Today 16.1

$\boxed{L24}$

Today 16.1

$\boxed{L24}$

Kinetics of rigid bodies

$\boxed{L24}$

<u>Today</u> 16.1

<u>Friday</u> 16.2

L24

Today 16.1

Friday 16.2

Constrained plane motion

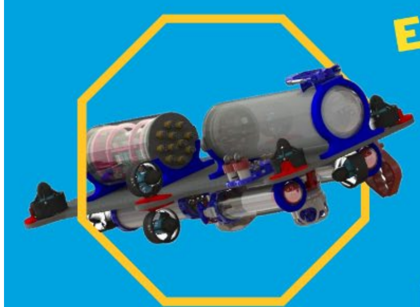
JOIN THE WAVE

ATTEND OUR 2020-2021
NEW MEMBER MEETING
VIA ZOOM
OCTOBER 21ST
6:00 PM
REGISTER THROUGH THE
QR CODE BELOW

Desert WAVE is a **W**omen in **A**utonomous **V**ehicle **E**ngineering team. They compete annually in RoboNation's International RoboSub competition, placing 2nd in the World Overall, 2020!

- Learn about autonomous systems
- Collaborate w/ industry mentors
- Build valuable technical skills
- Work w/ motivated women in STEM

NO EXPERIENCE REQUIRED



Desert WAVE
Women in Autonomous Vehicle Engineering

Si Se Puede

Open to all undergraduate and graduate students of any major across all ASU campuses

ASU

Some moments of inertia

Some moments of inertia

Item	$\bar{I}$
Rectangular plate	$\left(\frac{m}{12}\right)(w^2+L^2)$

Some moments of inertia

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Rectangular plate	$\left(\frac{m}{12}\right)(w^2+L^2)$
Slender rod	$\left(\frac{m}{12}\right)L^2$

$w \to 0$

Some moments of inertia

Item	$\bar{I}$
Rectangular plate	$\left(\frac{M}{12}\right)(W^2+L^2)$
Slender rod	$\left(\frac{M}{12}\right)L^2$
Thin pipe or hoop	MR^2

all mass located distance R from rotation axis

Some moments of inertia

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Rectangular plate	$\left(\frac{M}{12}\right)(W^2+L^2)$
Slender rod	$\left(\frac{M}{12}\right)L^2$
Thin pipe or hoop	MR^2
Cylinder or disk	$MR^2/2$

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Slender rod	$\left(\frac{M}{12}\right)L^2$
Thin pipe or hoop	MR^2
Cylinder or disk	$MR^2/2$
Sphere	$\left(\frac{2}{5}\right)MR^2$

For a system of particles

For a system of particles [or rigid body]

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$$\sum \vec{F} = m\vec{\bar{a}}$$

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$$\Sigma \vec{F} = m\vec{\bar{a}} \quad \& \quad \Sigma \vec{M} = \dot{\vec{H}}_G$$

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From book: "The system of the external forces and moments is equipollent to the system consisting of the vector $m\bar{\vec{a}}$ attached to G and the couple of moment $\dot{\vec{H}}_G$."

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For plane motion of a rigid body

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For plane motion of a rigid body we have

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What if we wanted to sum moments about a point other than G?

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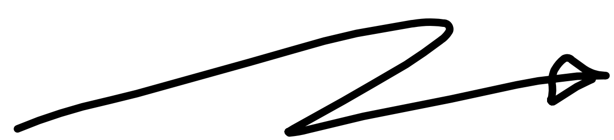
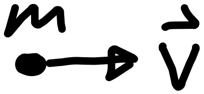
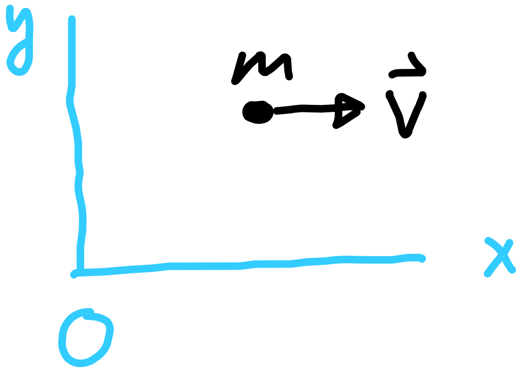
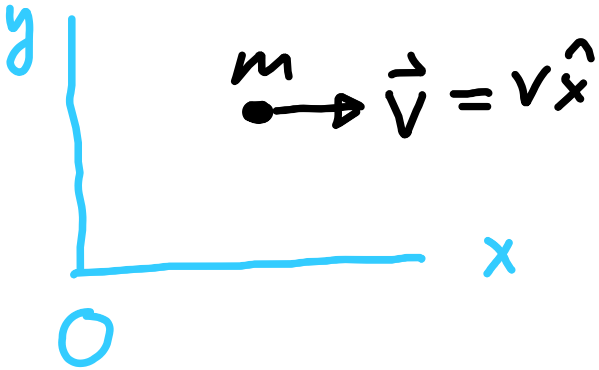
What if we wanted to sum moments about a point other than G?

First we will look at a point particle

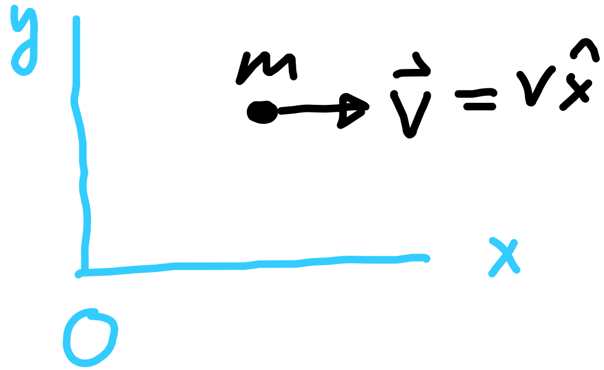



m
$\vec{V}$

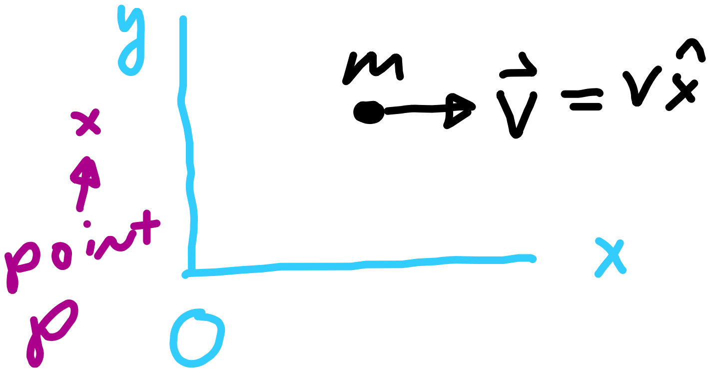

y
m
$\vec{V}$
x
O



$$\vec{v} = v\hat{x}$$



$\vec{L} = m\vec{v}$ what about the angular momentum?



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For point p

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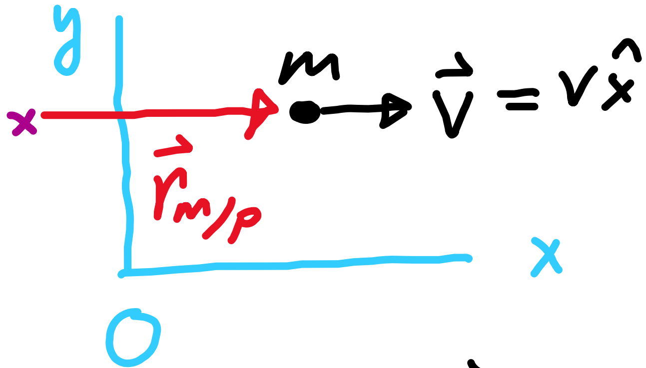
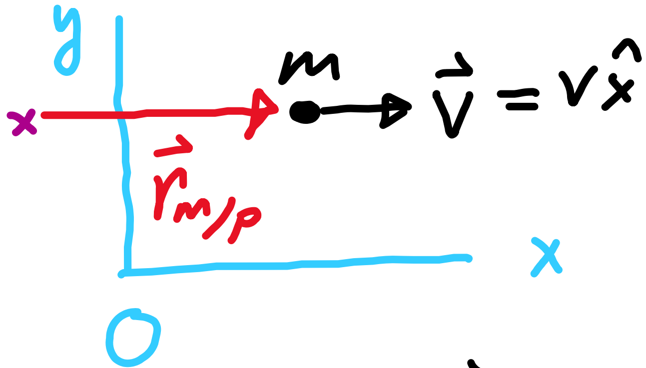


For point p:

$$\vec{H}_p = \vec{r}_{m/p} \times \vec{L}$$



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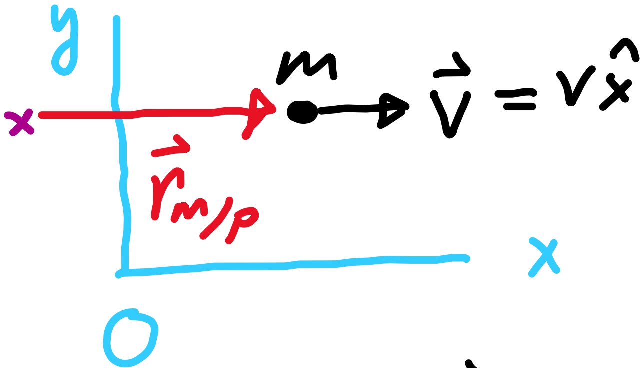
$$\vec{H}_p = \vec{r}_{m/p} \times \vec{L} = (r_{m/p}\hat{x}) \times (mv\hat{x})$$



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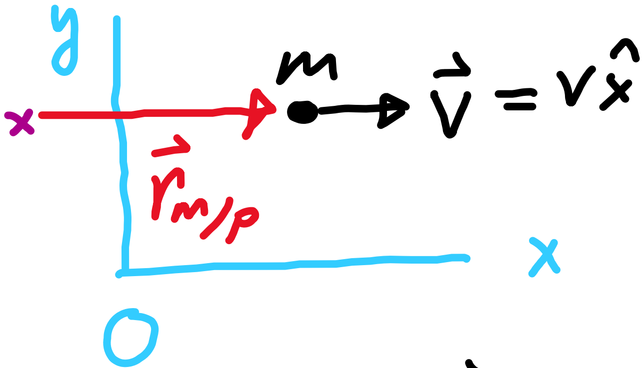

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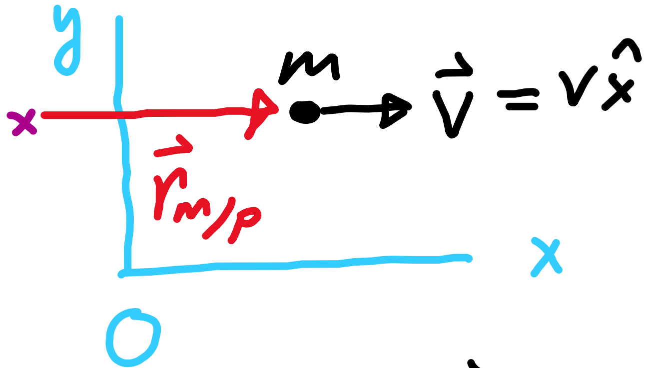

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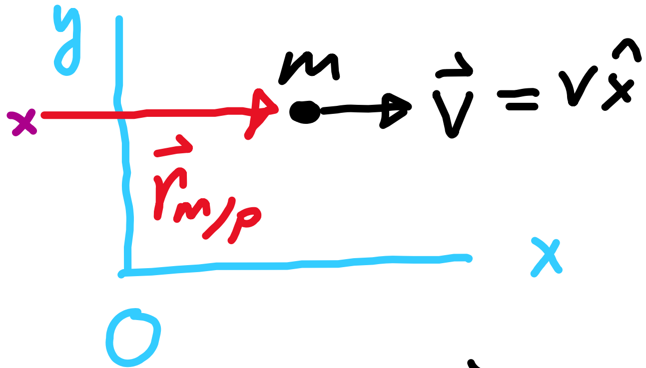

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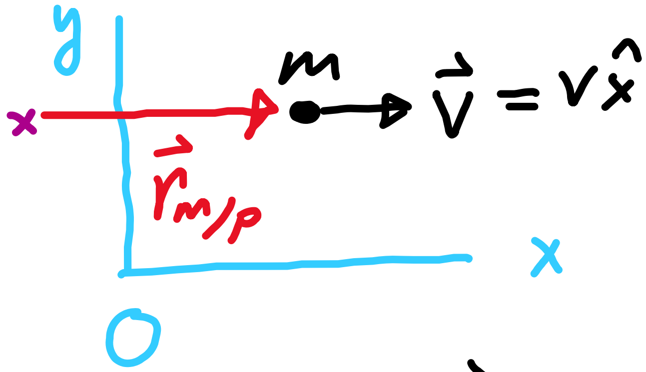

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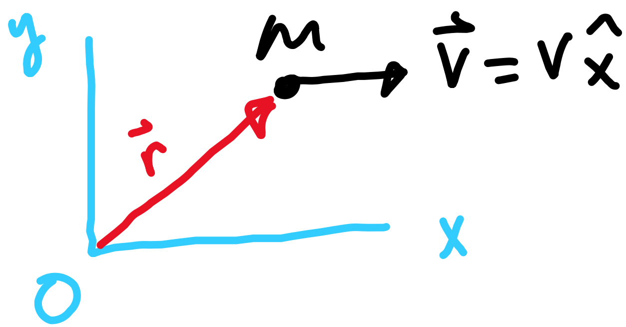

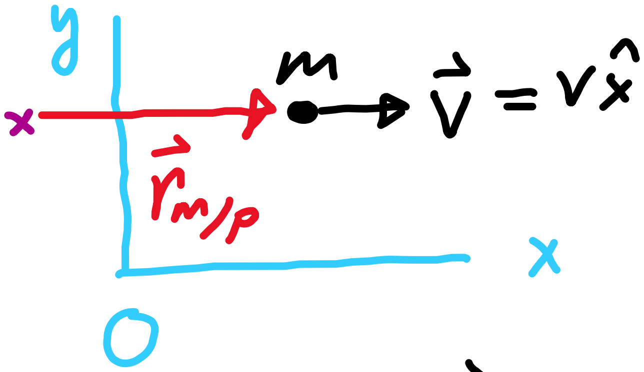

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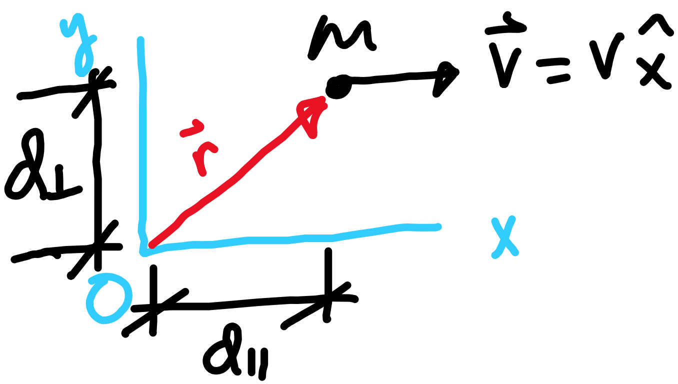

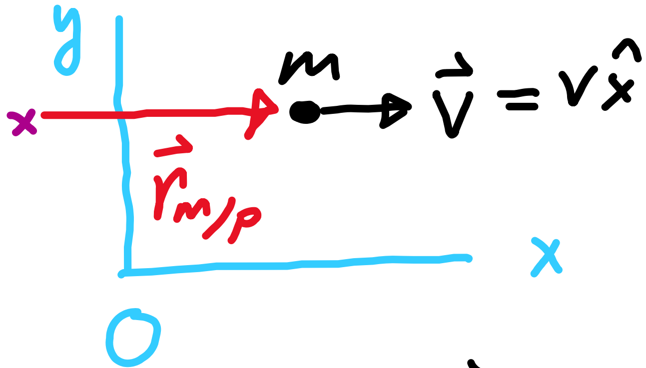

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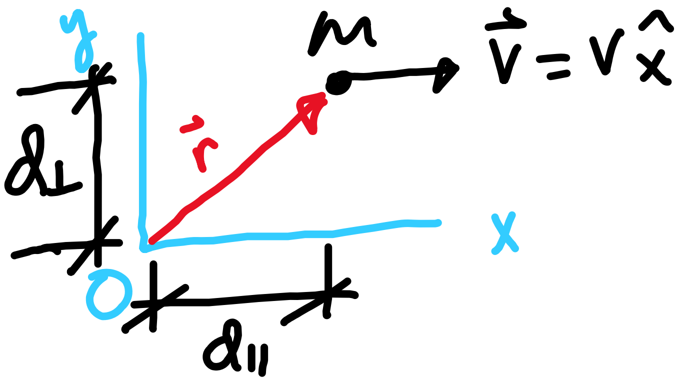
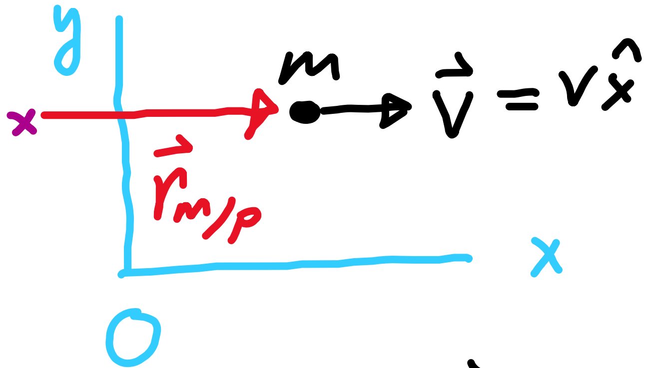
What about $\vec{H}_O$? As before $\vec{L} = mv\hat{x}$





