Today 16.2

$\boxed{L25}$

Today 16.2

$\boxed{L25}$

constrained motion

Today 16.2

Monday 16.2 & 17.1

$\boxed{L25}$

Today 16.2

Monday 16.2 & 17.1

$\boxed{L25}$

Energy methods for rigid bodies

$\bar{k} \equiv$ Radius of gyration

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$\bar{I} = m\bar{k}^2$

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$\bar{I} = m\bar{k}^2 \Rightarrow \bar{k}^2 = \bar{I}/m$

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Parallel axis theorem

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So far we have only used moments of inertia about the center-of-mass position G ($\bar{I}$). What about the moment of inertia about other points?

Before we determine the moment of inertia about points other than G, it will be useful to take another look at the center-of-mass for continuos mass distributions

From previous we have $m\vec{\bar{r}} = \sum m_i \vec{r}_i$

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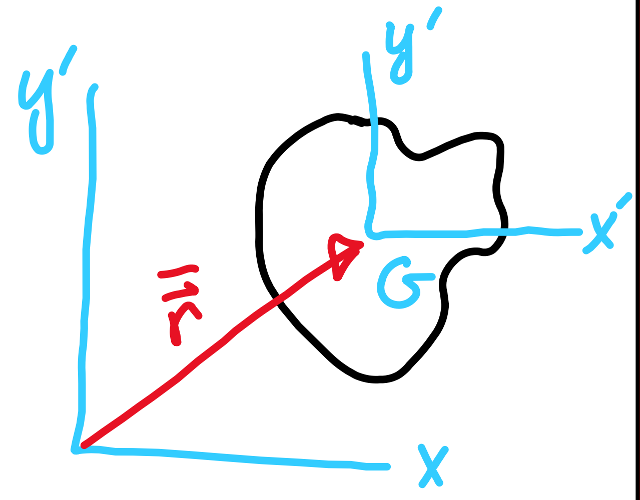
If mass is uniform over area A



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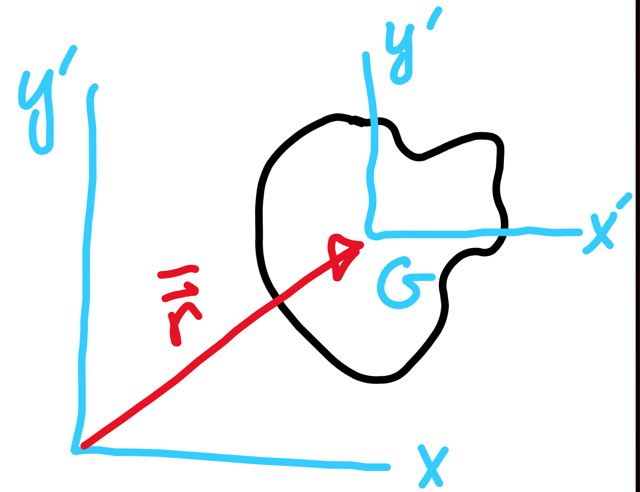

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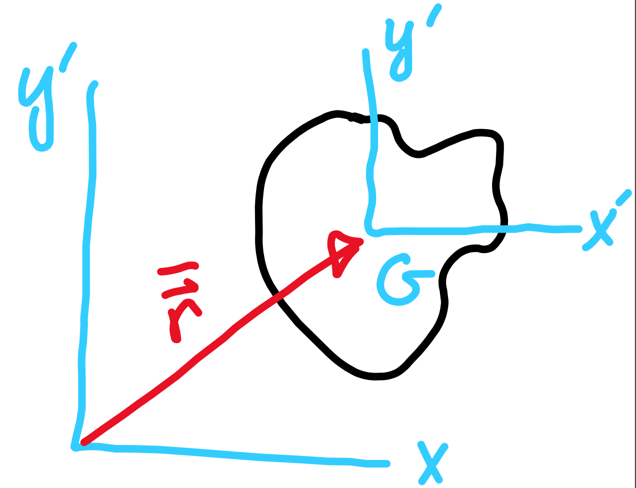

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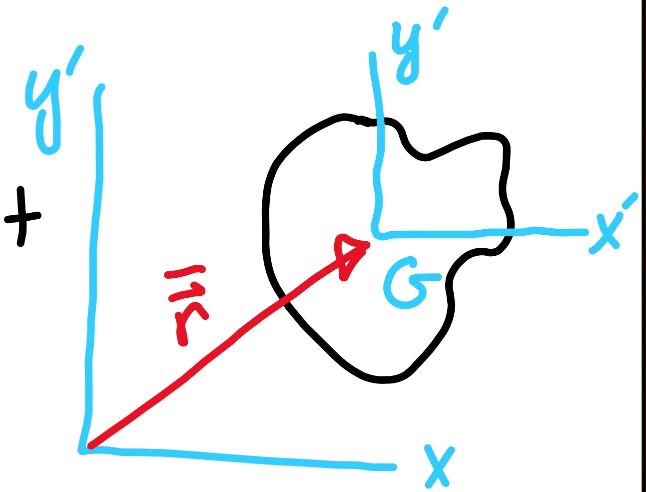

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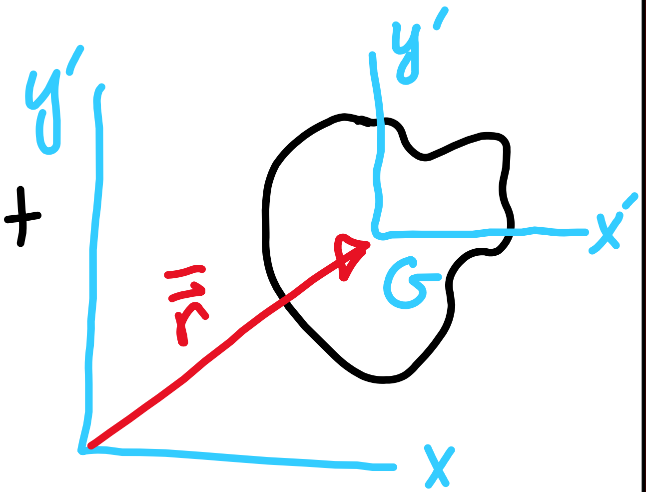

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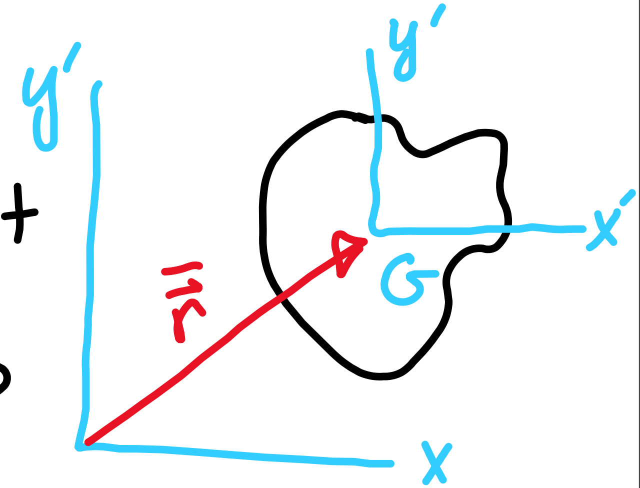

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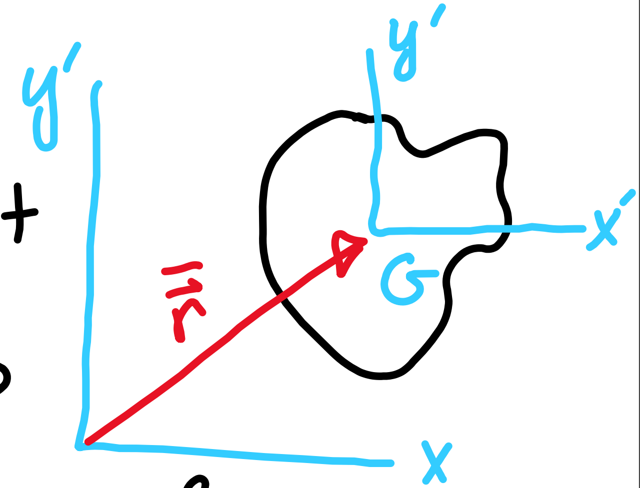

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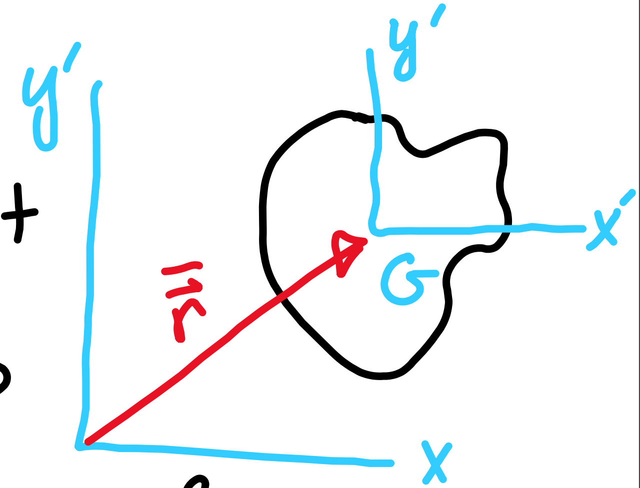

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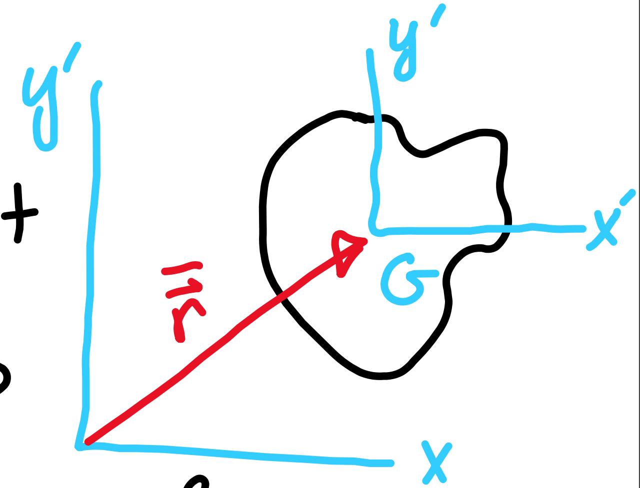

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Parallel axis theorem

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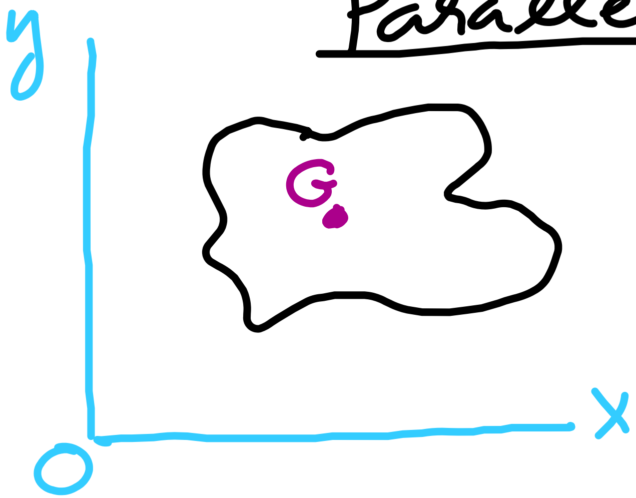

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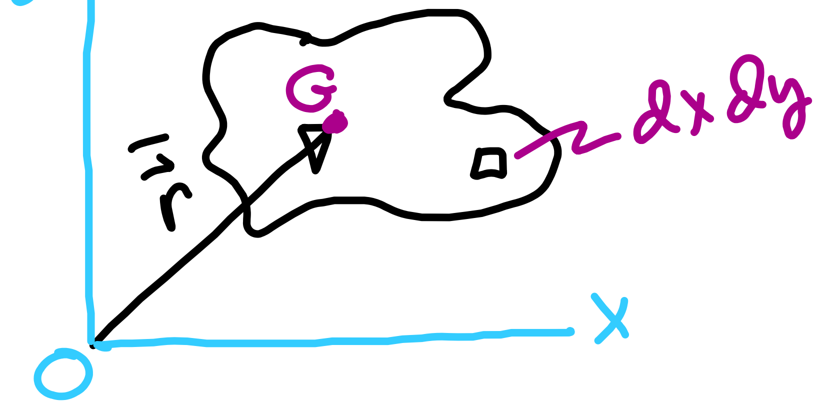

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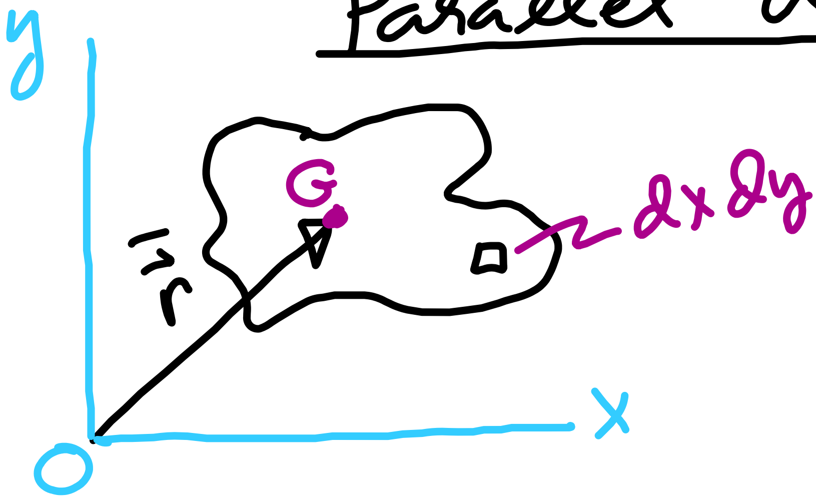

