Today 17.2

$\boxed{L29}$

ASU

Today 17.2

Wednesday Review

L29

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Wednesday Review

Friday Exam #3

Last time we saw that

$$\int_{t_I}^{t_F} \sum \vec{M}_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\omega_I$$

Last time we saw that

$\int_{t_I}^{t_F} \sum \vec{M}_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\omega_I$ & for a rotation about some fixed point P

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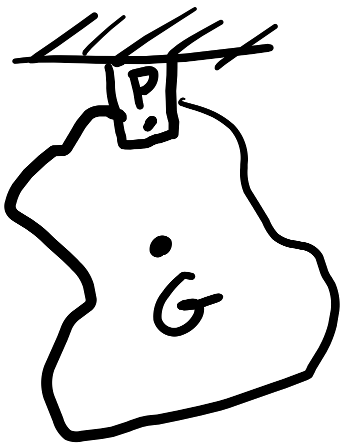
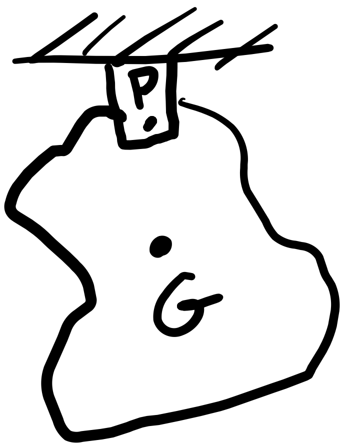
& for a rotation about some fixed point P



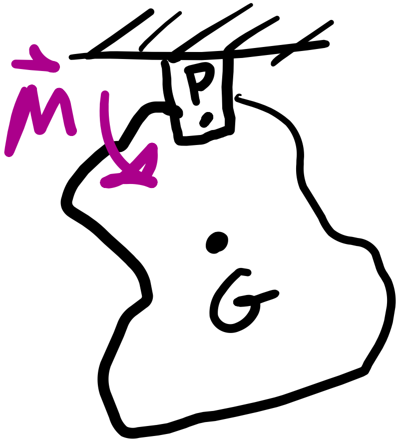
Last time we saw that

$$\int_{t_I}^{t_F} \sum \vec{M}_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\omega_I$$ & for a rotation about some fixed point P with torque $\vec{M}$



Last time we saw that

$\int_{t_I}^{t_F} \sum \vec{M}_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\omega_I$ & for a rotation about some fixed point P with torque $\vec{M}_P$ as shown



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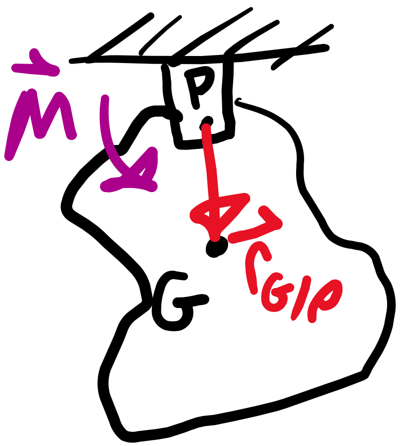

$$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a}$$

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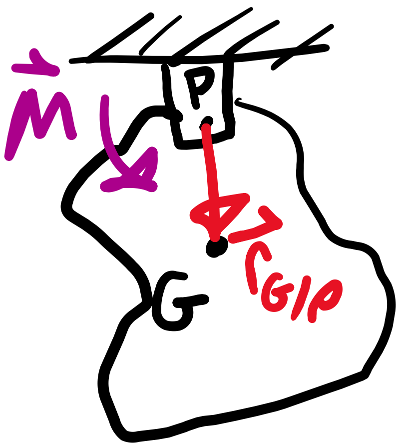

$$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a} \quad \&$$

$$\bar{a} = a_n \hat{e}_n + a_t \hat{e}_t$$

Last time we saw that

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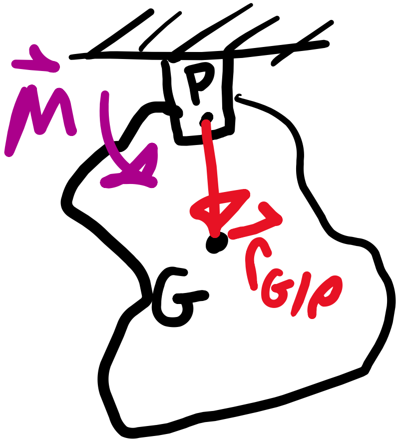

$$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a} \quad \&$$

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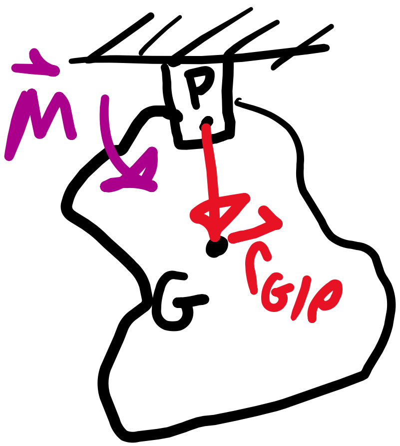

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So $\vec{M}_P = \bar{I}\vec{\alpha} + m r_{G/P}^2 \vec{\alpha}$

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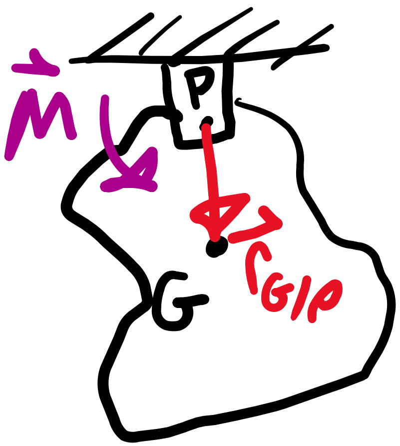

$$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a} \quad \&$$

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So $\vec{M}_P = \bar{I}\vec{\alpha} + m r_{G/P}^2 \vec{\alpha}$ but

$$\bar{I} + m r_{G/P}^2 = I_P$$

Last time we saw that

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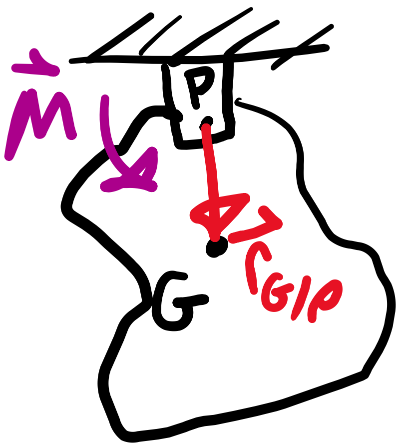

$$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a} \quad \&$$

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So $\vec{M}_P = \bar{I}\vec{\alpha} + m r_{G/P}^2 \vec{\alpha}$ but

$\bar{I} + m r_{G/P}^2 = I_P$ so $\vec{M}_P = I_P \vec{\alpha}$

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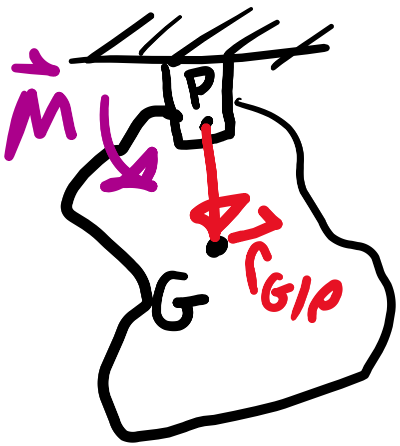

$\vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{a}$ &

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$\bar{I} + m r_{G/P}^2 = I_P$ so $\vec{M}_P = I_P \vec{\alpha}$

$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_P \, dt = I_P \int \vec{\alpha} \, dt$

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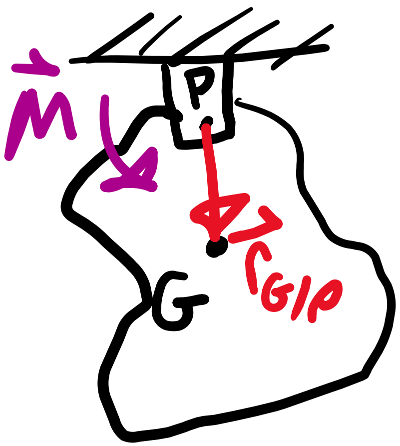

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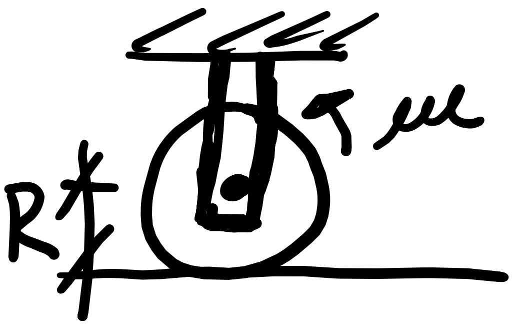
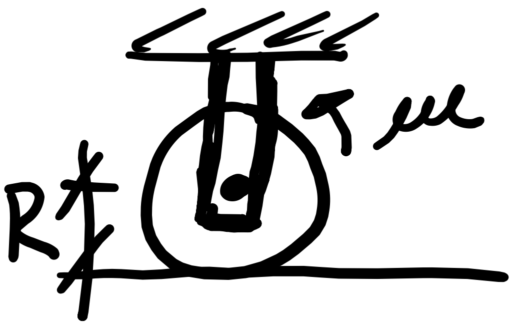
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_P \, dt = I_P \int \vec{\alpha}\, dt \Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_P \, dt = I_P\omega_F - I_P\omega_I$$

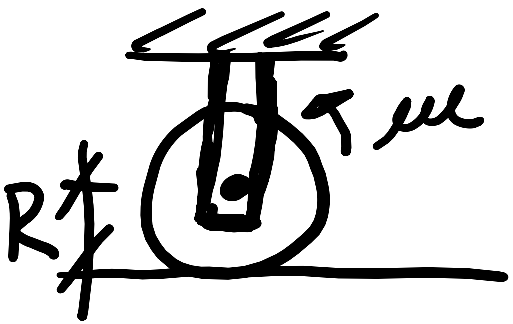
__Example__: Rotating disk brought into contact with rough surface.



Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$



Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = \cancel{0}$, at what time does the disk stop rotating?



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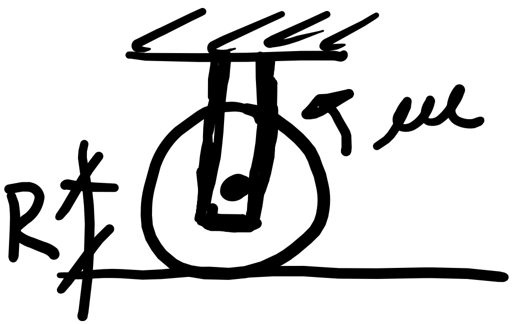

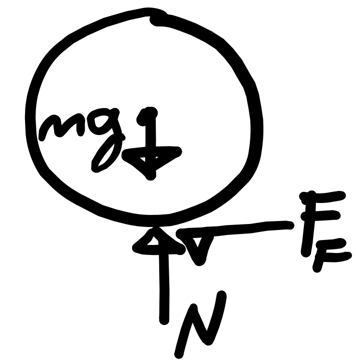

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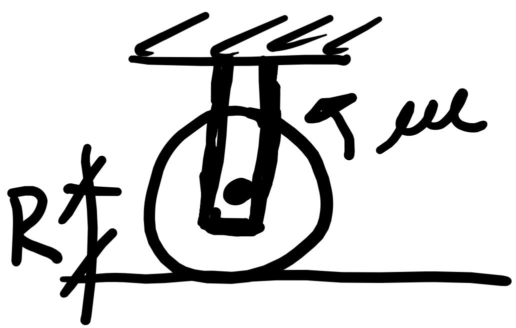
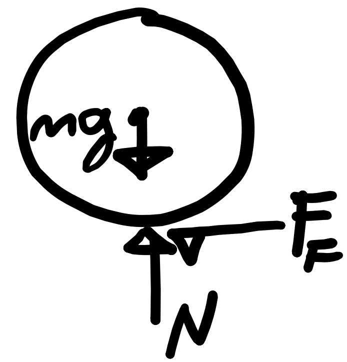


$F_F = N\mu_K$



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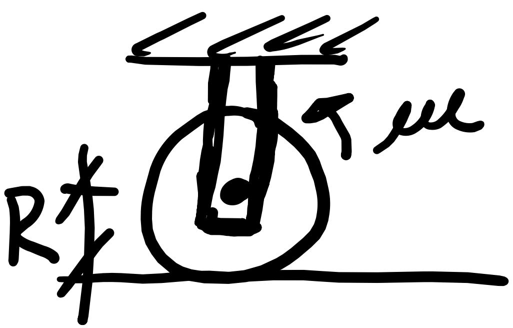

$F_F = N\mu_K$

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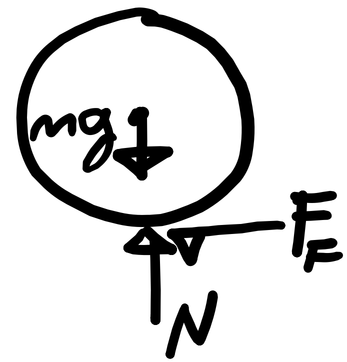

