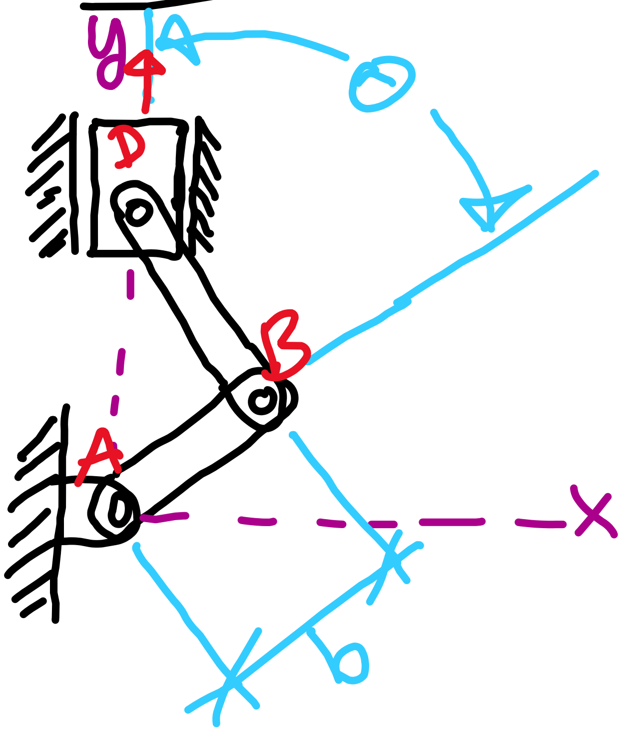


Notes on 15.62 : Given $\theta = 60^\circ$



Notes on 15.62:

Given $\theta = 60°$, $b = 60mm$



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Given $\theta = 60°$, $b = 60mm$

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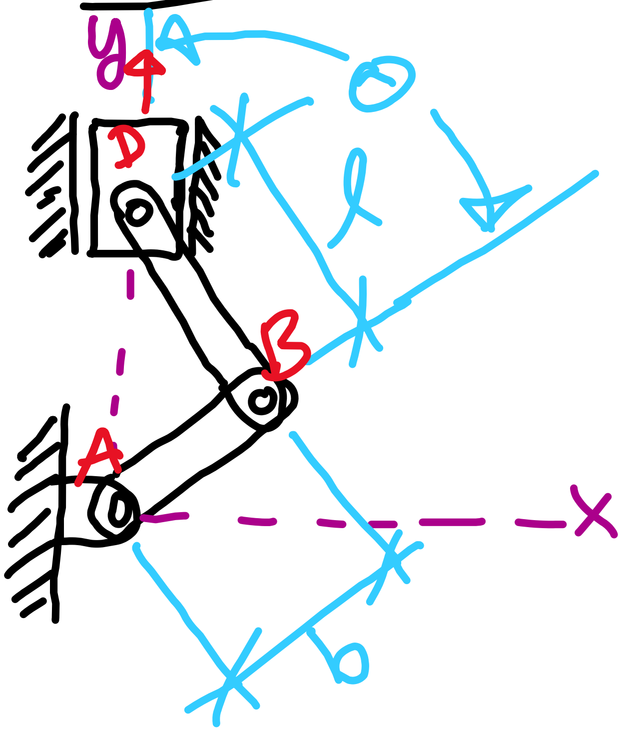

Notes on 15.62:

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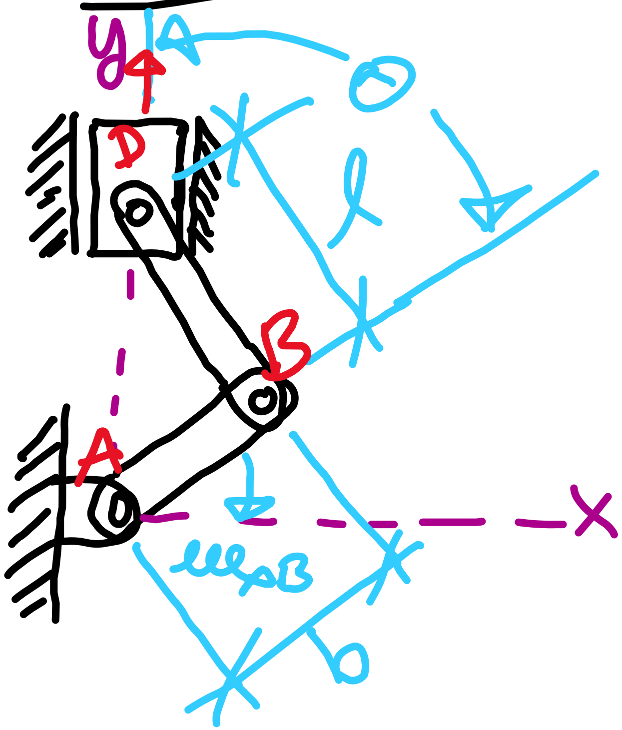

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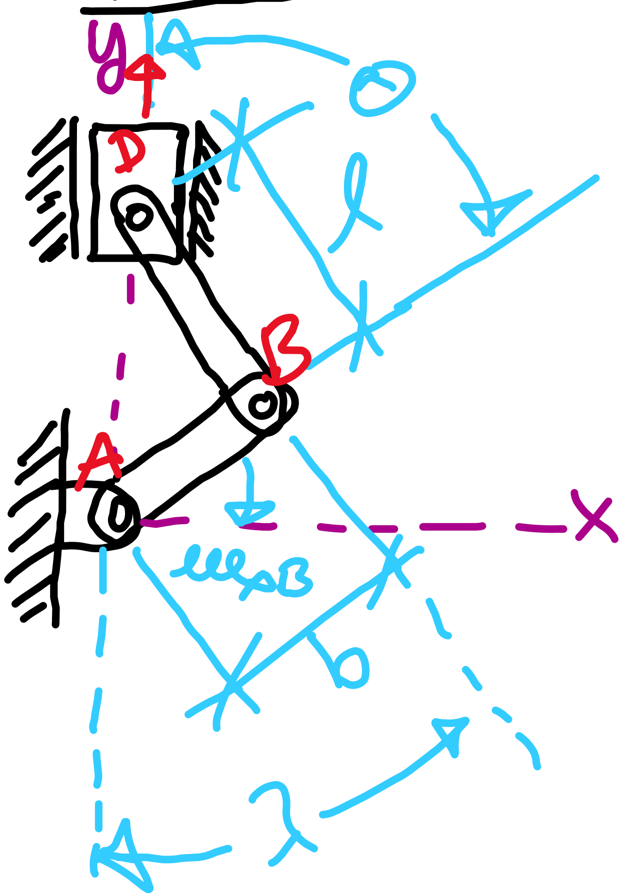

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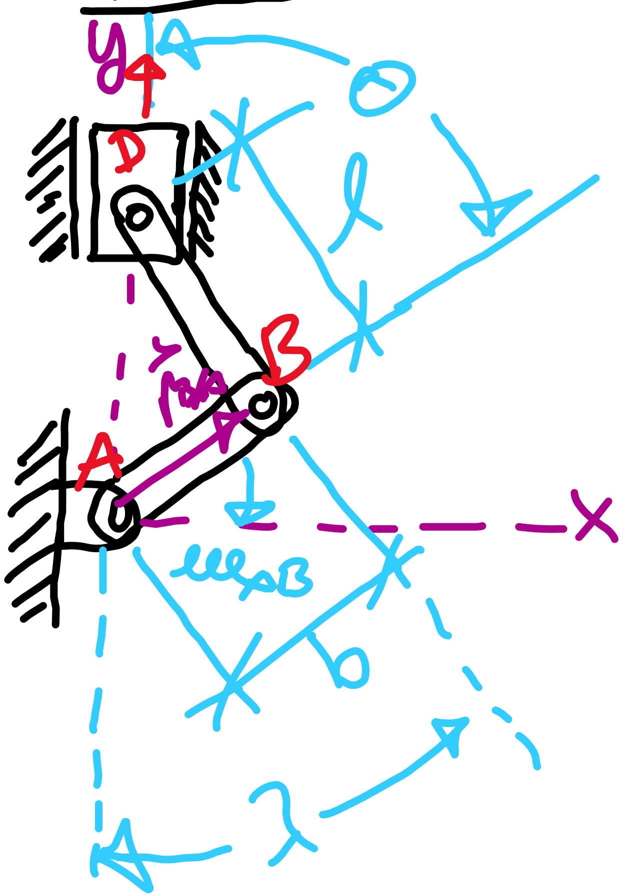

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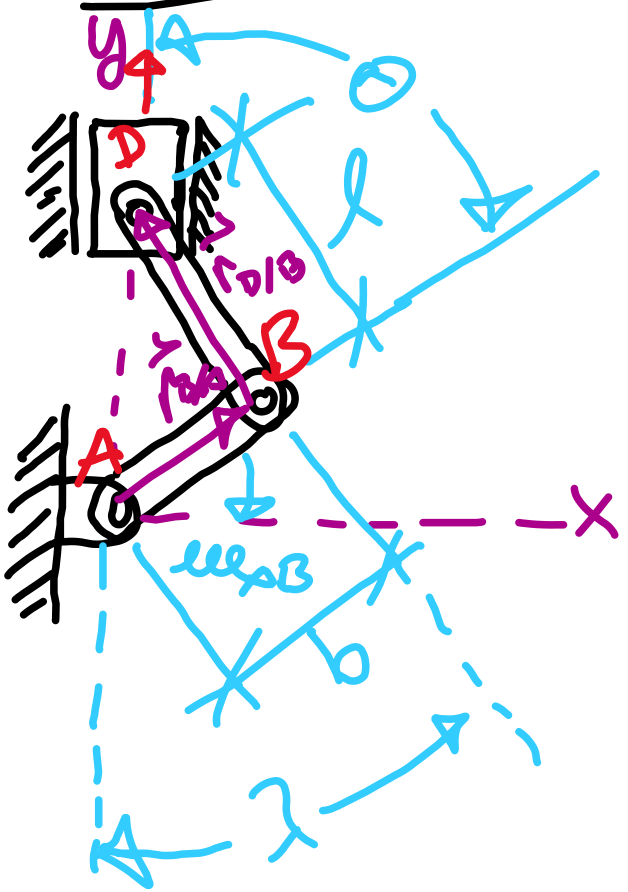

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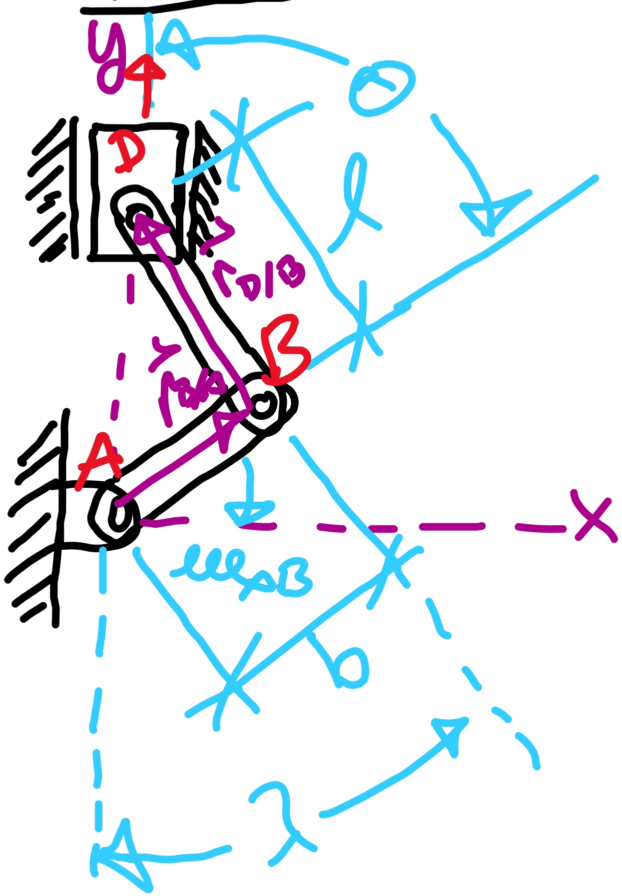

