Today: 16.2 & 17.1

$\boxed{L26}$

Today: 16.2 & 17.1

$\boxed{\text{L26}}$

Constrained plane motion

L26

Today: 16.2 & 17.1

Constrained plane motion

Energy methods for rigid bodies

Today: 16.2 & 17.1

$\boxed{L26}$

Wednesday 17.1

L26

Today: 16.2 & 17.1

Wednesday 17.1

Friday 17.1 & 17.2

L26

Today: 16.2 & 17.1

Wednesday 17.1

Friday 17.1 & 17.2

Momentum methods

ASU

L26

Today: 16.2 & 17.1

Wednesday 17.1

Friday 17.1 & 17.2

Monday Nov 2nd: 17.2

ASU

L26

Today: 16.2 & 17.1

Wednesday 17.1

Friday 17.1 & 17.2

Monday Nov 2nd: 17.2

Wednesday Nov 4th: Review

L26

Today: 16.2 & 17.1

Wednesday 17.1

Friday 17.1 & 17.2

Monday Nov 2nd: 17.2

Wednesday Nov. 4th: Review

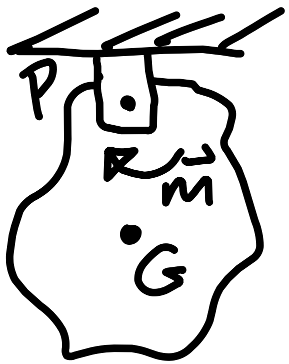
Friday Nov. 6th: Exam #3

ASU

Last time, we saw that for a

fixed point rotation of a rigid body

Last time, we saw that for a <u>fixed point rotation</u> of a rigid body



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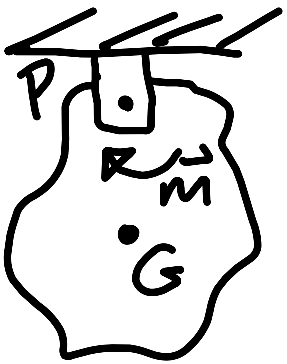

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$

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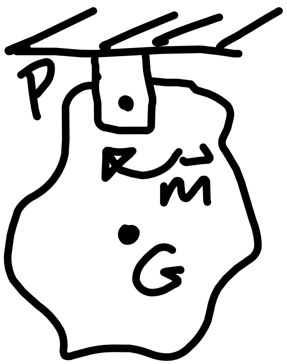
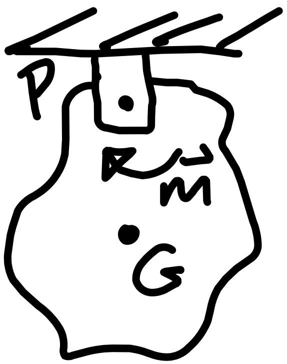


$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ <u>or</u>

$$\sum \vec{M}_P = I_P \vec{\alpha},$$

Last time, we saw that for a <u>fixed point rotation</u> of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

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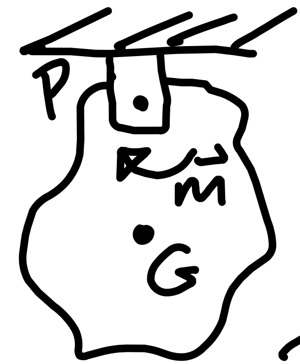

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For a fixed point of rotation it is,

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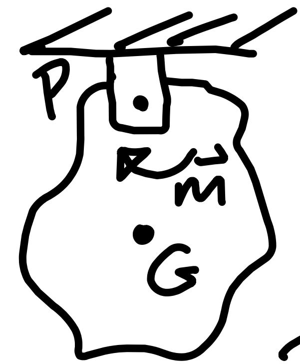

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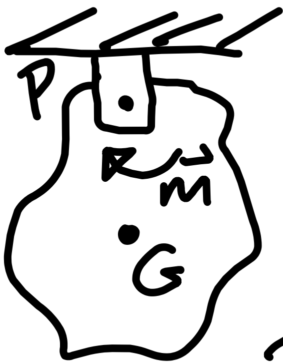

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

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For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$

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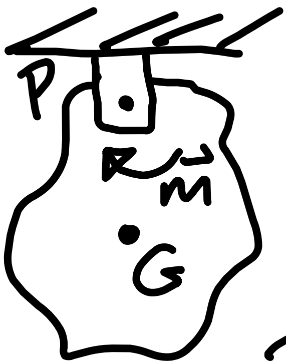

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

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For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\sum \vec{M}_P = I_P \vec{\alpha}$

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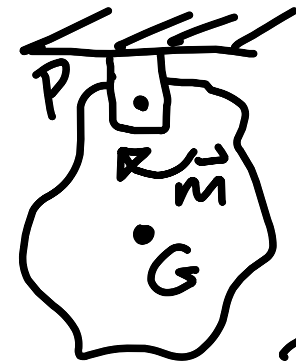

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ <u>or</u>

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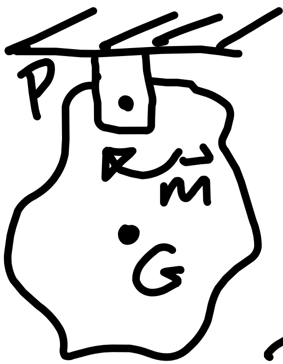
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P \vec{\alpha}$ and use the form $\sum \vec{M}_P = I_P \vec{\alpha}$. However,

Last time, we saw that for a <u>fixed point rotation</u> of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \text{or}$$

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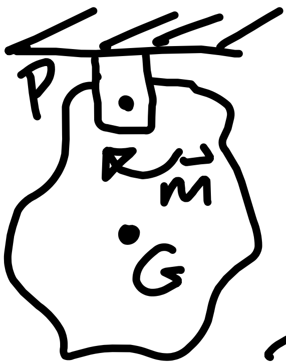
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\sum \vec{M}_P = I_P\vec{\alpha}$. However, for problems that do not have a fixed point of rotation, Last time, we saw that for a **fixed point rotation** of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \text{or}$$

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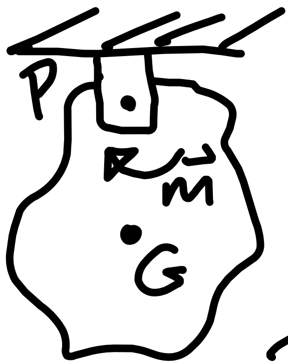
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\sum \vec{M}_P = I_P\vec{\alpha}$. However, for problems that do not have a fixed point of rotation, the other form

$$\left\{ \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \right\}$$

Last time, we saw that for a <u>fixed point rotation</u> of a rigid body



$$\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

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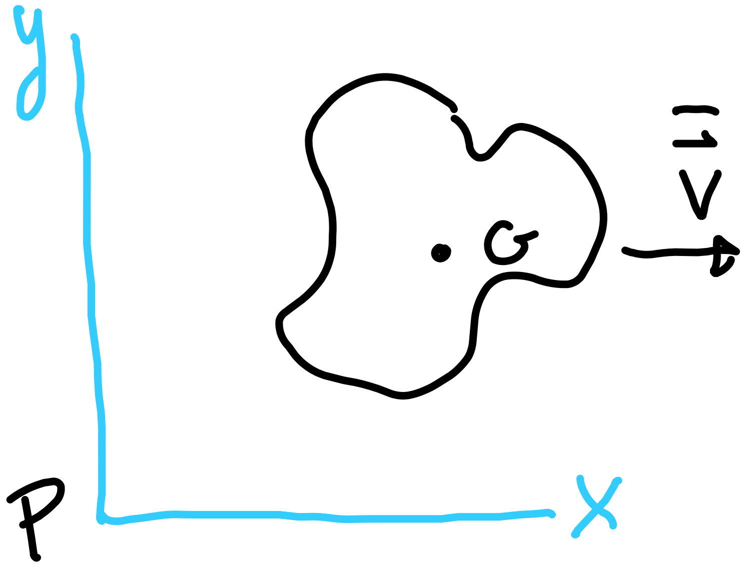
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\Sigma \vec{M}_P = I_P \vec{\alpha}$. However, for problems that do not have a fixed point of rotation, the other form $\{\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}\}$ must be used

<u>Example</u>: rigid body in plane motion that is translating & rotating.

Example: rigid body in plane motion that is translating & rotating.



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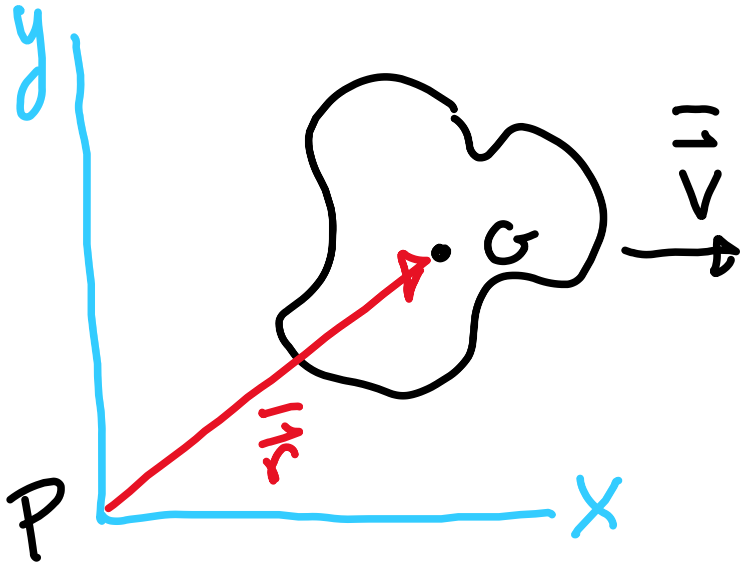

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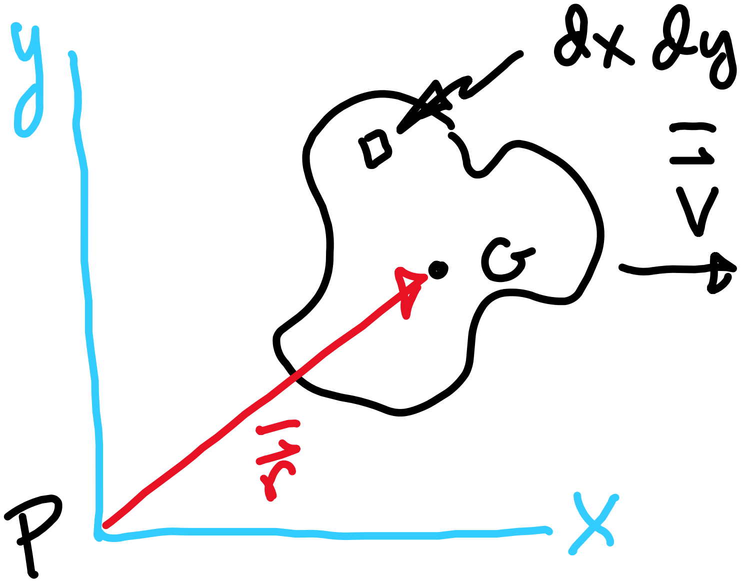