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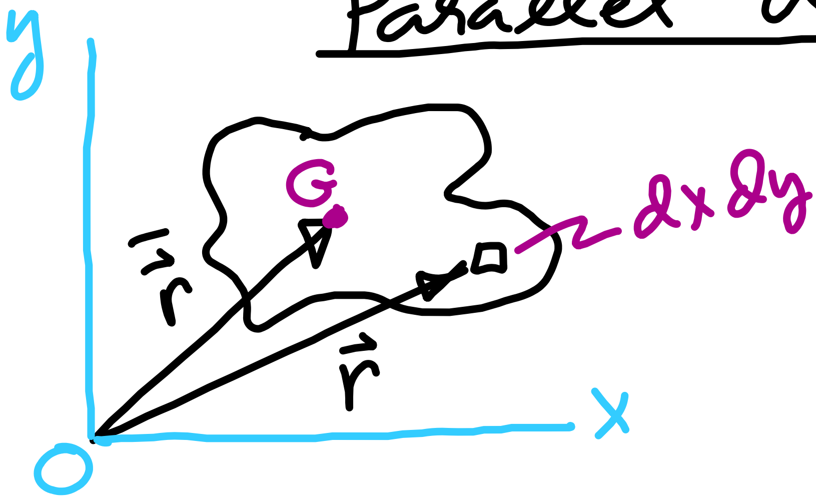

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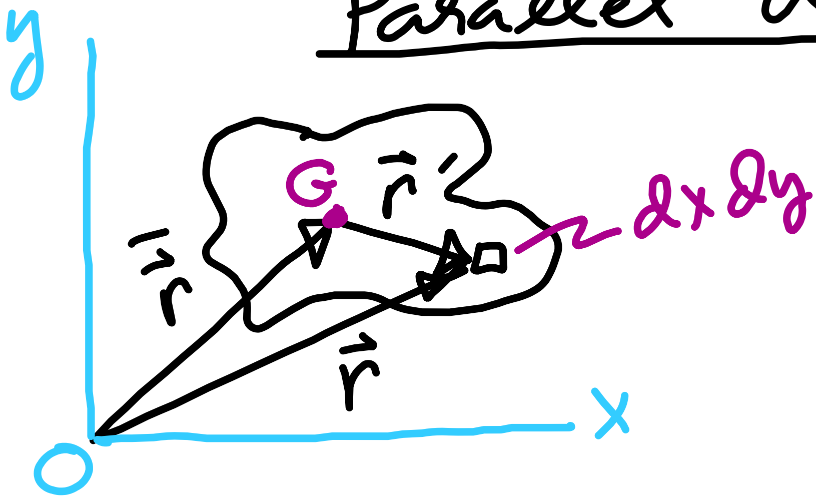

here $\vec{r} = \vec{\bar{r}} + \vec{r}\,'$

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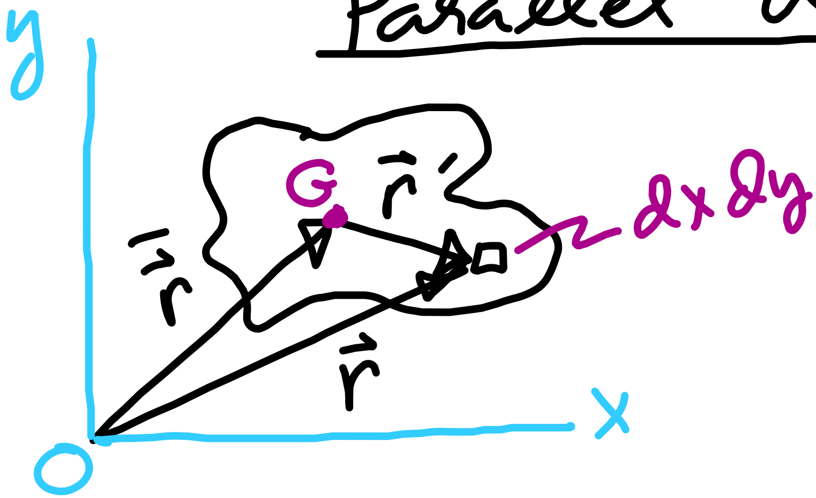

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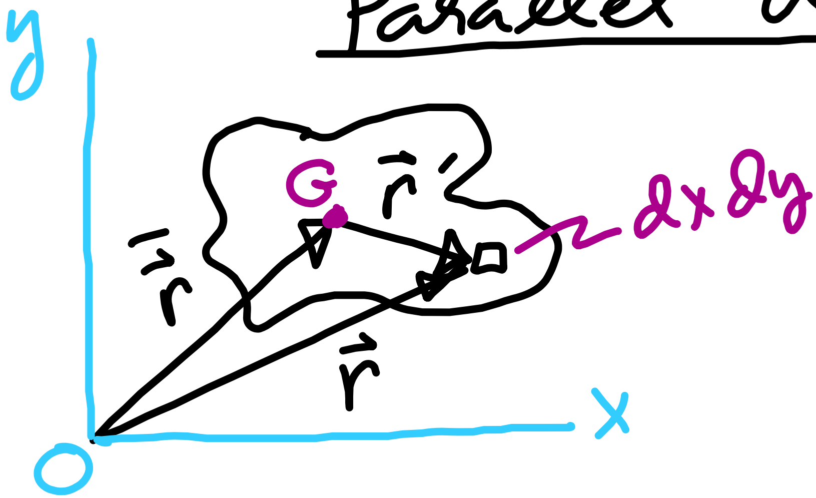

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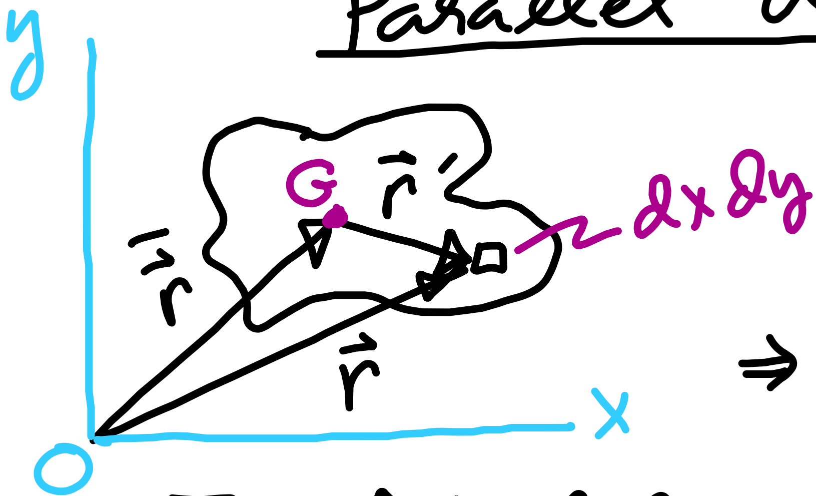

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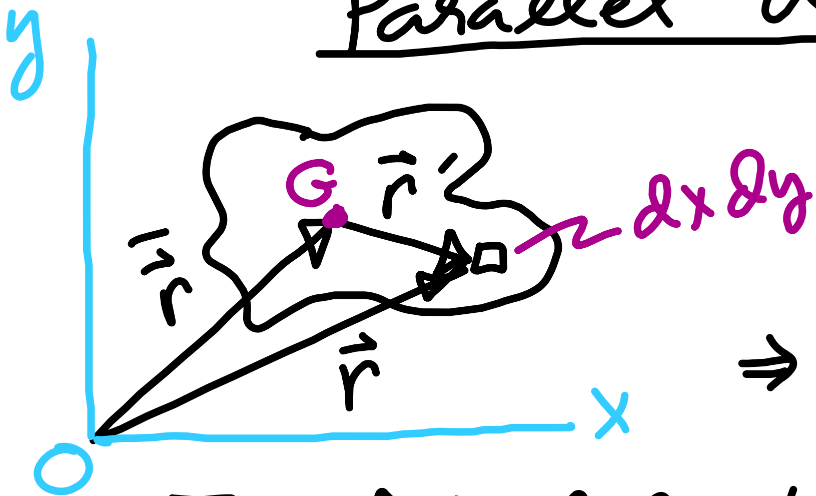

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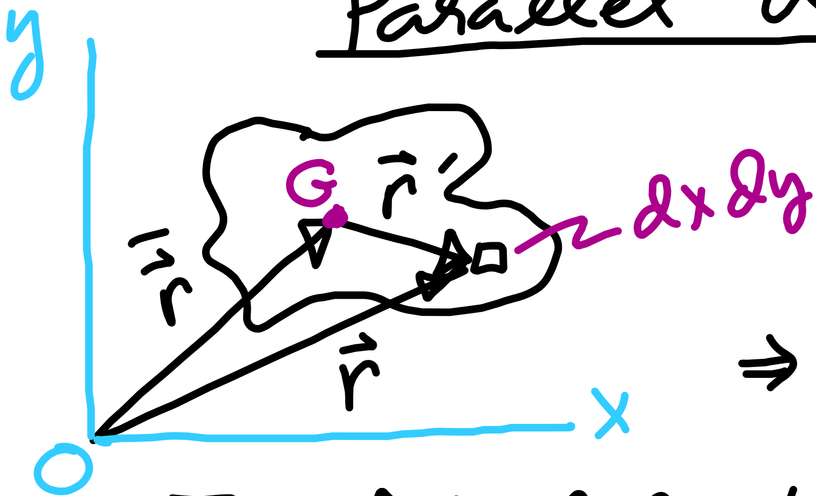

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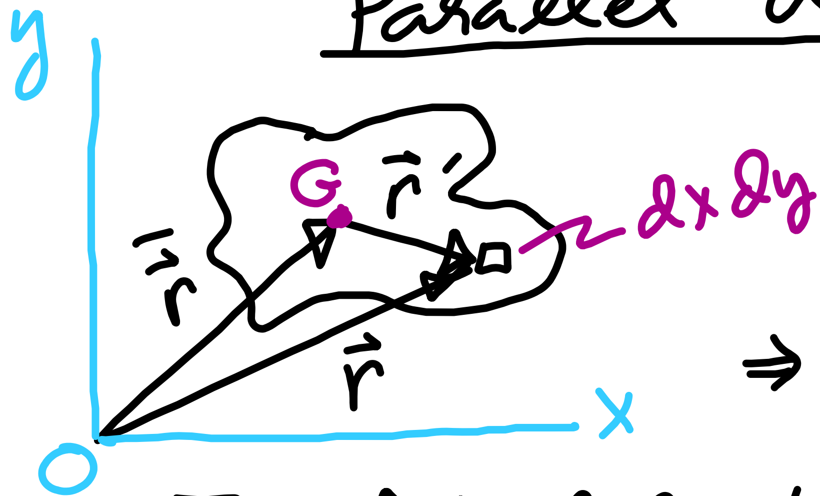

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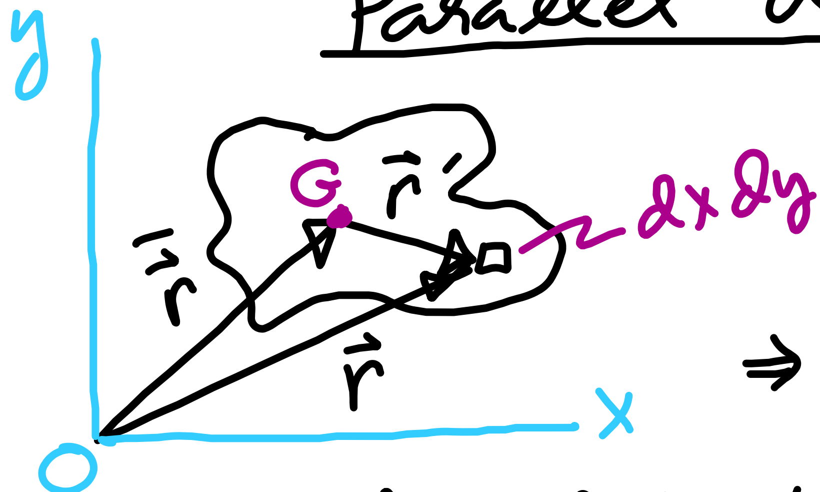

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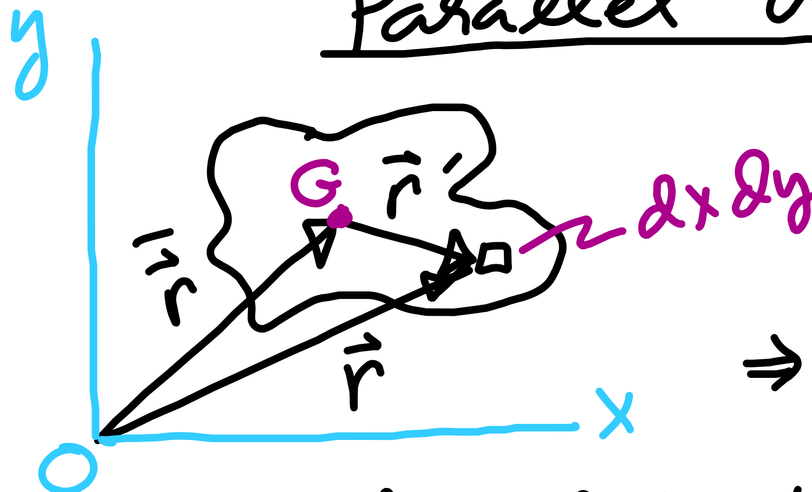

