Today 16.1

$\boxed{L23}$

ASU

Today 16.1

$\boxed{L\,23}$

Kinetics of a rigid body

Today 16.1

Wednesday 16.1

$\boxed{L23}$

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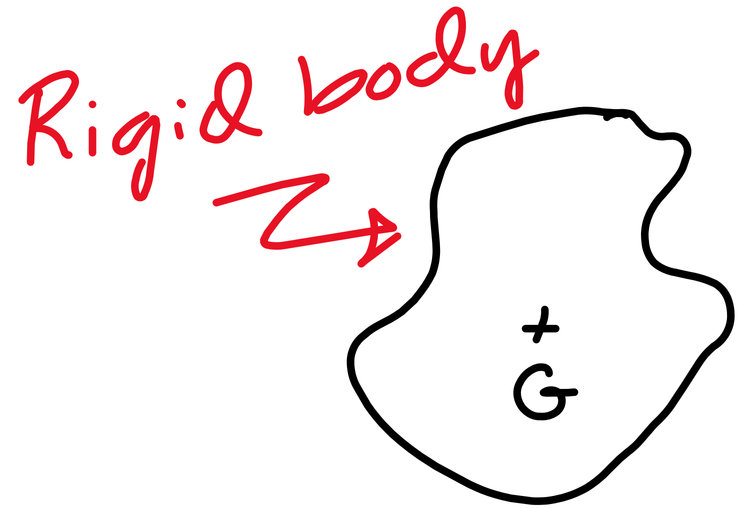
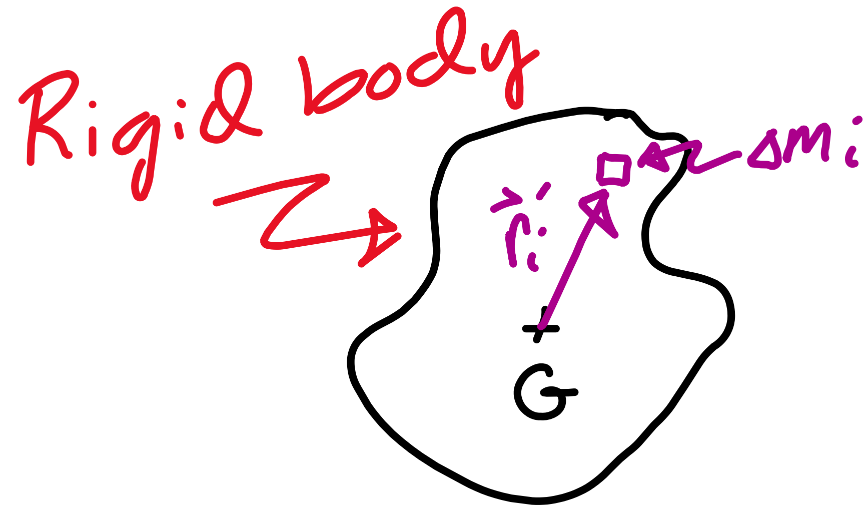
<u>Note</u>: We are going to take the rigid body as being made up of an ∞ number of point particles


Rigid body
G

We can write

$$\vec{H}_G = \sum_i \left(\vec{r}_i{}' \times \Delta m_i \vec{V}_i{}'\right)$$



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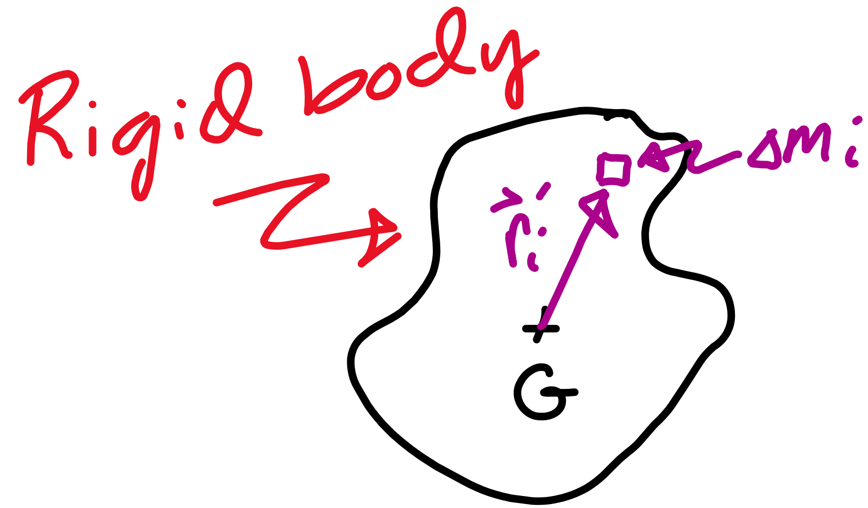
For plane motion

$$\vec{r}_i' \times \vec{V}_i' = r_i v_i$$



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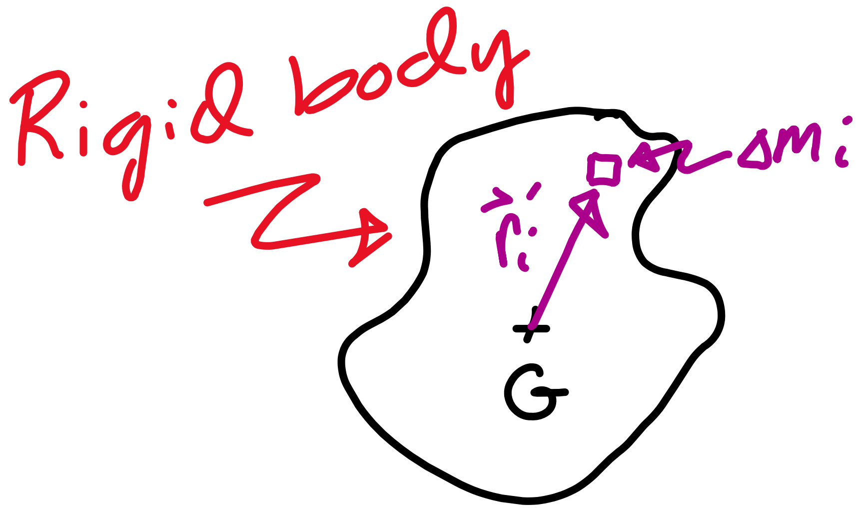

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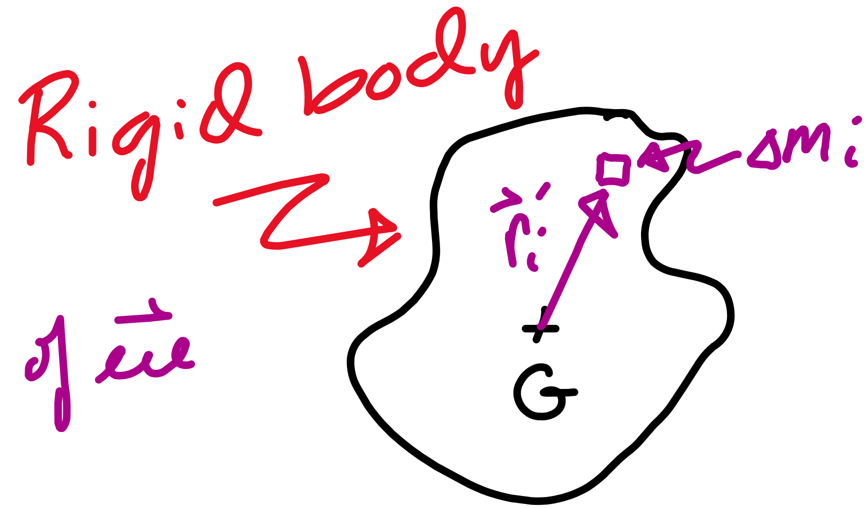

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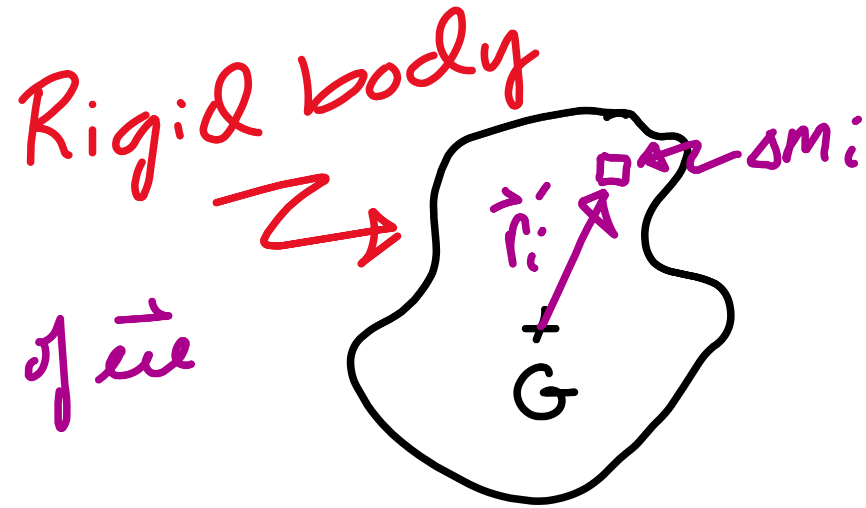

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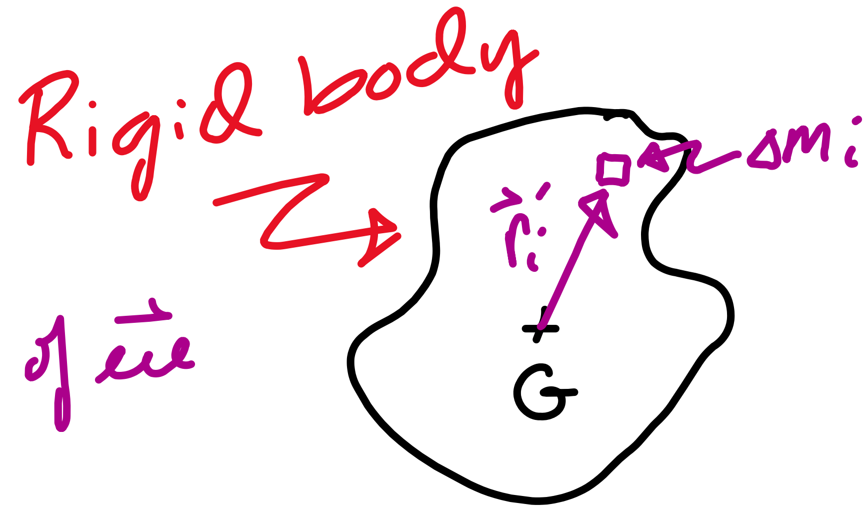

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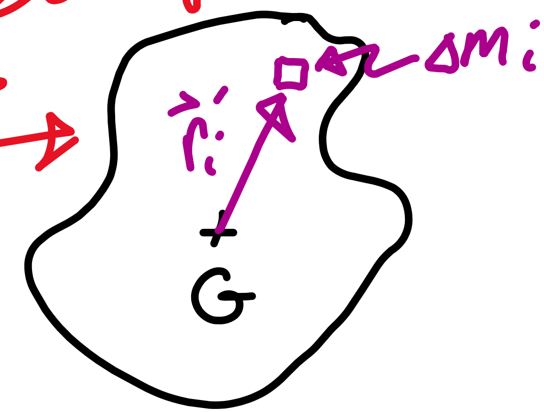
Rigid body



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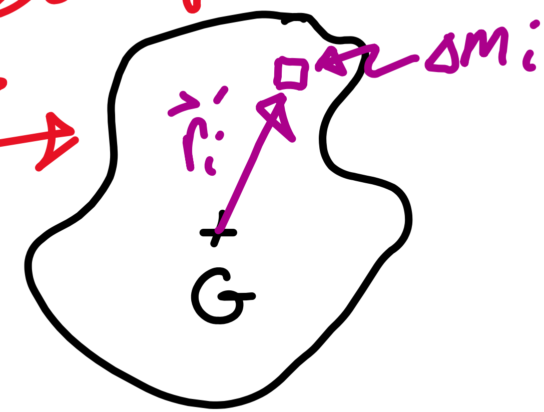

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$\bar{I} \equiv$ Moment of inertia about G

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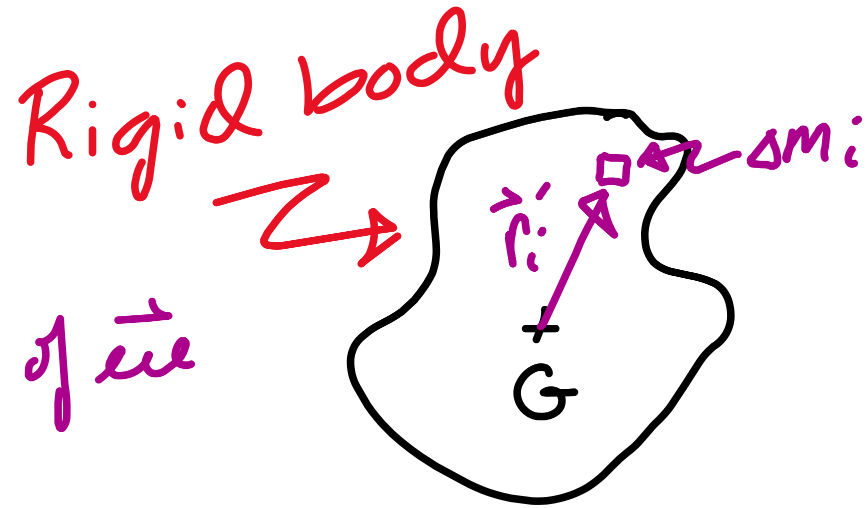