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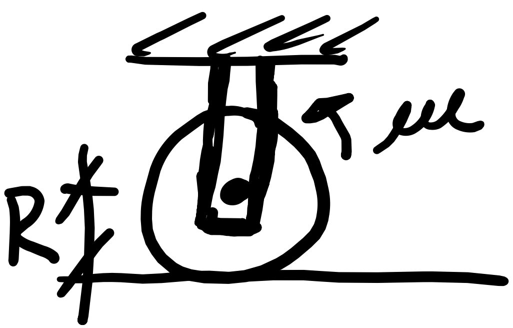
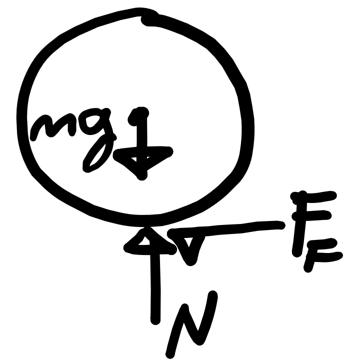


$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \; N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$



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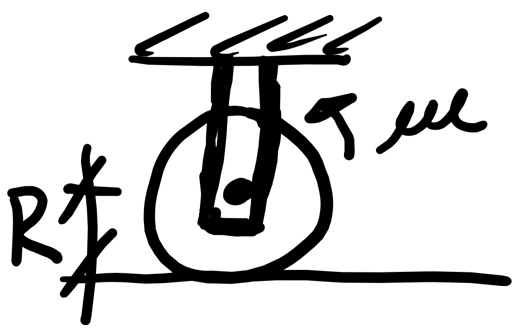

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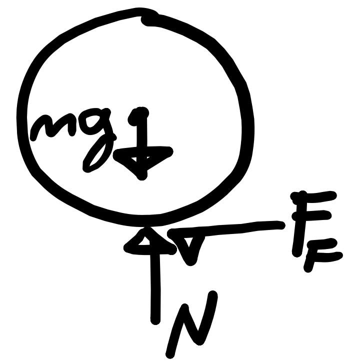
$$\& \ \vec{M}_G = RF_F \curvearrowright$$



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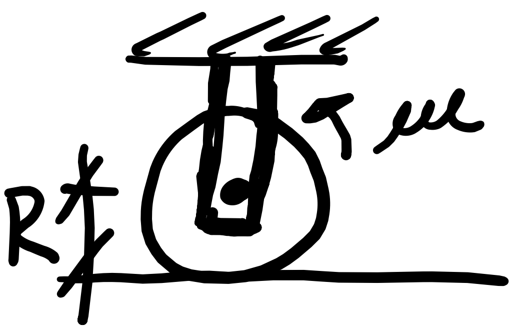

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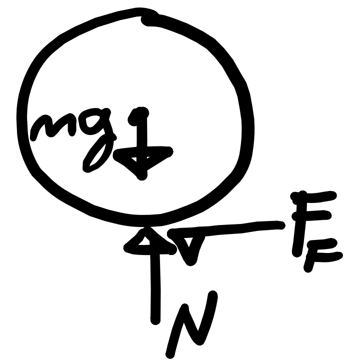

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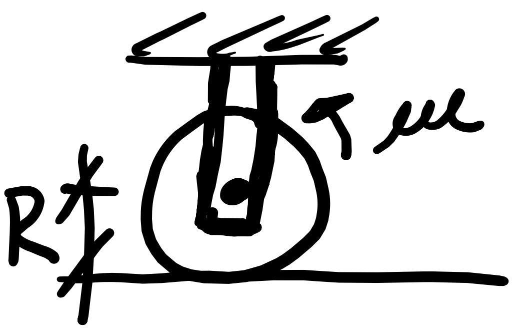
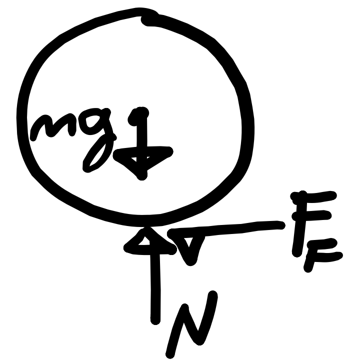


NOW



$$\left.\begin{aligned} F_F &= N\mu_k \\ \& \; N &= mg \end{aligned}\right\} \Rightarrow F_F = mg\mu_k$$

$$\& \; \vec{M}_G = RF_F \circlearrowright = Rmg\mu_k \circlearrowright$$

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\vec{\omega}_I$$

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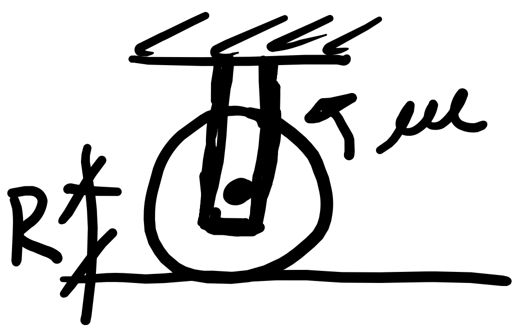

$F_F = N\mu_k$
$\& \; N = mg$ $\Rightarrow F_F = mg\mu_k$

$\& \; \vec{M}_G = R F_F \circlearrowright = R m g \mu_k \circlearrowright$

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\vec{\omega}_I$$

with $\vec{\omega}_F = 0$

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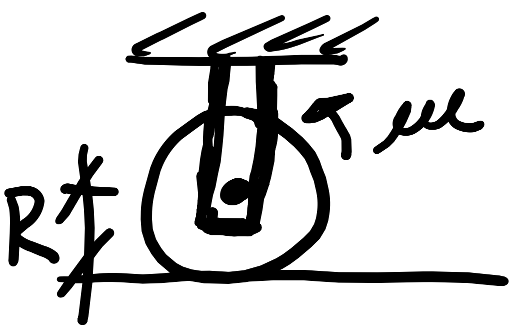

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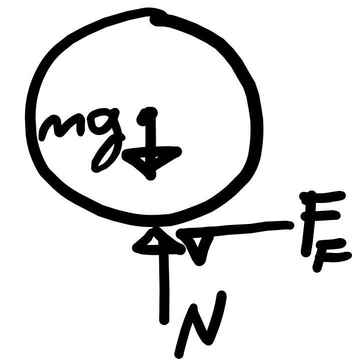

$$\& \; \vec{M}_G = R F_F \circlearrowright = R mg \mu_k \circlearrowright$$

$$\int_{\cancel{t_I}\;0}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\vec{\omega}_F}^{\,0} - \bar{I}\vec{\omega}_I$$

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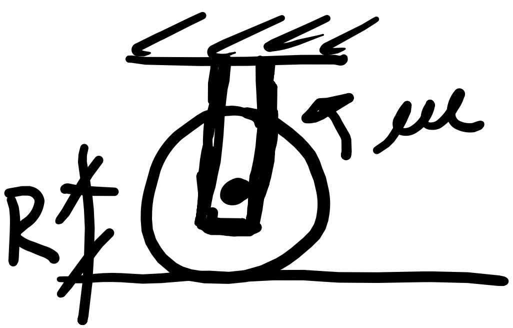

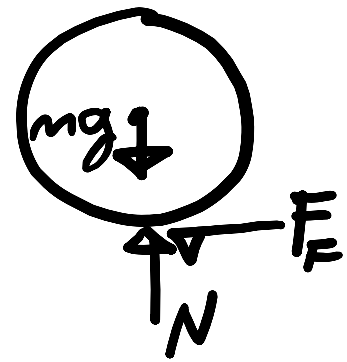

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$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\omega_F} - \bar{I}\omega_I \Rightarrow$$

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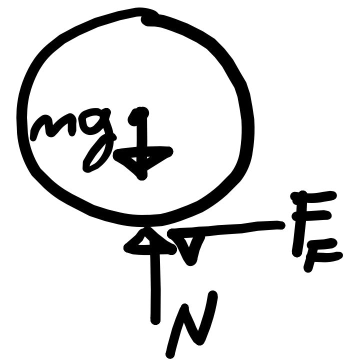
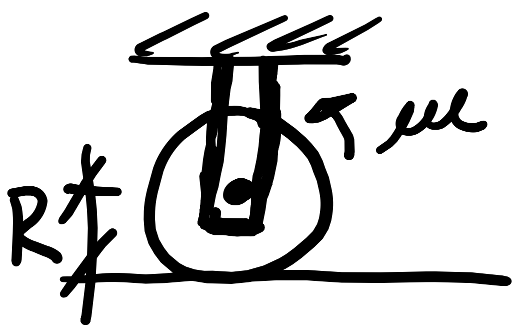




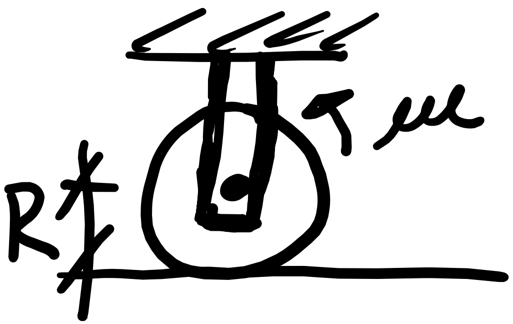
$F_F = N\mu_k$
& $N = mg$ $\Big\} \Rightarrow F_F = mg\mu_k$

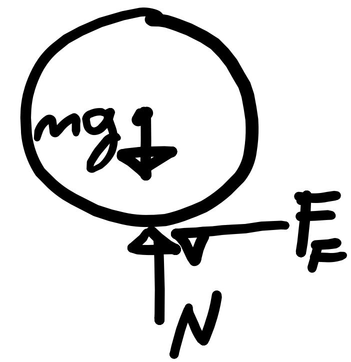
& $\vec{M}_G = RF_F \circlearrowright = Rmg\mu_k \circlearrowright$

Now $\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\omega_F - \bar{I}\omega_I \Rightarrow$

$Rmg\mu_k t_F = \bar{I}\omega_I$ But $\bar{I} = \frac{1}{2}MR^2$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?





$$\left.\begin{aligned} F_F &= N\mu_k \\ \& \; N &= mg \end{aligned}\right\} \Rightarrow F_F = mg\mu_k$$

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Now

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\omega_F - \bar{I}\omega_I \Rightarrow$$

But $\bar{I} = \frac{1}{2}MR^2$ so

$$Rmg\mu_k t_F = \bar{I}\omega_I$$

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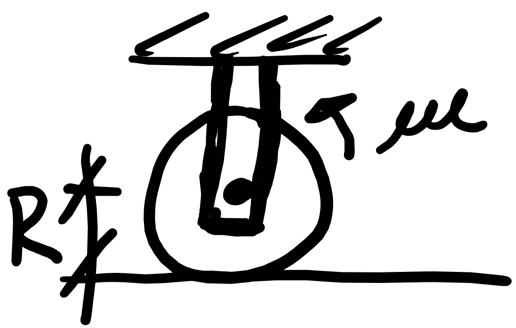

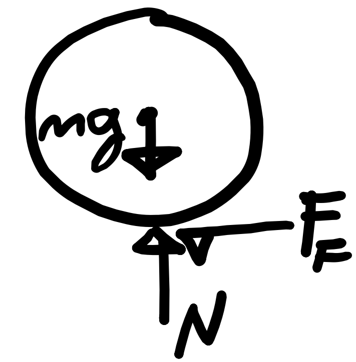

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Now

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\omega_F - \bar{I}\omega_I \Rightarrow$$

$\omega_F = 0$, $t_I = 0$

But $\bar{I} = \frac{1}{2}MR^2$ so

$$R m g \mu_K t_F = \bar{I}\omega_I$$

$$R m g \mu_K t_F = \frac{1}{2}MR^2\omega_I \Rightarrow \boxed{t_F = \frac{R\omega_I}{2g\mu_K}}$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt.