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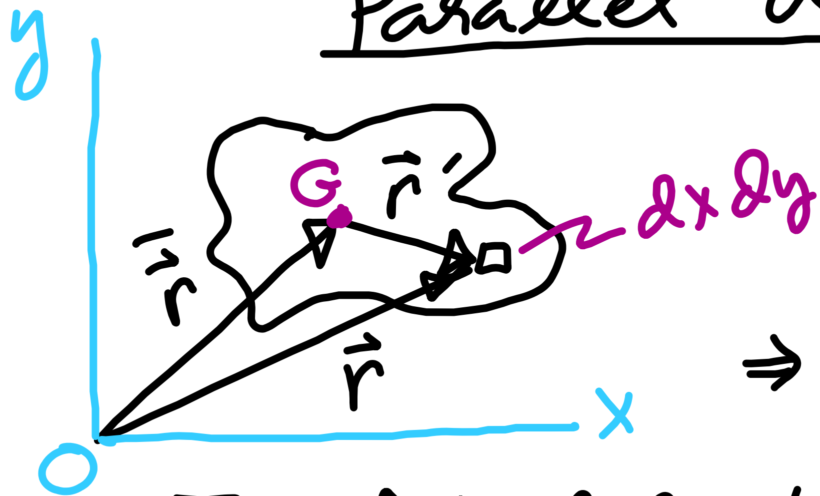
Parallel axis theorem



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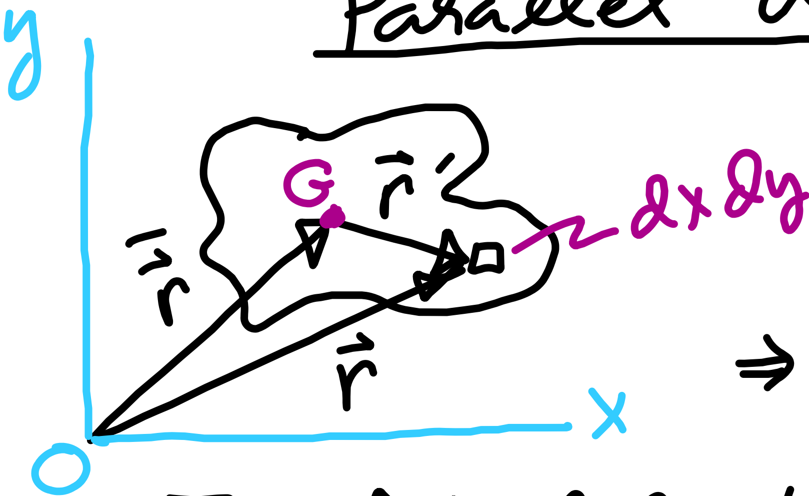
Parallel axis theorem



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$$\boxed{I_o = \bar I + M\bar r^2}$$

ASU

Last time we saw that

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$

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Let an object rotate about some fixed point

p

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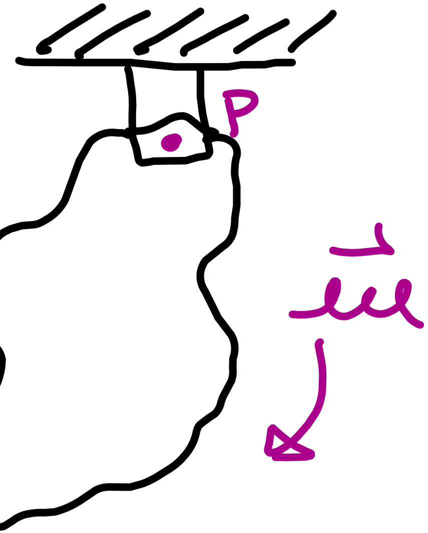

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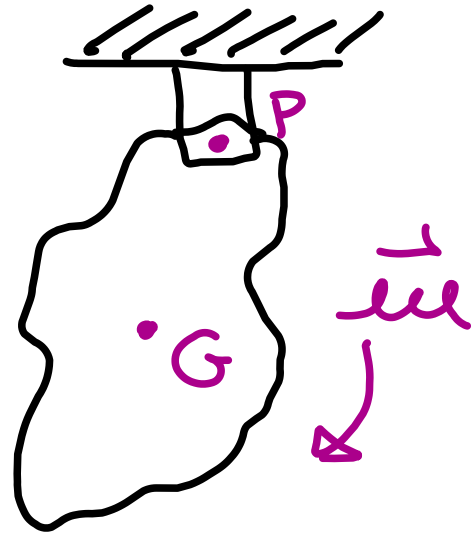

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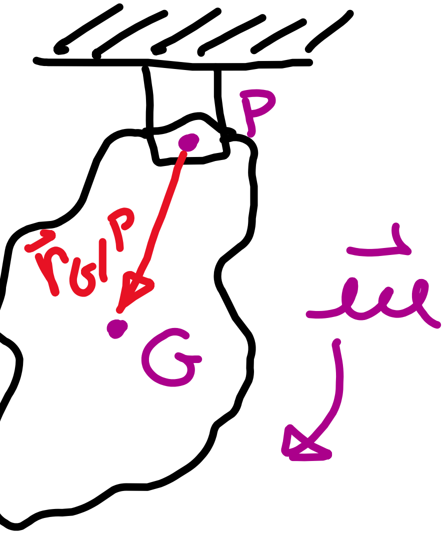

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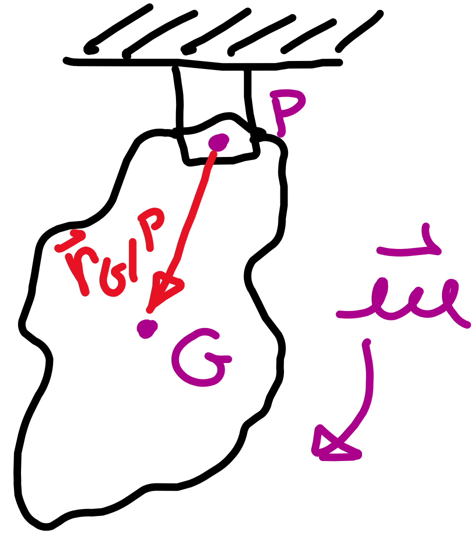

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$\Sigma \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

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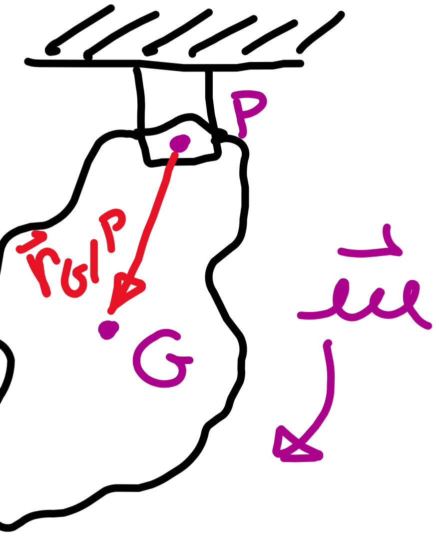

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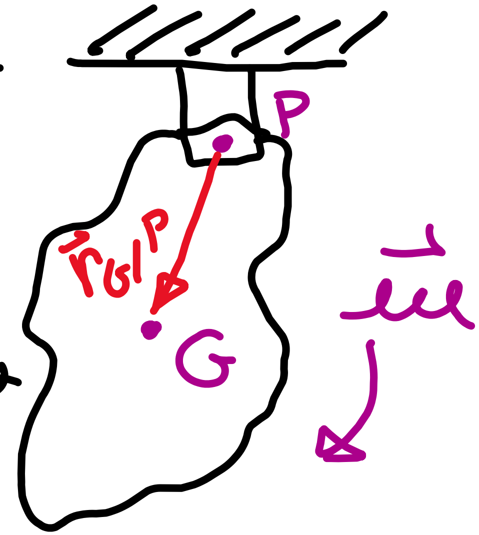

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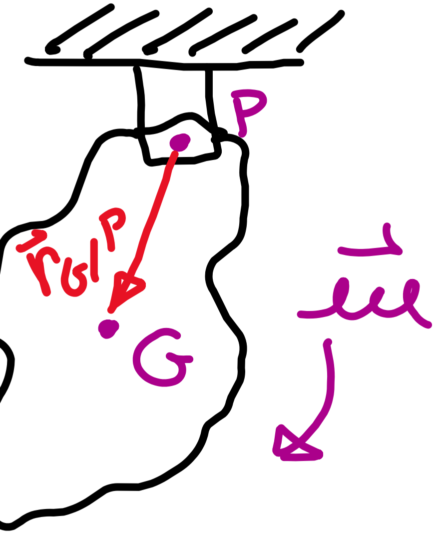

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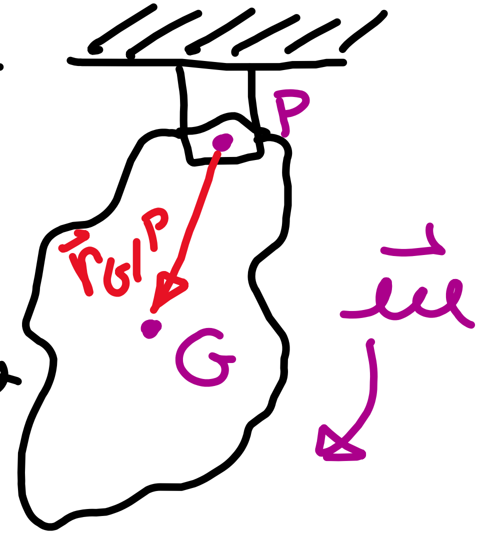

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& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$

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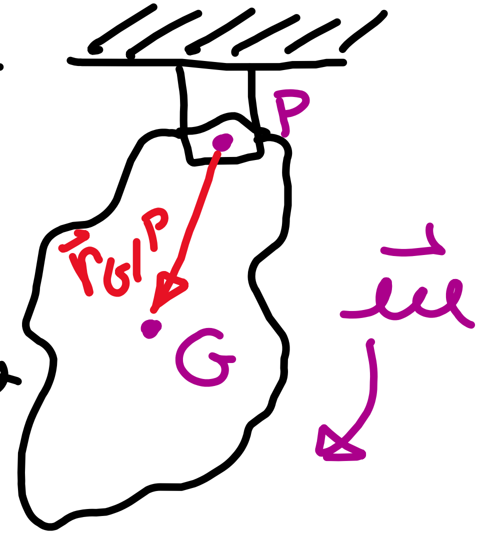
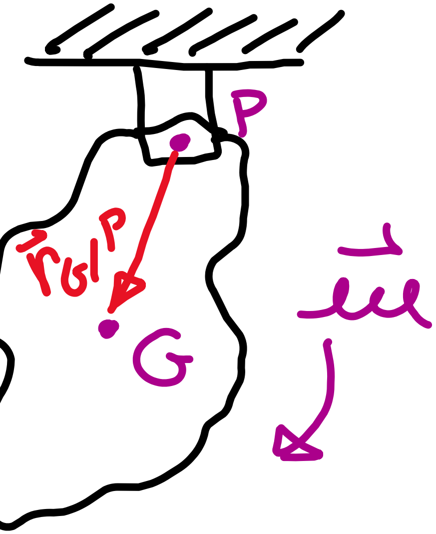
$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$



Let an object rotate about some fixed point P.



$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

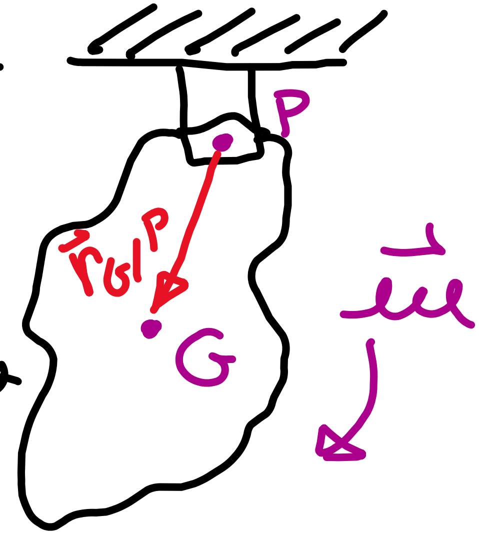
$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$

Let an object rotate about some fixed point



P. $\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

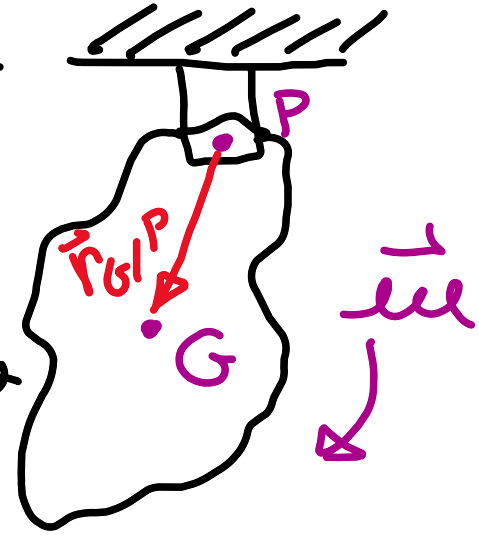
$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2 \vec{\alpha}$