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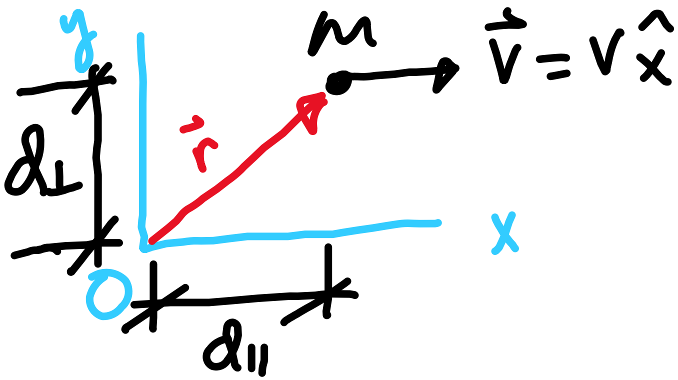
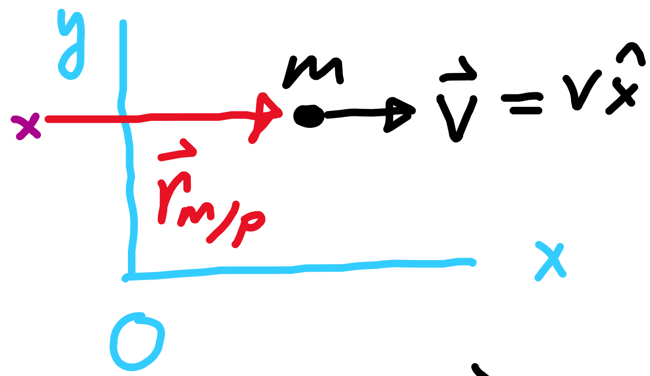


But now $\vec{H}_O = \vec{r} \times \vec{L}$



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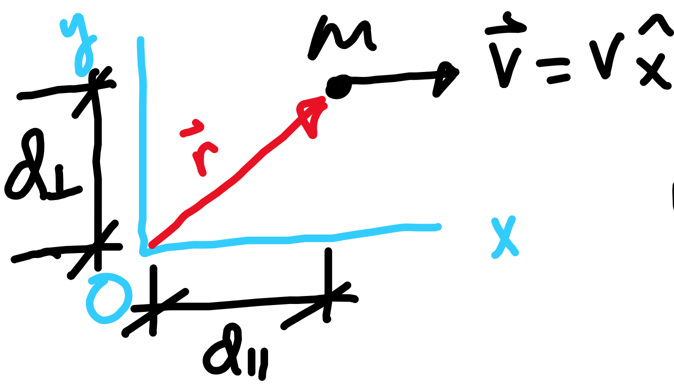

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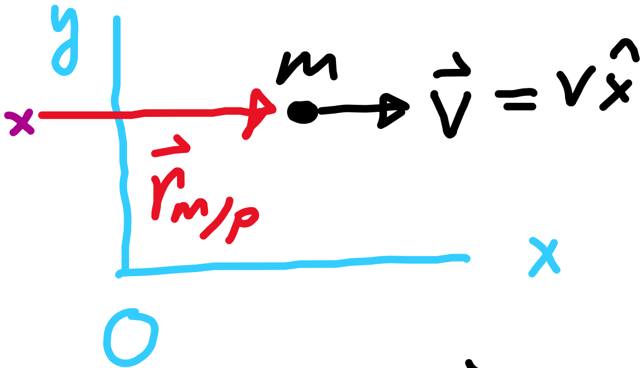
with $\vec{r} = d_\parallel \hat{x} + d_\perp \hat{y}$



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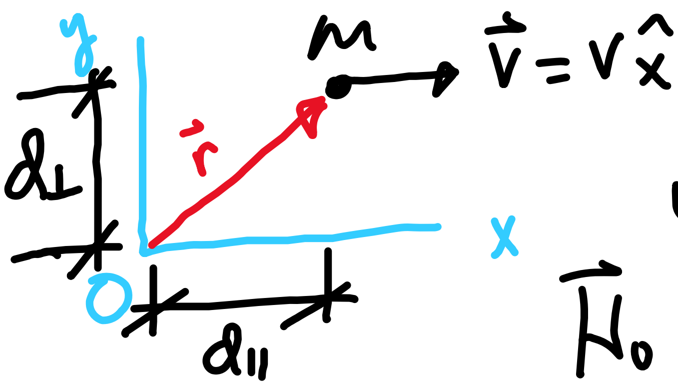

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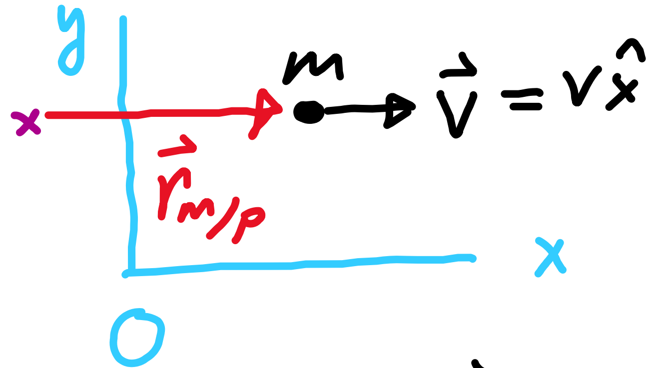
$$\vec{H}_o = (d_\parallel \hat{x}) \times (mv\hat{x}) +$$



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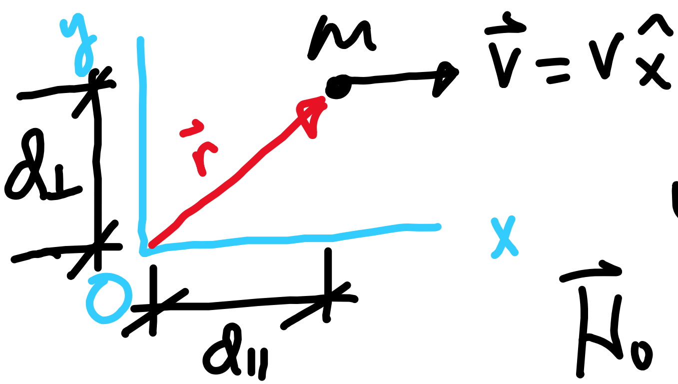

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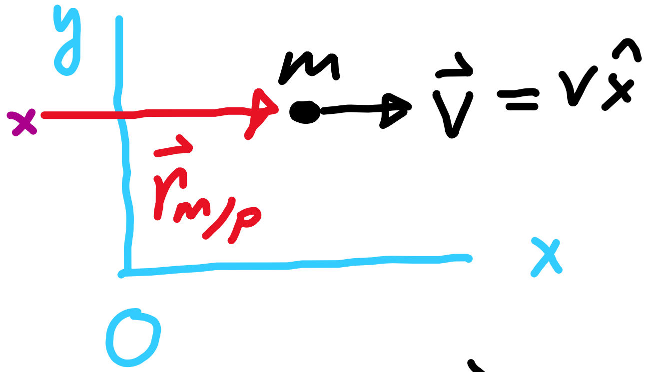

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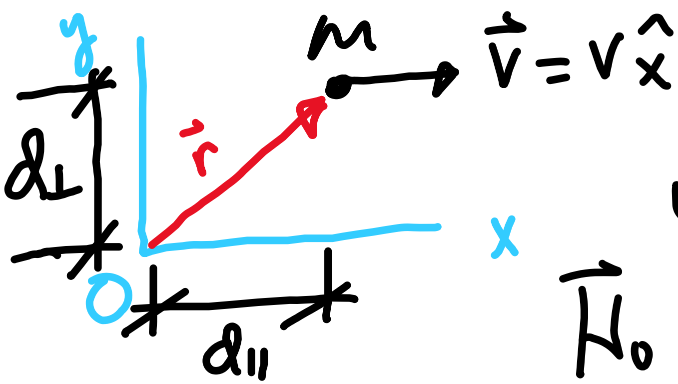

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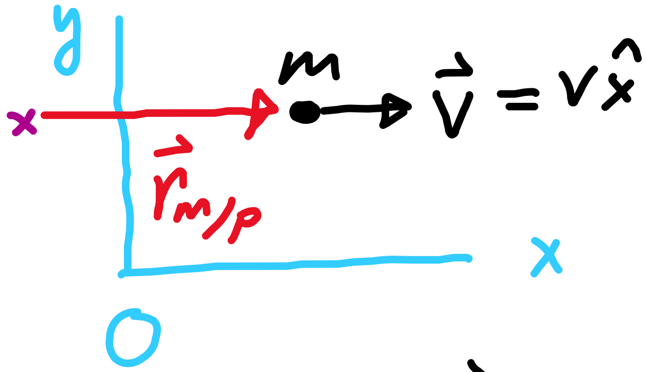

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For point p:

$$\vec{H}_p = \vec{r}_{m/p} \times \vec{L} = (r_{m/p}\hat{x}) \times (mv\hat{x}) = \emptyset$$

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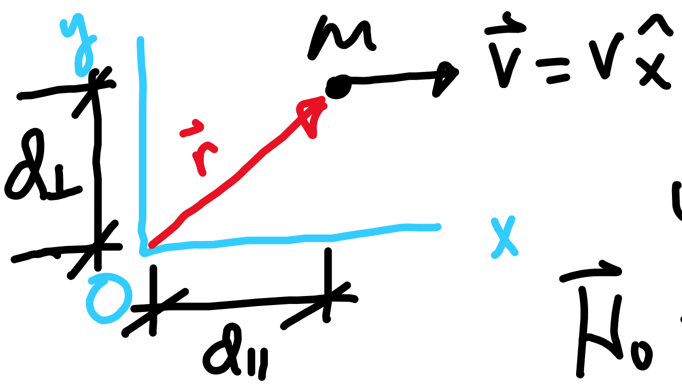

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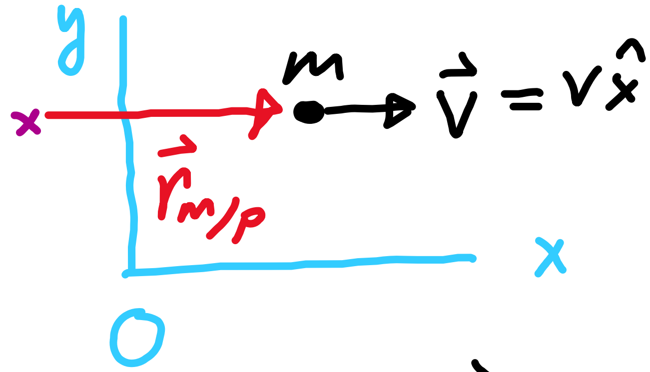
$$\Rightarrow \vec{H}_o = d_\perp m v(-\hat{z})$$



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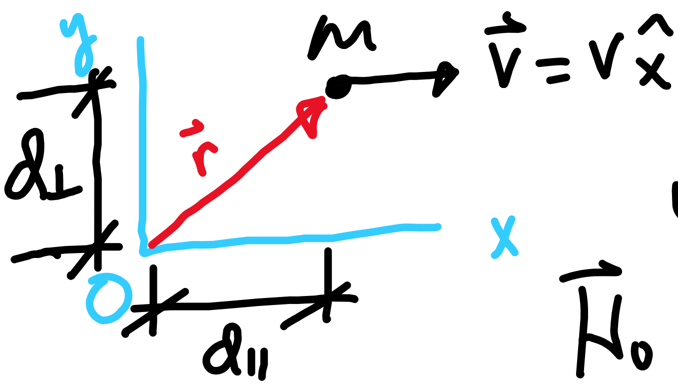
$$\vec{H}_p = \vec{r}_{m/p} \times \vec{L} = (r_{m/p}\hat{x}) \times (mv\hat{x}) = 0$$

$\Rightarrow$ No angular momentum [as expected].

What about $\vec{H}_o$? As before $\vec{L} = mv\hat{x}$



But now $\vec{H}_o = \vec{r} \times \vec{L}$

with $\vec{r} = d_\parallel \hat{x} + d_\perp \hat{y}$ So

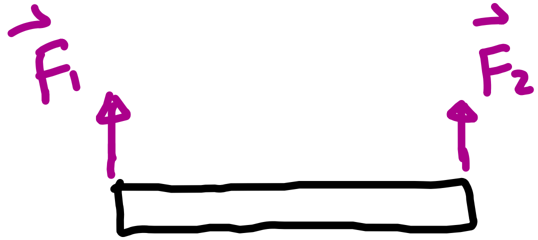
$$\vec{H}_o = \cancel{(d_\parallel \hat{x}) \times (mv\hat{x})} + (d_\perp \hat{y}) \times (mv\hat{y})$$

$$\Rightarrow \vec{H}_o = d_\perp m v(-\hat{z})$$

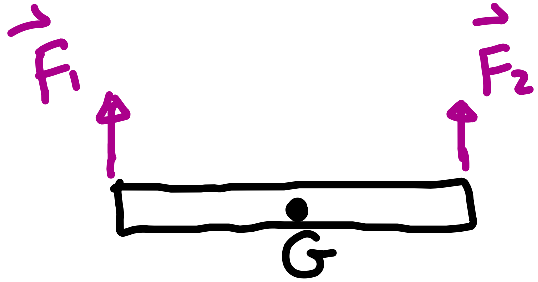
In this case $\vec{H}$ is <u>not</u> zero

Slender rod with some forces

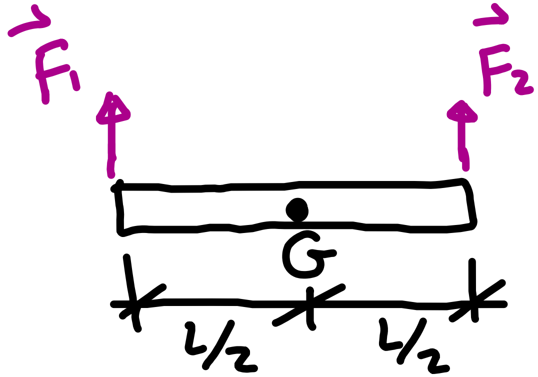
Slender rod with some forces



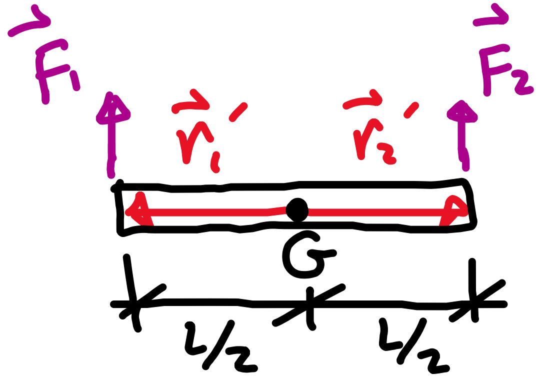
Slender rod with some forces



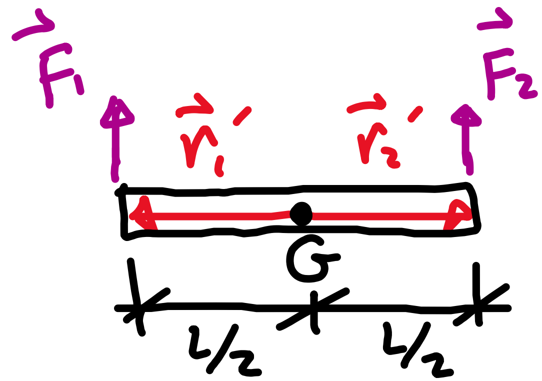
Slender rod with some forces



Slender rod with some forces

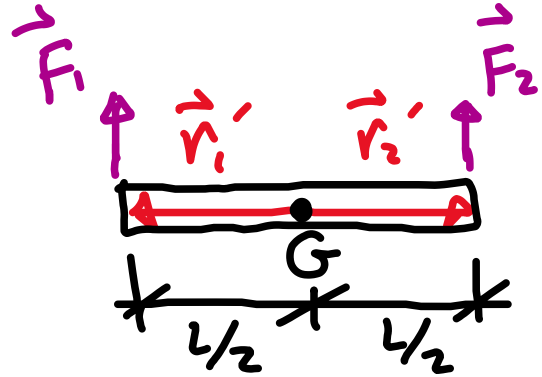


Slender rod with some forces



$$\sum \vec{F} = m\vec{\bar{a}}$$

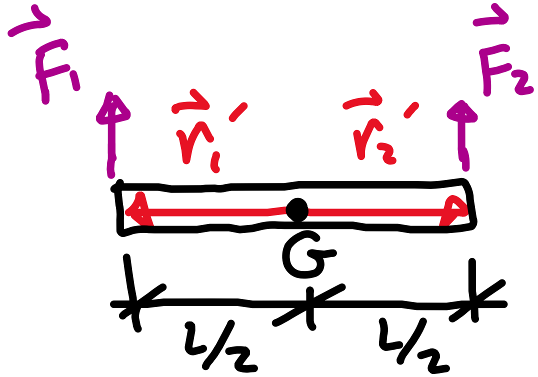
Slender rod with some forces



$$\sum \vec{F} = m\vec{\bar{a}} \ \&$$

$$\sum \vec{M}_G = \vec{r}_1' \times \vec{F}_1 +$$

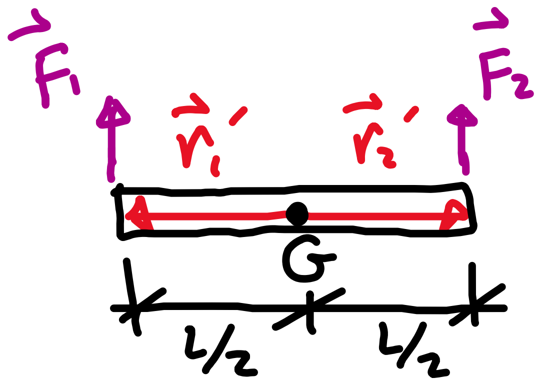
Slender rod with some forces



$$\Sigma \vec{F} = m\vec{\bar{a}} \;\&$$

$$\Sigma \vec{M}_G = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2$$

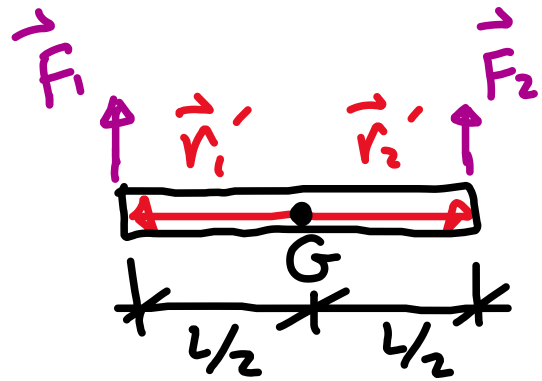
Slender rod with some forces



$$\sum \vec{F} = m\bar{\vec{a}} \;\; \&$$

$$\sum \vec{M}_G = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2 = \bar{I}\vec{\alpha}$$

