Today 19.1

$\boxed{L31}$

Today 19.1

L31

Vibrations without damping

Today 19.1

$\boxed{L31}$

Wednesday Holiday

Today 19.1

$\boxed{L31}$

Wednesday Holiday ☺

$\boxed{L31}$

Today 19.1

Wednesday Holiday ☺

Friday 19.1-19.2

L31

Today 19.1

Wednesday Holiday ☺

Friday 19.1-19.2

Free vibrations of rigid bodies

ASU

L31

Today 19.1

Wednesday Holiday

Friday 19.1-19.2

Important dates :

$\boxed{L31}$

Today 19.1

Wednesday Holiday ☺

Friday 19.1-19.2

Important dates:

* Friday Nov. 27th no class

L31

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Friday 19.1-19.2

Important dates:

* Friday Nov. 27th: no class
* Monday Nov. 30th: Exam 4

ASU

L31

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Friday 19.1-19.2

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ASU

L31

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Wednesday Holiday ☺

Friday 19.1-19.2

Important dates:

* Friday Nov. 27^{th}: no class
* Monday Nov. 30^{th}: Exam 4
* Wednesday Dec. 2^{nd}: Day of Reckoning
* Friday Dec. 4^{th}: Final exam

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ASU

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Now $f(t) = 3\sin(t) + 5\cos(t)$

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$$f_n = C\sin(\omega t + \phi)$$

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Anatomy of a wave:

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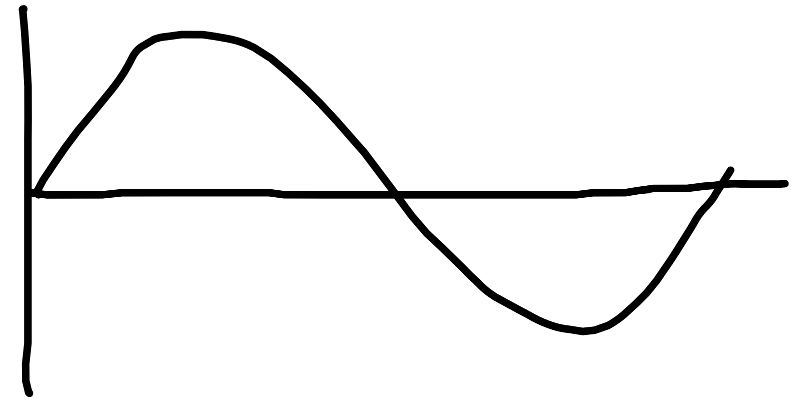
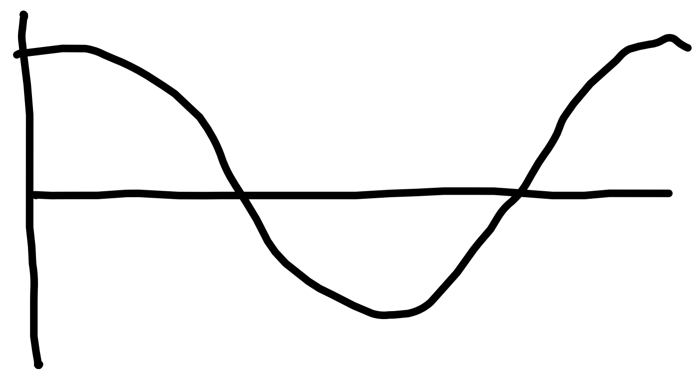
Let subscript "n" denote "natural" so that $\omega_n \equiv$ natural circular frequence