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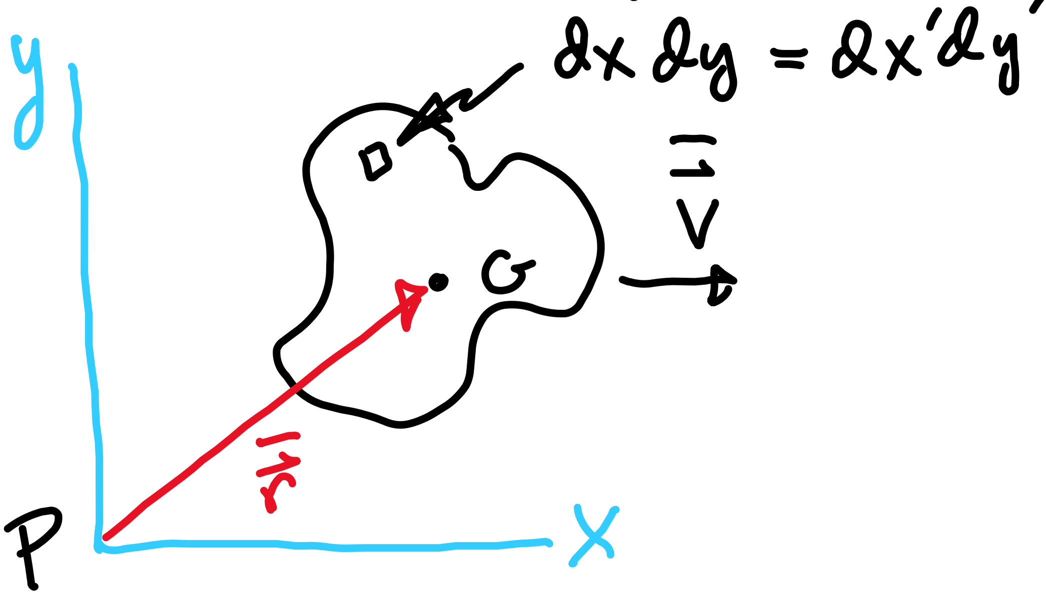

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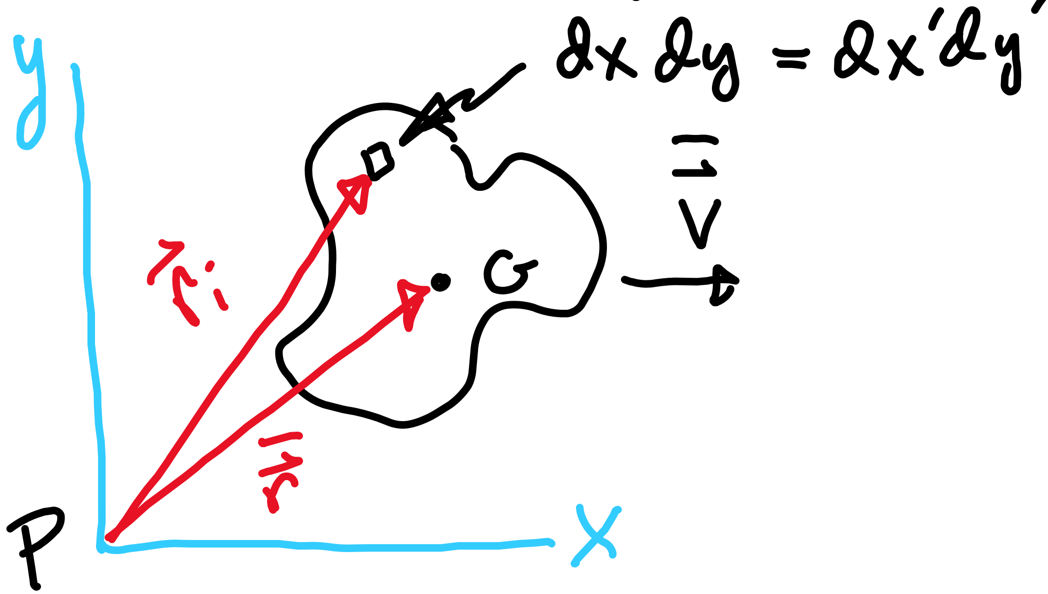
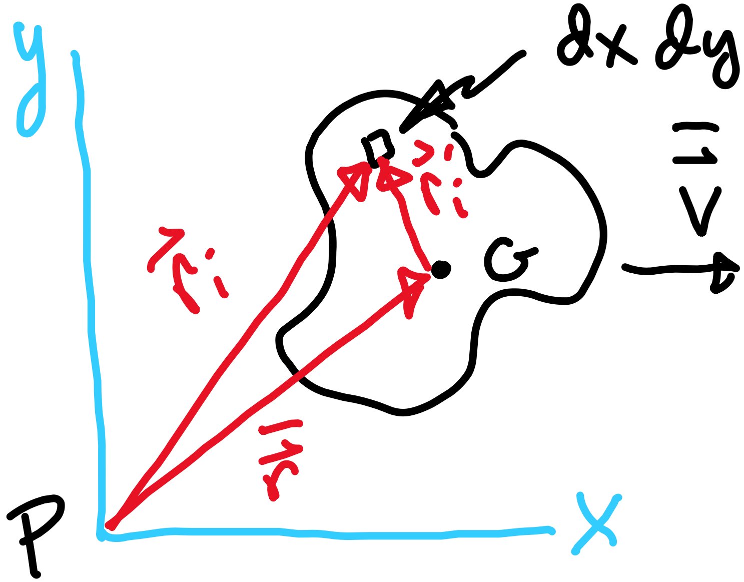


Example: rigid body in plane motion that is translating & rotating.

$dx\,dy = dx'dy'$



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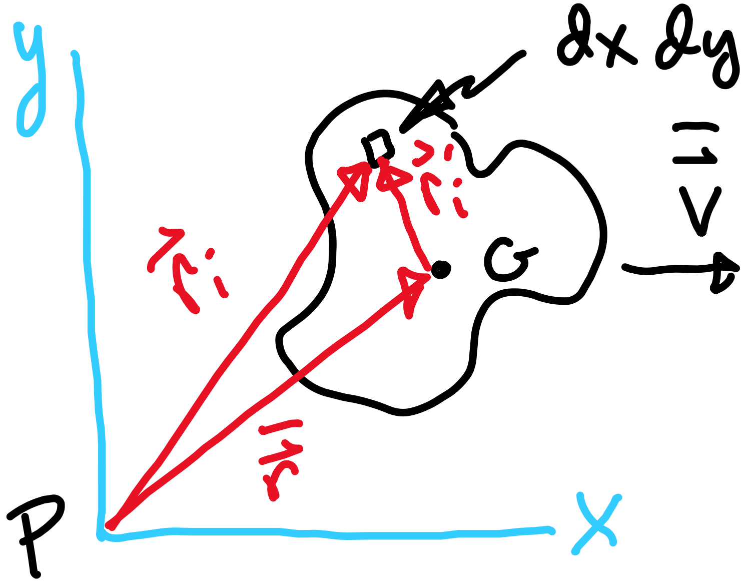

The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{V}_i\right) \Delta x_i \Delta y_i$$

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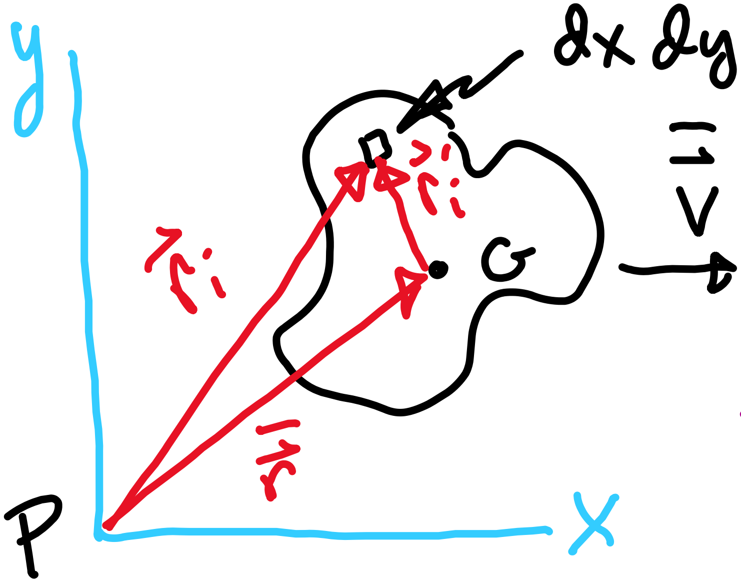

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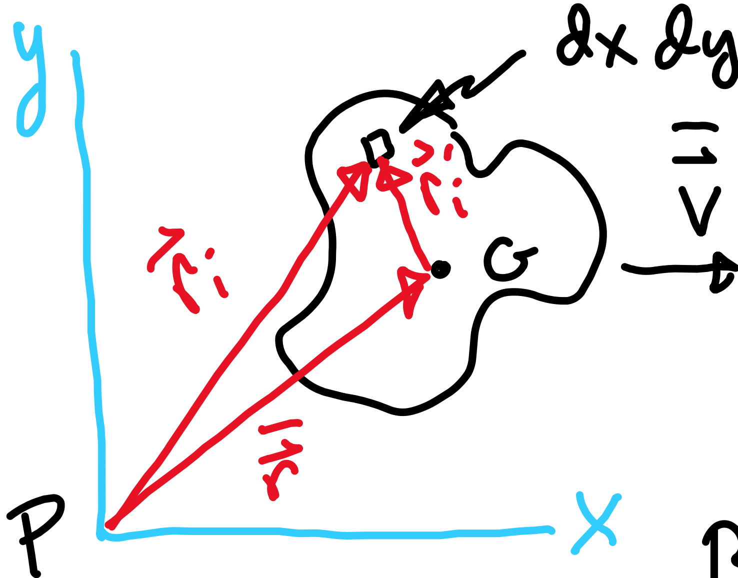

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But $\vec{r} = \vec{\bar{r}} + \vec{r}\,'$

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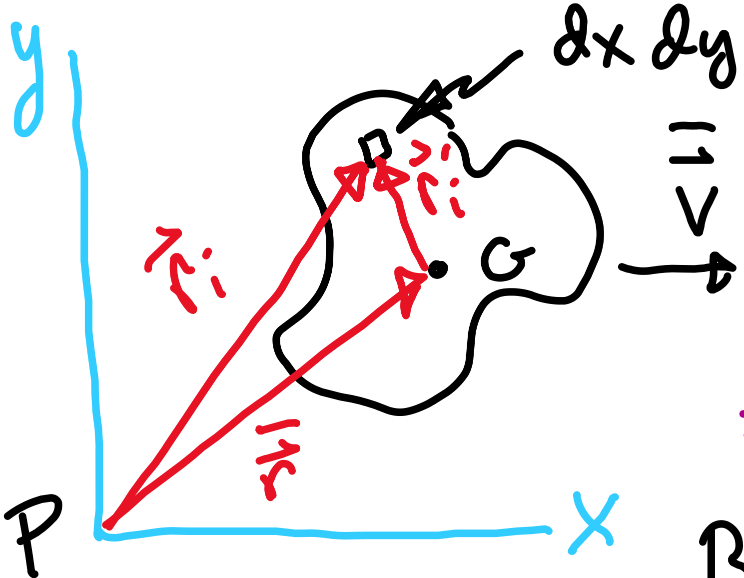