<u>Example</u> Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}

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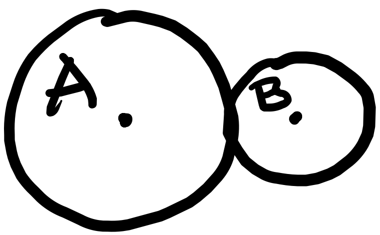
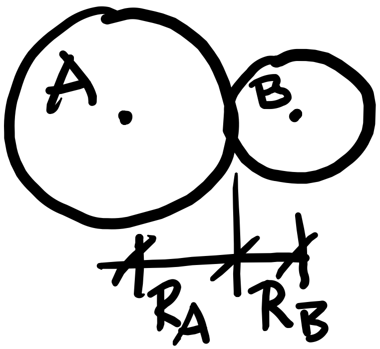
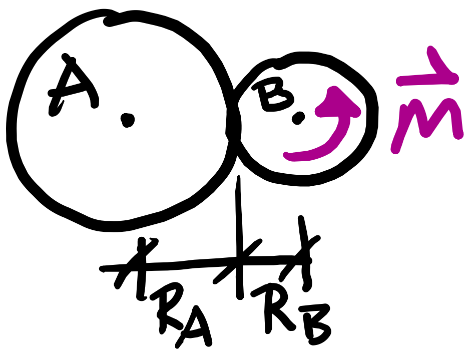


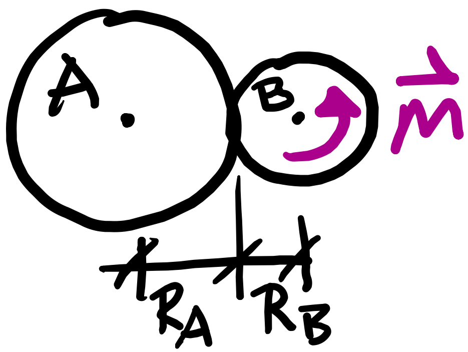
Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



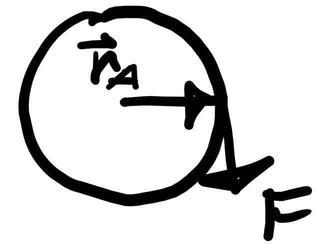
Example: Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



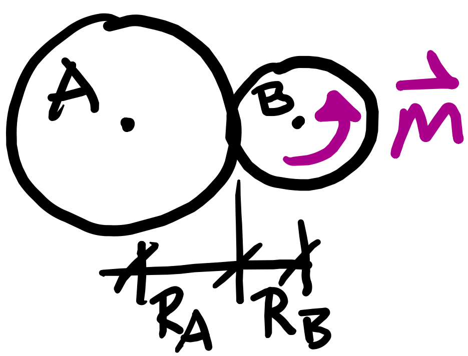
Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



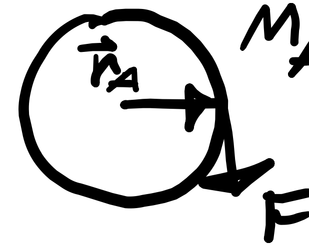
Gear A



Example: Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}

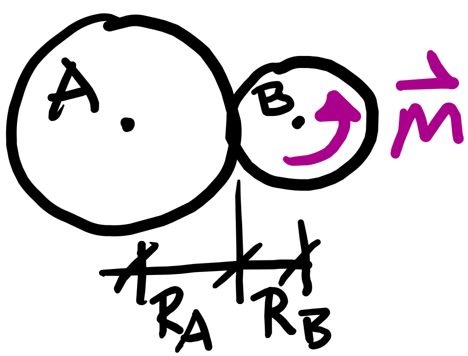


Gear A



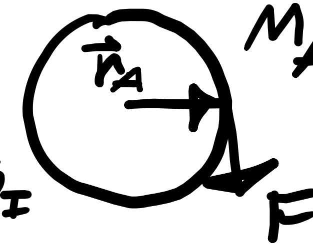
$\vec{M}_A = R_A F \circlearrowright$

Example: Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



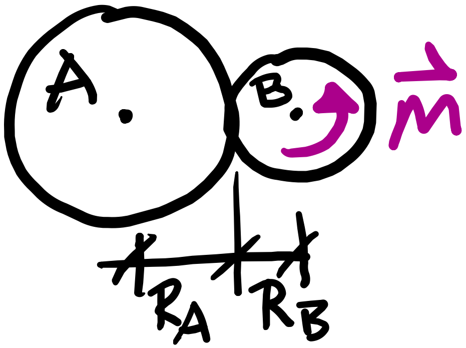
Gear A

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \bar{I}\vec{\omega}_I$$



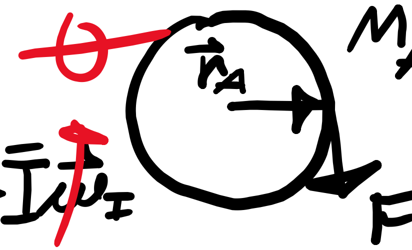
$\vec{M}_A = R_A F$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



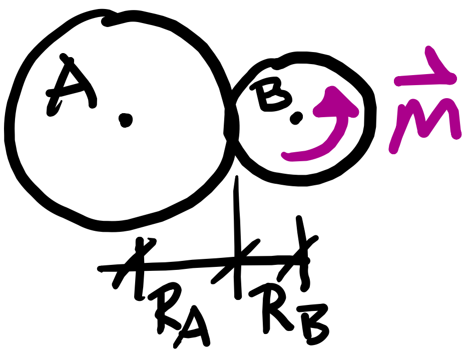
Gear A

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A\, dt = \bar{I}\vec{\omega}_F - \cancel{\bar{I}\vec{\omega}_I}$$

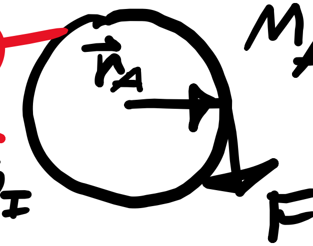


$\vec{M}_A = R_A F$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A

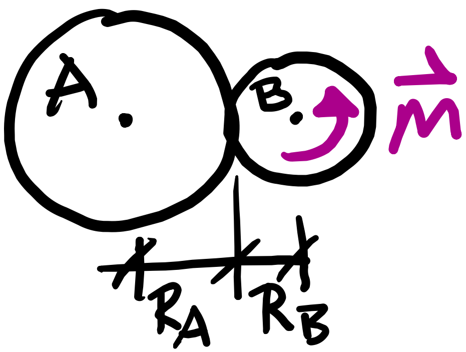


$\vec{M}_A = R_A F$ ↻

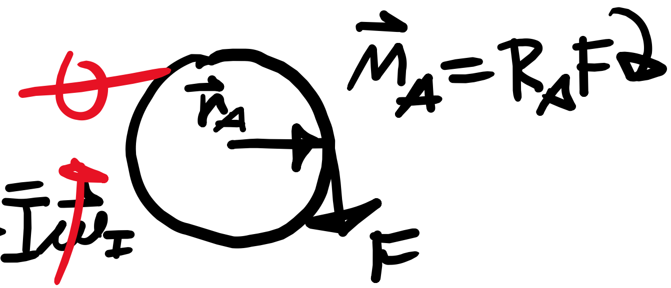
$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}^{\,0}$$

$$\Rightarrow R_A F t = I_A \omega_F$$

Example: Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



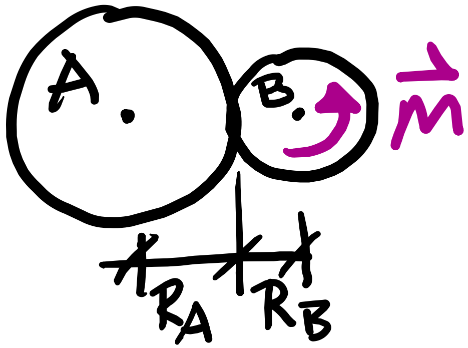
Gear A



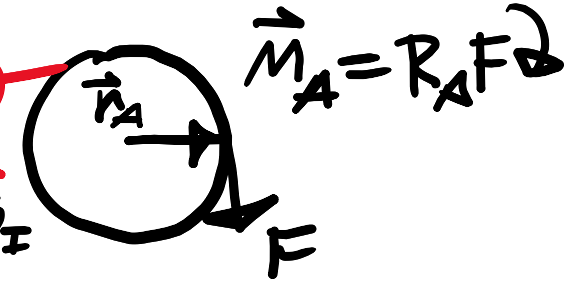
$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A

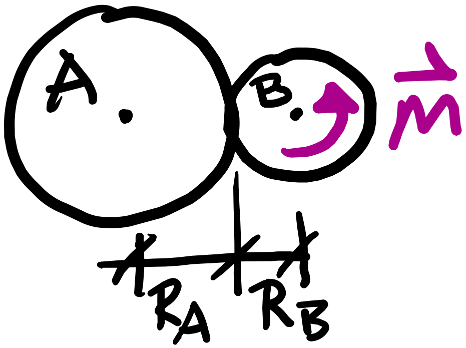


$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

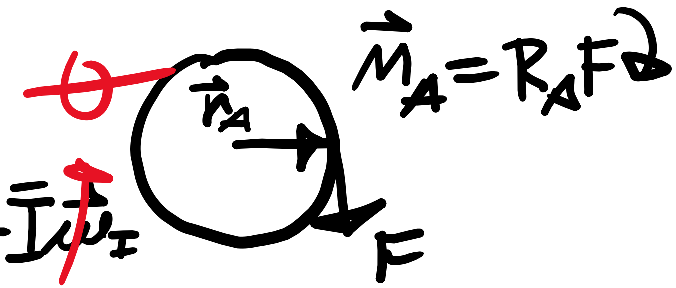
$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Gear B

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A

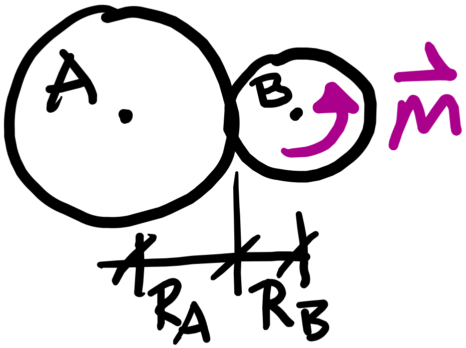


$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - I\cancel{\vec{\omega}_I}$$

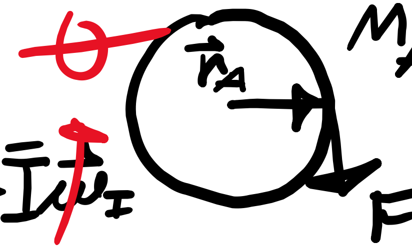
$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Gear B

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$

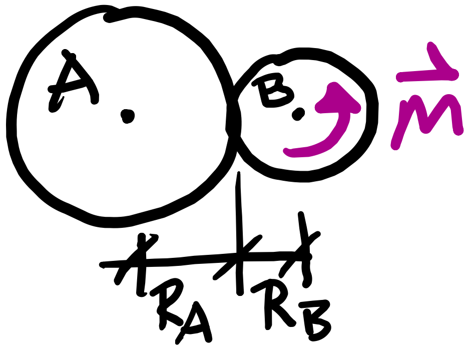
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - I\cancel{\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

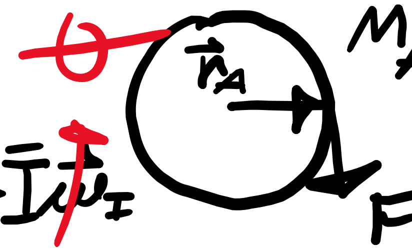
Gear B

$$\sum \vec{M}_B = (M - R_B F)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$

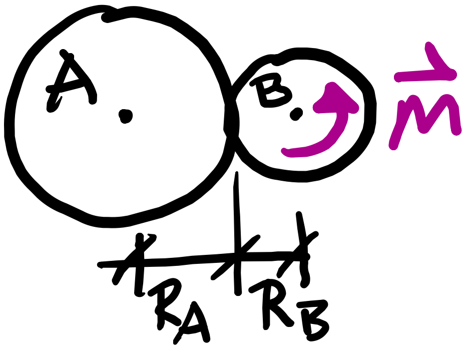
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\omega_F - \cancel{\bar{I}\omega_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

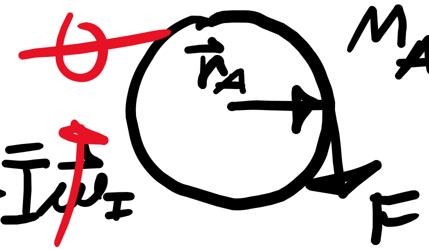
Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$

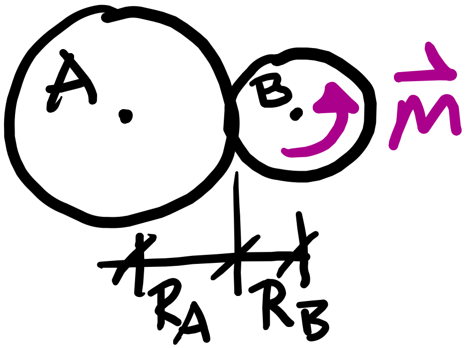
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\omega_F - \cancel{\bar{I}\omega_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \quad (1)$$

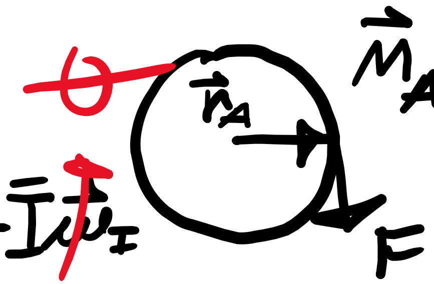
Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \quad (2)$$

Example: Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$$\vec{M}_A = R_A F$$

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \bar{I}\cancel{\omega_I}$$

$$\Rightarrow R_A F t = \bar{I}_A \omega_F \qquad (1)$$

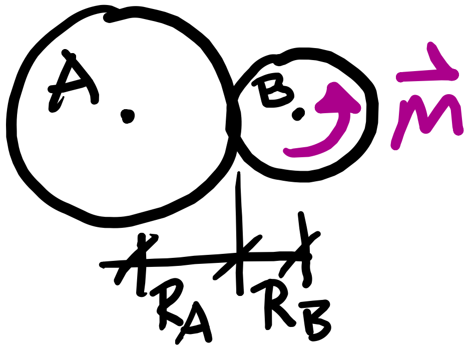
Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \qquad (2)$$

