Today Review

L30

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Friday Exam 3

Angular velocity $\equiv$ eee

Angular velocity $\equiv \omega$ & $\omega = \frac{d\theta}{dt}$

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We can write the velocity of a point on rigid body undergoing a pure rotation as $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ begins on the axis of rotation & ends at the point of interest.

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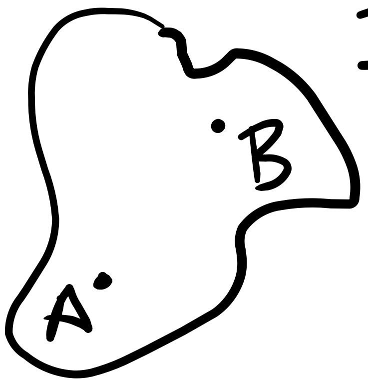
Also $\alpha = \omega \frac{d\omega}{d\theta}$

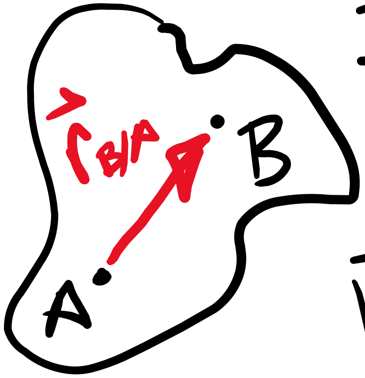
We can write the velocity of a point on rigid body undergoing a pure rotation as $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ begins on the axis of rotation & ends at the point of interest.

The acceleration can be written as

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times [\vec{\omega} \times \vec{r}]$$

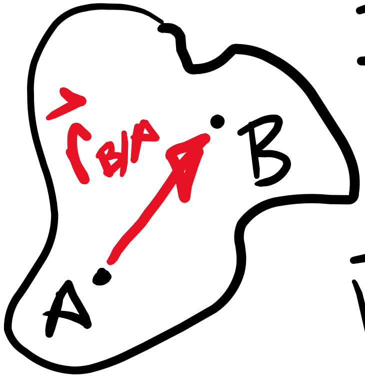


If points A & B are on a rigid body in plane motion



If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$



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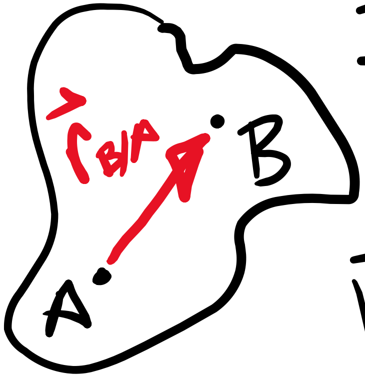

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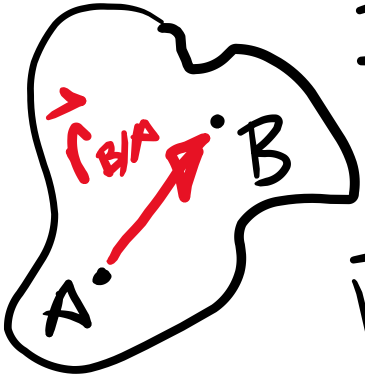
Rotating disk



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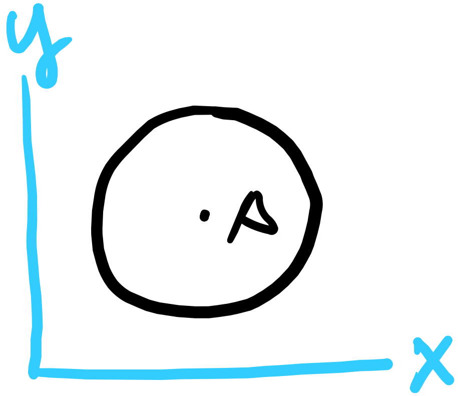

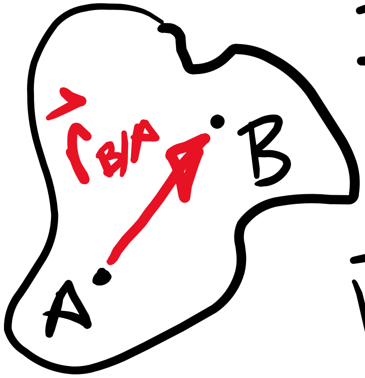

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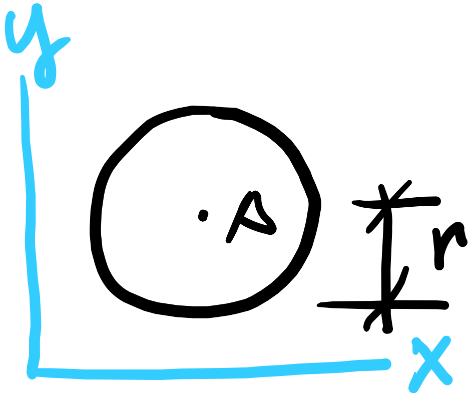

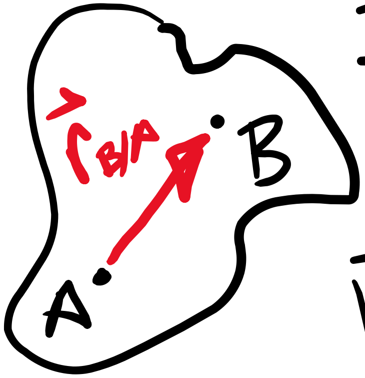

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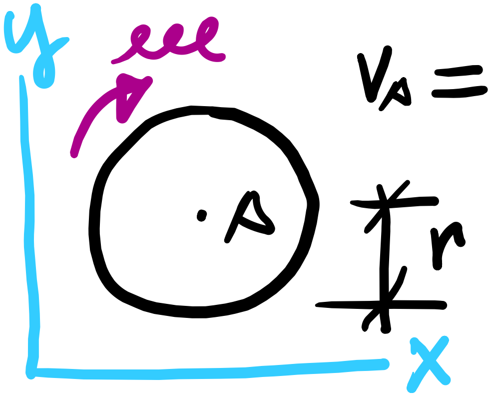
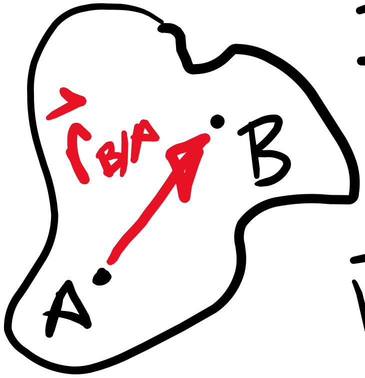


$V_A = 0$



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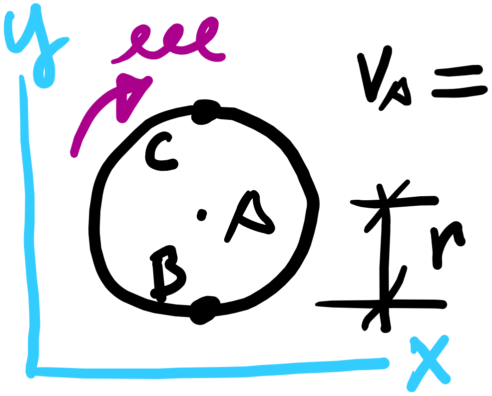

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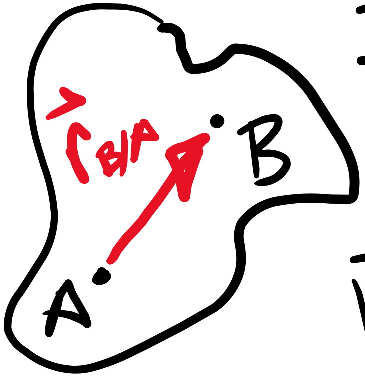

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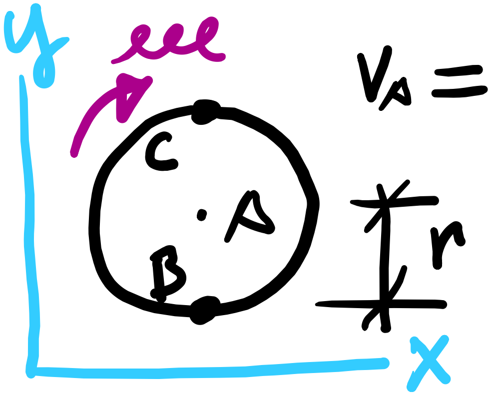
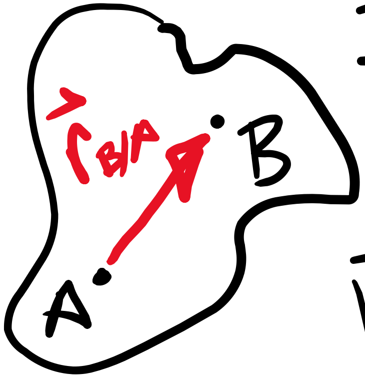


$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$



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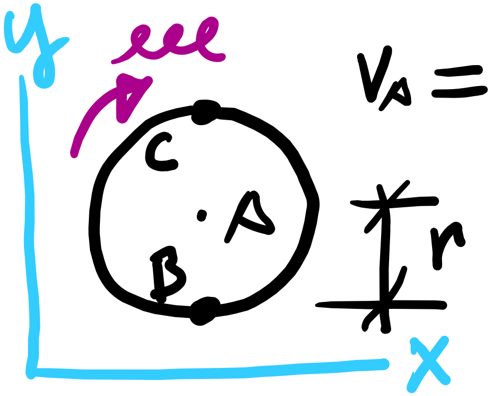

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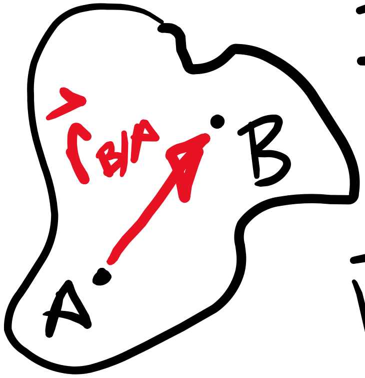

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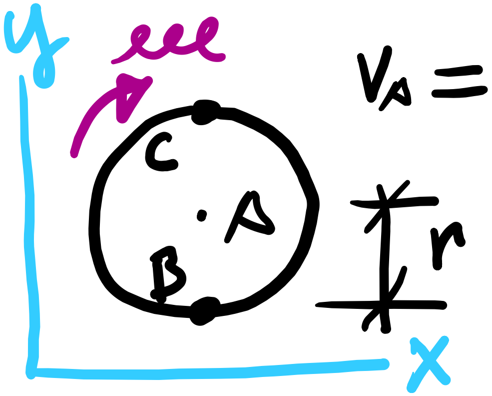

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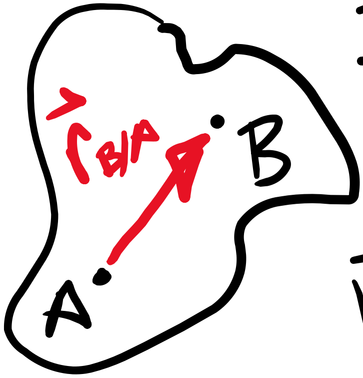

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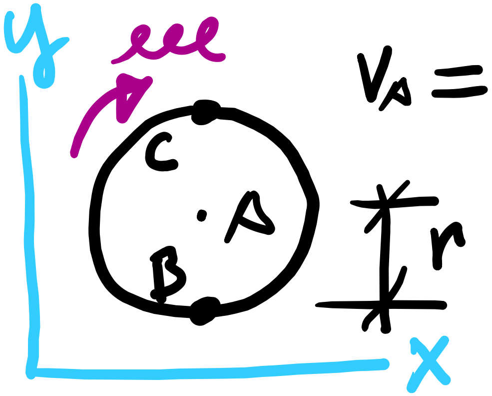

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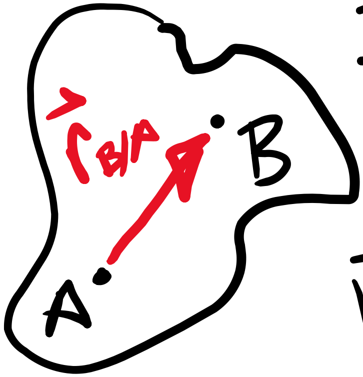

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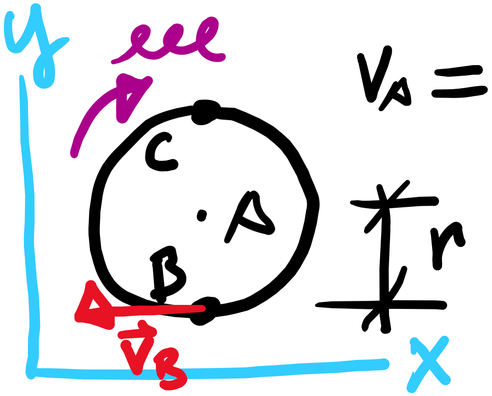

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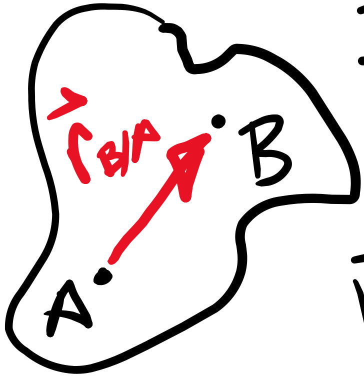

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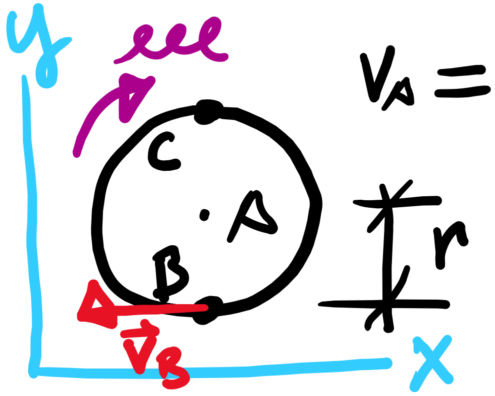

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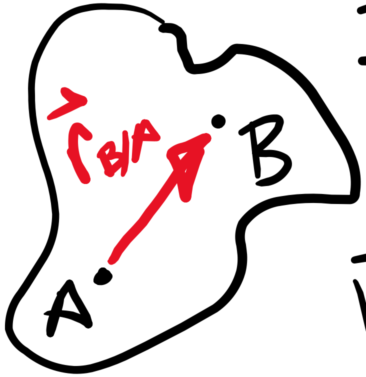
If disk is rolling without slipping

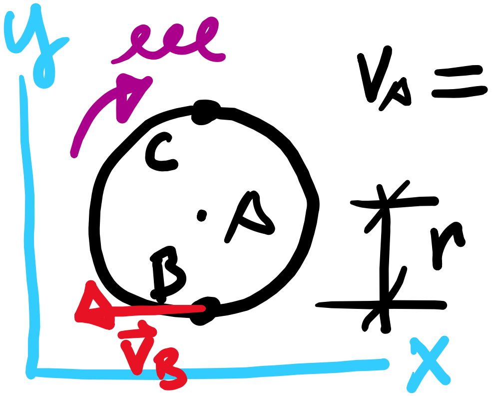
$\vec{V}_A = r\omega\hat{x}$



If points A & B are on a rigid body in plane motion

$$\vec V_{B/A} = \vec\omega \times \vec r_{B/A} \Rightarrow |\vec V_{B/A}| = \omega r_{B/A}$$

Rotating disk



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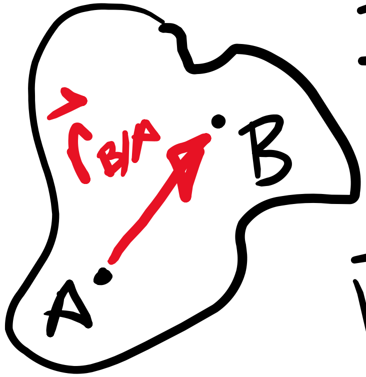
If disk is rolling without slipping