Slender rod with some forces

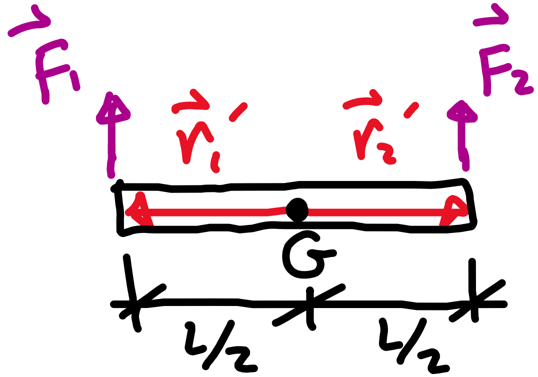


$$\sum \vec{F} = m\vec{\bar{a}} \;\; \&$$

$$\sum \vec{M}_G = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2 = \bar{I}\vec{\alpha}$$

& same as

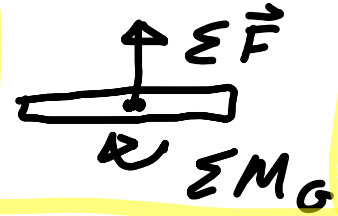
Slender rod with some forces



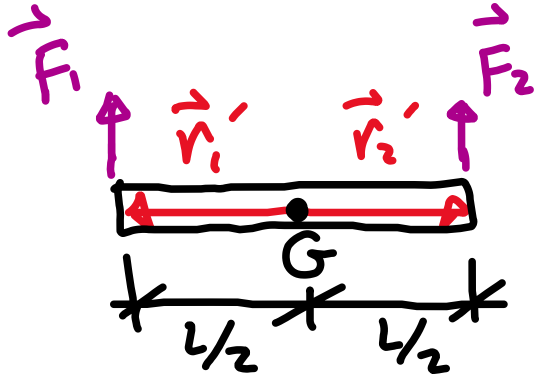
$$\sum \vec F = m\bar{\vec a} \;\;\&$$

$$\sum \vec M_G = \vec r_1' \times \vec F_1 + \vec r_2' \times \vec F_2 = \bar I \vec \alpha$$

& same as



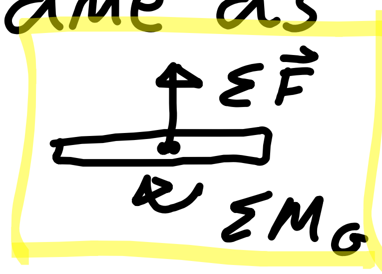
Slender rod with some forces



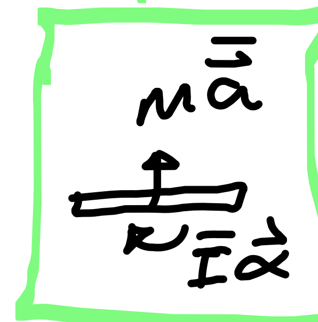
$$\sum \vec{F} = m\bar{\vec{a}} \quad \&$$

$$\sum \vec{M}_G = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2 = \bar{I}\vec{\alpha}$$

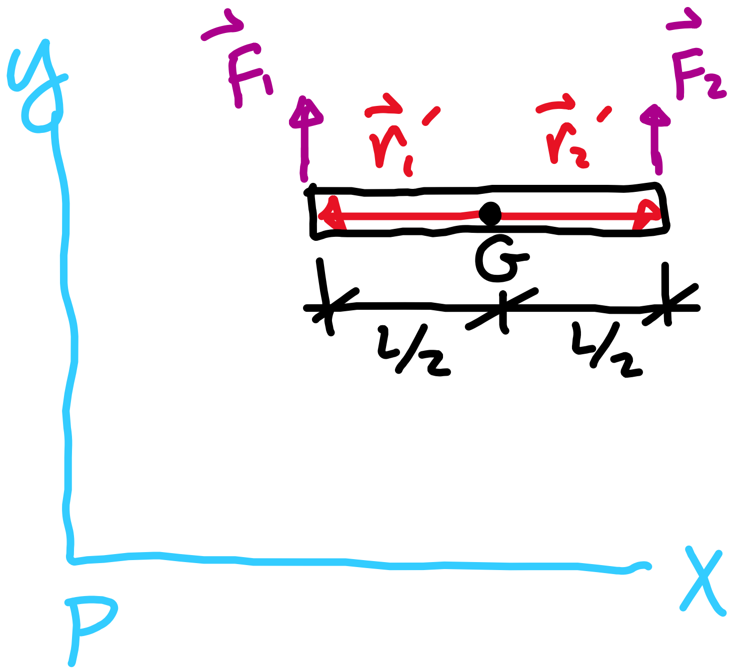
& same as



&



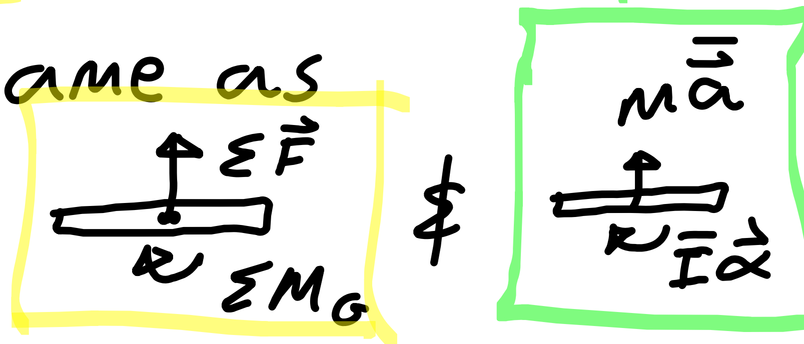
Slender rod with some forces



$\sum \vec{F} = m\vec{\bar{a}}$ &

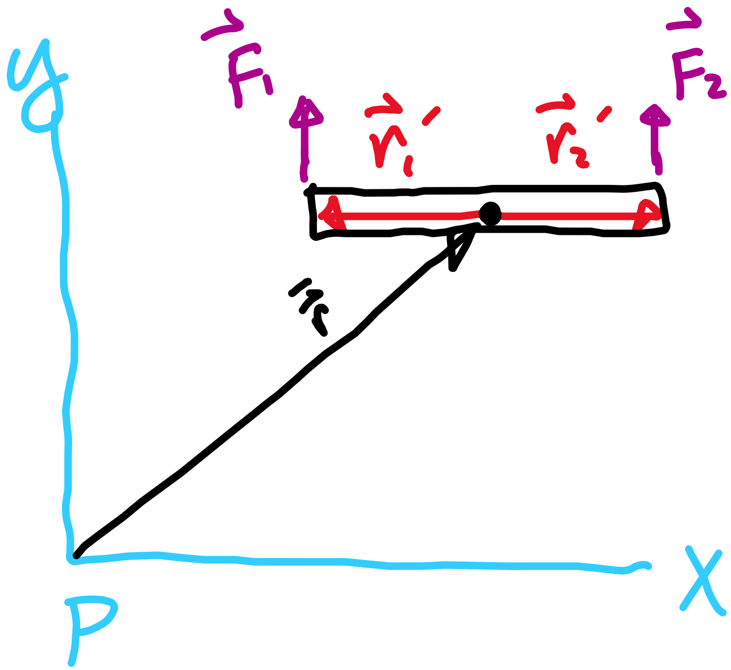
$\sum \vec{M}_G = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2 = \bar{I}\vec{\alpha}$

& same as

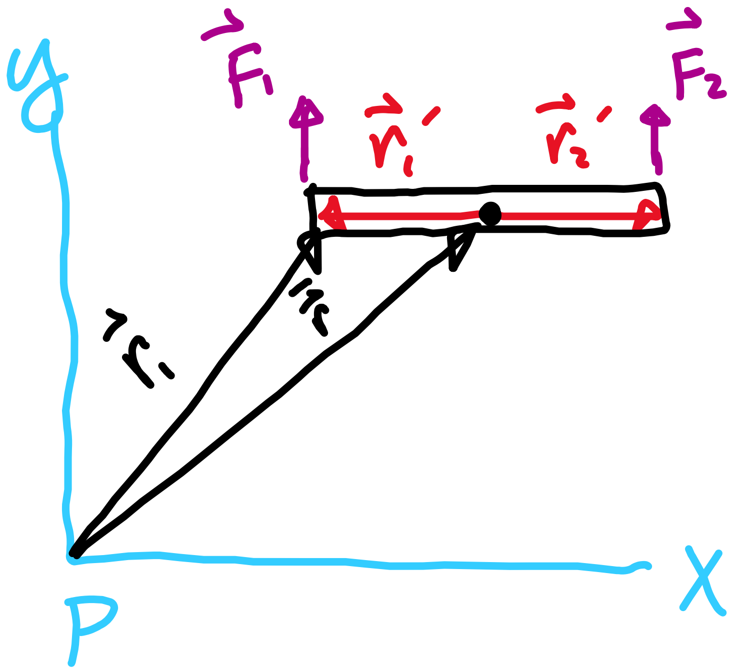


What if we want moments about P?

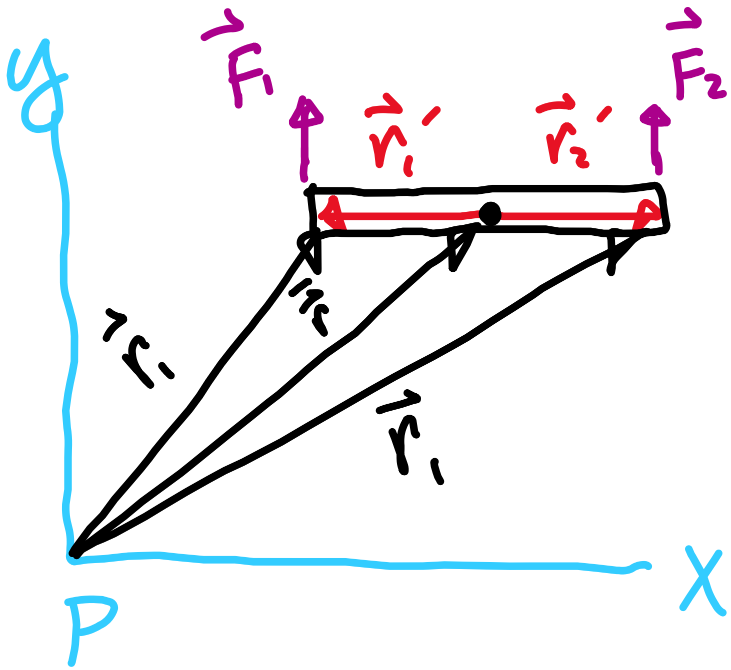
Slender rod with some forces



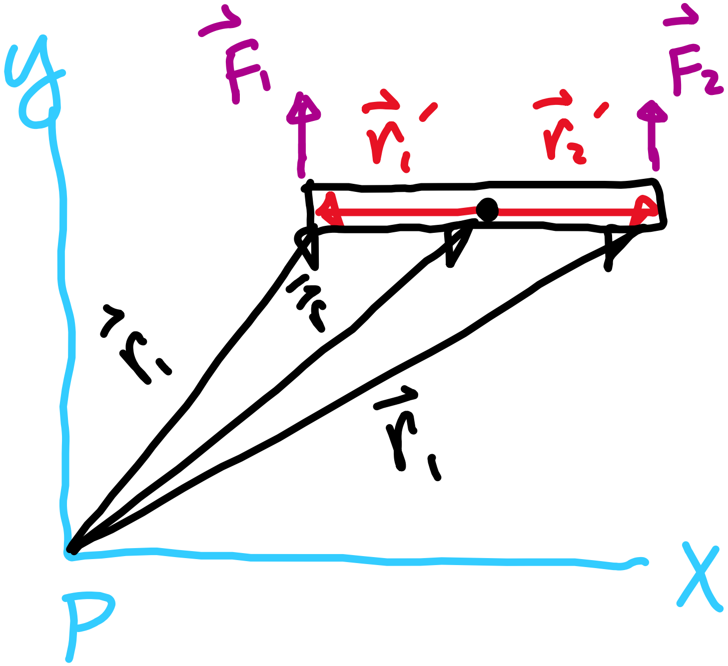
Slender rod with some forces



Slender rod with some forces

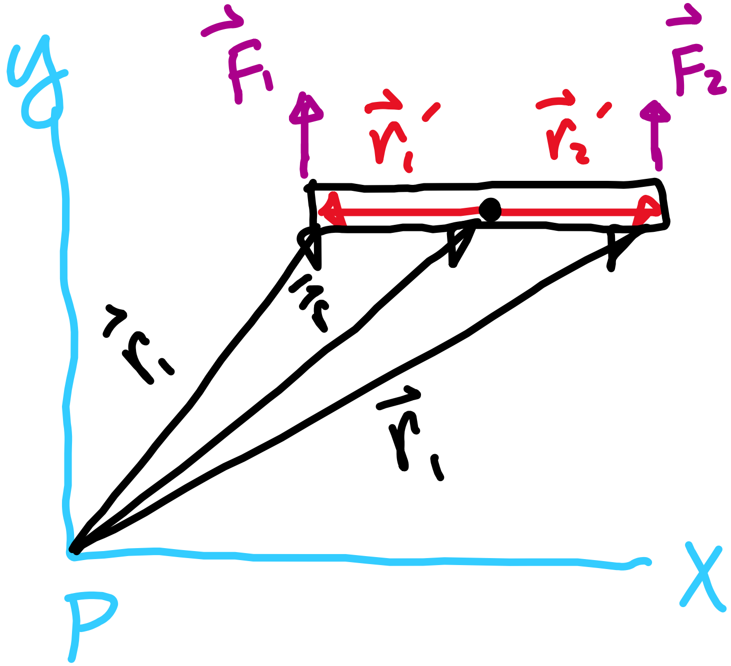


Slender rod with some forces



Note: $\vec{r}_1 = \vec{\bar{r}} + \vec{r}_1{}'$

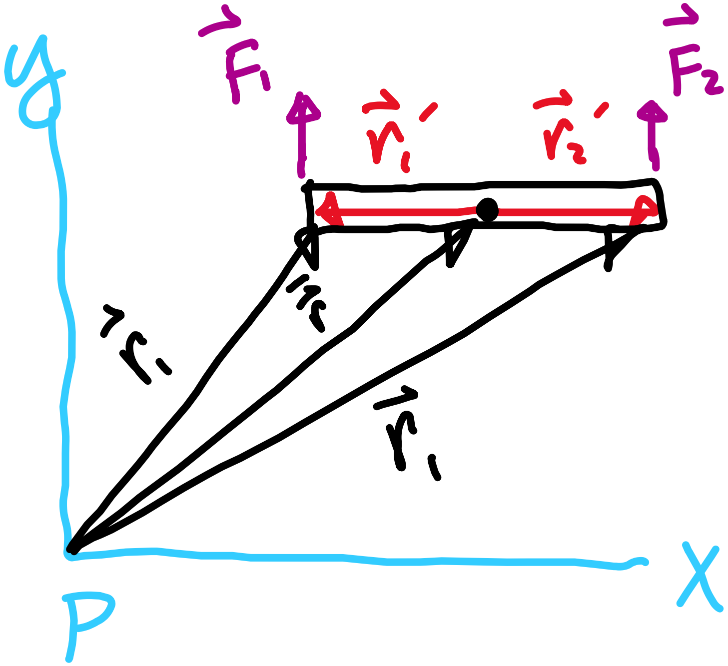
Slender rod with some forces



Note: $\vec{r}_1 = \vec{r} + \vec{r}_1{}'$ &

$\vec{r}_2 = \vec{r} + \vec{r}_2{}'$

Slender rod with some forces

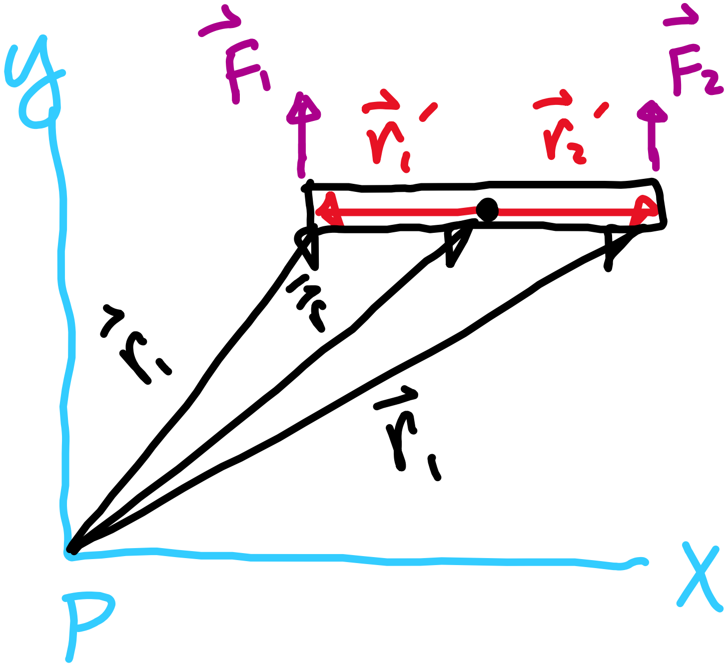


Note: $\vec{r}_1 = \vec{\bar{r}} + \vec{r}_1{}'$ &

$\vec{r}_2 = \vec{\bar{r}} + \vec{r}_2{}'$

Now $\sum \vec{M}_P = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

Slender rod with some forces



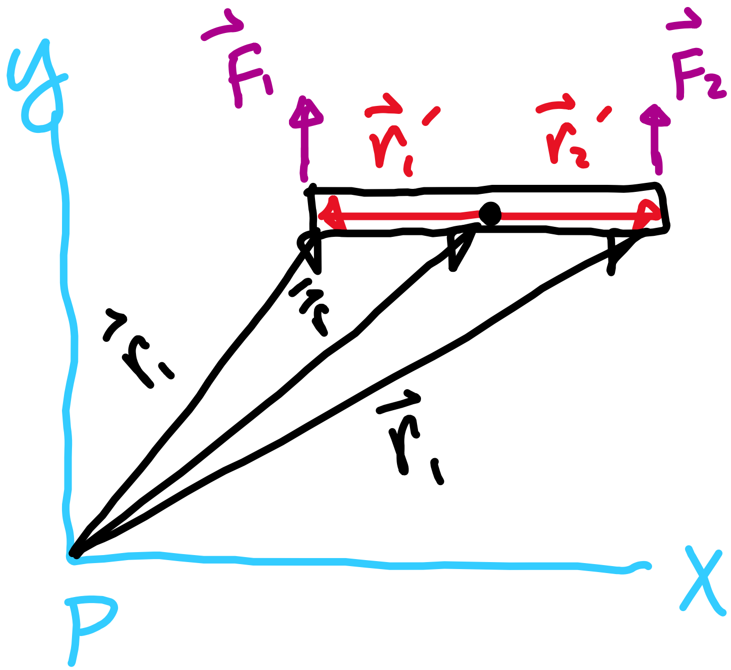
Note: $\vec r_1 = \vec{\bar r} + \vec r_1'$ &