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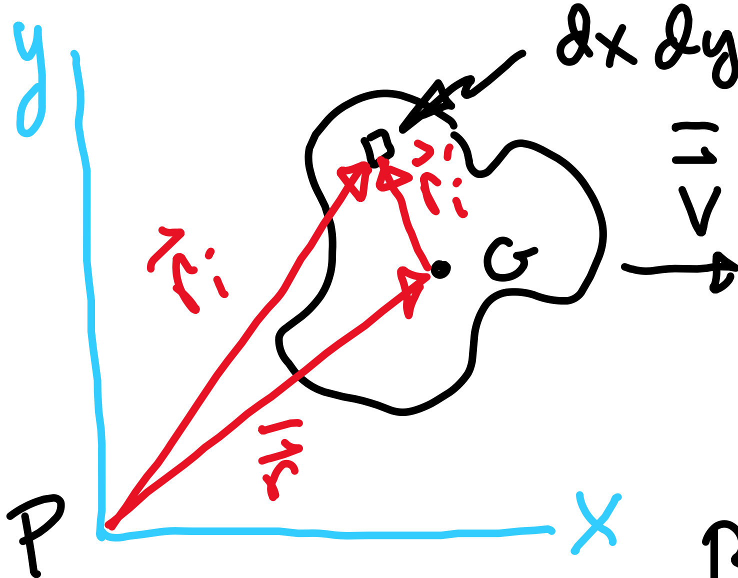

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But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right) \iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' +$

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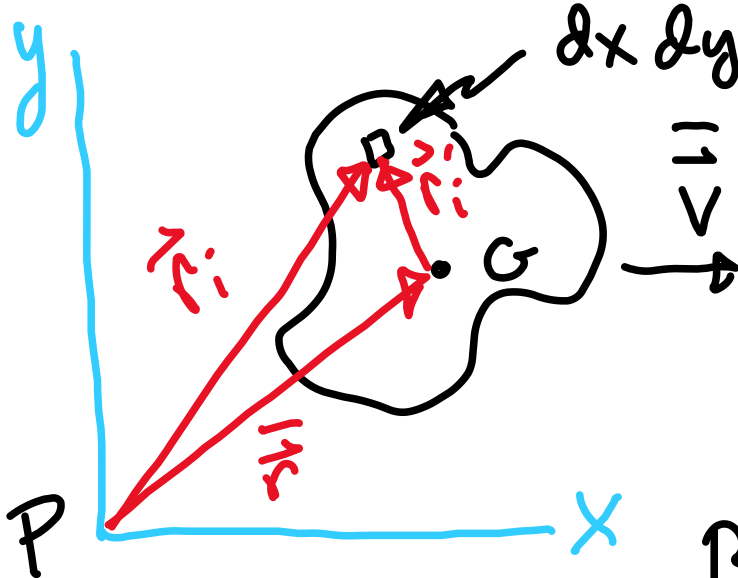

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The angular momentum about P is

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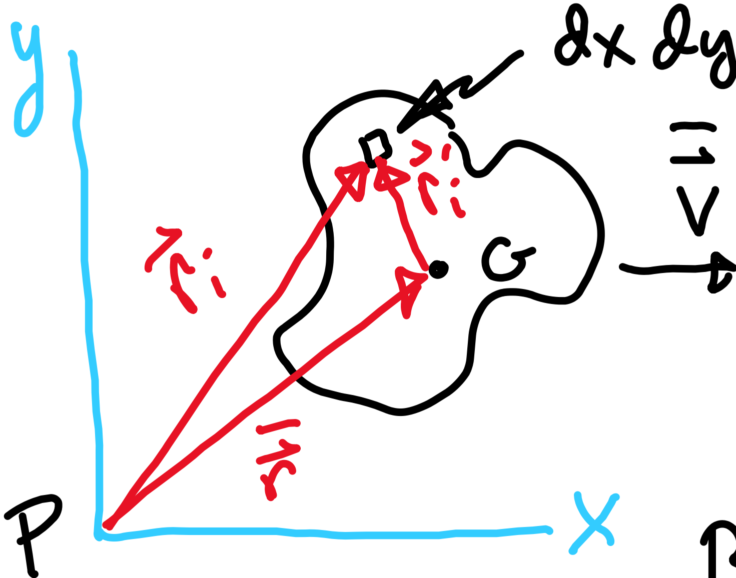

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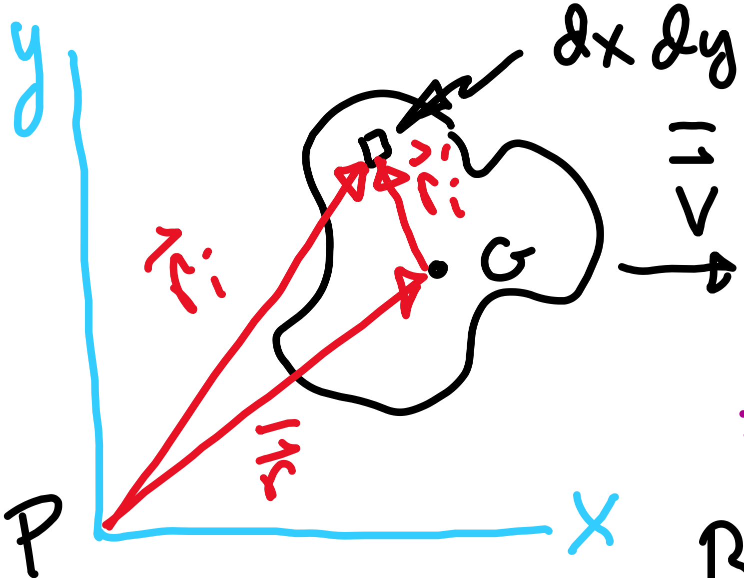
Example: rigid body in plane motion that is translating & rotating.

$dx\,dy = dx'\,dy'$



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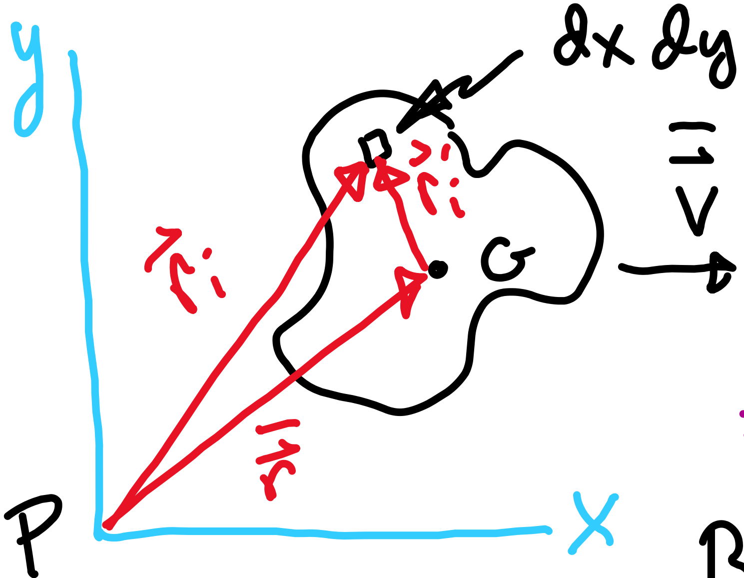

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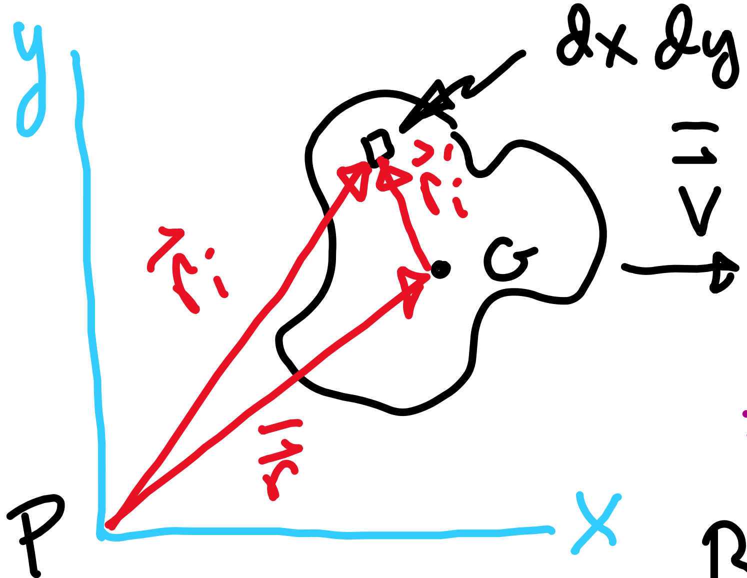
But $\vec{r} = \vec{\bar{r}} + \vec{r}\,'$ & $\vec{v} = \vec{\bar{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \vec{\bar{r}}\times\vec{\bar{v}}\,dx'dy' + \left(\frac{m}{A}\right)\iint \vec{\bar{r}}\times\vec{v}\,'\,dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,'\times\vec{\bar{v}}\,dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec{r}\,'\times\vec{v}\,'\,dx'dy' = \left(\frac{m}{A}\right)\vec{\bar{r}}\times\vec{\bar{v}}\iint dx'dy' +$

Example: rigid body in plane motion that is translating & rotating.



$dx\,dy = dx'dy'$

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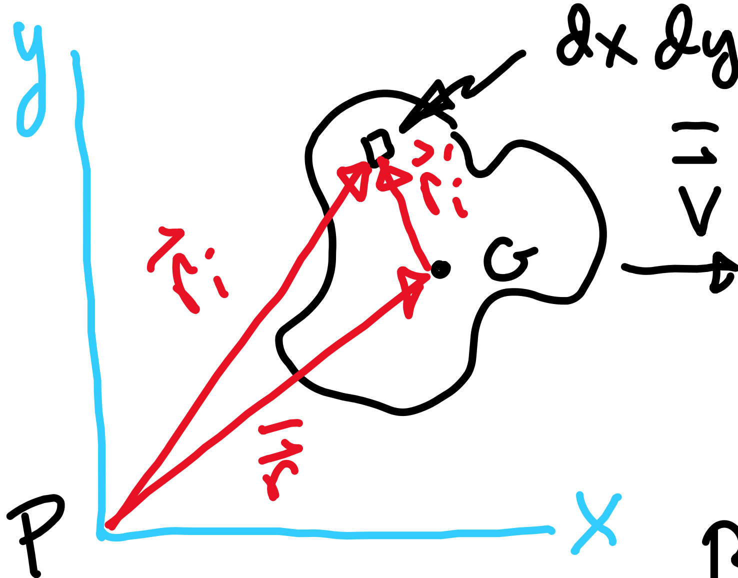
So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \underline{\left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,' dx'dy'} + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \underline{\left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,' dx'dy'} +$

Example: rigid body in plane motion that is translating & rotating.



$dx\,dy = dx'dy'$

The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum (\vec{r}_i \times \tfrac{m}{A} \vec{v}_i)\,\Delta x_i \Delta y_i$$

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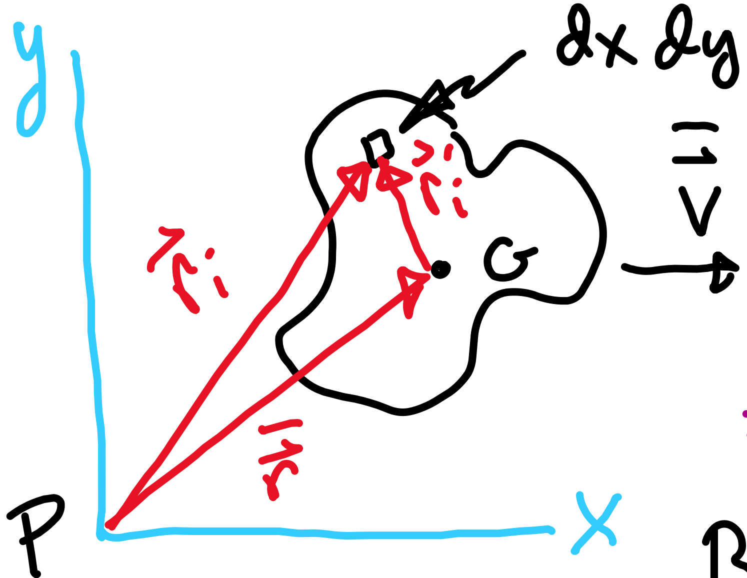
But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\tfrac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\tfrac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'\, dx'dy' + \left(\tfrac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$
$\left(\tfrac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'\, dx'dy' = \left(\tfrac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\tfrac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,'\, dx'dy' +$
$\left(\tfrac{m}{A}\right)\left[\iint \vec{r}\,'\, dx'dy'\right] \times \bar{\vec{v}}\,' +$

Example: rigid body in plane motion that is translating & rotating.