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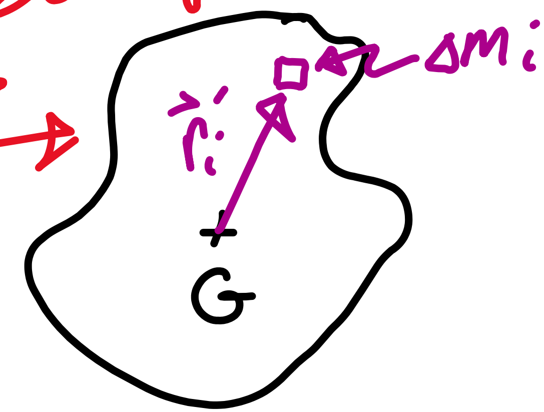

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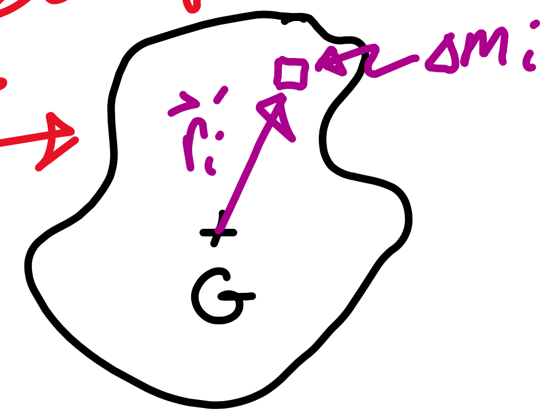

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If mass is uniform over 2-d surface

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If mass is uniform over 2-d surface with area A

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ASU

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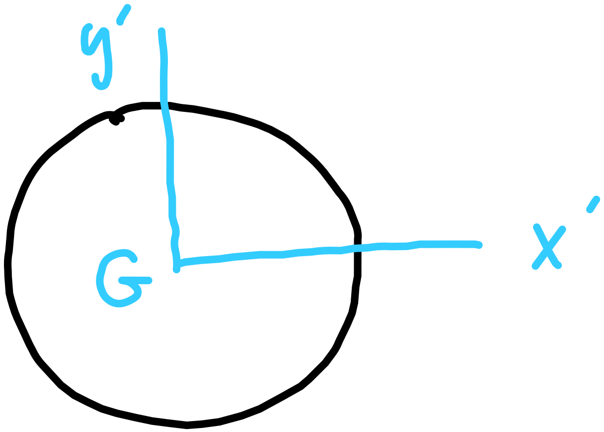
For polar coordinates

$$dx' dy' \to r' dr' d\theta'$$

Uniform disk

Uniform disk



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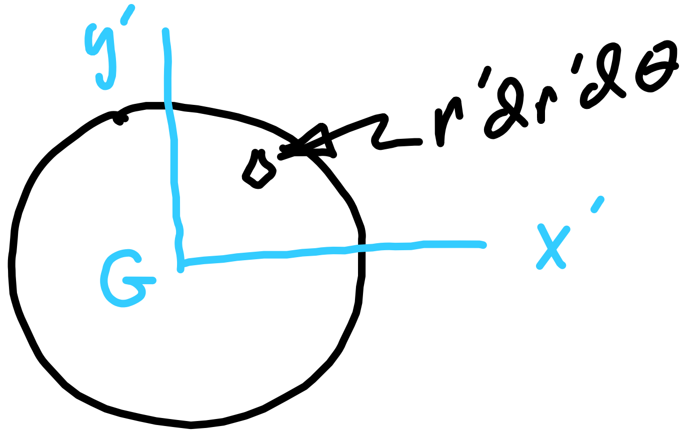
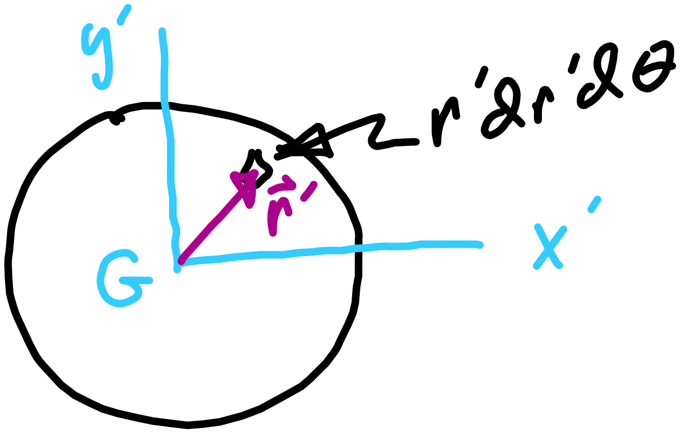
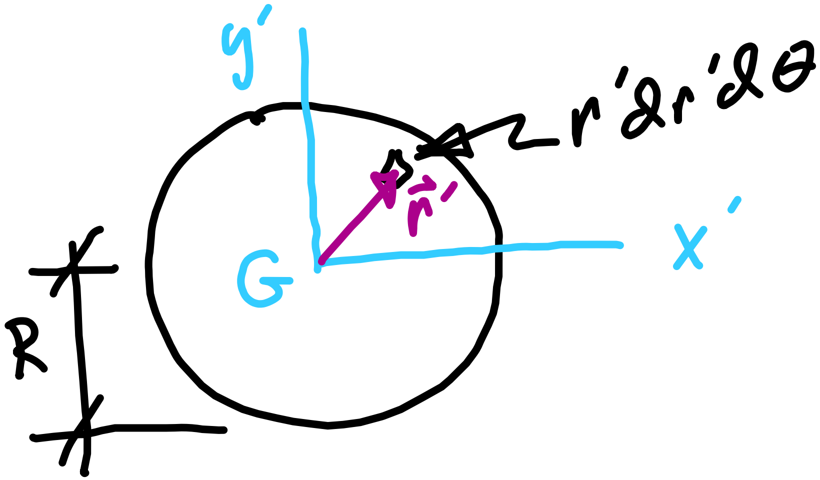


Uniform disk

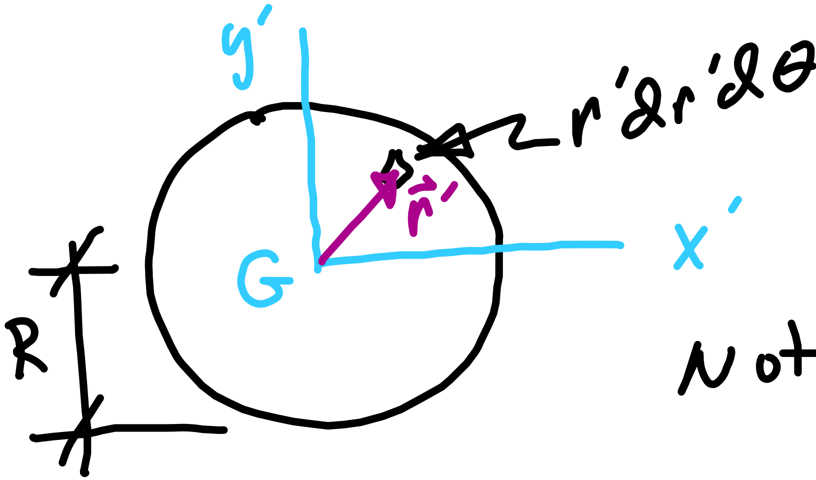


Uniform disk



$$\bar{I} = \left(\frac{M}{A}\right)\int_0^R\int_0^{2\pi}(r')^2\,r'\,dr'\,d\theta'$$

Uniform disk



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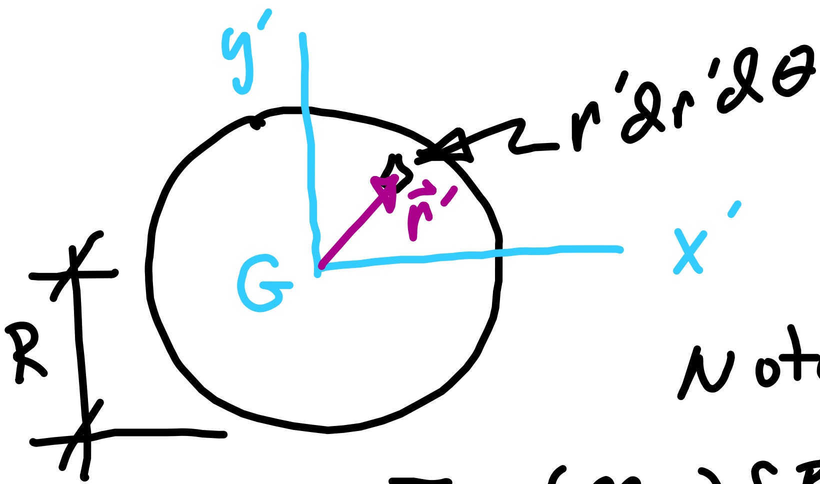
Note: $A = \pi R^2$

Uniform disk



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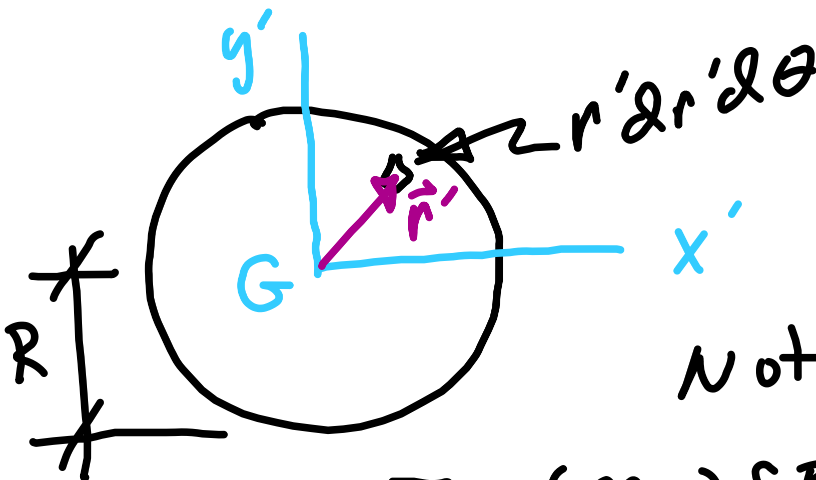
Uniform disk



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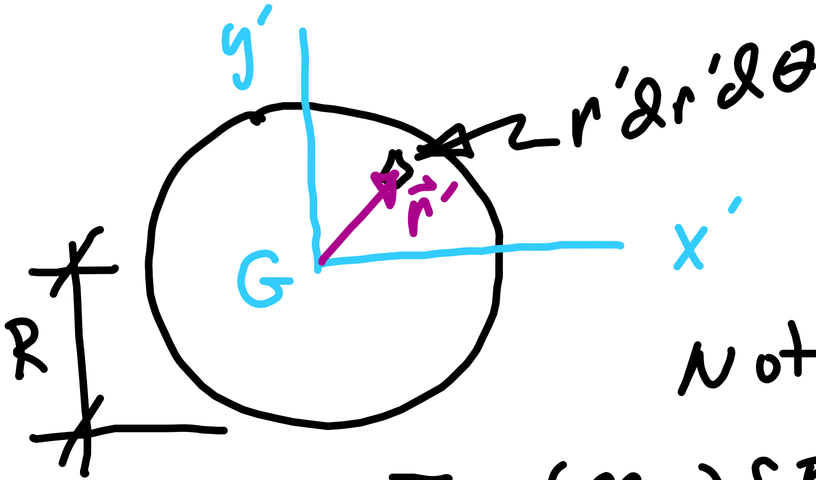

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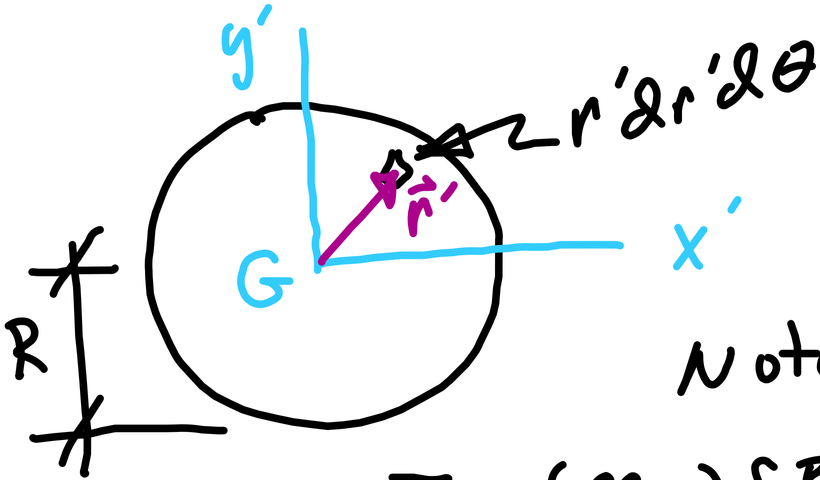
$$\Rightarrow \quad \bar{I} = \left(\frac{M}{2R^2}\right) r'^4\Big|_0^R$$