Let an object rotate about some fixed point



P. $\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

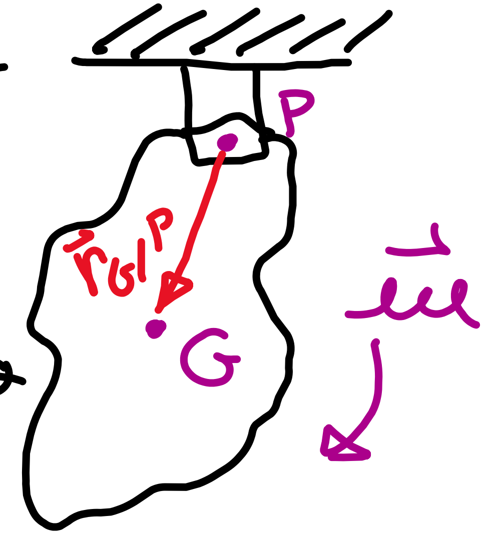
then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2\vec{\alpha}$ so

$$\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha}$$

Let an object rotate about some fixed point



P. $\boxed{\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}}$ But

$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

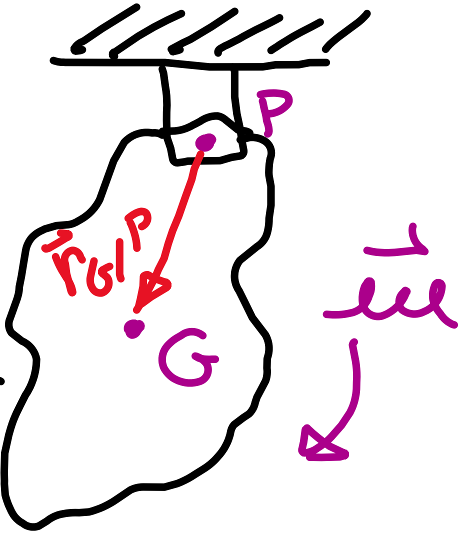
then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t\hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2\vec{\alpha}$ so

$\sum \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha}$ <u>or</u> $\boxed{\sum \vec{M}_P = I_P \vec{\alpha}}$

Let an object rotate about some fixed point P.



$$\boxed{\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}}$$

But $\bar{\vec{a}} = \bar{a}_n\hat{e}_n + \bar{a}_t\hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\bar{\vec{a}} = \vec{r}_{G/P} \times m\bar{a}_t\hat{e}_t$

& since $\bar{\vec{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\bar{\vec{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\bar{\vec{a}} = m\bar{r}^2\vec{\alpha}$ so

$$\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha} \quad \text{or} \quad \boxed{\Sigma \vec{M}_P = I_P\vec{\alpha}}$$

So $\underline{\bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} = I_P\vec{\alpha}}$

We have now seen that, for a fixed point of rotation

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$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

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$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

$$\& \quad \sum \vec{M}_P = I_P \vec{\alpha}, \text{ where } I_P = \bar{I} + m\bar{r}^2$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ &

$\bar{r} = r_{G/P}$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ &

$\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time!!

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ &

$\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time !!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} \\ \end{cases}$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ &

$\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time!!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \\ \text{or} \quad \sum \vec{M}_P = I_P \vec{\alpha} \end{cases}$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

$$\& \quad \sum \vec{M}_P = I_P\vec{\alpha}, \text{ where } I_P = \bar{I} + m\bar{r}^2 \ \&$$

$\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time!!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} \\ \text{or} \quad \sum \vec{M}_P = I_P\vec{\alpha} \end{cases}$$

$$\text{Bad}: \sum \vec{M}_P = I_P\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}}$$

& $\sum \vec{M}_p = I_p \vec{\alpha}$, where $I_p = \bar{I} + m\bar{r}^2$ &

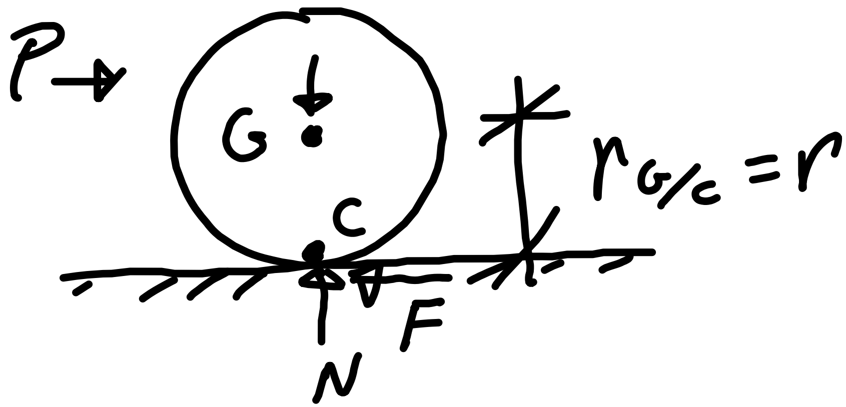
$\bar{r} = r_{G/p}$

So, do not use **both** types of expressions at the same time!!

$$\text{Good}\begin{cases} \sum \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}} \\ \quad \text{or} \quad \sum \vec{M}_p = I_p \vec{\alpha} \end{cases}$$

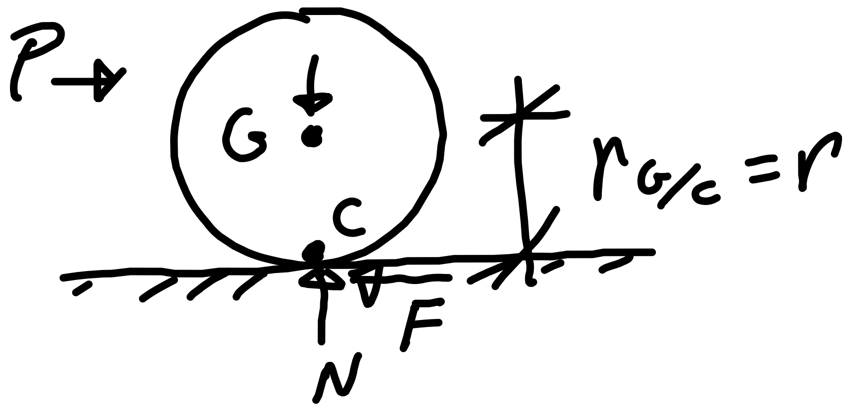
$$\text{Bad}: \sum \vec{M}_p = \underbrace{I_p \vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}}}_{\text{¡¡WRONG!!}}$$

16.92



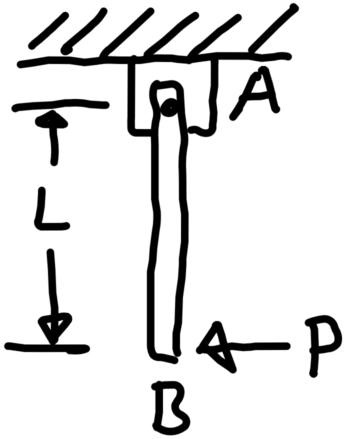
$$\circlearrowleft \sum M_C = \bar{I}\vec{\alpha} + \vec{r}_{G/C} \times m\vec{\bar{a}}$$

Notes on 16.92

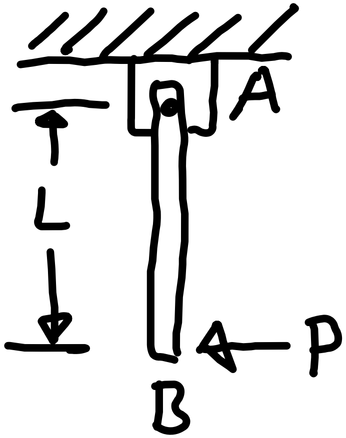


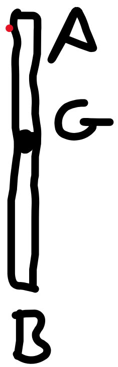
$$\circlearrowleft \sum M_C = \underbrace{\bar{I}\vec{\alpha} + \vec{r}_{G/C} \times m\vec{\bar{a}}}_{\text{Just need to convert this to } (\bar{I}+mr^2)\vec{\alpha} = I_C\vec{\alpha}}$$