$\vec V_A = r\omega\hat x$ & we still have $\vec V_{B/A} = r\omega(-\hat x)$



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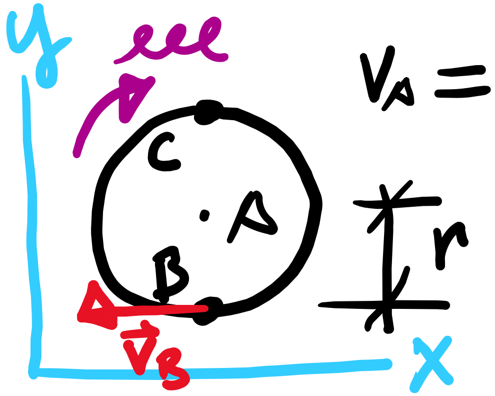
$$\vec V_{B/A} = \vec\omega \times \vec r_{B/A} \Rightarrow |\vec V_{B/A}| = \omega r_{B/A}$$

Rotating disk



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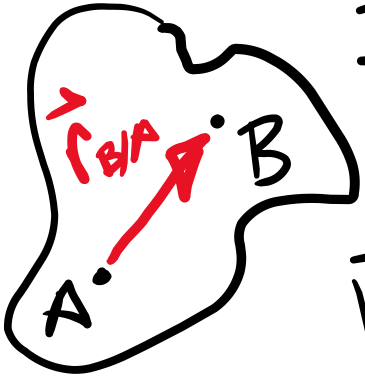
If disk is rolling without slipping

$\vec V_A = r\omega\hat x$ & we still have $\vec V_{B/A} = r\omega(-\hat x)$

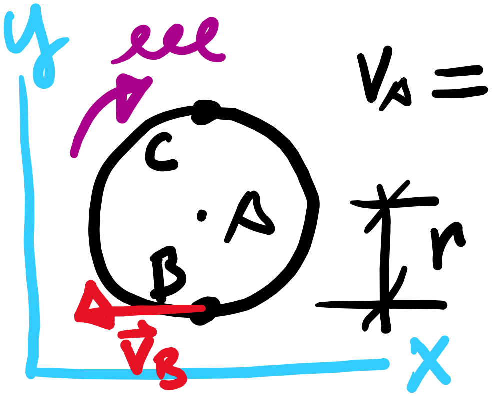
And this gives us $\vec V_B = \vec V_{B/A} + \vec V_A$



If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk



$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

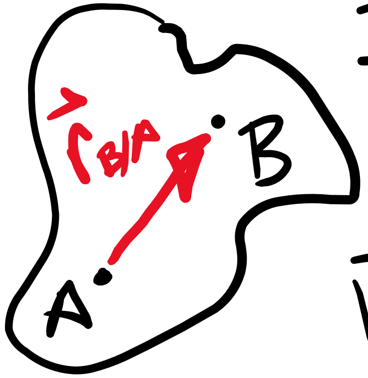
$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancelto{0}{\vec{V}_A} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

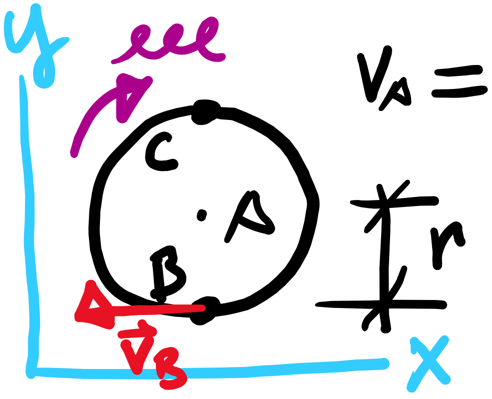
And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = r\omega\hat{x} - r\omega\hat{x}$



If points A & B are on a rigid body in plane motion

$$\vec V_{B/A} = \vec\omega \times \vec r_{B/A} \Rightarrow |\vec V_{B/A}| = \omega r_{B/A}$$

Rotating disk



$V_A = 0$

$\Rightarrow \vec V_{B/A} = r\omega(-\hat x)$

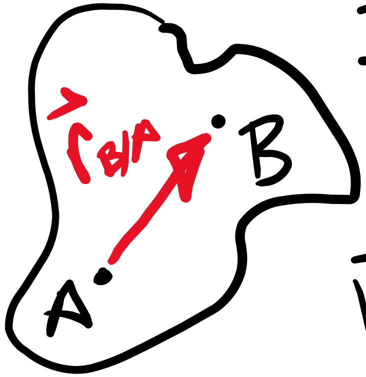
$\Rightarrow \vec V_B = \vec V_{B/A} + \cancel{\vec V_A}\ \cancel{0} \Rightarrow$

$\vec V_B = r\omega(-\hat x)$

If disk is rolling without slipping

$\vec V_A = r\omega\hat x$ & we still have $\vec V_{B/A} = r\omega(-\hat x)$

And this gives us $\vec V_B = \vec V_{B/A} + \vec V_A = r\omega\hat x - r\omega\hat x = 0$



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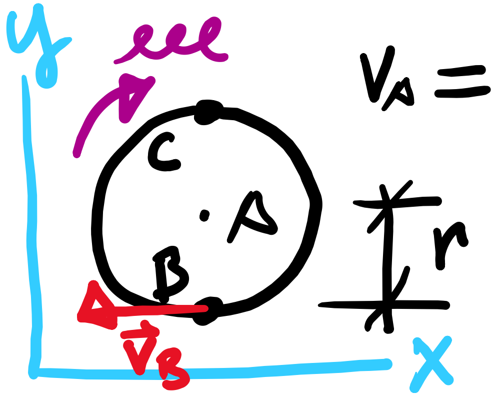
Rotating disk



$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancel{\vec{V}_A}\ \cancel{\vec{0}} \Rightarrow$

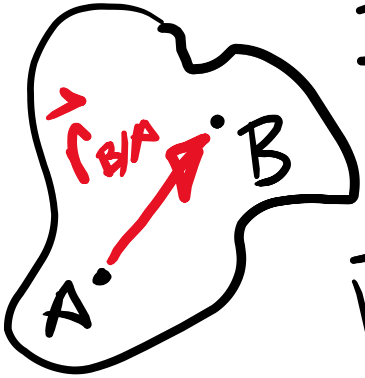
$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = r\omega\hat{x} - r\omega\hat{x} = 0$

So $\vec{V}_B$ is at rest at this moment in time!

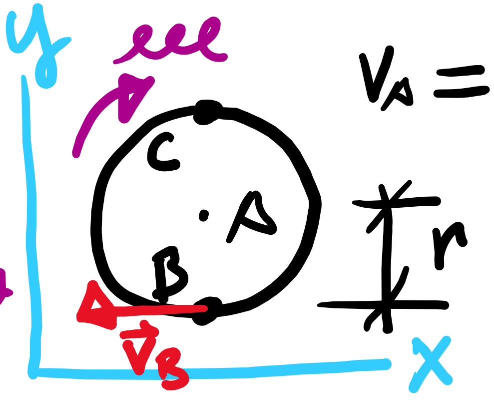


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?

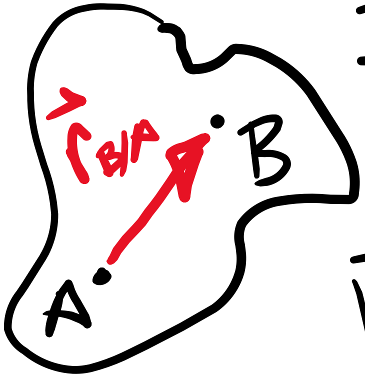


$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancel{\vec{V}_A} \xrightarrow{} \cancel{0} \Rightarrow$

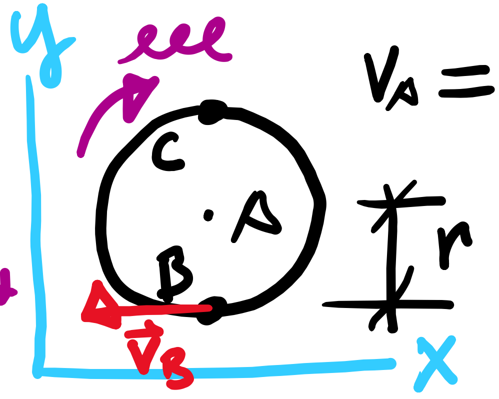
$\vec{V}_B = r\omega(-\hat{x})$



If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk



What about point C at top of disk?

$V_A = 0$

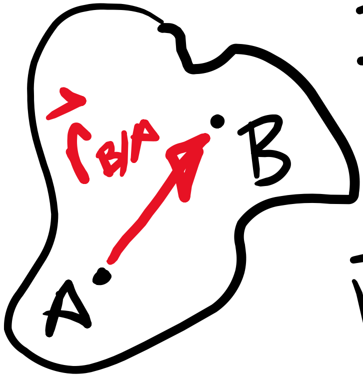
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancel{\vec{V}_A} \cancel{0} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$$V_c = \vec{V}_{c/A} + \vec{V}_A$$



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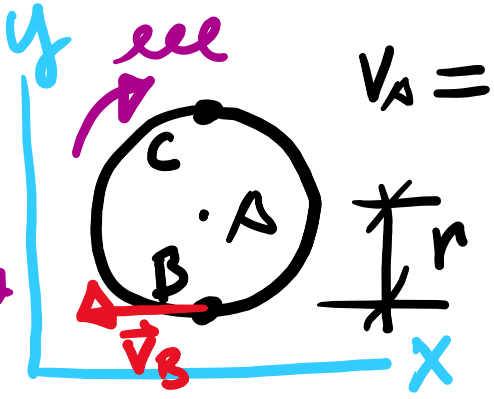


$V_A = 0$

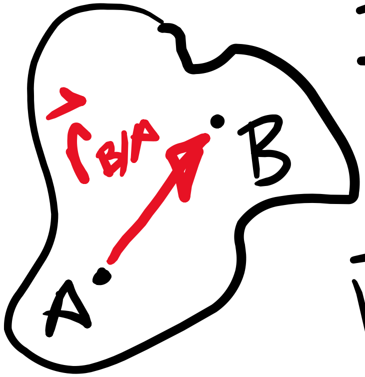
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \overset{\cancel{0}}{\cancel{\vec{V}_A}} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$V_c = \vec{V}_{C/A} + \vec{V}_A$, but $\vec{V}_{C/A} = r\omega\hat{x}$

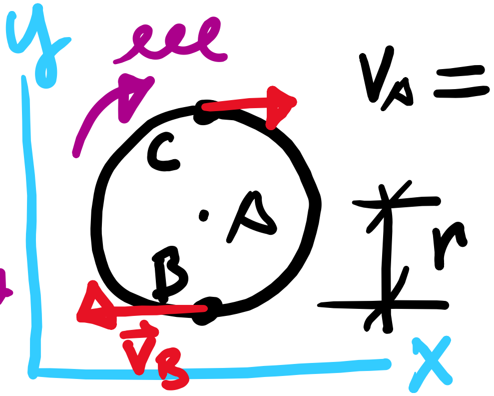


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = 0$

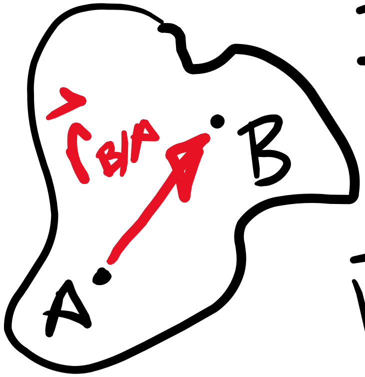
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \overset{0}{\cancel{\vec{V}_A}} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$V_c = \vec{V}_{c/A} + \vec{V}_A$, but $\vec{V}_{c/A} = r\omega\hat{x}$

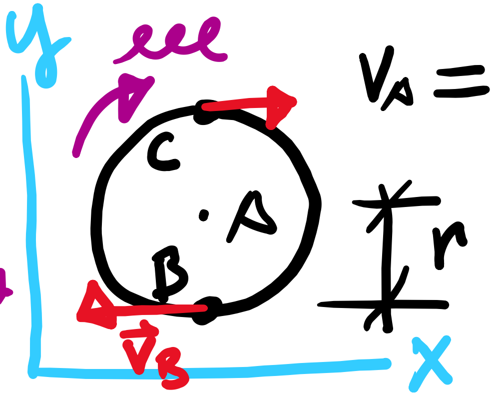


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

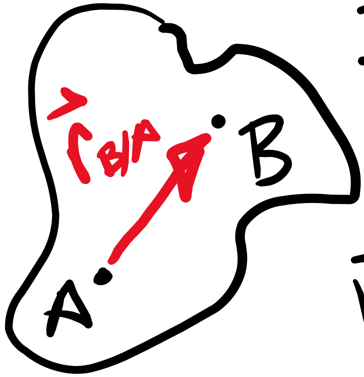
$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \overset{0}{\cancel{\vec{V}_A}} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A$, but $\vec{V}_{C/A} = r\omega\hat{x}$ So

$\vec{V}_C = r\omega\hat{x} + r\omega\hat{x}$

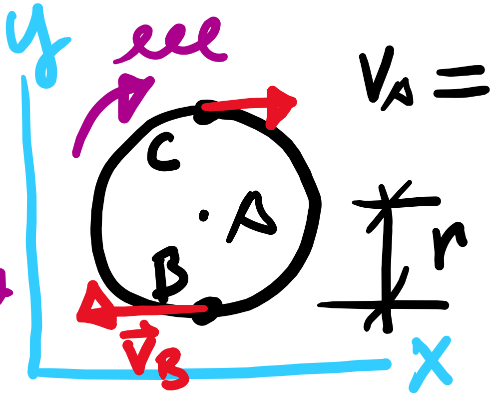


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = \cancel{0}$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancel{\vec{V}_A}\ \cancel{0} \Rightarrow$

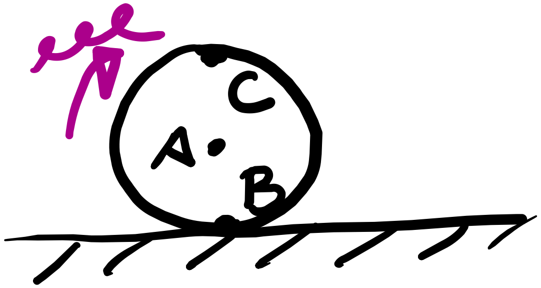
$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

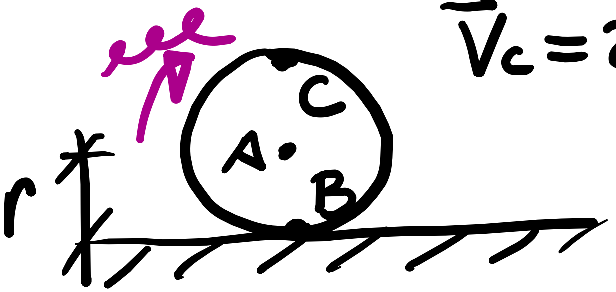
$\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A$, but $\vec{V}_{C/A} = r\omega\hat{x}$ So

$\vec{V}_C = r\omega\hat{x} + r\omega\hat{x} \Rightarrow \vec{V}_C = 2r\omega\hat{x}$

Wheel rolling with no slipping

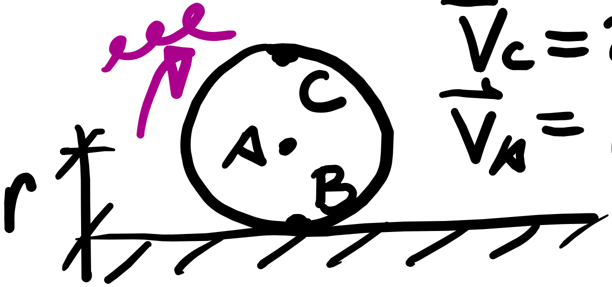


Wheel rolling with no slipping



$\vec{V}_c = 2r eee \hat{x}$

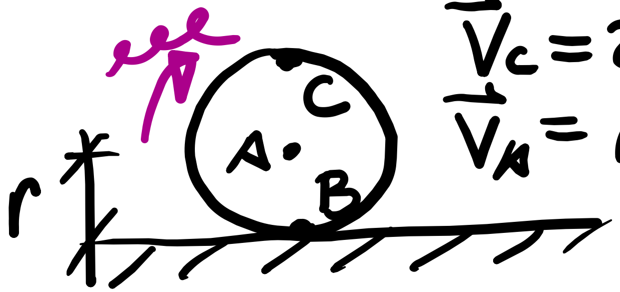
Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\,\hat{x}$

