Today 17.1 & 17.2

$\boxed{L28}$

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$\boxed{L28}$

Energy methods for rigid bodies

Today 17.1 & 17.2

$\boxed{L28}$

Energy methods for rigid bodies

Momentum methods

Today 17.1 & 17.2

$\boxed{L28}$

Monday 17.2

Today 17.1 & 17.2

$\boxed{L28}$

Monday 17.2

Wednesday Review

Today 17.1 & 17.2 $\boxed{\text{L28}}$

Monday 17.2

Wednesday Review

Friday Nov 6th Exam #3

We found that no work was performed by static friction for wheel rolling without slipping

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$$U = \int \vec{F}_F \cdot d\vec{l}_c = \int \vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\, dt \Rightarrow$$

$$U = \int \vec{F}_F \cdot \vec{V}_c\, dt$$

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$$U = \int \vec{F}_F \cdot \vec{V}_c\, dt$$ But roll No slip

$$\Rightarrow \quad \vec{V}_c = \vec{V}_{surface}$$

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$$U = \int \vec{F}_F \cdot \vec{V}_c\, dt$$ But roll No slip

$$\Rightarrow \vec{V}_c = \vec{V}_{surface} = 0$$

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$\Rightarrow \vec{V}_c = \vec{V}_{surface} = 0$ So $U = 0$

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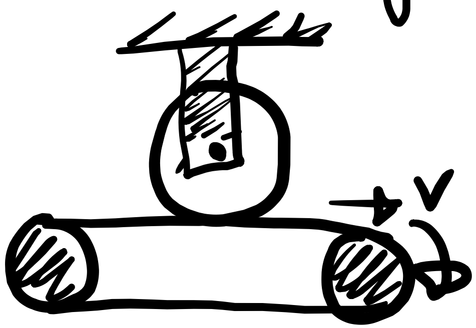

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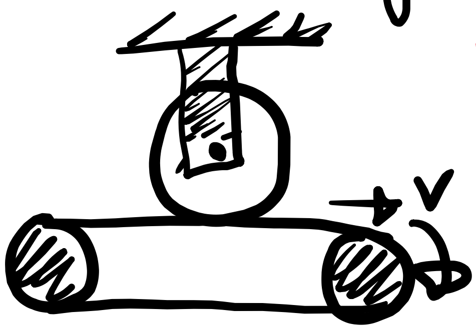

What about case where surface is moving?

We found that No work was performed by static friction for wheel rolling without slipping **On fixed surface**

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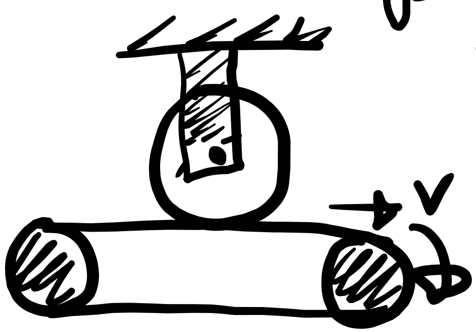

$$\vec{V}_c\neq 0$$

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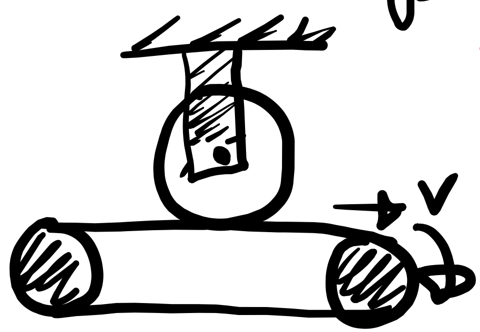

$\vec{v}_c \neq 0$ &

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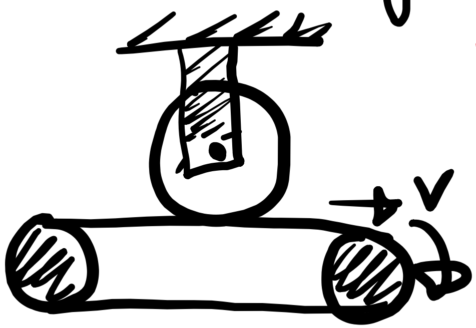

What about case where surface is moving? Here

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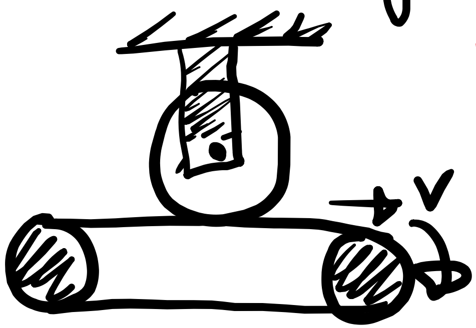

What about case where surface is moving? Here

$\vec{V}_c \neq 0$ &

$$U = \int \vec{F}_F \cdot \vec{V}\, dt \neq 0$$

work performed by this frictional force

Last time we saw that

$$T_I + V_I + U^{NC}_{I \to F} = T_F + V_F$$

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$$T_I + V_I + U^{NC}_{I\to F} = T_F + V_F, \text{ where } T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}M\bar{v}^2$$

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$T_I + V_I + U^{NC}_{I \to F} = T_F + V_F$, where $T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}M\bar{v}^2$

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$T = \frac{1}{2}I_P\omega^2$

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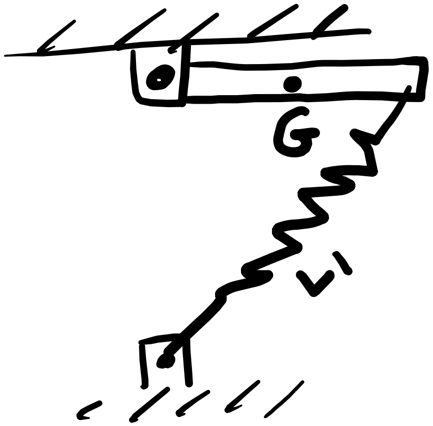
$$T_I + V_I + U^{NC}_{I\to F} = T_F + V_F, \text{ where } T = \tfrac{1}{2}\bar{I}\omega^2 + \tfrac{1}{2}m\bar{v}^2$$

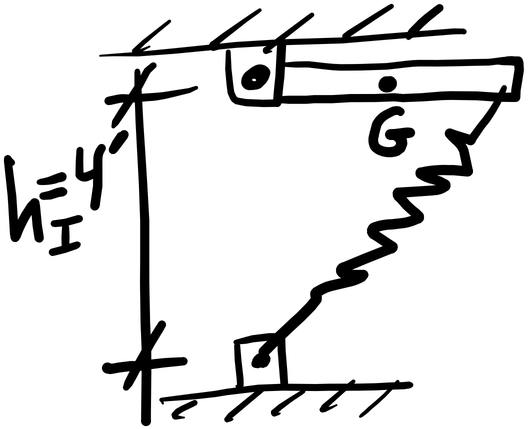
and if there is a fixed point of rotation about P, we could write

$$T = \tfrac{1}{2}I_P\omega^2, \text{ where } I = \bar{I} + m r_{G/P}^2$$

Example : Find K.E. when rod is vertical



Example : Find K.E. when rod is vertical



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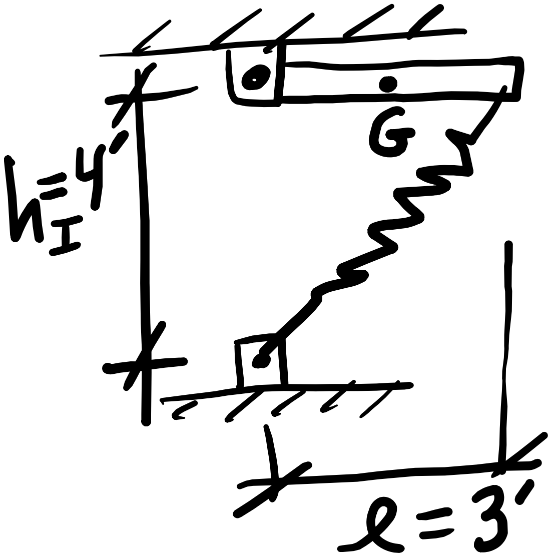

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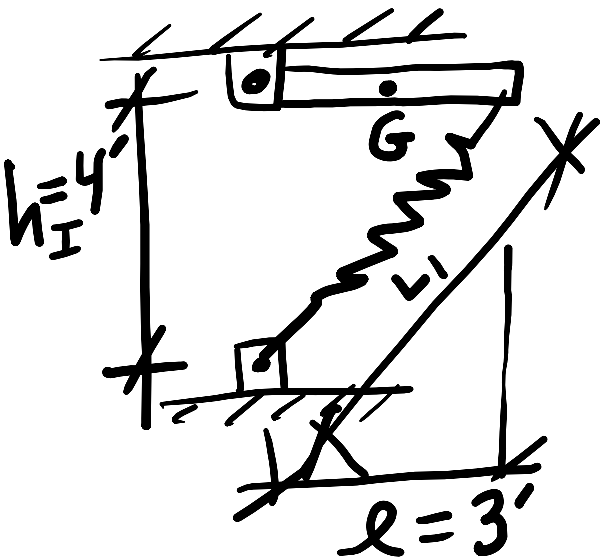