Uniform disk



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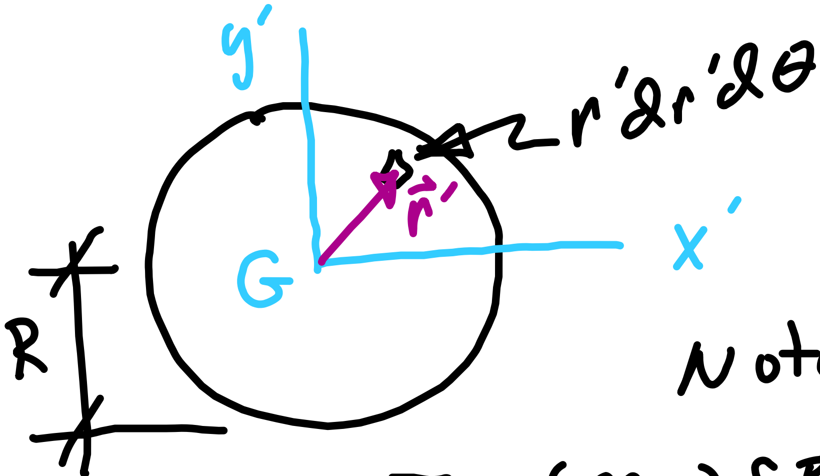

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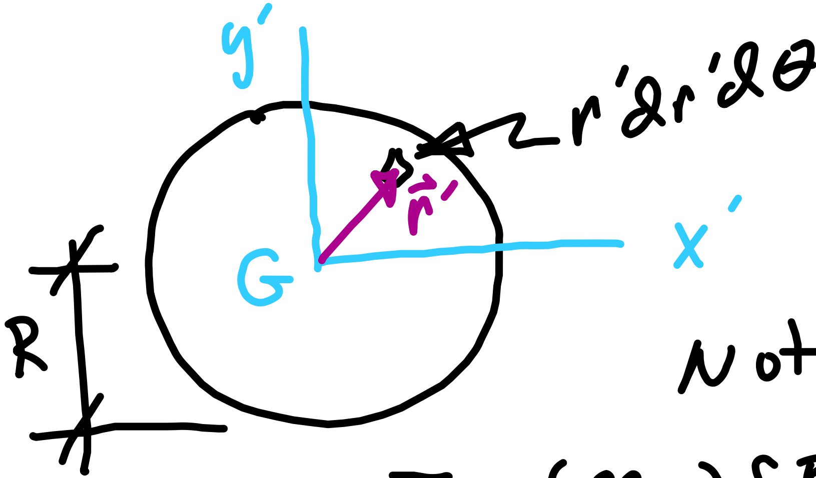
Uniform disk



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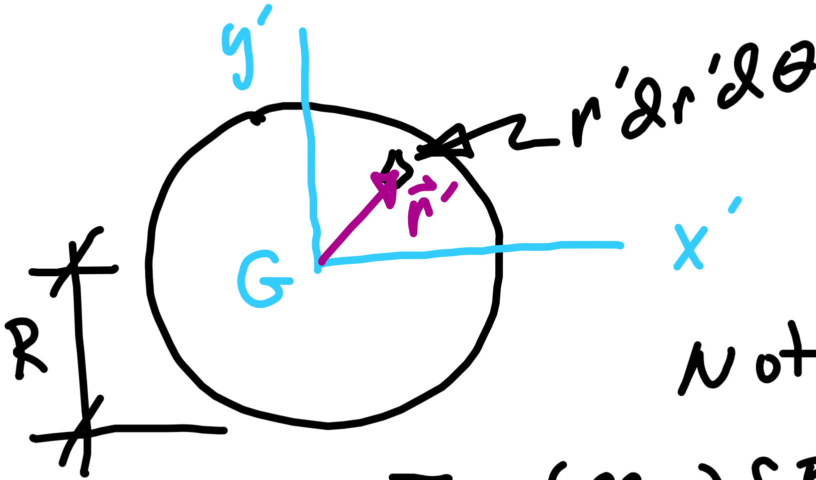
Uniform disk



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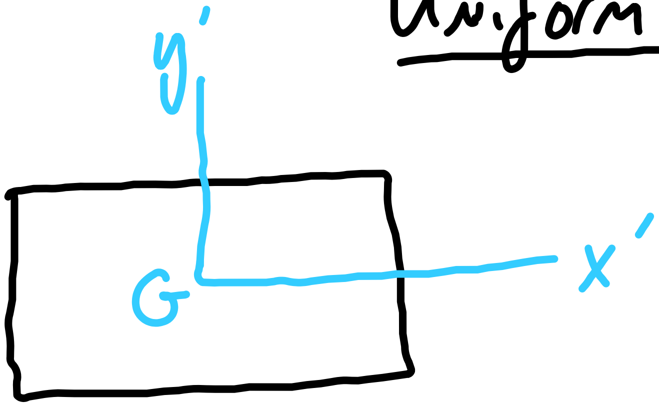
$$\Rightarrow \quad \bar{I} = \frac{mR^2}{2} \quad \text{so}$$

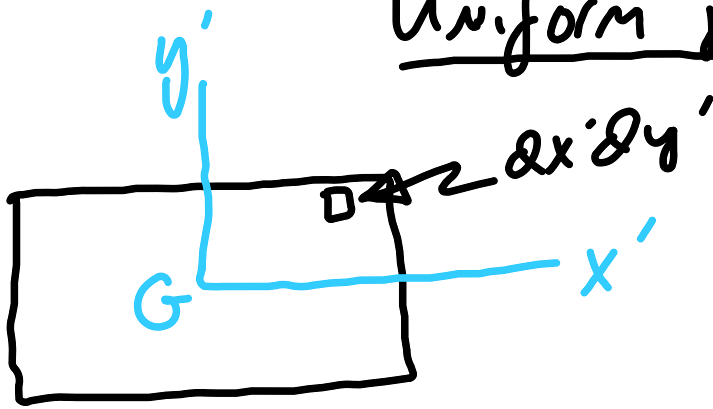
$$\vec{H}_G = \bar{I}\vec{\omega} \Rightarrow \vec{H}_G = \left(\frac{mR^2}{2}\right)\vec{\omega}$$

Uniform plate

Uniform plate



Uniform plate



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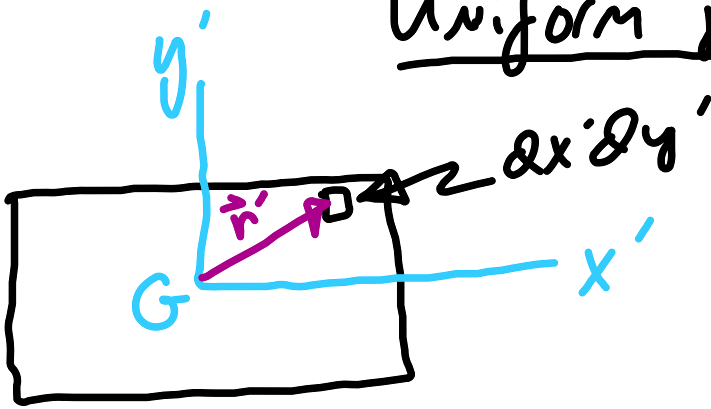

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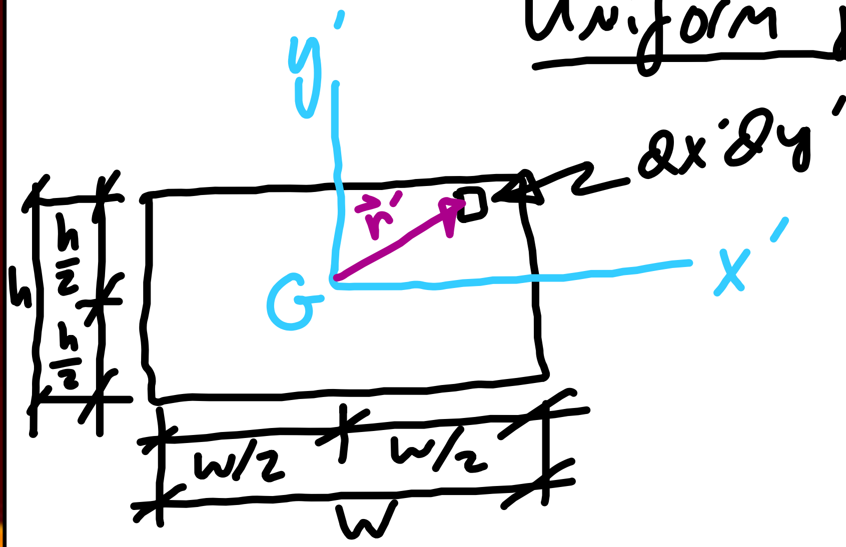
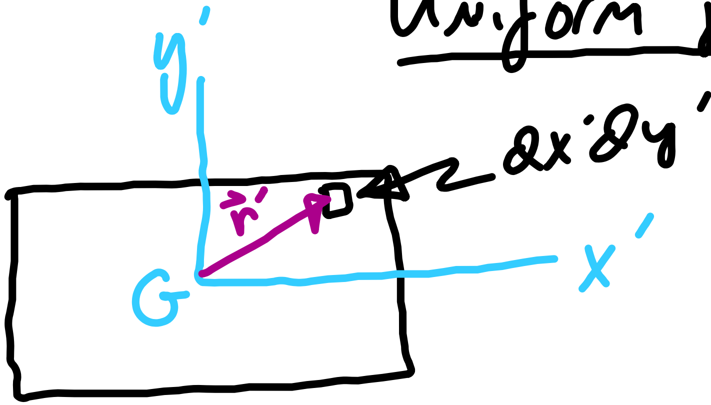
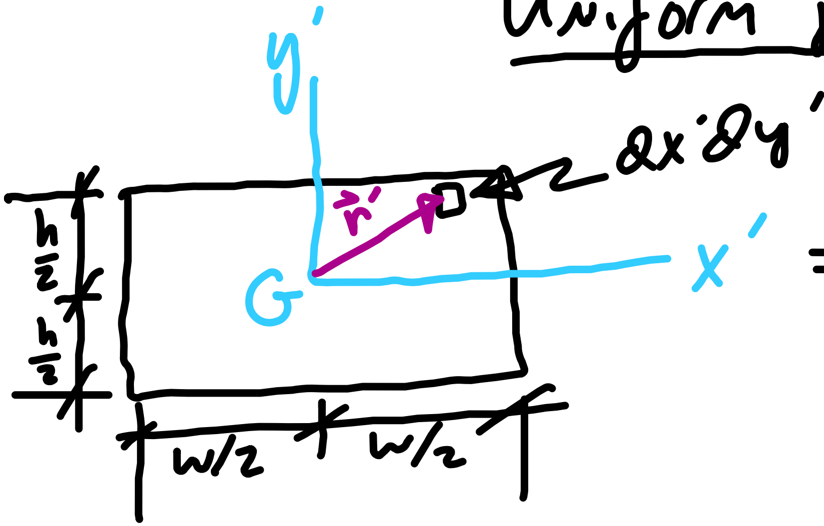


Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right) \iint r'^2 \, dx' dy'$$

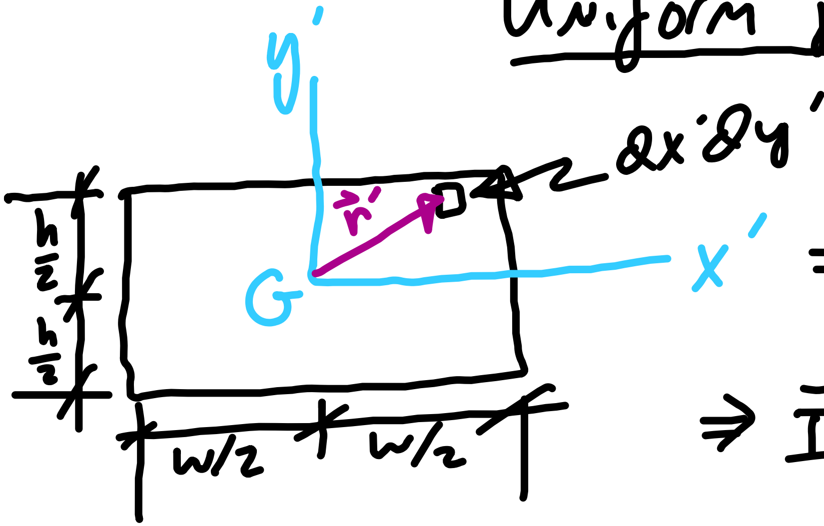
Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right) \iint r'^2 \, dx' dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-h/2}^{h/2} \int_{-w/2}^{w/2} (x'^2 + y'^2) \, dx' dy'$$