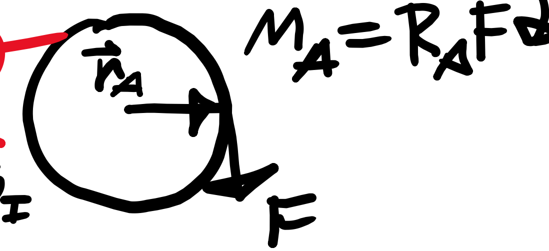
From equation 1 we have

$$F \Delta t = \frac{\bar{I}_A \omega_{AF}}{R_A}$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = \bar{I}\omega_F - \cancel{\bar{I}\omega_I}$$

$$\Rightarrow R_A F t = \bar{I}_A \omega_F \quad (1)$$

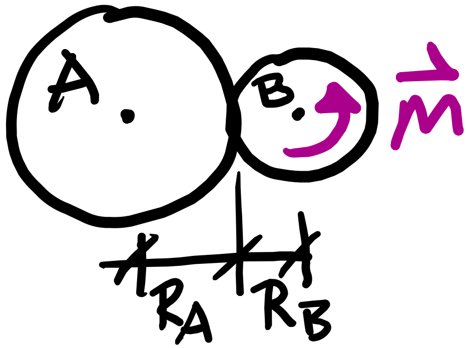
Gear B

$$\Sigma \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \quad (2)$$

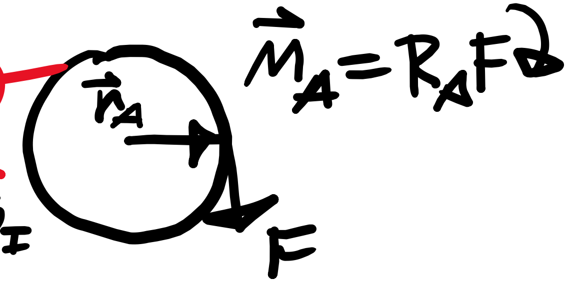
From equation 1 we have

$$F \Delta t = \frac{\bar{I}_A \omega_{AF}}{R_A} \quad (3)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \cancel{\bar{I}\omega_I}$$

$$\Rightarrow R_A F t = \bar{I}_A \omega_F \quad (1)$$

Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \quad (2)$$

From equation 1 we have

$$F \Delta t = \frac{\bar{I}_A \omega_{AF}}{R_A} \quad (3)$$

From previous slide

From previous slide

$$R_A F \Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

From previous slide

$$R_A F\Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

$$\& \quad F\Delta t = \bar{I}_A \omega_{AF}/R_A \quad (3)$$

ASU

From previous slide

$R_A F \Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t (M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

& $F \Delta t = \boxed{\bar{I}_A \omega_{AF} / R_A}$ (3) Eqn 3 into eqn 2

From previous slide

$$R_A F\Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

$$\& \quad F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A} \quad (3)$$

Eqn 3 into eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

From previous slide

$R_A F\Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t(M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

& $F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A}$ (3) Eqn 3 into

eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

But $R_A \omega_A = R_B \omega_B$

From previous slide

$R_A F \Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t (M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

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eqn 2 gives us $M \Delta t - \bar{I}_A \omega_{AF} \left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A} \omega_{BF}$

ASU

From previous slide

$$R_A F\Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

$$\& \quad F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A} \quad (3)$$

Eqn 3 into eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A}\omega_{BF}$

$$\Rightarrow \quad M\Delta t - \bar{I}_A \omega_{BF}\left(\frac{R_B}{R_A}\right)^2 = \bar{I}_B \omega_{BF}$$

From previous slide

$R_A F\Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t(M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

& $F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A}$ (3) Eqn 3 into eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A}\omega_{BF}$

$\Rightarrow \quad M\Delta t - \bar{I}_A \omega_{BF}\left(\frac{R_B}{R_A}\right)^2 = \bar{I}_B \omega_{BF} \quad \Rightarrow$

$$M\Delta t = \left[\bar{I}_A\left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right]\omega_{BF}$$

From previous slide

$$R_A F\Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \text{\&} \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

$$\text{\&} \quad F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A} \quad (3)$$

Eqn 3 into eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A}\omega_{BF}$

$$\Rightarrow \quad M\Delta t - \bar{I}_A \omega_{BF}\left(\frac{R_B}{R_A}\right)^2 = \bar{I}_B \omega_{BF} \quad \Rightarrow$$

$$M\Delta t = \left[\bar{I}_A\left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right]\omega_{BF} = \left[\frac{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}{R_A^2}\right]\omega_{BF}$$

From previous slide

$R_A F\Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t(M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

& $F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A}$ (3) Eqn 3 into

eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

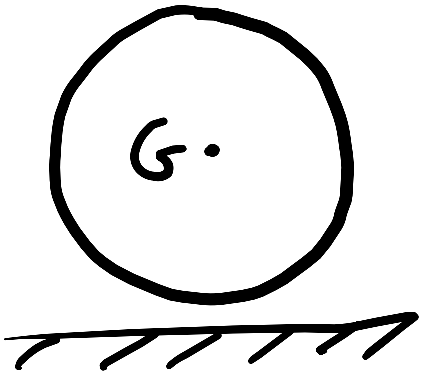
But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A}\omega_{BF}$

$\Rightarrow M\Delta t - \bar{I}_A \omega_{BF}\left(\frac{R_B}{R_A}\right)^2 = \bar{I}_B \omega_{BF} \Rightarrow$

$$M\Delta t = \left[\bar{I}_A\left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right]\omega_{BF} = \left[\frac{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}{R_A^2}\right]\omega_{BF}$$

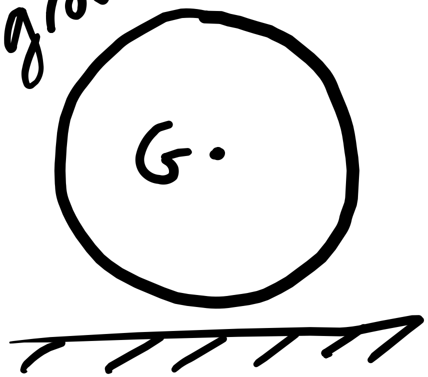
$$\Rightarrow \boxed{\omega_{BF} = \left[\frac{M\Delta t R_A^2}{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}\right]}$$

$\boxed{17.54}$



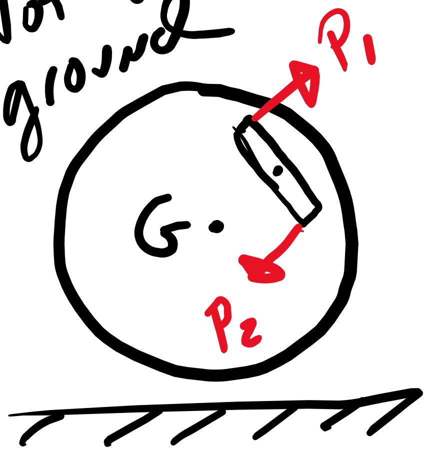
$\boxed{17.54}$

Not on ground



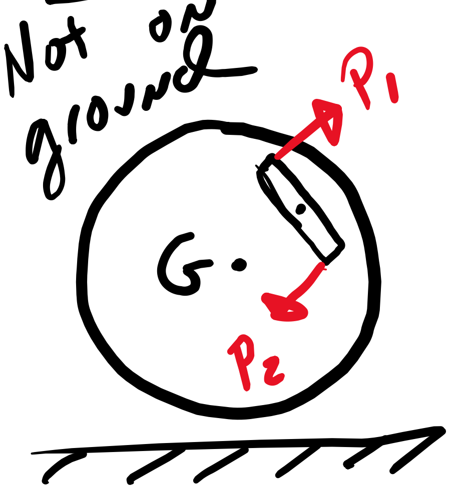
$\boxed{17.54}$

Not on ground



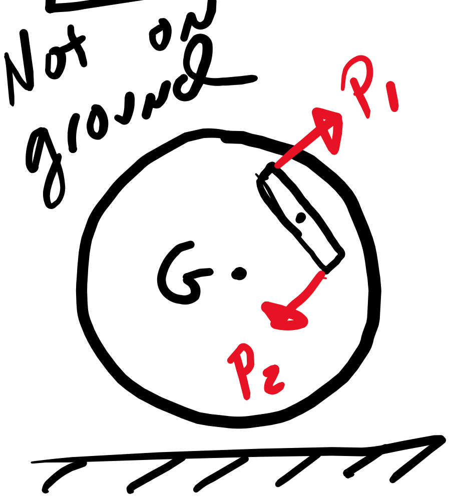
$\boxed{17.54}$

$\vec{P}_1 = -\vec{P}_2$



$\boxed{17.54}$

$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$



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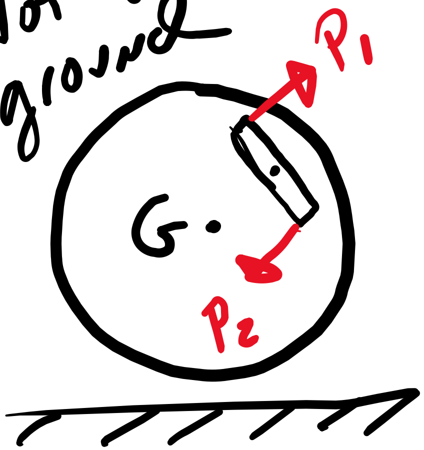
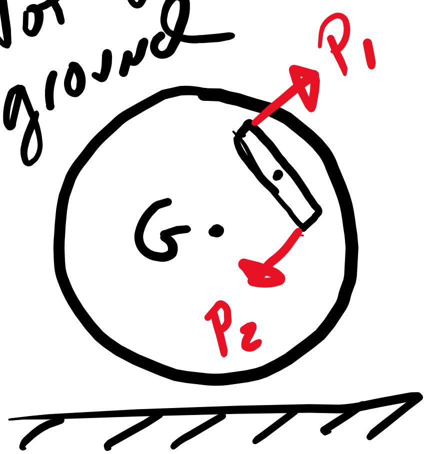


$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G

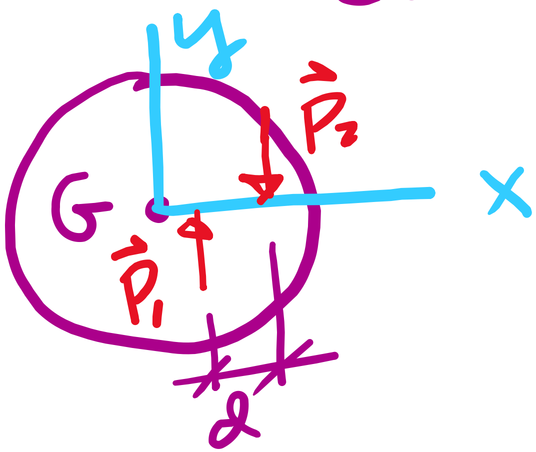
$\boxed{17.54}$

Not on ground

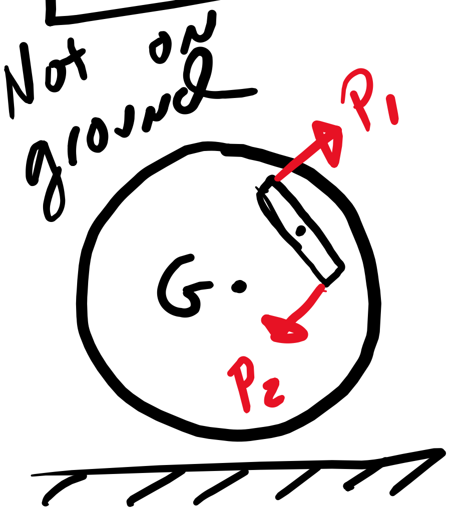


$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G

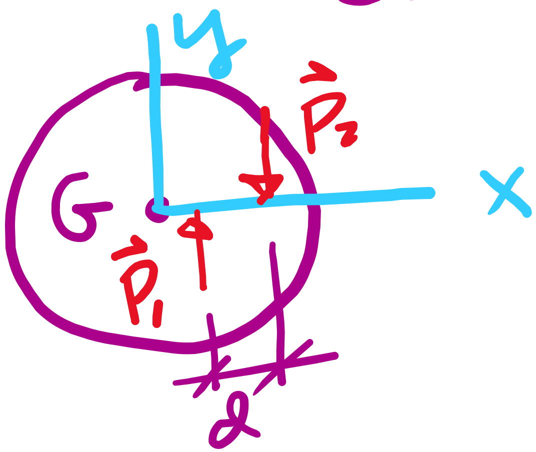
First, look at an easier problem:



$\boxed{17.54}$



$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G First, look at an easier problem:



$$\Sigma \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$

$\boxed{17.54}$

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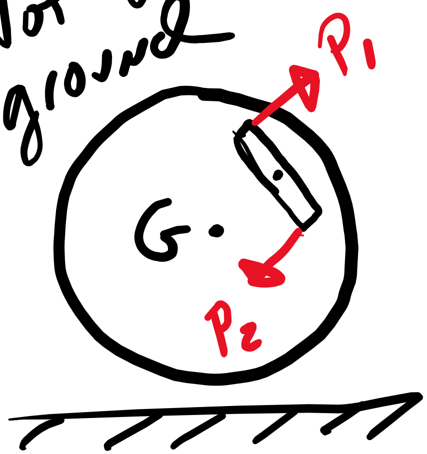
Not on ground