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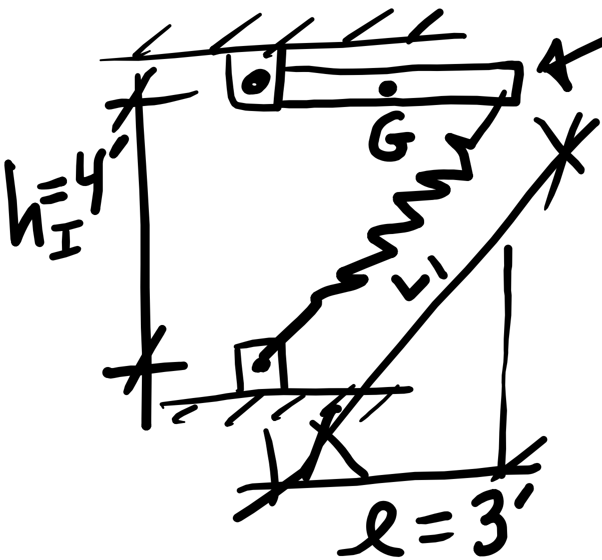
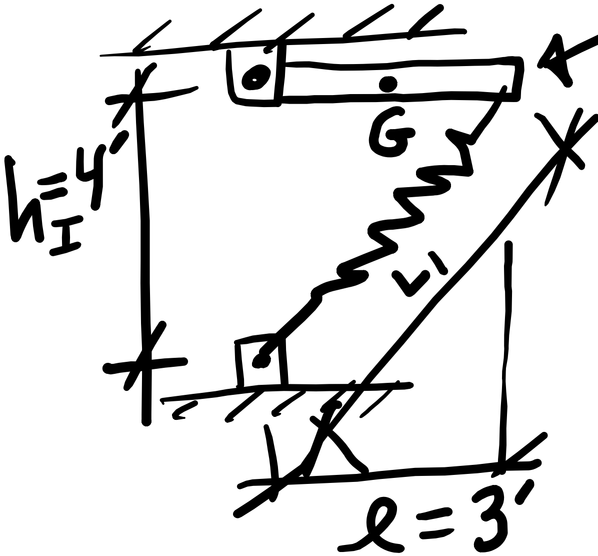


Slender rod of weight $w = 2\,lb$ rotates about fixed point A.

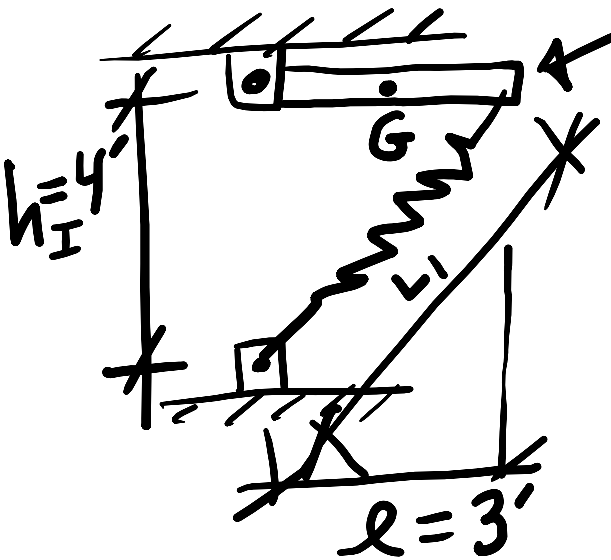
Example: Find K.E. when rod is vertical



Slender rod of weight $w = 2\,lb$ rotates about fixed point A.

Natural length of spring $L_0 = 2\,ft$

Example: Find K.E. when rod is vertical

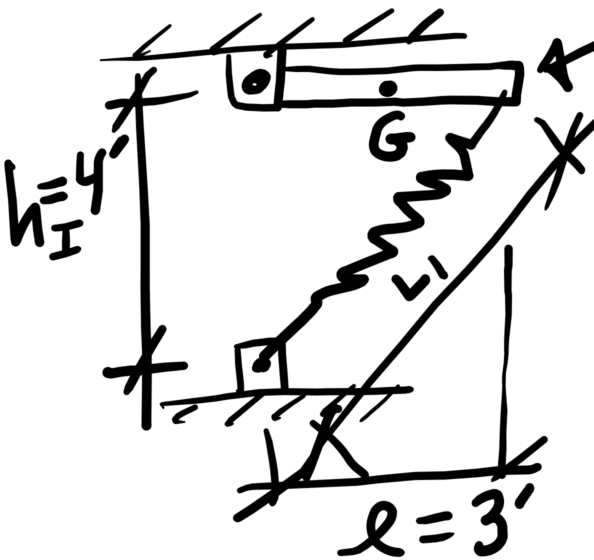


Slender rod of weight $w = 2\,lb$ rotates about fixed point A.

Natural length of spring $L_0 = 2\,ft$

Spring constant $k = 0.75\,lb/ft$

Example: Find K.E. when rod is vertical



Slender rod of weight $w = 2\,lb$ rotates about fixed point A. Natural length of spring $L_0 = 2\,ft$. Spring constant $k = 0.75\,lb/ft$. No friction.

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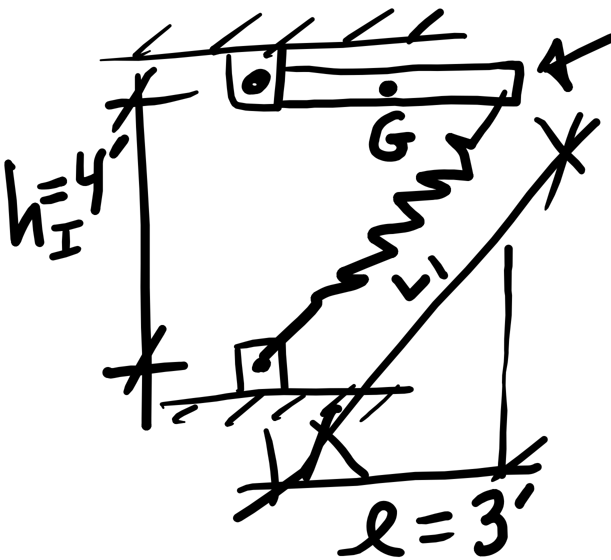
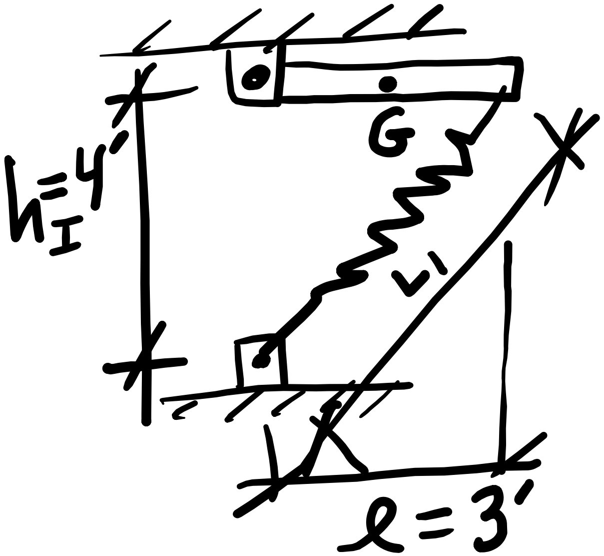


Slender rod of weight $w = 2\,lb$ rotates about fixed point A. Natural length of spring $L_0 = 2\,ft$. Spring constant $k = 0.75\,lb/ft$. No friction. Starts at rest

Example: Find K.E. when rod is vertical

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$



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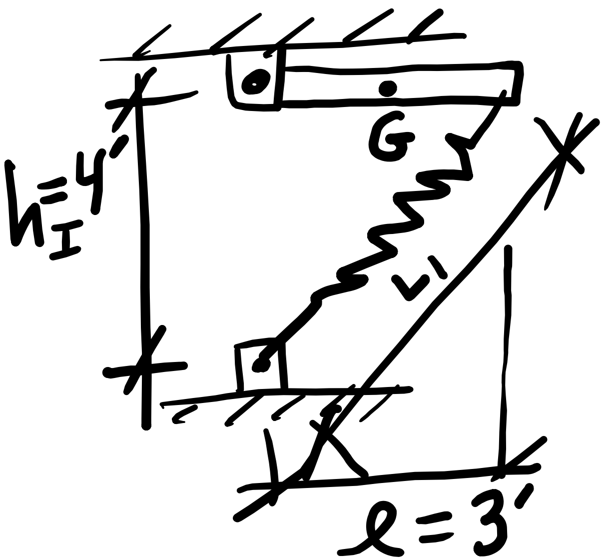
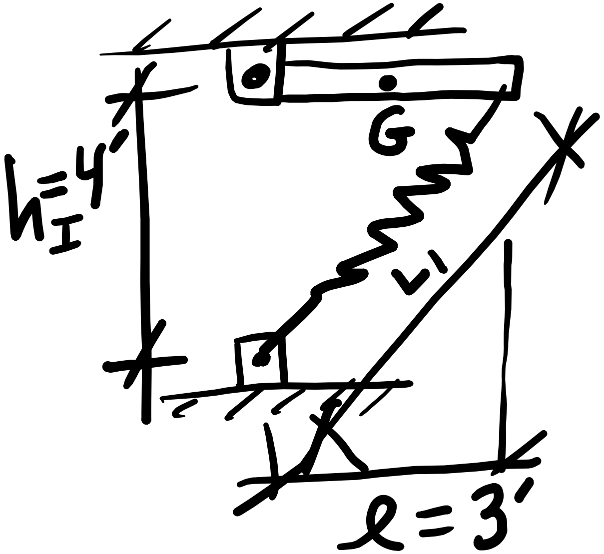
Example: Find K.E. when rod is vertical



$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

$$T_1 + V_1 + U_{1\to 2}^{nc} = T_2 + V_2$$

Example : Find K.E. when rod is vertical



$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$

$\cancel{T_1}^{0} + V_1 + U_{1\to 2}^{nc} = T_2 + V_2$

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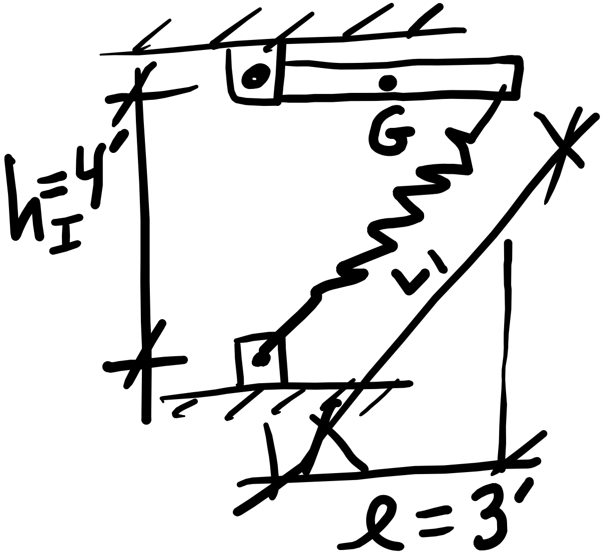
$W = 2\,lb,\; L_0 = 2\,ft,\; k = 0.75\,lb/ft$