Uniform plate

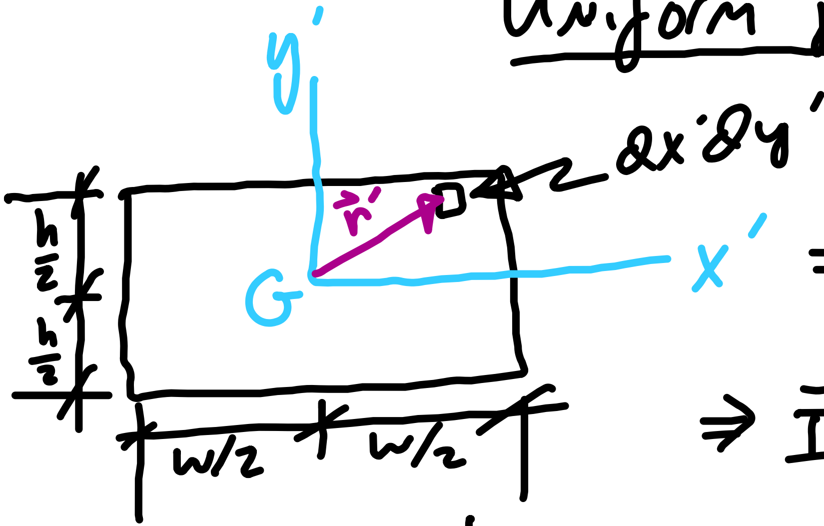


$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2 \, dx' dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x'^2 + y'^2)\, dx' dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3} + y'^2 x'\right)\Big|_{-\frac{w}{2}}^{w/2} dy'$$

Uniform plate



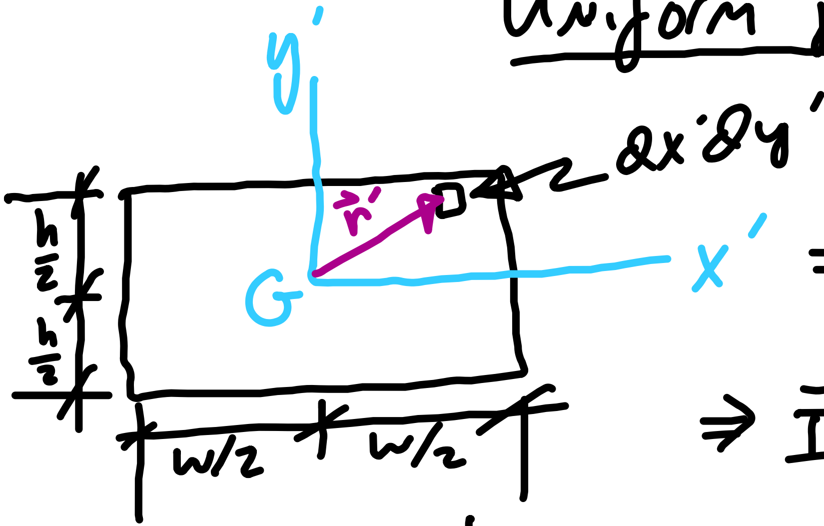
$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\,dx'dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}\left(x'^2+y'^2\right)dx'dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Big|_{-\frac{w}{2}}^{w/2} dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{w^3}{24}+\frac{w}{2}y'^2-\left(-\frac{w^3}{24}\right)-\left(-\frac{w}{2}\right)y'^2\right]dy'$$

Uniform plate



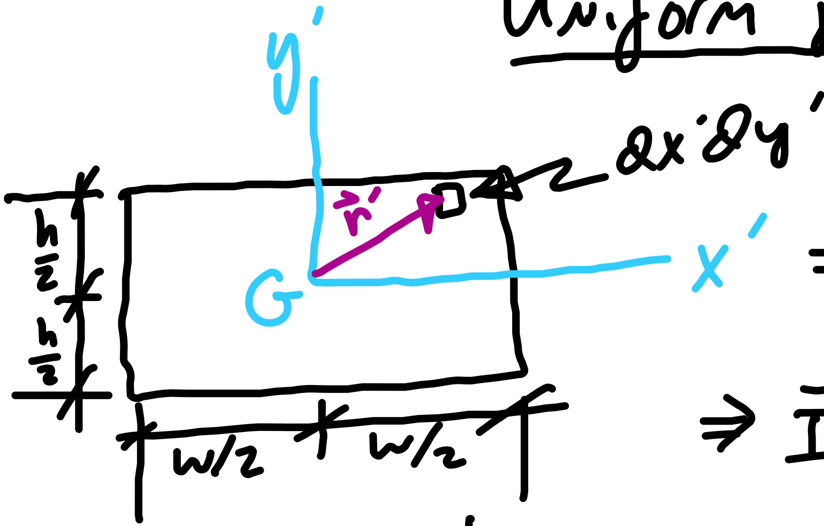
$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\,dx'\,dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}\left(x'^2+y'^2\right)dx'\,dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Big|_{-\frac{w}{2}}^{w/2}dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + w y'^2\right)dy'$$

Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\, dx'\,dy'$$

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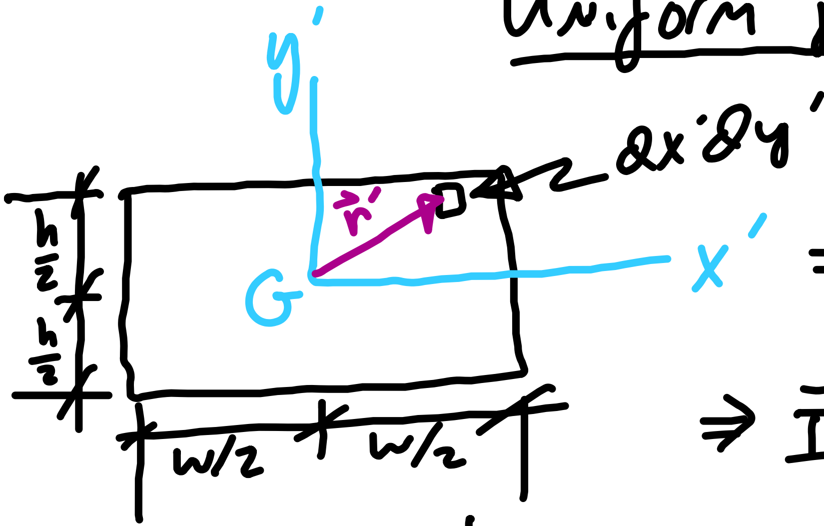
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + w y'^2\right)dy' = \left(\frac{m}{A}\right)\left[\frac{w^3}{12}y' + \frac{w y'^3}{3}\right]\Bigg|_{-h/2}^{h/2}$$

Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\,dx'\,dy'$$

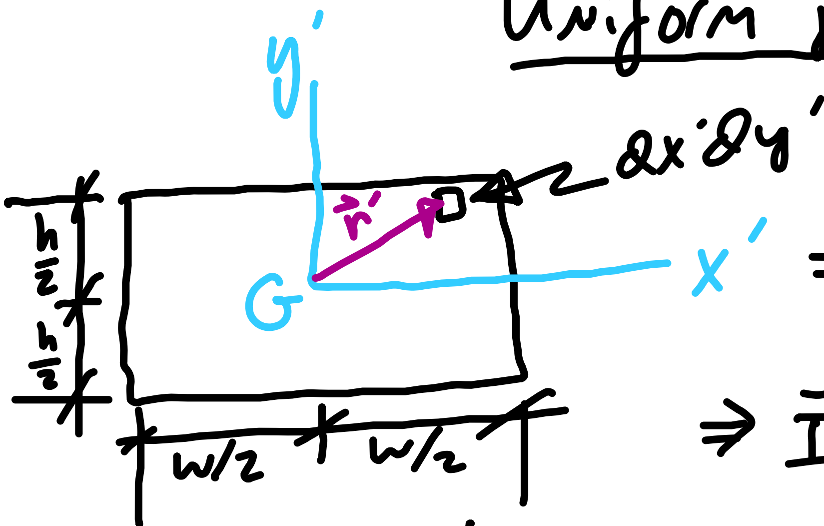
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x'^2+y'^2)\,dx'\,dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Big|_{-\frac{w}{2}}^{w/2}dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + wy'^2\right)dy' = \left(\frac{m}{A}\right)\left[\frac{w^3}{12}y' + \frac{wy'^3}{3}\right]\Big|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\left[\frac{w^3}{12}\left(\frac{h}{2}\right) + \frac{w}{3}\left(\frac{h^3}{8}\right) - \frac{w^3}{12}\left(-\frac{h}{2}\right) - \frac{w}{3}\left(-\frac{h^3}{8}\right)\right]$$

Uniform plate



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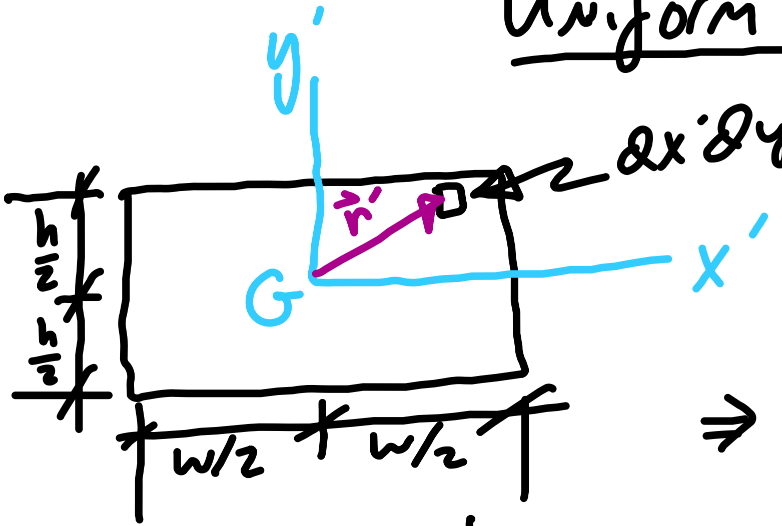
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\frac{x'^3}{3} + y'^2 x'\right) \Big|_{-\frac{w}{2}}^{\frac{w}{2}} dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(w^3/12 + w y'^2\right) dy' = \left(\frac{m}{A}\right) \left[\frac{w^3}{12} y' + \frac{w y'^3}{3}\right] \Big|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \left[\frac{w^3 h}{12} + \frac{w h^3}{12}\right]$$

Uniform plate



$$\bar{I} = \left(\frac{M}{A}\right)\iint r'^2\,dx'dy'$$

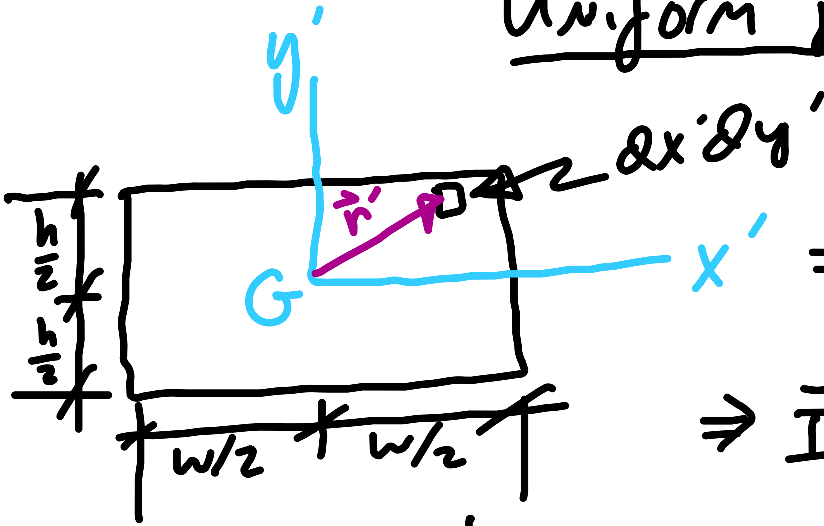
$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x'^2+y'^2)\,dx'dy'$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Big|_{-\frac{w}{2}}^{w/2}dy'$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + wy'^2\right)dy' = \left(\frac{M}{A}\right)\left[\frac{w^3}{12}y' + \frac{wy'^3}{3}\right]\Big|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\left[\frac{w^3h}{12} + \frac{wh^3}{12}\right] = \left(\frac{M}{A}\right)\left(\frac{wh}{12}\right)\left[w^2+h^2\right]$$

Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right) \iint r'^2 \, dx' dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-h/2}^{h/2} \int_{-w/2}^{w/2} (x'^2 + y'^2) \, dx' dy'$$