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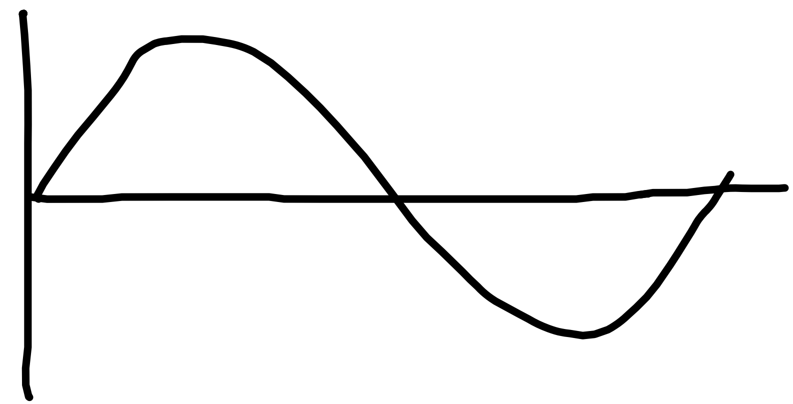

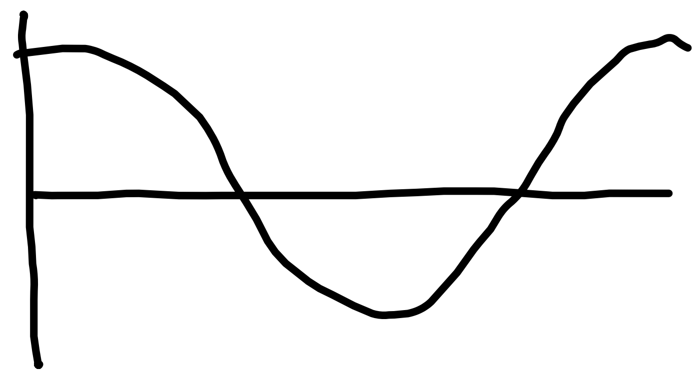

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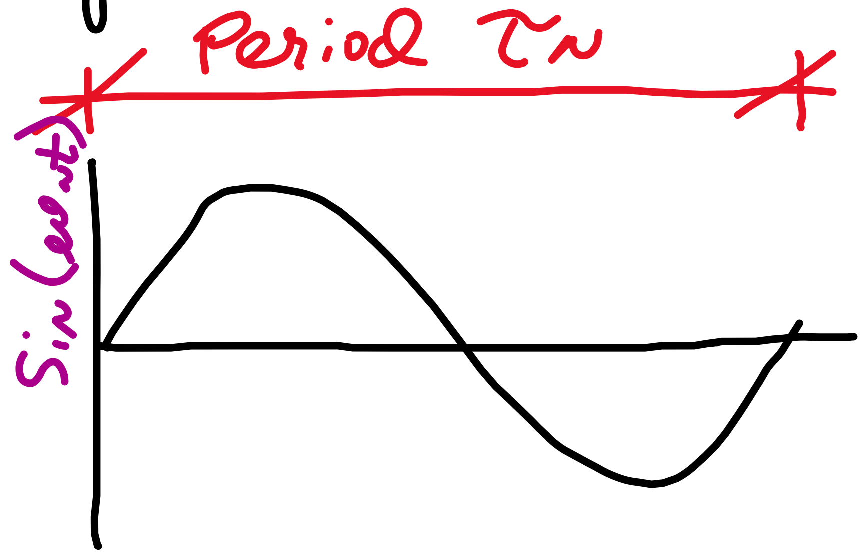