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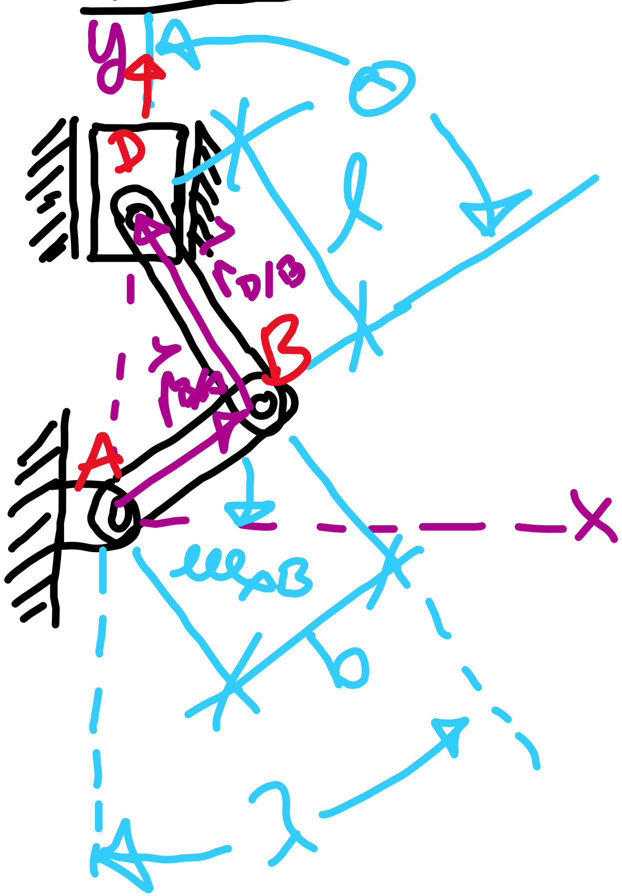

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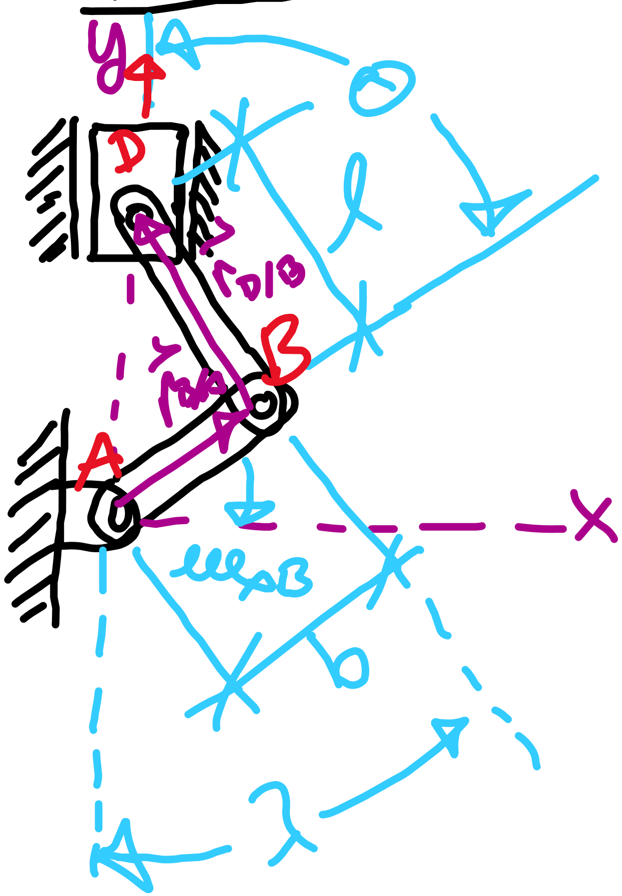

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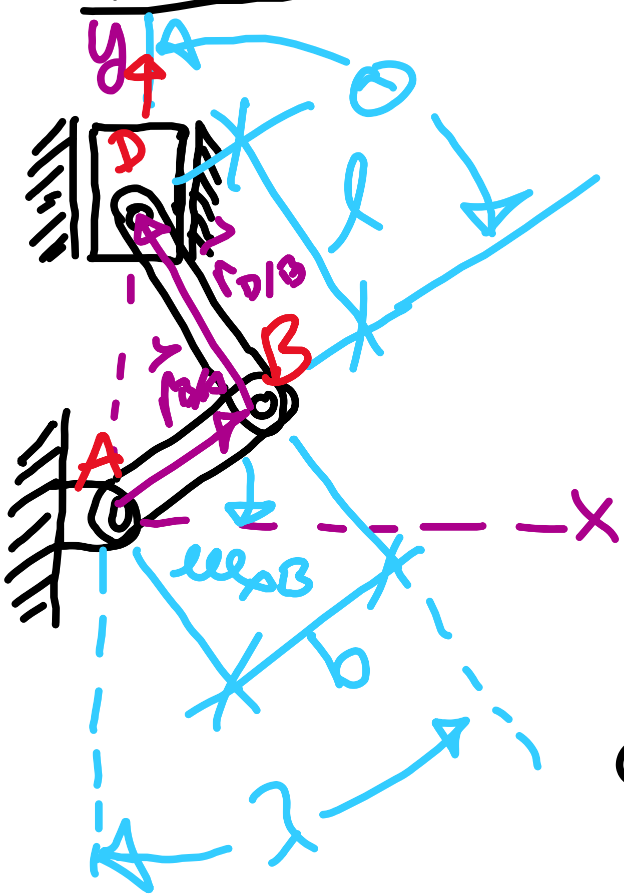

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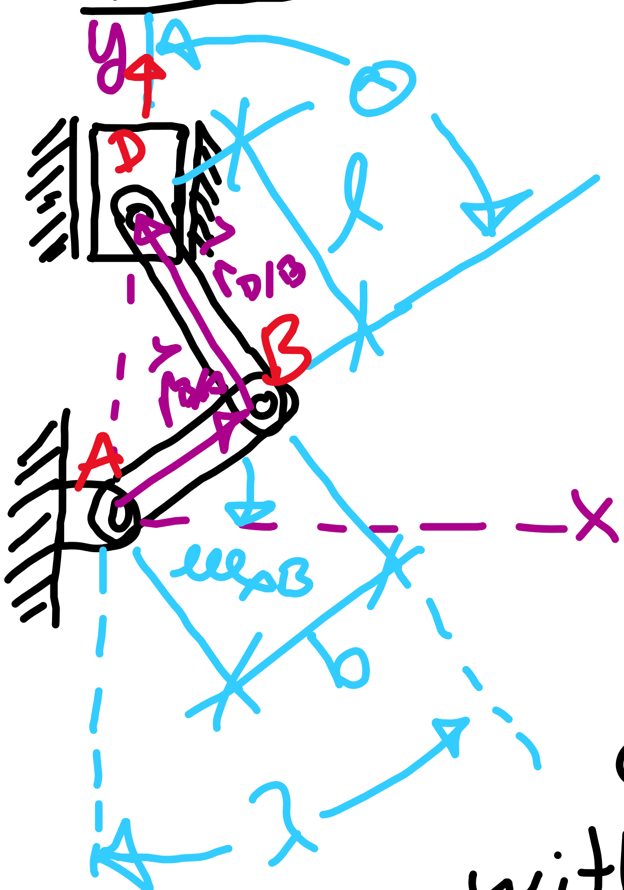

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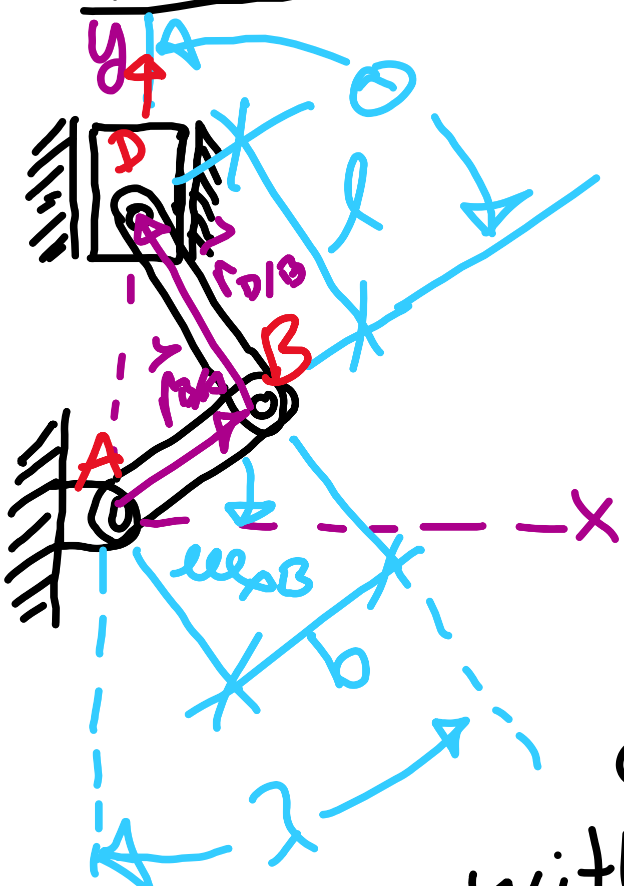

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Your eqn for $\vec{V}_D$ will have 3 unknowns $(\lambda, \vec{V}_D, \omega_{DB})$

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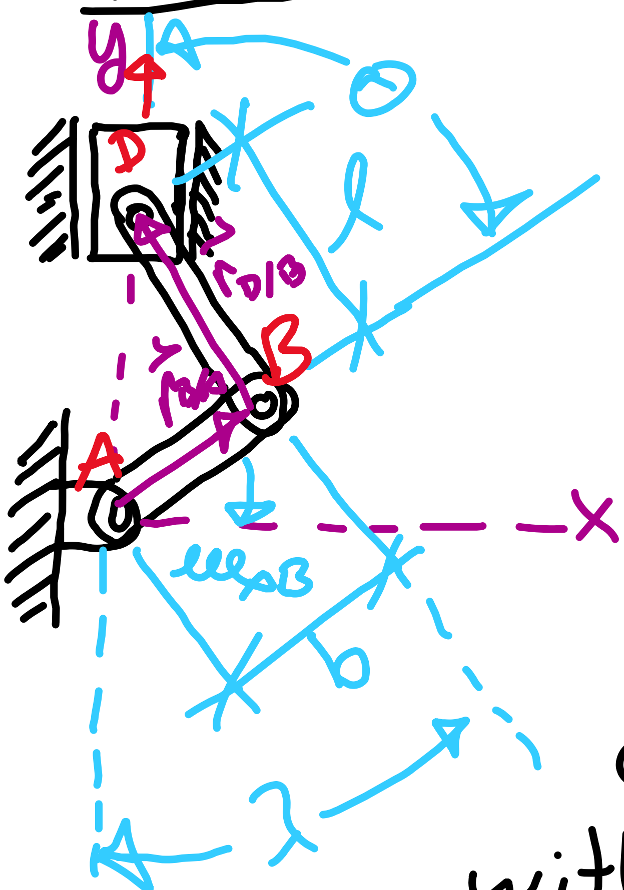

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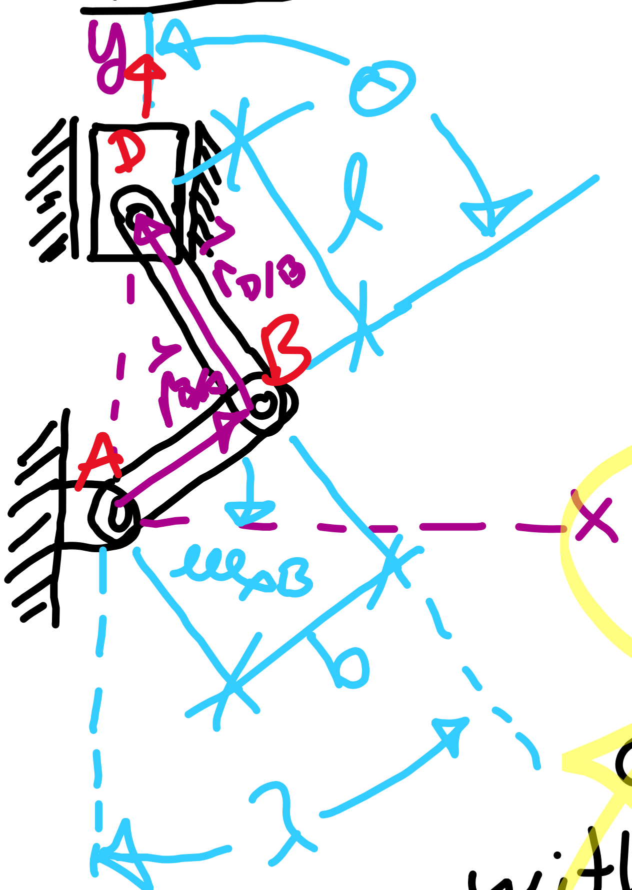