$dx\,dy = dx'dy'$



The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{v}_i\right)\Delta x_i \Delta y_i$$

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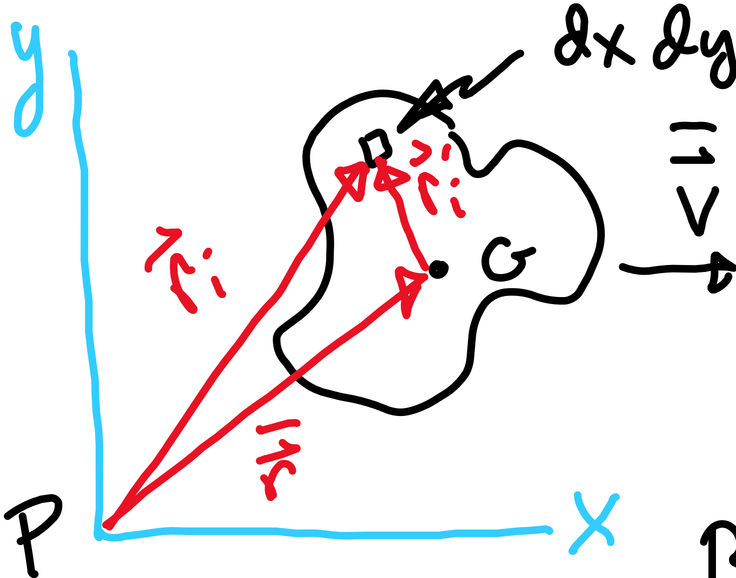
So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,'dx'dy' +$

$\left(\frac{m}{A}\right)\left[\iint \vec{r}\,'dx'dy'\right] \times \bar{\vec{v}} + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' =$

Example: rigid body in plane motion that is translating & rotating.



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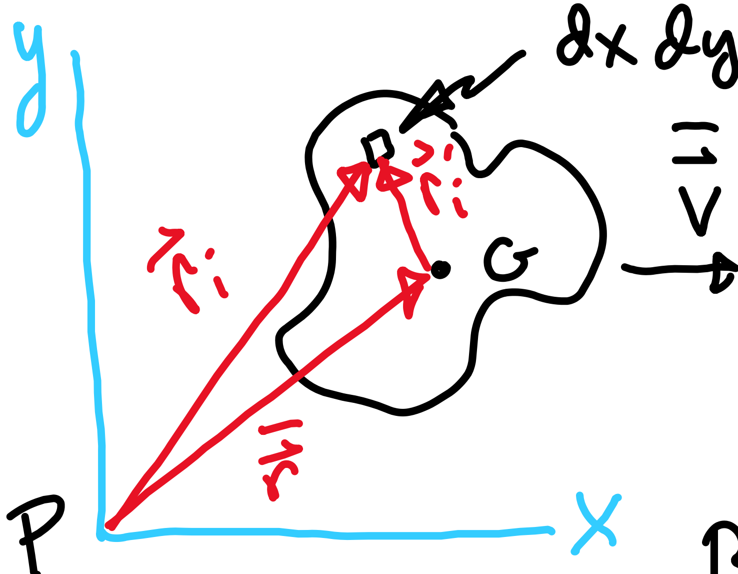
But $\vec r = \bar{\vec r} + \vec r\,'$ & $\vec v = \bar{\vec v} + \vec v\,'$

So
$$\vec H_P = \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \bar{\vec v}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \vec v\,' dx'dy' + \left(\frac{m}{A}\right)\iint \vec r\,' \times \bar{\vec v}\, dx'dy' +$$
$$\left(\frac{m}{A}\right)\iint \vec r\,' \times \vec v\,' dx'dy' = \left(\frac{m}{A}\right)\bar{\vec r} \times \bar{\vec v} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec r} \times \iint \vec v\,' dx'dy' +$$
$$\left(\frac{m}{A}\right)\left[\iint \vec r\,' dx'dy'\right] \times \bar{\vec v} + \left(\frac{m}{A}\right)\iint \vec r\,' \times \vec v\,' dx'dy' = m\,\bar{\vec r} \times \bar{\vec v} +$$

Example: rigid body in plane motion that is translating & rotating.



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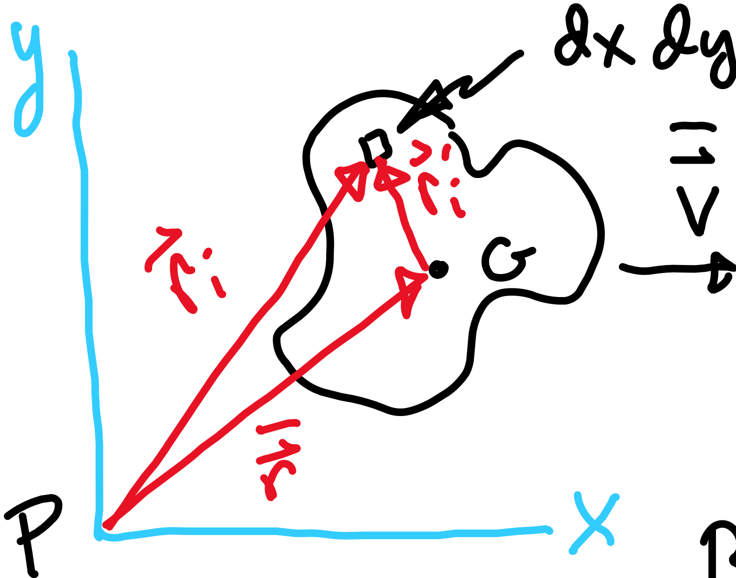
Example: rigid body in plane motion that is translating & rotating.



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$\left(\frac{m}{A}\right)\iint \vec r' \times \vec v'\, dx'dy' = \left(\frac{m}{A}\right)\bar{\vec r} \times \bar{\vec v} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec r} \times \iint \vec v'\, dx'dy' +$
$\left(\frac{m}{A}\right)\left[\iint \vec r'\, dx'dy'\right] \times \bar{\vec v} + \left(\frac{m}{A}\right)\iint \vec r' \times \vec v'\, dx'dy' = m\,\bar{\vec r} \times \bar{\vec v} + 0 + 0 +$

<u>Example</u>: rigid body in plane motion that is translating & rotating.

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$$\vec{H}_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{v}_i\right) \Delta x_i \Delta y_i$$

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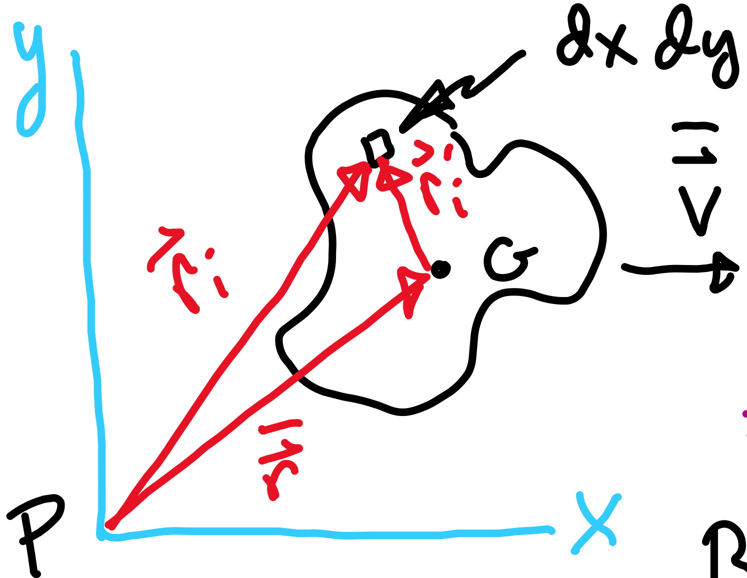
Example: rigid body in plane motion that is translating & rotating.



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$$\Rightarrow \boxed{\vec{H}_P = m\bar{\vec{r}} \times \bar{\vec{v}} + \vec{H}_G}$$

ASU

From previous slide

From previous slide

$$\vec{H}_P = m\vec{r} \times \vec{v} + \vec{H}_G$$

From previous slide

$$\vec{H}_P = m\vec{r}\times\vec{v} + \vec{H}_G \quad \text{so} \quad \dot{\vec{H}}_P = m\frac{d}{dt}\left[\vec{r}\times\vec{v}\right] + \dot{\vec{H}}_G$$

From previous slide

$\vec{H}_P = m\bar{\vec{r}} \times \bar{\vec{v}} + \vec{H}_G$ so $\dot{\vec{H}}_P = m\frac{d}{dt}\left[\bar{\vec{r}} \times \bar{\vec{v}}\right] + \dot{\vec{H}}_G$

$= m\bar{\vec{v}} \times \bar{\vec{v}} +$

From previous slide

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$$= m\vec{v}\times\vec{v} + m\,\vec{r}\times\vec{a} +$$

From previous slide

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ASU

From previous slide

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From previous slide

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then

From previous slide

$$\vec{H}_P = m\bar{\vec{r}}\times\bar{\vec{v}} + \vec{H}_G \quad \text{so} \quad \dot{\vec{H}}_P = m\frac{d}{dt}\left[\bar{\vec{r}}\times\bar{\vec{v}}\right] + \dot{\vec{H}}_G$$

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then $\sum \vec{M}_P = \dot{\vec{H}}_P$ becomes

From previous slide

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From previous slide

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For this case

From previous slide

$\vec{H}_P = m\bar{\vec{r}}\times\bar{\vec{v}} + \vec{H}_G$ so $\dot{\vec{H}}_P = m\frac{d}{dt}\left[\bar{\vec{r}}\times\bar{\vec{v}}\right] + \dot{\vec{H}}_G$

$= \cancel{m\bar{\vec{v}}\times\bar{\vec{v}}} + m\bar{\vec{r}}\times\bar{\vec{a}} + \dot{\vec{H}}_G$ & since $\dot{\vec{H}}_G = \bar{I}\vec{\alpha}$

then $\sum \vec{M}_P = \dot{\vec{H}}_P$ becomes

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For this case our moments [torques]

From previous slide

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For this case our moments [torques] can cause rotational motion about the center-of-mass G

From previous slide

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$$= \cancelto{0}{m\bar{\vec{v}}\times\bar{\vec{v}}} + m\bar{\vec{r}}\times\bar{\vec{a}} + \dot{\vec{H}}_G \quad \text{\& since} \quad \dot{\vec{H}}_G = \bar{I}\vec{\alpha}$$

then $\sum \vec{M}_P = \dot{\vec{H}}_P$ becomes

$$\boxed{\sum \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{\vec{r}}\times\bar{\vec{a}}}$$

For this case our moments [torques] can cause rotational motion about the center-of-mass G & linear acceleration of the body

Remember,

Remember, for our car problem (16.3),

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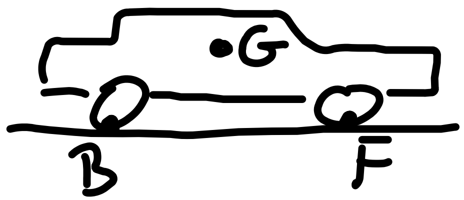

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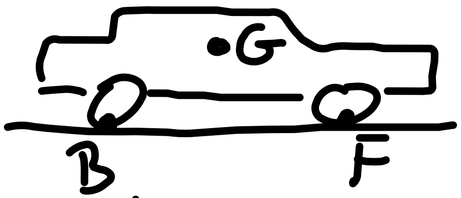

we could determine the acceleration of the car using sum of torques about points B or F while taking $\alpha_{car} = 0$.