Example

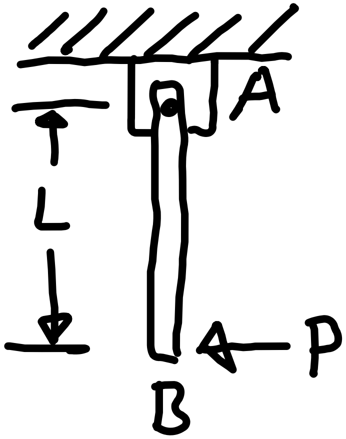


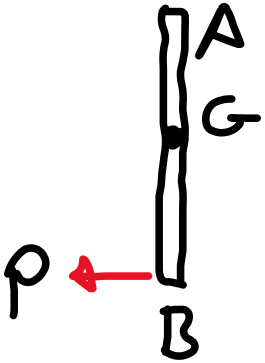
Example



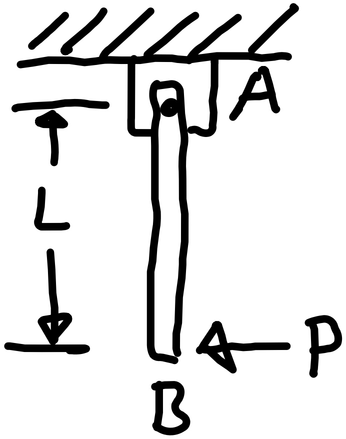


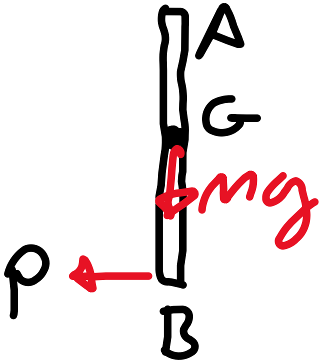
Example



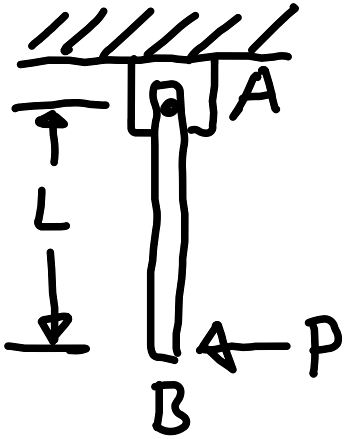


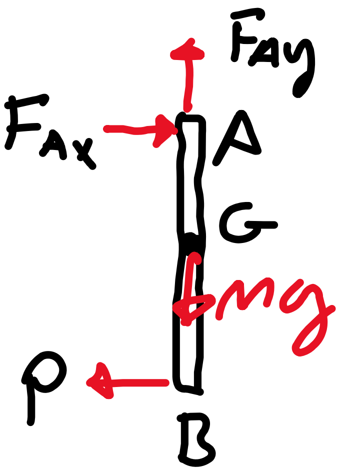
Example



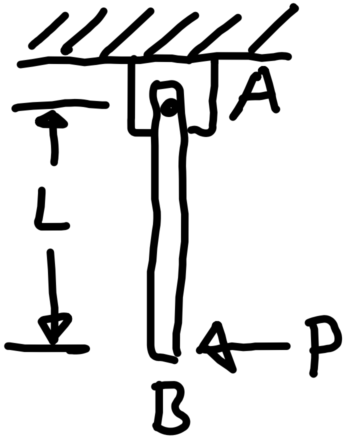


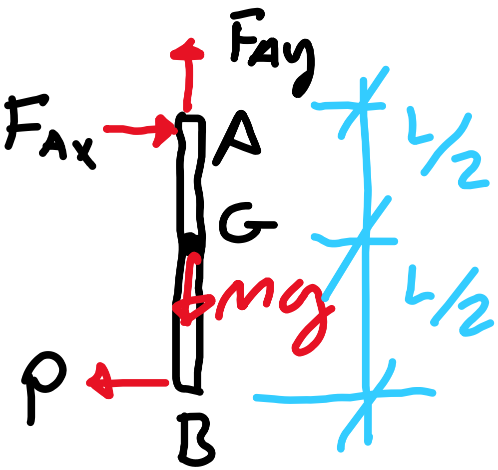
Example





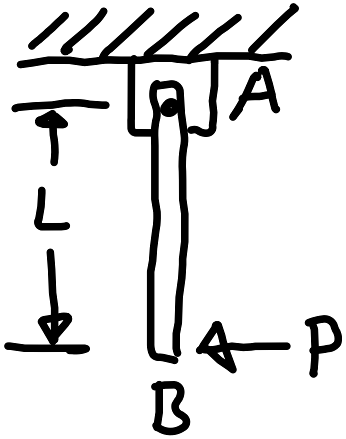
Example

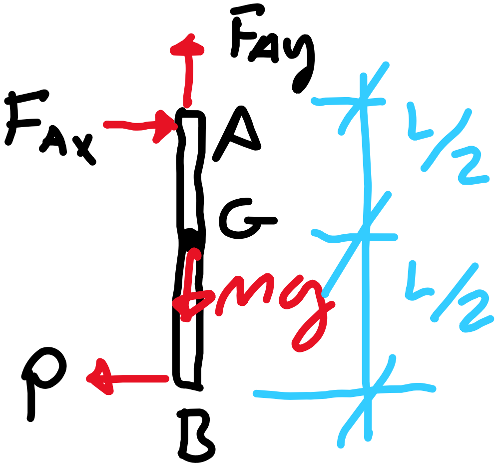




Example

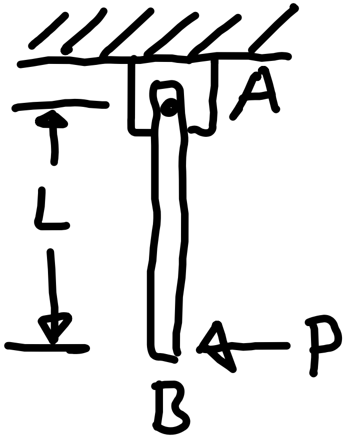
$L = 36\,in$

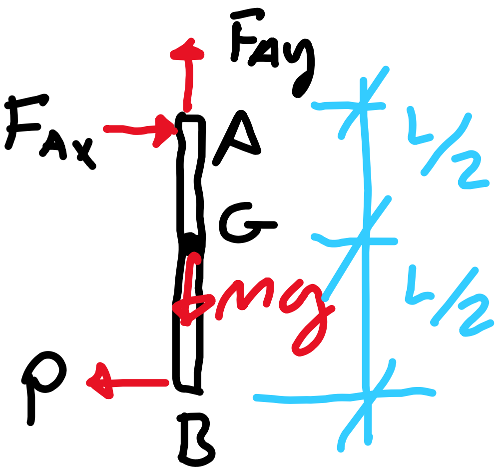




Example

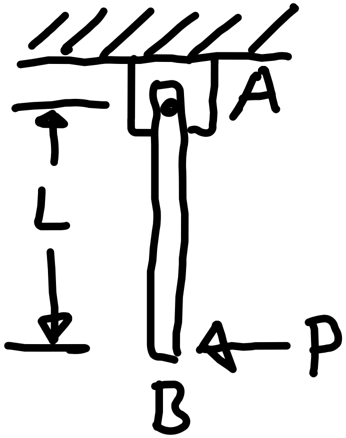
$L = 36\,in, \; w = 4\,lb$

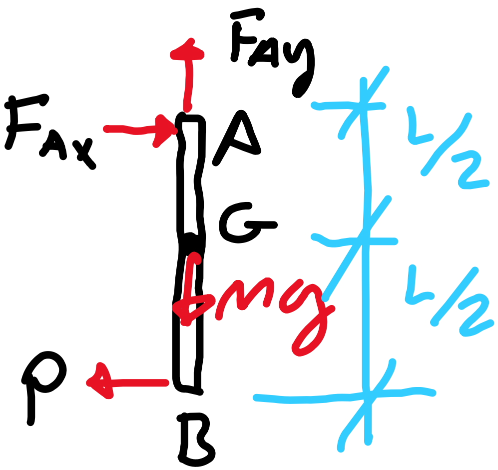




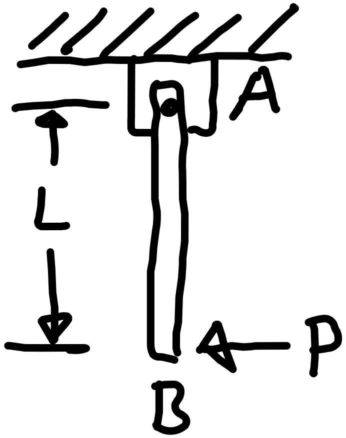
Example

$L = 36\,in$, $w = 4\,lb$, $P = 1.5\,lb$



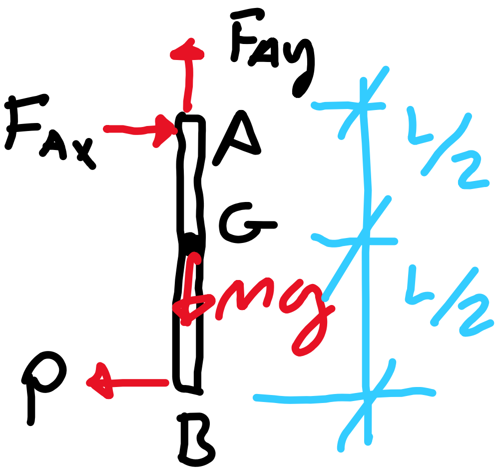


Example



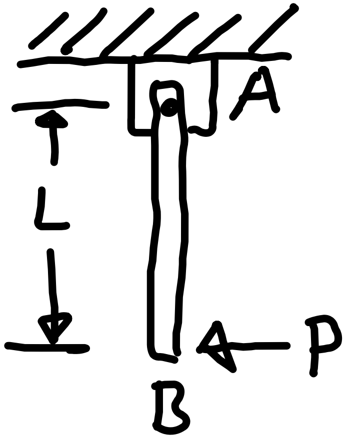
$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

Find α:

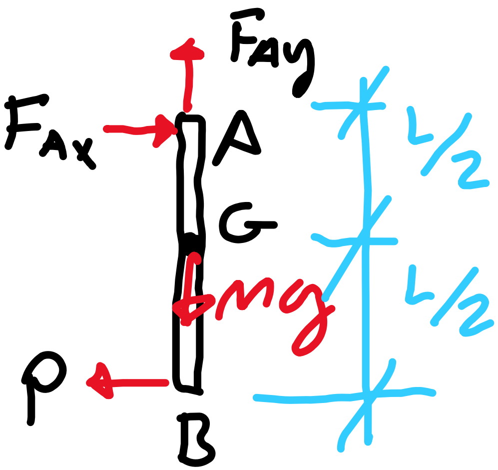


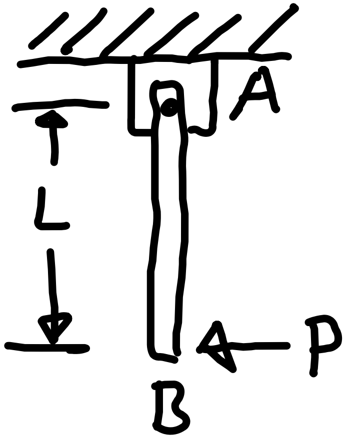
Example

$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$



Find α: $\circlearrowleft \sum M_A = I_A \alpha$



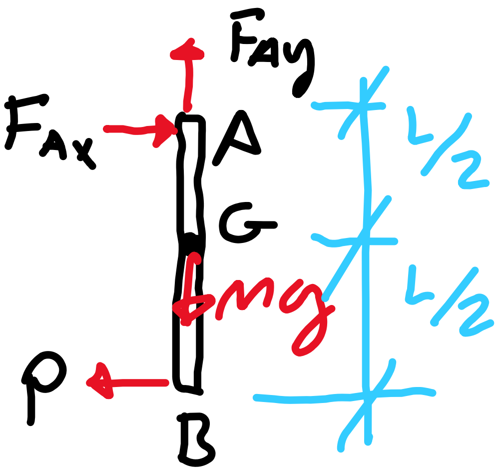


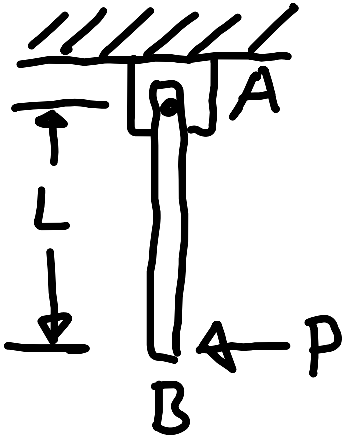
Example

$L = 36\,in,\ W = 4\,lb,\ P = 1.5\,lb$

Find α: $\circlearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$



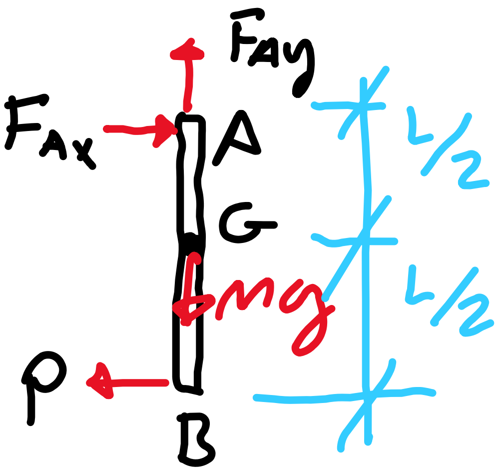


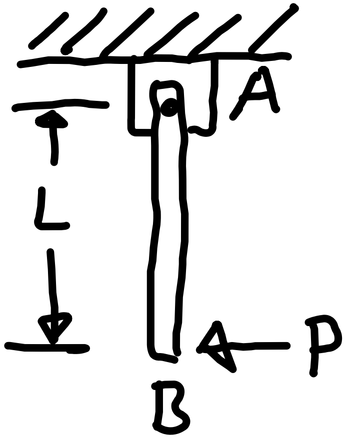
Example

$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

Find α: $\circlearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$





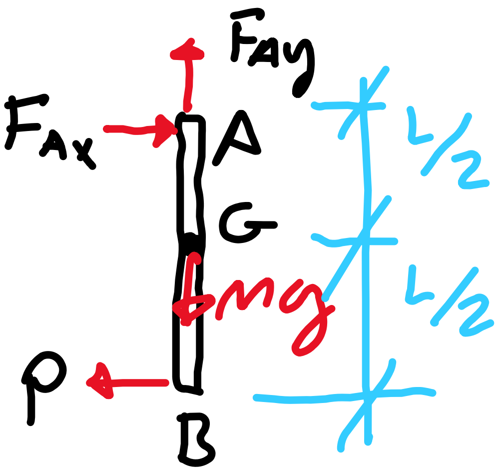
Example

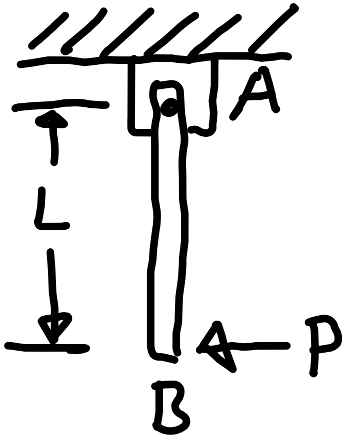
$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

Find α: $\circlearrowright \sum M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

& $\bar{I} = m\frac{L^2}{12}$





Example

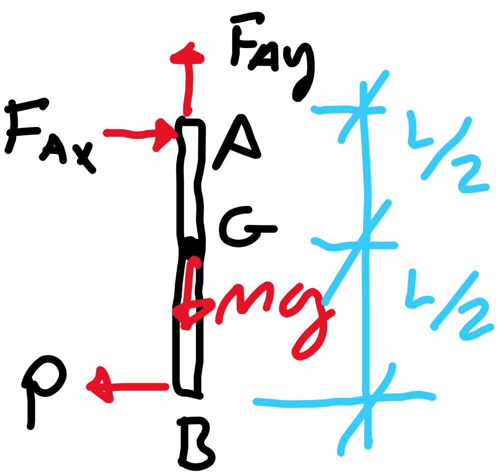
$L = 36\text{in}$, $w = 4\text{lb}$, $P = 1.5\text{lb}$

Find α: $\circlearrowleft \sum M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

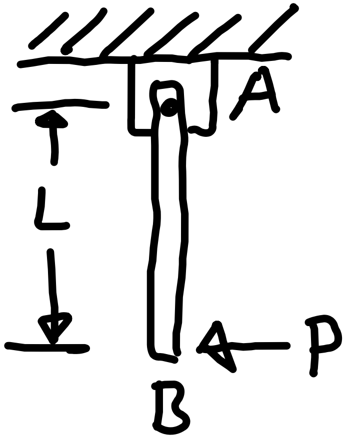
$\&\ \bar{I} = m\frac{L^2}{12} \Rightarrow$

$I_A = m\frac{L^2}{12} + m\frac{L^2}{4}$



ASU

Example



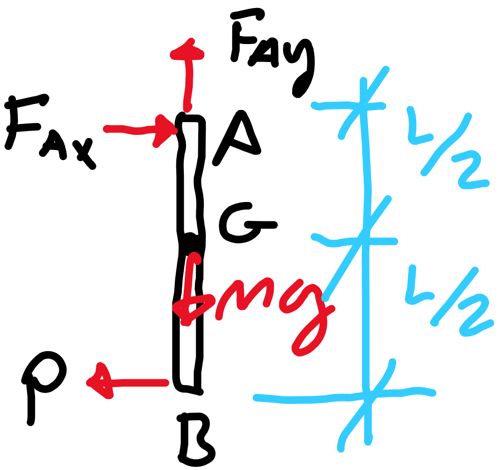
$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

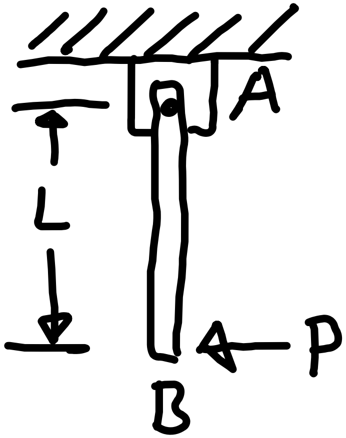
Find α: $\circlearrowright \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

$\&\ \bar{I} = m\frac{L^2}{12} \quad \Rightarrow$

$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3)$





Example

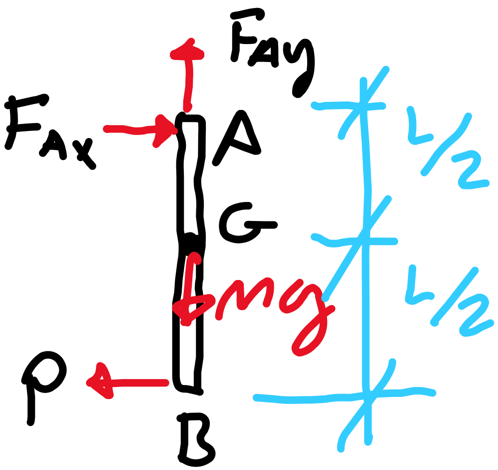
$L = 36\,\text{in},\ W = 4\,\text{lb},\ P = 1.5\,\text{lb}$

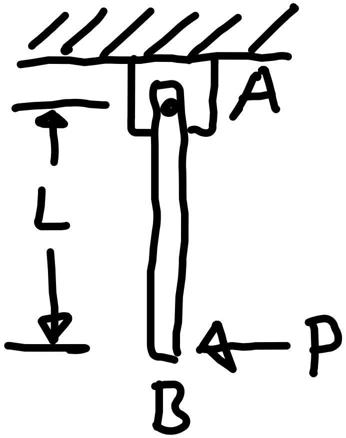
Find α: $\circlearrowright \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