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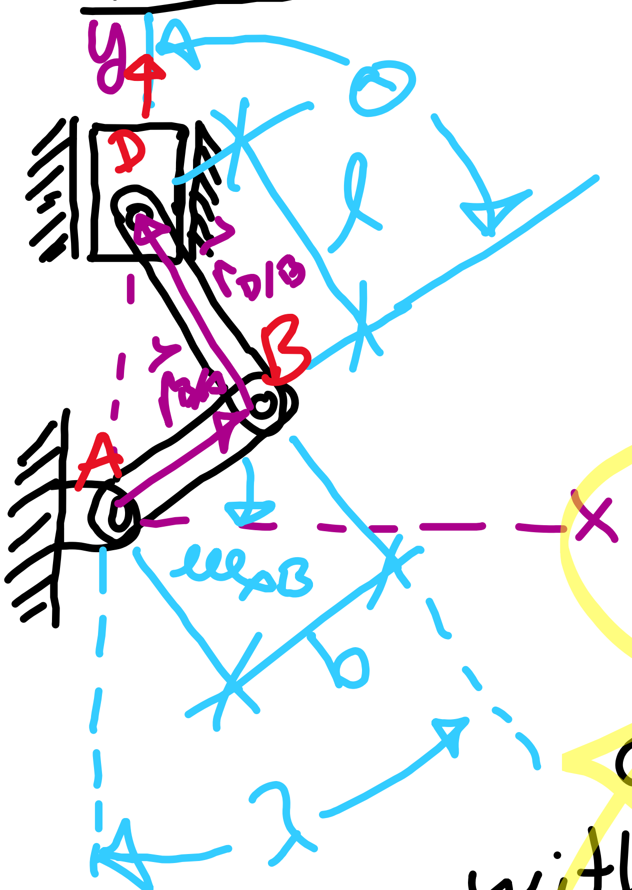

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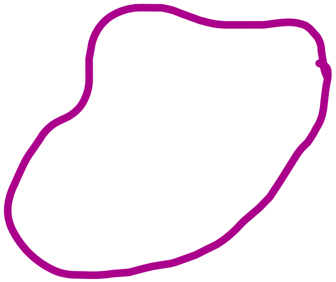
Your eqn for $\vec V_D$ will have 3 unknowns ($\lambda, \vec V_D, \omega_{DB}$) so you need two more equations. Here are the other two. When you set $V_{Dx} = 0$ you can get ω_{DB} and then use that for $\vec V_D = V_{Dy}\,\hat y$.

Instantaneous center of rotation

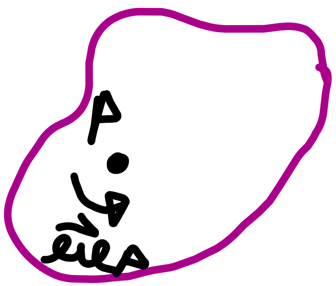
Instantaneous center of rotation

Take a rigid body



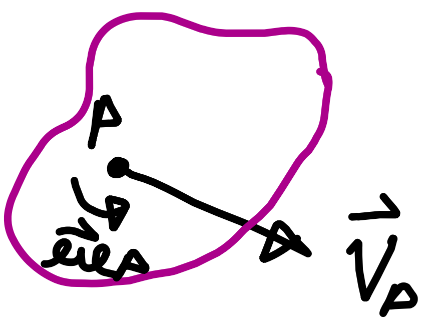
Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A



Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A that has a velocity $\vec{V}_A$



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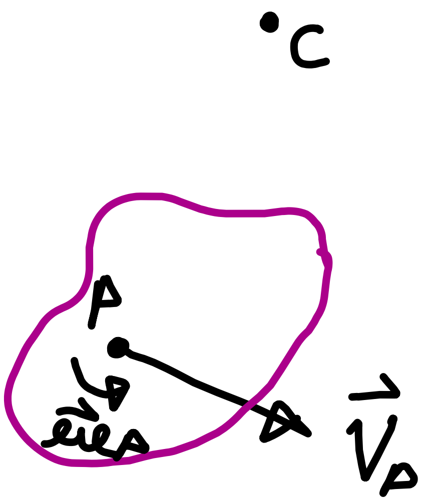

For that point A there exists another point C

Instantaneous center of rotation

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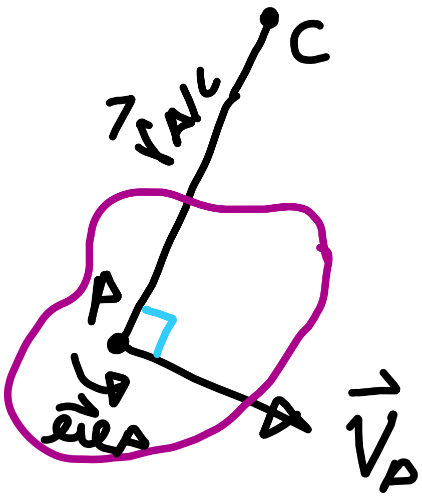

For that point A there exists another point C such that

$$\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$$

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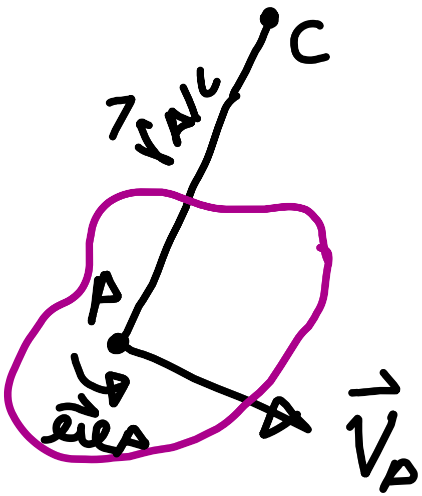


For that point A there exists another point C such that

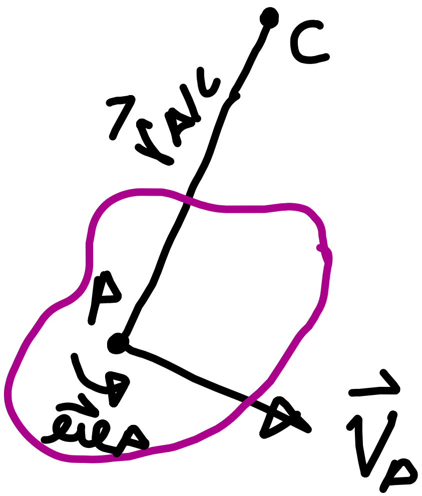
$$\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$$

Note:

since $\vec{r}_{A/C} \cdot \vec{V}_A = \vec{r}_{A/C} \cdot \underbrace{(\vec{\omega}_A \times \vec{r}_{A/C})}_{\text{perpendicular to both } \vec{\omega}_A \text{ \& } \vec{r}_{A/C}}$

Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A that has a velocity $\vec{V}_A$.



For that point A there exists another point C such that

$$\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$$

Note:

since $\vec{r}_{A/C} \cdot \vec{V}_A = \vec{r}_{A/C} \cdot (\vec{\omega}_A \times \vec{r}_{A/C}) = 0$

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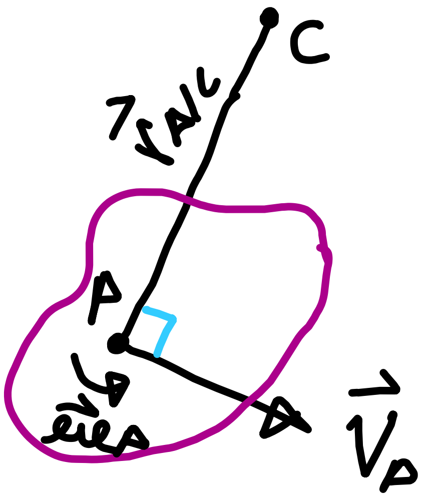

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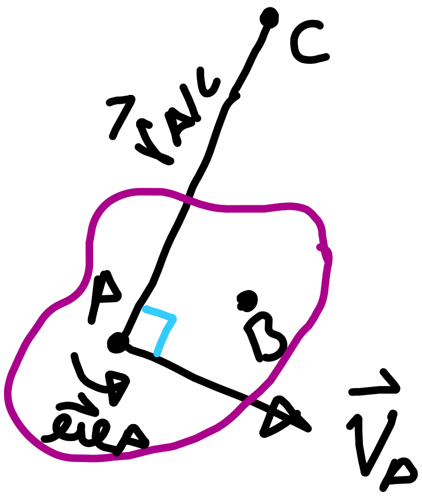
$\vec{V}_A$ & $\vec{r}_{A/C}$ are perpendicular. What about some other point on that rigid body?

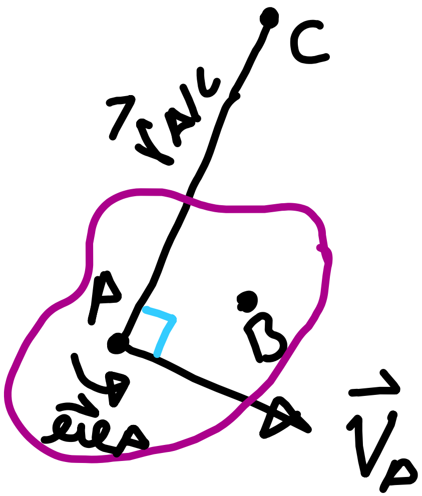
Instantaneous center of rotation

Take some point B on the rigid body



Instantaneous center of rotation

Take some point B on the rigid body. We have $\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{v}_B$:



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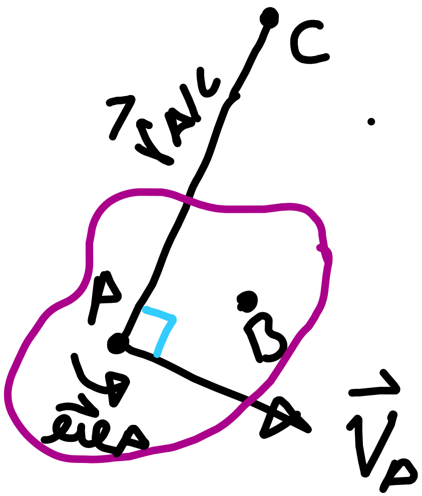
Take some point B on the rigid body. We have $\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{V}_B$:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$



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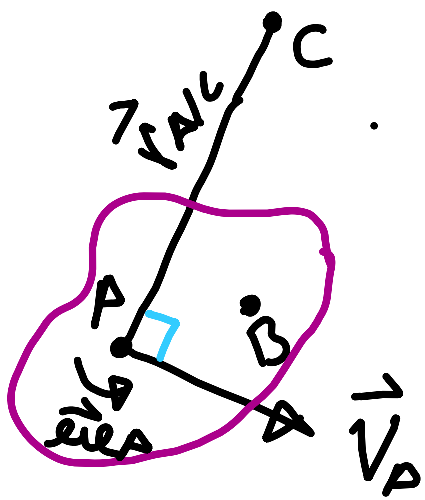

$$\vec{V}_B = \underbrace{\vec{V}_{B/A}}_{\text{relative velocity of B about point A}} + \vec{V}_A$$

Instantaneous center of rotation

Take some point B on the rigid body. We have $\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{v}_B$:

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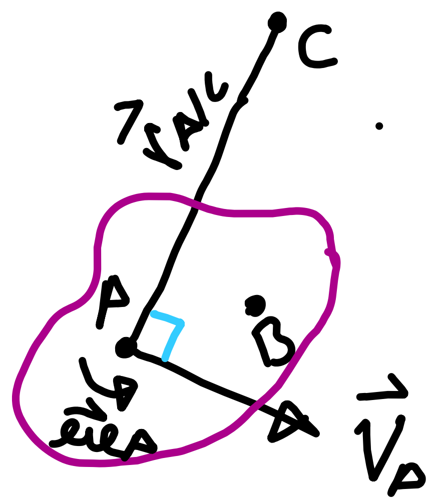

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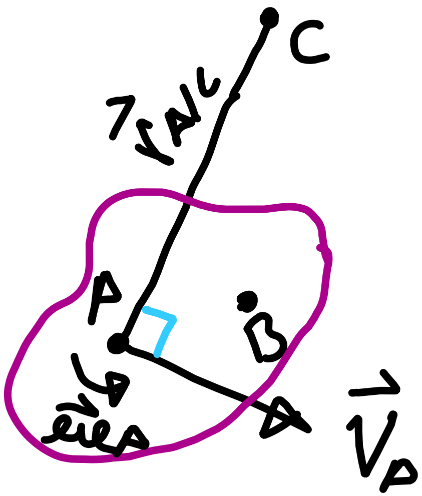

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = \vec{\omega}_A \times \vec{r}_{B/A} + \vec{\omega}_A \times \vec{r}_{A/C}$$

$$= \vec{\omega}_A \times (\vec{r}_{B/A} + \vec{r}_{A/C})$$

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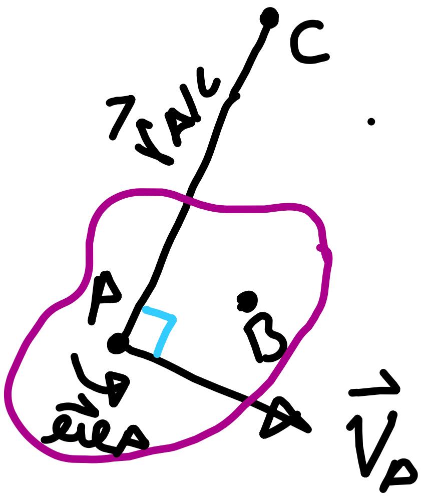