$\vec{V}_A = r\omega\,\hat{x}$

Wheel rolling with no slipping

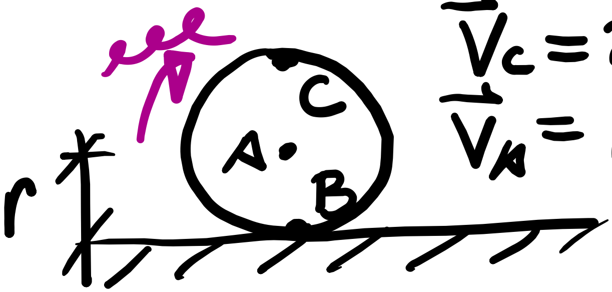


$\vec{V}_C = 2r\omega\,\hat{x}$

$\vec{V}_A = r\omega\,\hat{x}$

& $\vec{V}_B = 0$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

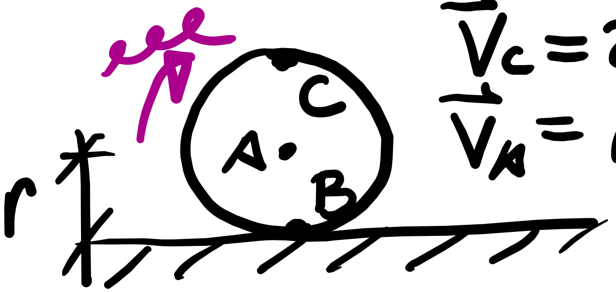
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write

$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

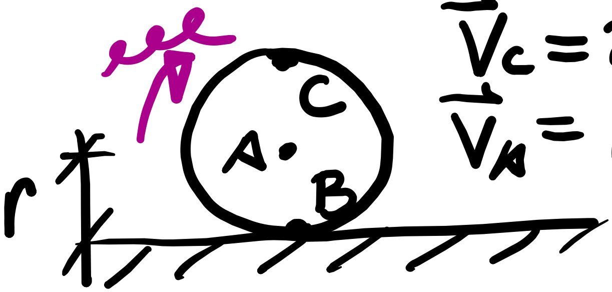
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write

$$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$$

Wheel rolling with no slipping



$\vec{V}_c = 2r\omega\hat{x}$

$\vec{V}_A = r\omega\hat{x}$

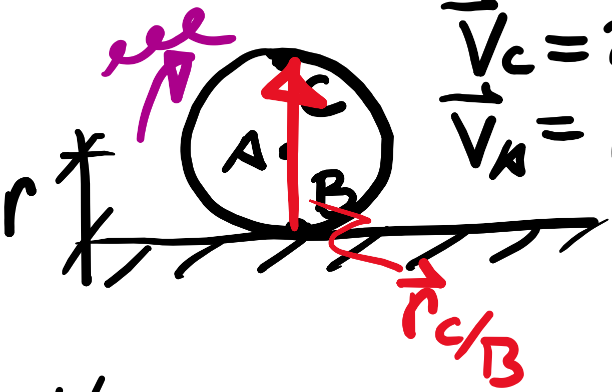
& $\vec{V}_B = 0$

We can now write

$\vec{V}_{c/B} = \vec{V}_c - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using

$V_{c/B} = r_{c/B}\omega$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

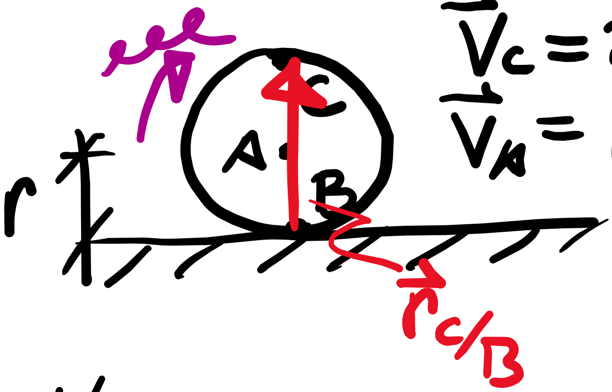
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write $\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using

$V_{C/B} = r_{C/B}\,\omega$

Wheel rolling with no slipping



$\vec{V}_c = 2r\omega\hat{x}$

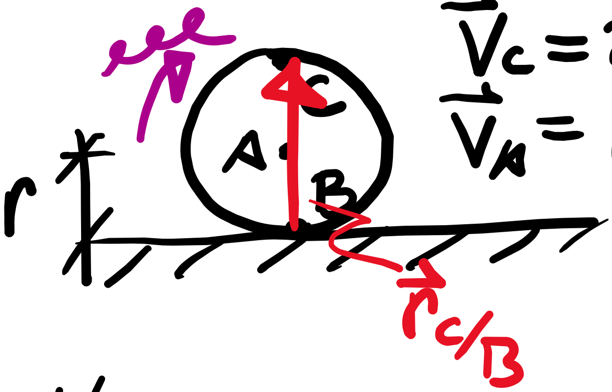
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write $\vec{V}_{c/B} = \vec{V}_c - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using

$V_{c/B} = r_{c/B}\omega = 2r\omega$

Wheel rolling with no slipping



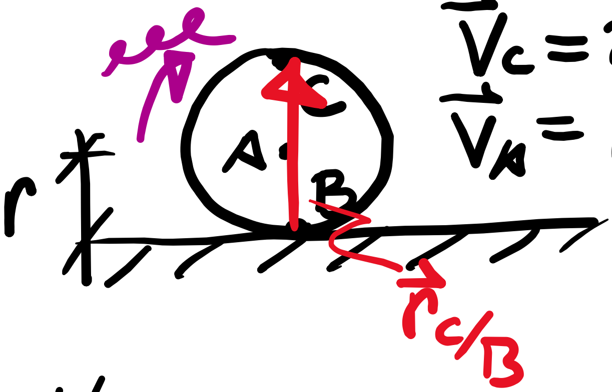
$\vec V_C = 2r\omega\hat x$

$\vec V_A = r\omega\hat x$

& $\vec V_B = 0$

We can now write $\vec V_{C/B} = \vec V_C - \vec V_B = 2r\omega\hat x$. However, we could have just written this down using $V_{C/B} = r_{C/B}\omega = 2r\omega$, since "The angular velocity is independent of the reference point"

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

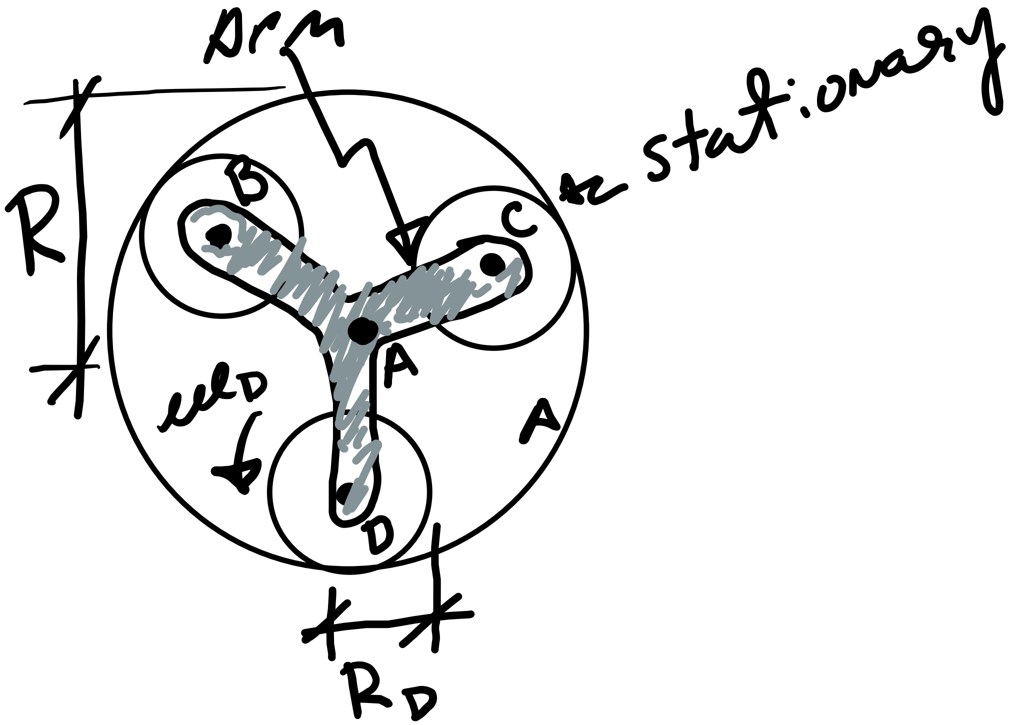
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

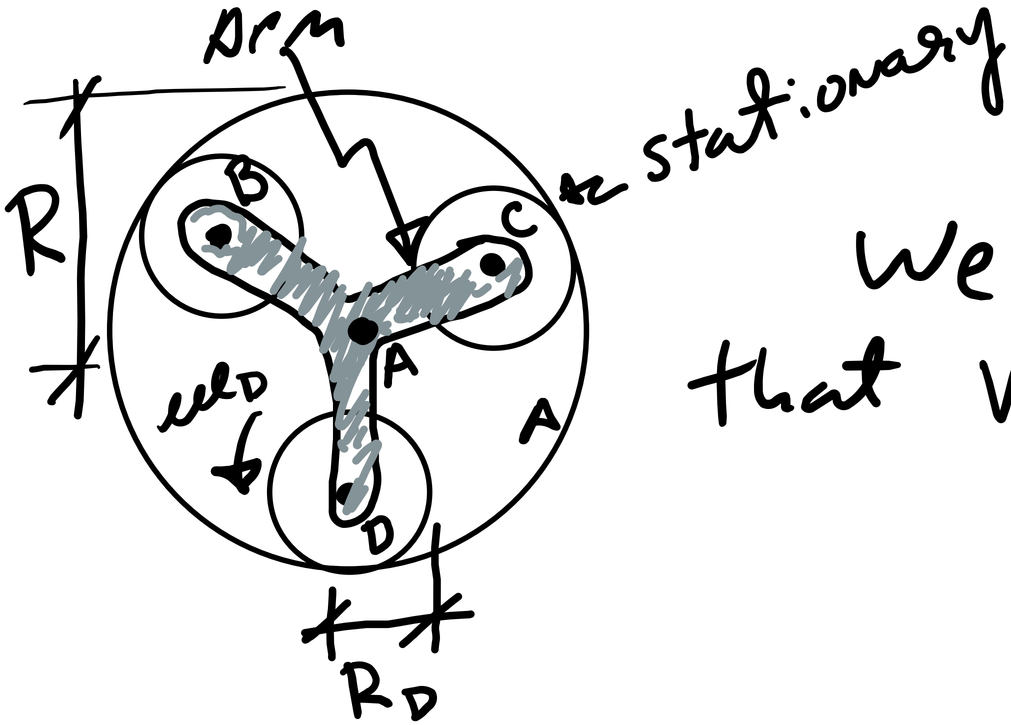
We can now write

$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using $V_{C/B} = r_{C/B}\omega = 2r\omega$, since "The angular velocity is independent of the reference point" & $r_{C/B} = 2r$

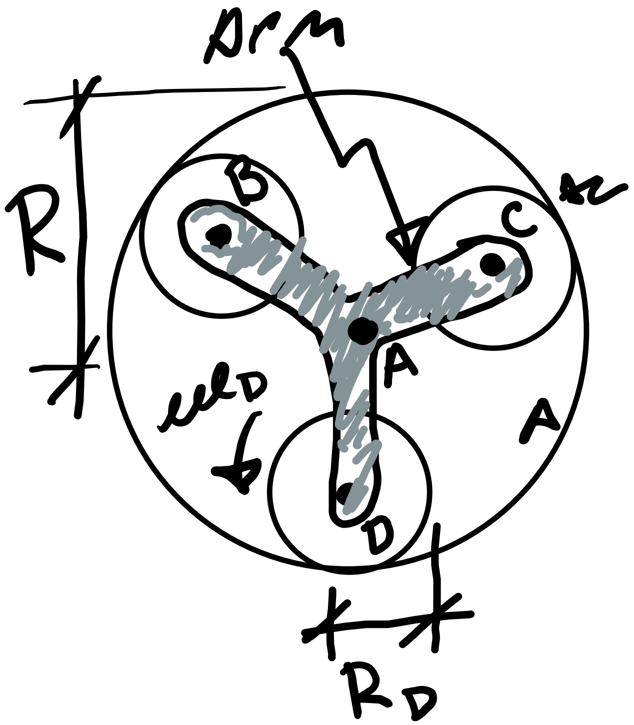
Example: Find ω_{ARM}



Example: Find ω_{ARM}

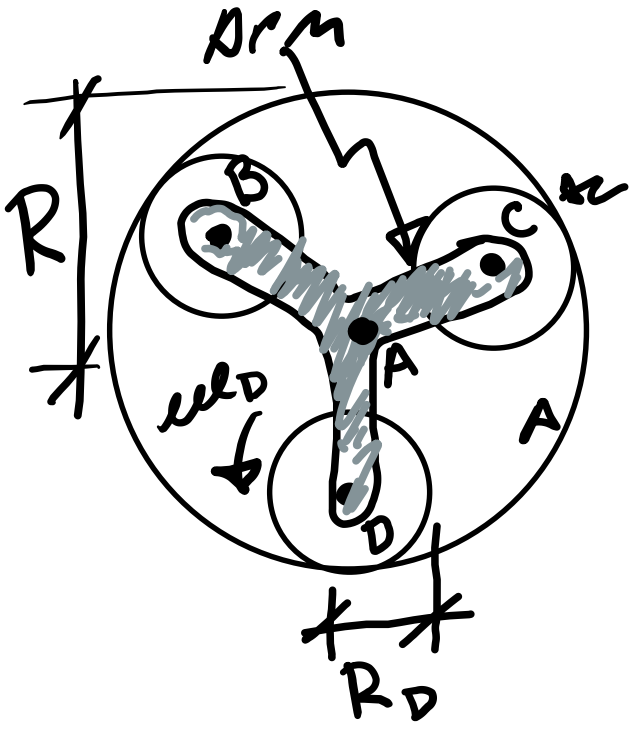


We want $V_{D/A}$ & know that $V_A = 0$



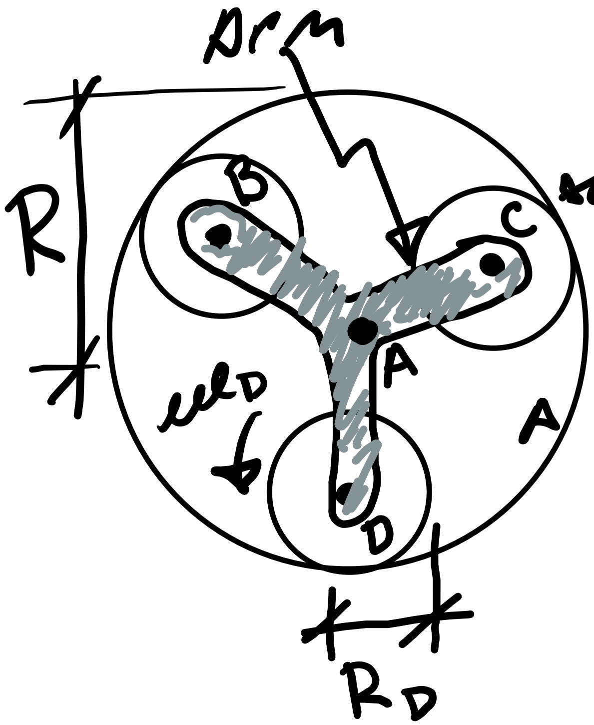
<u>Example</u>: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$