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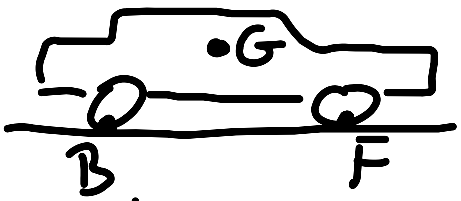

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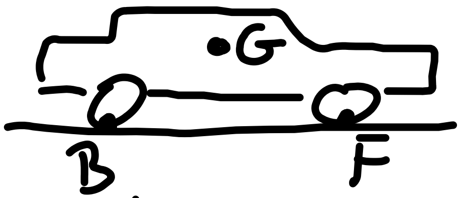

we could determine the acceleration of the car using sum of torques about points B or F while taking $\alpha_{car} = 0$. For a sum about B, we end up with $\Sigma \vec{M}_B = m\vec{r}_{G/B} \times \vec{a}$

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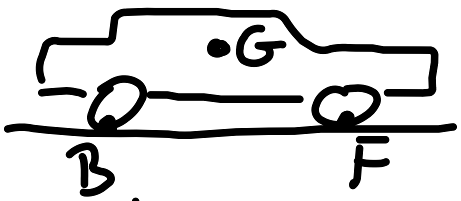

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When we have a fixed point of rotation about some point P,

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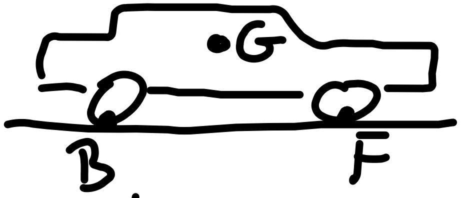

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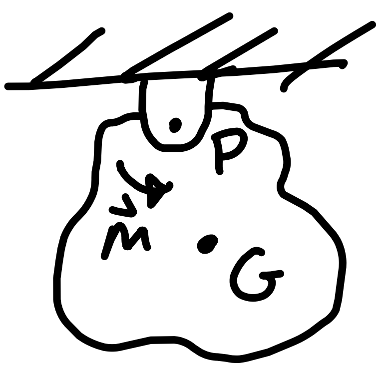

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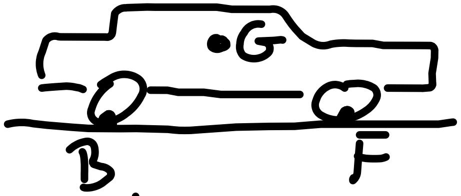
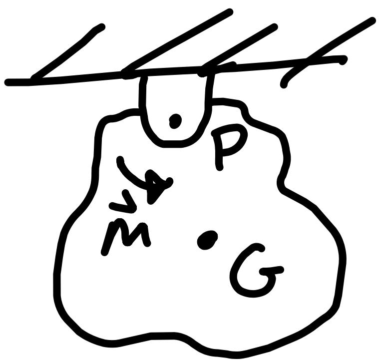


we could determine the acceleration of the car using sum of torques about points B or F while taking $\alpha_{car} = 0$. For a sum about B, we end up with $\Sigma \vec{M}_B = m \vec{r}_{G/B} \times \vec{a}$

When we have a fixed point of rotation about some point P, we can modify our expression to read



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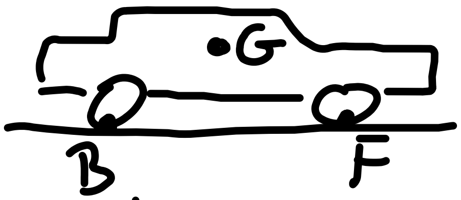

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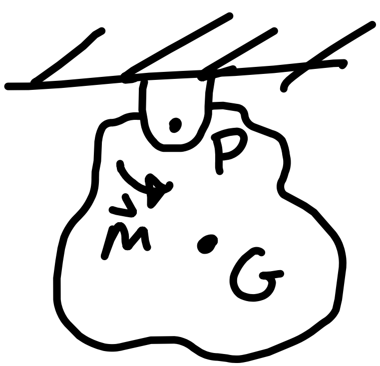

$$\Sigma \vec{M}_P = I_P \vec{\alpha},$$

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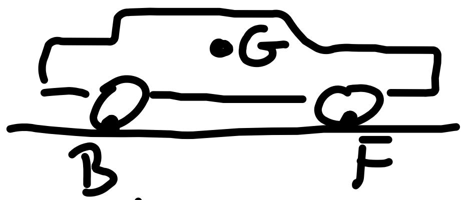

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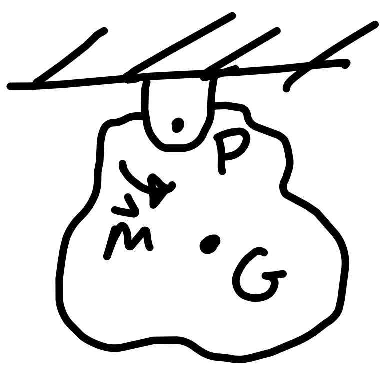

$$\Sigma \vec{M}_P = I_P \vec{\alpha}, \text{ where}$$

$$I_P = \bar{I} + m r_{G/P}^2$$

Work and energy

Work and energy

$U_{1\to 2} = T_2 - T_1$,

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$U_{1\to 2} = T_2 - T_1$, where $U_{1\to 2} \equiv$ Work on object from position 1 to position 2

Work and energy

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Work and energy

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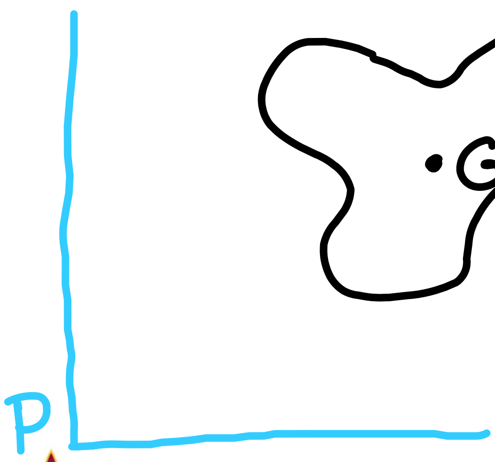

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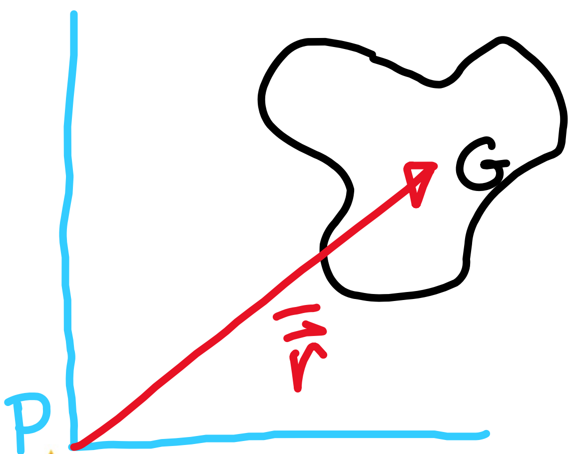

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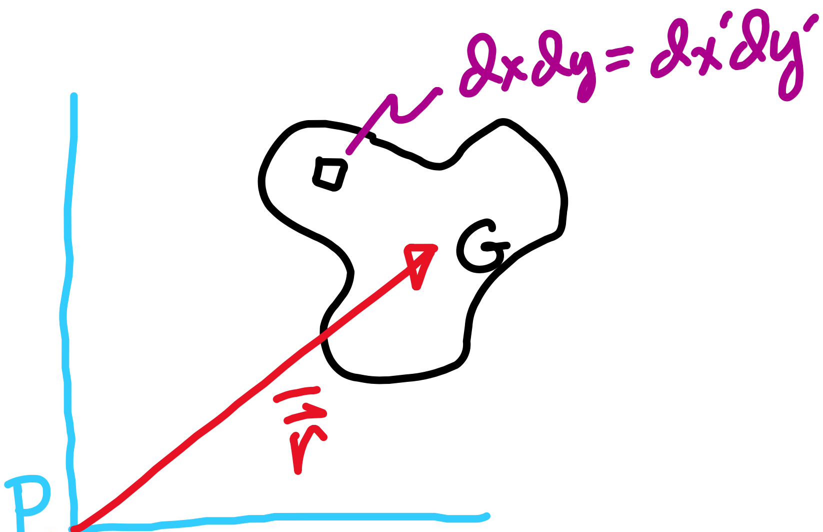

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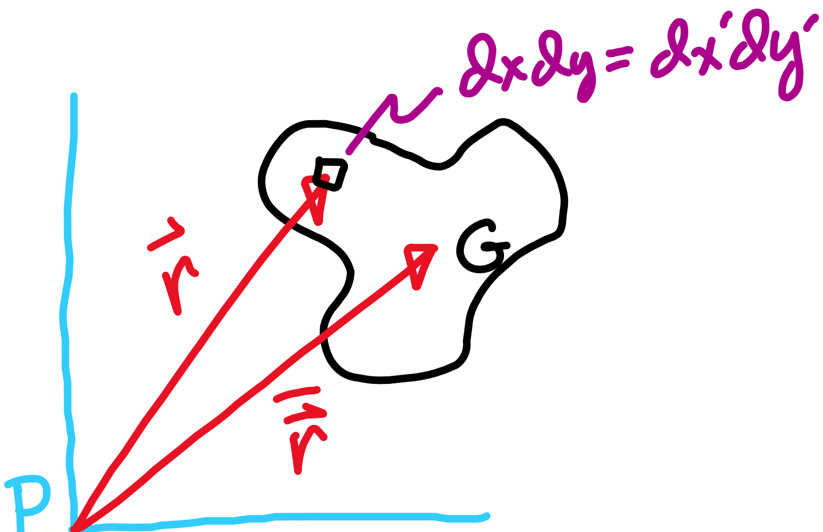

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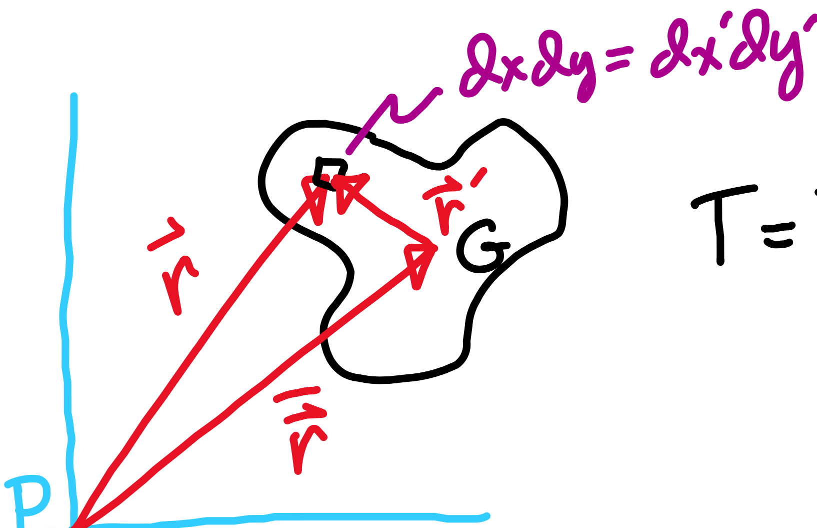

$$T = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{1}{2}\left(\frac{m}{A}\right)\sum v_i^2 \Delta x_i \Delta y_i$$