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to2}^{nc}} = T_2 + V_2$$

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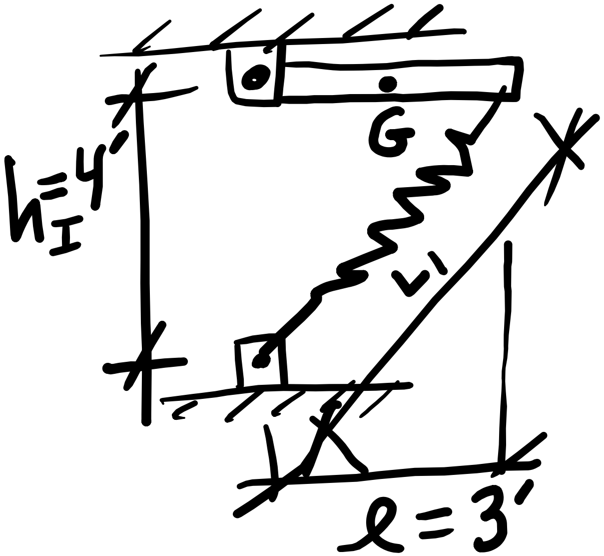

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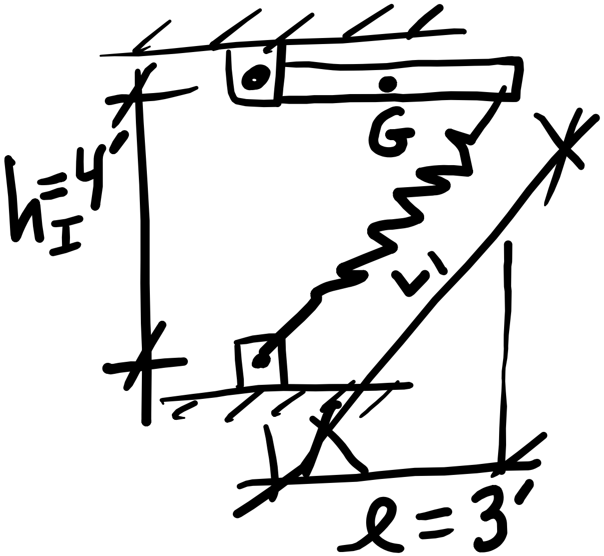

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$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{nc}}^{0} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2,$

where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$

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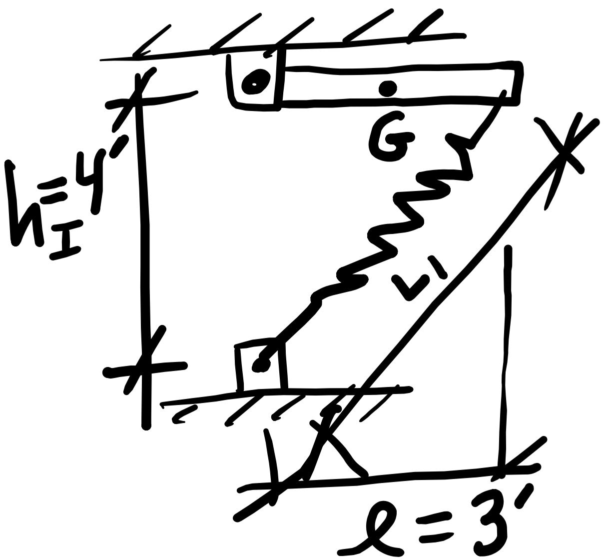

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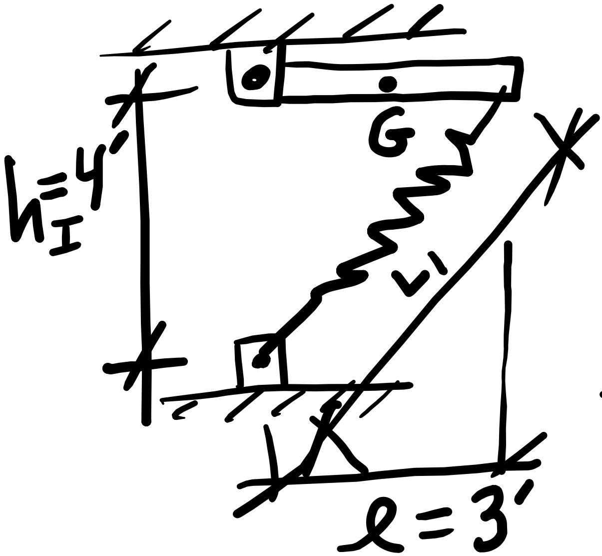

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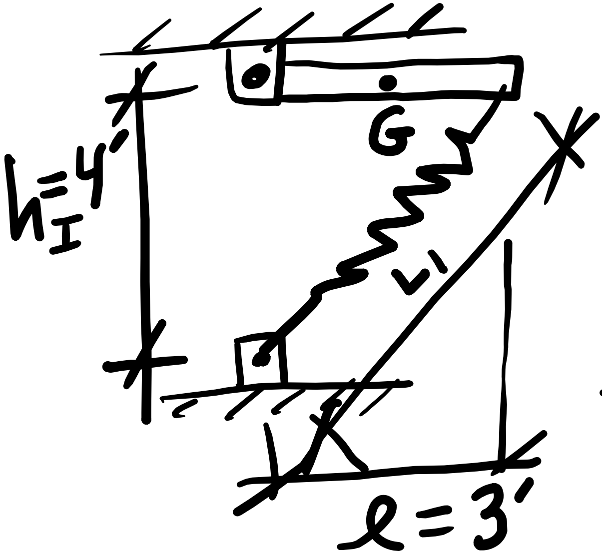
where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$ & $V_2 = \frac{1}{2}kx_2^2 + mgy_2$

$$\Rightarrow T_2 = \frac{1}{2}k(x_1^2 - x_2^2) + mg(y_1 - y_2)$$

Example: Find K.E. when rod is vertical



$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

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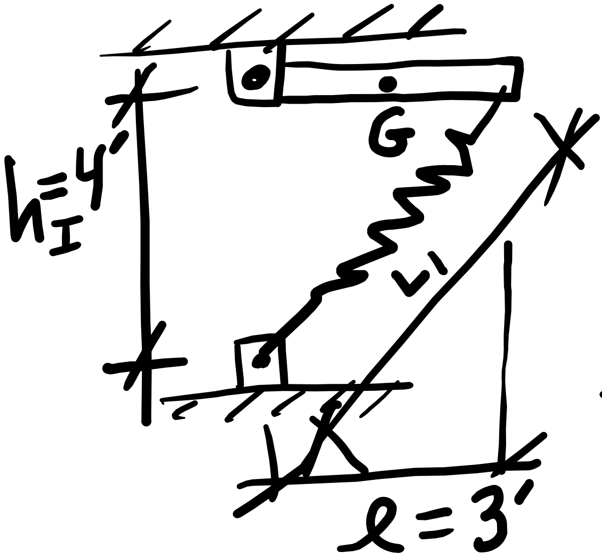
$y_1 - y_2 = \ell/2 = \frac{3}{2}\,ft$ & $L_1 = L_0 + x_1$ &

$L_1 = \sqrt{3^2 + 4^2}\,ft = 5\,ft$

Example: Find K.E. when rod is vertical



$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

$\cancel{T_1} + V_1 + \cancel{U_{1\to 2}^{nc}} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2$,

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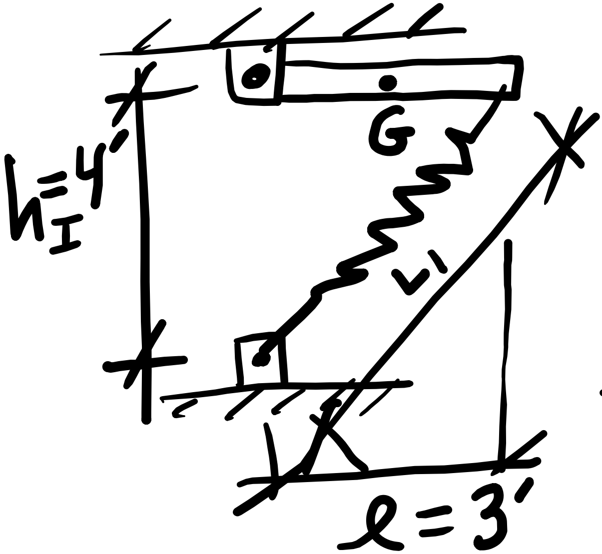
Example: Find K.E. when rod is vertical

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$



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$L_2 = h - l = (4-3)\,ft = 1\,ft$

Example: Find K.E. when rod is vertical

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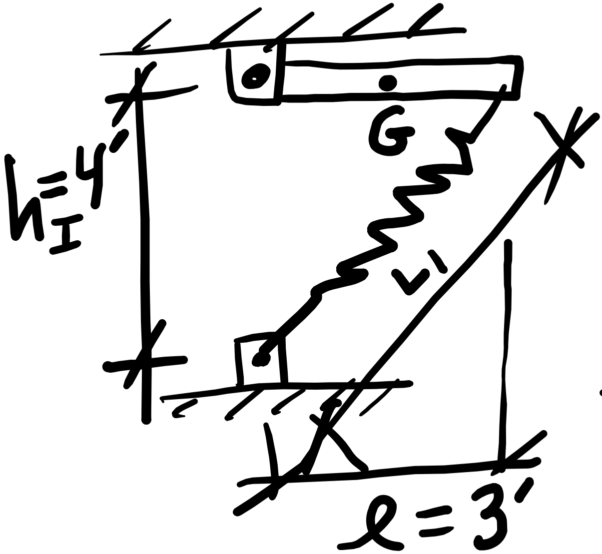