Example: Find ω_{ARM}

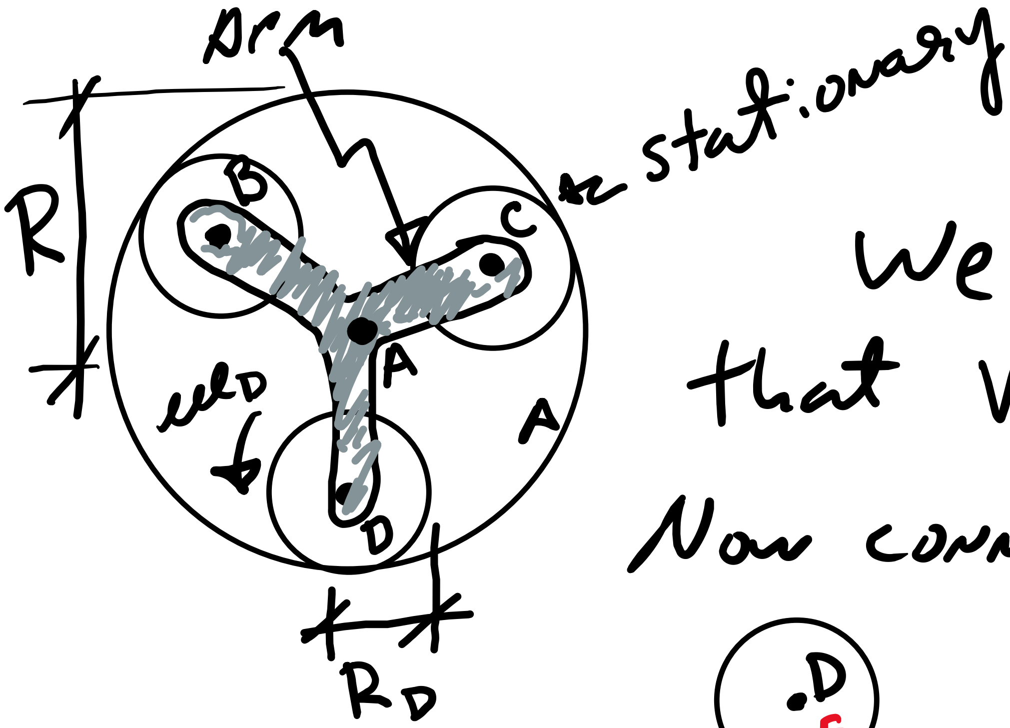
We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

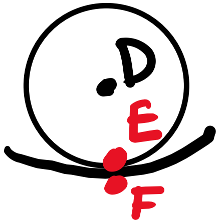
Now connect the dots:



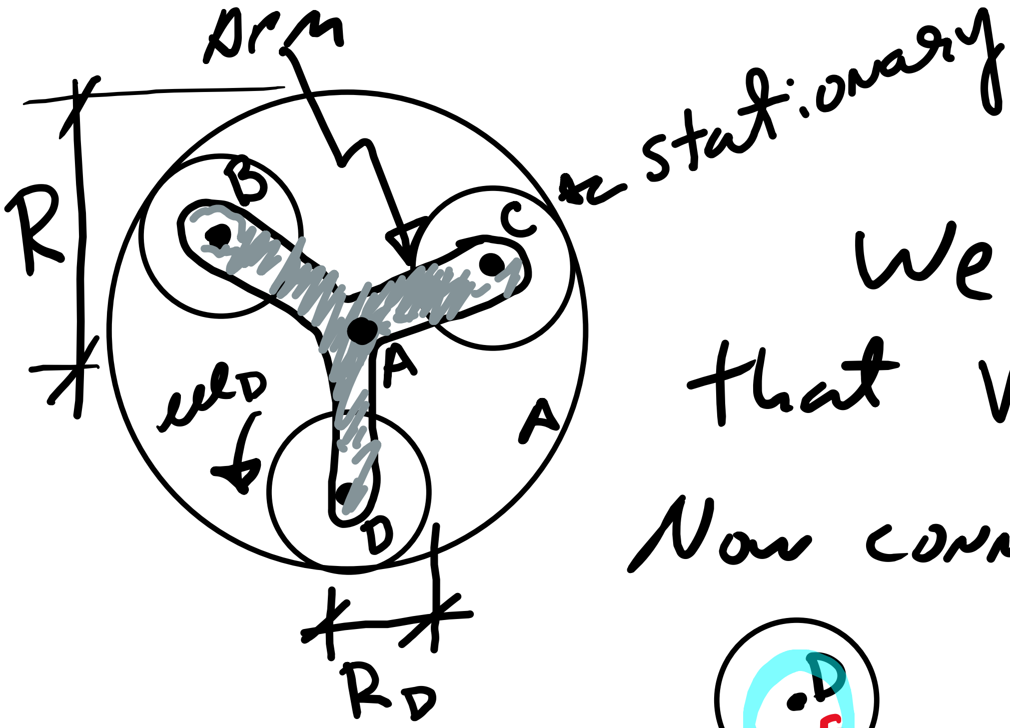
Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots



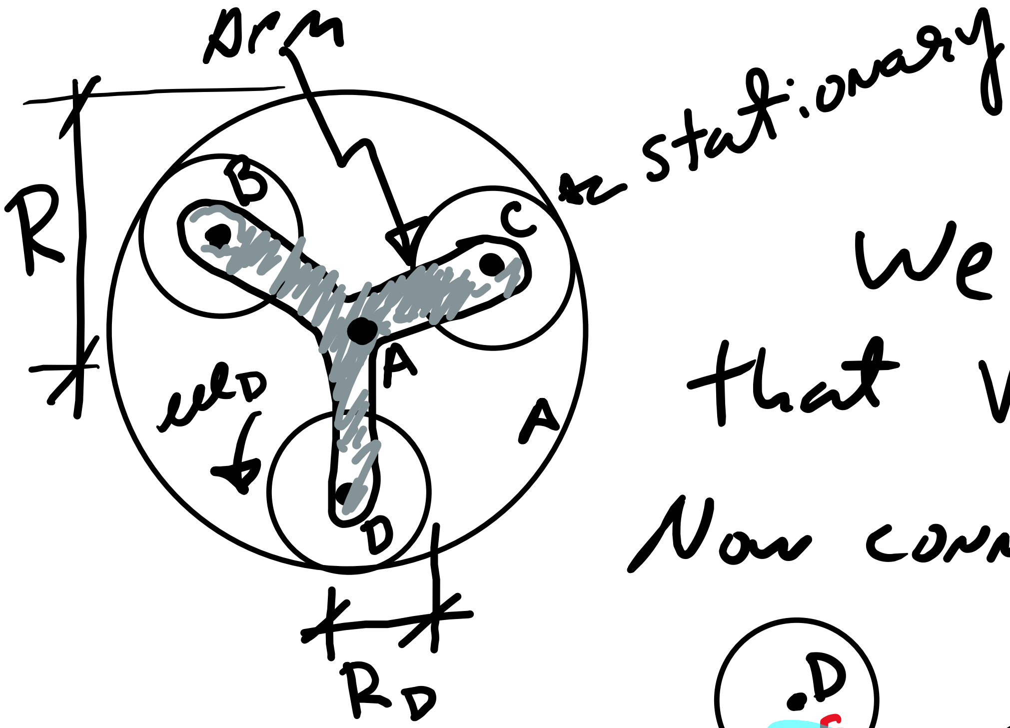
Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

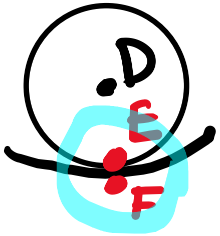
$$\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E$$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

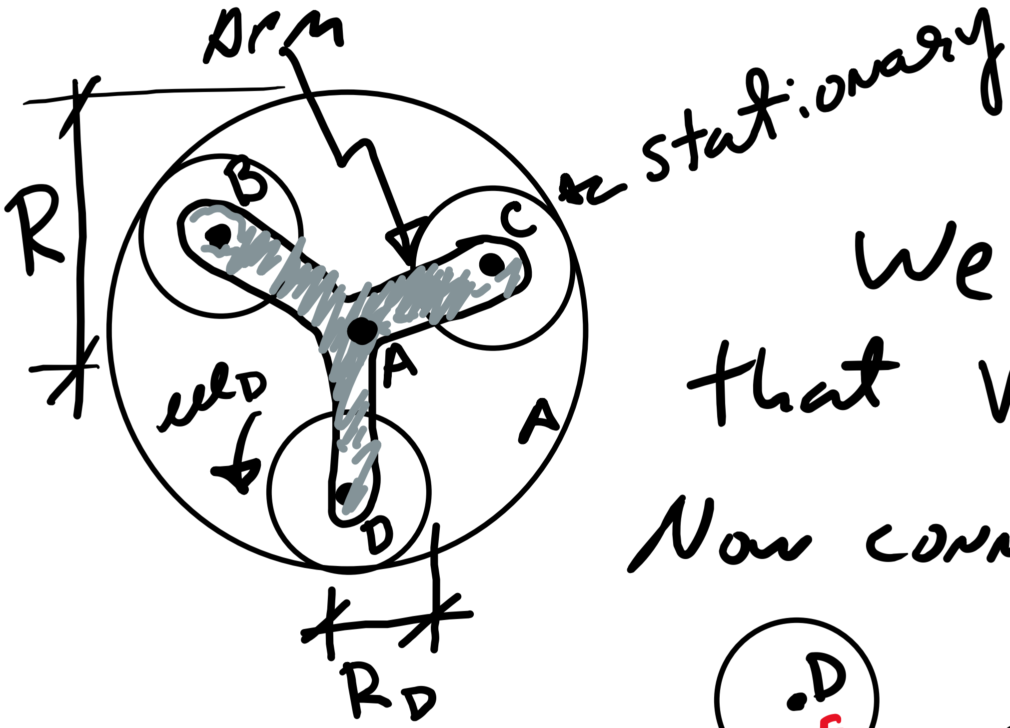
Now connect the dots



$$\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E$$

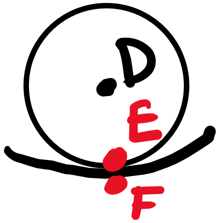
& $\vec{V}_E = \vec{V}_{E/F} + \vec{V}_F$

Example: Find ω_{ARM}

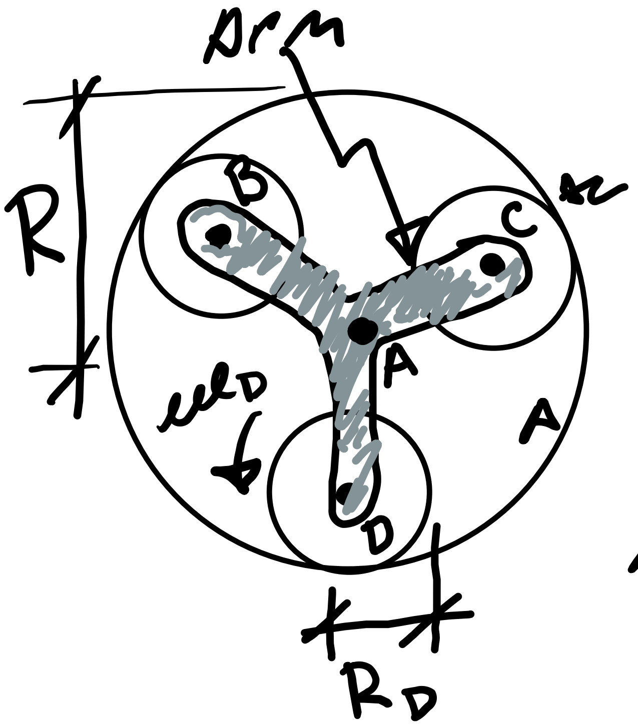


We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots



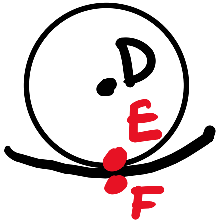
$$\left.\begin{aligned} \vec{V}_D &= \vec{V}_{D/E} + \vec{V}_E \\ \& \ \vec{V}_E &= \vec{V}_{E/F} + \vec{V}_F \end{aligned}\right\} \text{But } V_F = 0$$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

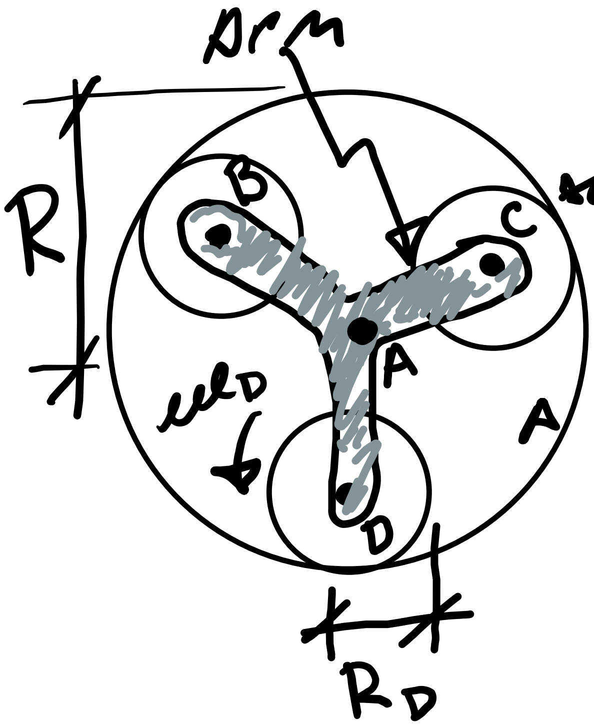
Now connect the dots



$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E \\ \& \ \vec{V}_E = \vec{V}_{E/F} + \vec{V}_F\end{array}\right\}$$

But $V_F = 0$ & $V_{E/F} = 0$

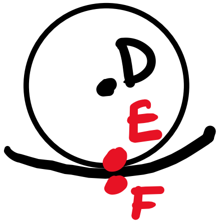
Roll no slip



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

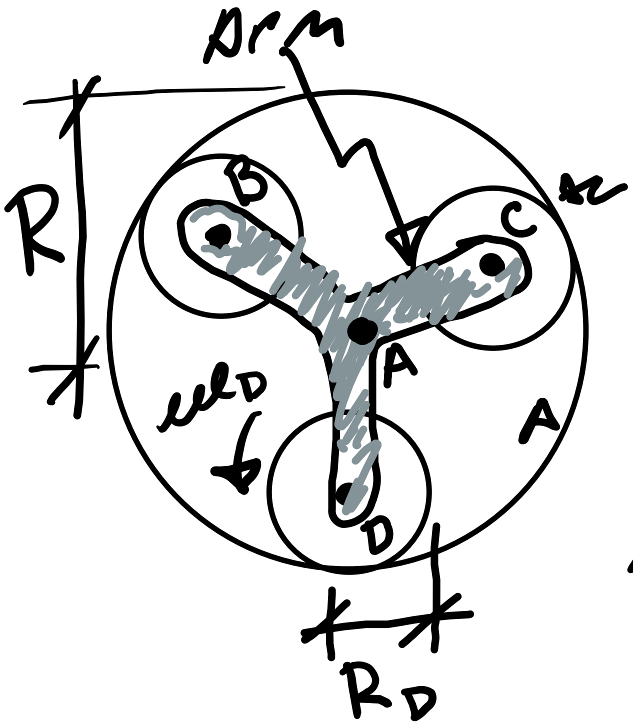


$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/E} + \vec{V}_E \\ \text{\& } \overset{0}{\cancel{\vec{V}_E}} &= \vec{V}_{E/F} + \vec{V}_F\end{aligned}\right\}$$