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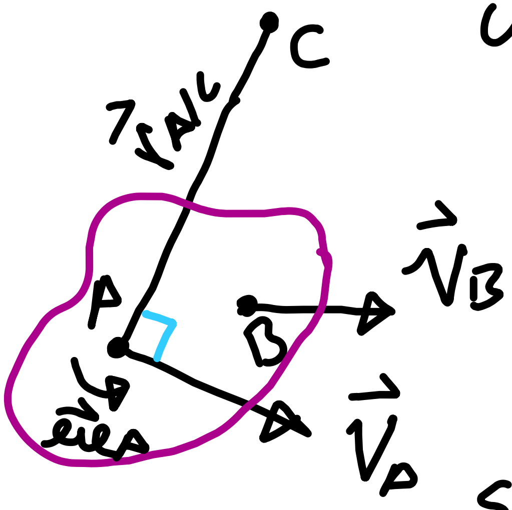

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So $\vec{v}_B =$

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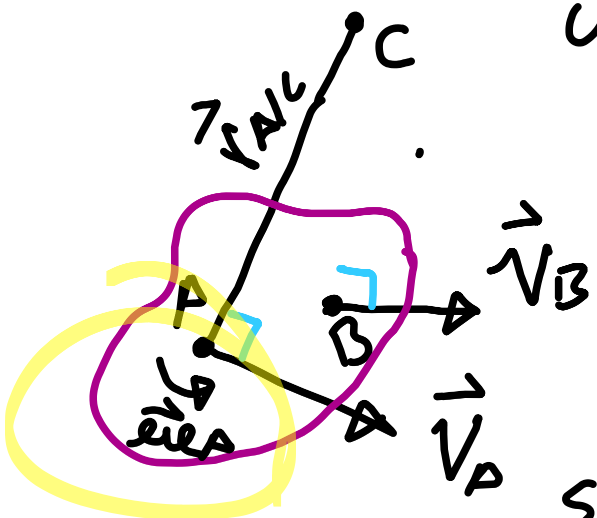

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So $\vec{v}_B = \vec{\omega}_A \times$

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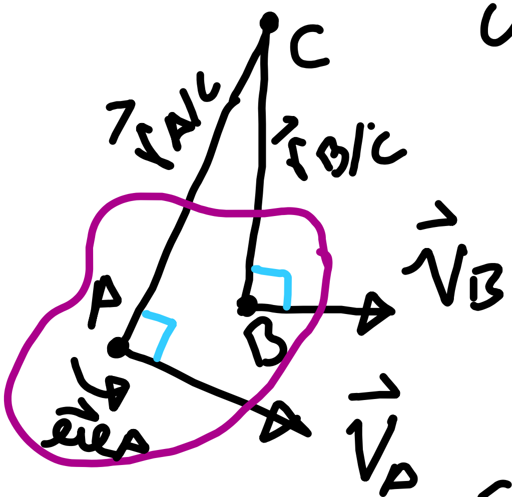

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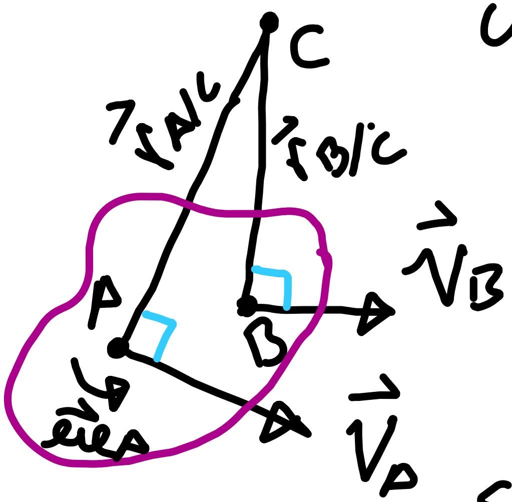
so $\vec{v}_B = \vec{\omega}_A \times \vec{r}_{B/C}$

Instantaneous center of rotation

Take some point B on the rigid body. We have $\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ &



want to construct $\vec{v}_B$:

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So $\vec{v}_B = \vec{\omega}_A \times \vec{r}_{B/C}$ We now have $\vec{v}_B$ constructed as a rotation about point C.

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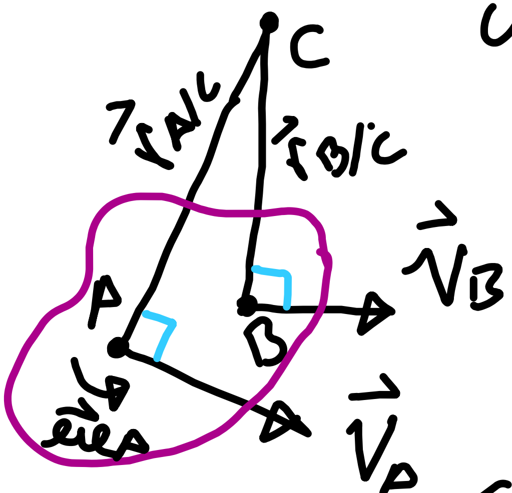

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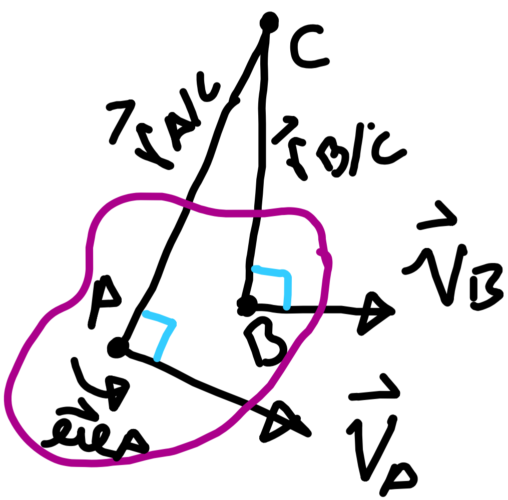
So $\vec{v}_B = \vec{\omega}_A \times \vec{r}_{B/C}$ We now have $\vec{v}_B$ constructed as a rotation about point C. Since B was arbitrary, this will work for <u>any</u> point on the rigid body!

ASU

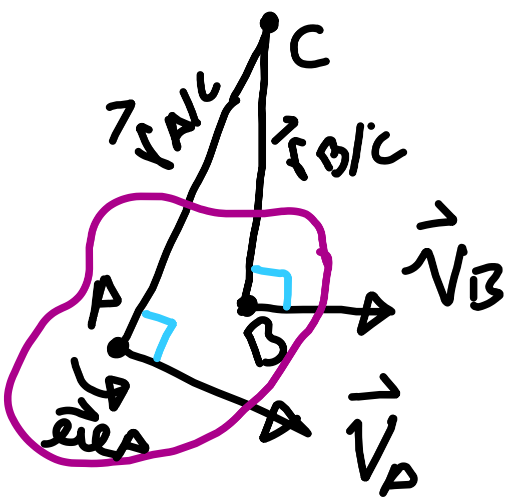
Instantaneous center of rotation

∃ for this one instance in time



Instantaneous center of rotation

For this one instance in time we have a single center of rotation for all points on the rigid body.



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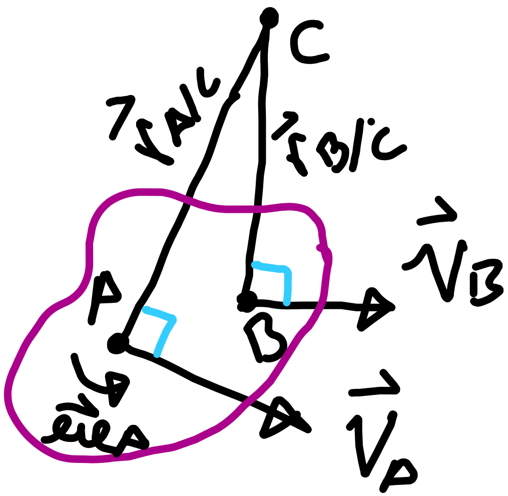
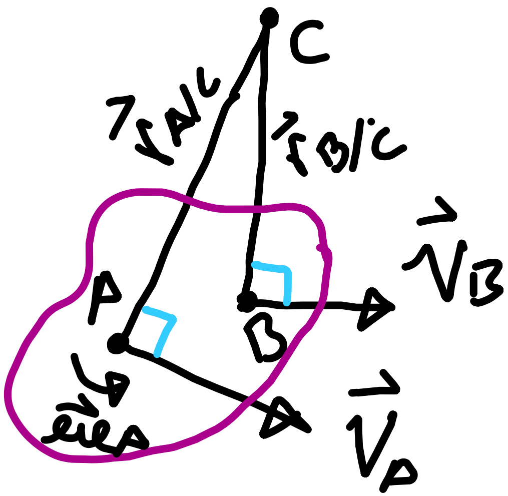


Instantaneous center of rotation

For this one instance in time we have a single center of rotation for all points on the rigid body. As with point A, the velocity of point B is perpendicular to the line connecting it to point C



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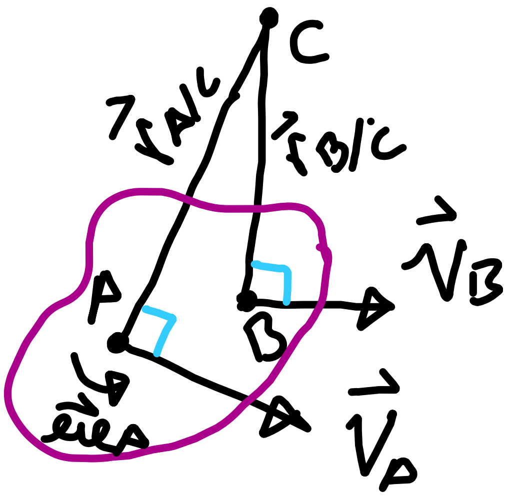

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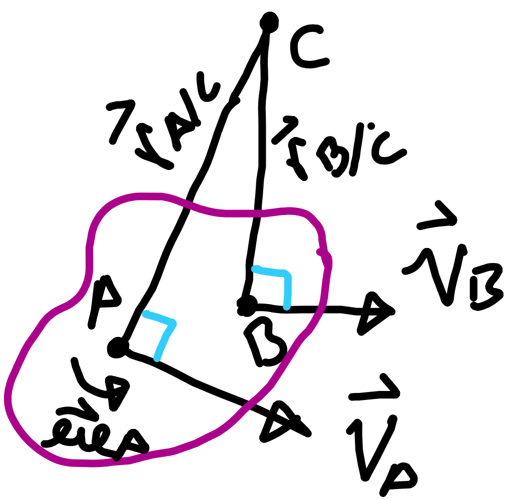

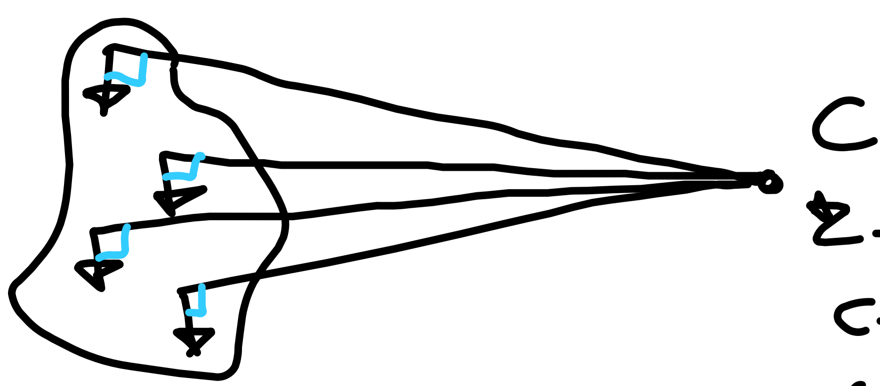

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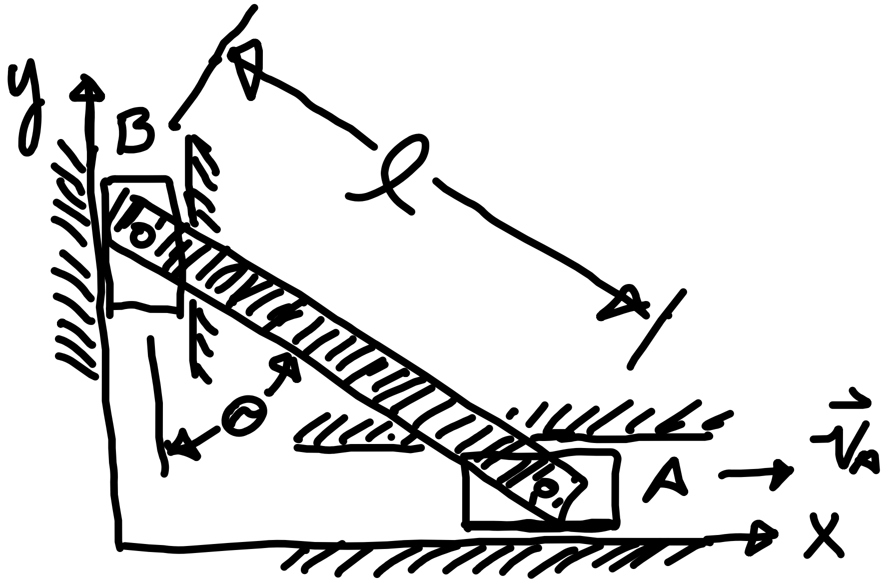