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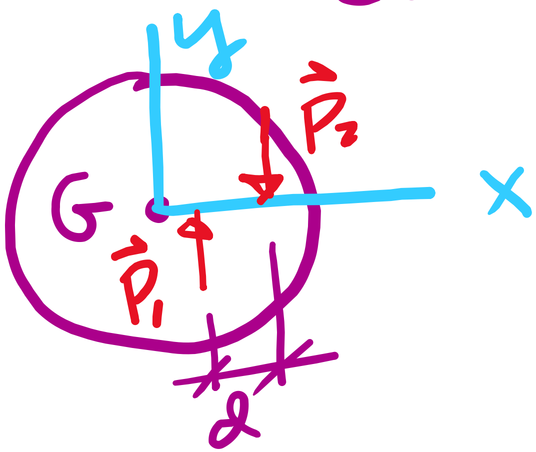
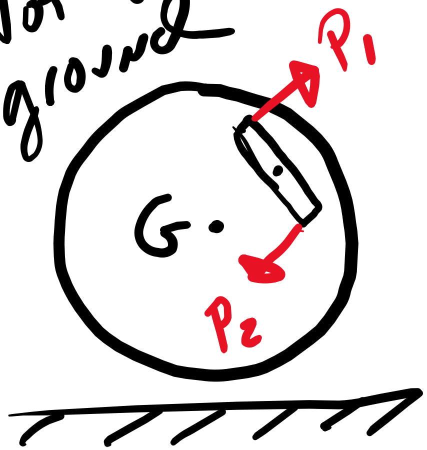


$$\Sigma \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$
$$= P(r_1 - r_2)\hat{z}$$

$\boxed{17.54}$

Not on ground



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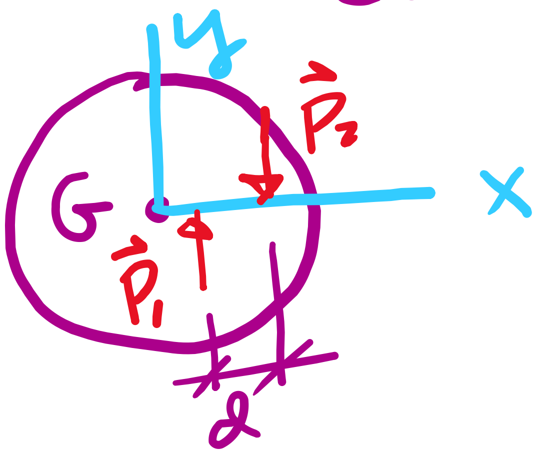

$$\Sigma \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$
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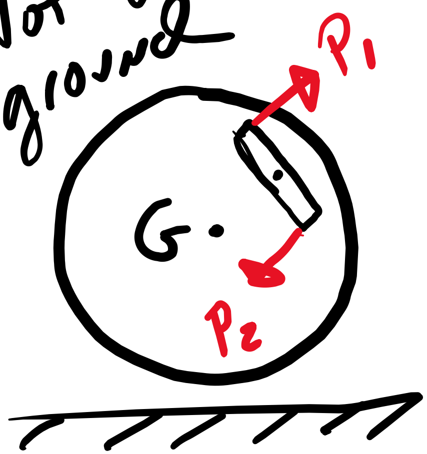

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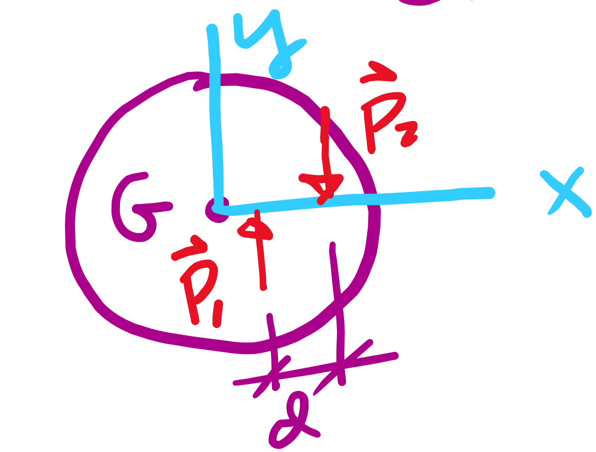

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$$= P(r_1 - r_2)\hat{z}$$
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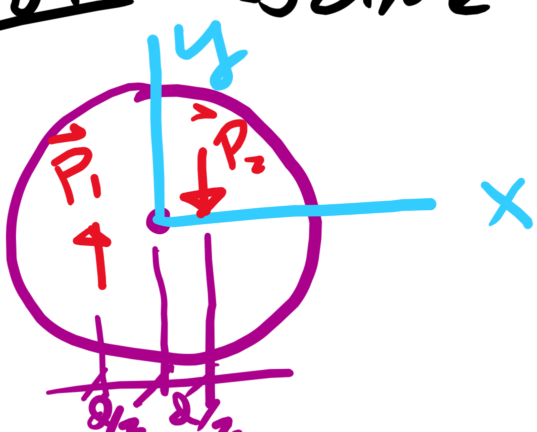
Note: Same as if geometry was



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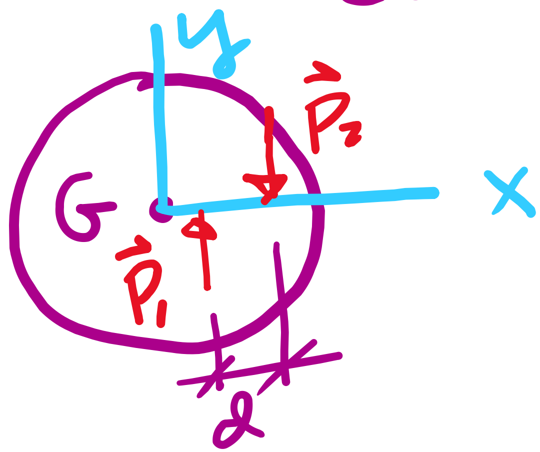

$$\Sigma \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$
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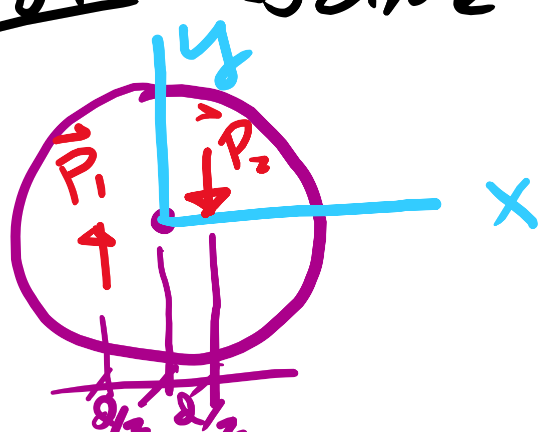
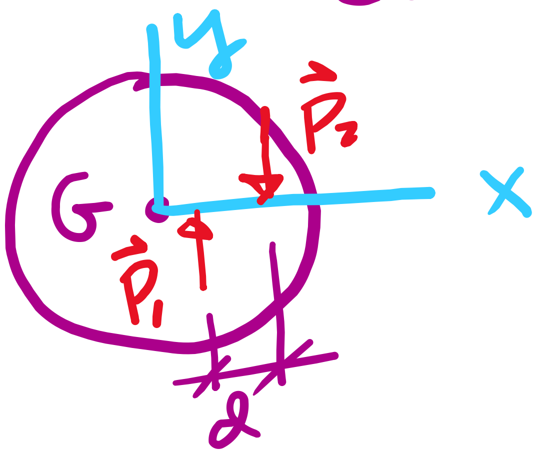


$$\Sigma \vec{M}_G = -\frac{d}{2} P \hat{z} - \frac{d}{2} P \hat{z}$$

$\boxed{17.54}$

Not on ground

$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G. First, look at an easier problem:



$$\sum \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$
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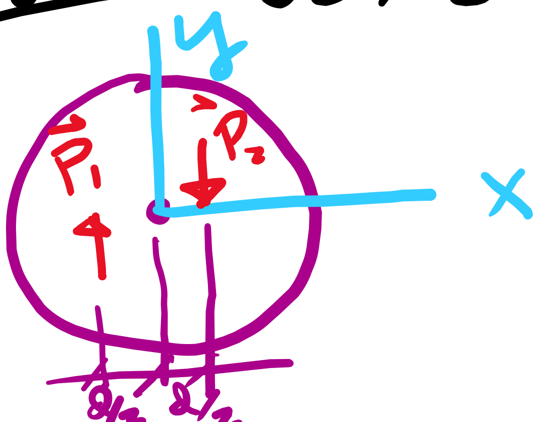
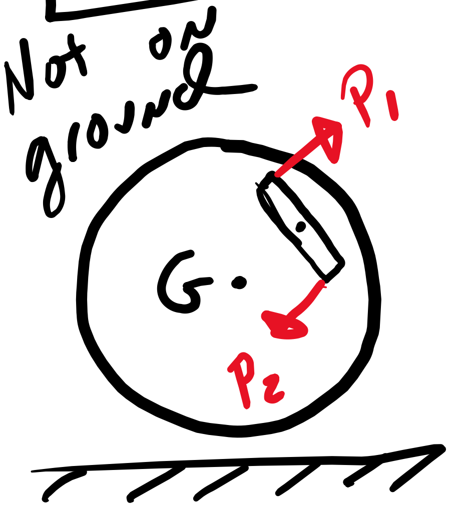


$$\sum \vec{M}_G = -\frac{d}{2}P\hat{z} - \frac{d}{2}P\hat{z} = -Pd\hat{z}$$

$\boxed{17.54}$



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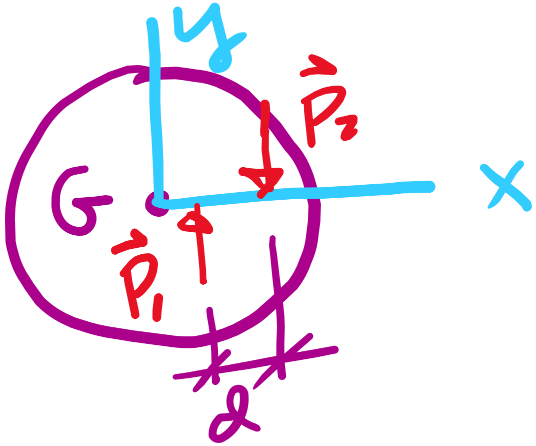

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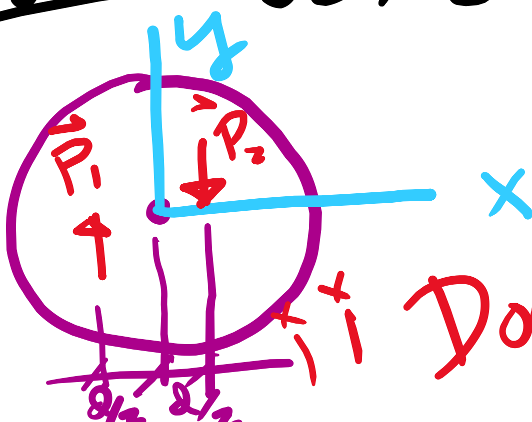
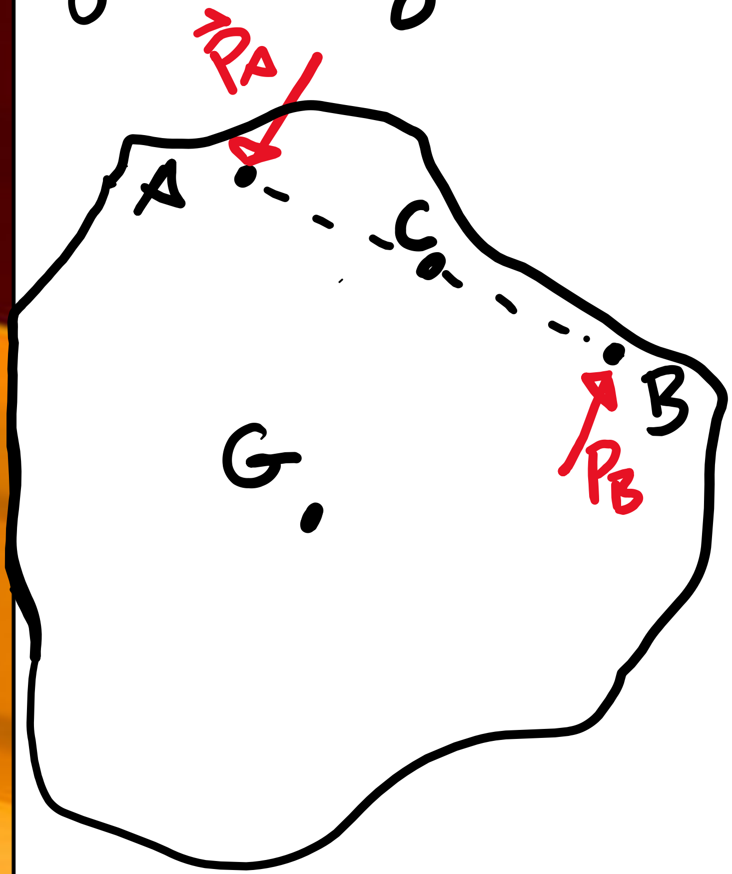


$$\sum \vec{M}_G = -\frac{d}{2}P\hat{z} - \frac{d}{2}P\hat{z} = -Pd\hat{z}$$

!! Does not seem to matter where couple is located on rigid body !!