$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2,$$

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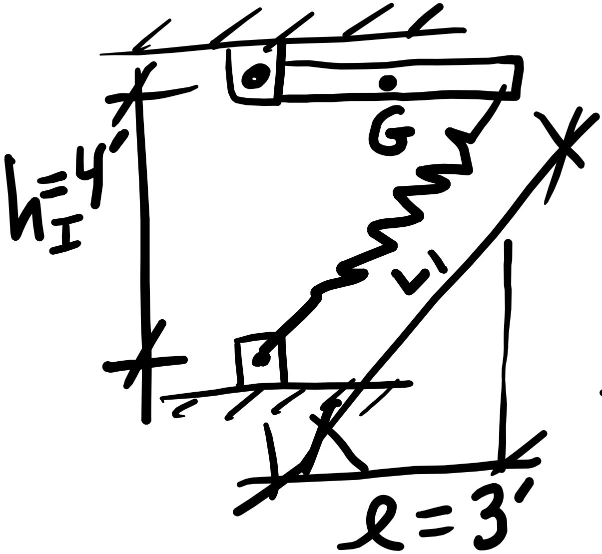
Example : Find K.E. when rod is vertical



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$\Rightarrow T_2 = \frac{1}{2}k(x_1^2 - x_2^2) + mg(y_1 - y_2)$, here

$y_1 - y_2 = l/2 = \frac{3}{2}\,ft$ & $L_1 = L_0 + x_1$ &

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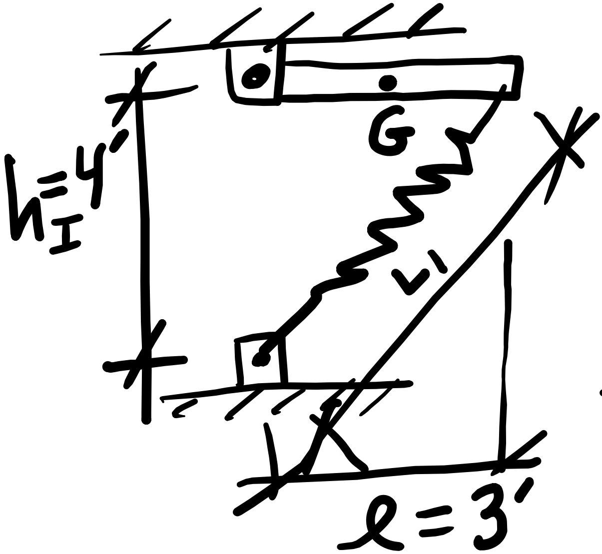
So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right]lb\cdot ft$

Example: Find K.E. when rod is vertical

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$



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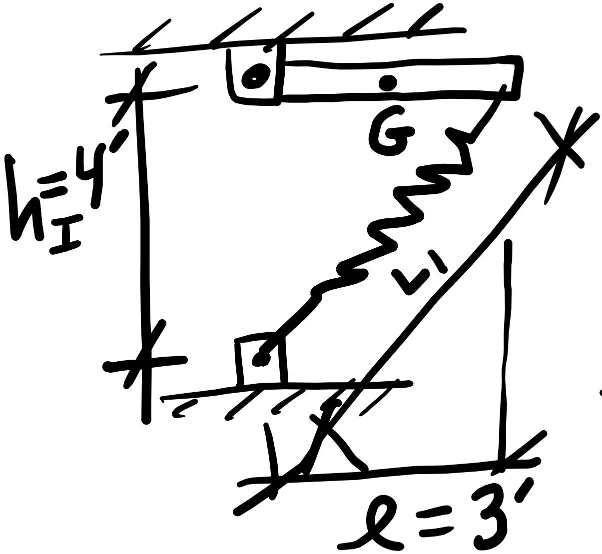
$L_1 = \sqrt{3^2 + 4^2}\,ft = 5\,ft \Rightarrow x_1 = L_1 - L_0 = 5\,ft - 2\,ft = 3\,ft$ &

$L_2 = h - l = (4-3)\,ft = 1\,ft \Rightarrow x_2 = L_2 - L_0 = (1-2)\,ft = -1\,ft$

So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb\cdot ft = \left[\frac{3}{8} * 8 + 3\right] lb\cdot ft = \boxed{6\,lb\cdot ft}$

Example: Find K.E. when rod is vertical

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$



$$\cancel{T_1} + V_1 + \cancel{U_{1\to 2}^{nc}} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2,$$

where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$ & $V_2 = \frac{1}{2}kx_2^2 + mgy_2$

$\Rightarrow T_2 = \frac{1}{2}k(x_1^2 - x_2^2) + mg(y_1 - y_2)$, here

$y_1 - y_2 = l/2 = \frac{3}{2}\,ft$ & $L_1 = L_0 + x_1$ &

$L_1 = \sqrt{3^2 + 4^2}\,ft = 5\,ft \Rightarrow x_1 = L_1 - L_0 = 5\,ft - 2\,ft = 3\,ft$ &

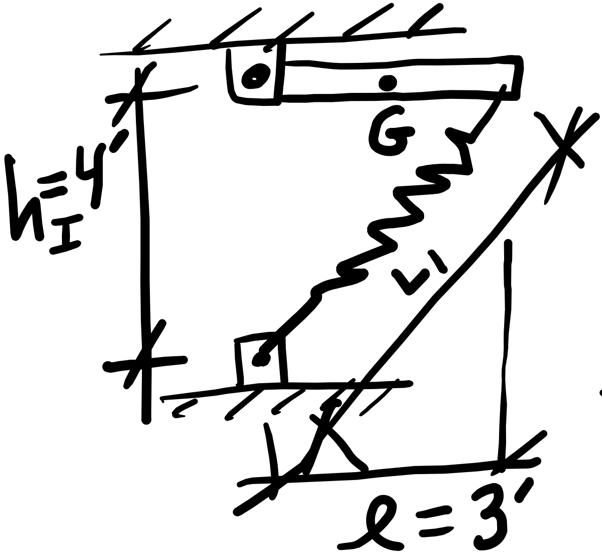
$L_2 = h - l = (4-3)\,ft = 1\,ft \Rightarrow x_2 = L_2 - L_0 = (1-2)\,ft = -1\,ft$

So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb\cdot ft = \left[\frac{3}{8} * 8 + 3\right] lb\cdot ft = \boxed{6\,lb\cdot ft}$

If we wanted to find ω_2 we could use

$$T_2 = \frac{1}{2} I_A \omega_2^2$$

Example: Find K.E. when rod is vertical



$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

$\cancel{T_1} + V_1 + \cancel{U_{1\to2}^{nc}} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2$,

where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$ & $V_2 = \frac{1}{2}kx_2^2 + mgy_2$

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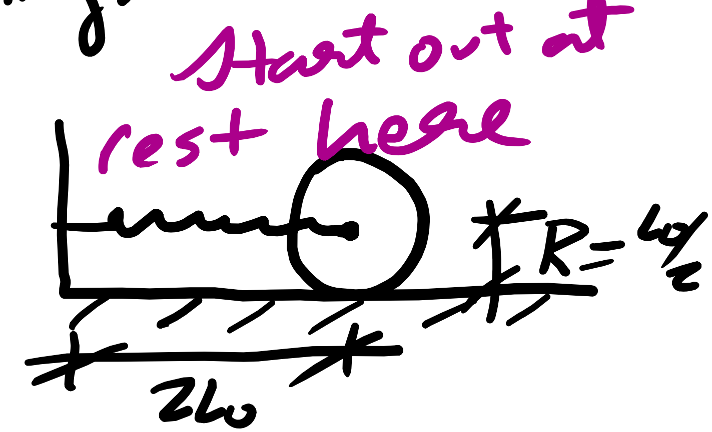
So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb \cdot ft = \left[\frac{3}{8} * 8 + 3\right] lb \cdot ft = \boxed{6\,lb \cdot ft}$

If we wanted to find ω_2 we could use

$$T_2 = \frac{1}{2} I_A \omega_2^2 \Rightarrow \omega_2 = \sqrt{\frac{2T_2}{I_A}}$$

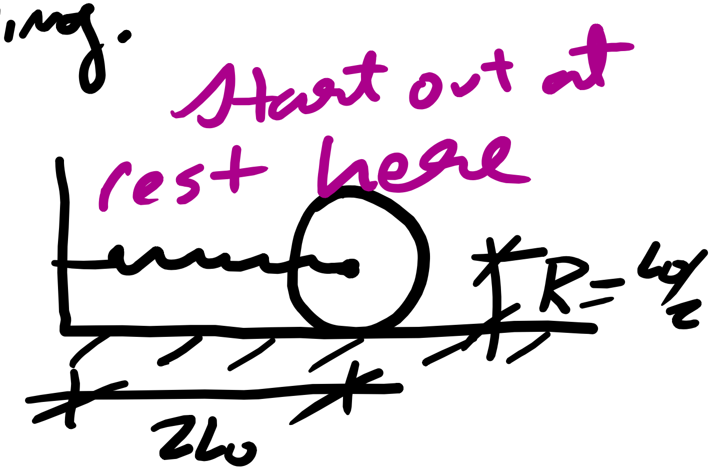
Example: Spring attached to axel of disk that rolls w/out slipping.

Example: Spring attached to axel of disk that rolls w/out slipping.



Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall? $\times$



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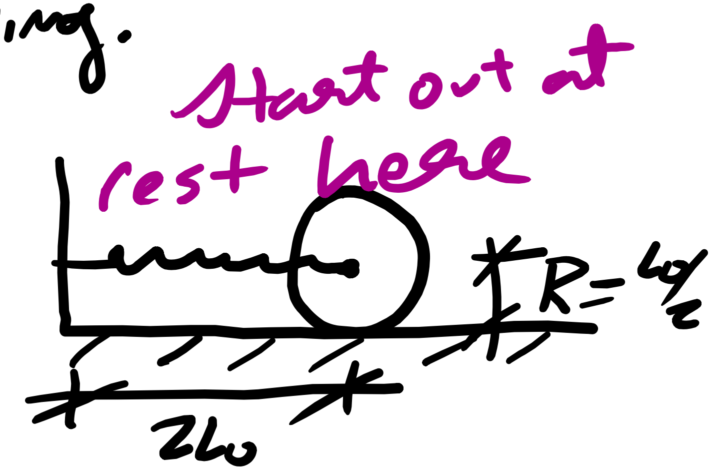
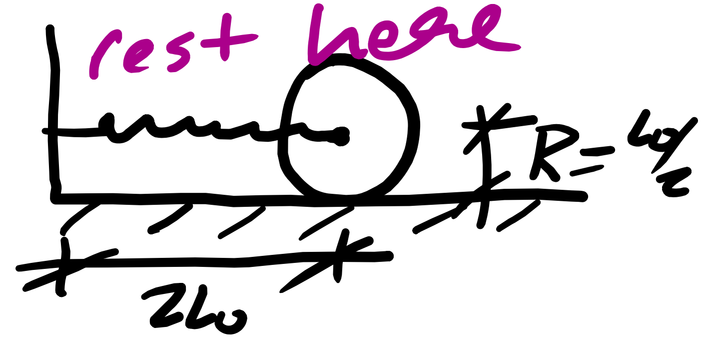


What is $\bar{v}$ when disk hits wall?

$$T_1 + V_1 + U_{1\to 2}^{nc} = T_2 + V_2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

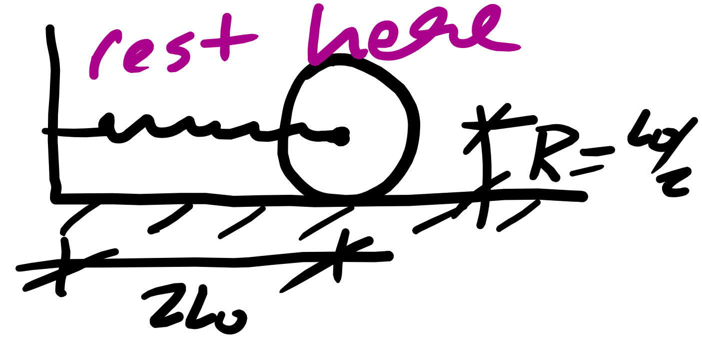


$$\cancel{T_1} + V_1 + U_{1\to 2}^{nc} = T_2 + V_2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

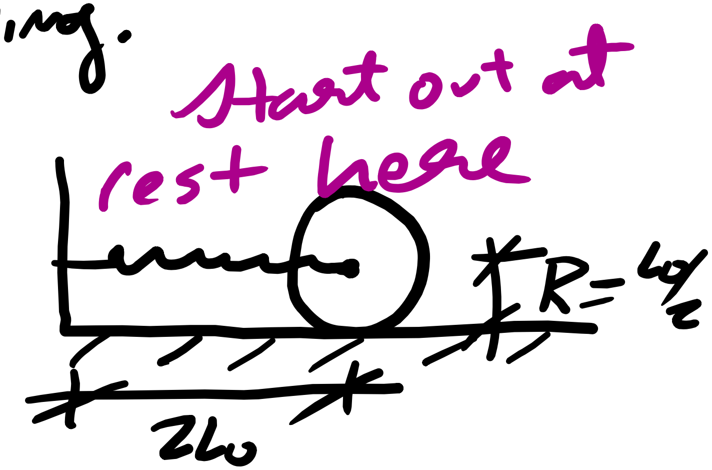
Start out at rest here



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U^{nc}_{1\to 2}} = T_2 + V_2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

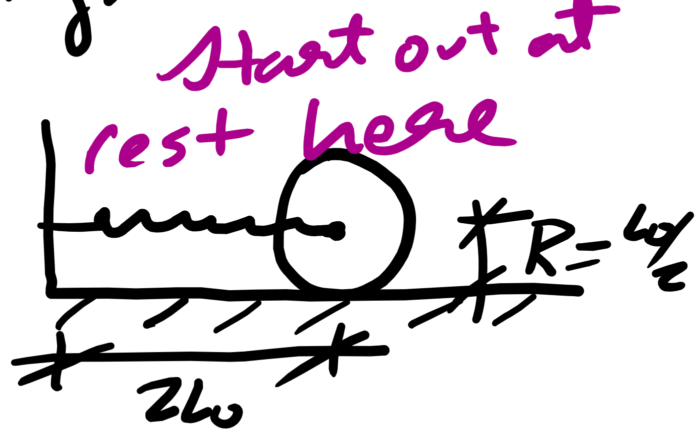
What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{\,0} + V_1 + \cancel{U_{1\to2}^{nc}}^{\,0} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

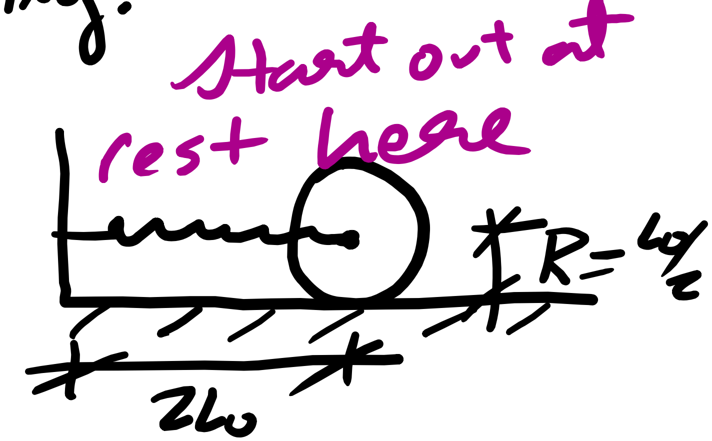


$\cancel{T_1} + V_1 + \cancel{U_{1\to2}^{nc}} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

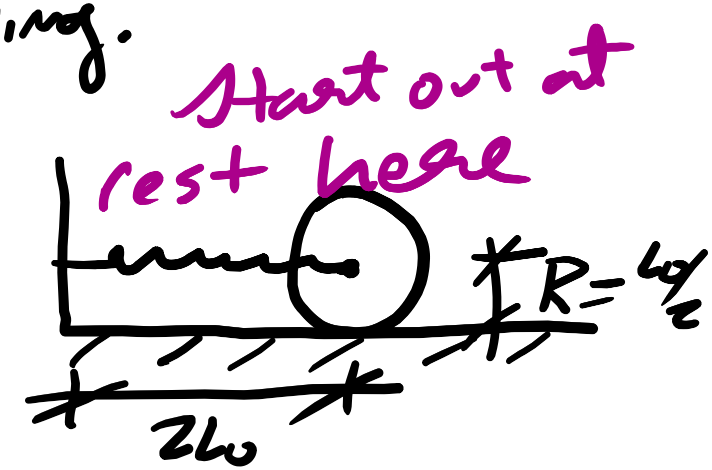


$$\cancel{T_1} + V_1 + \cancel{U^{nc}_{1\to 2}} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

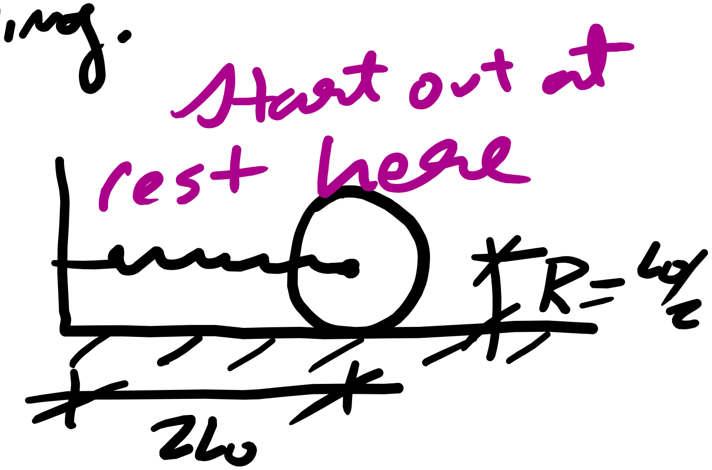


$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2\,,\quad V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2,\quad V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to2}^{nc}}^{0} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

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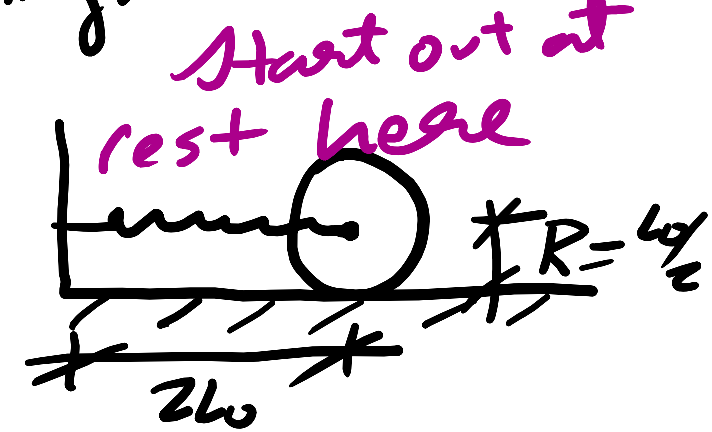
But $\bar{I} = \tfrac{1}{2}mR^2$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