But $V_F = 0$

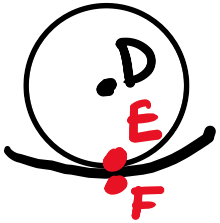
& $V_{E/F} = 0$

Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

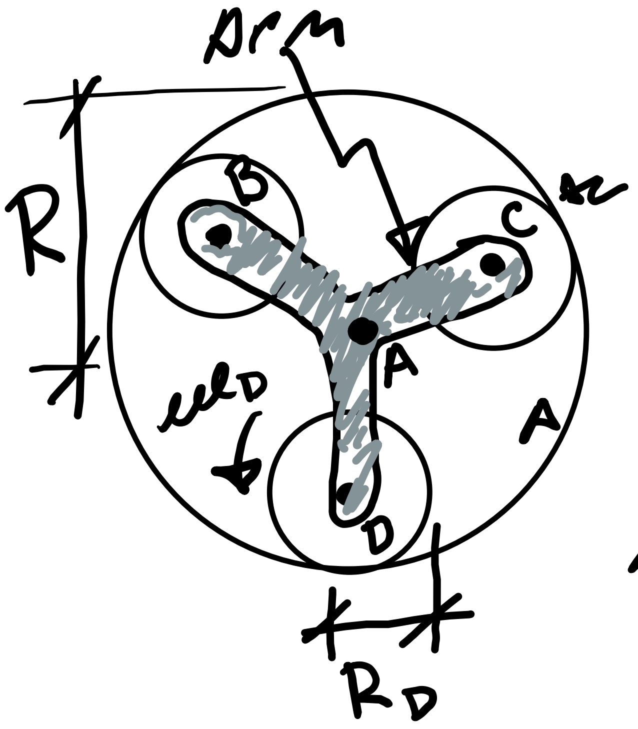


$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

$$\& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

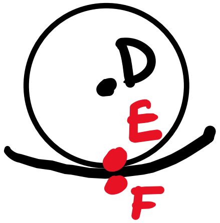
But $V_F = 0$ & $V_{E/F} = 0$

Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

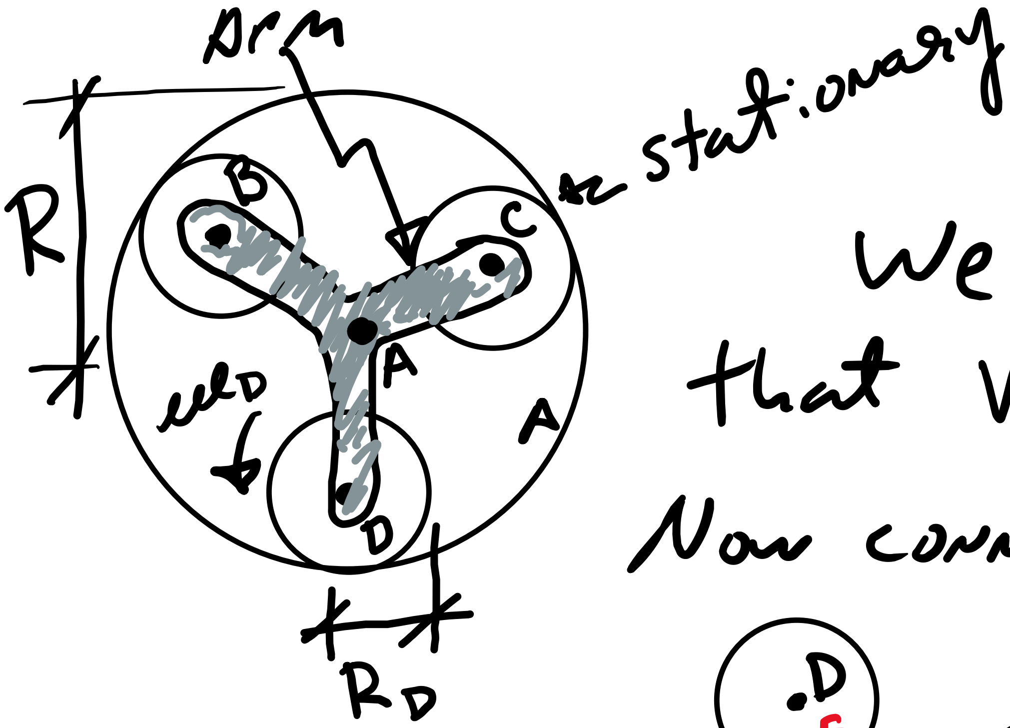


$$\vec{V}_D = \vec{V}_{D/E} + \cancelto{0}{\vec{V}_E}$$

$$\& \ \cancelto{0}{\vec{V}_E} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

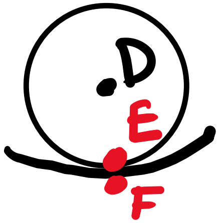
So $\vec{V}_D = \vec{V}_{D/E}$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots



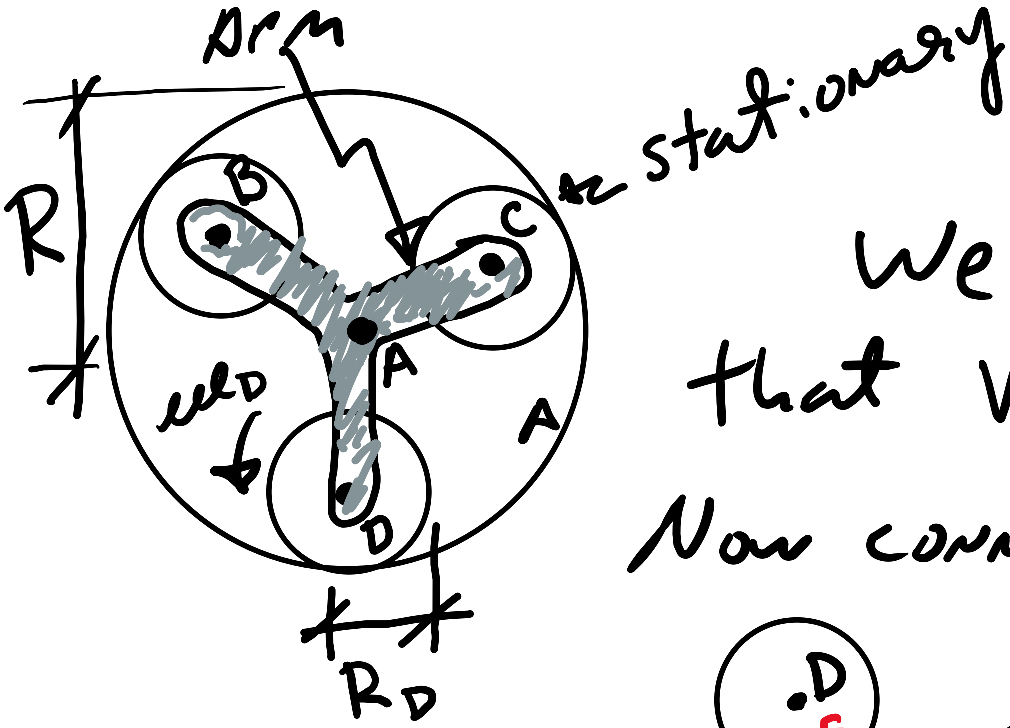
$$\vec{V}_D = \vec{V}_{D/E} + \cancelto{0}{\vec{V}_E}$$

$$\& \ \cancelto{0}{\vec{V}_E} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

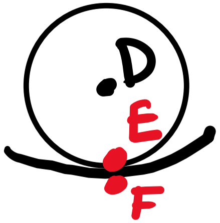
so $\vec{V}_D = \vec{V}_{D/E}$ (2)

Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots



$$\vec{V}_D = \vec{V}_{D/E} + \cancel{\vec{V}_E}^{\,0}$$

$$\& \ \cancel{\vec{V}_E}^{\,0} = \vec{V}_{E/F} + \vec{V}_F$$

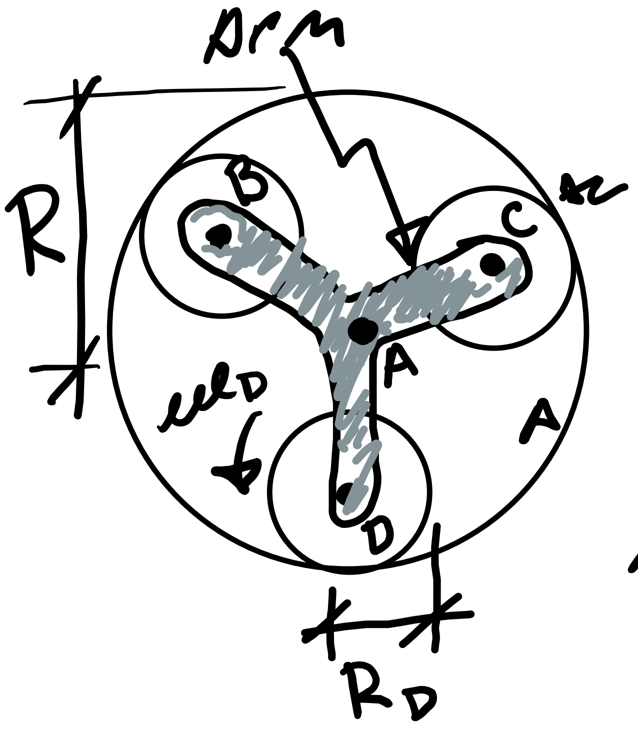
But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)

Eqns 1&2 give us

$$\vec{V}_{D/E} = \vec{V}_{D/A}$$

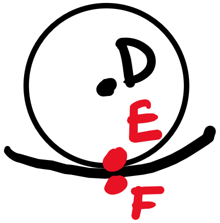
Example: Find ω_{ARM}



stationary

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

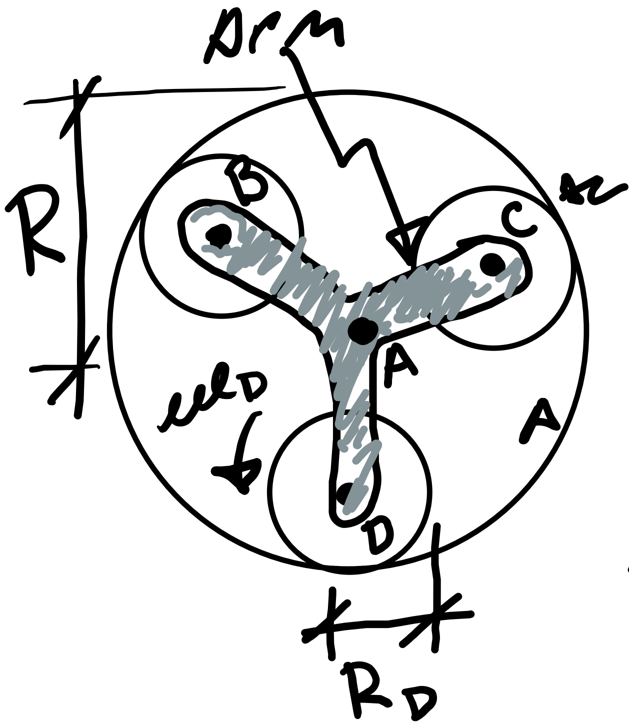
Now connect the dots



$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/E} + \cancelto{0}{\vec{V}_E} \\ \& \ \cancelto{0}{\vec{V}_E} &= \vec{V}_{E/F} + \vec{V}_F\end{aligned}\right\} \text{But } V_F = 0 \ \& \ V_{E/F} = 0$$

So $\vec{V}_D = \vec{V}_{D/E}$ (2) Eqns 1&2 give us

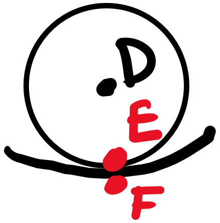
$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D \omega_D$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

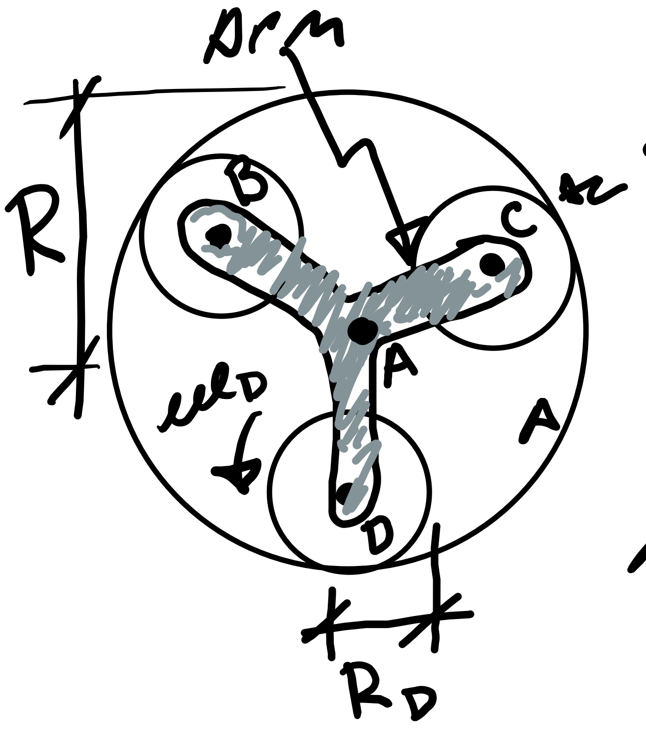


$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}\\ \& \ \overset{0}{\cancel{\vec{V}_E}} &= \vec{V}_{E/F} + \vec{V}_F\end{aligned}\right\}$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)   Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D\omega_D$ & $V_{D/A} = r_{D/A}\omega_{ARM}$



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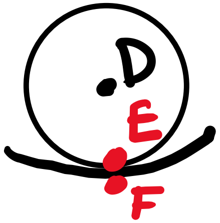

$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

$$\text{\& } \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)   Eqns 1 & 2 give us

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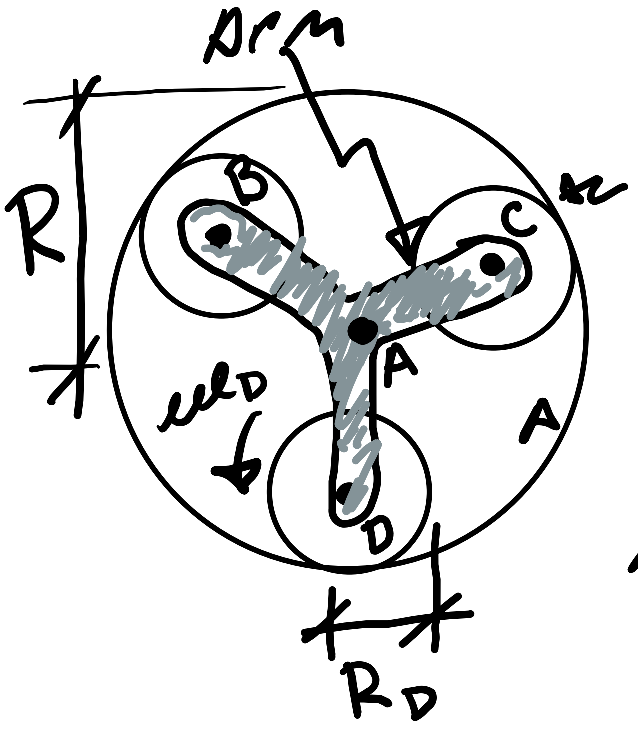
So $\omega_{ARM}(R - R_D) = \omega_D R_D$



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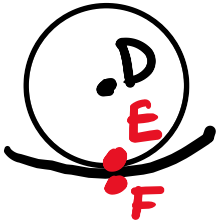

$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

$$\& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2) Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D \omega_D$ & $V_{D/A} = r_{D/A}\omega_{ARM}$

So $\omega_{ARM}(R-R_D) = \omega_D R_D \Rightarrow \boxed{\omega_{ARM} = \frac{R_D \omega_D}{(R-R_D)}}$

The method of connecting the dots should always work, The method of connecting the dots should always work, however, if you feel confident in your understanding, The method of connecting the dots should always work, however, if you feel confident in your understanding, you could perform the previous problem quickly.

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The more complicated the problem

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The more complicated the problem, the more likely you will need to connect the dots methodically

Kinetics :

Kinetics : $\sum \vec{F} = m\vec{a}$ & $\sum \vec{M}_G = \dot{\vec{H}}_G$,

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For rigid body :

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where $\vec{M}=\vec{r}\times\vec{F}$ & $\vec{H}=\vec{r}\times\vec{L}$ & $\vec{L}=m\vec{v}$

For rigid body : $\vec{H}_G=I\vec{\omega}$

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For rigid body : $\vec{H}_G = I\vec{\omega} \Rightarrow \dot{\vec{H}}_G = I\vec{\alpha}$

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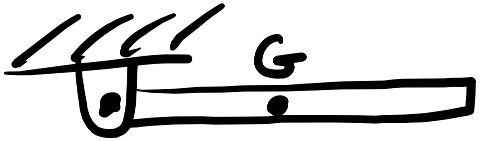
$\Rightarrow \sum \vec{M}_G = \bar{I}\vec{\alpha}$ For rotation about fixed point C: $\sum \vec{M}_C = I_C\vec{\alpha}$, where

$I_C = \bar{I} + m r_{G/C}^2$

Example problem:



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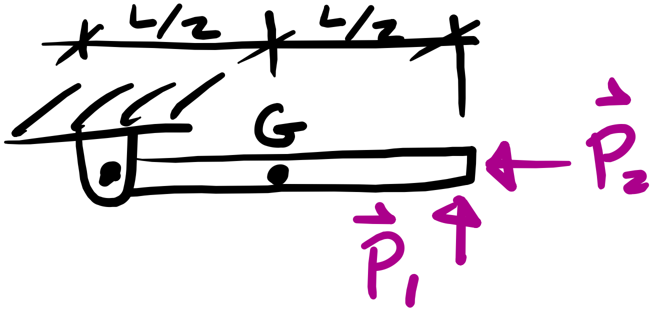

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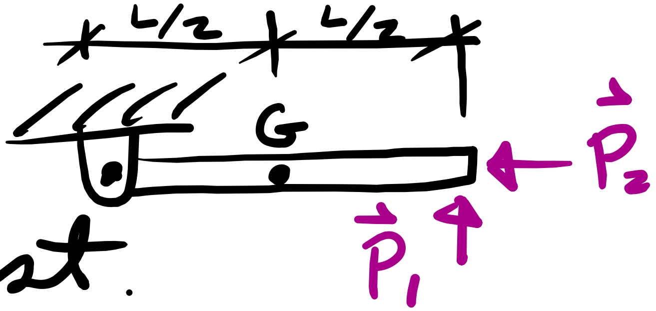

Arm is initially at rest.