& $\bar{I} = m\frac{L^2}{12} \quad \Rightarrow$

$$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3) = \frac{mL^2}{3}$$





Example

$L = 36\,\text{in}, \; w = 4\,\text{lb}, \; P = 1.5\,\text{lb}$

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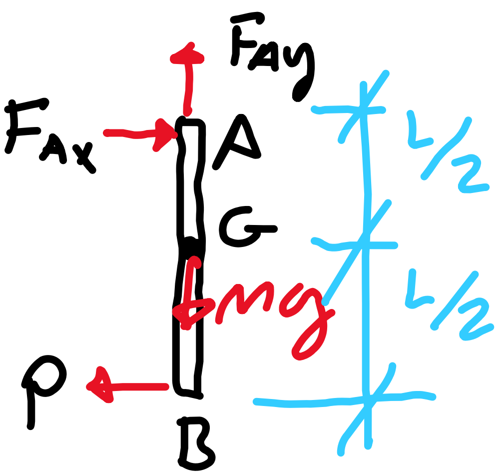
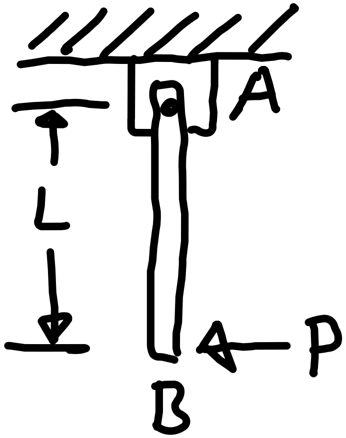


Now $\quad PL = \left(mL^2/3\right)\alpha$

Example



$$L = 36\,\text{in}, \quad w = 4\,\text{lb}, \quad P = 1.5\,\text{lb}$$

<u>Find α:</u> $\circlearrowleft \Sigma M_A = I_A \alpha$

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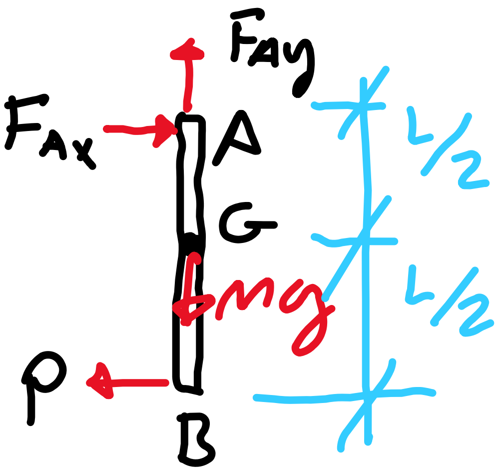
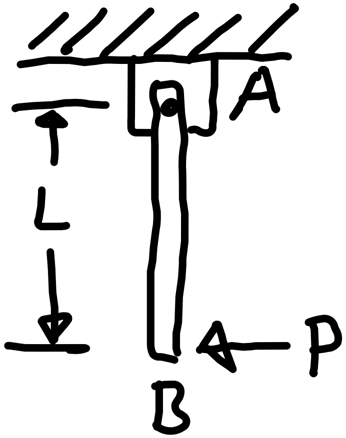


$$\text{Now} \quad PL = \left(mL^2/3\right)\alpha \Rightarrow$$

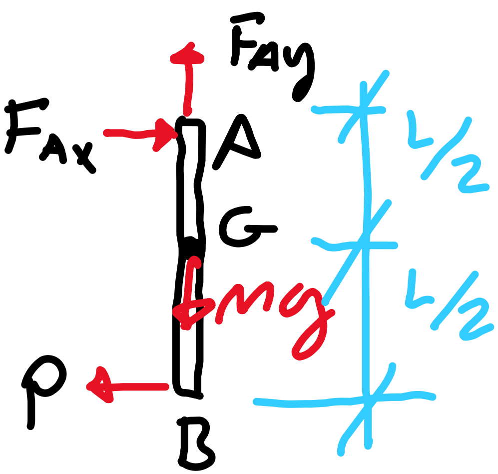
$$\alpha = \frac{3P}{mL}$$



Example

$3\,ft = 36\,in$

$L = 36\,in$, $w = 4\,lb$, $P = 1.5\,lb$



Find α: $\circlearrowleft \sum M_A = I_A \alpha$

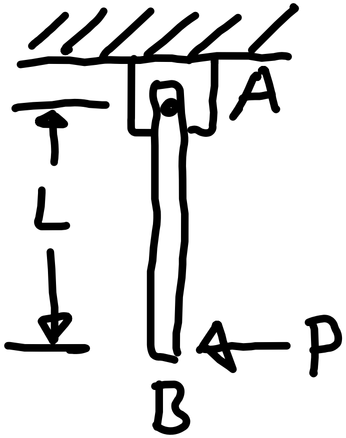
$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

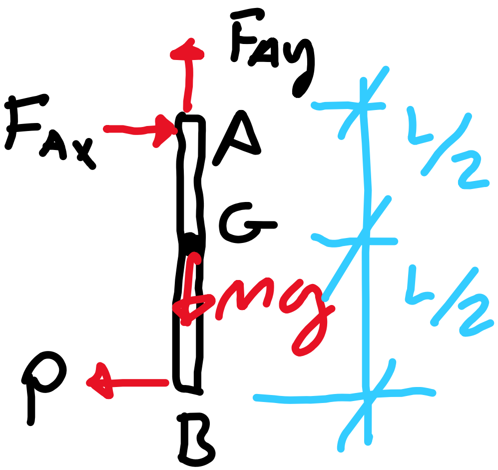
& $\bar{I} = m\frac{L^2}{12} \Rightarrow$

$$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3) = \frac{mL^2}{3}$$

Now $PL = \left(mL^2/3\right)\alpha \Rightarrow$

$$\alpha = \frac{3P}{mL} = \left(\frac{3 * \frac{3}{2}}{(4/32.2)3}\right)\frac{rad}{s^2}$$





Example

$3\ ft = 36 in$

$L = 36 in, \; w = 4 lb, \; P = 1.5 lb$

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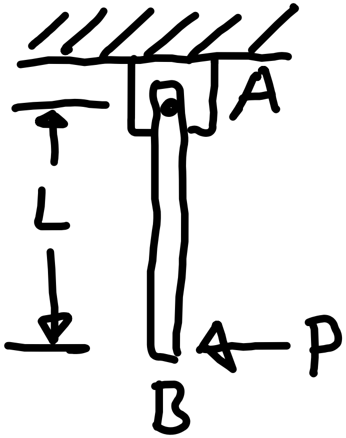
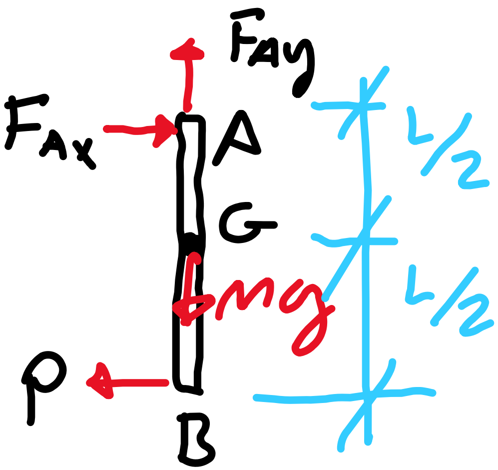
$$\text{Now} \quad PL = \left(mL^2/3\right)\alpha \Rightarrow$$

$$\alpha = \frac{3P}{mL} = \left(\frac{3 * \frac{3}{2}}{(4/32.2)3}\right)\frac{rad}{s^2}$$

$$\Rightarrow \quad \alpha = \left(\frac{3}{8}\right)32.2\frac{rad}{s^2}$$





Example

$L = 36\,in$, $w = 4\,lb$, $P = 1.5\,lb$

$3\,ft = 36\,in$

Find α: $\circlearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

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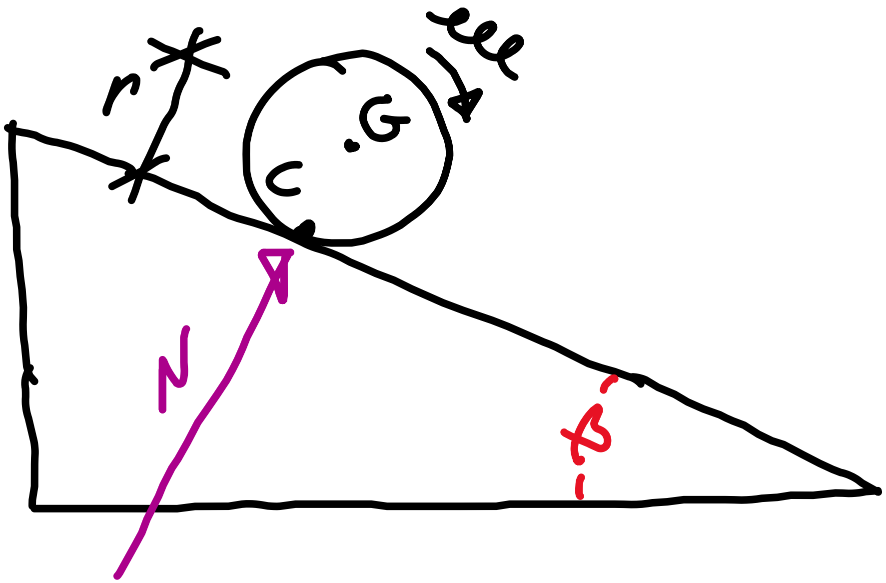
Now $PL = \left(mL^2/3\right)\alpha \Rightarrow$

$$\alpha = \frac{3P}{mL} = \left(\frac{3 * \frac{3}{2}}{(4/32.2)3}\right)\frac{rad}{s^2}$$

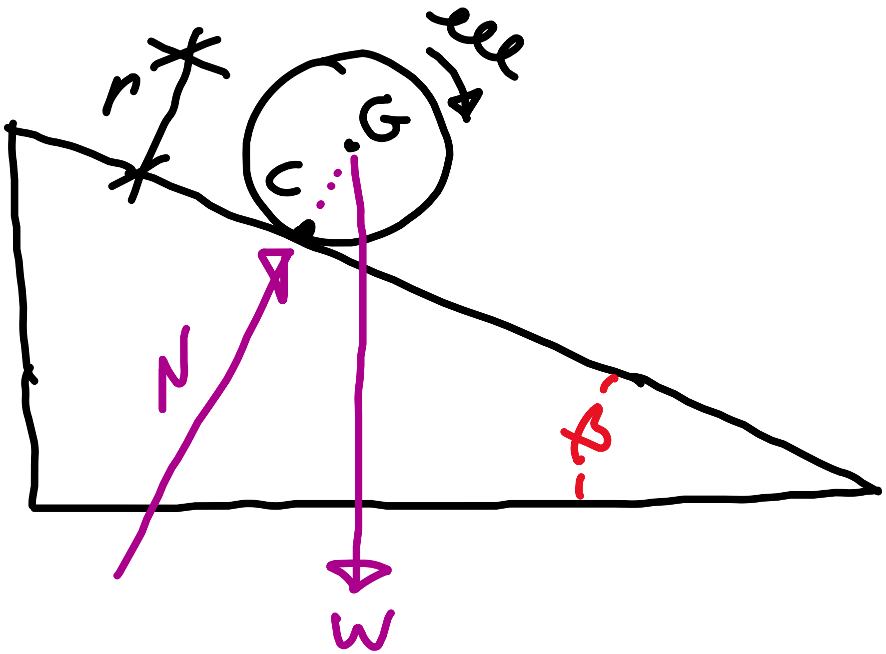
$$\Rightarrow \quad \alpha = \left(\frac{3}{8}\right)32.2\frac{rad}{s^2} = 12.08\,rad/s^2$$