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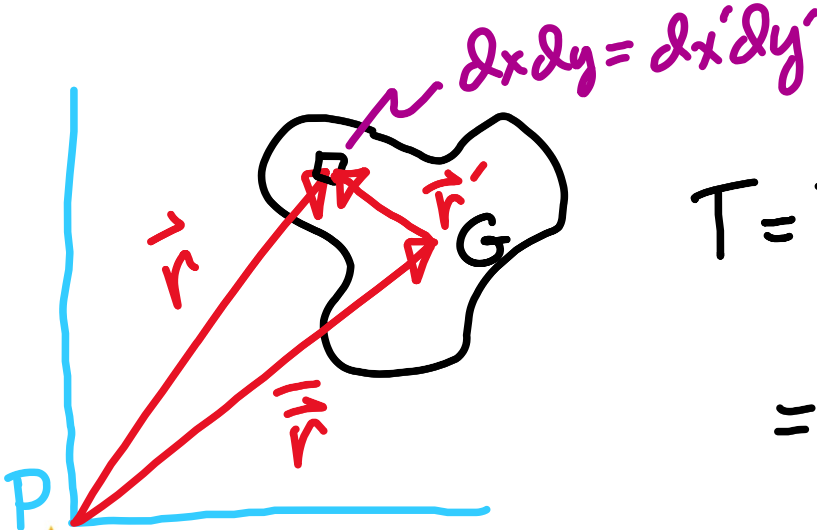

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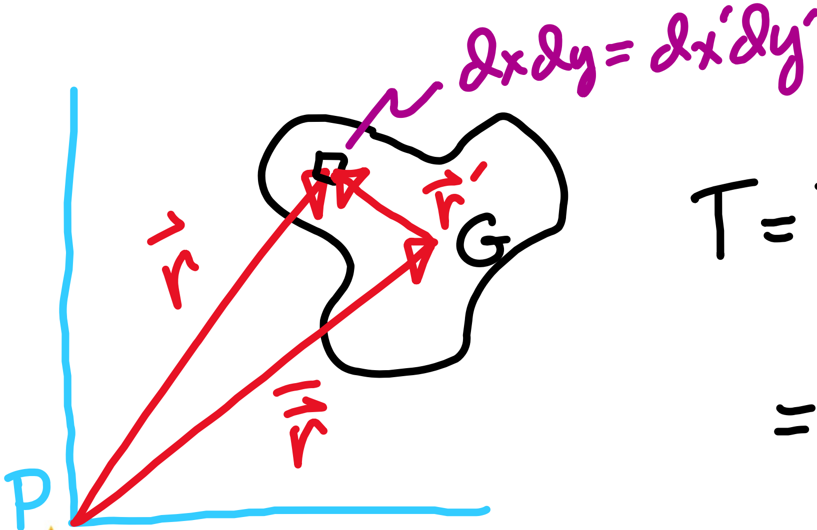

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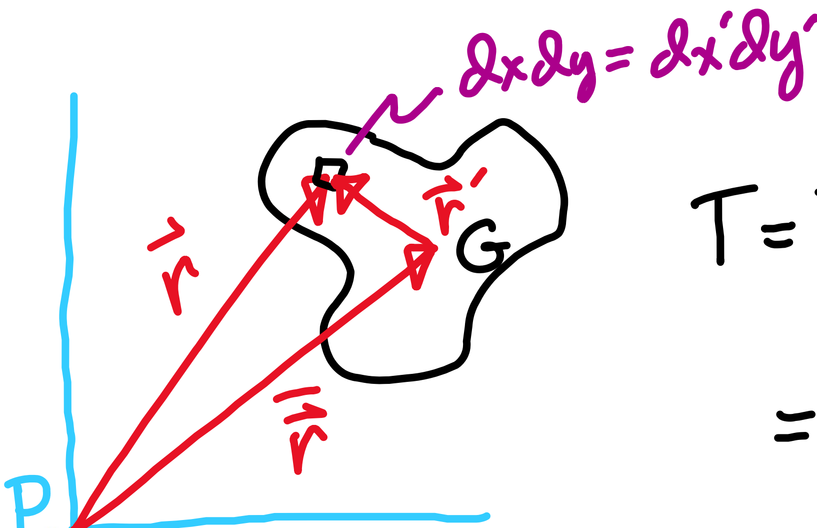

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From previous slide $T = \frac{\mu}{A} \iint v^2 \, dx' \, dy'$

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$$\vec{V} = \bar{V} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{V} + \vec{V}') \cdot (\bar{V} + \vec{V}')$$

From previous slide $T = \frac{M}{A}\int\int \frac{1}{2} V^2 \, dx' \, dy'$ &

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From previous slide $T = \rho_\infty \iint \frac{1}{2} V^2 \, dx' dy'$ &

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$$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}' \sim 0$$

$$T = \left(\frac{1}{2}\rho_\infty\right) \iint \left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'\right) dx' dy'$$

From previous slide $T = \frac{\rho}{A}\iint \frac{1}{2} V^2 \, dx' \, dy'$ &

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Constant over $dx'dy'$ integration

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx'\, dy'$ &

$\vec{V} = \vec{\bar{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\vec{\bar{V}} + \vec{V}') \cdot (\vec{\bar{V}} + \vec{V}') \Rightarrow$

$V^2 = \bar{V}^2 + V'^2 + 2\vec{\bar{V}} \cdot \vec{V}'$ so

$T = \left(\frac{1}{2}\frac{m}{A}\right) \iint (\bar{V}^2 + V'^2 + 2\vec{\bar{V}} \cdot \vec{V}') \, dx'\, dy' \Rightarrow$

$T = \left(\frac{1}{2}\frac{m}{A}\right) \bar{V}^2 \iint dx'\, dy' +$

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$$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint\left(\bar{V}^2 + V'^2 + 2\vec{\bar{V}}\cdot\vec{V}'\right) dx' dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{V}^2 \iint dx' dy' + \left(\frac{m}{A}\right)\iint V'^2 \, dx' dy'$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx' dy'$ &

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From previous slide $T=\frac{m}{A}\iint \frac{1}{2} v^2\, dx'\, dy'$ &

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$$\Rightarrow T = \frac{1}{2} m \bar{V}^2 + \left(\frac{m}{A}\right)\iint V'^2 \, dx' dy'$$

From previous slide $T = \frac{M}{A}\iint \frac{1}{2} v^2 \, dx' dy'$ &

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we can rewrite $v'^2 = r'^2 \dot{\theta}^2$

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$$= \frac{1}{2}\left[m r_{G/P}^2 + \bar{I}\right]\omega^2$$

ASU

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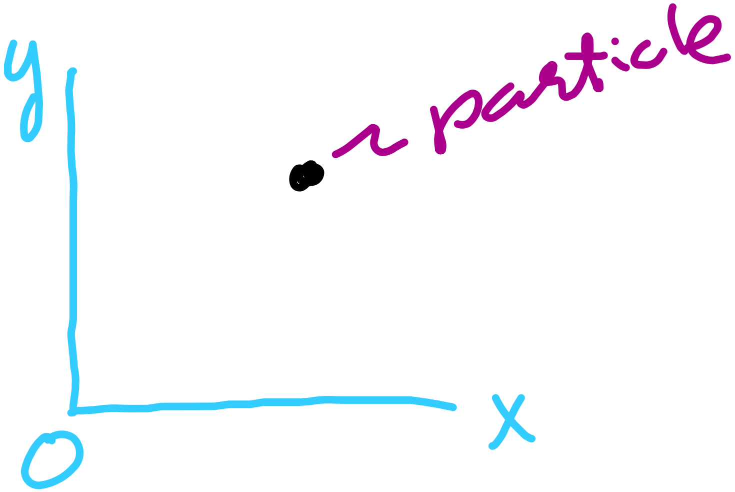
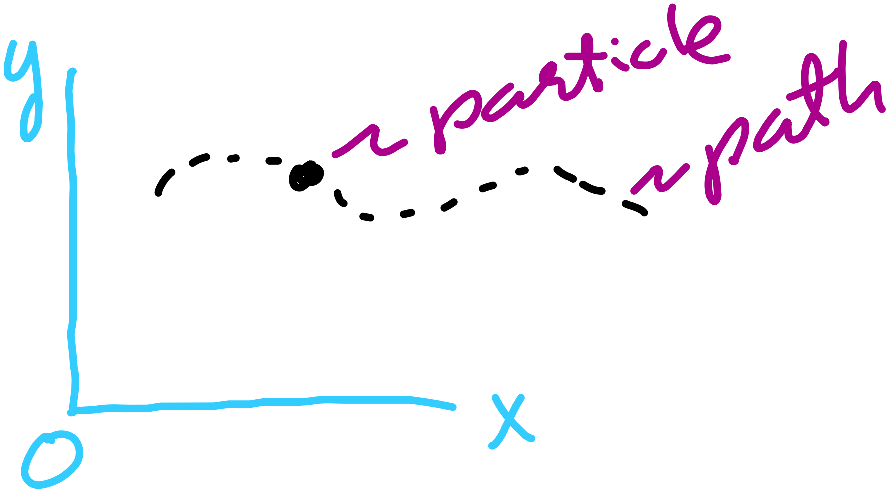