$\vec r_2 = \vec{\bar r} + \vec r_2'$

Now $\Sigma \vec M_P = \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2$

$= \vec{\bar r} \times \vec F_1 + \vec r_1' \times \vec F_1 + \vec{\bar r} \times \vec F_2 + \vec r_2' \times \vec F_2$

Slender rod with some forces



Note: $\vec r_1 = \vec{\bar r} + \vec r_1{}'$ &

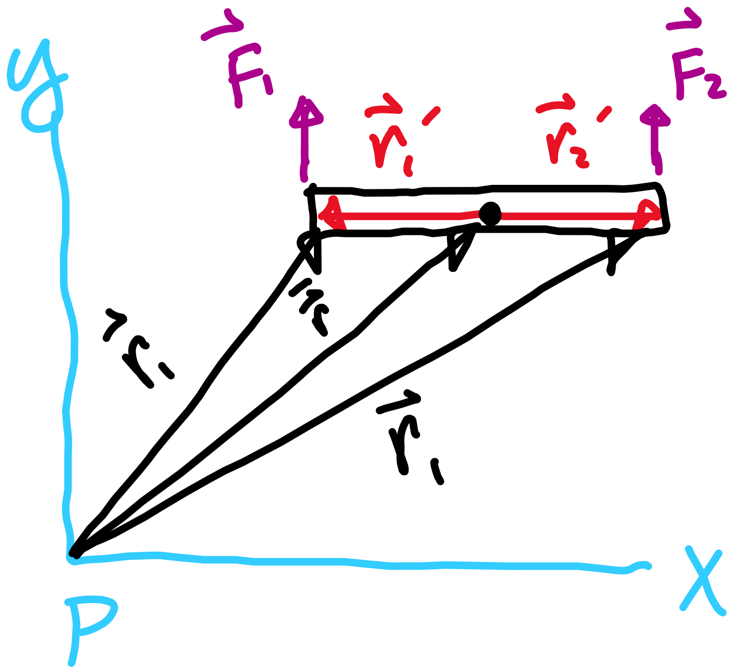
$\vec r_2 = \vec{\bar r} + \vec r_2{}'$

Now $\sum \vec M_P = \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2$

$$= \vec{\bar r} \times \vec F_1 + \vec r_1{}' \times \vec F_1 + \vec{\bar r} \times \vec F_2 + \vec r_2{}' \times \vec F_2$$

$$= \vec{\bar r} \times \sum \vec F + [\vec r_1{}' \times \vec F_1 + \vec r_2{}' \times \vec F_2]$$

Slender rod with some forces



Note: $\vec{r}_1 = \vec{\bar{r}} + \vec{r}_1{}'$ &

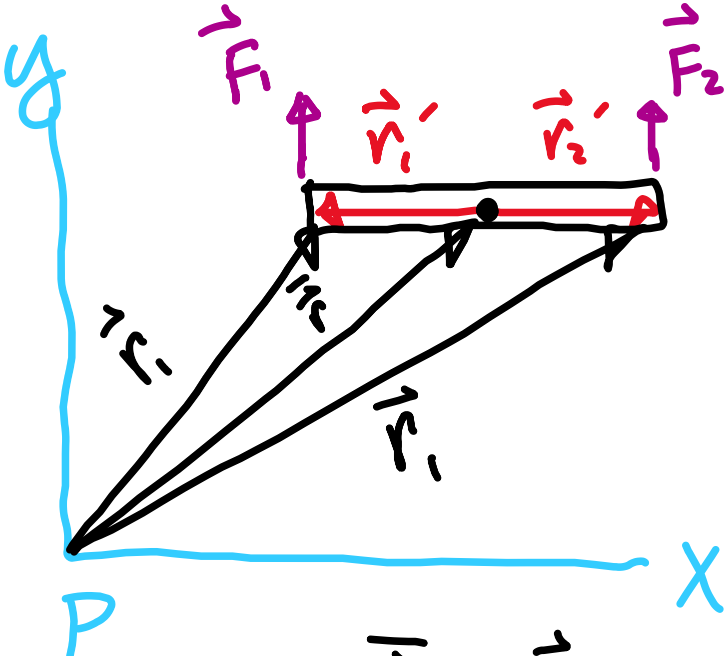
$\vec{r}_2 = \vec{\bar{r}} + \vec{r}_2{}'$

Now $\Sigma \vec{M}_P = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$= \vec{\bar{r}} \times \vec{F}_1 + \vec{r}_1{}' \times \vec{F}_1 + \vec{\bar{r}} \times \vec{F}_2 + \vec{r}_2{}' \times \vec{F}_2$

$$= \vec{\bar{r}} \times \Sigma \vec{F} + \underbrace{\left[\vec{r}_1{}' \times \vec{F}_1 + \vec{r}_2{}' \times \vec{F}_2\right]}_{\text{Same as before}}$$

Slender rod with some forces



Note: $\vec{r}_1 = \vec{\bar{r}} + \vec{r}_1{}'$ &

$\vec{r}_2 = \vec{\bar{r}} + \vec{r}_2{}'$

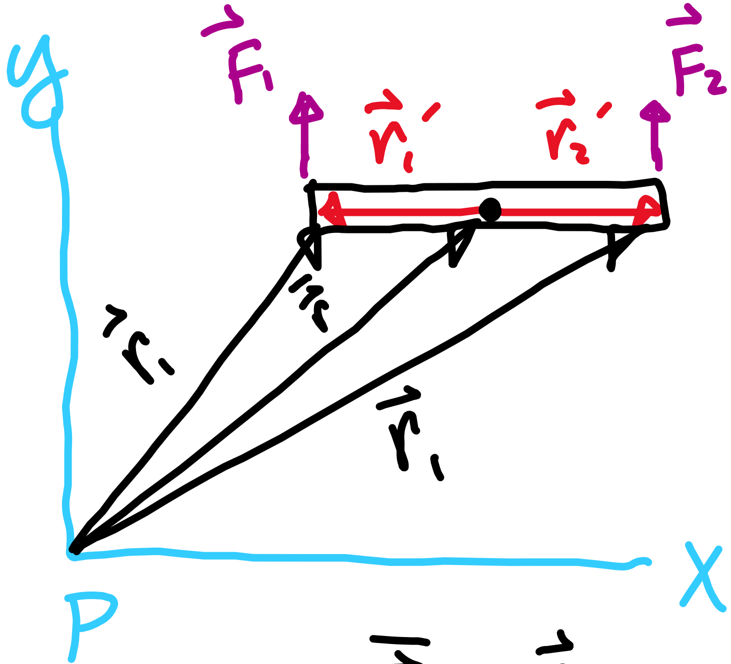
Now $\Sigma \vec{M}_P = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$= \vec{\bar{r}} \times \vec{F}_1 + \vec{r}_1{}' \times \vec{F}_1 + \vec{\bar{r}} \times \vec{F}_2 + \vec{r}_2{}' \times \vec{F}_2$

$$= \vec{\bar{r}} \times \Sigma \vec{F} + \underbrace{\left[\vec{r}_1{}' \times \vec{F}_1 + \vec{r}_2{}' \times \vec{F}_2\right]}_{\text{Same as before}}$$

Note: $\vec{\bar{r}} = \vec{r}_{G/P}$

Slender rod with some forces



Note: $\vec r_1 = \vec{\bar r} + \vec r_1{}'$ &

$\vec r_2 = \vec{\bar r} + \vec r_2{}'$

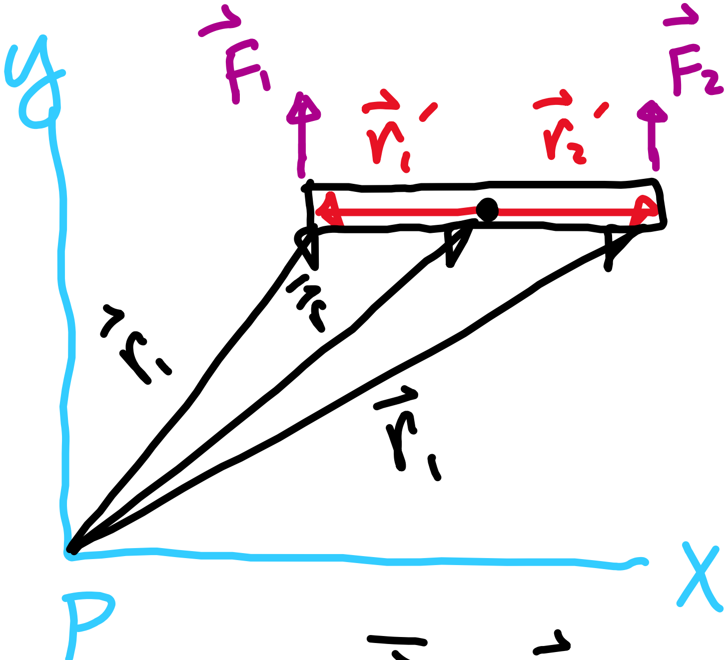
Now $\sum \vec M_P = \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2$

$= \vec{\bar r} \times \vec F_1 + \vec r_1{}' \times \vec F_1 + \vec{\bar r} \times \vec F_2 + \vec r_2{}' \times \vec F_2$

$= \vec{\bar r} \times \sum \vec F + \underbrace{\left[\vec r_1{}' \times \vec F_1 + \vec r_2{}' \times \vec F_2\right]}_{\text{Same as before}}$

Note: $\vec{\bar r} = \vec r_{G/P}$ so $\vec M_P = \vec r_{G/P} \times \sum \vec F + \vec M_G$

Slender rod with some forces



Note: $\vec r_1 = \bar{\vec r} + \vec r_1'$ &

$\vec r_2 = \bar{\vec r} + \vec r_2'$

Now $\Sigma \vec M_P = \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2$

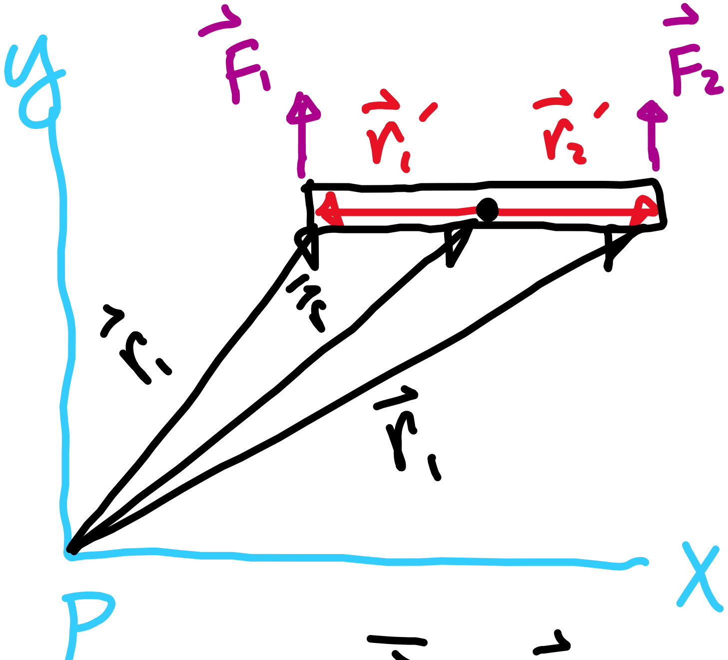
$= \bar{\vec r} \times \vec F_1 + \vec r_1' \times \vec F_1 + \bar{\vec r} \times \vec F_2 + \vec r_2' \times \vec F_2$

$= \bar{\vec r} \times \Sigma \vec F + \underbrace{[\vec r_1' \times \vec F_1 + \vec r_2' \times \vec F_2]}_{\text{Same as before}}$

Note: $\bar{\vec r} = \vec r_{G/P}$ so $\vec M_P = \vec r_{G/P} \times \Sigma \vec F + \vec M_G$

& since $\Sigma \vec F = m\bar{\vec a}$

Slender rod with some forces



Note: $\vec{r}_1 = \vec{\bar{r}} + \vec{r}_1'$ &

$\vec{r}_2 = \vec{\bar{r}} + \vec{r}_2'$

Now $\sum \vec{M}_P = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

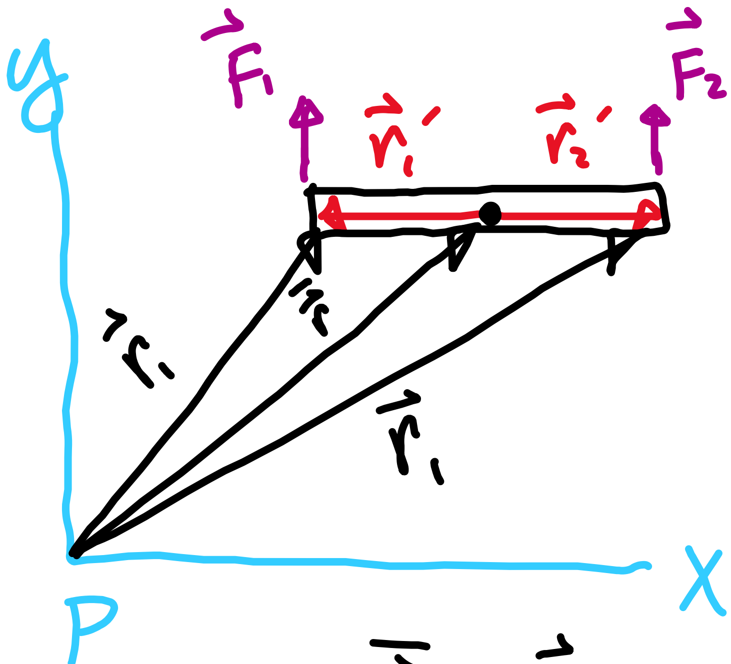
$= \vec{\bar{r}} \times \vec{F}_1 + \vec{r}_1' \times \vec{F}_1 + \vec{\bar{r}} \times \vec{F}_2 + \vec{r}_2' \times \vec{F}_2$

$= \vec{\bar{r}} \times \sum \vec{F} + \underbrace{[\vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2]}_{\text{Same as before}}$

Note: $\vec{\bar{r}} = \vec{r}_{G/P}$ So $\vec{M}_P = \vec{r}_{G/P} \times \sum \vec{F} + \vec{M}_G$

& Since $\sum \vec{F} = m\vec{\bar{a}}$ & $\vec{M}_G = \bar{I}\vec{\alpha}$

Slender rod with some forces



Note: $\vec{r}_1 = \bar{\vec{r}} + \vec{r}_1'$ &

$\vec{r}_2 = \bar{\vec{r}} + \vec{r}_2'$

Now $\Sigma \vec{M}_P = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$= \bar{\vec{r}} \times \vec{F}_1 + \vec{r}_1' \times \vec{F}_1 + \bar{\vec{r}} \times \vec{F}_2 + \vec{r}_2' \times \vec{F}_2$

$= \bar{\vec{r}} \times \Sigma \vec{F} + \underbrace{[\vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2]}_{\text{Same as before}}$

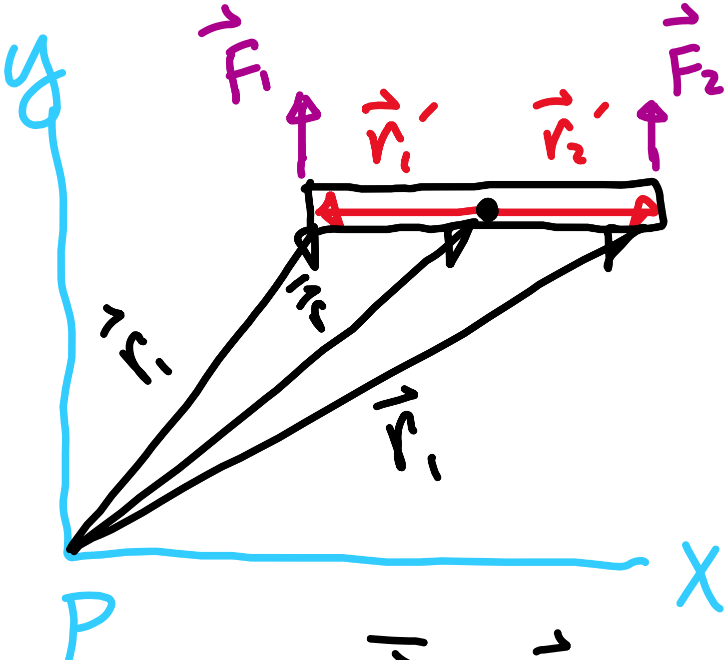
Note: $\bar{\vec{r}} = \vec{r}_{G/P}$ so $\vec{M}_P = \vec{r}_{G/P} \times \Sigma \vec{F} + \vec{M}_G$

& since $\Sigma \vec{F} = m\bar{\vec{a}}$ & $\vec{M}_G = \bar{I}\vec{\alpha}$ then

$$\boxed{\vec{M}_P = \underbrace{\vec{r}_{G/P} \times m\bar{\vec{a}}}_{\text{New piece}} + \bar{I}\vec{\alpha}}$$

ASU

Slender rod with some forces



Note: $\vec r_1 = \vec{\bar r} + \vec r_1'$ &

$\vec r_2 = \vec{\bar r} + \vec r_2'$

Now $\Sigma \vec M_P = \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2$

$$= \vec{\bar r}\times\vec F_1 + \vec r_1'\times \vec F_1 + \vec{\bar r}\times\vec F_2 + \vec r_2'\times\vec F_2$$

$$= \vec{\bar r}\times\Sigma\vec F + \underbrace{[\vec r_1'\times\vec F_1 + \vec r_2'\times\vec F_2]}_{\text{Same as before}}$$

Note: $\vec{\bar r} = \vec r_{G/P}$ so $\vec M_P = \vec r_{G/P}\times\Sigma\vec F + \vec M_G$

& since $\Sigma\vec F = m\vec{\bar a}$ & $\vec M_G = \bar I\vec\alpha$ then

$$\boxed{\vec M_P = \underbrace{\vec r_{G/P}\times m\vec{\bar a}}_{\text{New piece}} + \bar I\vec\alpha}$$