Example

Given ℓ, $\vec{V}_A = V_A \hat{x}$ &
θ

Find $\vec{V}_B$ & ω_{ARM}:

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Given l, $\vec{V}_A = V_A \hat{x}$ & θ

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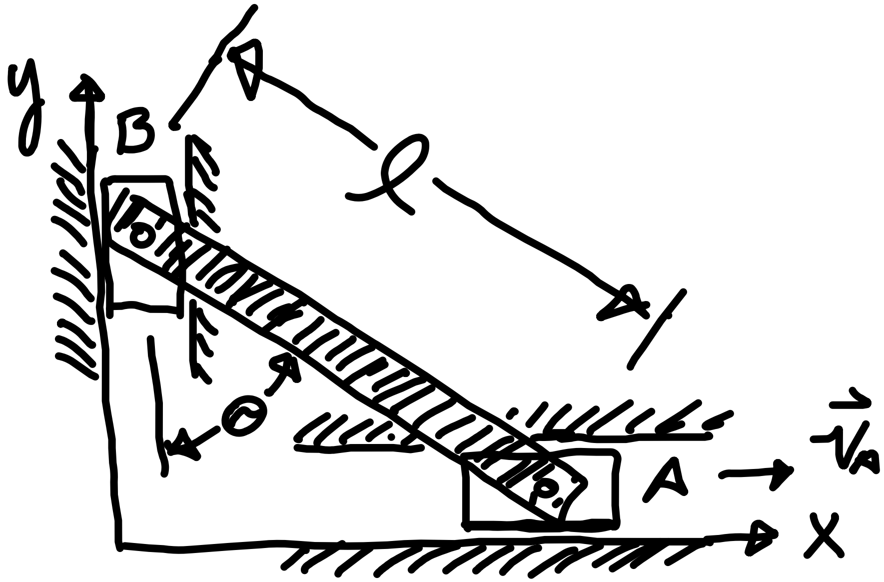

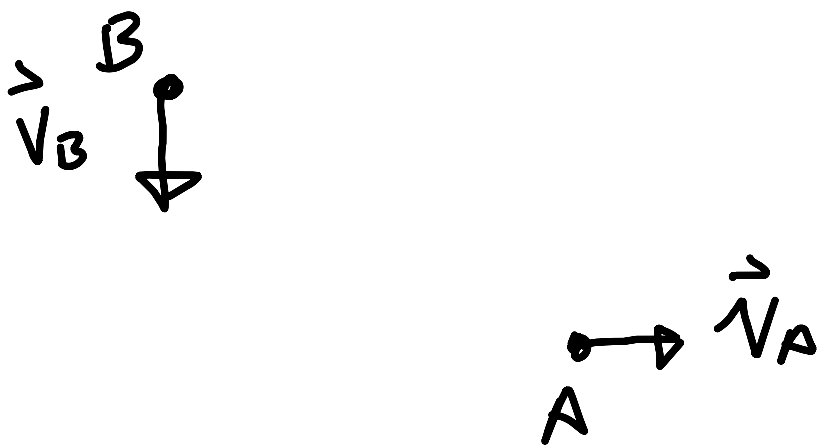

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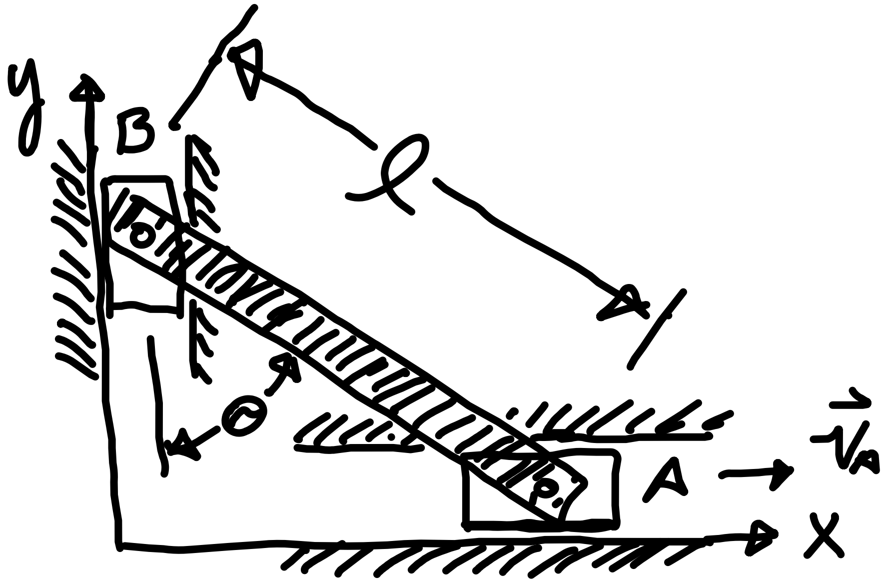

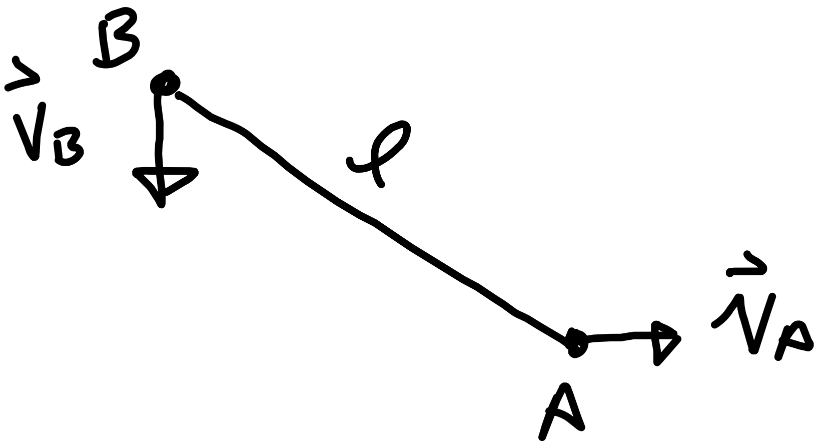

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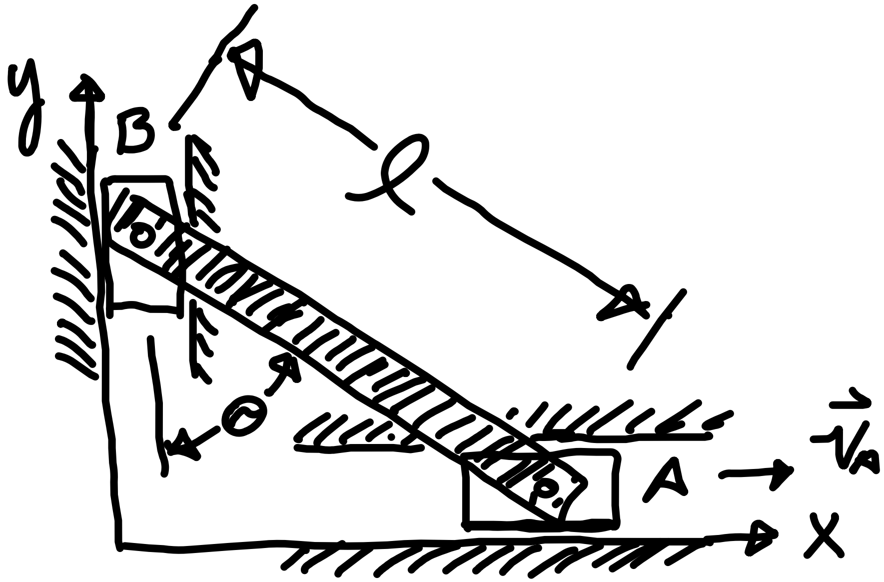

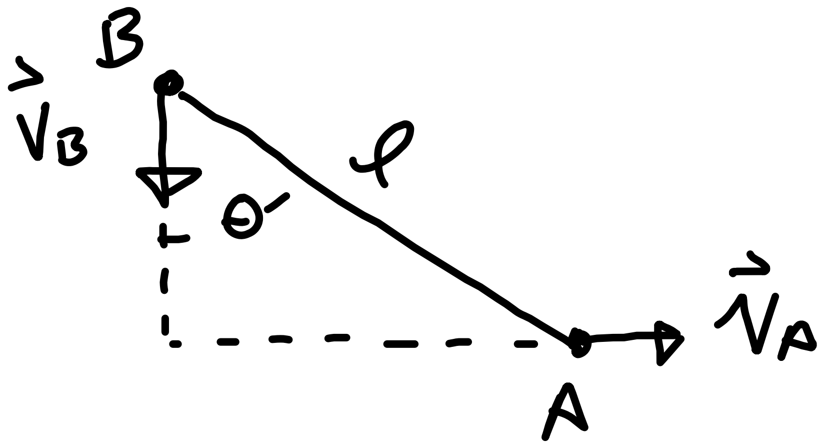

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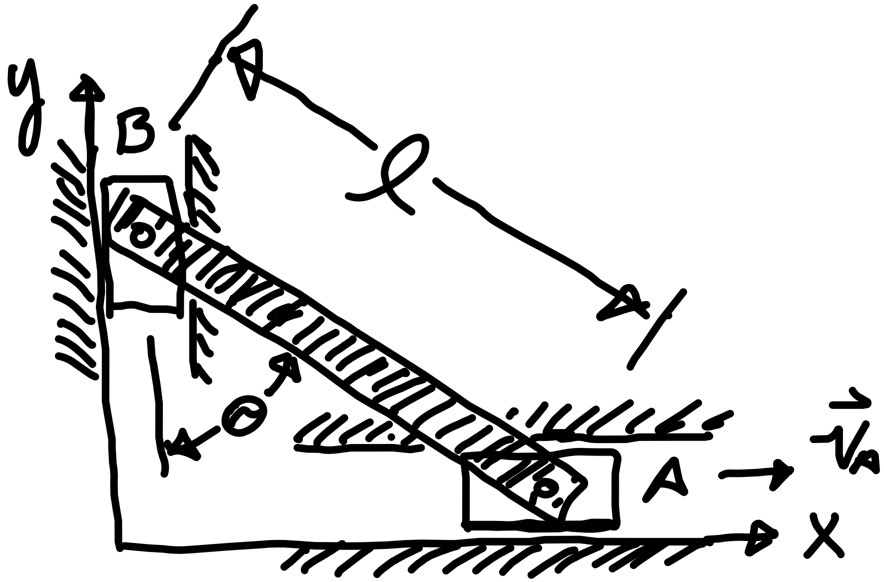