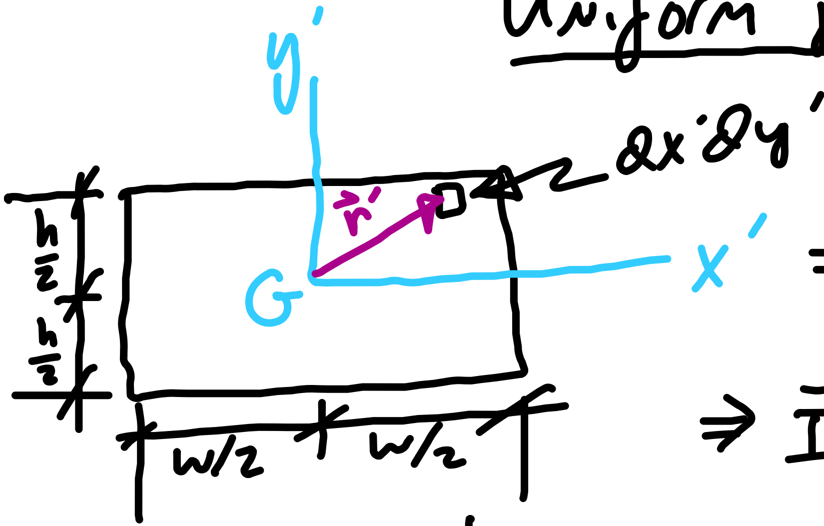
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\frac{x'^3}{3} + y'^2 x'\right) \Big|_{-\frac{w}{2}}^{\frac{w}{2}} dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(w^3/12 + w y'^2\right) dy' = \left(\frac{m}{A}\right) \left[\frac{w^3}{12} y' + \frac{w y'^3}{3}\right] \Big|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right) \left[\frac{w^3 h}{12} + \frac{w h^3}{12}\right] = \left(\frac{m}{A}\right) \left(\frac{wh}{12}\right) \left[w^2 + h^2\right]$$

But $A = wh$

Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\, dx'\, dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}\left(x'^2+y'^2\right) dx'\, dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Bigg|_{-\frac{w}{2}}^{w/2} dy'$$

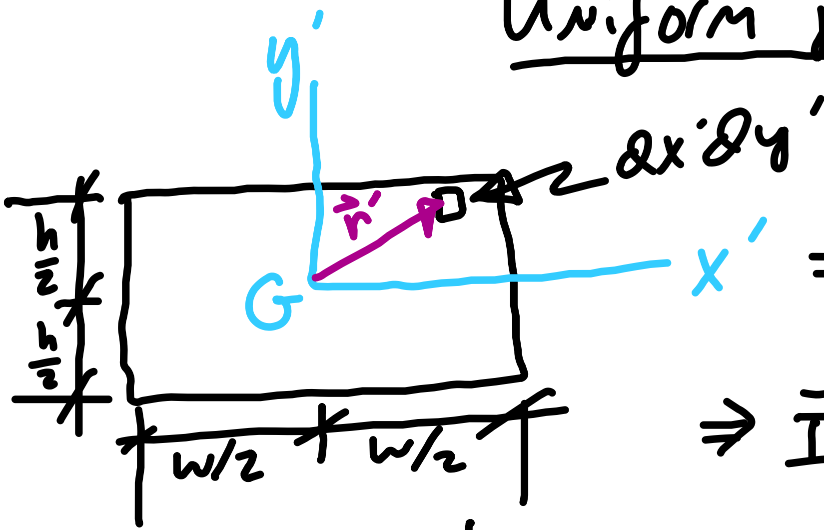
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + wy'^2\right)dy' = \left(\frac{m}{A}\right)\left[\frac{w^3}{12}y' + \frac{wy'^3}{3}\right]\Bigg|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\left[\frac{w^3h}{12} + \frac{wh^3}{12}\right] = \left(\frac{m}{A}\right)\left(\frac{wh}{12}\right)\left[w^2+h^2\right]$$

But $A = wh$ so $\boxed{\bar{I} = \frac{m}{12}\left[w^2+h^2\right]}$

ASU

Uniform plate



$$\bar{I} = \left(\frac{M}{A}\right)\iint r'^2\, dx'\, dy'$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2} (x'^2 + y'^2)\, dx'\, dy'$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-\frac{h}{2}}^{h/2} \left(\frac{x'^3}{3} + y'^2 x'\right)\Big|_{-\frac{w}{2}}^{w/2} dy'$$

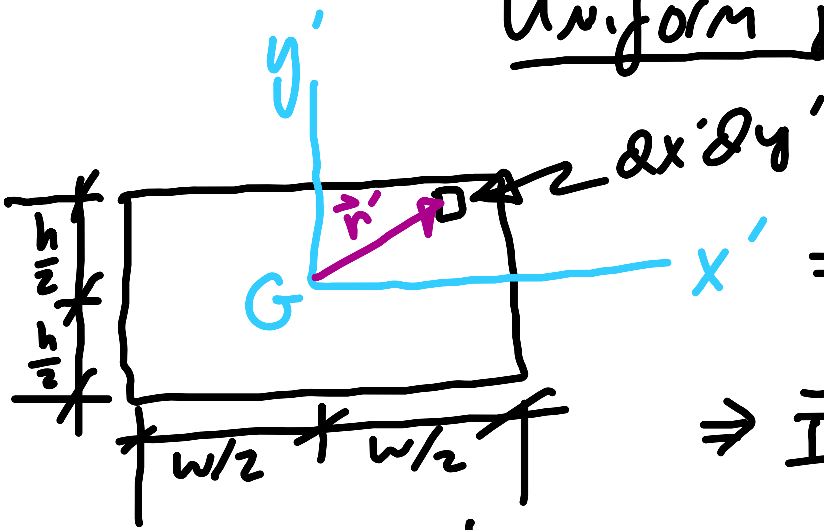
$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(w^3/12 + w y'^2\right) dy' = \left(\frac{M}{A}\right)\left[\frac{w^3}{12}y' + \frac{w y'^3}{3}\right]\Big|_{-h/2}^{h/2}$$

$$\Rightarrow \bar{I} = \left(\frac{M}{A}\right)\left[\frac{w^3 h}{12} + \frac{w h^3}{12}\right] = \left(\frac{M}{A}\right)\left(\frac{wh}{12}\right)\left[w^2 + h^2\right]$$

But $A = wh$ so $\boxed{\bar{I} = \frac{M}{12}\left[w^2 + h^2\right]}$

& $\vec{H}_G = \bar{I}\vec{\omega}$

Uniform plate



$$\bar{I} = \left(\frac{m}{A}\right)\iint r'^2\,dx'dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x'^2+y'^2)\,dx'dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{h/2}\left(\frac{x'^3}{3}+y'^2x'\right)\Big|_{-\frac{w}{2}}^{w/2}dy'$$

$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(w^3/12 + wy'^2\right)dy' = \left(\frac{m}{A}\right)\left[\frac{w^3}{12}y' + \frac{wy'^3}{3}\right]\Bigg|_{-h/2}^{h/2}$$

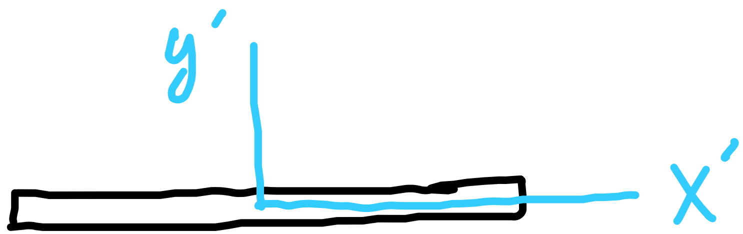
$$\Rightarrow \bar{I} = \left(\frac{m}{A}\right)\left[\frac{w^3h}{12} + \frac{wh^3}{12}\right] = \left(\frac{m}{A}\right)\left(\frac{wh}{12}\right)\left[w^2+h^2\right]$$

But $A = wh$ so $\boxed{\bar{I} = \frac{m}{12}\left[w^2+h^2\right]}$

& $\vec{H}_G = \bar{I}\vec{\omega} \Rightarrow \boxed{\vec{H}_G = \left(\frac{m}{12}\right)\left[w^2+h^2\right]\vec{\omega}}$

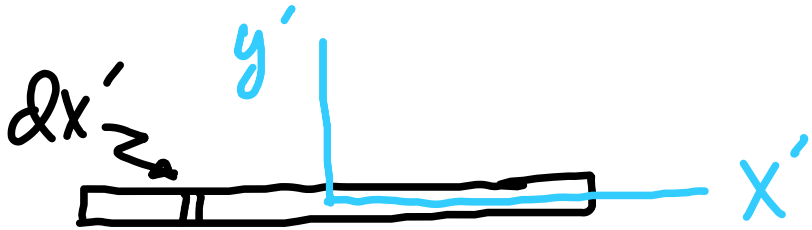
Slender rod

Slender rod



Only need 1D in this case

Slender rod



Only need 1d in this case

Slender rod

Only need 1d in this case

Slender rod

Only need 1D in this case

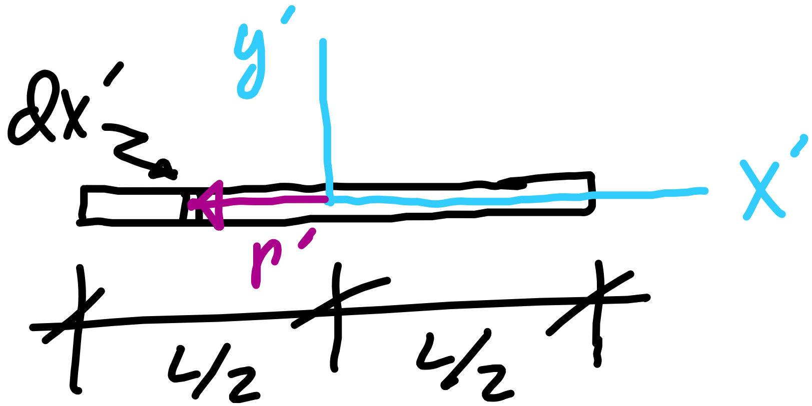
Here $\frac{M}{A}$

Slender rod

Only need 1D in this case

Here $\frac{M}{A} \to \frac{M}{L}$

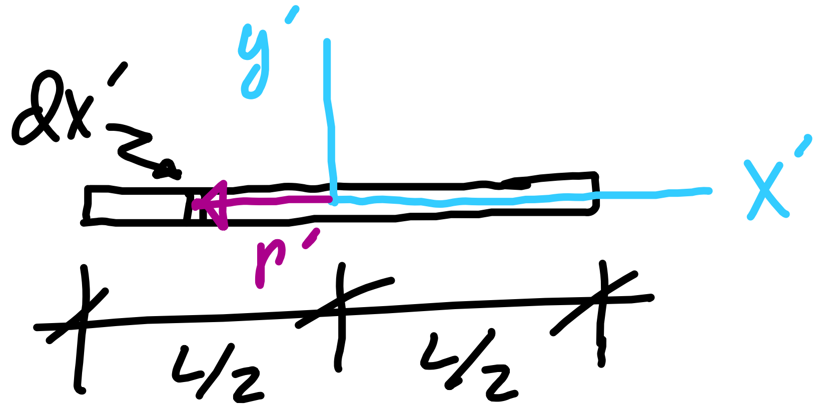
Slender rod



Only need 1d in this case

Here $\frac{M}{A} \to \frac{M}{L}$

Slender rod

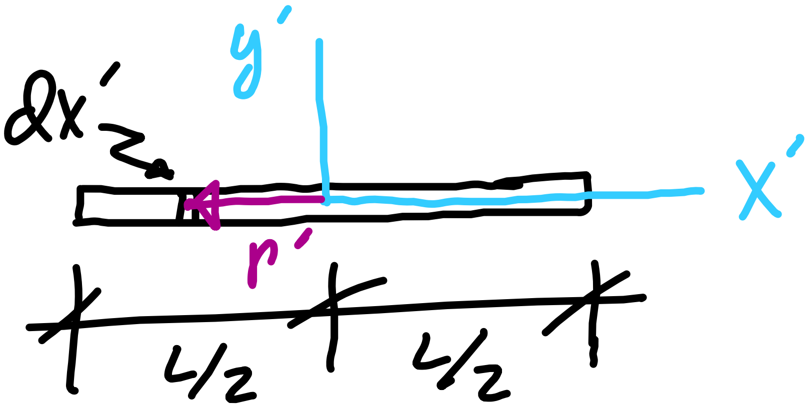


Only need 1d in this case

Here $\frac{M}{A} \to \frac{m}{L}$

Now $\bar{I} = \left(\frac{m}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx'$