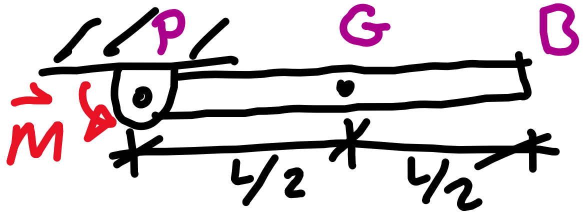
New piece → $\vec H_P$ of point particle of mass m at G

Example

Example

Example

L_x^g



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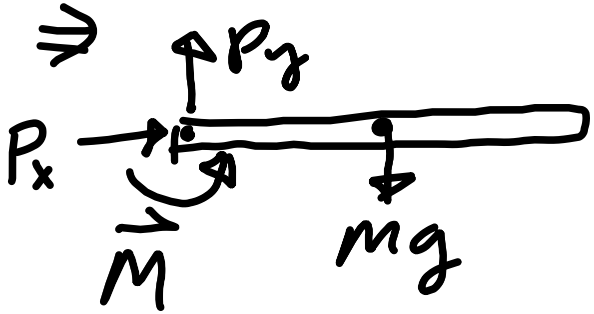
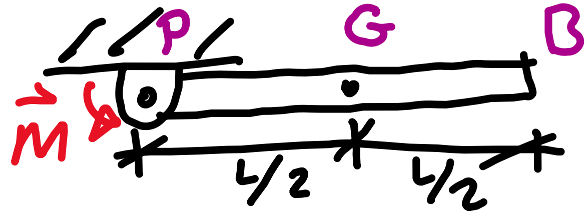
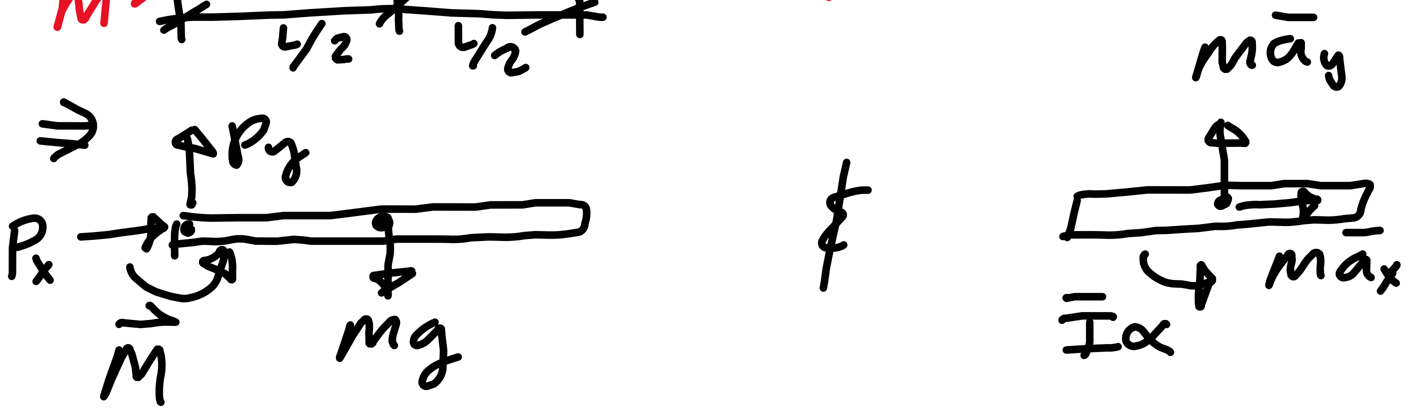
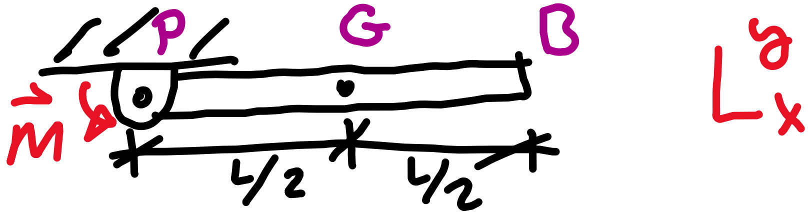


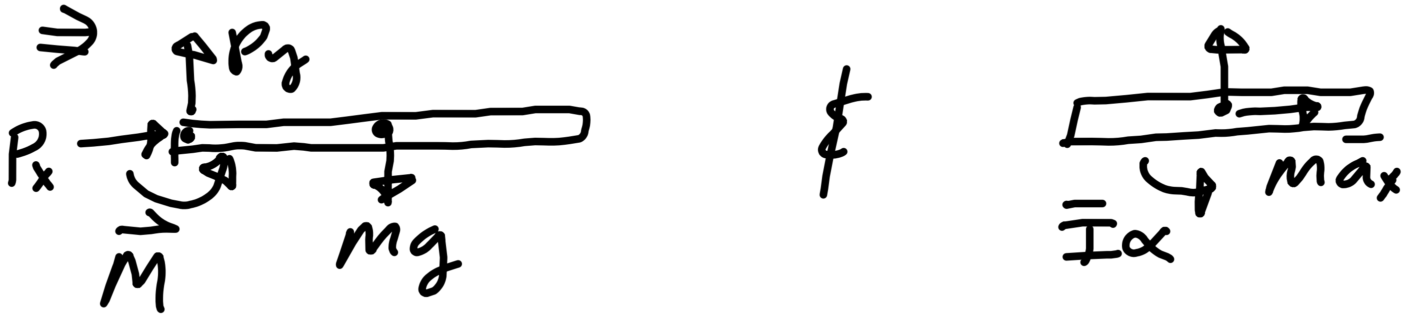
Example





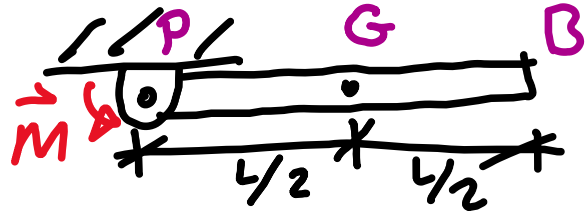
Example

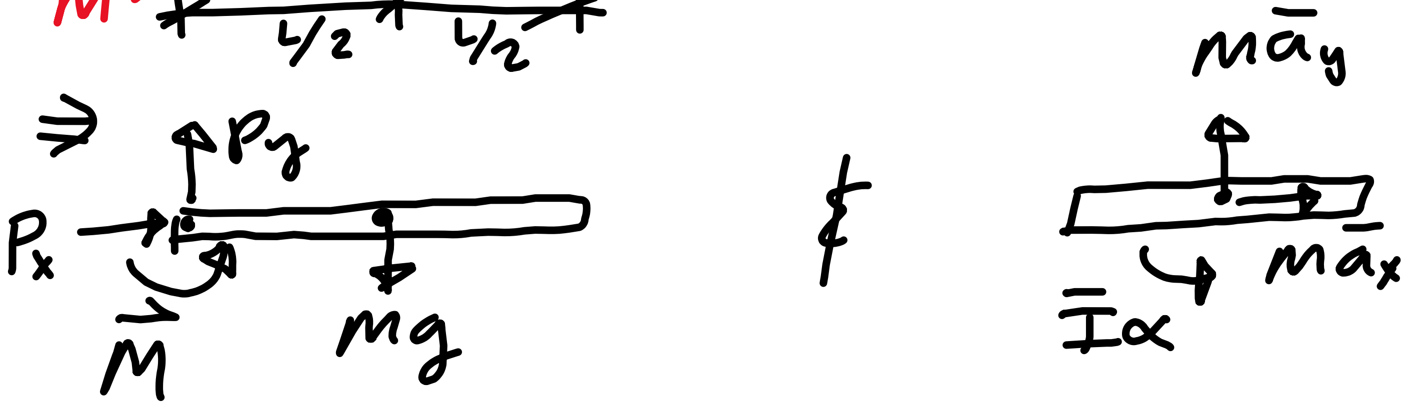




About G: $\circlearrowleft \Sigma M_G = \bar{I}\alpha$

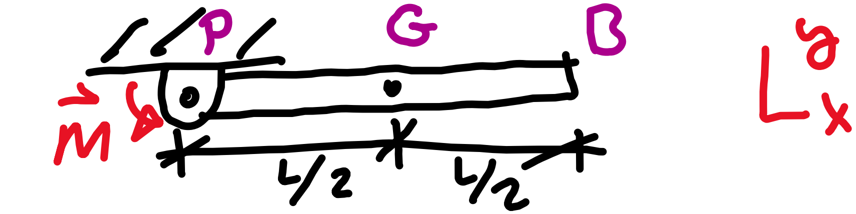
Example

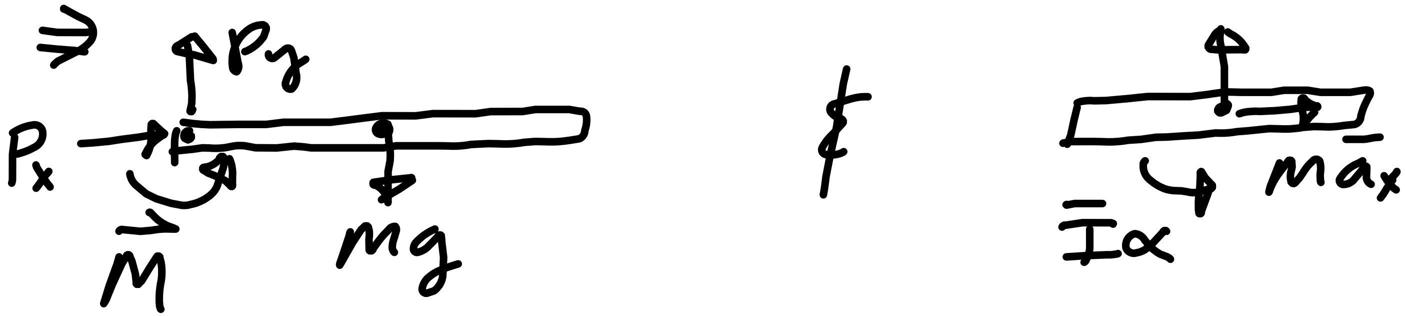




About G: $\circlearrowleft \sum M_G = \bar{I}\alpha \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha$

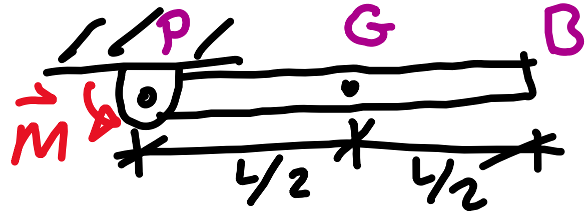
Example

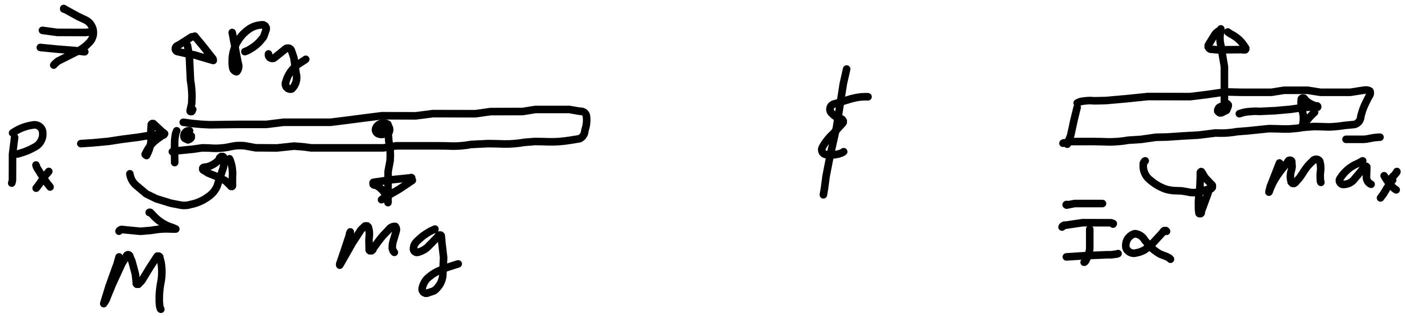




About G: $\circlearrowleft \Sigma M_G = \bar{I}\alpha \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha \quad (1)$

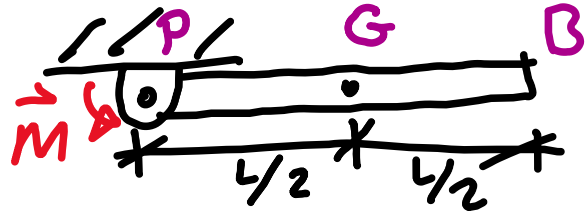
Example



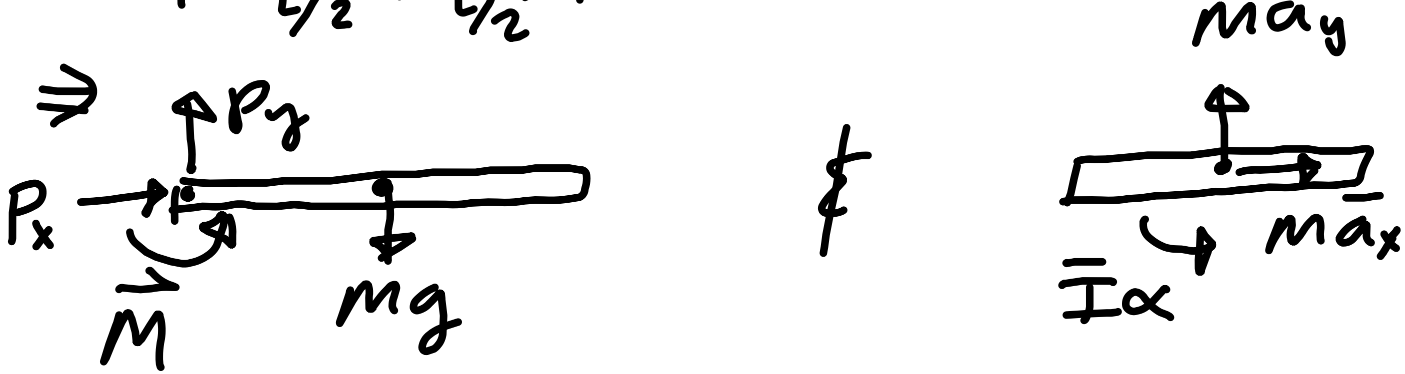


About G: $\circlearrowleft \Sigma M_G = \bar{I}\alpha \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha$ (1)

About P: $\circlearrowleft \Sigma M_P = \bar{I}\alpha + \vec{r}_{G/P} \times m\bar{\vec{a}}$

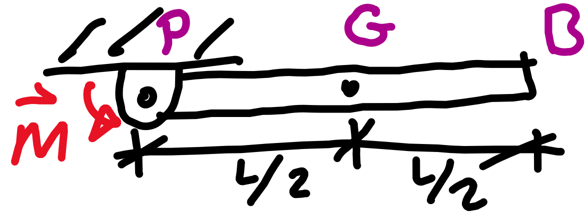


Example

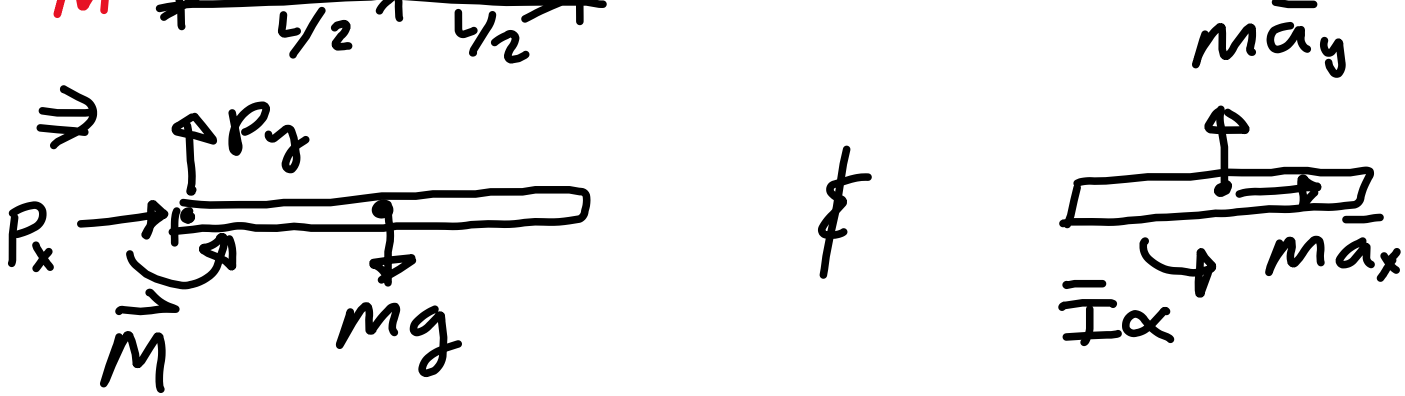


About G: $\circlearrowleft \sum M_G = \bar I \alpha \Rightarrow M - \frac{L}{2} P_y = \bar I \alpha$ (1)

About P: $\circlearrowleft \sum M_P = \bar I \alpha + \vec r_{G/P} \times m \bar a \Rightarrow M - \frac{L}{2} mg = \bar I \alpha + \frac{mL\bar a_y}{2}$



Example

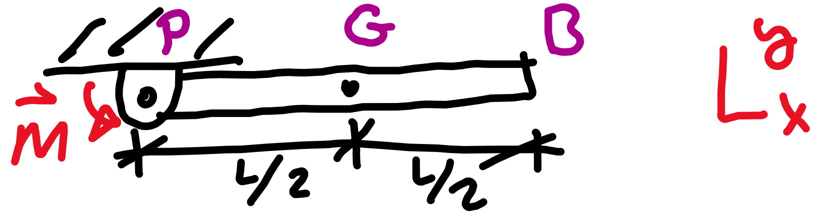


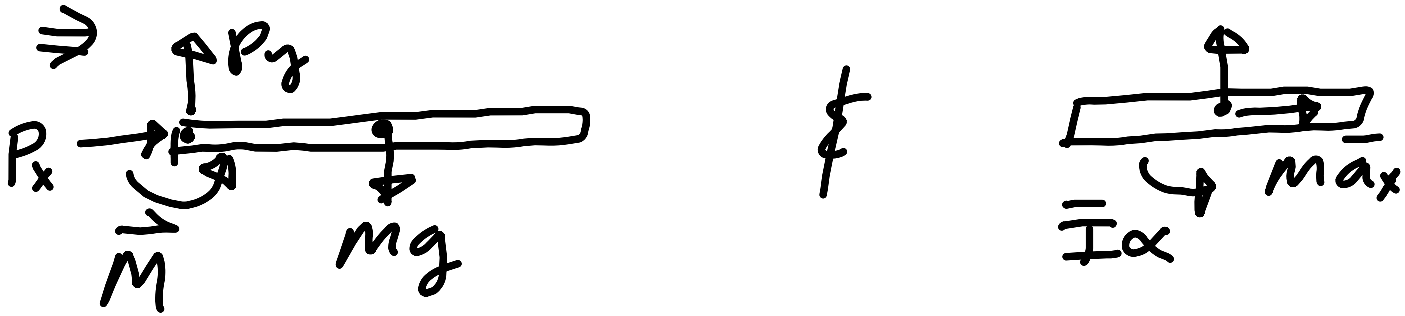
About G: $\circlearrowleft \Sigma M_G = \bar I \alpha \Rightarrow M - \frac{L}{2} P_y = \bar I \alpha \quad (1)$