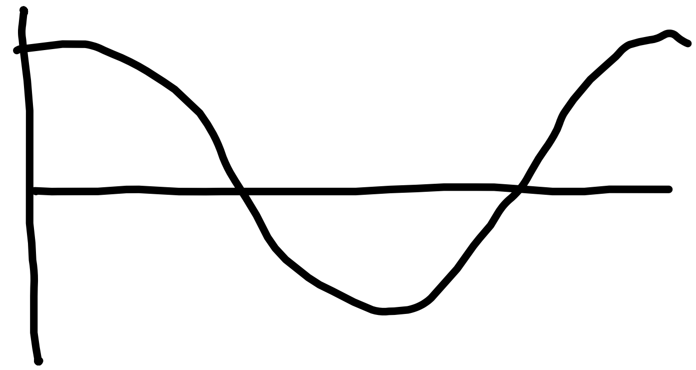

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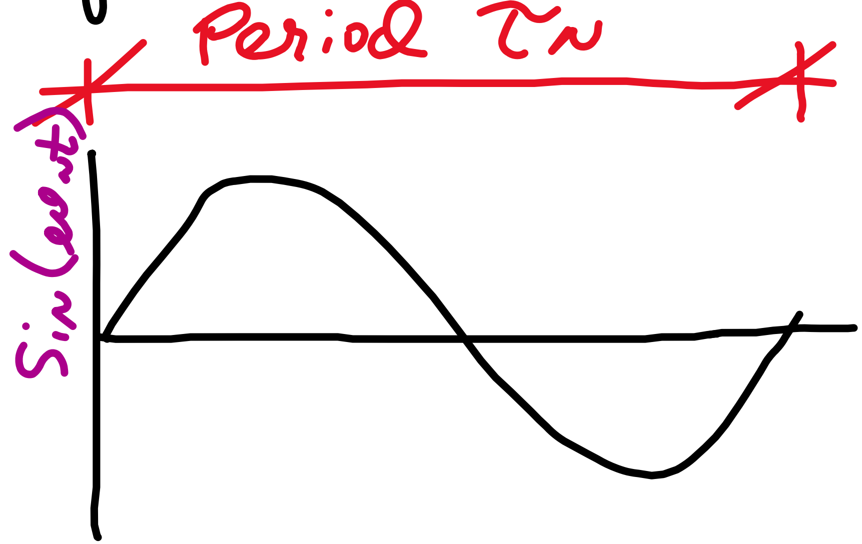