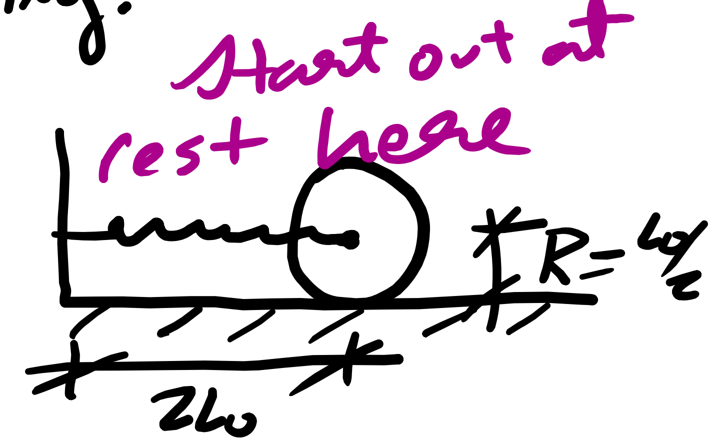
$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to2}^{nc}}^{0} = T_2 + V_2 \,, \quad V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 \,, \quad V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



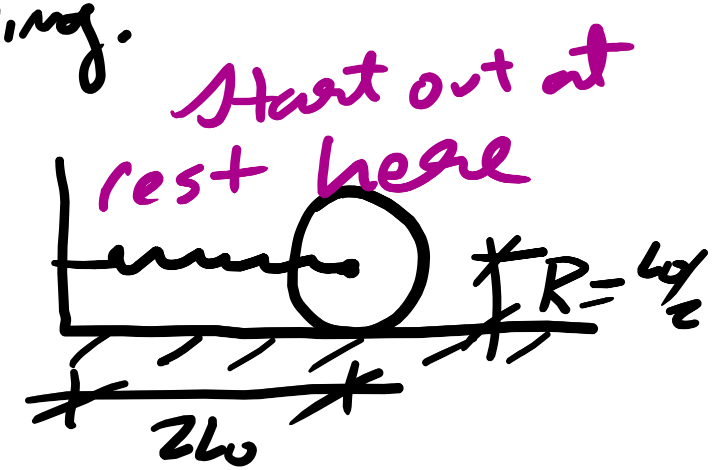
$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2} k x_1^2$$

$$T_2 = \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 \,, \quad V_2 = \tfrac{1}{2} k x_2^2 \Rightarrow \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$$

But $\bar{I} = \frac{1}{2} m R^2$ & $R\omega = \bar{v}$ so $\frac{1}{2} \bar{I} \omega^2 = \frac{1}{4} m \bar{v}^2$

<u>Example:</u> Spring attached to axel of disk that rolls w/out slipping.

<u>What is $\bar{v}$ when disk hits wall?</u>



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U^{nc}_{1\to 2}} = T_2 + V_2 \;, \quad V_1 = \tfrac{1}{2} k x_1^2$$

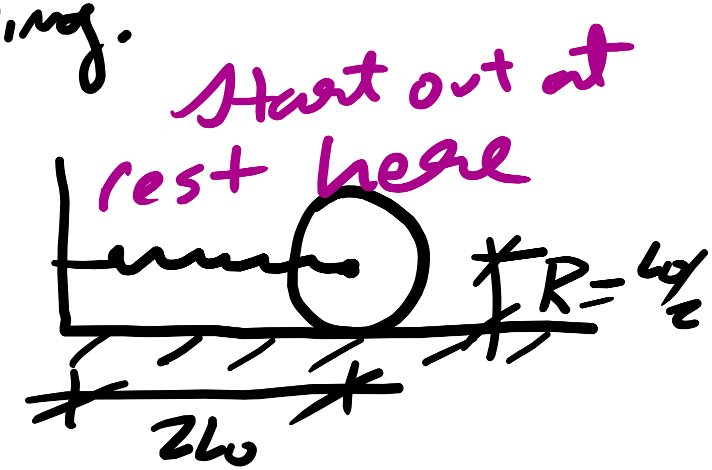
$$T_2 = \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 , \quad V_2 = \tfrac{1}{2} k x_2^2 \Rightarrow \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2} m R^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{4} m \bar{v}^2$

Now $\tfrac{1}{4} m \bar{v}^2 + \tfrac{1}{2} m \bar{v}^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2$, $V_1 = \frac{1}{2} k x_1^2$

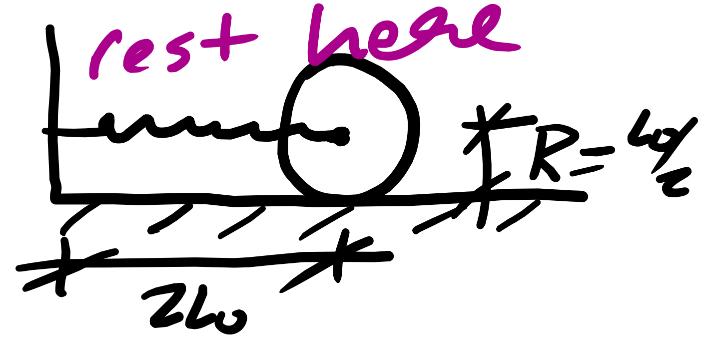
$T_2 = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$, $V_2 = \frac{1}{2} k x_2^2 \Rightarrow \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} k (x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2} m R^2$ & $R\omega = \bar{v}$ so $\frac{1}{2} \bar{I} \omega^2 = \frac{1}{4} m \bar{v}^2$

Now $\frac{1}{4} m \bar{v}^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2} k (x_1^2 - x_2^2) \Rightarrow \frac{3}{4} m \bar{v}^2 = \frac{1}{2} k (x_1^2 - x_2^2)$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{NC}_{1\to 2}}^{\,0} = T_2 + V_2\,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

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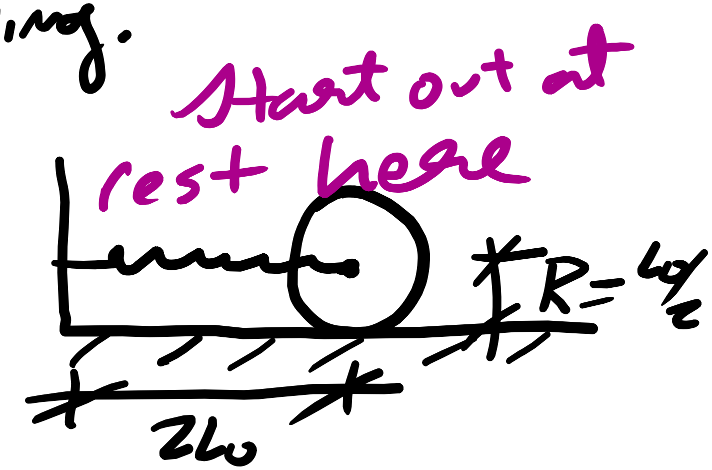
Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{nc}}^{0} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2} k x_1^2$$

$$T_2 = \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 \,, \quad V_2 = \tfrac{1}{2} k x_2^2 \Rightarrow \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$$

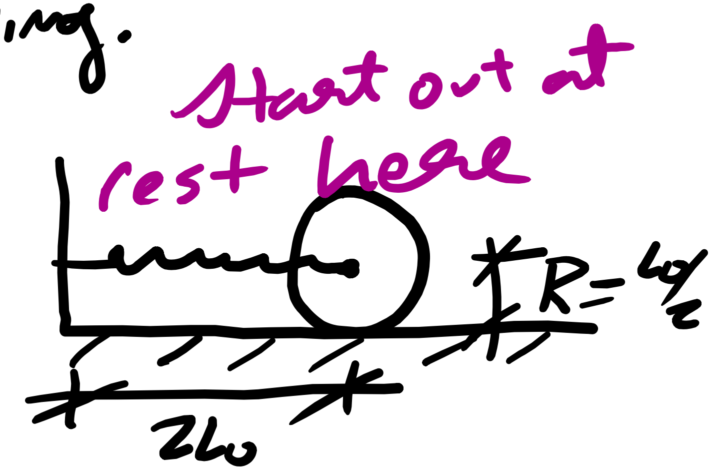
But $\bar{I} = \frac{1}{2} m R^2$ & $R\omega = \bar{v}$ so $\frac{1}{2} \bar{I} \omega^2 = \frac{1}{4} m \bar{v}^2$

Now $\frac{1}{4} m \bar{v}^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2} k (x_1^2 - x_2^2) \Rightarrow \frac{3}{4} m \bar{v}^2 = \frac{1}{2} k (x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?$_x$



$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{nc}}^{0} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

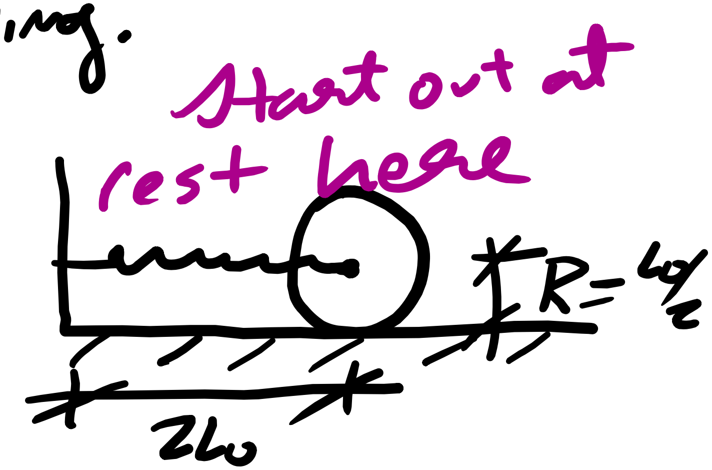
$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, $V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

$\cancel{T_1}^{\,0} + V_1 + \cancel{U_{1\to2}^{nc}}^{\,0} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, $V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