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown

A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown



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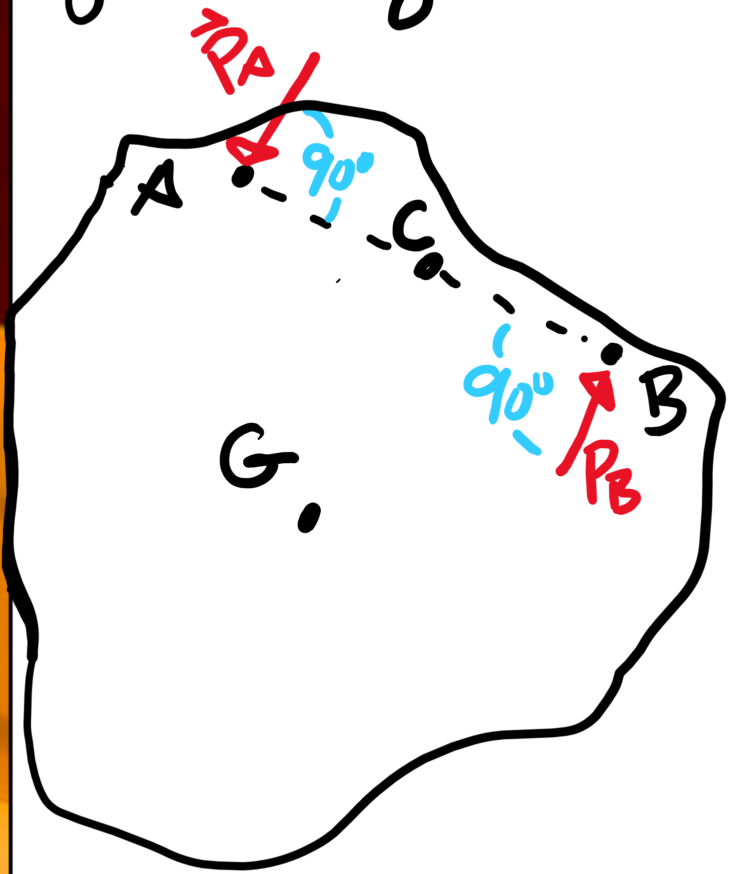

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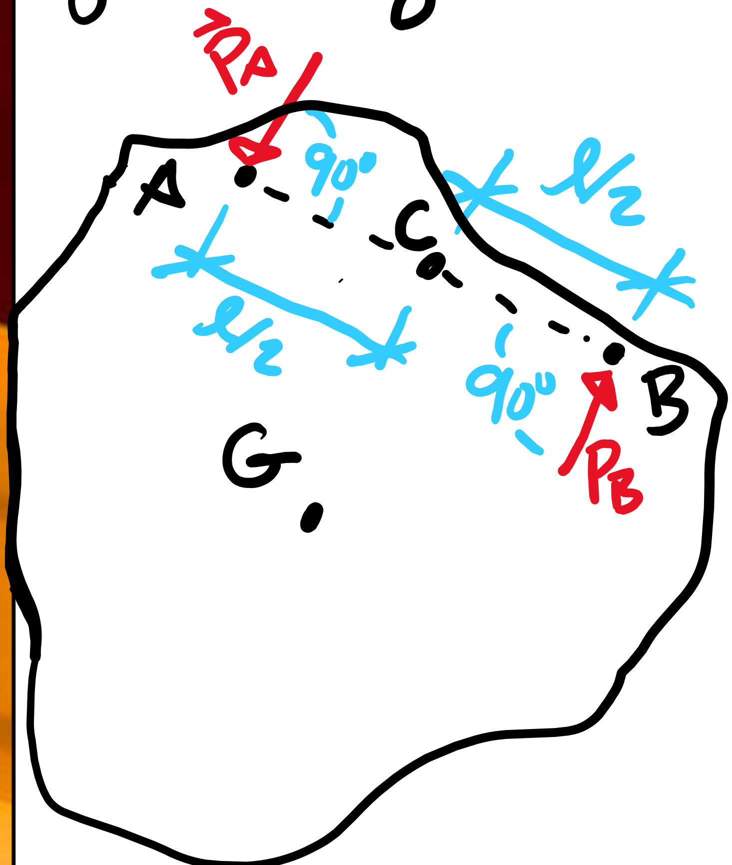

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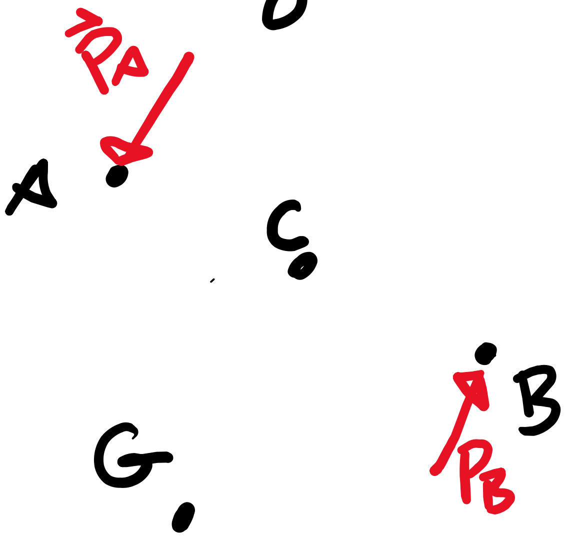

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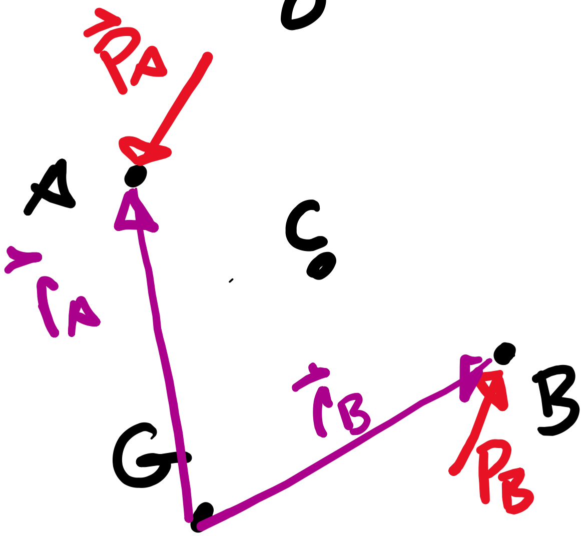

$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B$$

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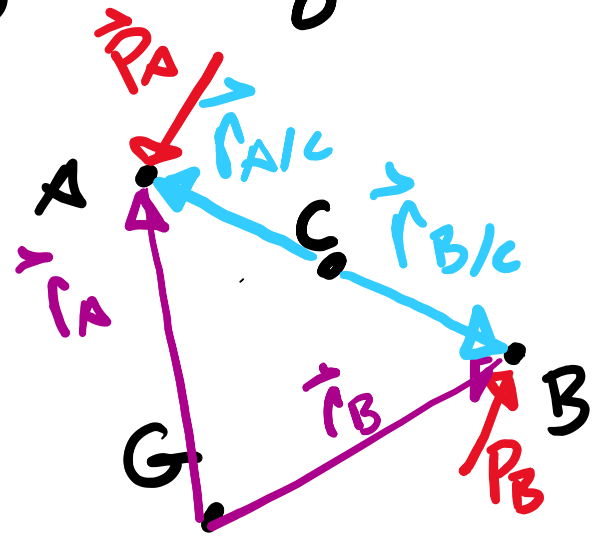
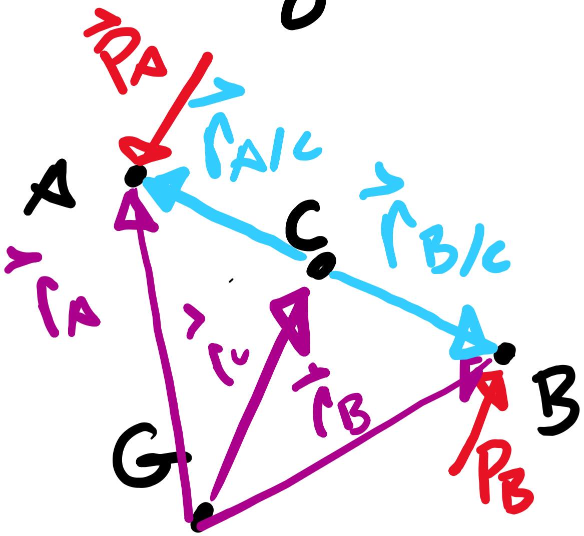


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown

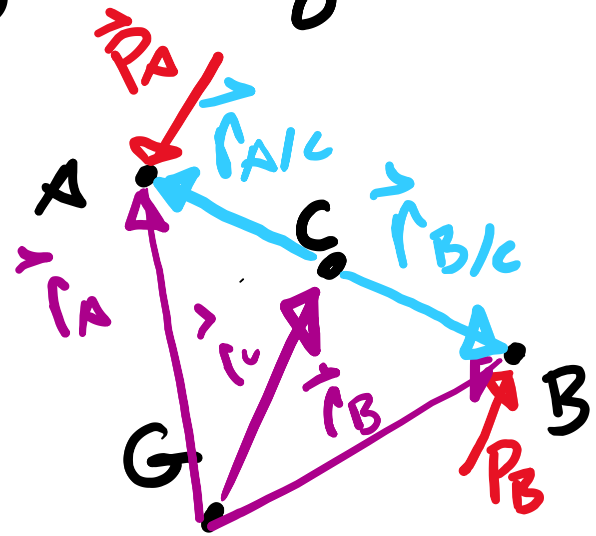


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



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$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

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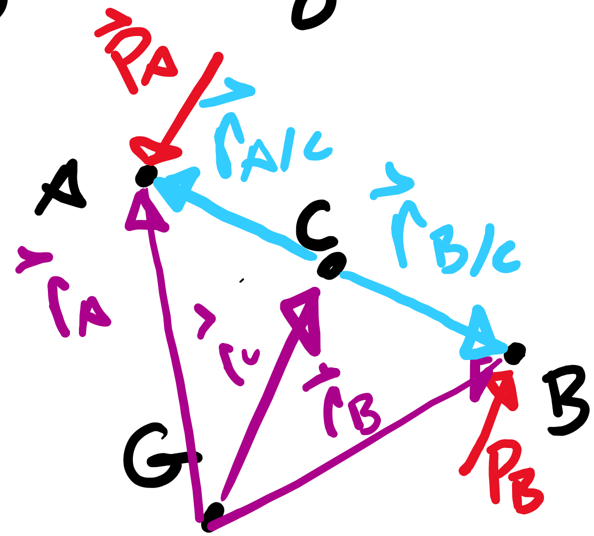


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

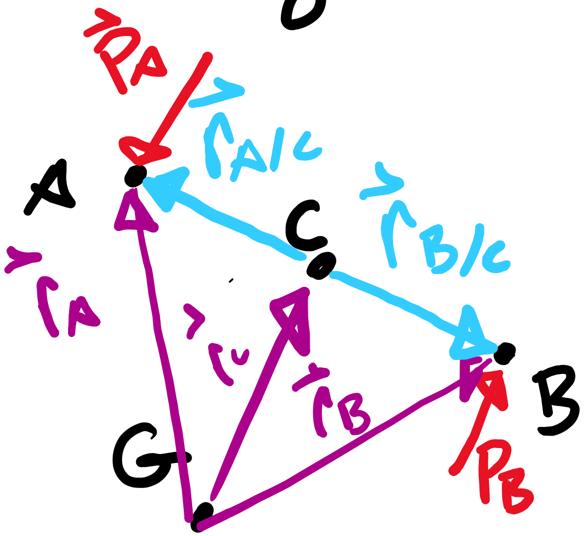
$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C \;\&\; \vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

So

$$\Sigma \vec{M}_C = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B$$

A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \ \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C \ \& \ \vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

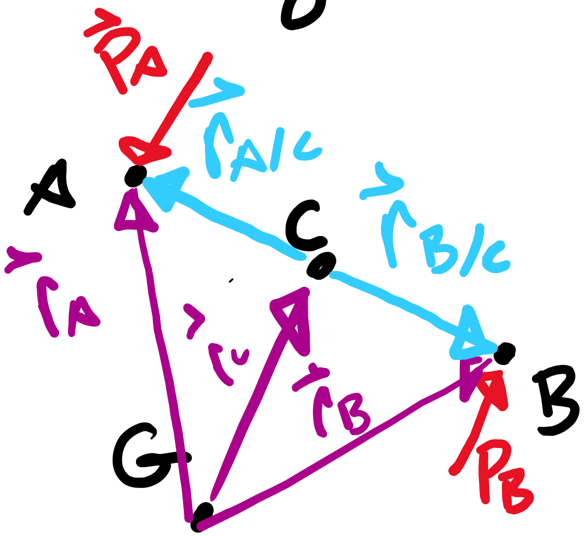
So

$$\Sigma \vec{M}_C = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B}_{\vec{M}_G} - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B \quad \Rightarrow$$

$$\Sigma \vec{M}_C = \vec{M}_G - \vec{r}_C \times (\vec{P}_A + \vec{P}_B)$$

ASU

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\sum \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \ \&$$

$$\sum \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

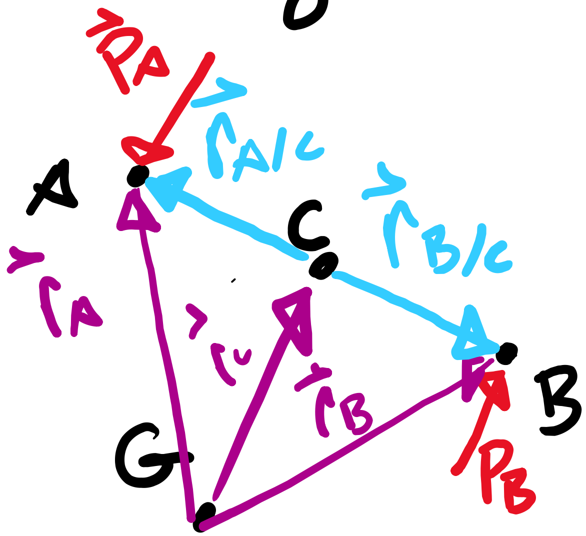
But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C \ \& \ \vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