particle
y
x
O


y
particle
path
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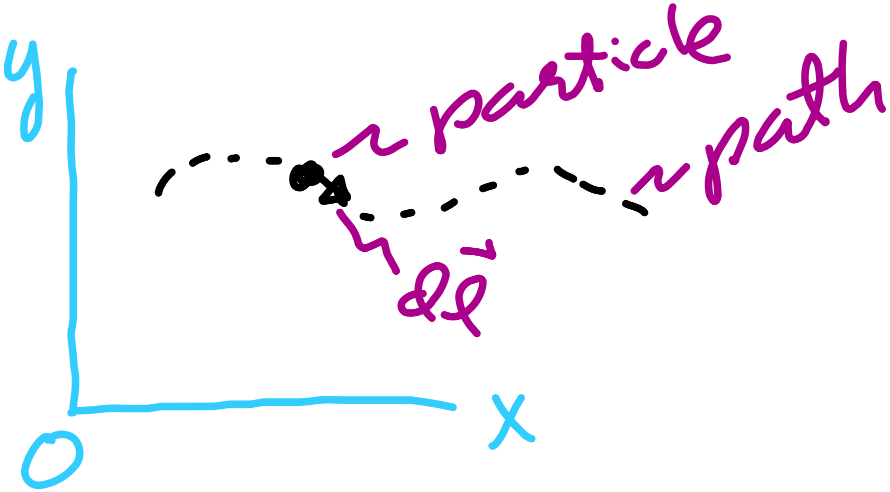
y
particle
path
$d\vec{\ell}$
O
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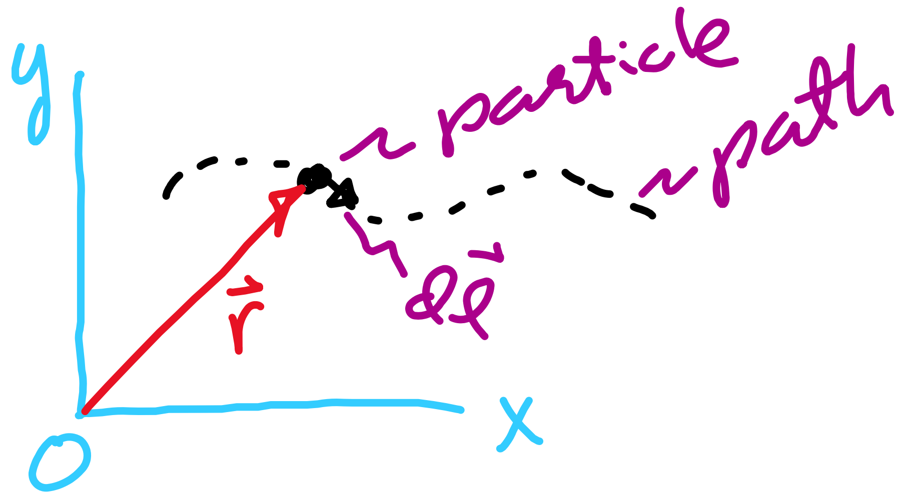
y
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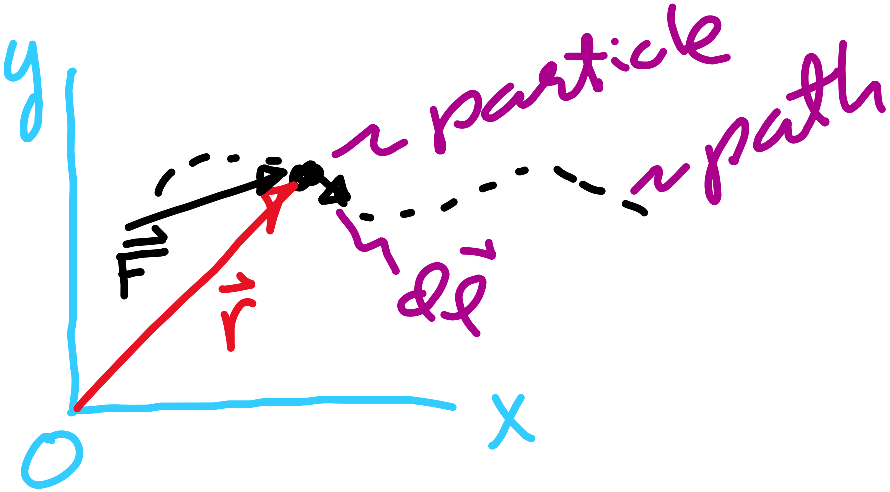
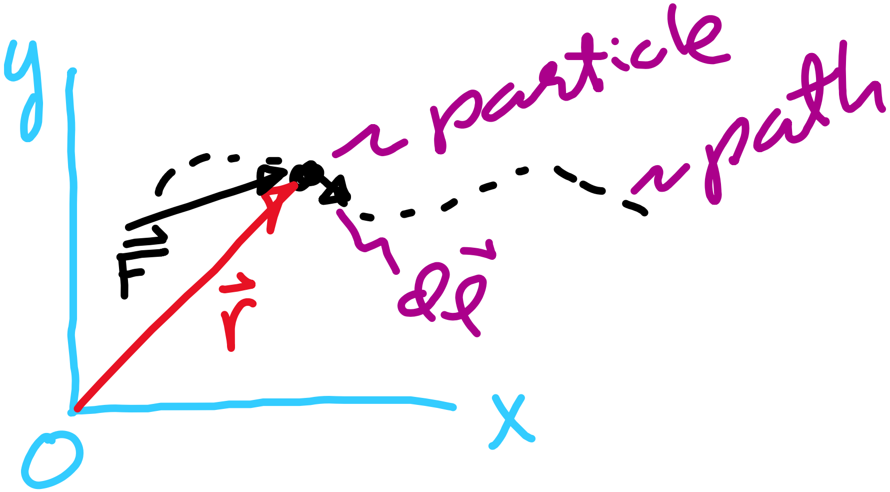
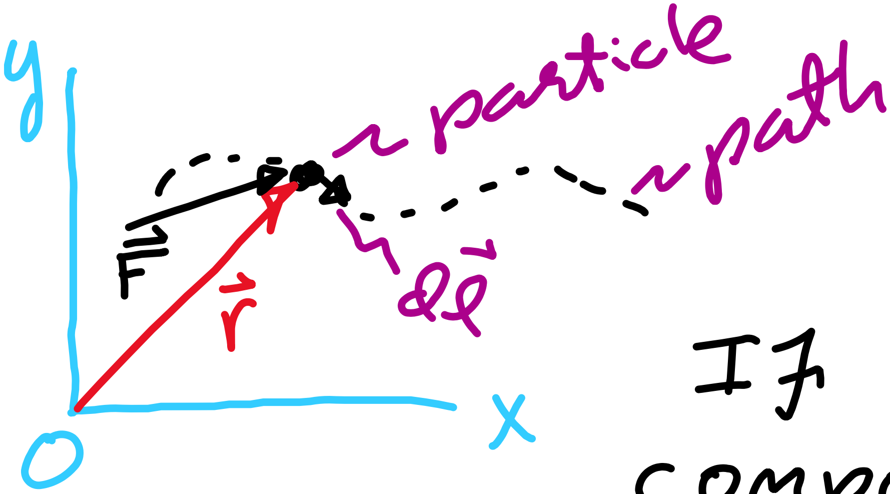
O
particle
path
$\vec{r}$
$d\vec{\ell}$



y
particle
path
$\vec{F}$
$\vec{r}$
$d\vec{\ell}$
O
x



A particle follows some path.



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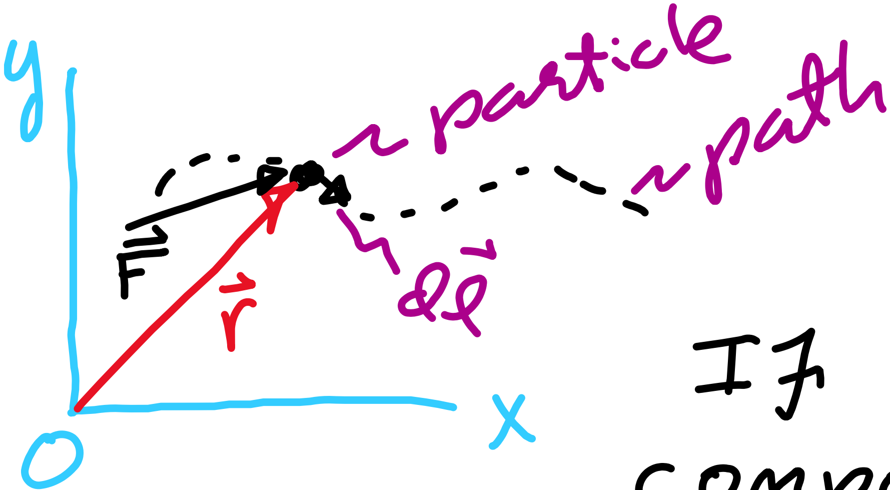
If we break up components in polar coordinates



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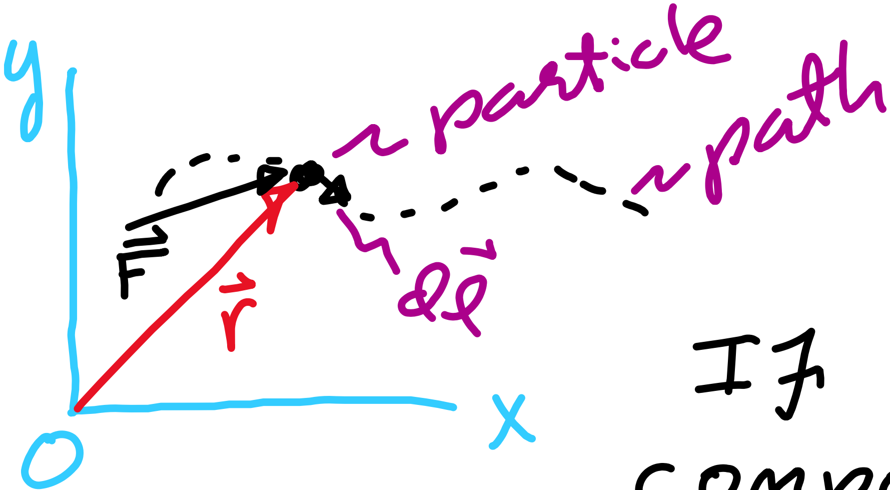

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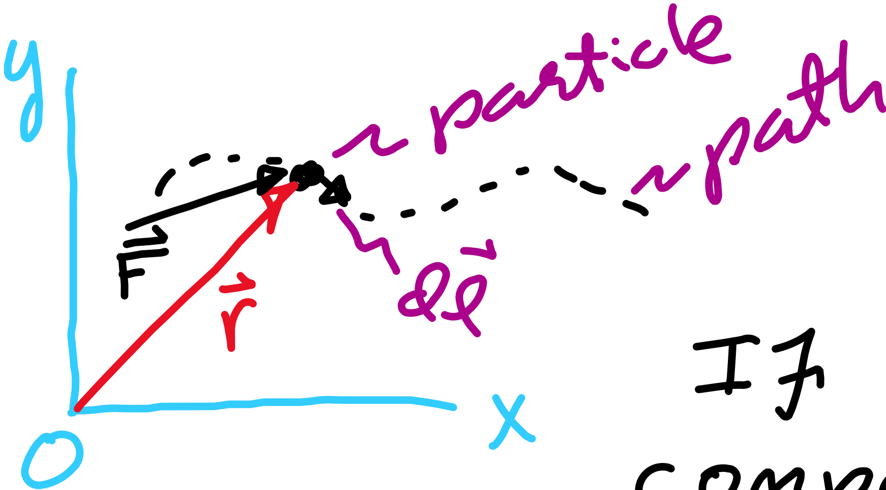

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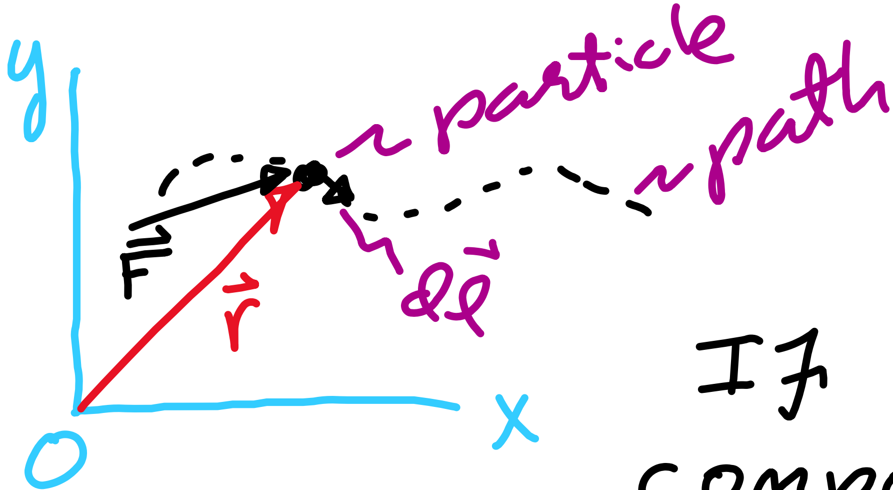
ASU