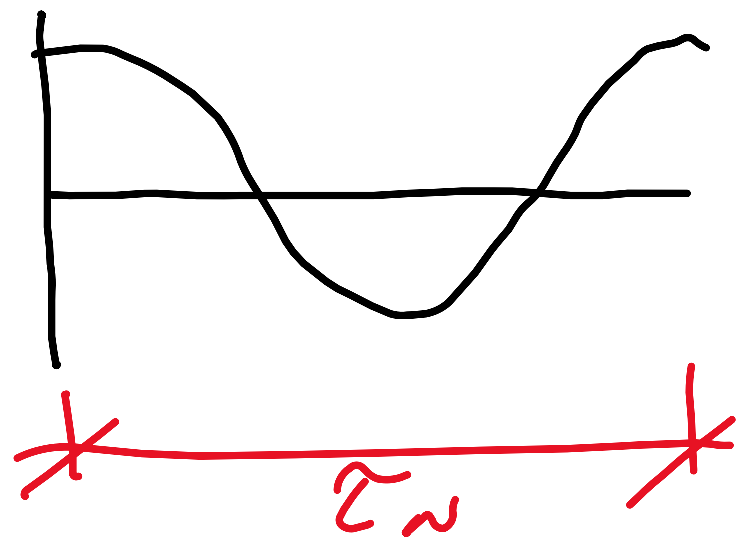

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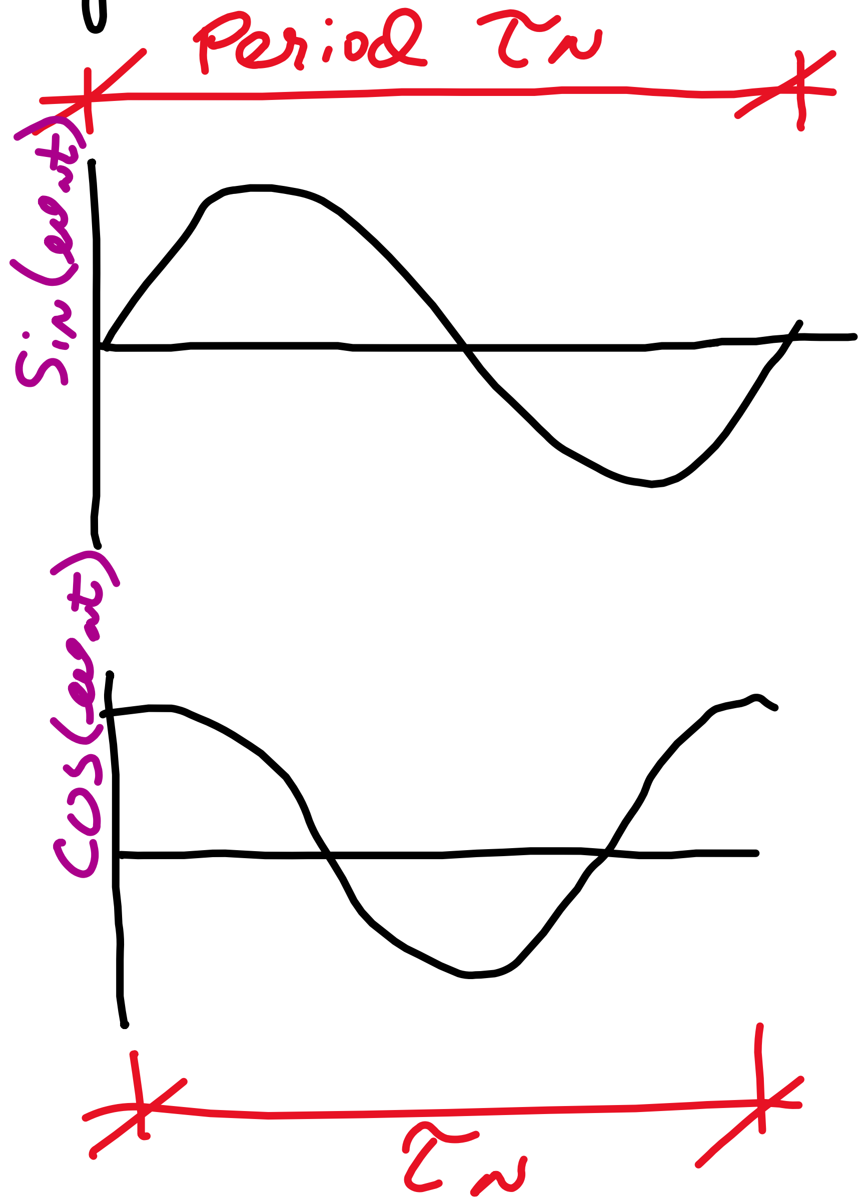

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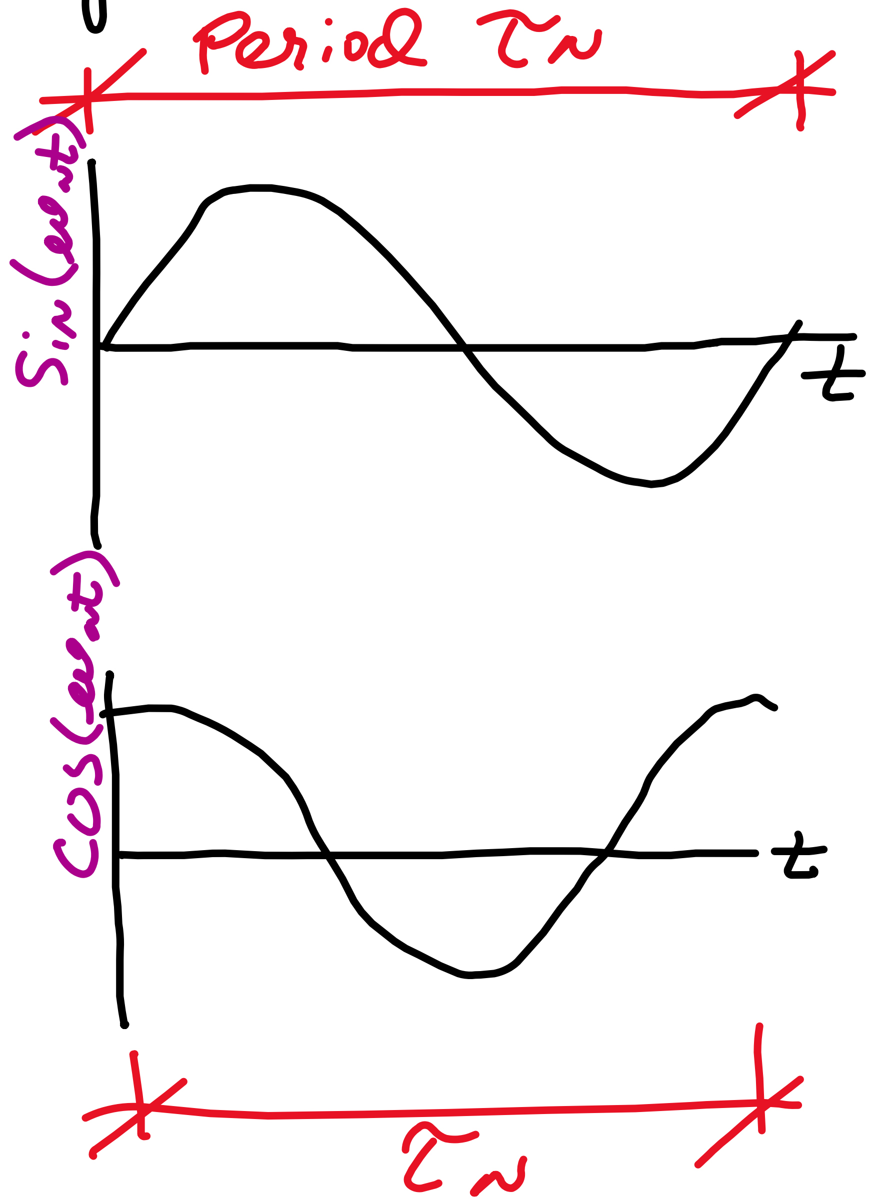