So

$$\sum \vec{M}_C = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B}_{} - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B \Rightarrow$$

$$\sum \vec{M}_C = \vec{M}_G - \vec{r}_C \times (\vec{P}_A + \vec{P}_B)$$ But $\vec{P}_A = -\vec{P}_B$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\sum \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

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But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

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$$\sum \vec{M}_C = \vec{M}_G - \cancel{\vec{r}_C \times (\vec{P}_A + \vec{P}_B)}$$ But $\vec{P}_A = -\vec{P}_B$

so

$$\boxed{\sum \vec{M}_C = \vec{M}_G}$$

ASU

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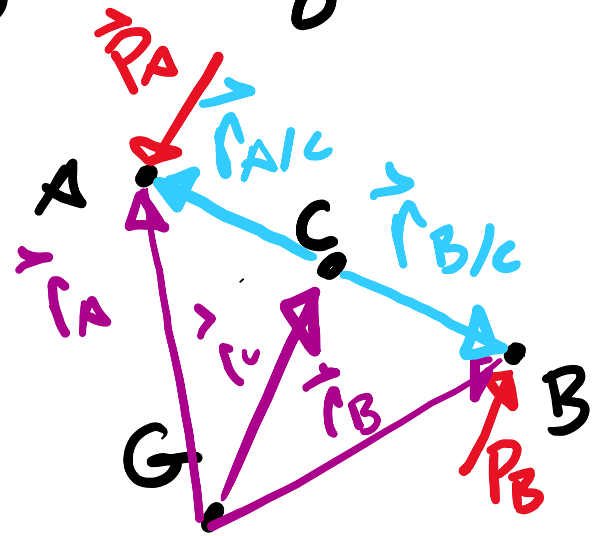

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But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

so

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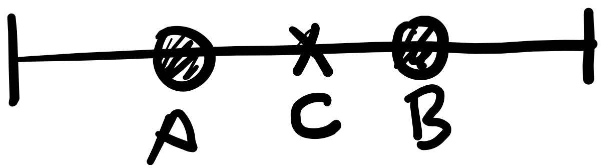
$$\Sigma \vec{M}_C = \vec{M}_G - \cancelto{0}{\vec{r}_C \times (\vec{P}_A + \vec{P}_B)}$$ But $\vec{P}_A = -\vec{P}_B$

so $$\Sigma \vec{M}_C = \vec{M}_G$$

Does <u>not</u> matter where couple is located on rigid body

Example Rod free to rotate about center

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod



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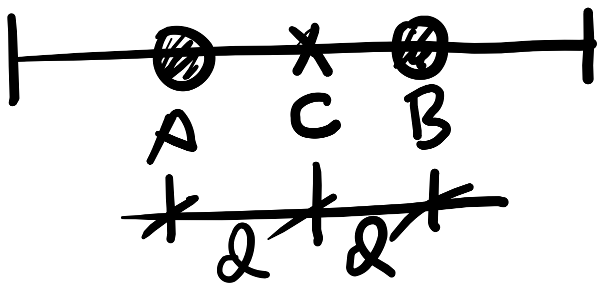

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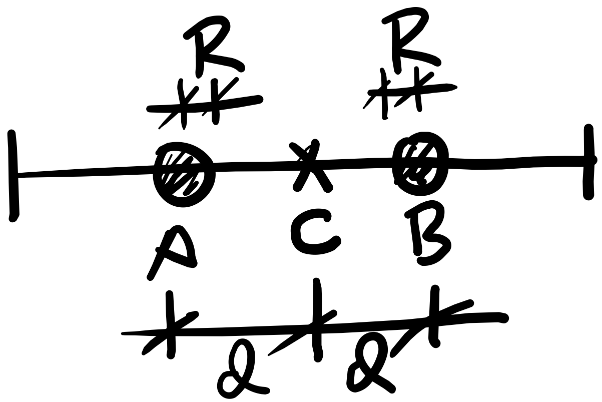

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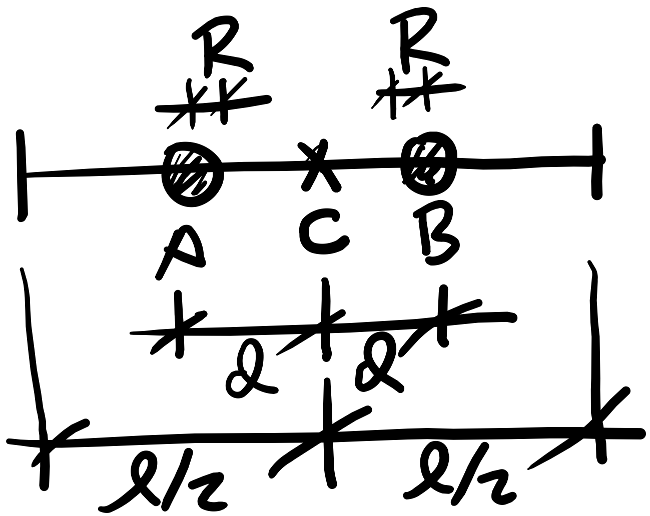
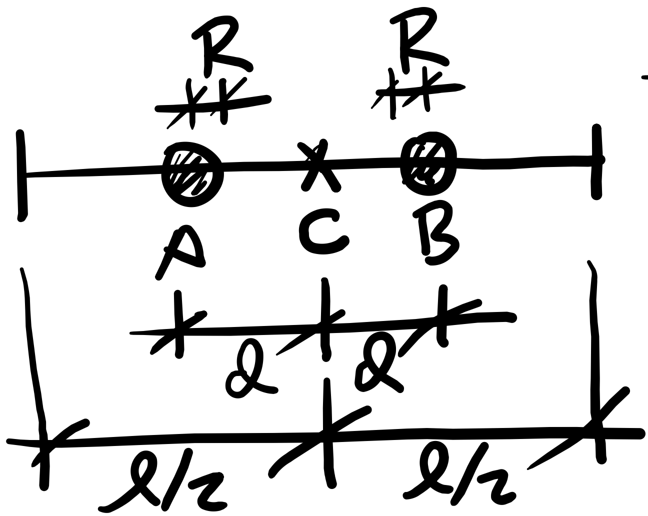


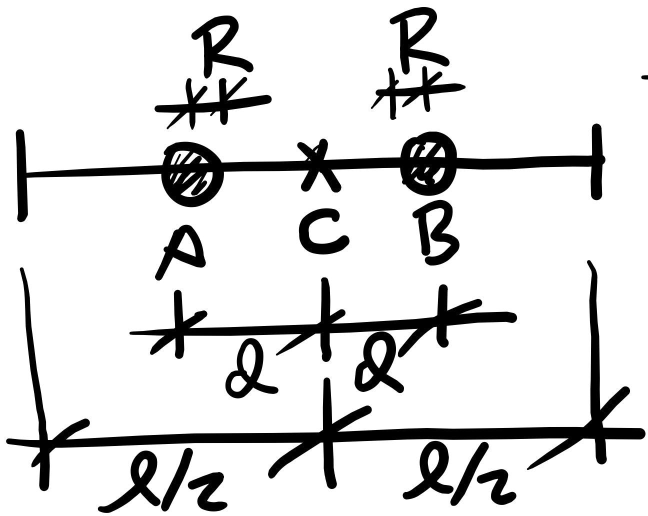
Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



m, l, I,

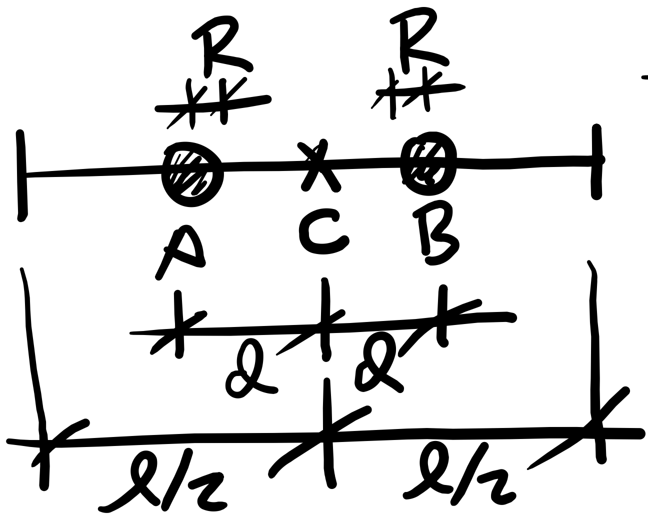
Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

ℓ_I, I_{rod},



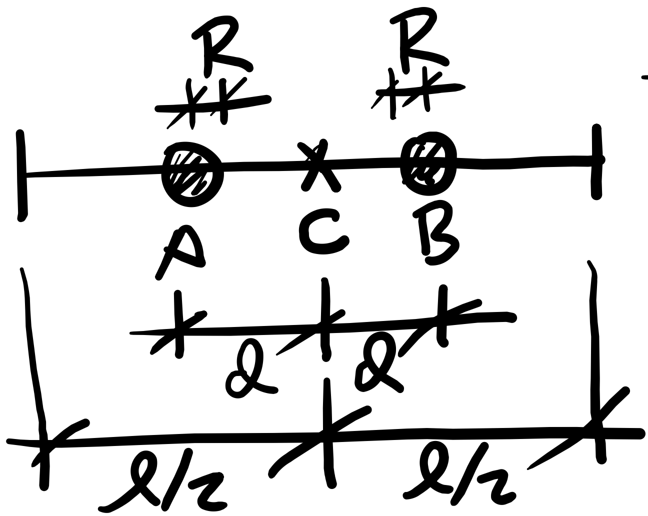
__Example__ Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

ω_I, I_{rod}, R,

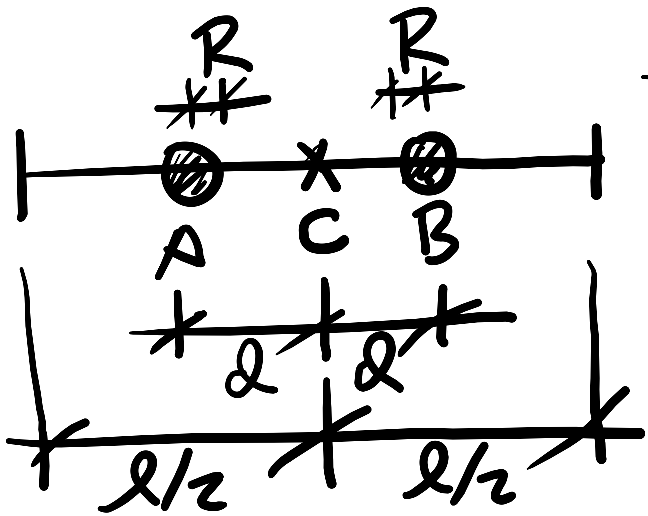


Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

ω_I, I_{rod}, R, d,



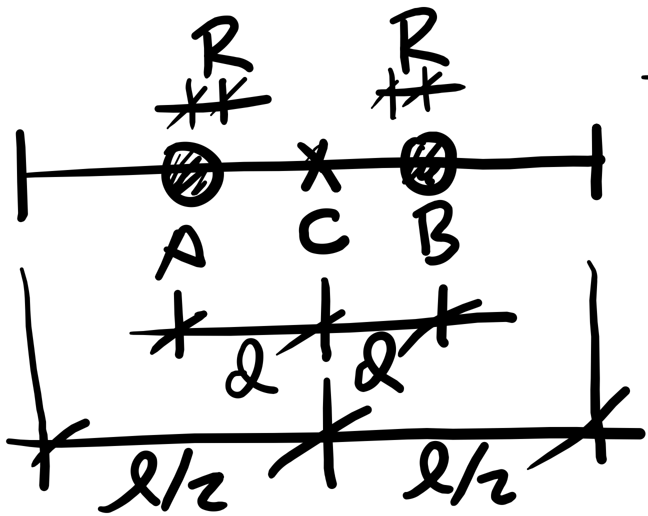
Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, I_{rod}, R, d, l,

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

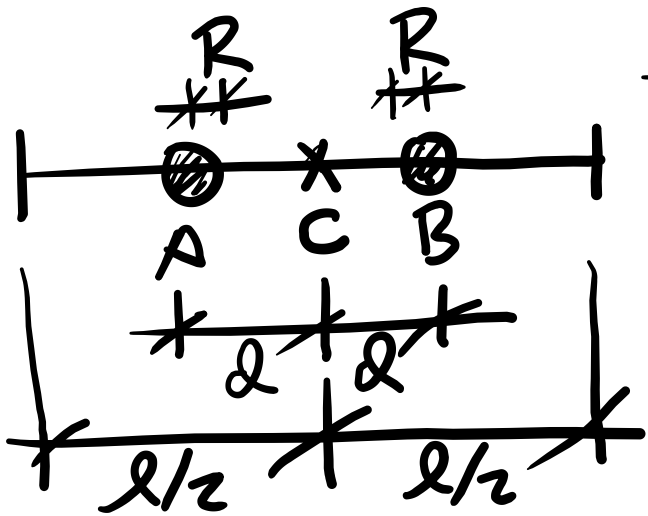
ω_I, $\bar{I}_{rod}$, R, d, ℓ, m_A & $m_A = m_B$