About P: $\circlearrowleft \Sigma M_P = \bar I \alpha + \vec r_{G/P} \times m\bar{\vec a} \Rightarrow M - \frac{L}{2} mg = \bar I \alpha + \frac{mL\bar a_y}{2}$

But $\Sigma F_y = m\bar a_y$

Example



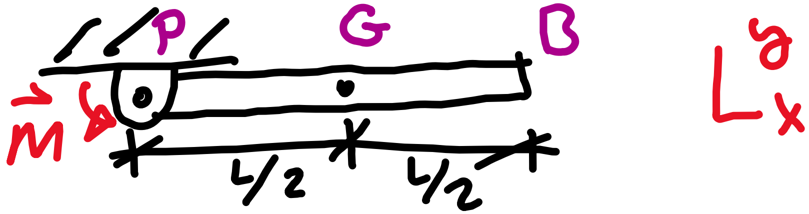


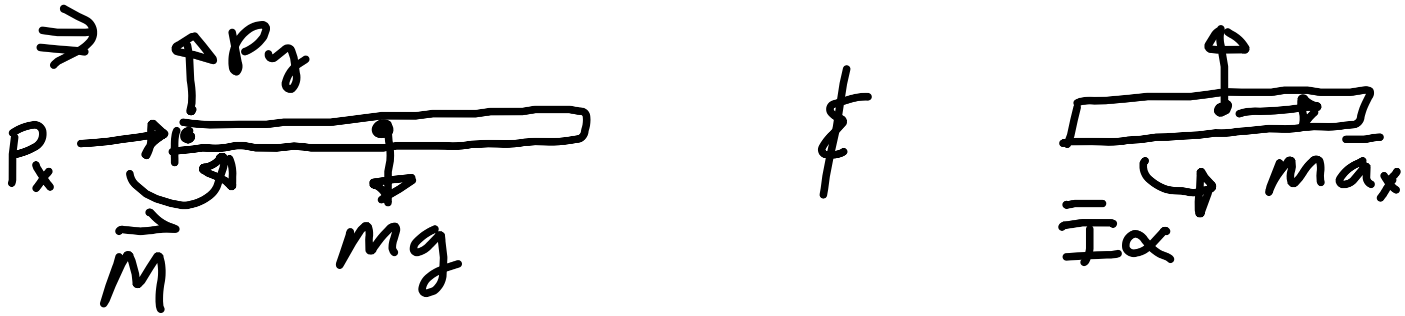
About G: $\sum M_G = \bar I\alpha \Rightarrow M - \frac{L}{2}P_y = \bar I\alpha$ (1)

About P: $\sum M_P = \bar I\alpha + \vec r_{G/P} \times m\bar{\vec a} \Rightarrow M - \frac{L}{2}mg = \bar I\alpha + \frac{mL\bar a_y}{2}$

But $\Sigma F_y = m\bar a_y \Rightarrow P_y - mg = m\bar a_y$

Example





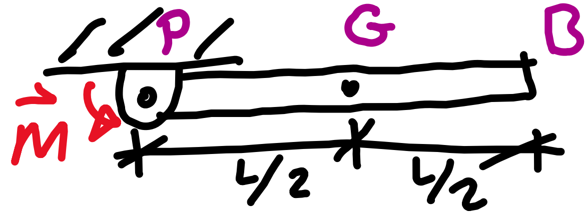
About G: $\sum M_G = \bar{I}\alpha \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha \quad (1)$

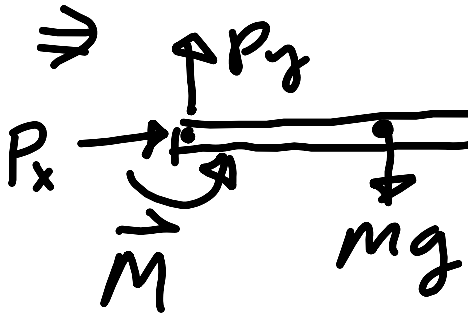
About P: $\sum M_P = \bar{I}\alpha + \vec{r}_{G/P} \times m\bar{\vec{a}} \Rightarrow M - \frac{L}{2}mg = \bar{I}\alpha + \frac{mL\bar{a}_y}{2}$

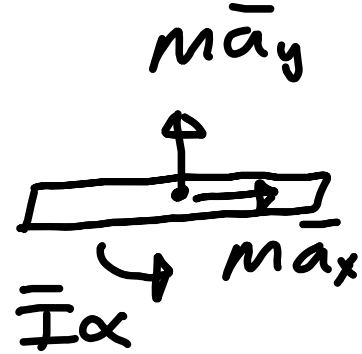
But $\sum F_y = m\bar{a}_y \Rightarrow P_y - mg = m\bar{a}_y$

$\Rightarrow M - \frac{L}{2}mg = \bar{I}\alpha + \frac{L}{2}(P_y - mg)$

Example





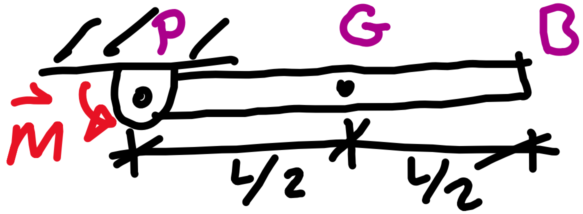


About G: $\sum M_G = \bar{I}\alpha \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha \quad (1)$

About P: $\sum M_P = \bar{I}\alpha + \vec{r}_{G/P} \times m\bar{\vec{a}} \Rightarrow M - \frac{L}{2}mg = \bar{I}\alpha + \frac{mL\bar{a}_y}{2}$

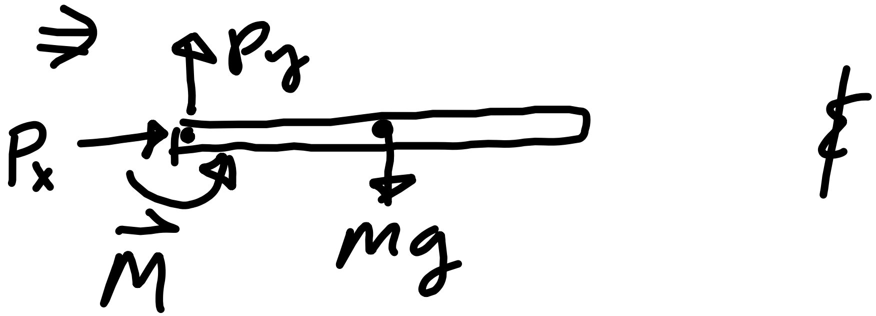
But $\sum F_y = m\bar{a}_y \Rightarrow P_y - mg = m\bar{a}_y$

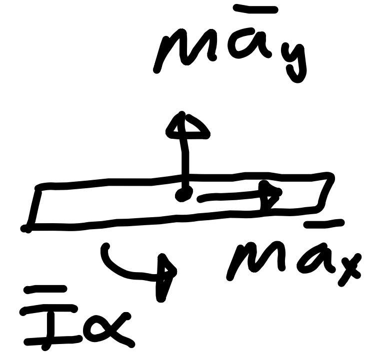
$$\Rightarrow M - \frac{L}{2}mg = \bar{I}\alpha + \frac{L}{2}(P_y - mg) \Rightarrow M - \frac{L}{2}P_y = \bar{I}\alpha$$



L_x^y

Example





About G: $\circlearrowleft \sum M_G = \bar I\alpha \Rightarrow M - \frac{L}{2}P_y = \bar I\alpha$ (1)

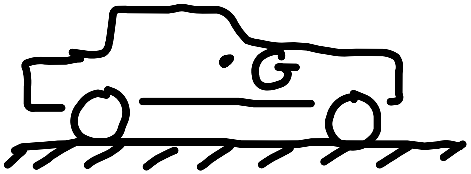
About P: $\circlearrowleft \sum M_P = \bar I\alpha + \vec r_{G/P} \times m\bar{\vec a} \Rightarrow M - \frac{L}{2}mg = \bar I\alpha + \frac{mL\bar a_y}{2}$

But $\sum F_y = m\bar a_y \Rightarrow P_y - mg = m\bar a_y$

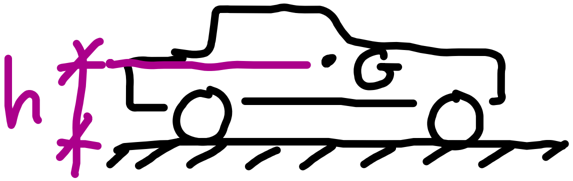
$$\Rightarrow M - \frac{L}{2}mg = \bar I\alpha + \frac{L}{2}(P_y - mg) \Rightarrow \boxed{M - \frac{L}{2}P_y = \bar I\alpha}$$

Same as equation 1

Notes on 16.3



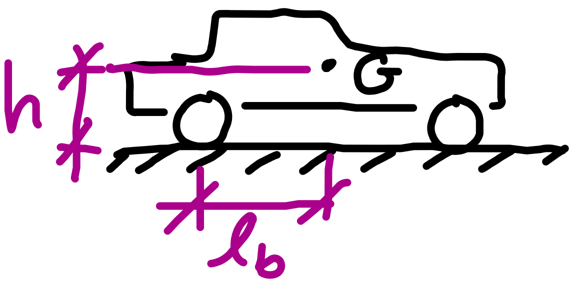
Notes on 16.3



$h = 20 in$

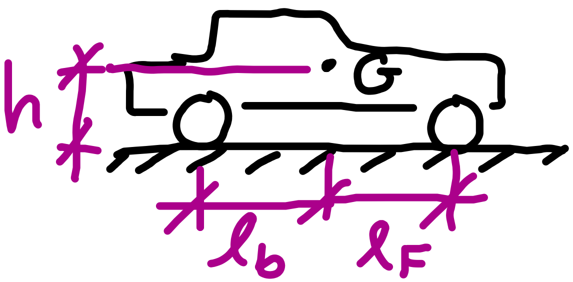
Notes on 16.3

$h = 20in, \ l_b = 60in$

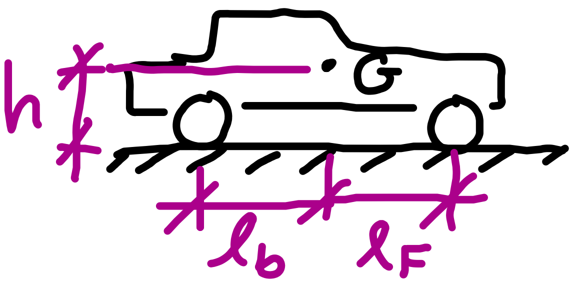


Notes on 16.3

$h = 20\text{in},\ l_b = 60\text{in},\ l_F = 40\text{in}$



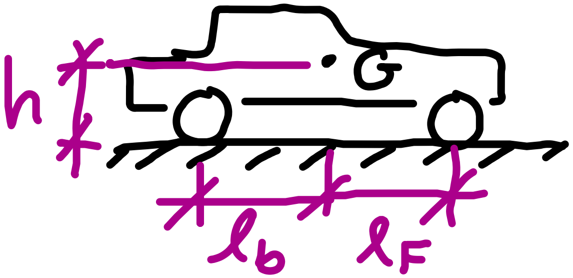
Notes on 16.3



$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

$\mu_K = 0.80$

Notes on 16.3



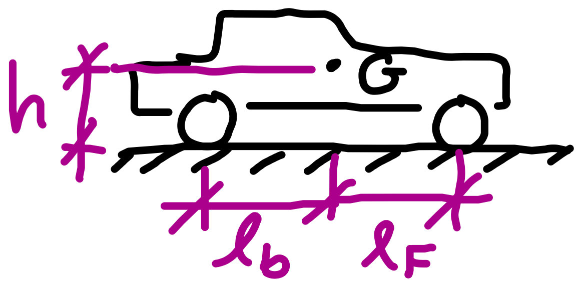
$h = 20\text{in}$, $l_b = 60\text{in}$, $l_F = 40\text{in}$

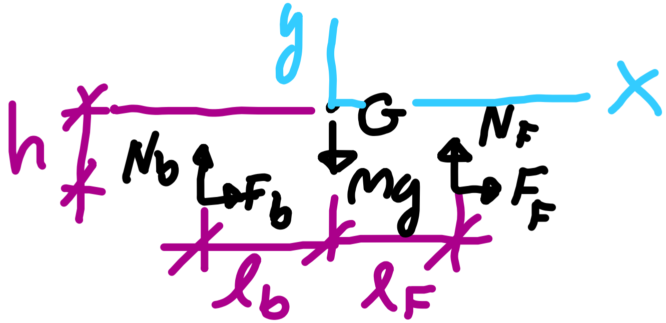
$\mu_K = 0.80$ Find a_{MAX}:

Notes on 16.3

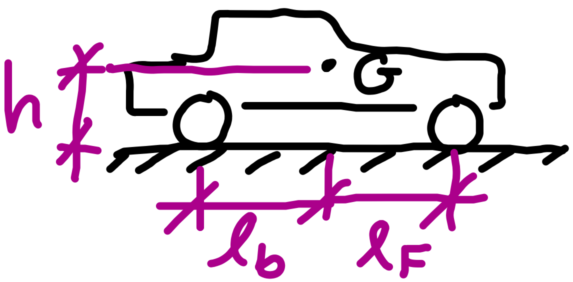
$h = 20in, \ell_b = 60in, \ell_F = 40in$

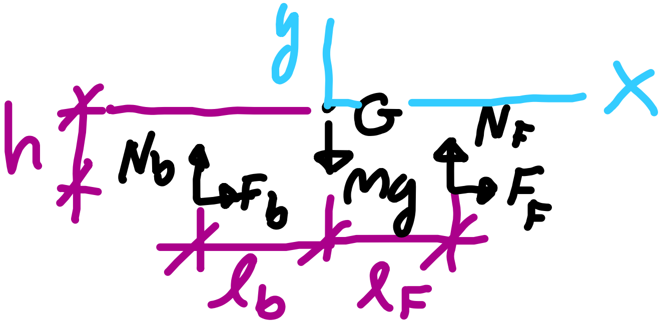
$\mu_K = 0.80$ Find a_{MAX}:





Notes on 16.3



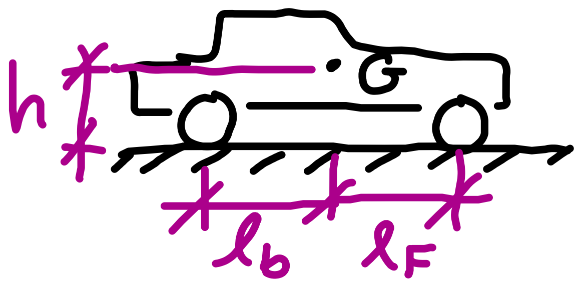


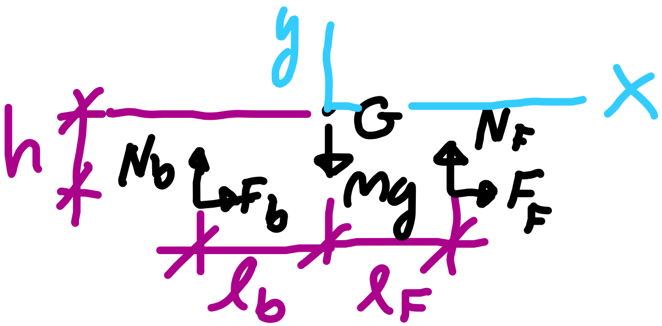
$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

$\mu_K = 0.80$ Find a_{MAX}:

4-wheel drive:

Notes on 16.3





$h = 20\,in,\ l_b = 60\,in,\ l_F = 40\,in$

$\mu_K = 0.80$ Find a_{MAX}:

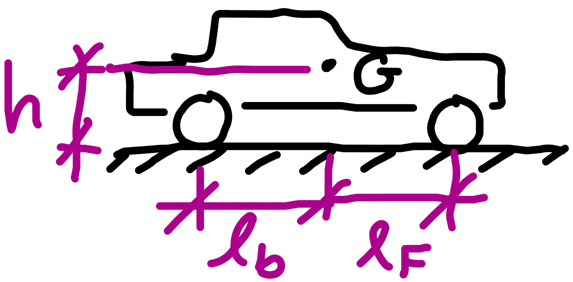
4-wheel drive:

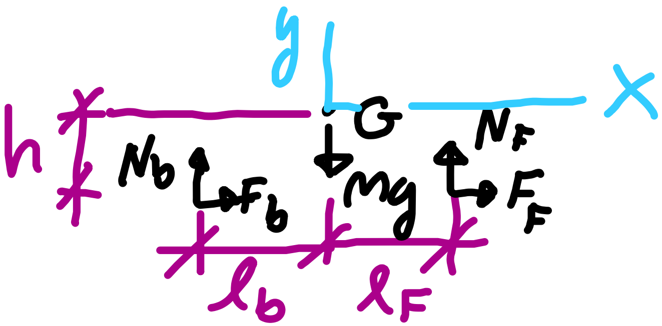
$$\sum F_y = 0 \Rightarrow N_B + N_F = mg$$

Notes on 16.3

$h = 20\text{in},\ \ell_b = 60\text{in},\ \ell_F = 40\text{in}$

$\mu_K = 0.80$ Find a_{max}:



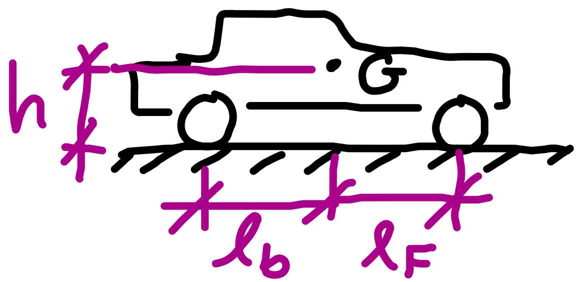


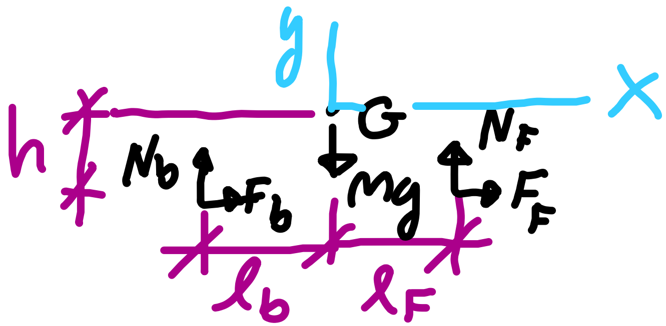
4-wheel drive:

$$\Sigma F_y = 0 \Rightarrow N_B + N_F = mg$$

$$\overset{+}{\rightarrow} \Sigma F_x = N_B \mu_s + N_F \mu_s$$

Notes on 16.3





$h = 20\,in,\ l_b = 60\,in,\ l_F = 40\,in$

$\mu_K = 0.80$ Find a_{max}:

4-wheel drive:

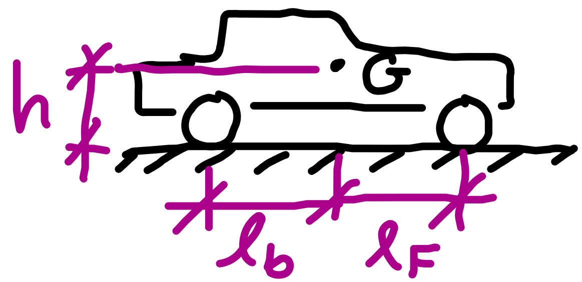
$$\Sigma F_y = 0 \Rightarrow N_B + N_F = mg$$

$$\Sigma F_x = N_B \mu_s + N_F \mu_s = m a_x$$

Notes on 16.3

$h = 20\,in,\ \ell_b = 60\,in,\ \ell_F = 40\,in$

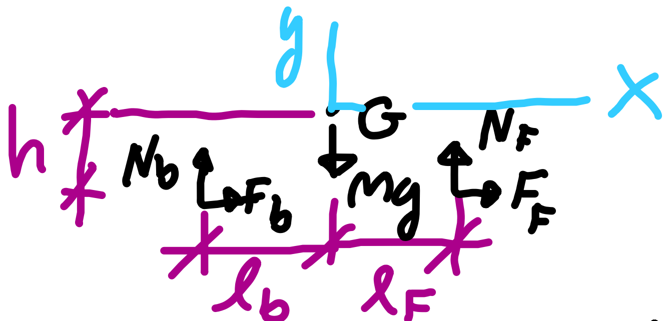
$\mu_K = 0.80$ Find a_{max}:



4-wheel drive:

$\Sigma F_y = 0 \Rightarrow N_B + N_F = mg$

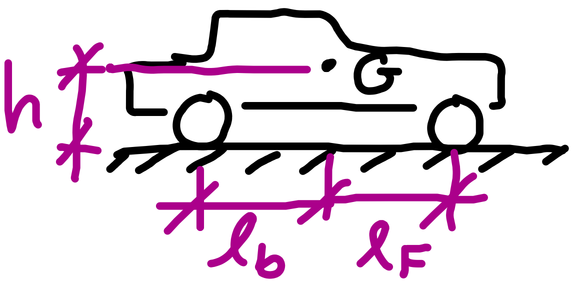
& $\Sigma F_x = N_B \mu_s + N_F \mu_s = m a_x$

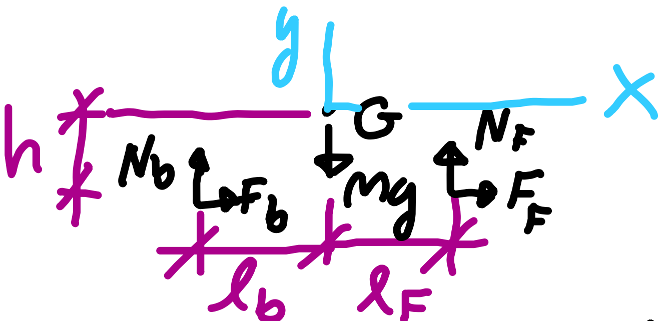


so just solve for a_x &

no need for moment analysis

Notes on 16.3





$h = 20\text{ in}, \ \ell_b = 60\text{ in}, \ \ell_F = 40\text{ in}$

$\mu_K = 0.80$ Find a_{max}:

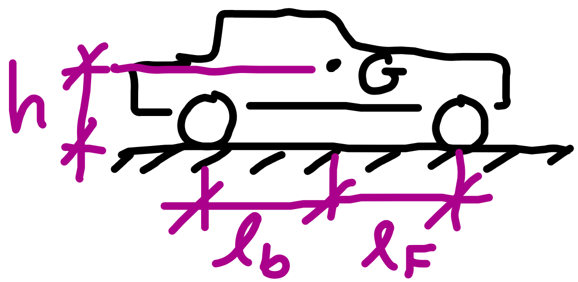
4-wheel drive:

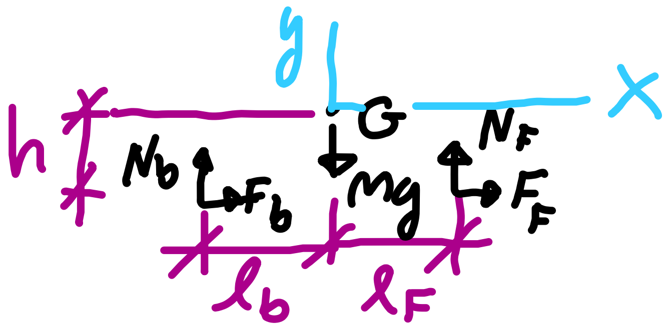
$$\Sigma F_y = 0 \Rightarrow N_B + N_F = mg$$

& $$\Sigma F_x = N_B \mu_s + N_F \mu_s = m a_x$$

so just solve for a_x & no need for moment analysis

Notes on 16.3



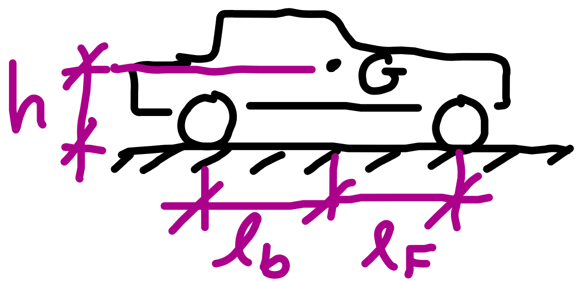


$h = 20\,in,\ l_b = 60\,in,\ l_F = 40\,in$

$\mu_K = 0.80$ <u>Find a_{MAX}</u>:

<u>Front wheel drive</u>:

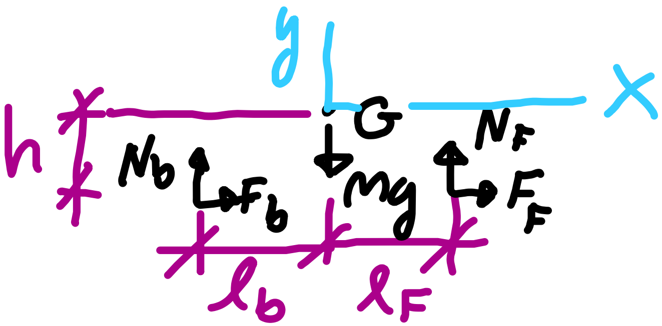
Notes on 16.3



$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

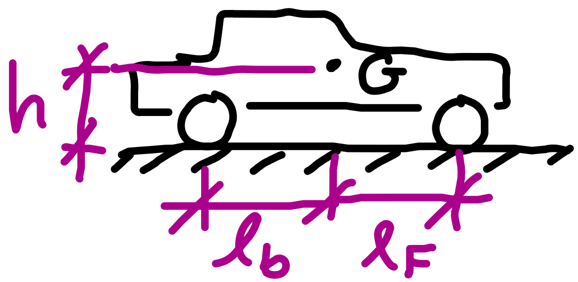
$\mu_K = 0.80$ Find a_{MAX}:

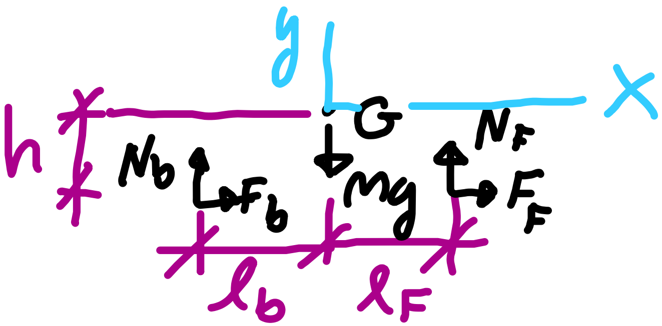
Front wheel drive:



NOTE: Rear wheel drive analysis is similar

Notes on 16.3





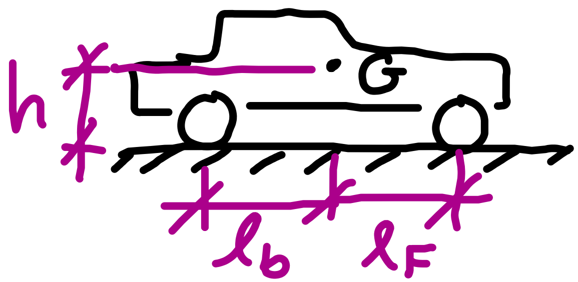
$h = 20\,in,\ l_b = 60\,in,\ l_F = 40\,in$

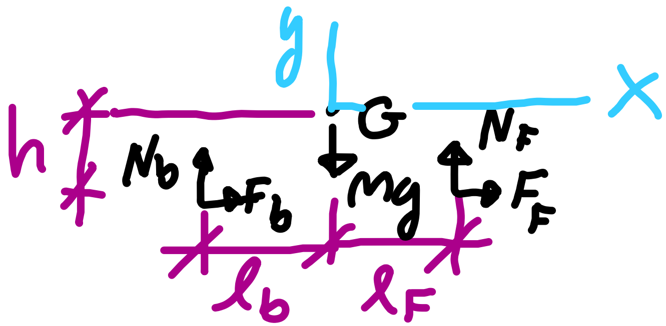
$\mu_K = 0.80$ Find a_{MAX} :

Front wheel drive :

$F_b = 0$ \& $F_F \neq 0$

Notes on 16.3





$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

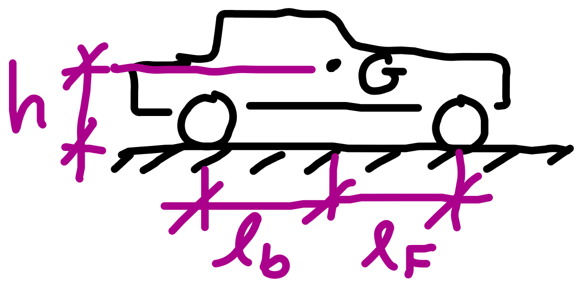
$\mu_K = 0.80$ Find a_{MAX} :

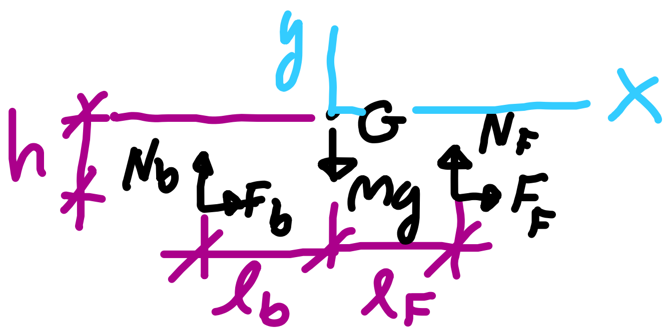
Front wheel drive :

$F_b = 0$ & $F_F \neq 0$

Here $\Sigma F_x = m a_x$

Notes on 16.3





$h = 20\,in,\ l_b = 60\,in,\ l_F = 40\,in$

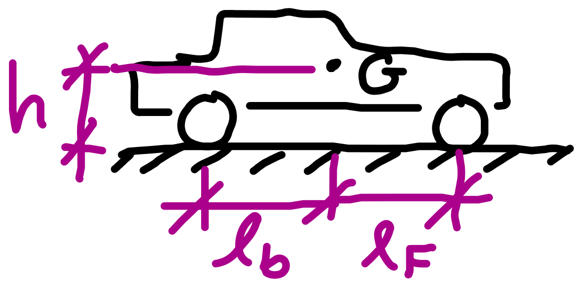
$\mu_K = 0.80$ Find a_{MAX}:

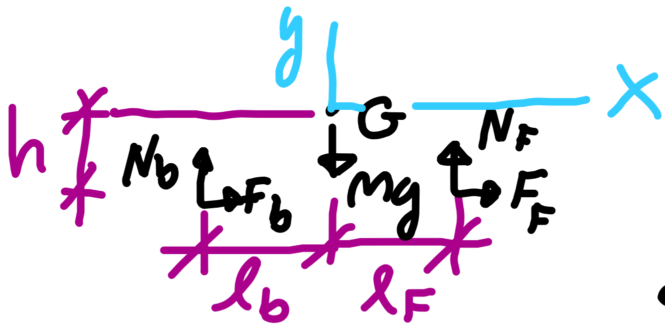
Front wheel drive:

$F_b = \theta$ & $F_F \neq \theta$

Here $\Sigma F_x = m a_x \Rightarrow N_F \mu_s = m a_x \quad (1)$

Notes on 16.3





$h = 20in, \ \ell_b = 60in, \ \ell_F = 40in$

$\mu_K = 0.80$ Find a_{max}:

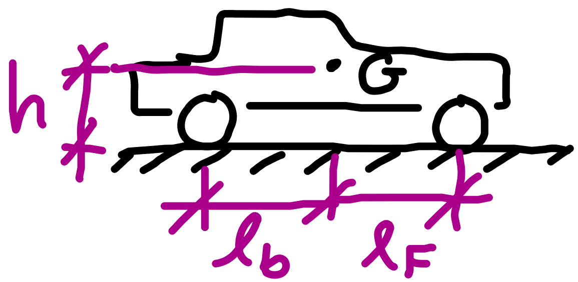
Front wheel drive:

$F_b = 0$ & $F_F \neq 0$

Here $\sum F_x = ma_x \Rightarrow N_F \mu_s = ma_x$ (1)

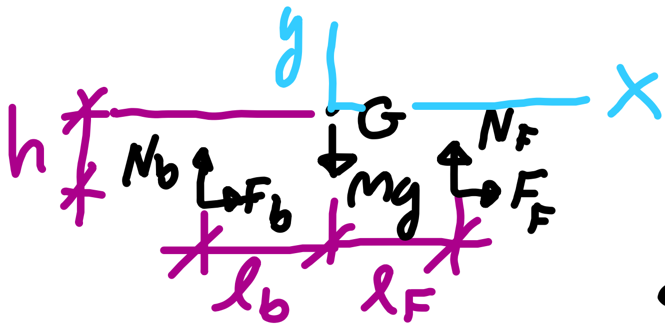
& $\sum F_y = 0$

Notes on 16.3



$h = 20\,in$, $\ell_b = 60\,in$, $\ell_F = 40\,in$

$\mu_K = 0.80$ Find a_{MAX} :



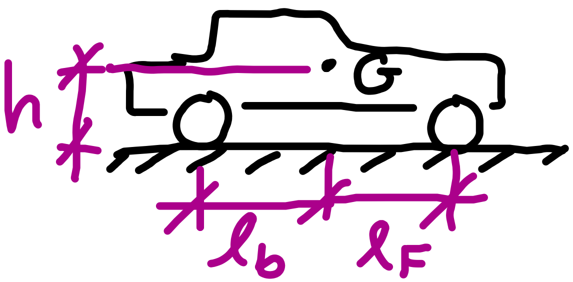
Front wheel drive:

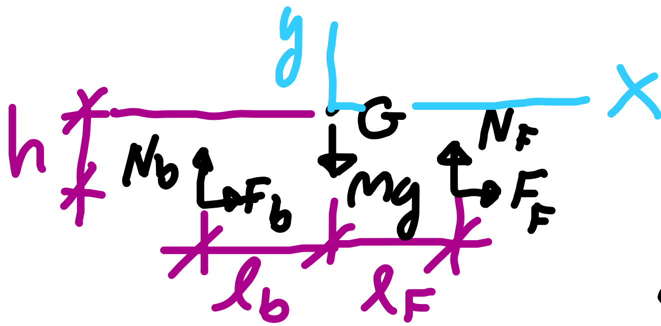
$$F_b = 0 \quad \& \quad F_F \neq 0$$

Here $\sum F_x = m a_x \Rightarrow N_F \mu_s = m a_x \quad (1)$

& $\sum F_y = 0 \Rightarrow N_B + N_F = mg \quad (2)$

Notes on 16.3





$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

$\mu_K = 0.80$ Find a_{max}:

Frontwheel drive:

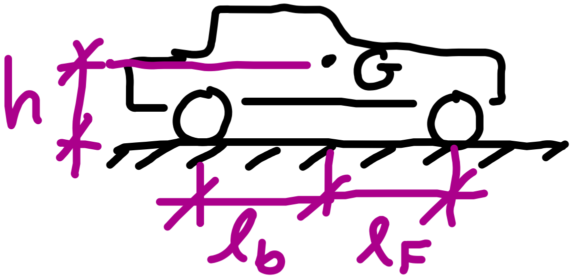
$F_b = 0$ & $F_F \neq 0$

Here $\Sigma F_x = m a_x \Rightarrow N_F \mu_s = m a_x$ (1)

& $\Sigma F_y = 0 \Rightarrow \boxed{N_B + N_F = mg}$ (2)

As before

Notes on 16.3



$h = 20\,in$, $l_b = 60\,in$, $l_F = 40\,in$

$\mu_K = 0.80$ Find a_{MAX}:

Front wheel drive:

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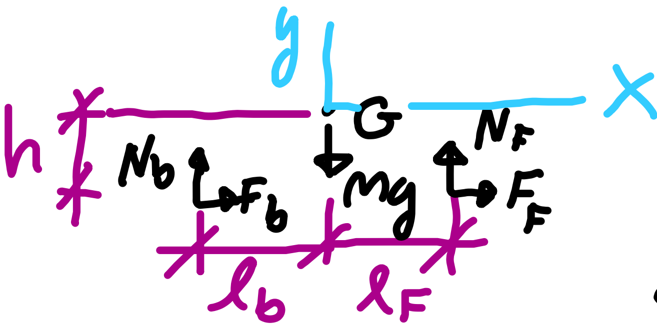

Here $\Sigma F_x = m a_x \Rightarrow N_F \mu_s = m a_x$ (1)

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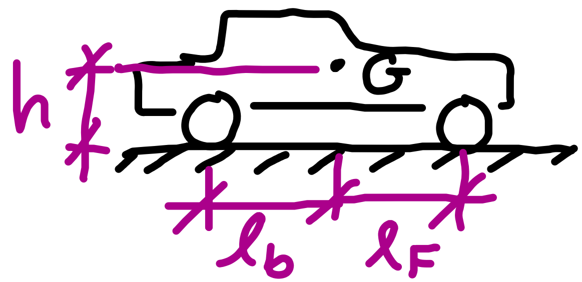
Now need moment analysis.