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Full wave when

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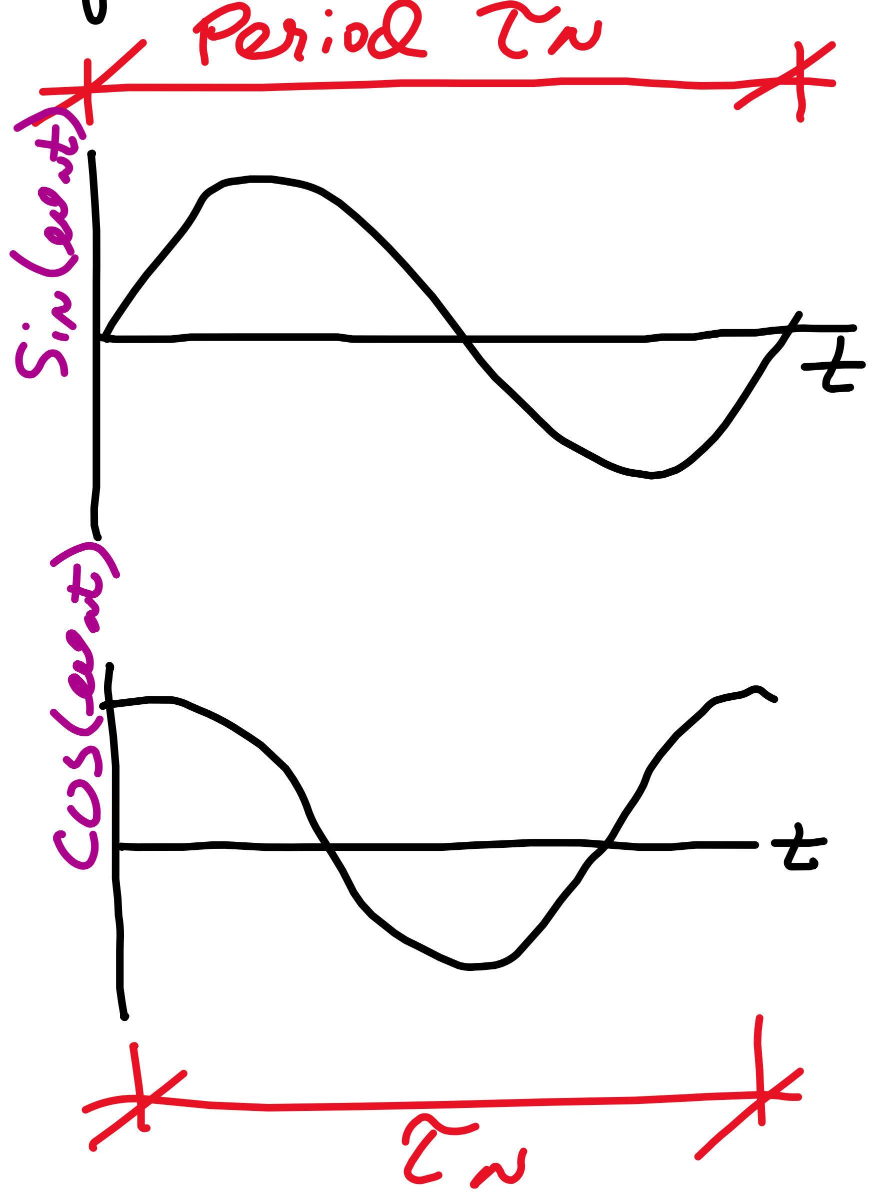