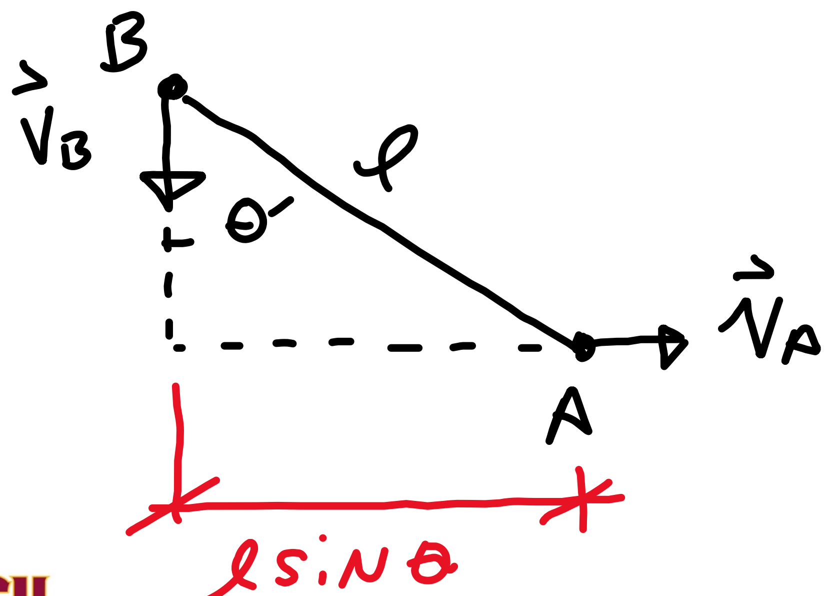

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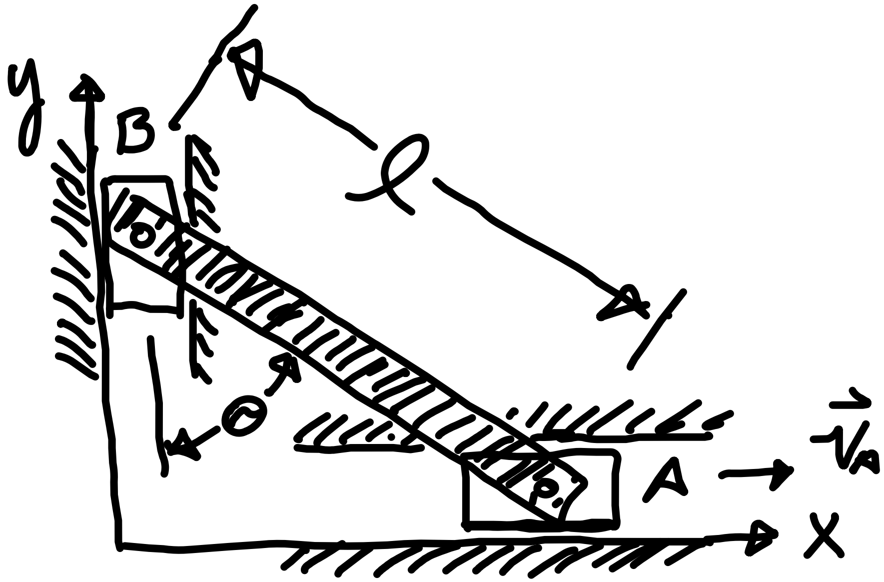

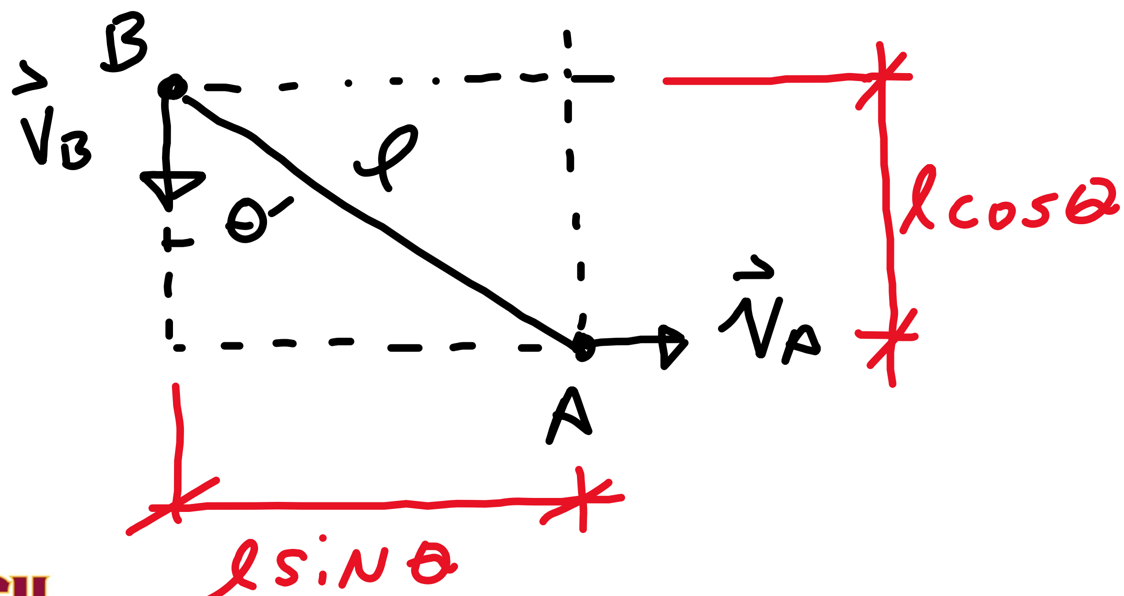

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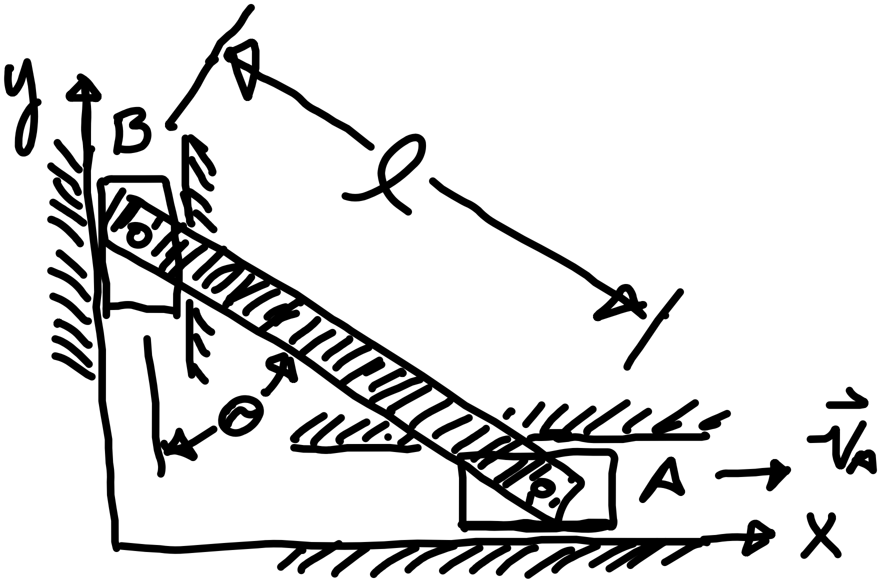
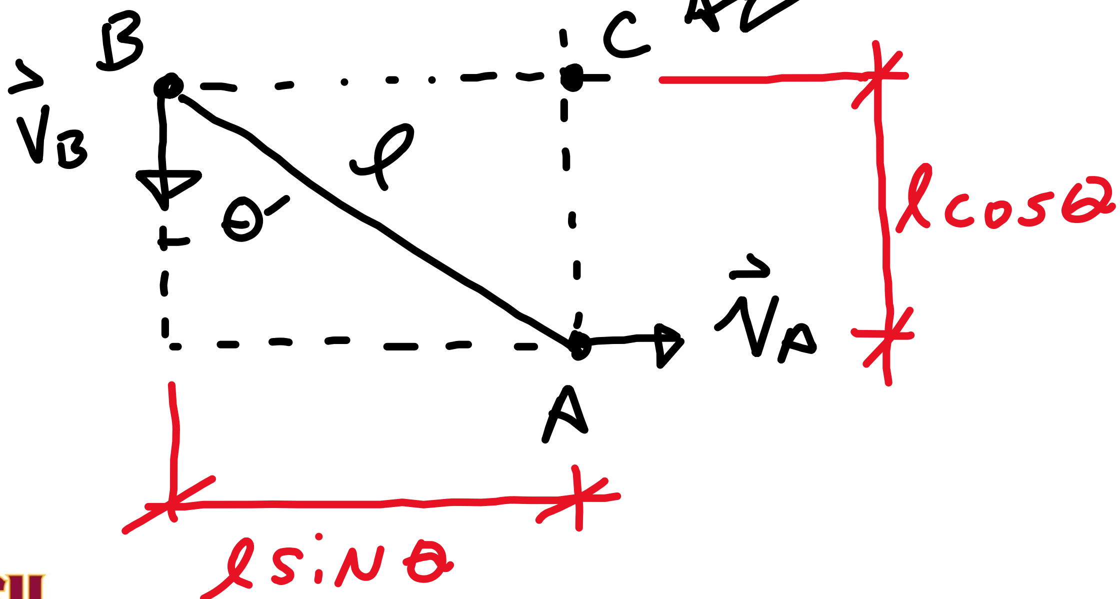


C ← instantaneous center of rotation



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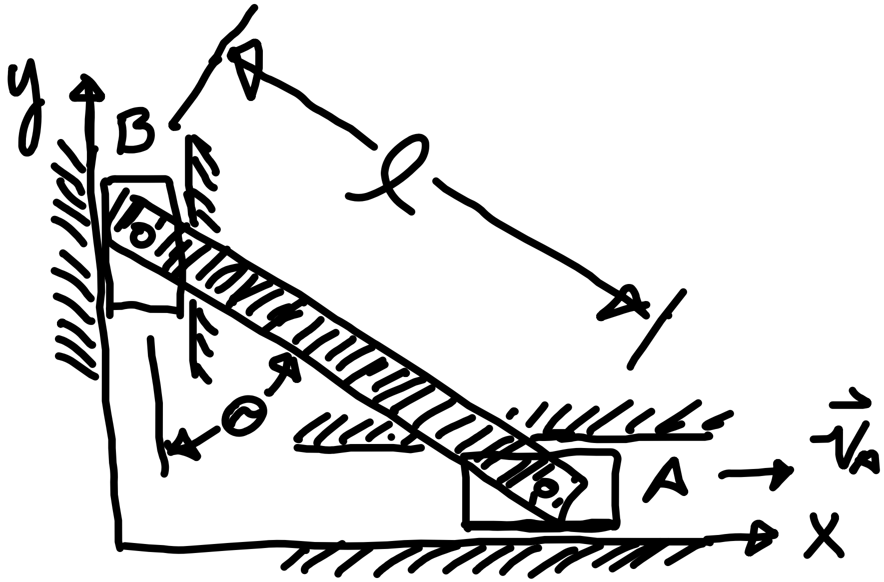