ASU

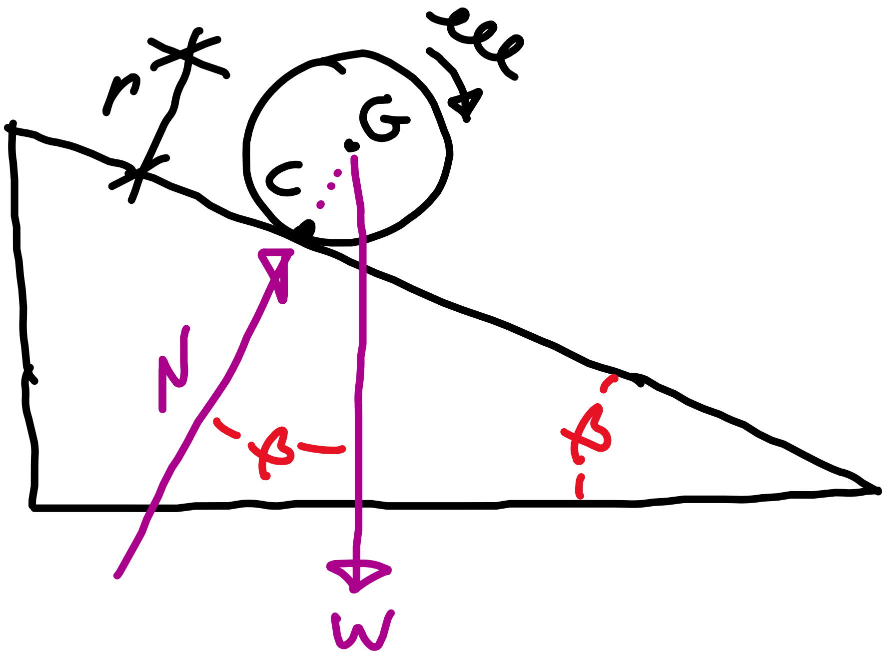
Example: Wheel on incline



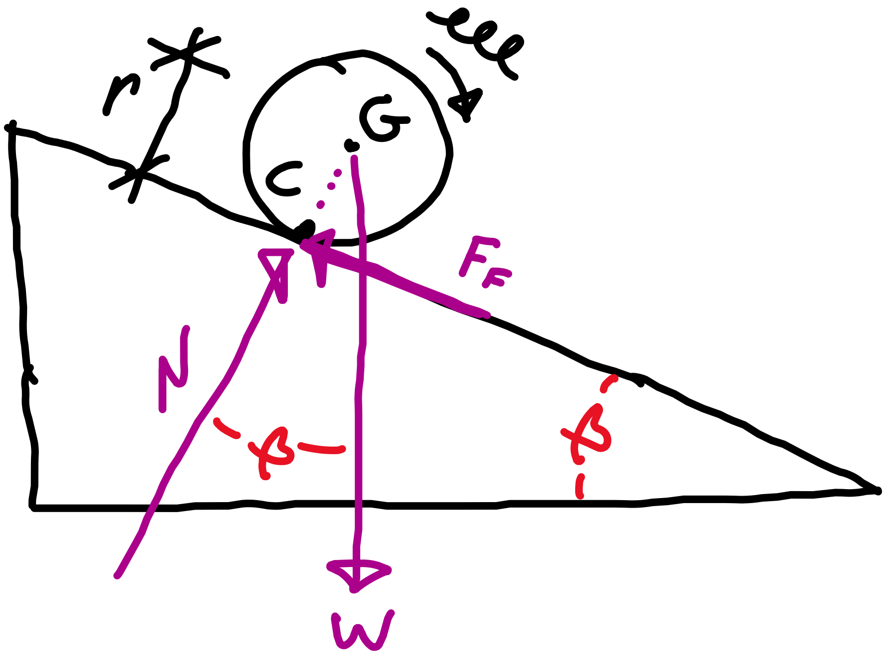
Example: Wheel on incline



Example: Wheel on incline

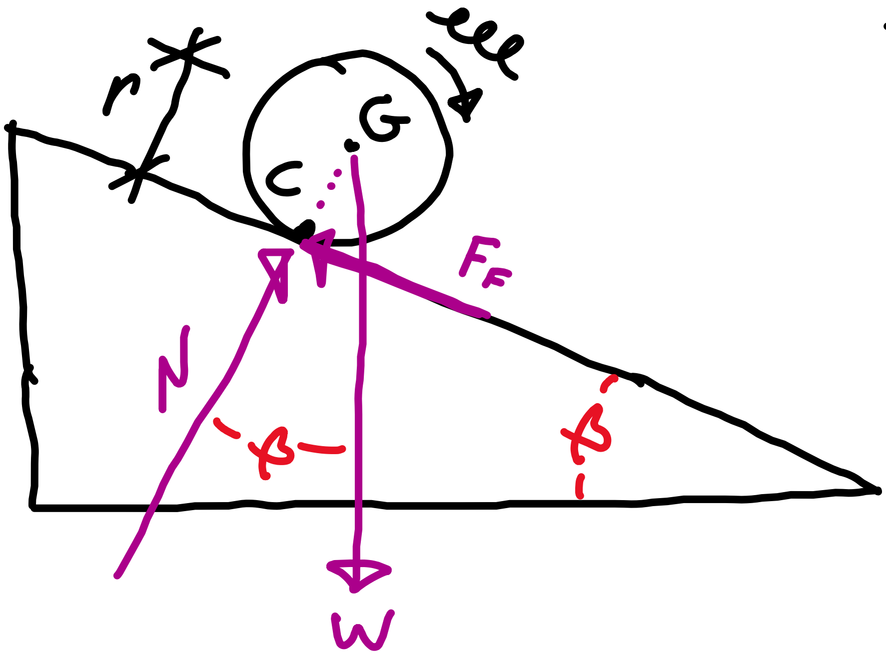


Example: Wheel on incline



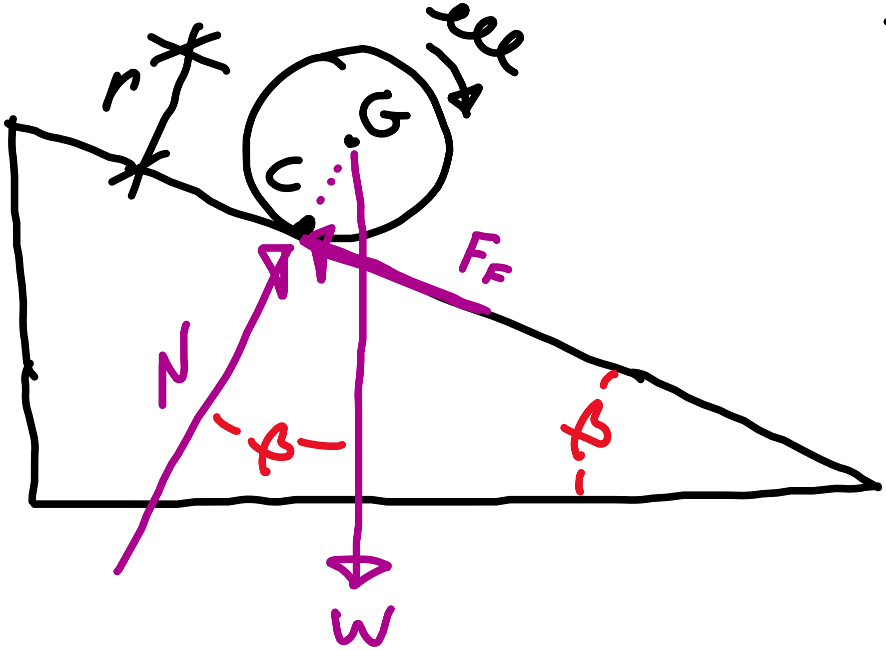
Example: Wheel on incline

Find α:



Example: Wheel on incline

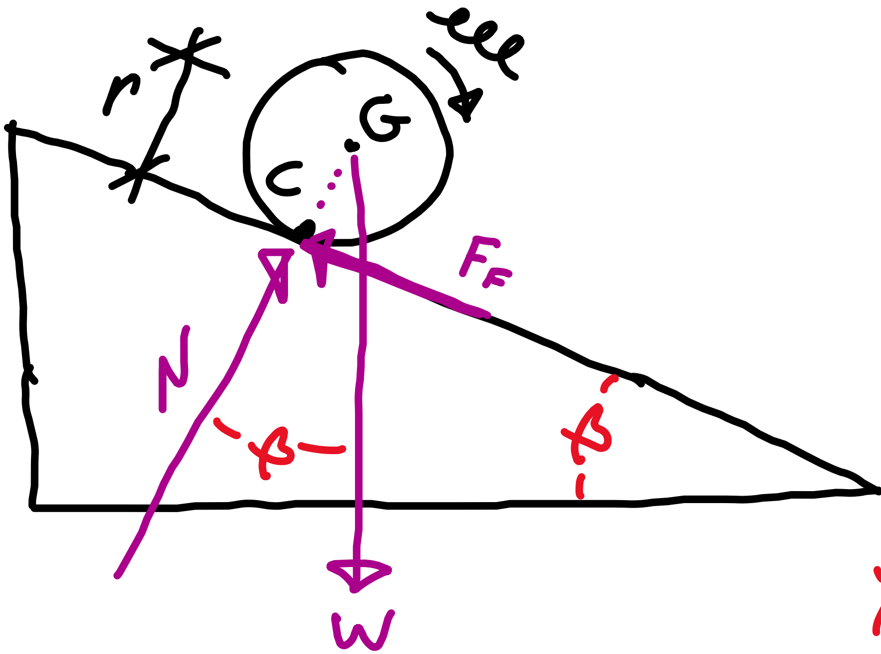
Find α:



Easiest to sum torques about point C

Example: Wheel on incline

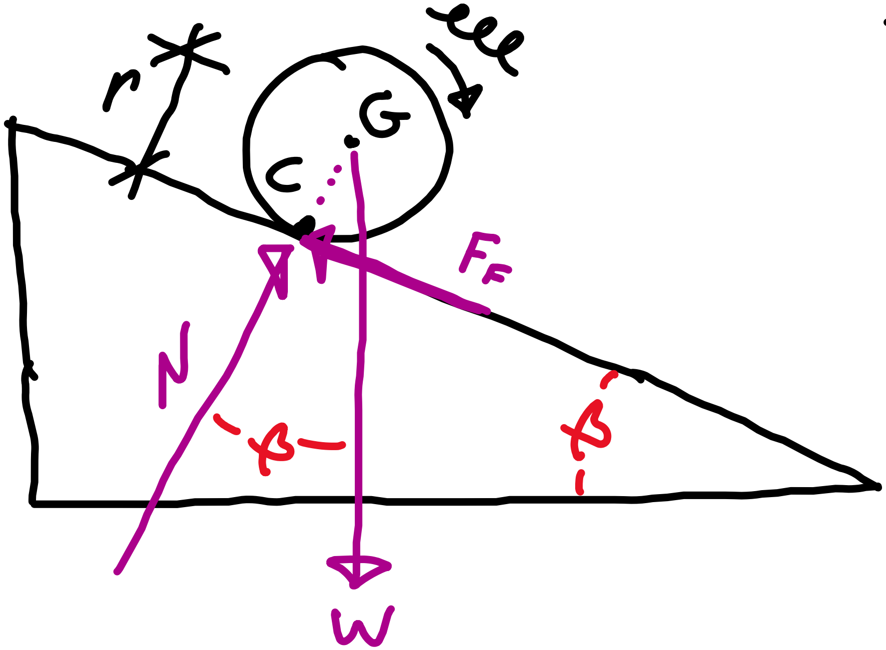
Find α:



Easiest to sum torques about point C [can neglect all forces pointing into (or out of) point C]

![ASU]

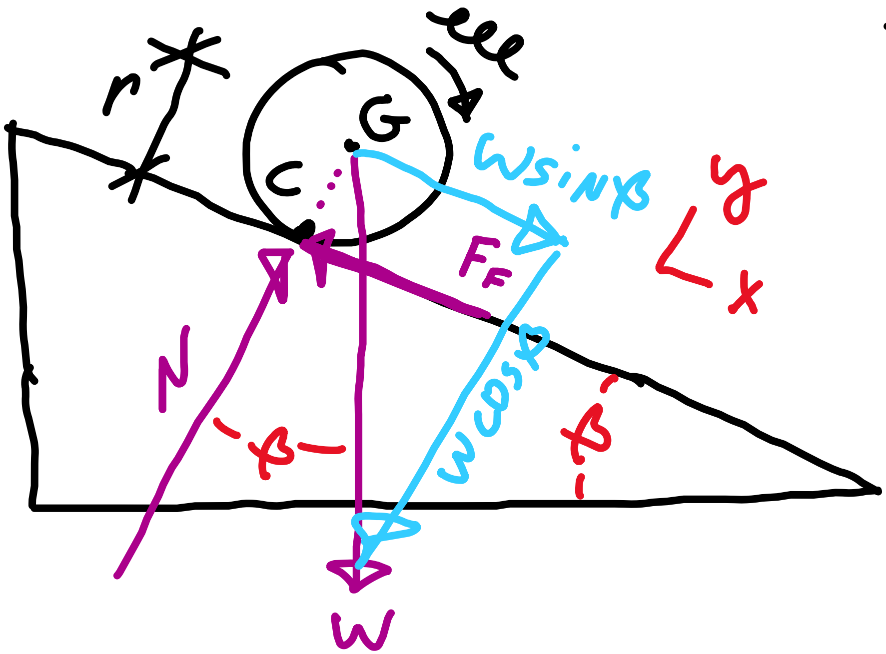
Example: Wheel on incline



Find α:

$$\curvearrowright \sum M_c = I_c \alpha$$

Example: Wheel on incline



Find α:

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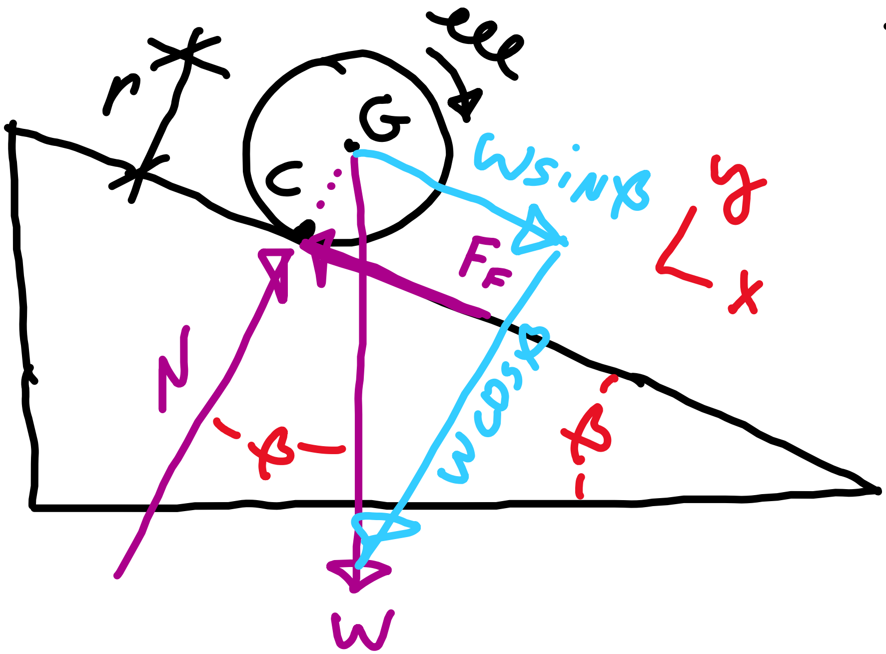
Example: Wheel on incline