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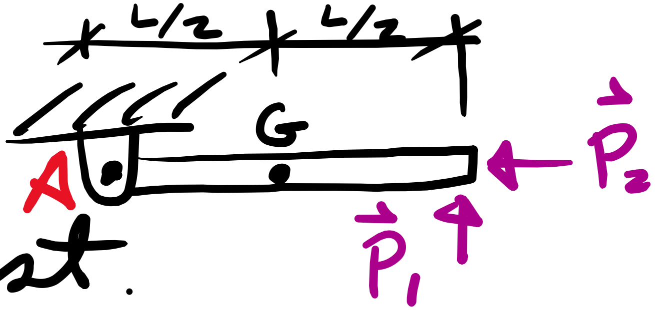
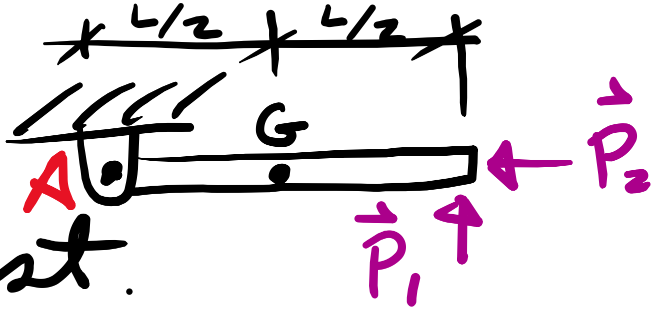


Arm is initially at rest.

Find reaction force at A

Example problem:



Arm is initially at rest.

Find reaction force at A & α_{ARM}

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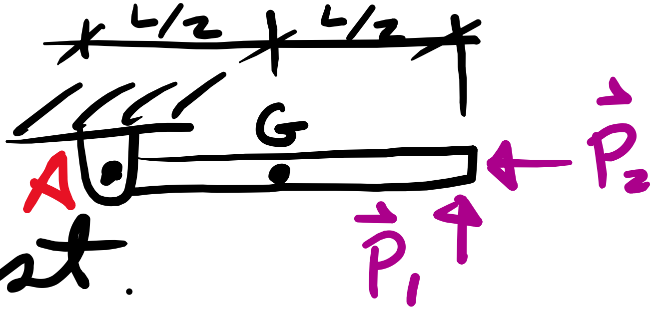

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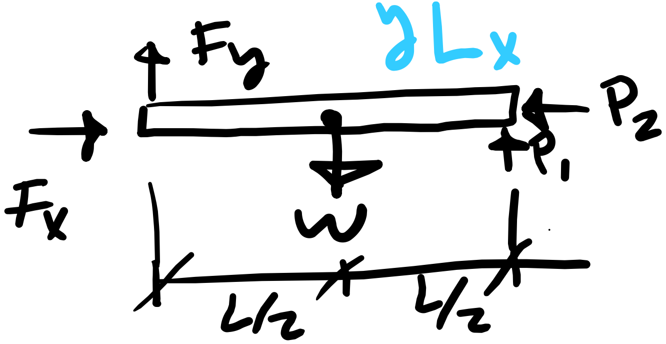

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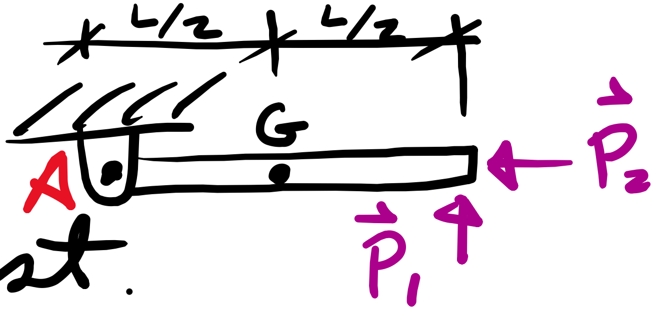

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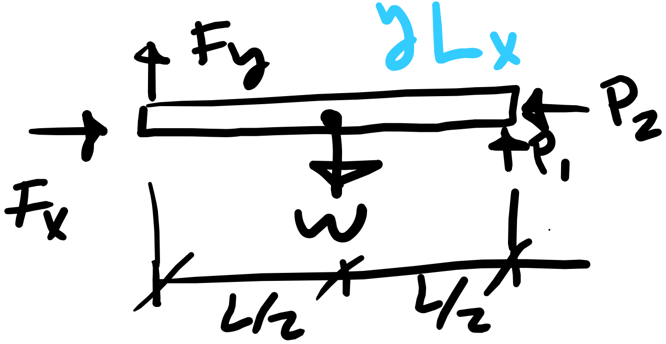

$$\Sigma F_x = m\bar{a}_x$$

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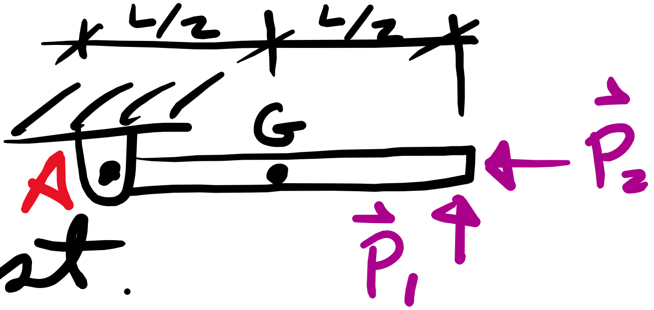

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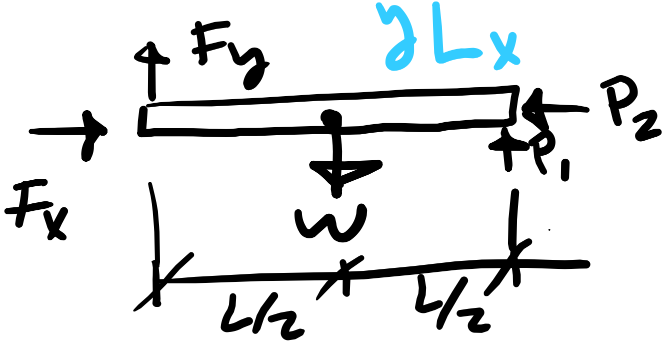

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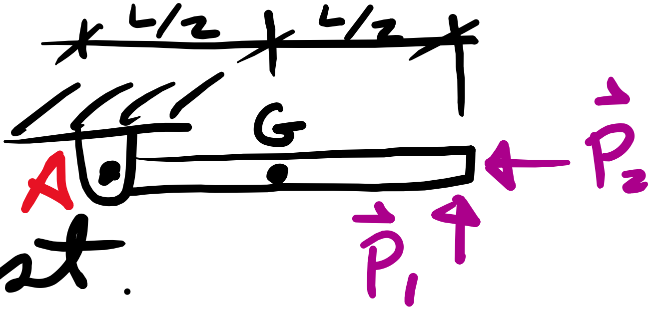

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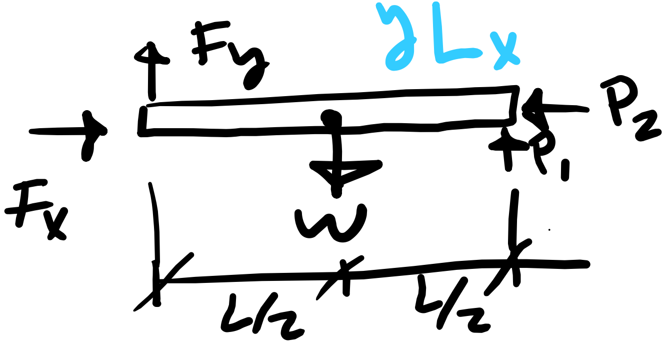
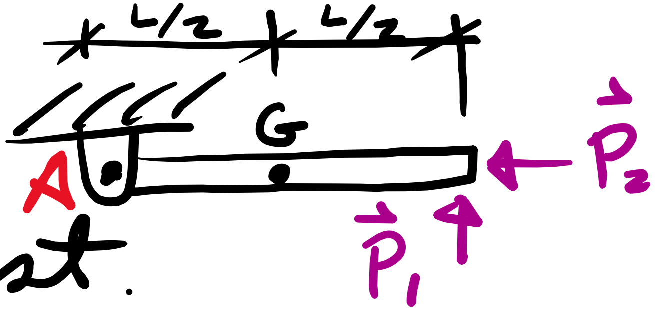


$$\sum F_x = m a_x \Rightarrow \boxed{F_x = P_2}$$

Example problem:



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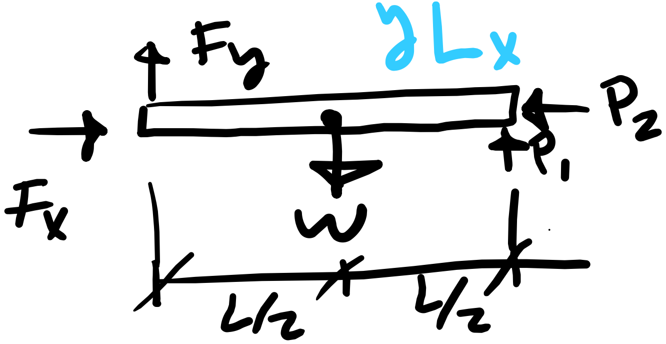

$$\sum F_x = m\cancel{a_x} \Rightarrow \boxed{F_x = P_2} \quad (1)$$

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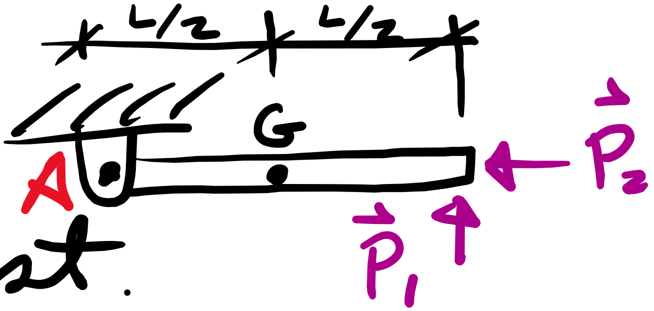

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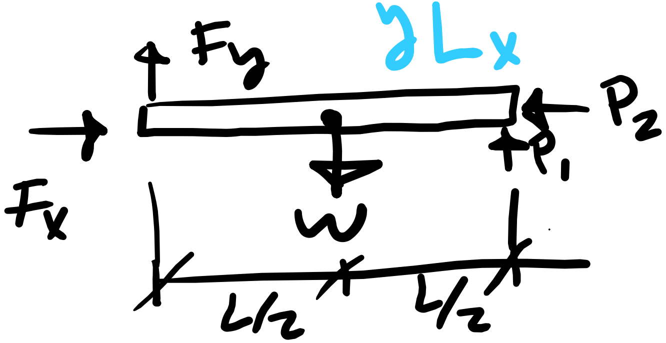

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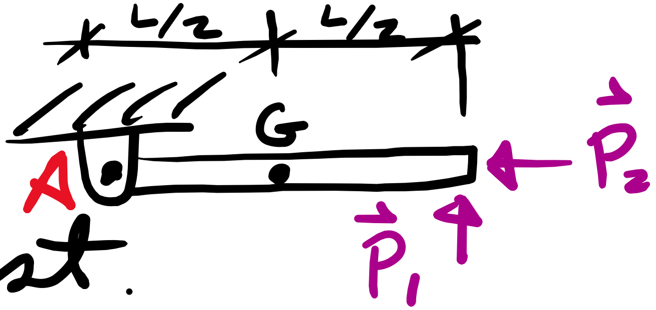

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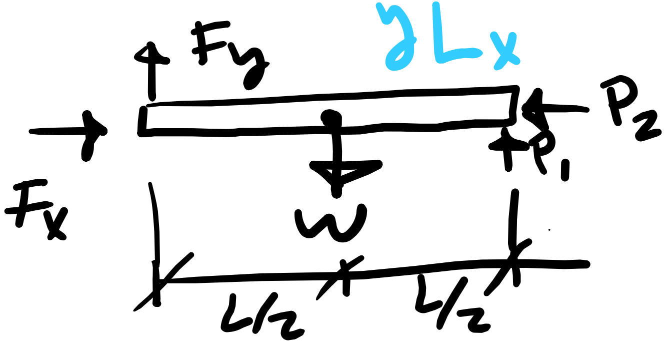

$\Sigma F_x = m\cancel{\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

$\Sigma F_y = m\bar{a}_y \Rightarrow F_y + P_1 - W = m\bar{a}_y$

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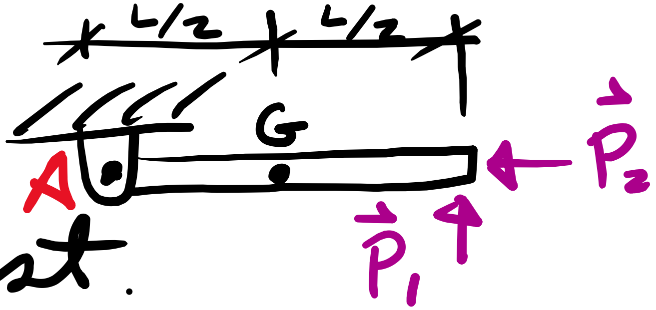

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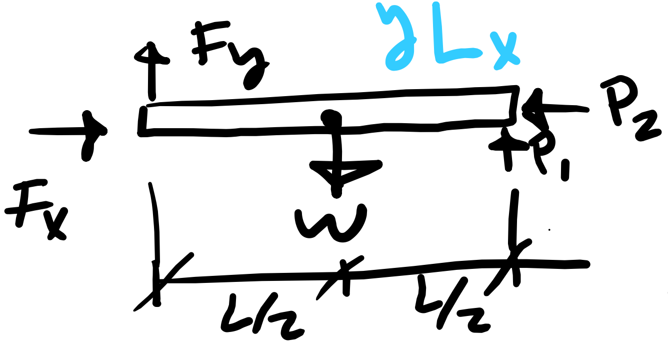

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But $\bar{a}_y = \alpha L/2$

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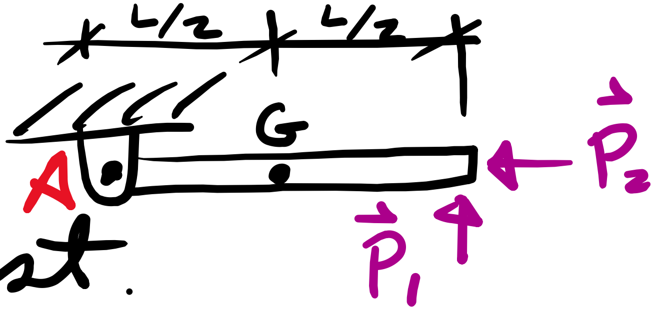

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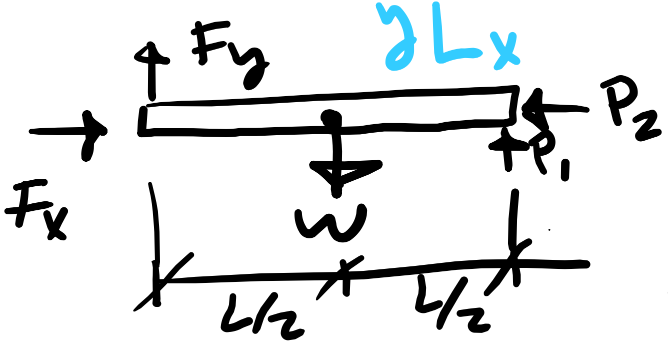