__Example__ Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

ω_I, I_{rod}, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod



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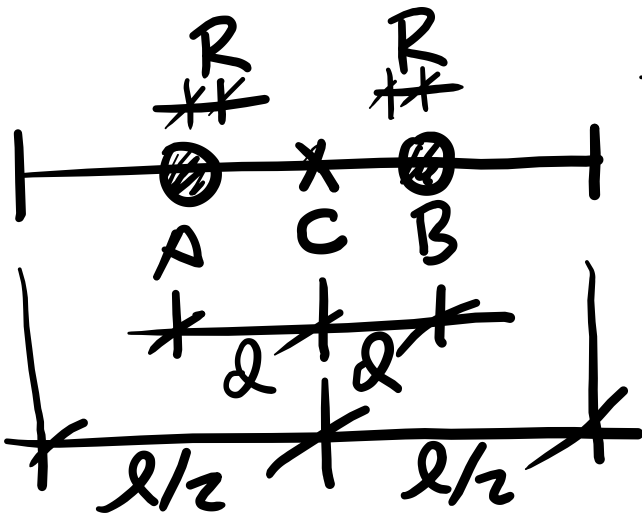
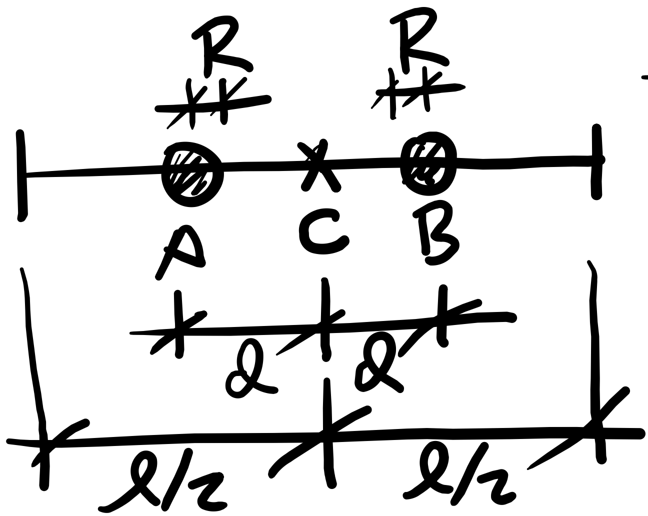


ω_I, I_{rod}, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod $\int \Sigma M dt = 0$

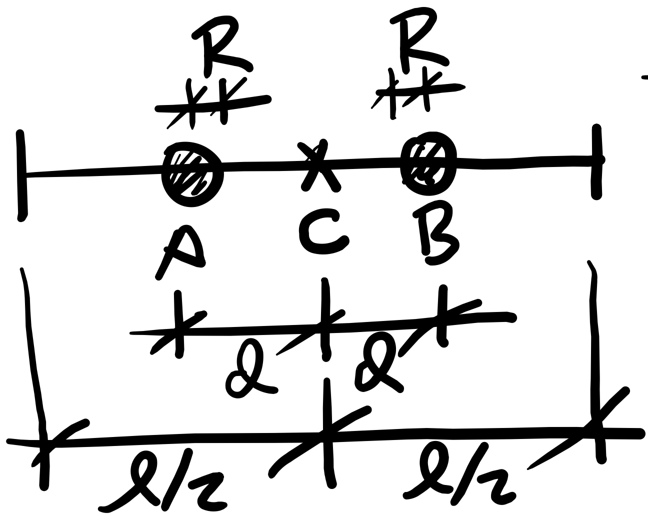
__Example__ Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, I_{rod}, R, d, ℓ, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod. $\int \Sigma M\, dt = 0$ so angular momentum is conserved

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, $\bar{I}_{rod}$, R, d, ℓ, M_A & $M_A = M_B$

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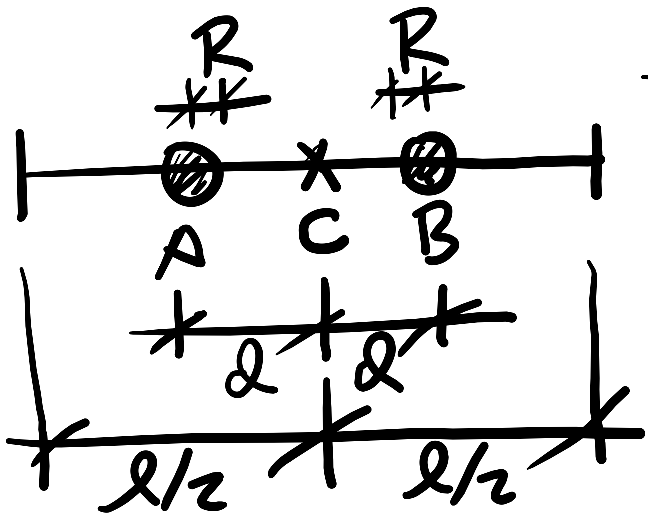

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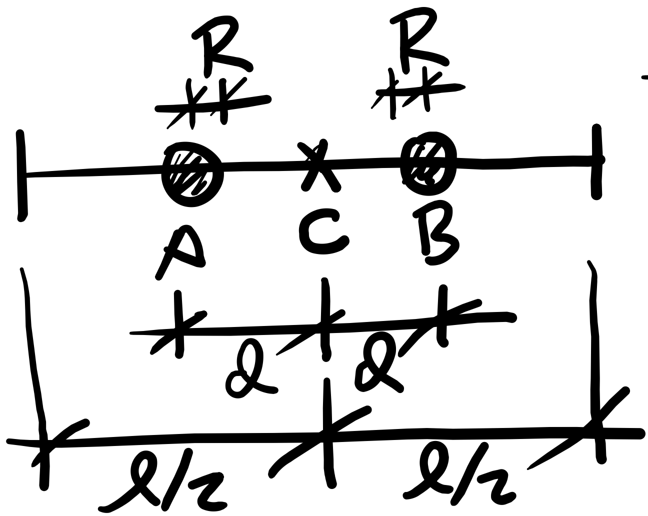

ω_I, I_{rod}, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod. $\int \Sigma M\, dt = 0$

So angular momentum is conserved $\Rightarrow \vec{H}_{CF} = \vec{H}_{CI}$

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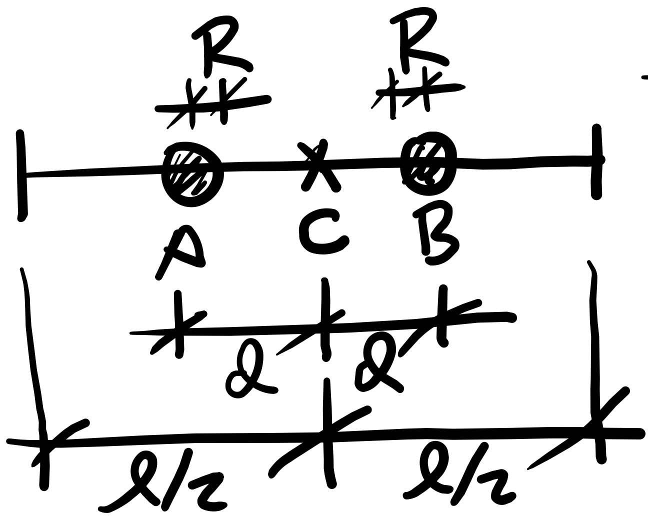

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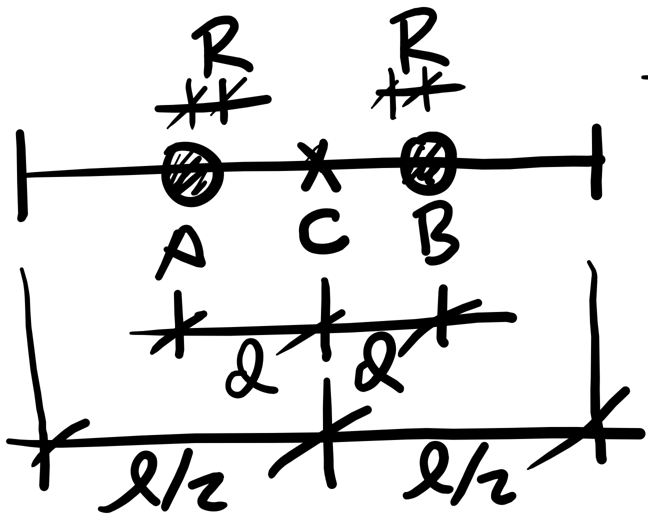

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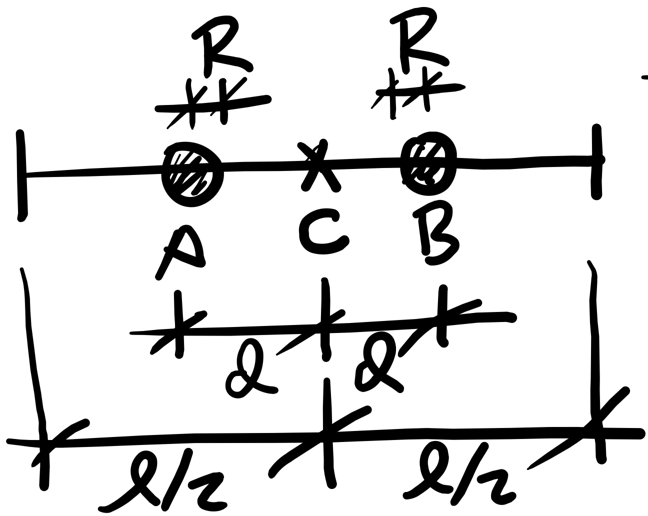

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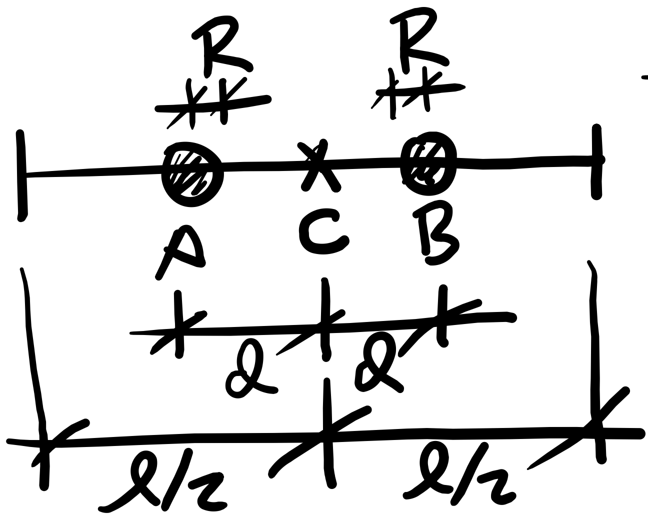
$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$

__Example__ Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

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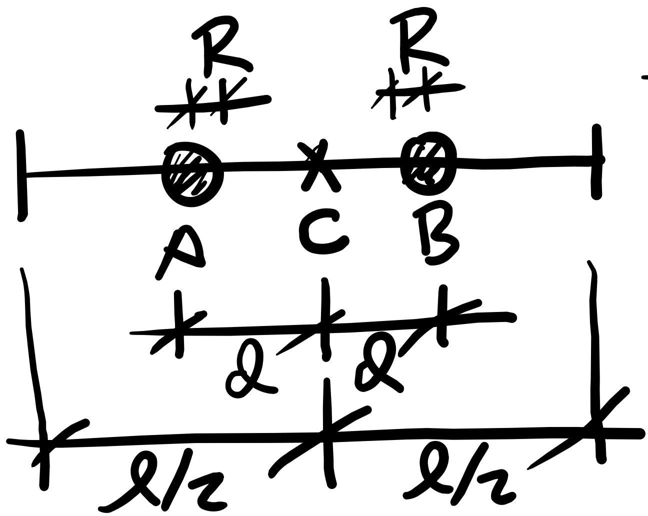
$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$ Now $H_{CF} = I_{CI} \Rightarrow$

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



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$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$ Now $H_{CF} = I_{CI} \Rightarrow$

$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2)\omega_F = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2)\omega_I$

From previous slide

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2\right)\omega_F =$$

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2\right)\omega_I$$

From previous slide

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$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2}\right]\omega_I$$

From previous slide

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since $d_F + R = \ell/2$

From previous slide

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2\right)\omega_F =$$

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2\right)\omega_I \Rightarrow$$

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since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

From previous slide

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2)\omega_F =$$

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2}\right]\omega_I \quad \&$$

since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

then

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A(\frac{\ell}{2} - R)^2}\right]\omega_I,$$

From previous slide

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2\right)\omega_F =$$

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2\right)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2}\right]\omega_I \quad \&$$

since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

then

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A\left(\frac{\ell}{2} - R\right)^2}\right]\omega_I \text{, where}$$

$$\bar{I}_A = \frac{2}{5}M_A R^2$$