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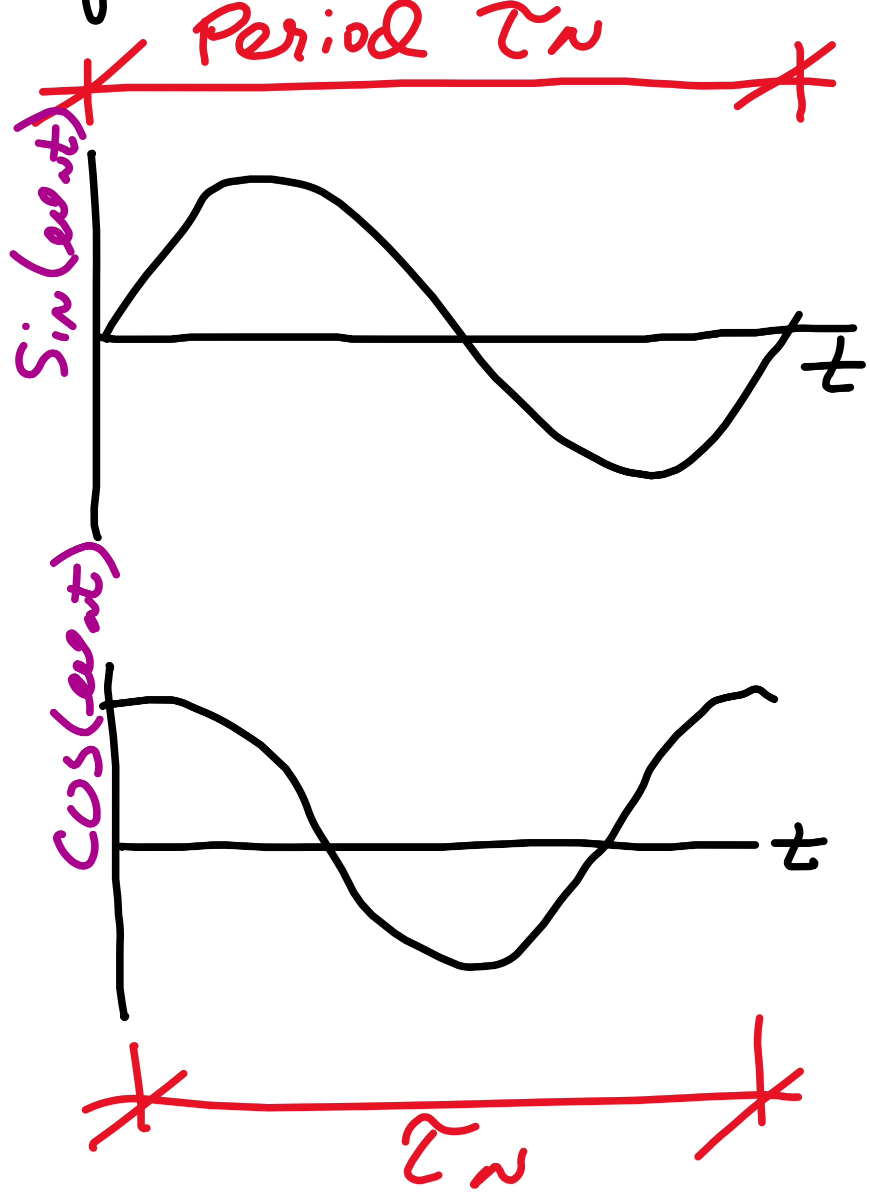