Slender rod

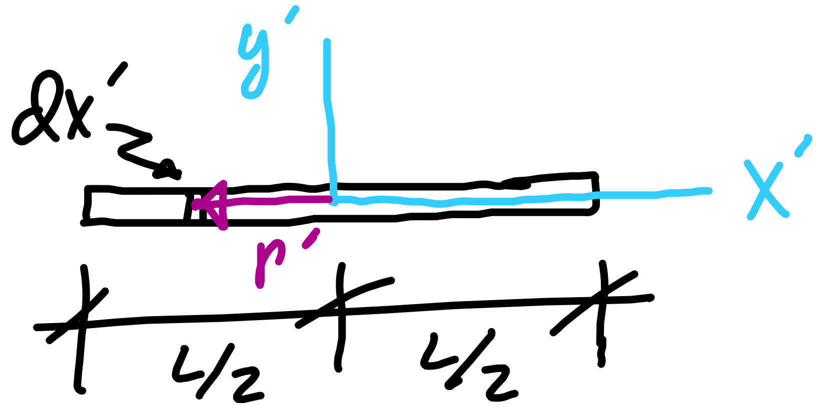


Only need 1d in this case

Here $\frac{M}{A} \to \frac{M}{L}$

Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Bigg|_{-L/2}^{L/2}$

Slender rod



Only need 1d in this case

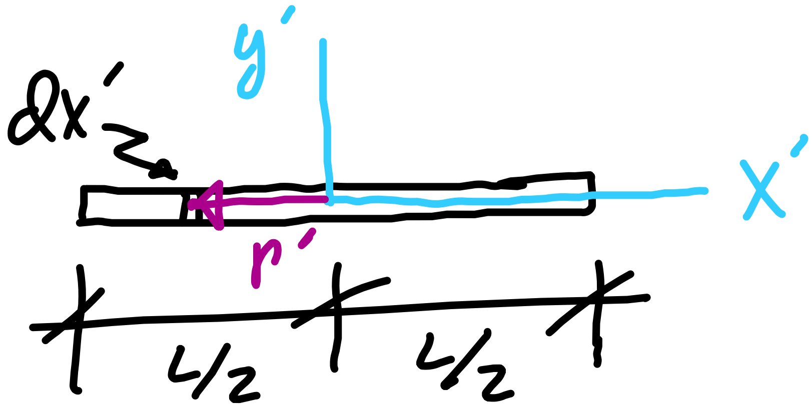
Here $\frac{M}{A} \rightarrow \frac{M}{L}$

Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Big|_{-L/2}^{L/2}$

$\Rightarrow \quad \bar{I} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right]$

Slender rod

Only need 1d in this case

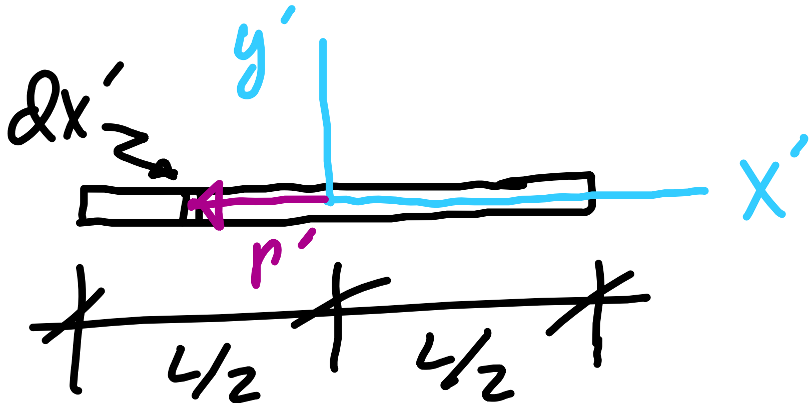


Here $\frac{M}{A} \rightarrow \frac{M}{L}$

Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Big|_{-L/2}^{L/2}$

$\Rightarrow \quad \bar{I} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{\bar{I} = \frac{ML^2}{12}}$

Slender rod



Only need 1d in this case

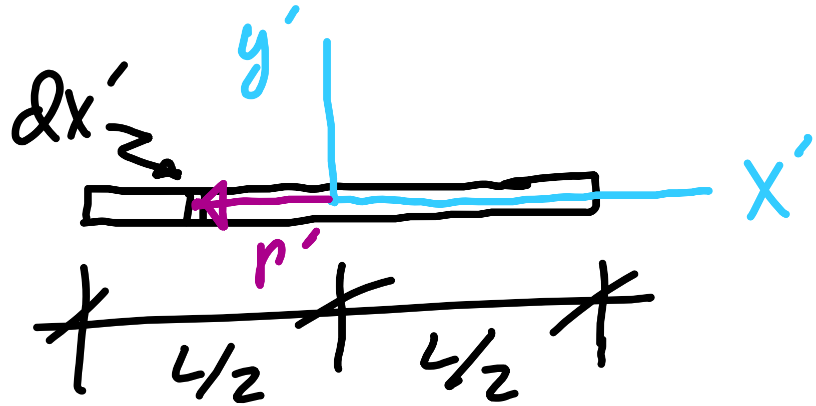
Here $\frac{M}{A} \rightarrow \frac{M}{L}$

Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Bigg|_{-L/2}^{L/2}$

$\Rightarrow \quad \bar{I} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{\bar{I} = \frac{ML^2}{12}}$

Compare to plate

Slender rod



Only need 1d in this case

Here $\frac{M}{A} \rightarrow \frac{M}{L}$

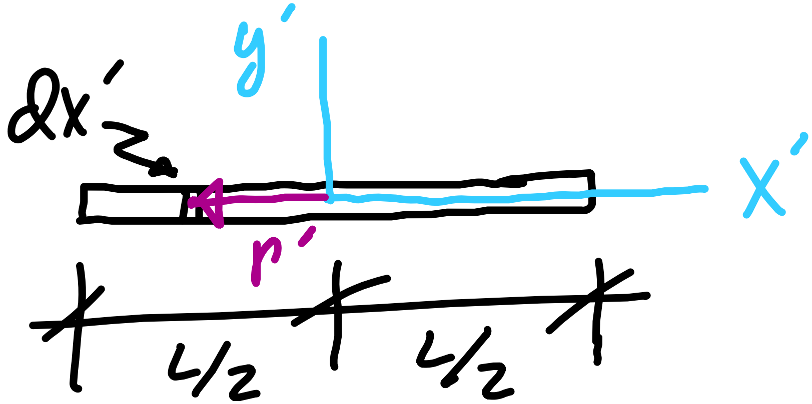
Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Big|_{-L/2}^{L/2}$

$\Rightarrow \quad \bar{I} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{\bar{I} = \frac{ML^2}{12}}$

Compare to plate

$\bar{I}_{plate} = \left(\frac{M}{12}\right)\left[L^2 + w^2\right]$

Slender rod



Only need 1D in this case

Here $\frac{M}{A} \to \frac{M}{L}$

Now $\bar{I} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x'^3}{3}\right]\Big|_{-L/2}^{L/2}$

$\Rightarrow \quad \bar{I} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{\bar{I} = \frac{ML^2}{12}}$

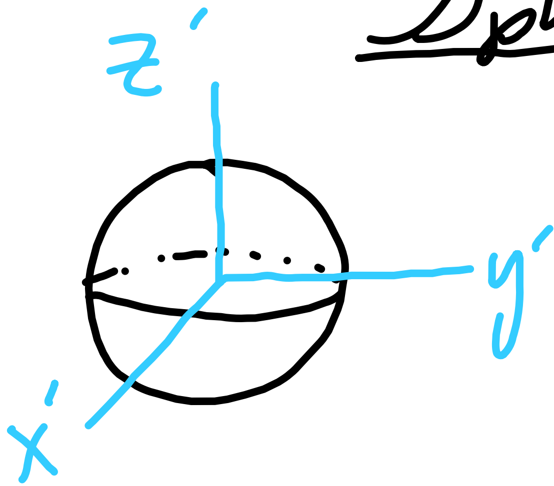
Compare to plate

$\bar{I}_{plate} = \left(\frac{M}{12}\right)\left[L^2 + w^2\right]$

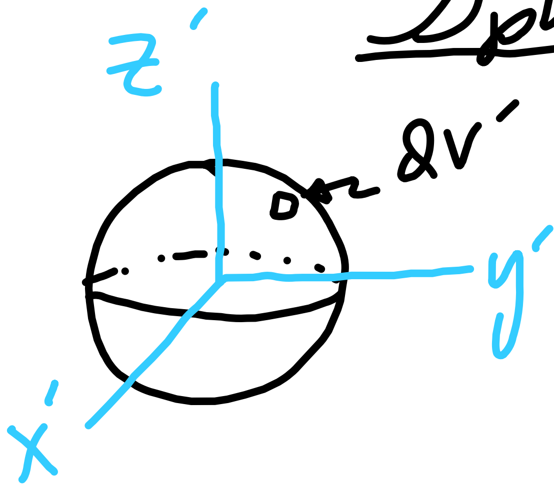
Slender rod is just a plate with $w \to 0$

Sphere

Sphere



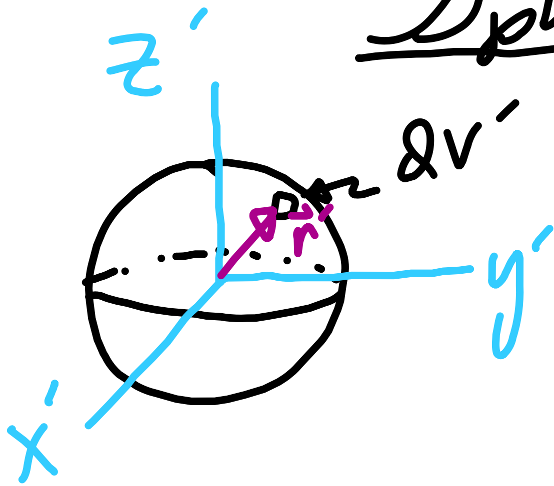
Sphere



Here we need 3d

where $dv' = (dr')(r'\sin\phi' d\phi')(r' d\theta')$

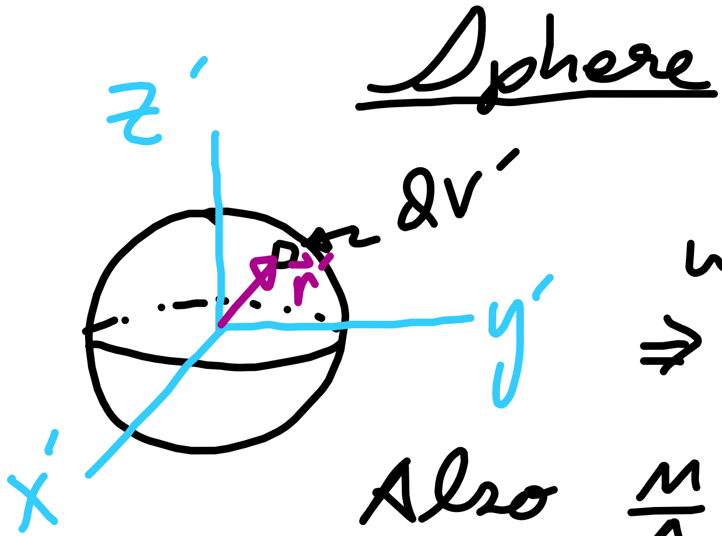
Sphere



Here we need 3d

where $dv' = (dr')(r'\sin\varphi' d\varphi')(r' d\theta')$

$\Rightarrow dv' = r'^2 \sin\varphi' \, dr' \, d\varphi' \, d\theta'$



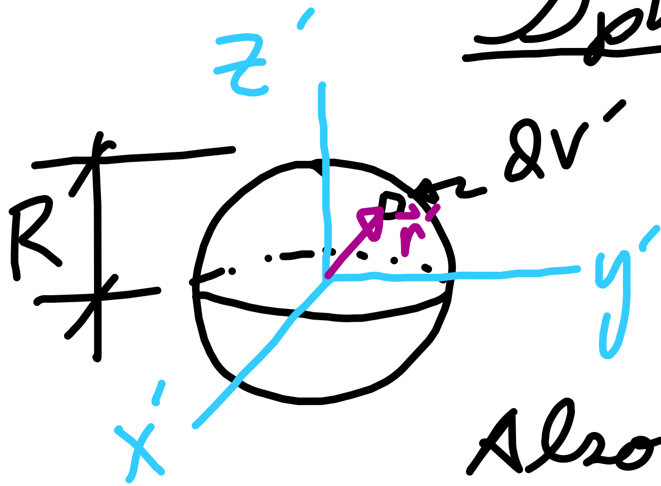
Here we need 3d

where $dv' = (dr')(r'\sin\varphi' d\varphi')(r' d\theta')$

$\Rightarrow dv' = r'^2 \sin\varphi' \, dr' \, d\varphi' \, d\theta'$

Also $\frac{M}{A} \to \frac{M}{V}$

Sphere



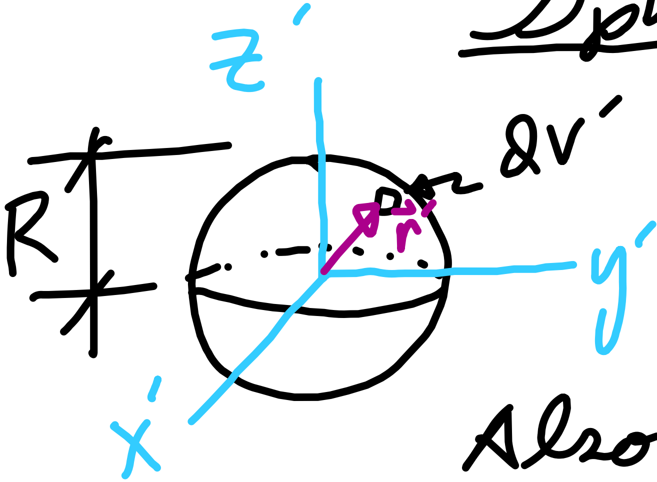
Here we need 3d

where $dV' = (dr')(r'\sin\phi' d\phi')(r' d\theta')$

$\Rightarrow dV' = r'^2 \sin\phi' \, dr' \, d\phi' \, d\theta'$

Also $\frac{M}{A} \to \frac{M}{V}$ & $V = \frac{4}{3}\pi R^3$

Sphere



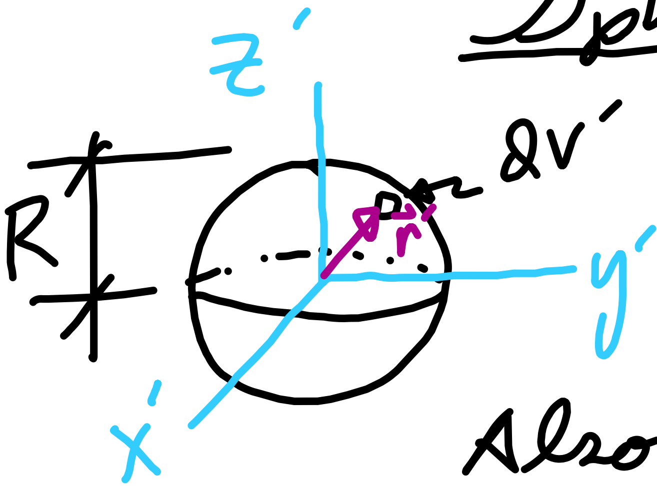
Here we need 3d where $dV' = (dr')(r'\sin\phi' d\phi')(r' d\theta')$