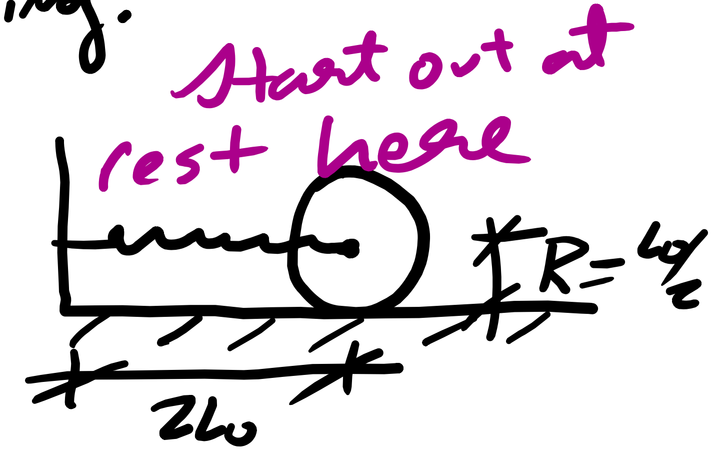
But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0 = -\frac{1}{2}L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U^{nc}_{1\to 2}} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, $V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

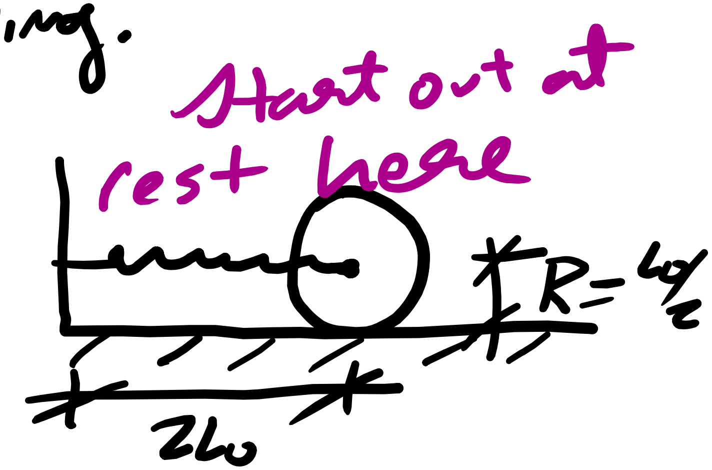
Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0 = -\frac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\frac{2}{3}\right)\left(\frac{k}{m}\right)\left(L_0^2 - \frac{1}{4}L_0^2\right)$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U^{nc}_{1\to 2}} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}M\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 \,, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}M\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2}MR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}M\bar{v}^2$

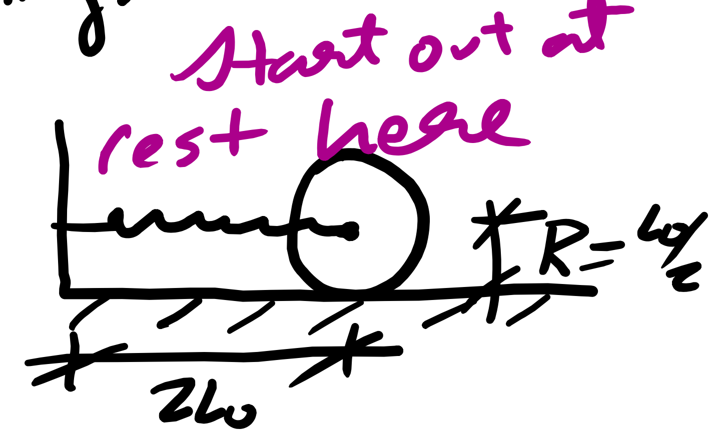
Now $\tfrac{1}{4}M\bar{v}^2 + \tfrac{1}{2}M\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}M\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \tfrac{1}{2}L_0 - L_0 = -\tfrac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\tfrac{2}{3}\right)\left(\tfrac{k}{m}\right)\left(L_0^2 - \tfrac{1}{4}L_0^2\right) = \tfrac{1}{2}\left(\tfrac{k}{m}\right)L_0^2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}m\bar{v}^2$

Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \tfrac{1}{2}L_0 - L_0 = -\tfrac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\tfrac{2}{3}\right)\left(\tfrac{k}{m}\right)\left(L_0^2 - \tfrac{1}{4}L_0^2\right) = \tfrac{1}{2}\left(\tfrac{k}{m}\right)L_0^2 \Rightarrow \boxed{\bar{v} = L_0\sqrt{\tfrac{k}{2m}}}$$

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We want the same sort of relationship for torque and angular momentum that we have for force & linear momentum.

We have $\sum \vec{M}_G = \bar{I}\,\vec{\alpha}$ from the previous chapter

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ASU

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So angular momentum is conserved when no resulting torque is applied to the rigid body

<u>Example</u>: Uniform sphere has horizontal velocity $\bar{v}_1$ along rough horizontal surface.

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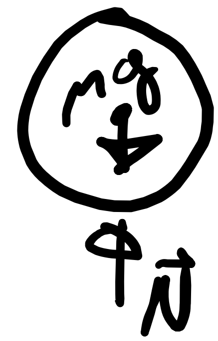
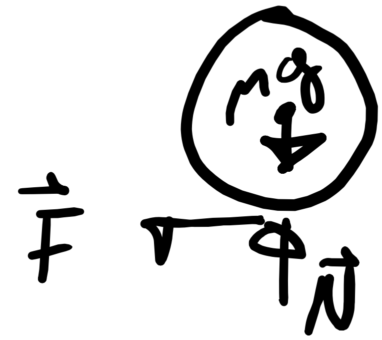
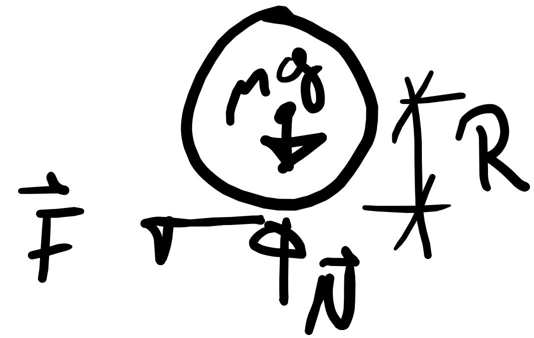
Example: Uniform sphere has horizontal velocity $\vec{v}_1$ along rough horizontal surface. Sphere is not rotating initially. Coefficient of kinetic friction is μ_k. What is time required for sphere to roll w/out slipping?

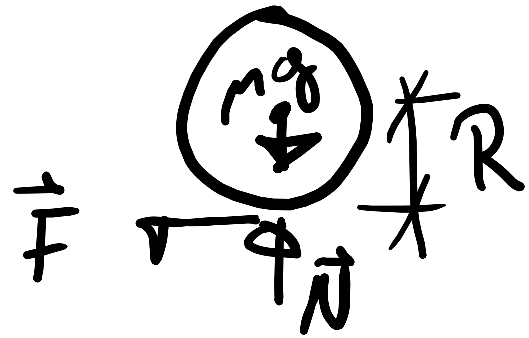

mg
N



$\vec{F}$
$m\vec{g}$
$\vec{N}$


mg
F
N
R

$F = N\mu_k$



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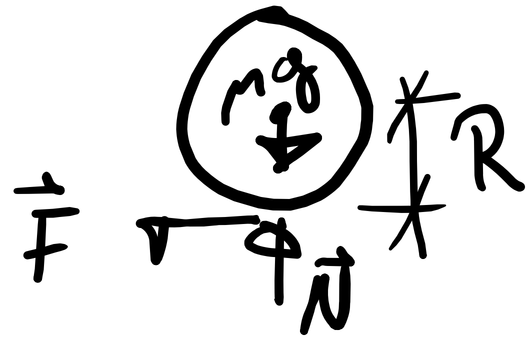

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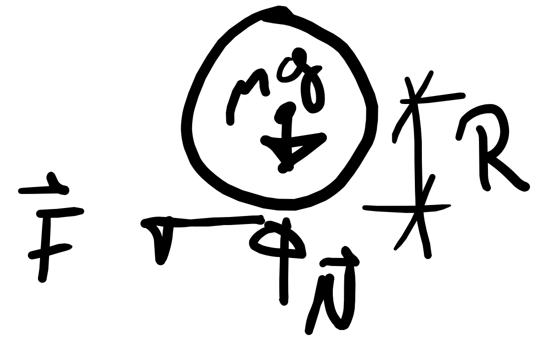

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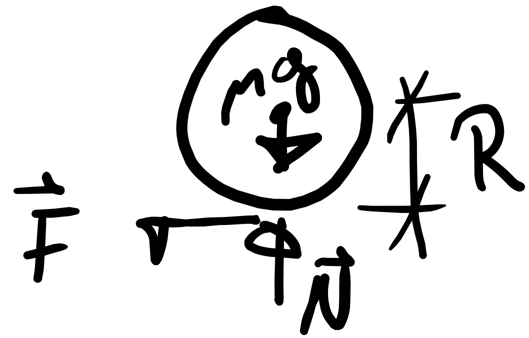

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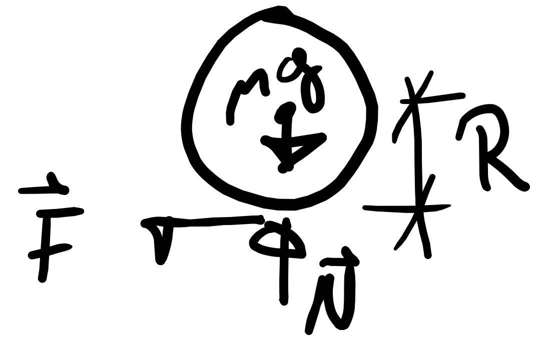

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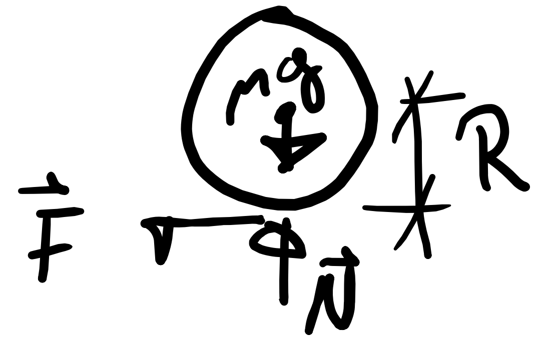