Notes on 16.3



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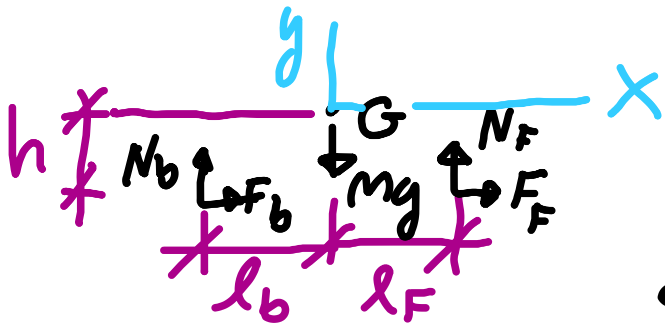

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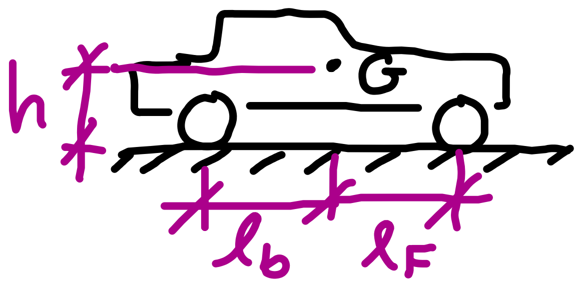
Now need moment analysis. I am choosing point B (where rear wheels touch ground) as a reference point for the moment analysis

Notes on 16.3



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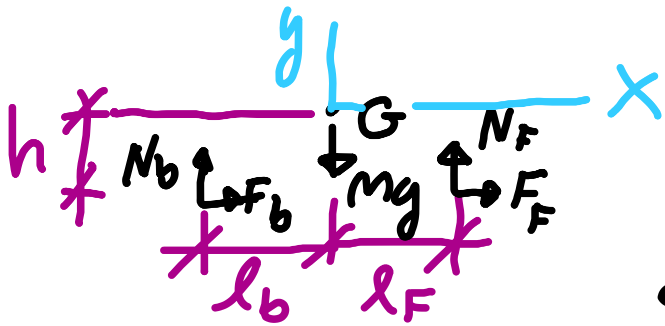

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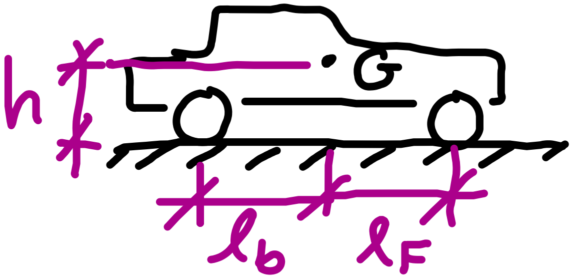

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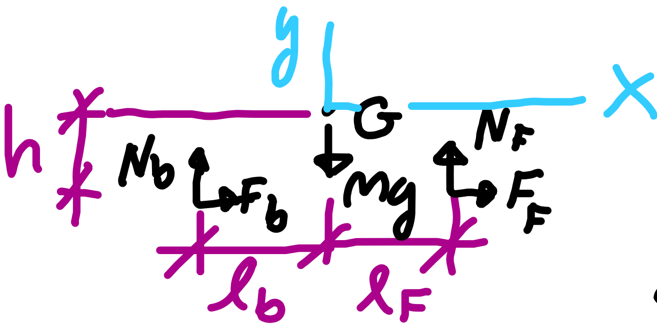

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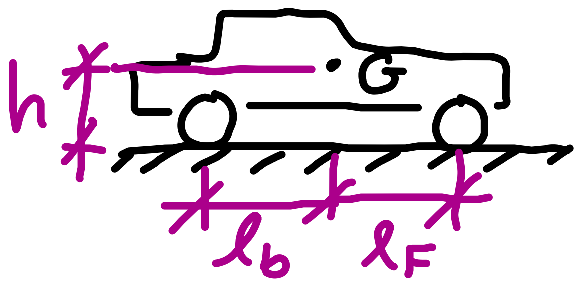
Notes on 16.3



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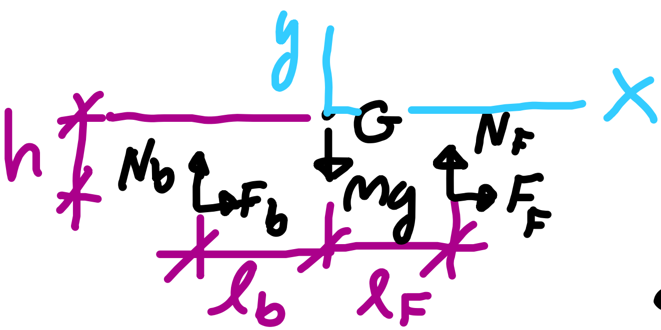

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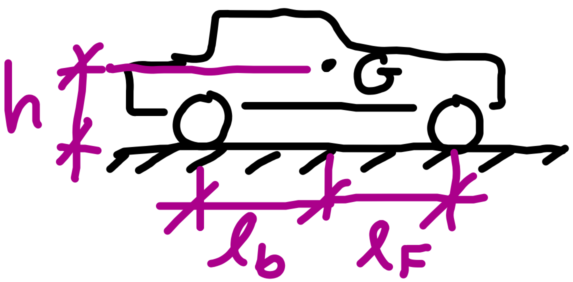

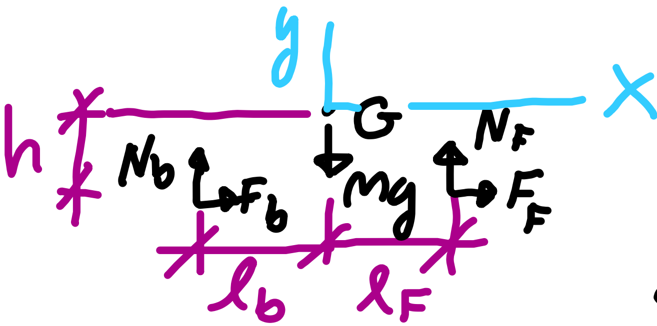

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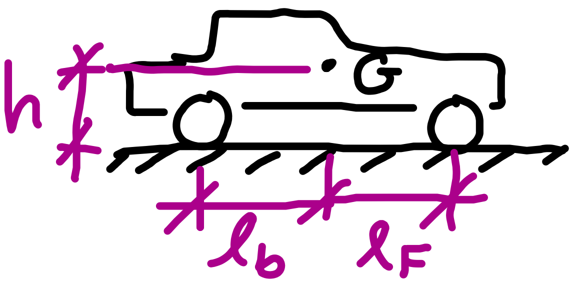

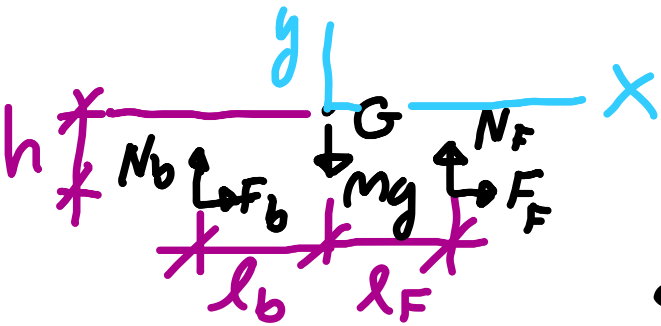

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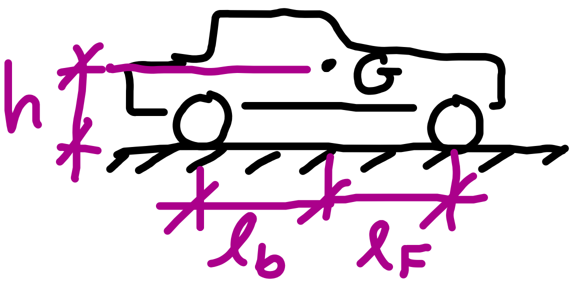
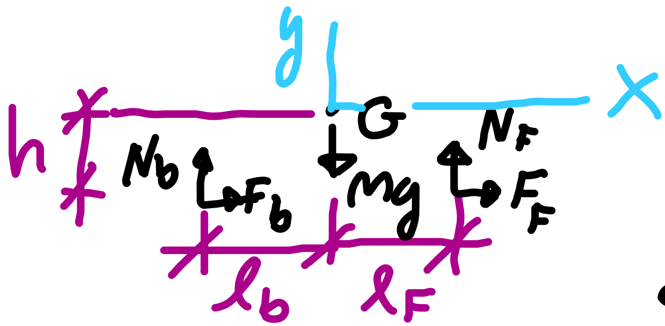
Notes on 16.3





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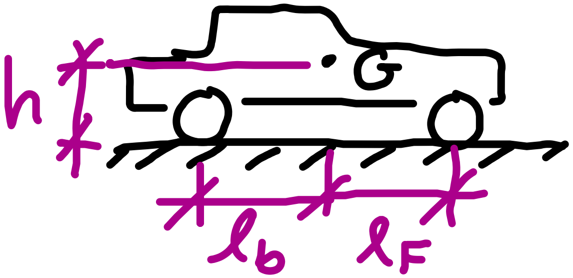

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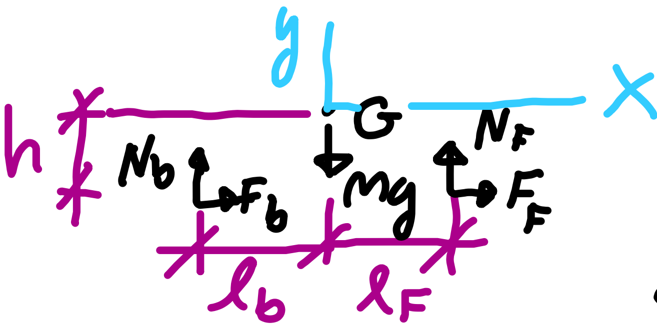

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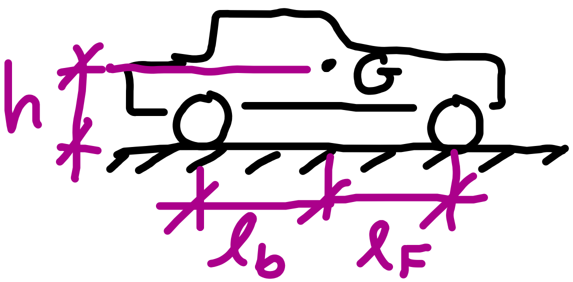
But eqn 1 says $N_F = \frac{m}{\mu_s} a_x$

Notes on 16.3



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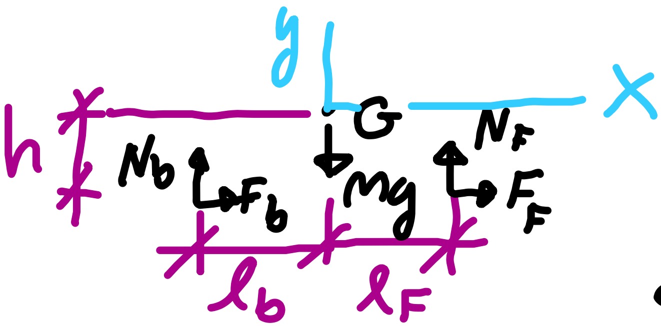

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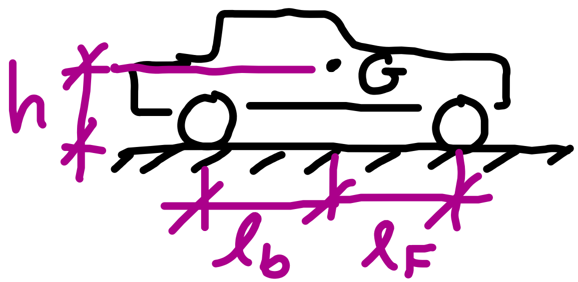

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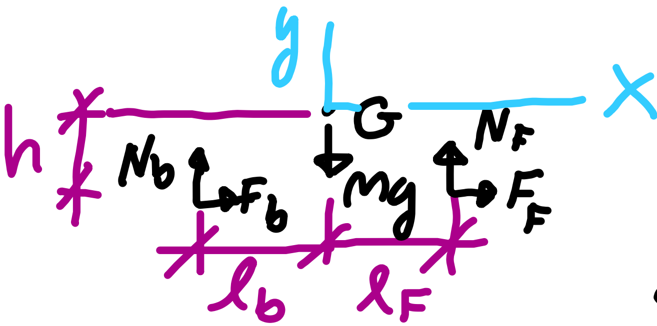

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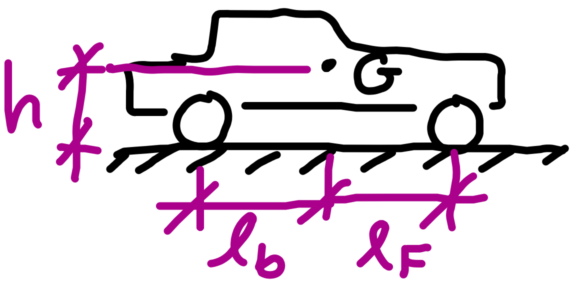

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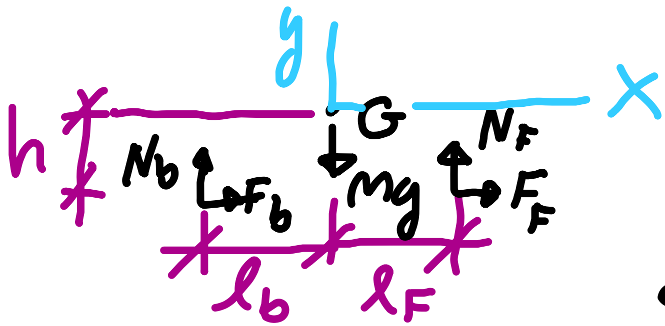

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So $l_b mg - (l_b + l_F)\left(\frac{m a_x}{\mu_s}\right) = m h \bar{a}_x \Rightarrow a_x\left(h + \frac{l_b + l_F}{\mu_s}\right) = l_b g$

$$\Rightarrow \quad a_x = \frac{l_b g}{h + (l_b + l_F)/\mu_s}$$

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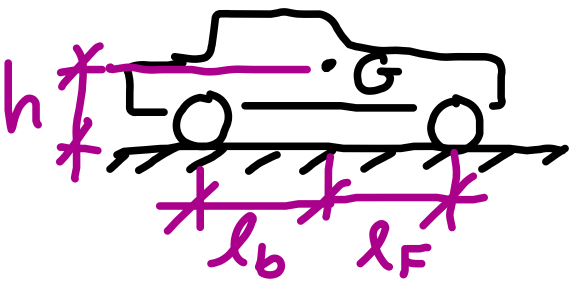

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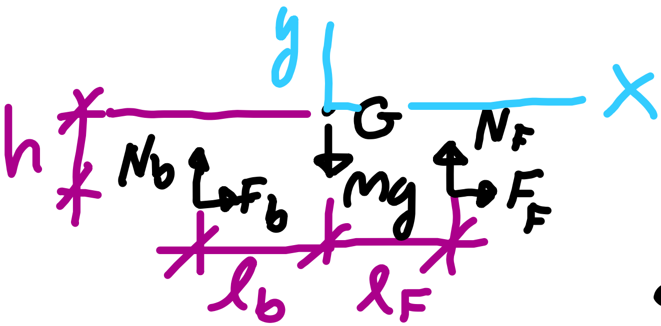

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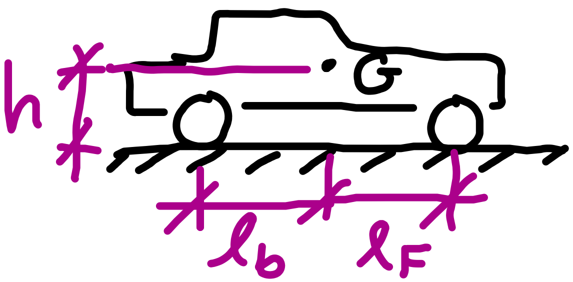

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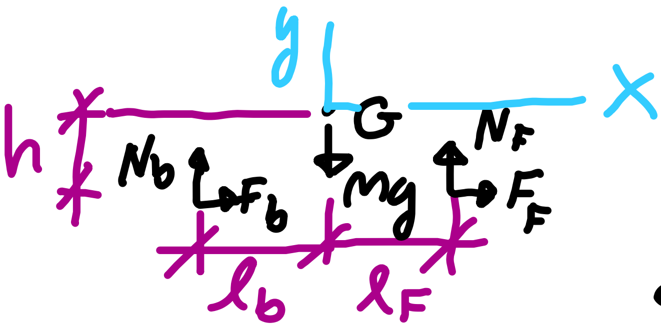

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$$\Rightarrow a_x = \frac{l_b g}{h + (l_b + l_F)/\mu_s} = \left[\frac{60 * 32.2\ ft/s^2}{20 + 100/0.8}\right] = 13.32\ \frac{ft}{s^2}$$