$\Rightarrow dV' = r'^2 \sin\phi' \, dr' \, d\phi' \, d\theta'$

Also $\frac{M}{A} \to \frac{M}{V}$ & $V = \frac{4}{3}\pi R^3$

Another modification is that we don't want to integrate over r'^2

Sphere



Here we need 3d

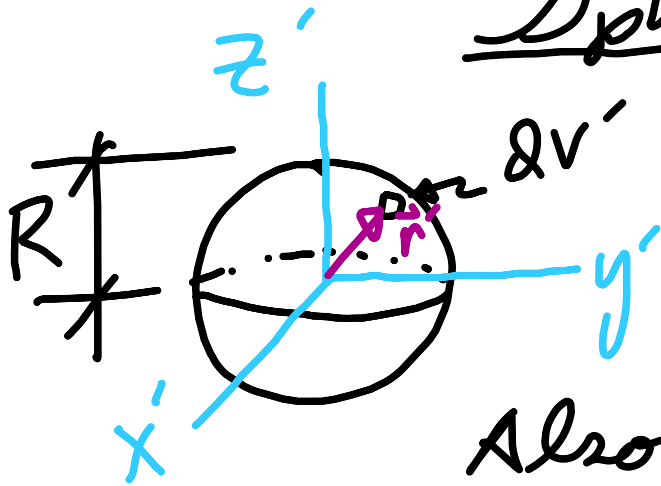
where $dV' = (dr')(r'\sin\phi' d\phi')(r' d\theta')$

$\Rightarrow dV' = r'^2 \sin\phi' \, dr' \, d\phi' \, d\theta'$

Also $\frac{M}{A} \to \frac{M}{V}$ & $V = \frac{4}{3}\pi R^3$

Another modification is that we don't want to integrate over r'^2 [as we do for 2d case]

Sphere



Here we need 3d

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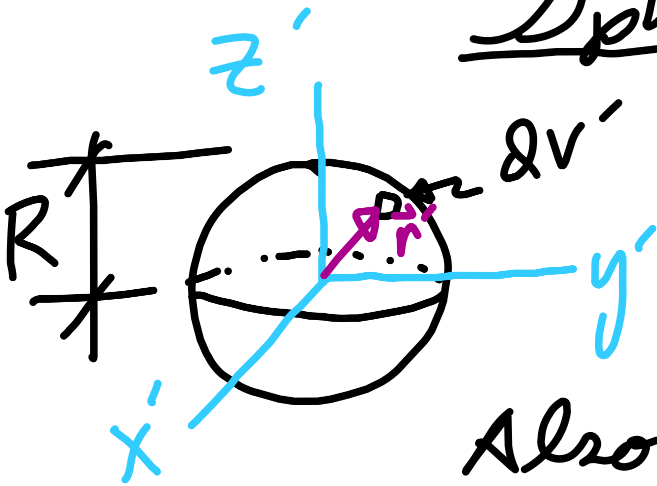

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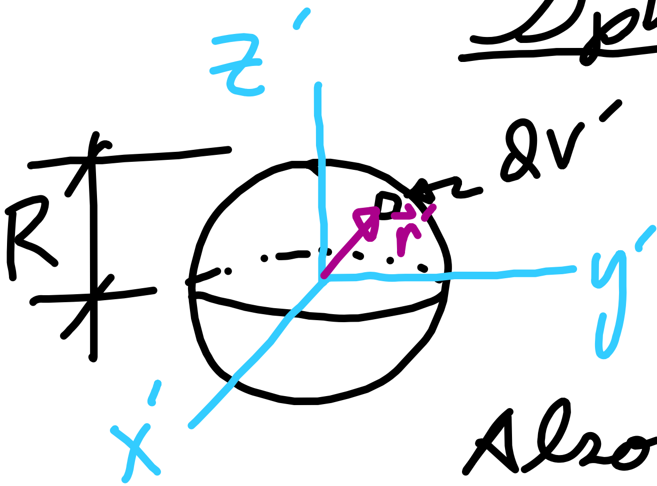

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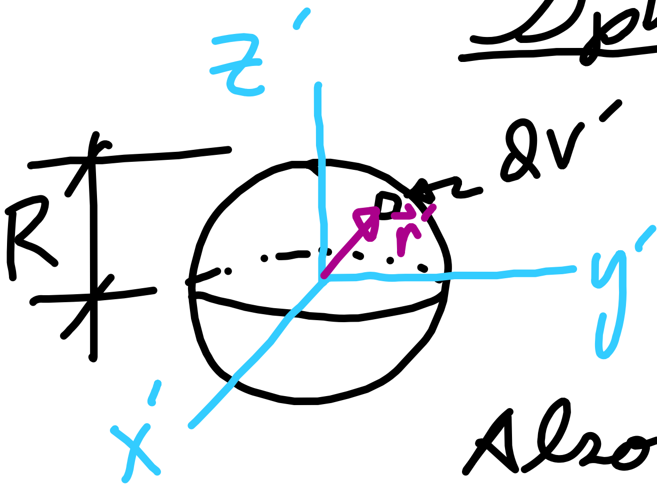

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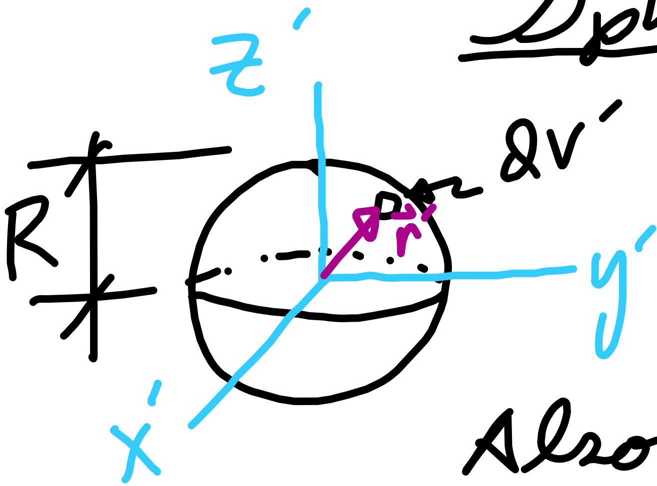

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Used Wolfram alpha to find $\int_0^\pi \sin^3\varphi \, d\varphi = 4/3$

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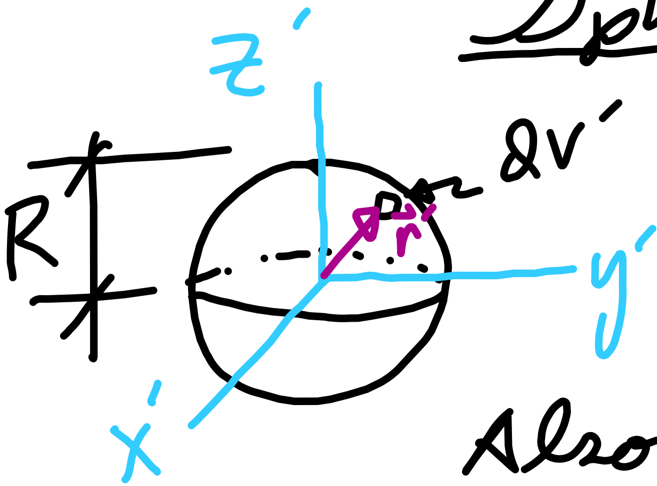

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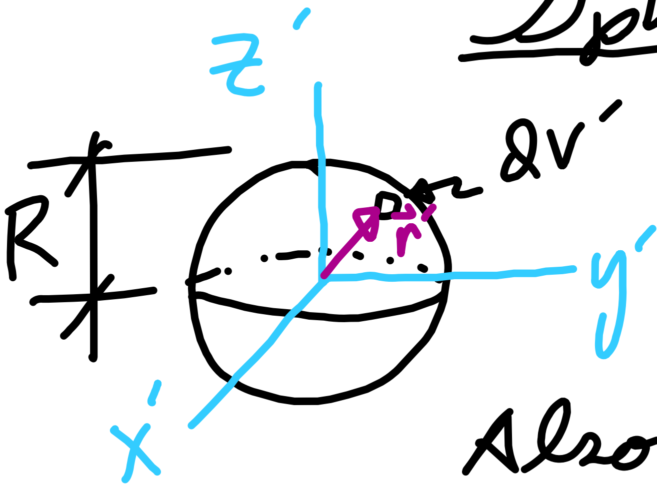
Sphere



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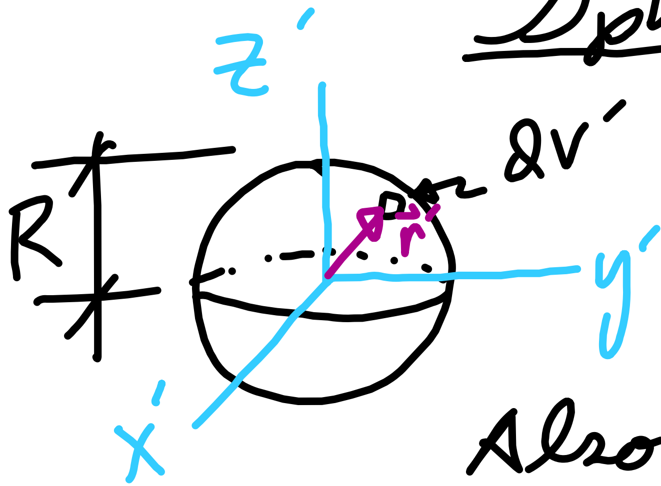

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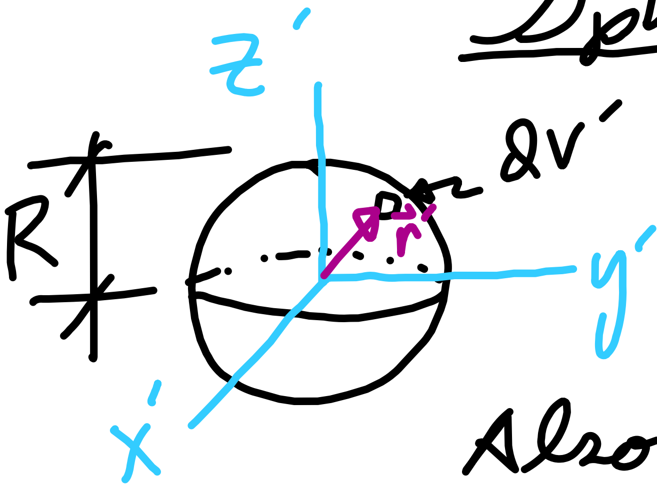
Sphere



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where $dv' = (dr')(r'\sin\varphi' d\varphi')(r' d\theta')$

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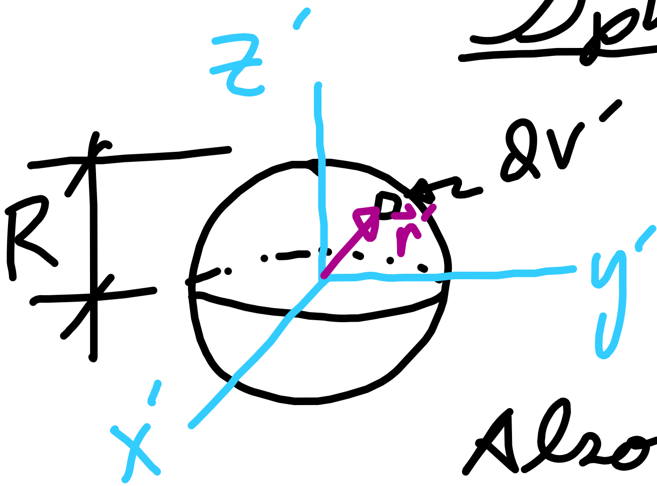
ASU