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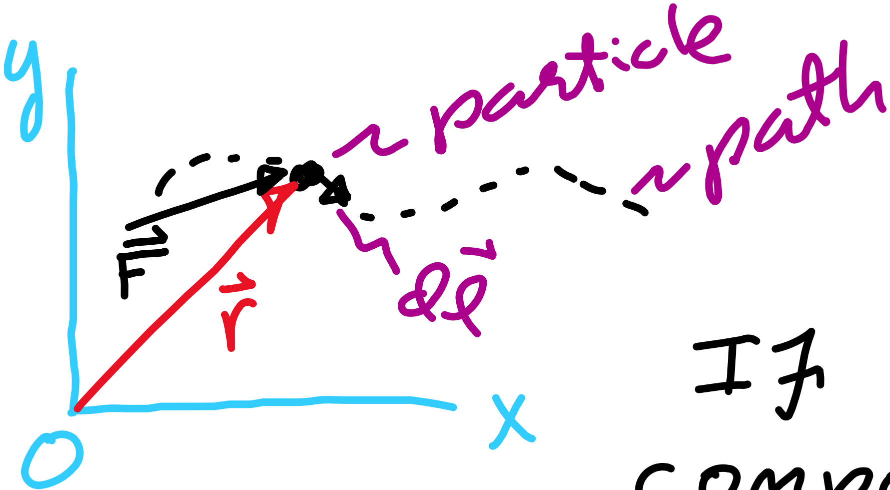

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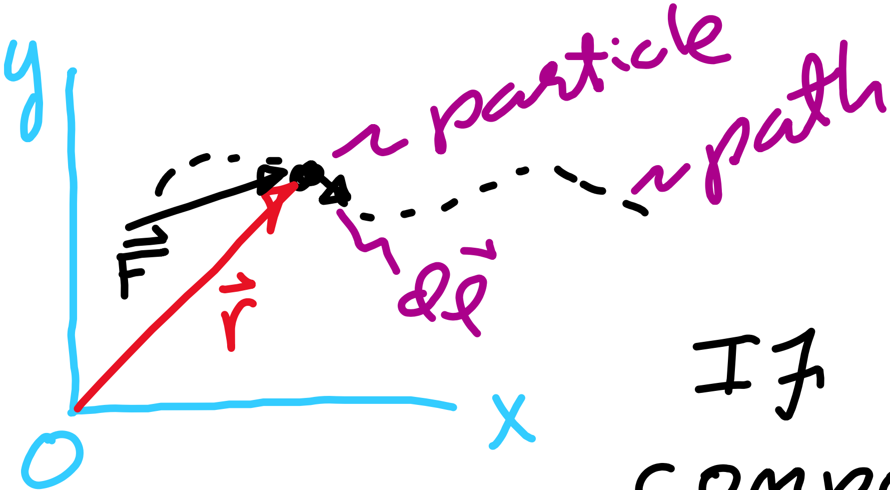

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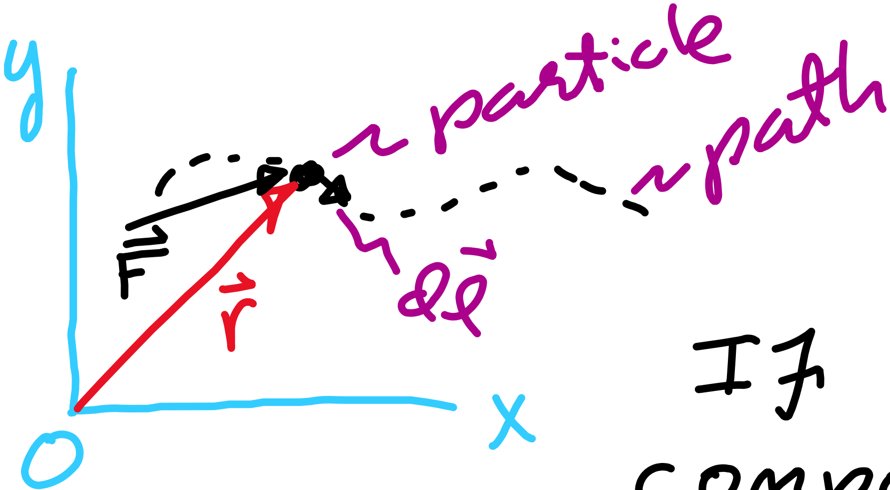

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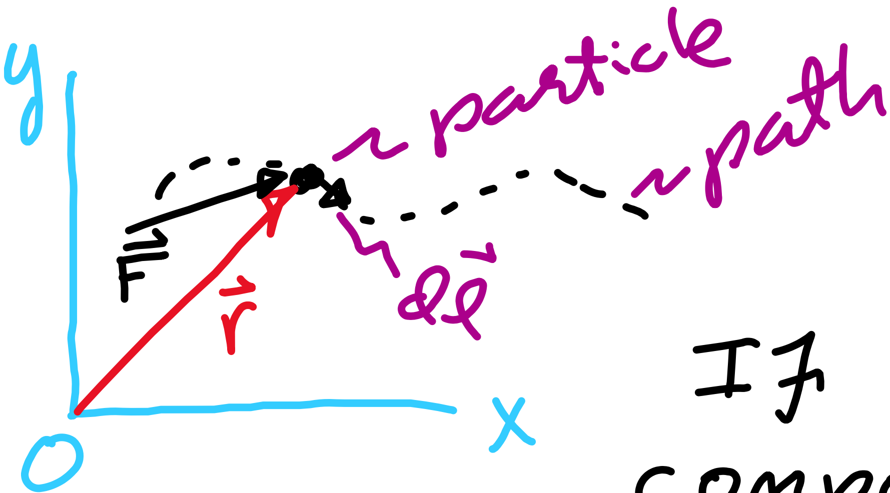

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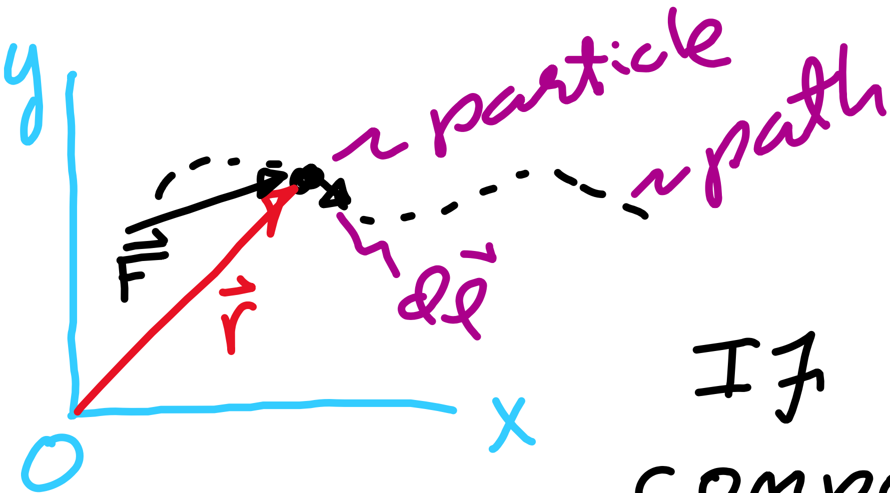

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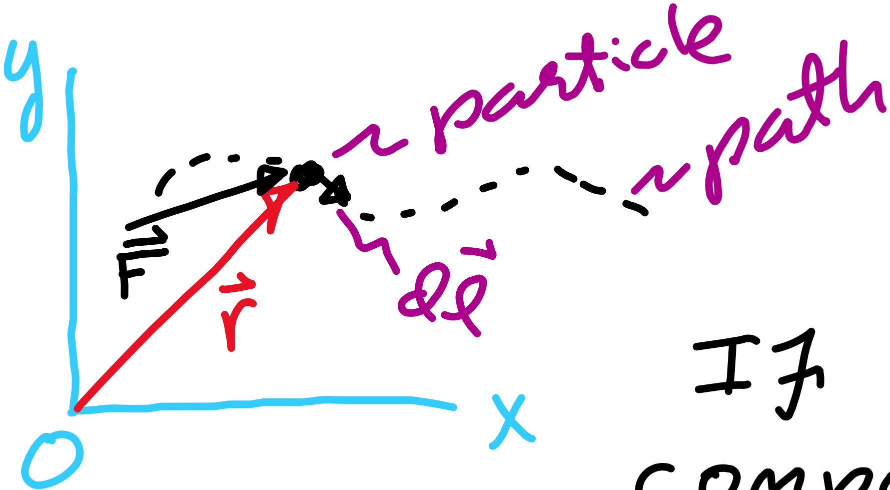

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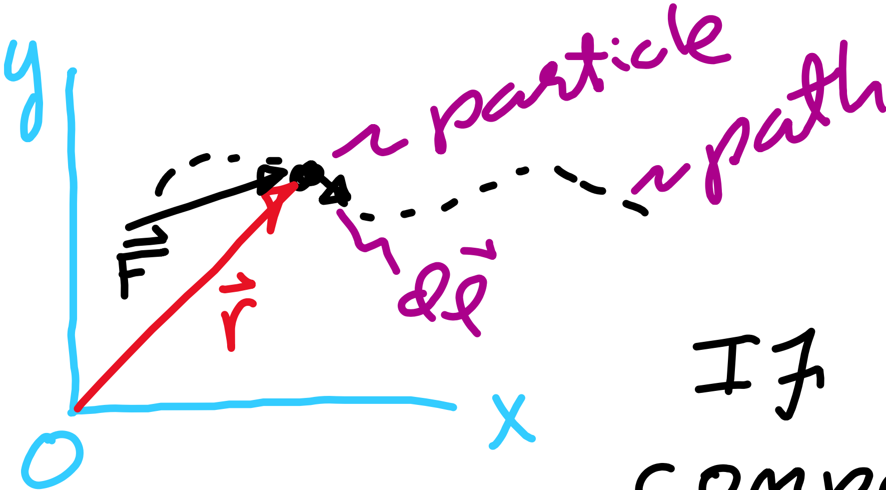

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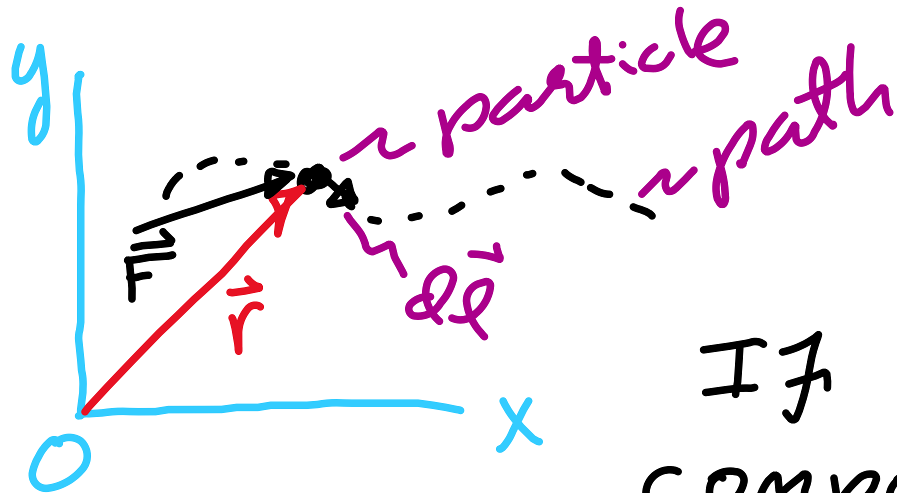
Part of work <u>NOT</u> associated with torque about point O



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$$U_{1\to2} = \underbrace{\int_{r_1}^{r_2} F_r\,dr}_{\text{Part of work NOT associated with torque about point O}} + \boxed{\int_{\theta_1}^{\theta_2} M_O\,d\theta} \rightarrow \text{Part of work associated with torque about O}$$

Part of work __NOT__ associated with torque about point O

Part of work associated with torque about O

For torques (moments) about point O

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In summary

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