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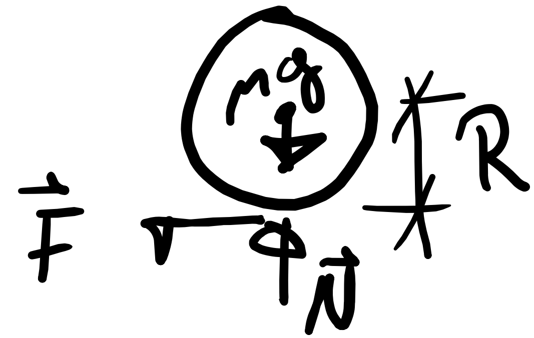

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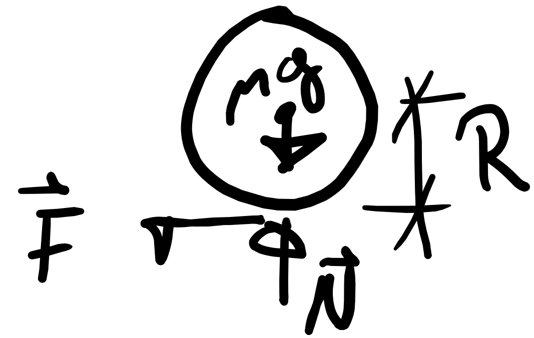

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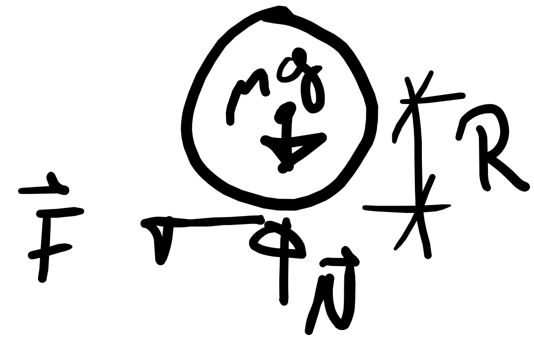

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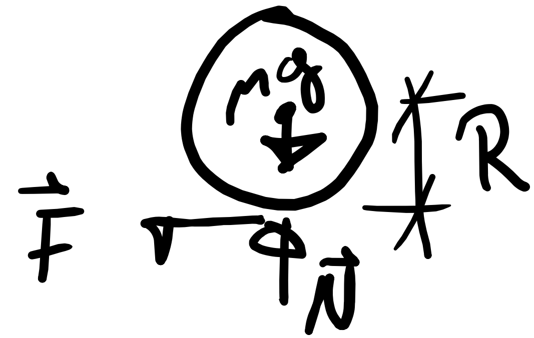

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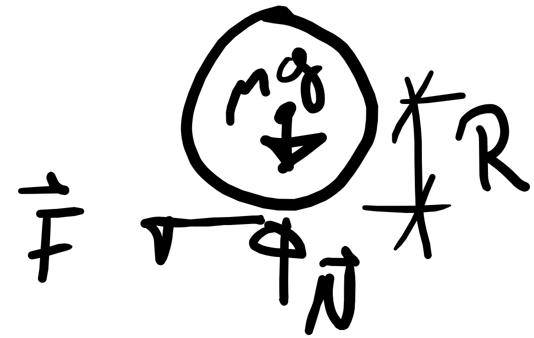

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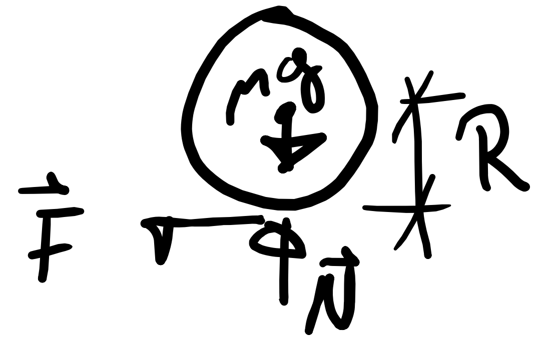

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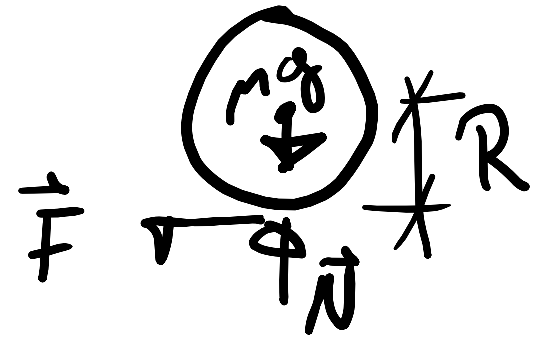

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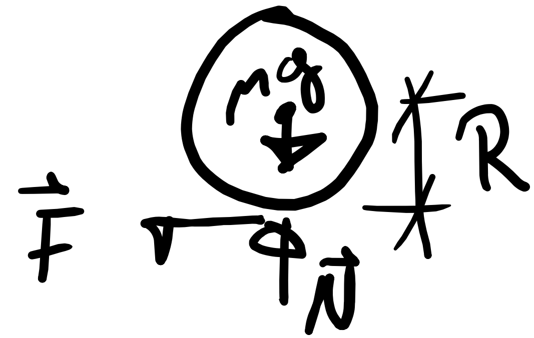

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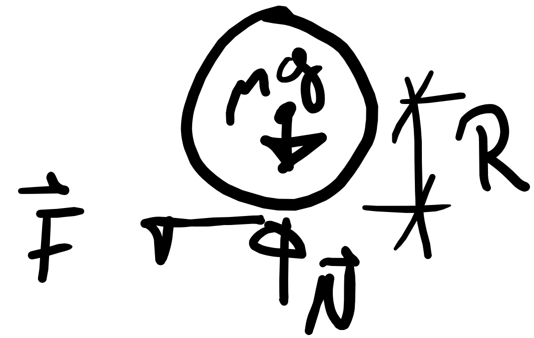

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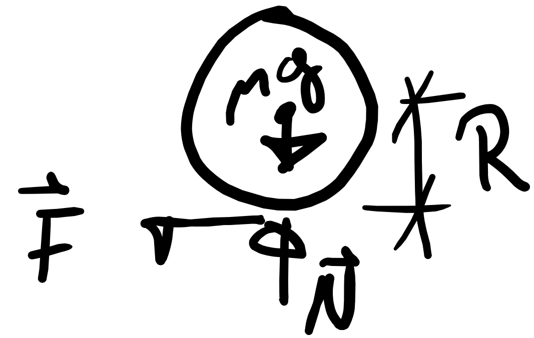

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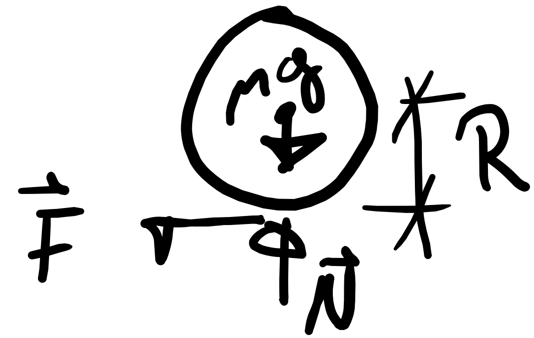

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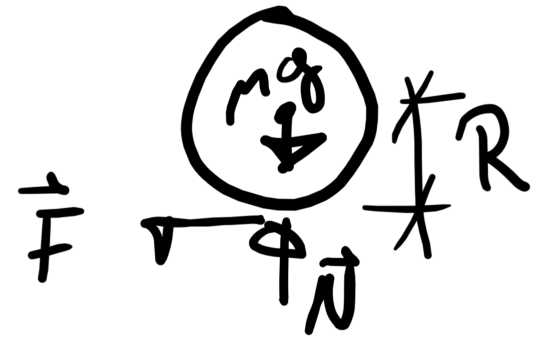

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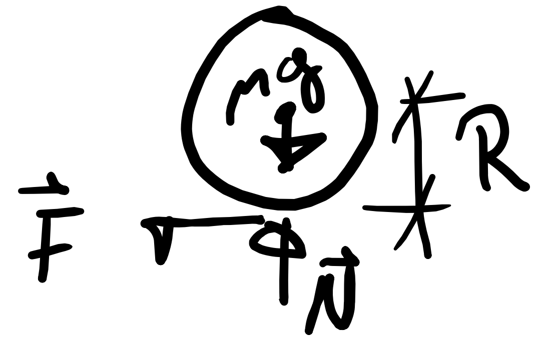

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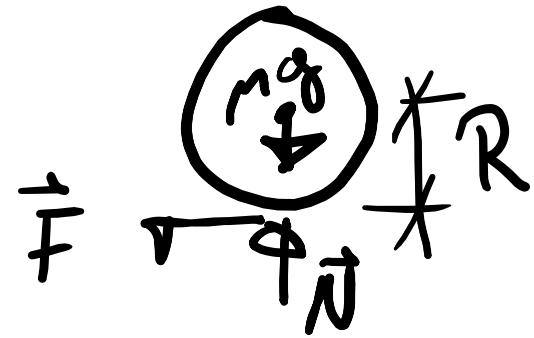

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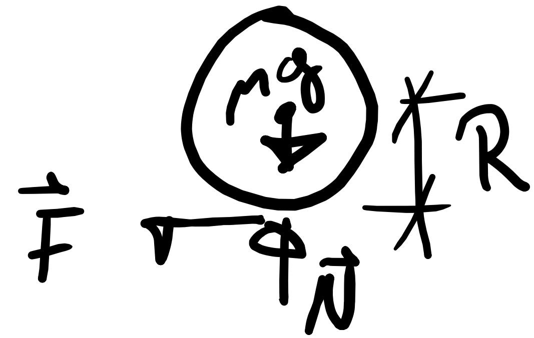

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