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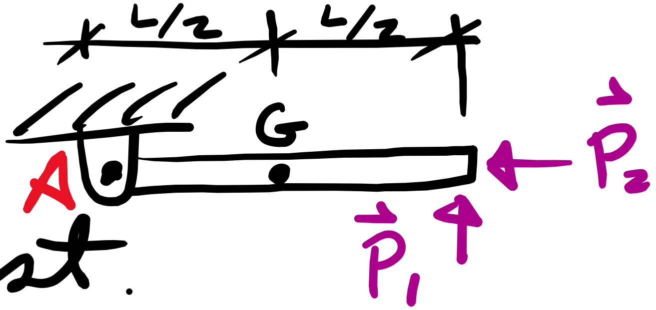

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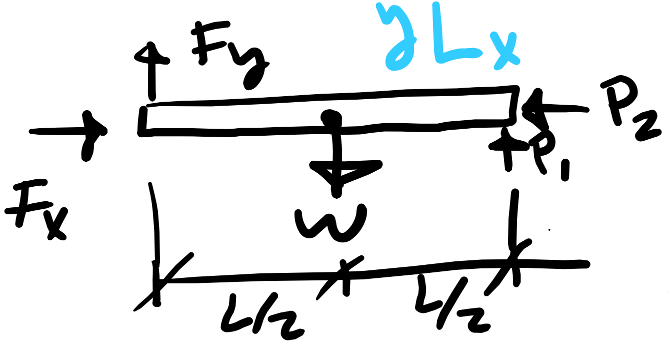

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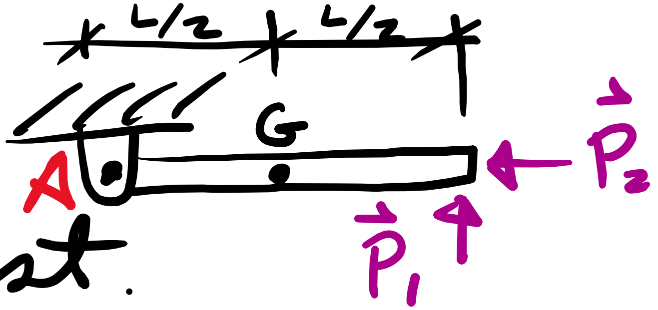
But $\bar{a}_y = \alpha L/2 \neq 0$

$F_y + P_1 - W = m(\frac{L}{2})\alpha$ (2)

Example problem:



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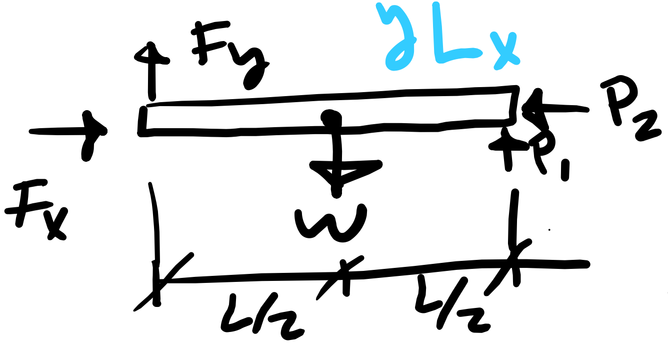

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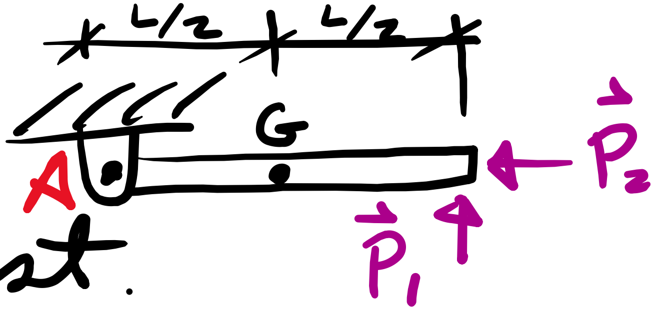
But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha \quad (2) \qquad \circlearrowleft \Sigma M_A = I_A \alpha$$

Example problem:



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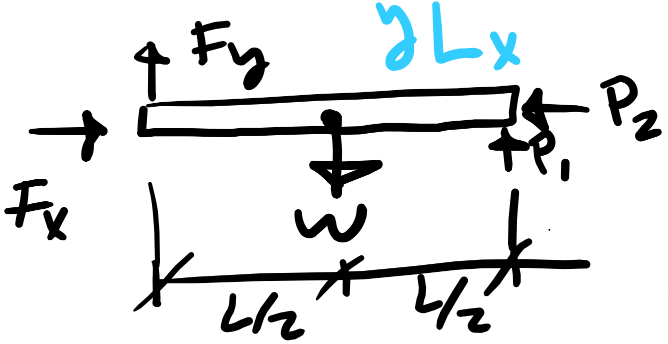

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But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha \quad (2) \qquad \circlearrowleft \Sigma M_A = I_A\alpha \Rightarrow$$

$$-W\frac{L}{2} + P_1 L = I_A\alpha$$

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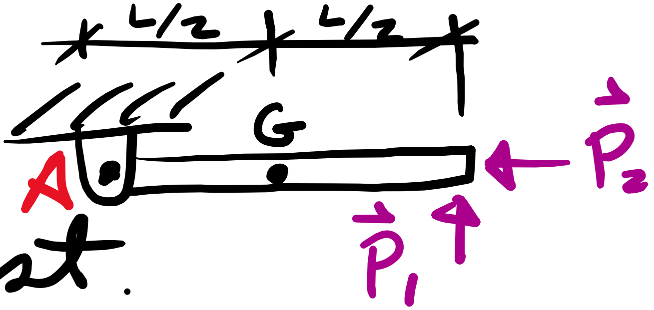

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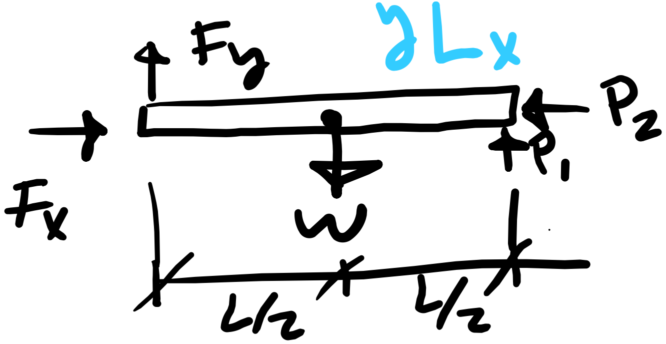

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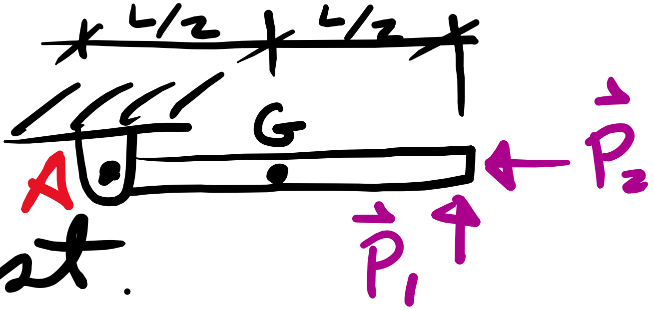
But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$ (2) $\quad \circlearrowleft \Sigma M_A = I_A\alpha \Rightarrow$

$-W\frac{L}{2} + P_1 L = I_A\alpha \Rightarrow \alpha = \dfrac{P_1 L - W\frac{L}{2}}{I_A}$

Example problem:



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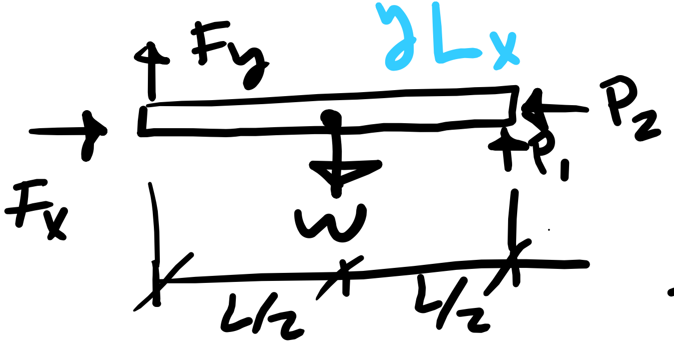

$$\Sigma F_x = m\bar{a}_x \Rightarrow \boxed{F_x = P_2} \quad (1)$$

$$\Sigma F_y = m\bar{a}_y \Rightarrow F_y + P_1 - W = m\bar{a}_y$$

But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha \quad (2) \qquad \circlearrowleft \Sigma M_A = I_A\alpha \Rightarrow$$

$$-W\frac{L}{2} + P_1 L = I_A\alpha \Rightarrow \alpha = \frac{P_1 L - W\frac{L}{2}}{I_A} \text{ But}$$

$$I_A = \frac{mL^2}{12} + m\frac{L^2}{4}$$

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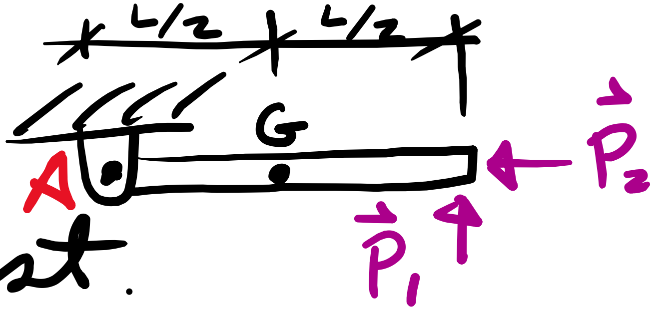

Arm is initially at rest.

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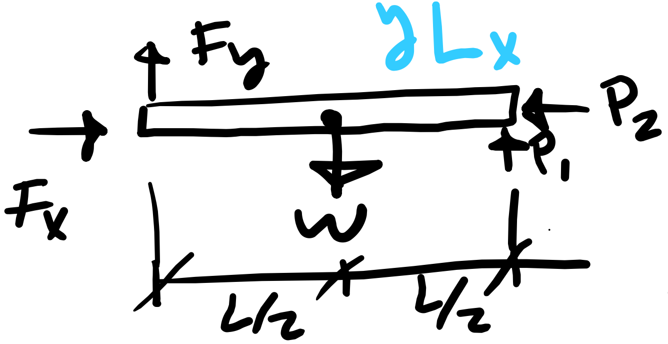

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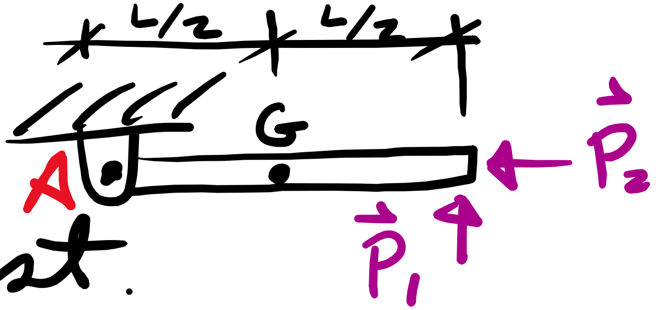

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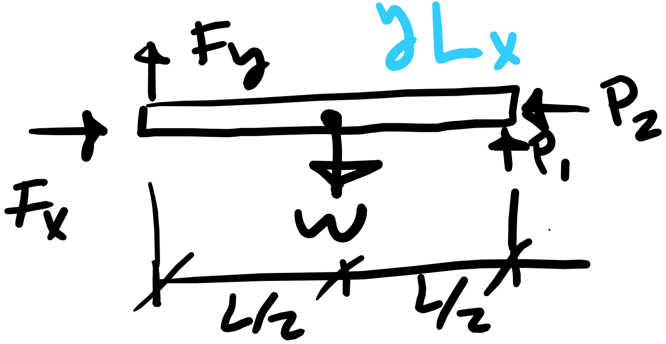

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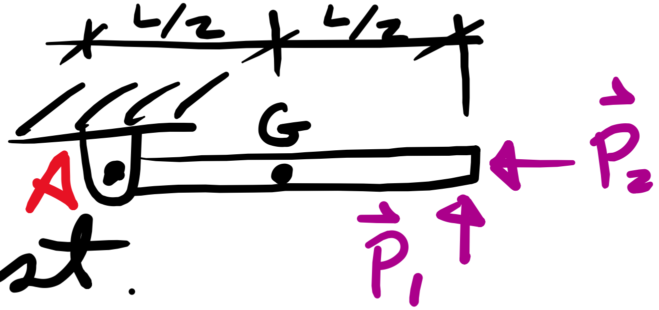

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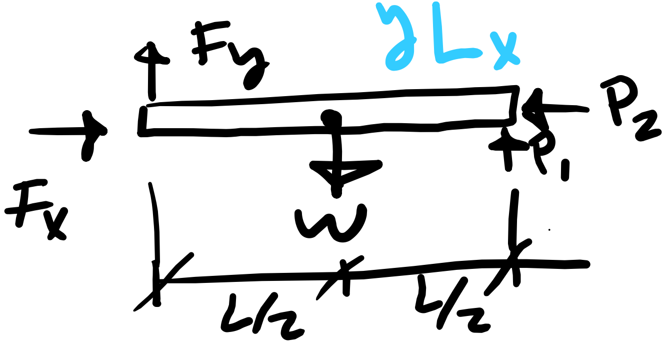

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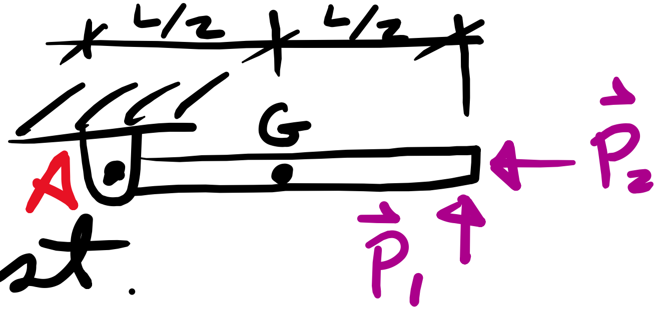

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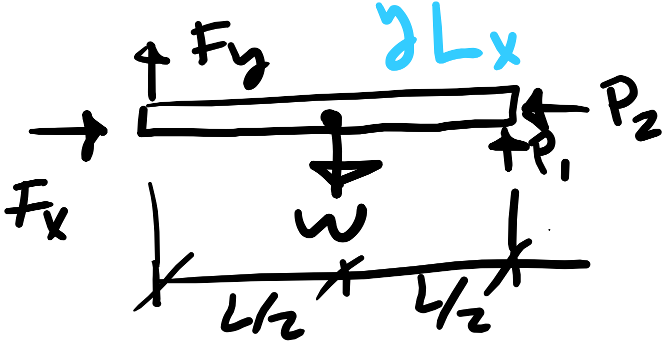

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$\Rightarrow \boxed{F_y = \frac{W}{4} + \frac{P_1}{2}}$

Work

Work $U_{1\rightarrow 2} = \Delta T,$

Work $U_{1\to 2} = \Delta T$, for moment couple

$$U_{1\to 2} = \int_{\theta_1}^{\theta_2} M\,d\theta$$

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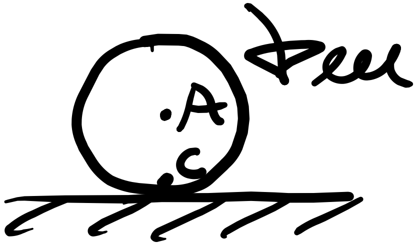

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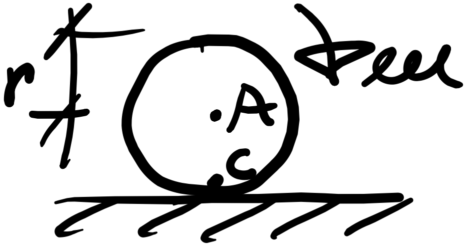

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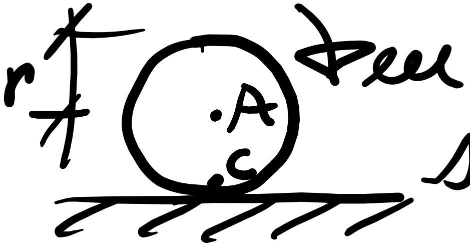

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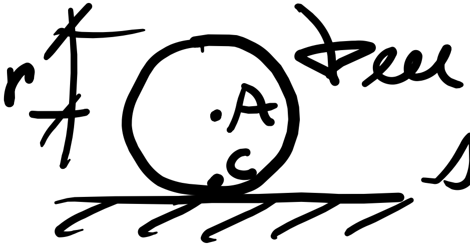