Sphere



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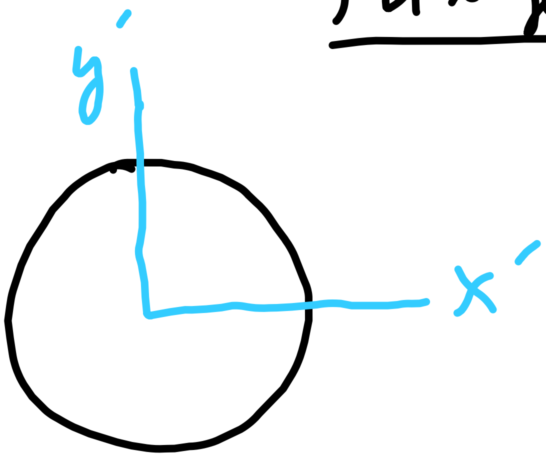
$\Rightarrow \boxed{I = \frac{2}{5}MR^2}$

Thin pipe or hoop

Thin pipe or hoop



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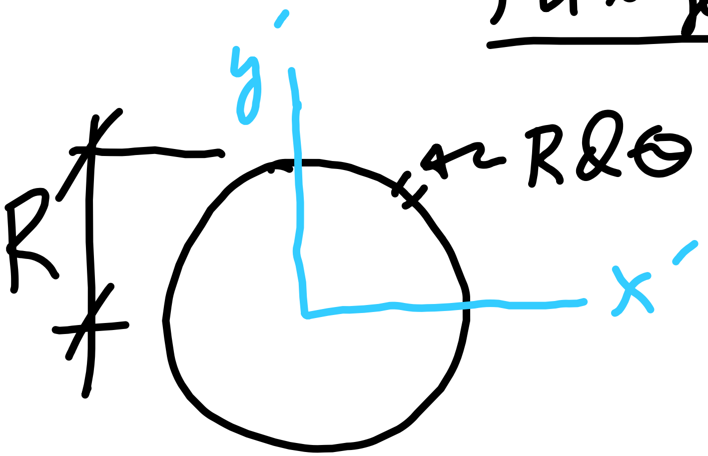
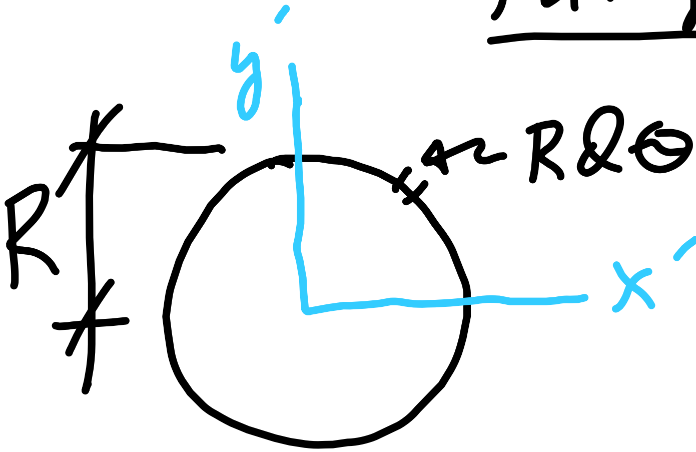


Just need I_d

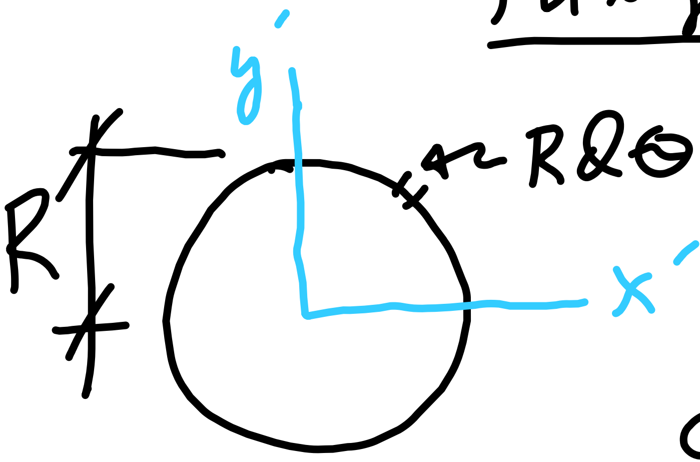
Thin pipe or hoop



Just need 1d

$$\oint \frac{M}{A} \to \frac{M}{C}$$

Thin pipe or hoop

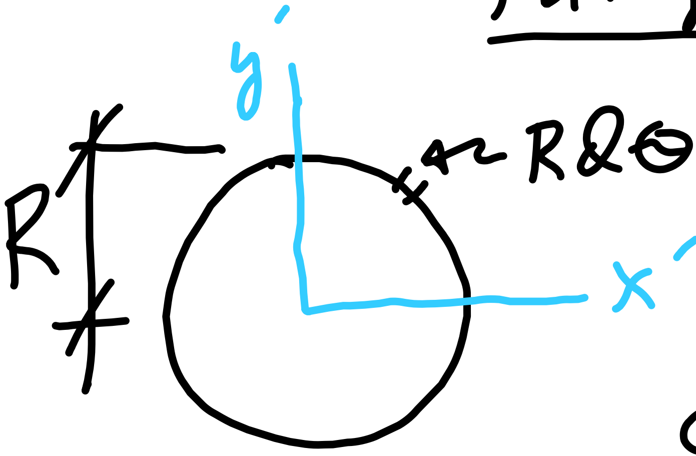


Just need 1d

$$\cancel{\iint} \ \frac{M}{A} \to \frac{M}{C}, \text{ where}$$

$$C = 2\pi R$$

Thin pipe or hoop

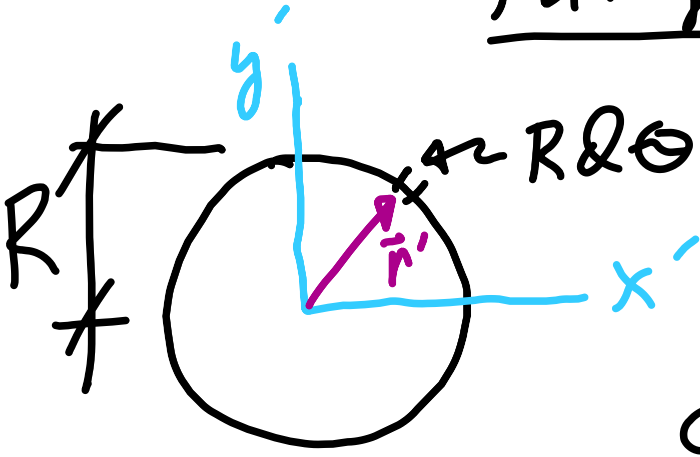


Just need Id

$f \quad \frac{M}{A} \to \frac{M}{C}$, where

$C = 2\pi R$ [circumference of circle]

Thin pipe or hoop

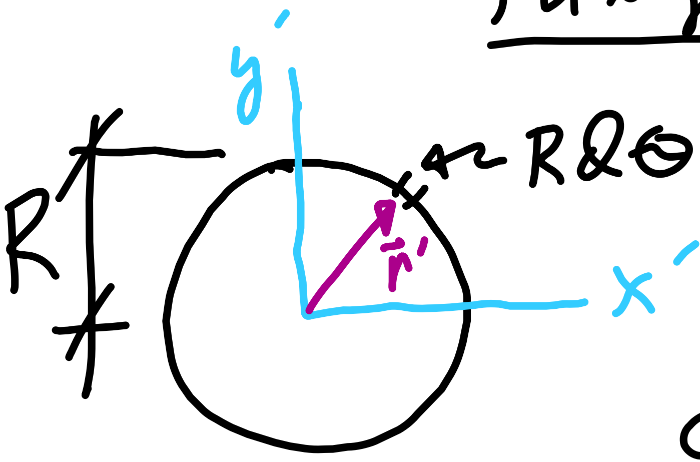


Just need 1d

$$\rho : \frac{M}{A} \to \frac{M}{C}, \text{ where}$$

$C = 2\pi R$ [circumference of circle]

Thin pipe or hoop



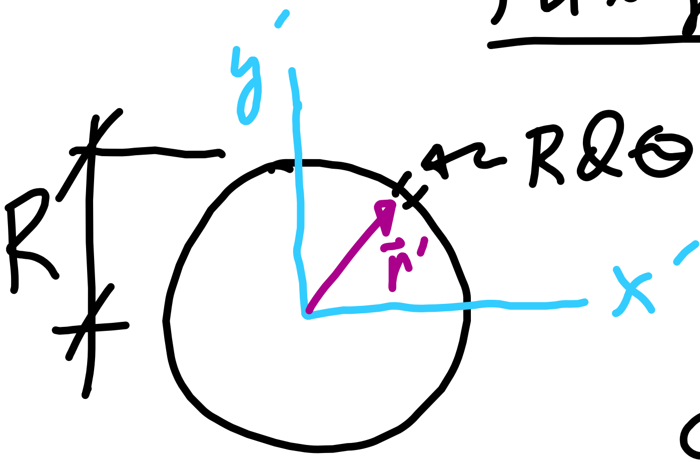
Just need 1d

$\oint \frac{M}{A} \to \frac{M}{C}$, where

$C = 2\pi R$ [circumference of circle]

Now $I = \left(\frac{M}{C}\right)\int_0^{2\pi} r'^2 R\,d\theta$

Thin pipe or hoop



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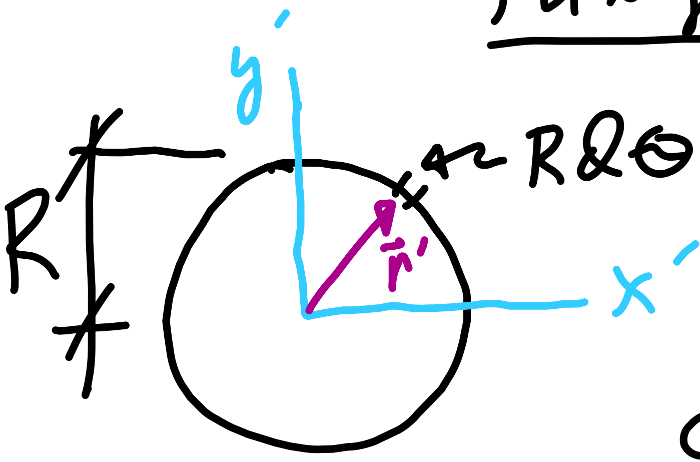
Now $\quad I = \left(\frac{M}{C}\right)\int_0^{2\pi} r'^2 R\,d\theta \quad$ But $r' = \text{const} = R$

Thin pipe or hoop



Just need 1d

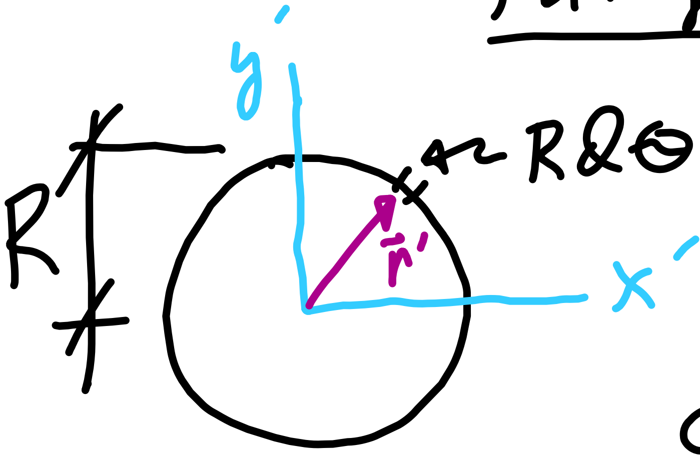
$\oint \quad \frac{M}{A} \rightarrow \frac{M}{C}$, where

$C = 2\pi R$ [circumference of circle]

Now $\quad I = \left(\frac{M}{C}\right)\int_0^{2\pi} r'^2 R\,d\Theta \quad$ But $r' = \text{const} = R$

so $\quad I = \frac{M}{C} R^2 \int_0^{2\pi} R\,d\Theta$

Thin pipe or hoop



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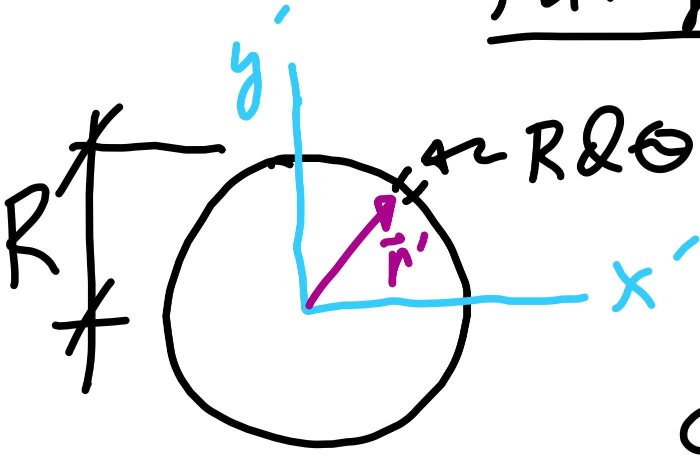

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