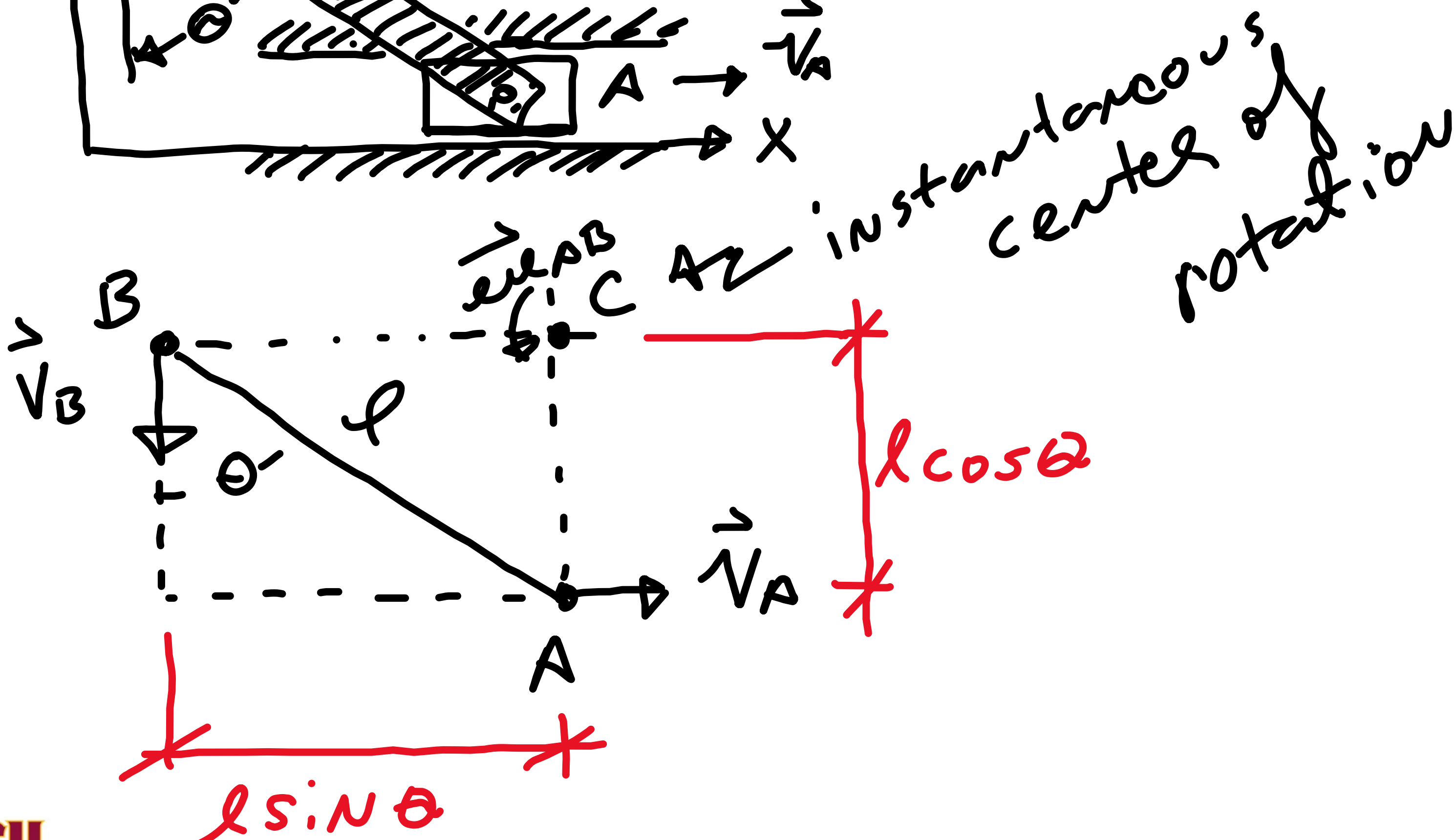

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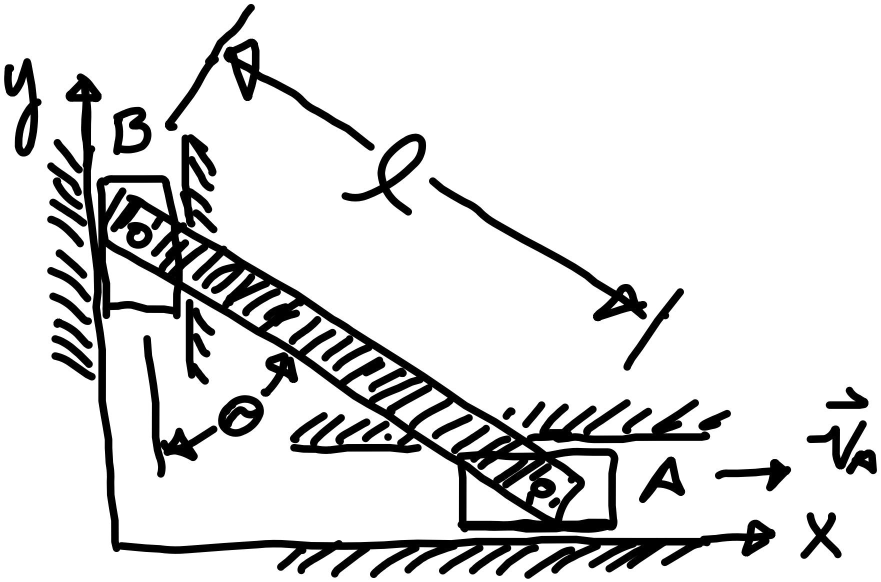

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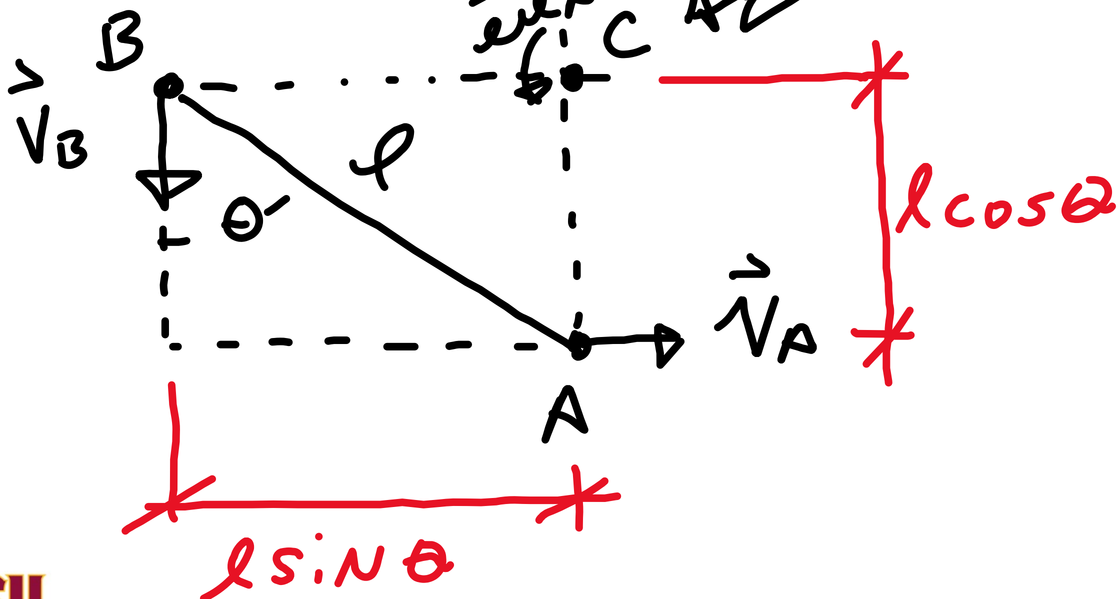

so

$V_A = \omega_{AB}\, l \cos\theta$

Example

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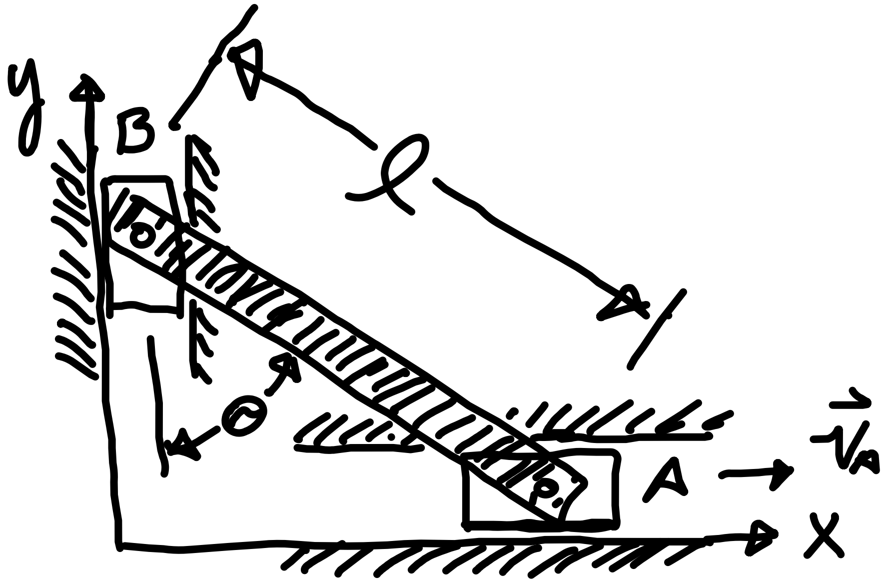
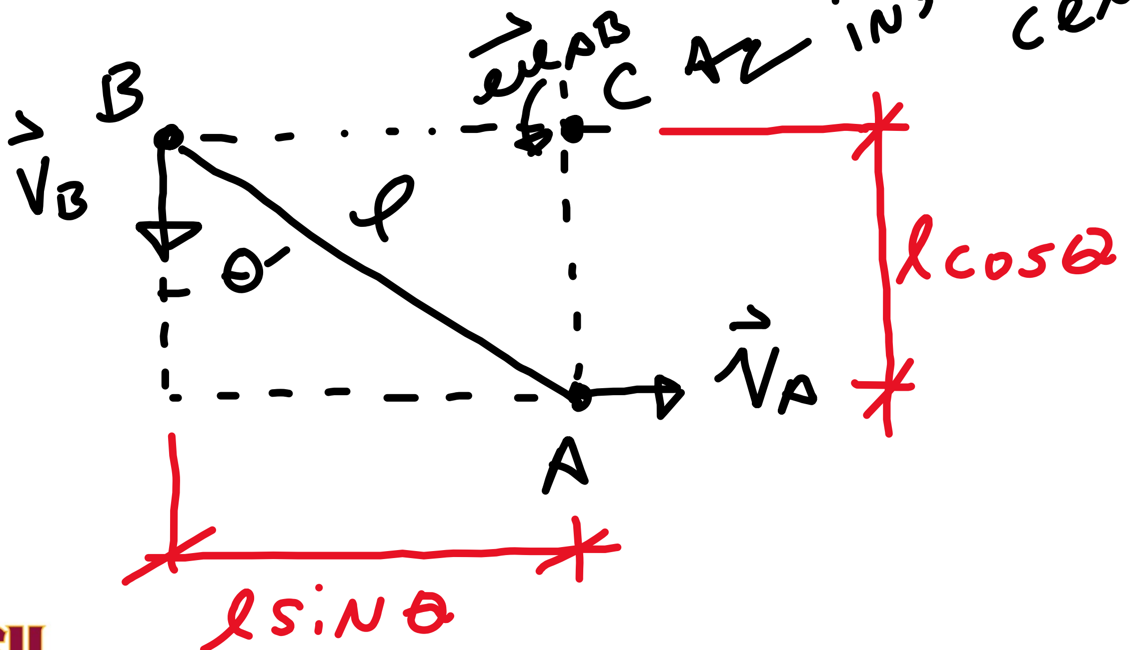


instantaneous center of rotation



so

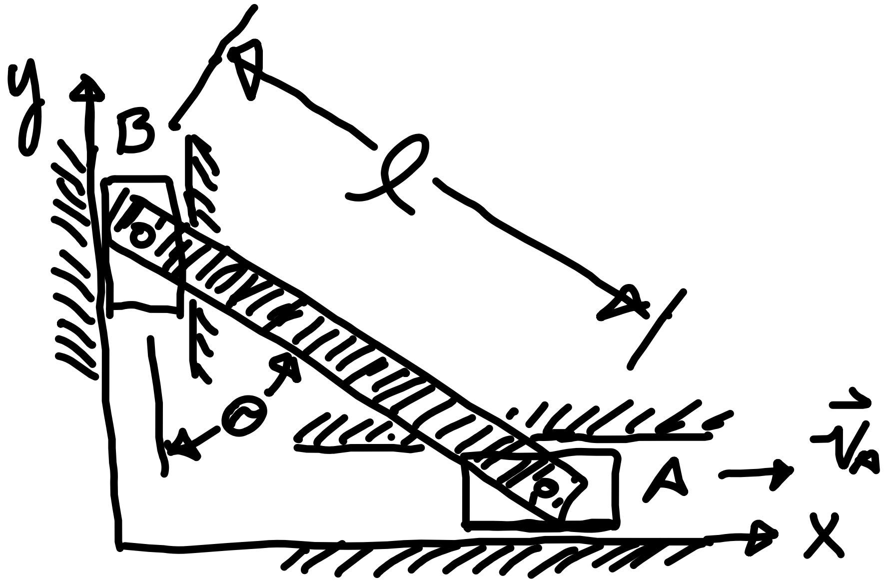
$V_A = \omega_{AB}\, l\cos\theta$

$$\Rightarrow \boxed{\omega_{AB} = \frac{V_A}{l\cos\theta}}$$

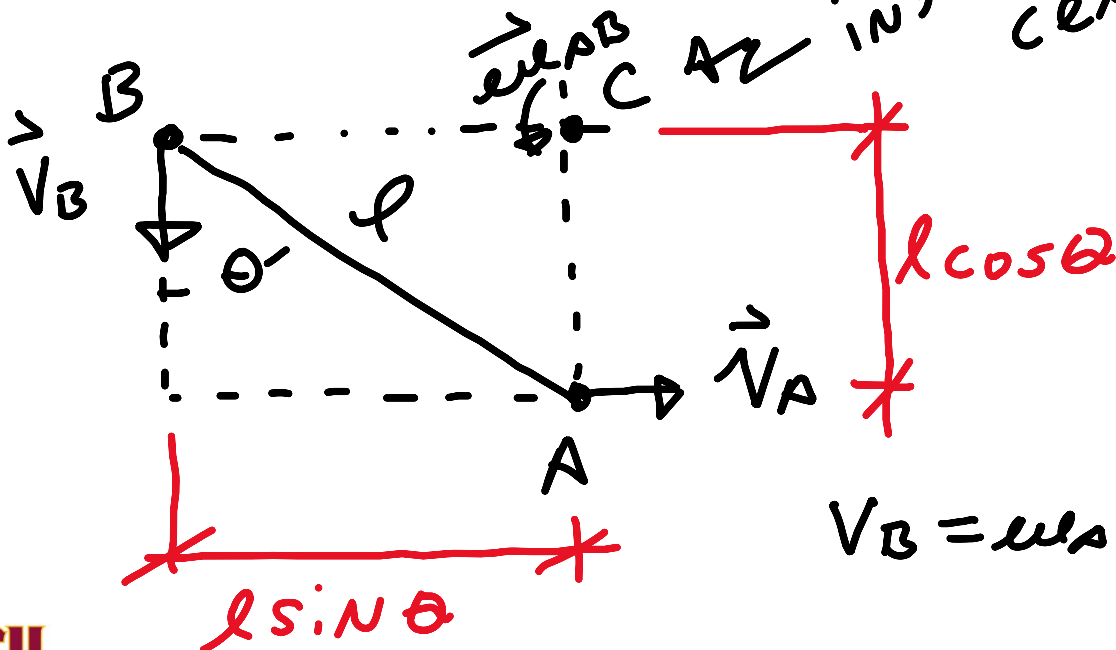
Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