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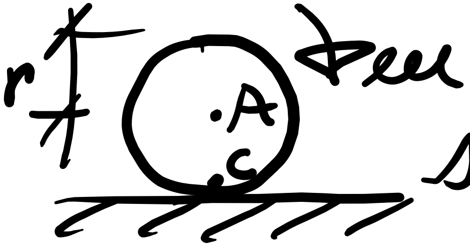

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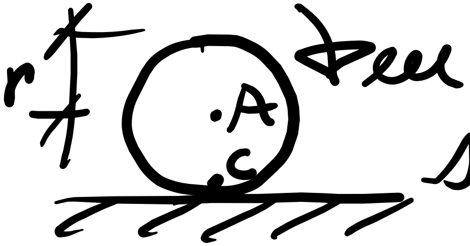

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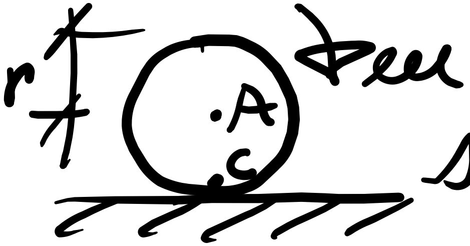

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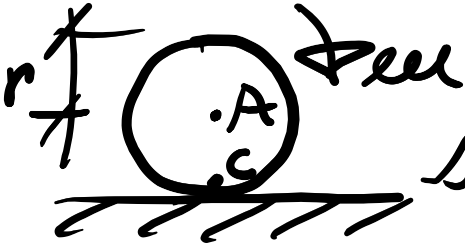

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[Same as before]

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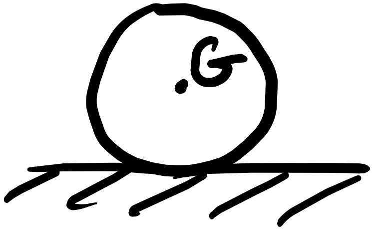

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$$T_I - M\frac{d}{r} = 0 \Rightarrow T_I = M\frac{d}{r} \Rightarrow \boxed{d = \frac{Tr}{M}}$$

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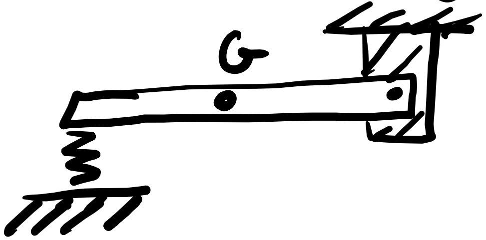

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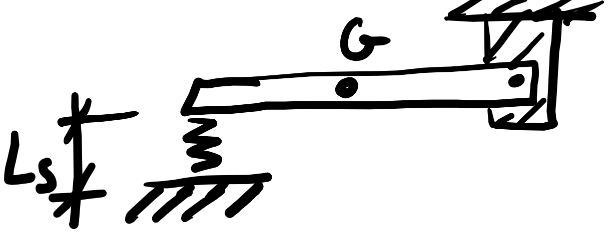

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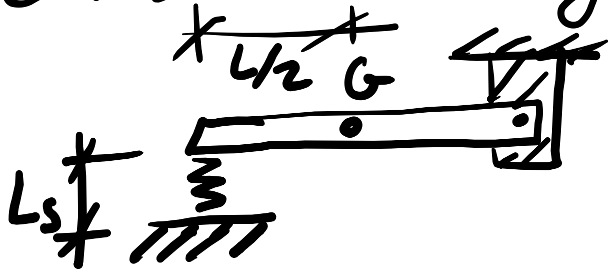

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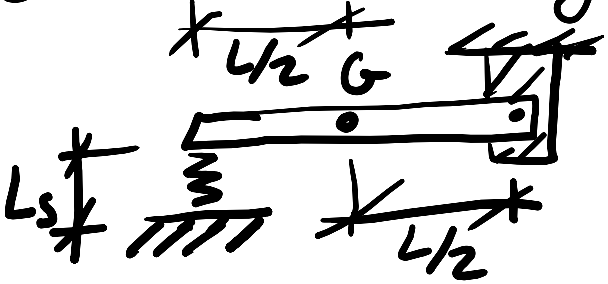

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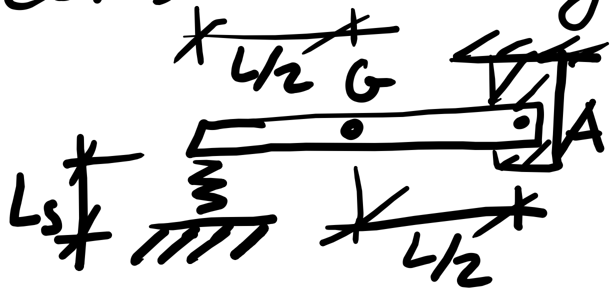

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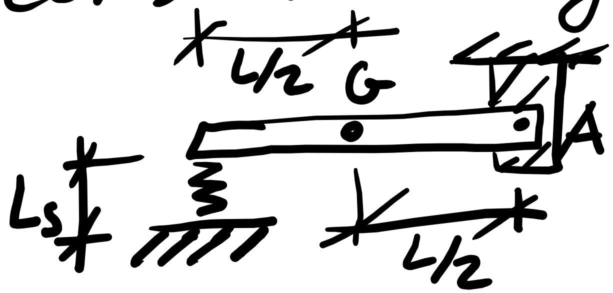

Mass of arm $= m$

Energy conservation

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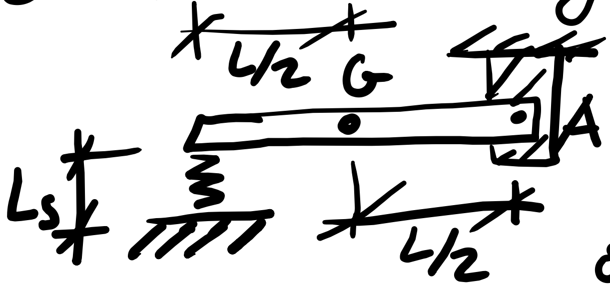

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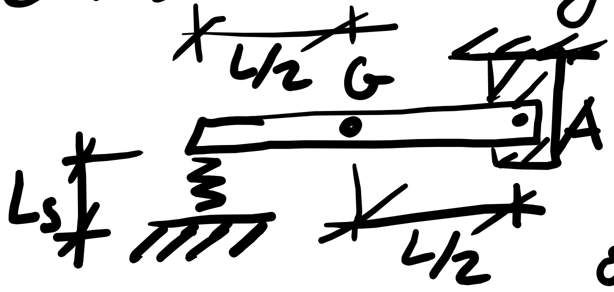

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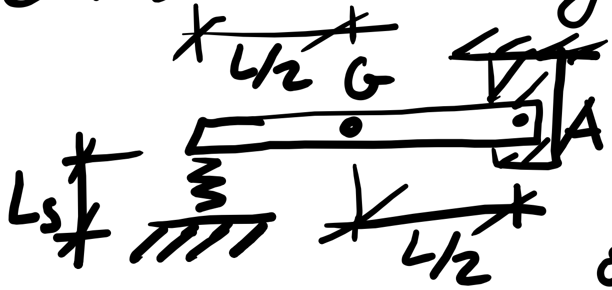

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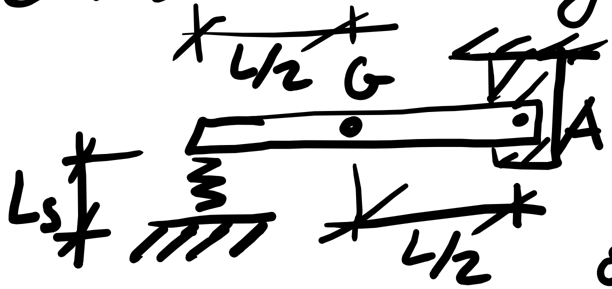

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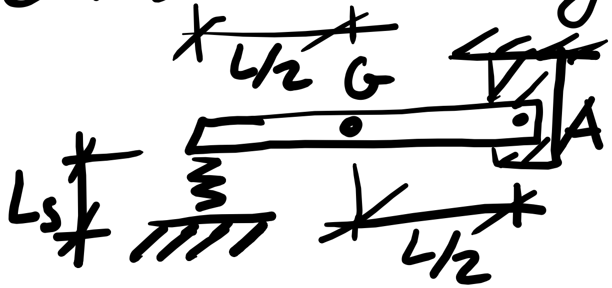

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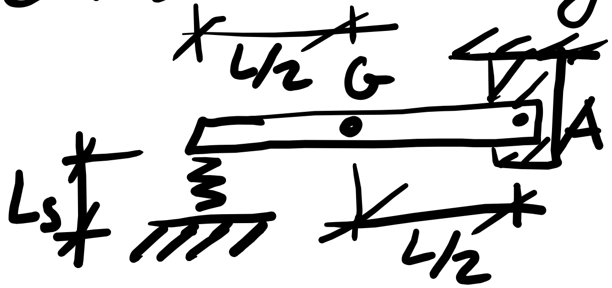

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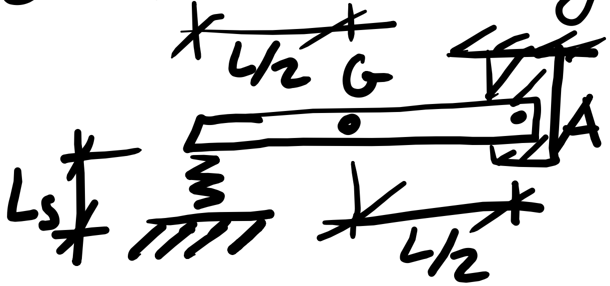

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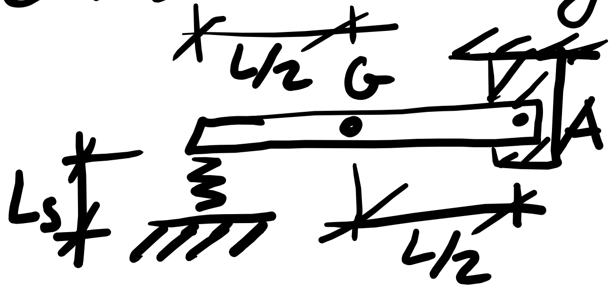

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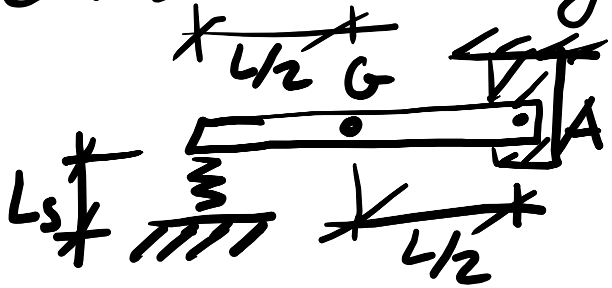

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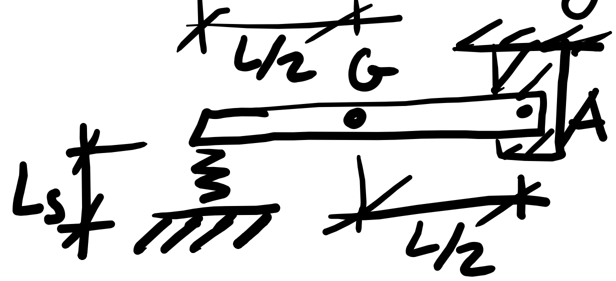

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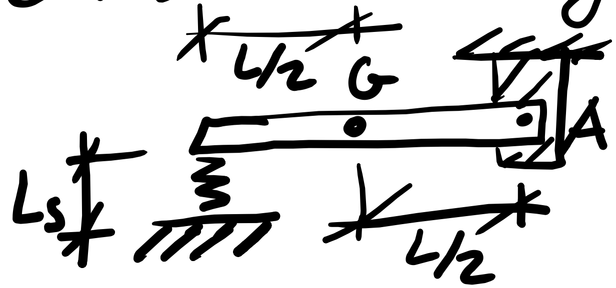

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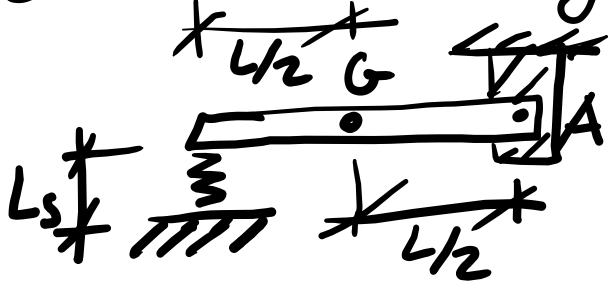

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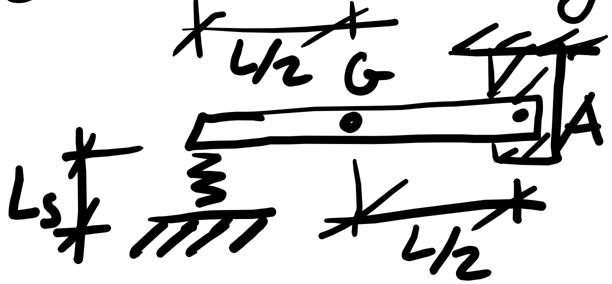

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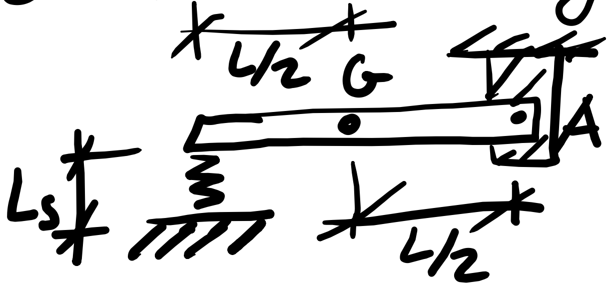

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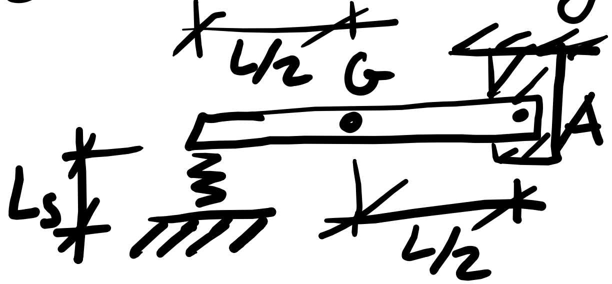

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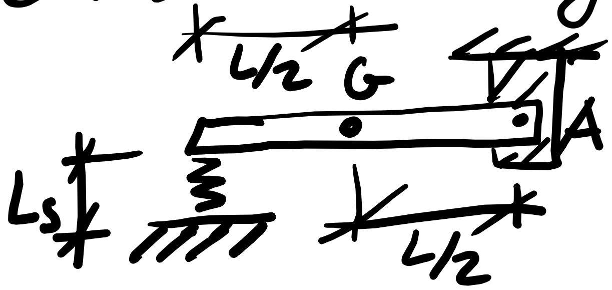

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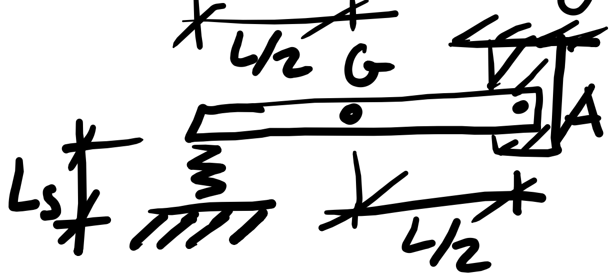

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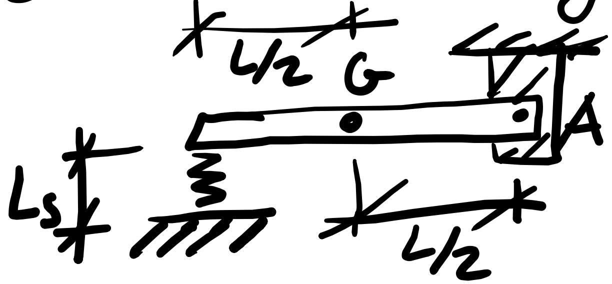

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Note: M is a brake so $M < 0$ if $\omega_I > 0$