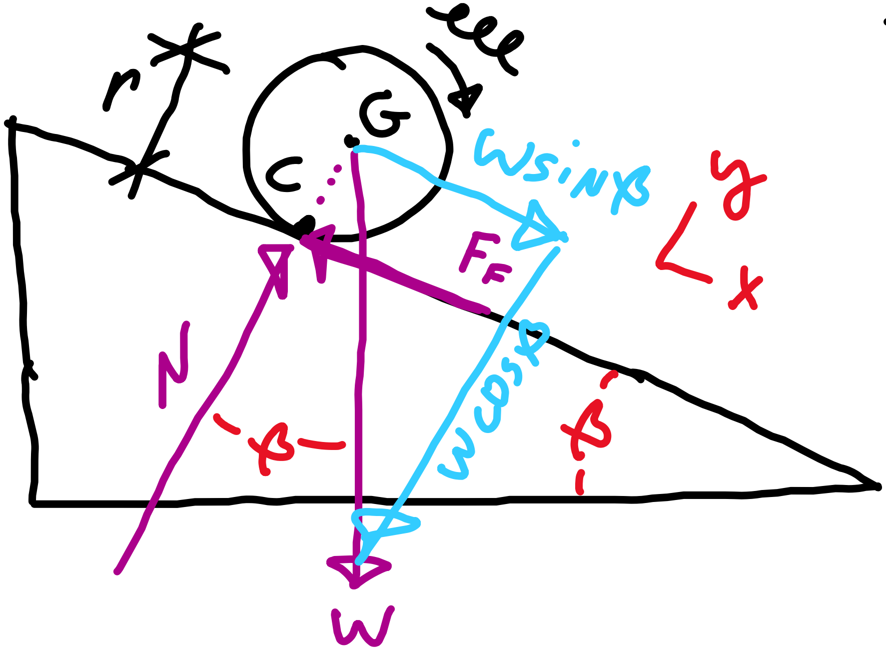


Find α:

$$\circlearrowright \sum M_c = I_c \alpha$$

$$\vec{w} = w\sin\beta\,\hat{x} - w\cos\beta\,\hat{y}$$

Example: Wheel on incline



Find α:

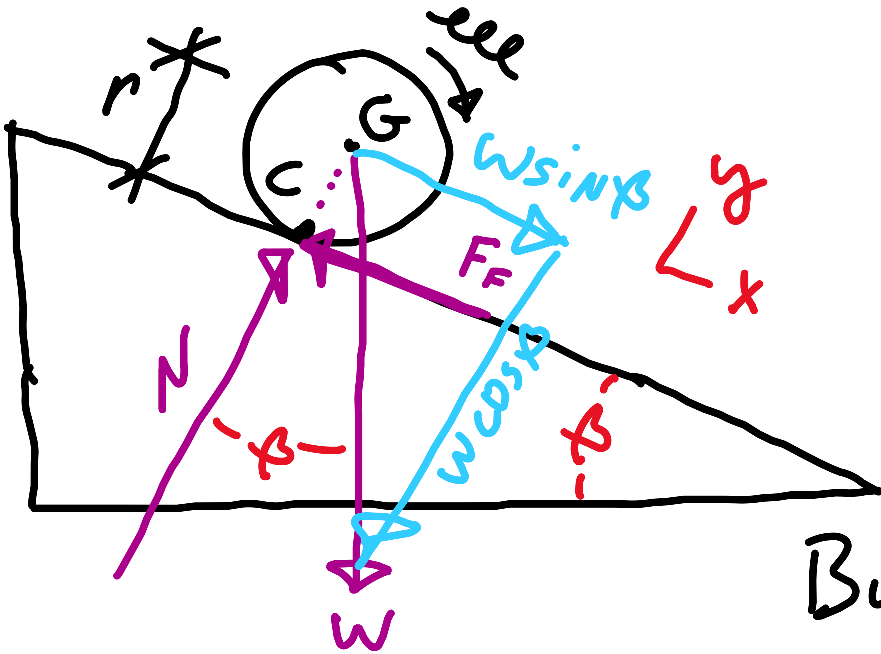
$$\circlearrowright \sum M_c = I_c \alpha$$

$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

$$\Rightarrow$$

$$W\cos\beta\, r = (\bar{I} + mr^2)\alpha$$

Example: Wheel on incline



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$$\curvearrowleft \sum M_c = I_c \alpha$$

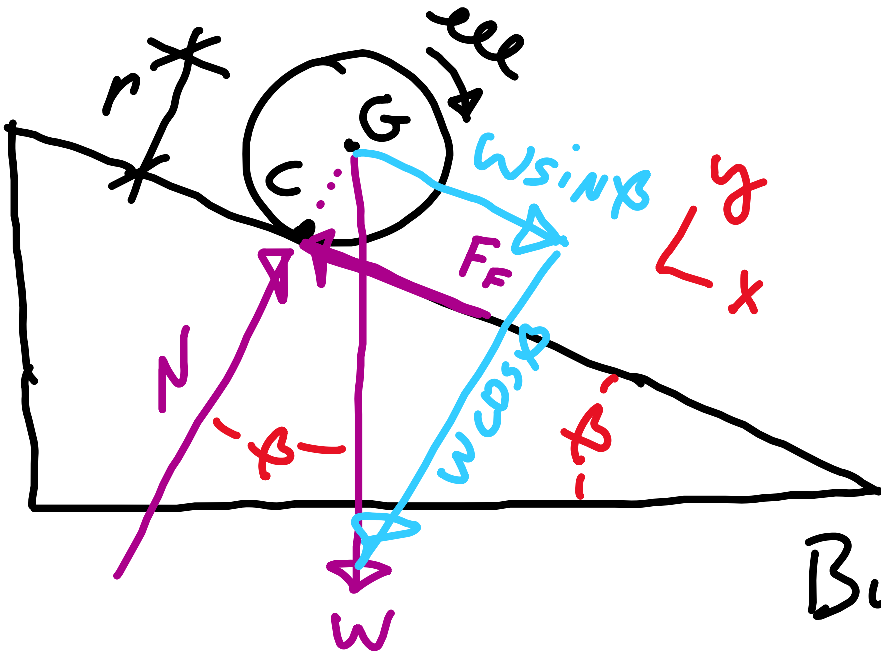
$$\vec{w} = w\sin\beta\,\hat{x} - w\cos\beta\,\hat{y}$$

$$\Rightarrow$$

$$w\cos\beta\, r = (\bar{I} + mr^2)\alpha$$

But $\bar{I} = mr^2/2$

Example: Wheel on incline



Find α:

$$\curvearrowright \sum M_c = I_c \alpha$$

$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

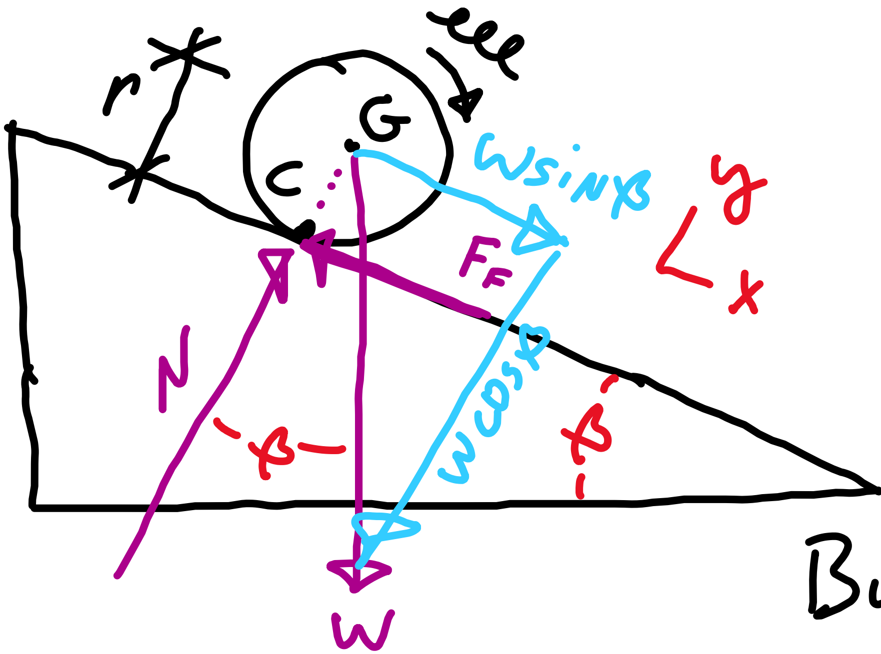
$$\Rightarrow$$

$$W\cos\beta\, r = (\bar{I} + mr^2)\alpha$$

But $\bar{I} = mr^2/2$ so

$$W\cos\beta\, r = \frac{3}{2} m r^2 \alpha$$

Example: Wheel on incline



Find α:

$$\circlearrowright \sum M_C = I_C \alpha$$

$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

$$\Rightarrow$$

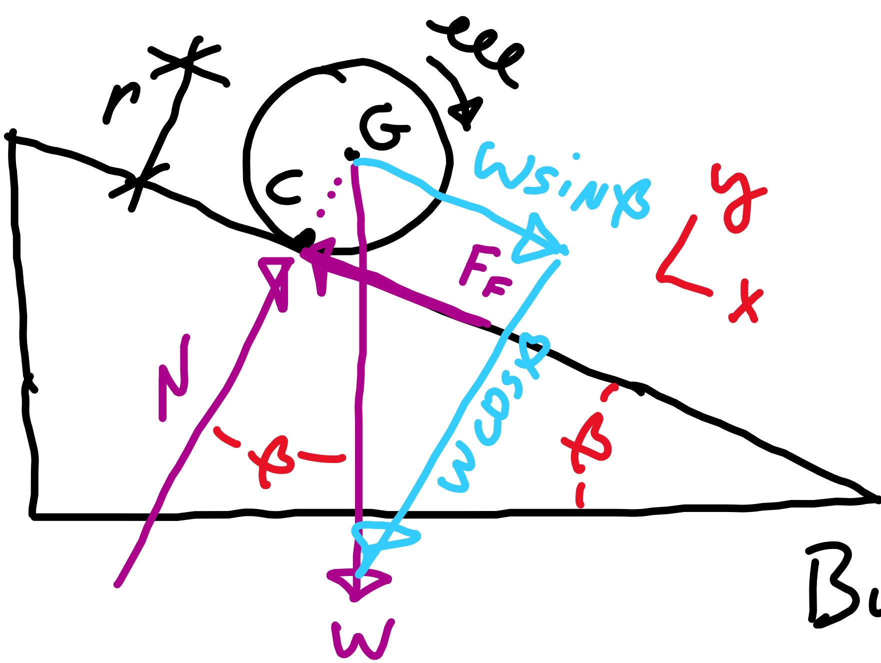
$$W\cos\beta\, r = (\bar{I} + mr^2)\alpha$$

But $\bar{I} = mr^2/2$ so

$$W\cos\beta\, r = \frac{3}{2} m r^2 \alpha \quad \Rightarrow$$

$$\alpha = \frac{2W\cos\beta}{3mr}$$

Example: Wheel on incline



Find α:

$$\circlearrowright \sum M_c = I_c \alpha$$

$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

$$\Rightarrow$$

$$W\cos\beta\, r = (\bar{I} + mr^2)\alpha$$

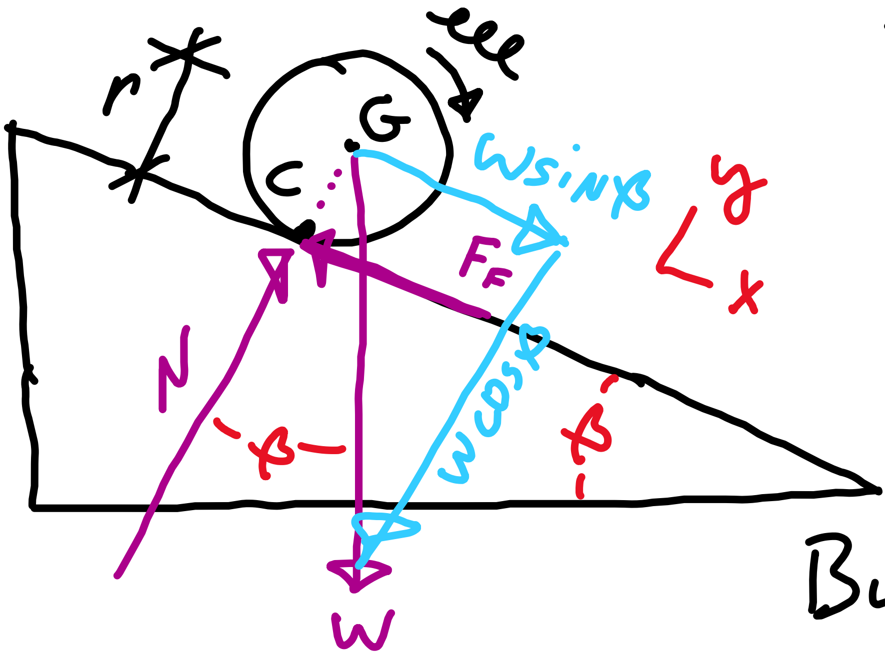
But $\bar{I} = mr^2/2$ so

$$W\cos\beta\, r = \frac{3}{2} m r^2 \alpha \quad \Rightarrow$$

$$\alpha = \frac{2W\cos\beta}{3mr} = \frac{2g\cos\beta}{3r}$$

ASU

Example: Wheel on incline



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