C A: instantaneous center of rotation



so

$V_A = \omega_{AB}\, l\cos\theta$

$\Rightarrow \boxed{\omega_{AB} = \frac{V_A}{l\cos\theta}}$

$V_B = \omega_{AB}\, l\sin\theta$

Example

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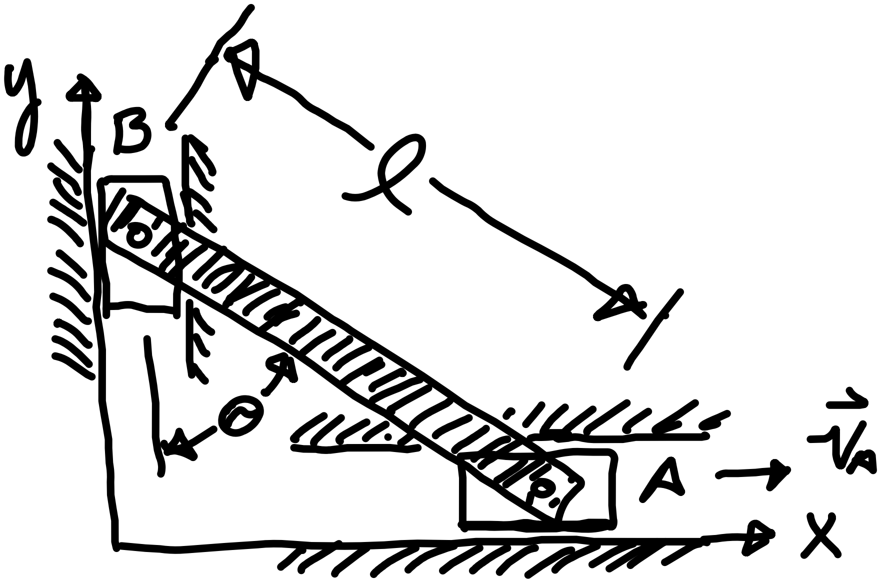
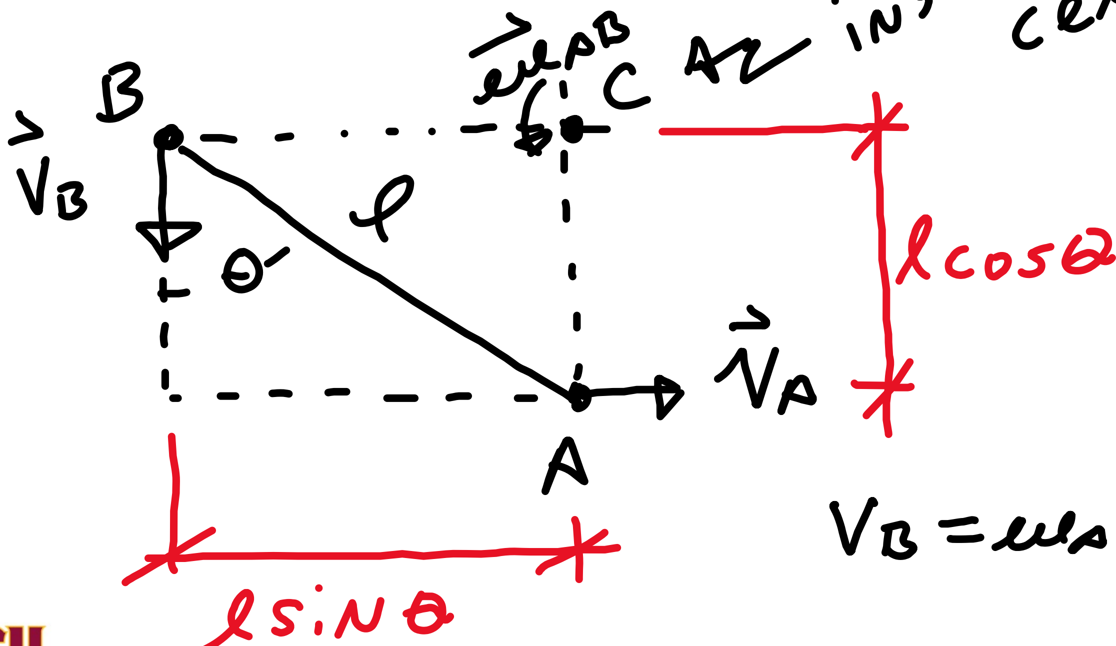
Find $\vec{V}_B$ & ω_{ARM}:





instantaneous center of rotation

so

$V_A = \omega_{AB}\, l\cos\theta$

$\Rightarrow \boxed{\omega_{AB} = \frac{V_A}{l\cos\theta}}$

$V_B = \omega_{AB}\, l\sin\theta = \frac{V_A}{l\cos\theta}\, l\sin\theta$

Example

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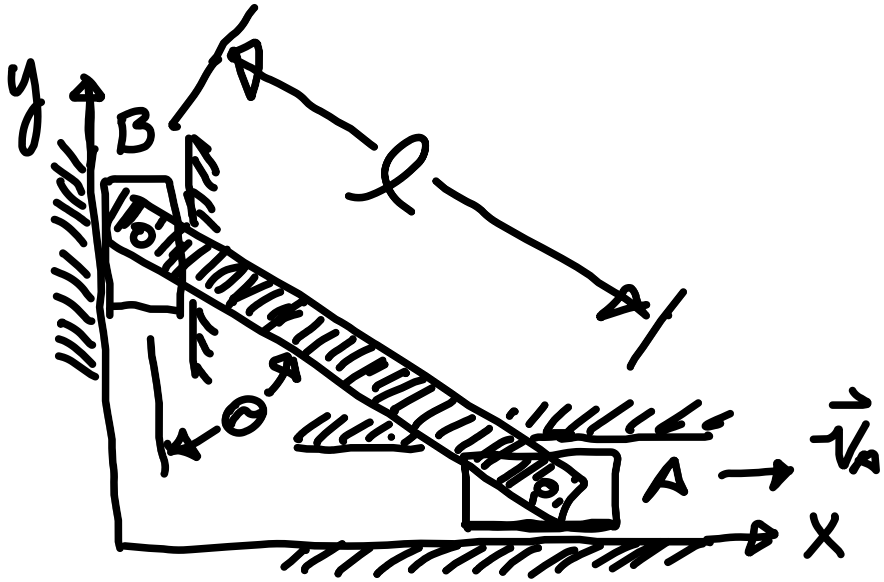

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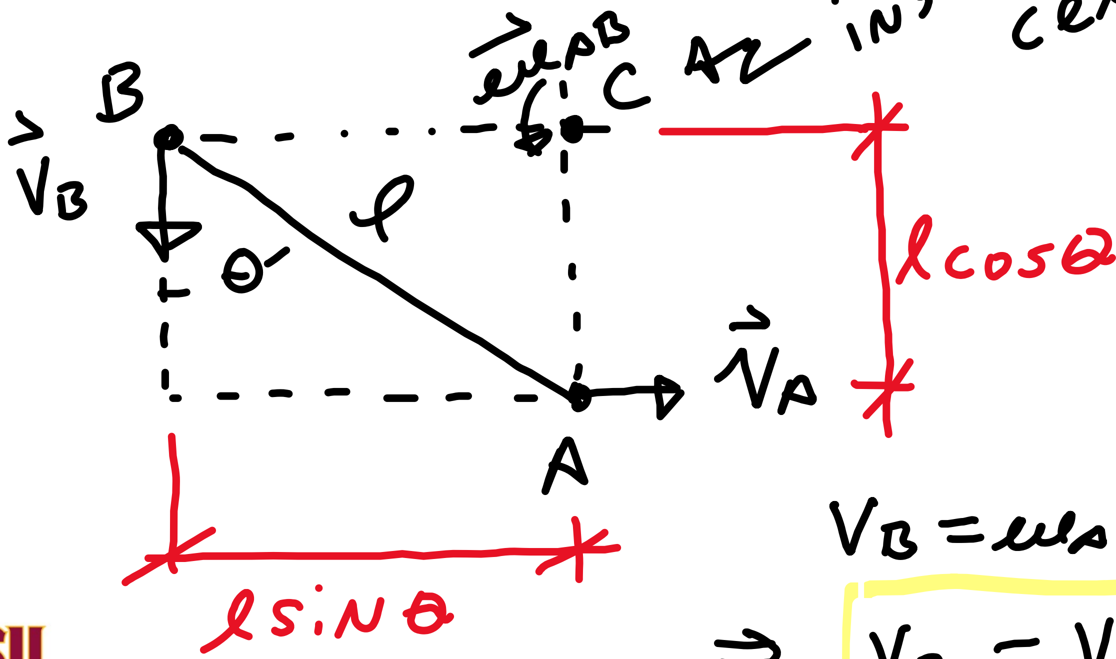


so

$$V_A = \omega_{AB}\, l \cos\theta$$

$$\Rightarrow \boxed{\omega_{AB} = \frac{V_A}{l\cos\theta}}$$

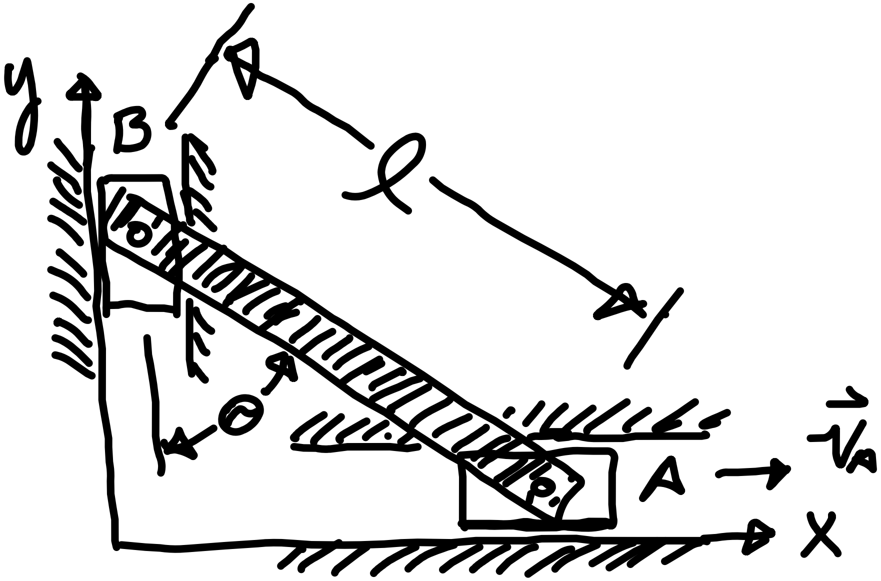
$$V_B = \omega_{AB}\, l \sin\theta = \frac{V_A}{l\cos\theta}\, l\sin\theta$$

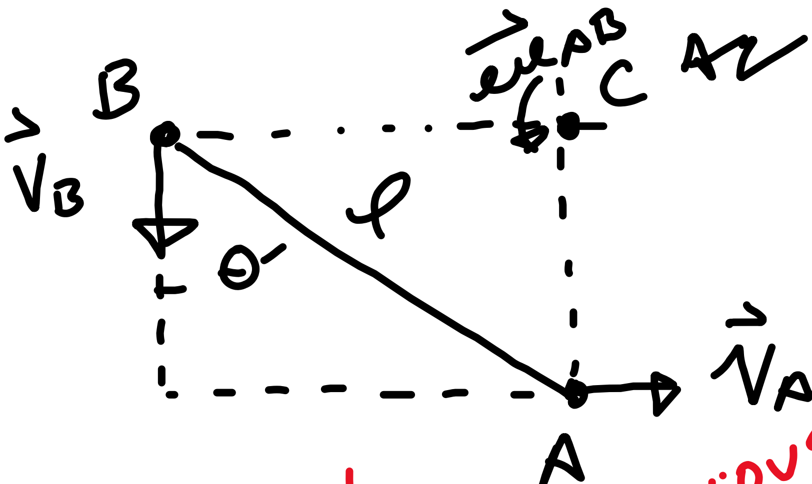
$$\Rightarrow \boxed{V_B = V_A \tan\theta}$$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:





C AZ instantaneous center of rotation

so

$V_A = \omega_{AB}\, l\cos\theta$

$\Rightarrow \boxed{\omega_{AB} = \frac{V_A}{l\cos\theta}}$

$V_B = \omega_{AB}\, l\sin\theta = \frac{V_A}{l\cos\theta}\, l\sin\theta$

Easier to solve than previous method $\Rightarrow$ $\boxed{V_B = V_A\tan\theta}$