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This means that

$v_{MAX} = \omega x_{MAX}$

ASU

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<u>Example</u>: mass hanging by a spring in equilibrium

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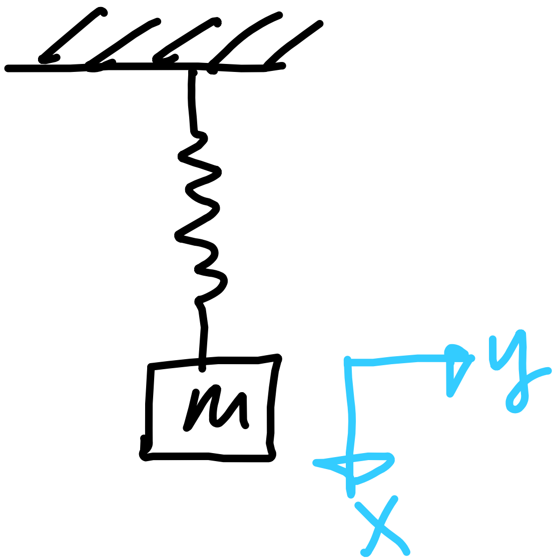

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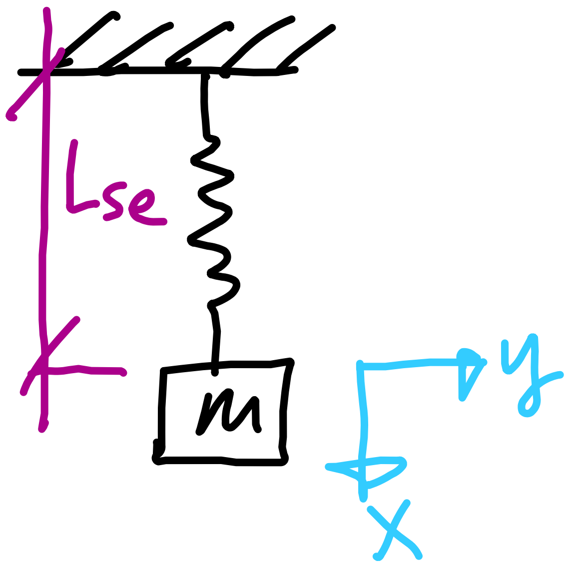
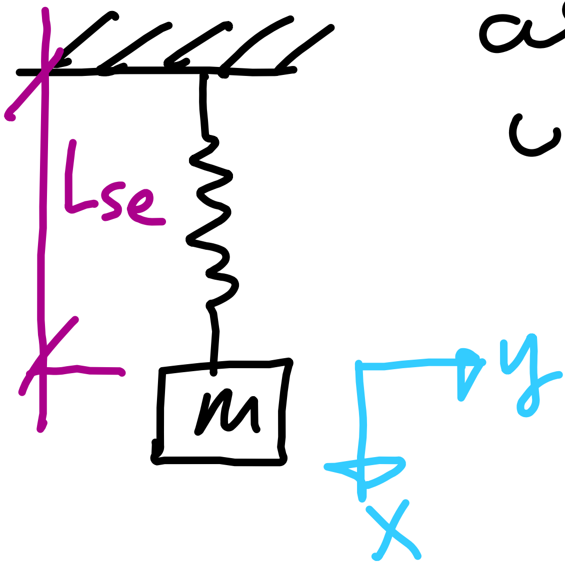


<u>Example</u>: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]. <u>Note</u>: we are going to change our usual notation a bit:



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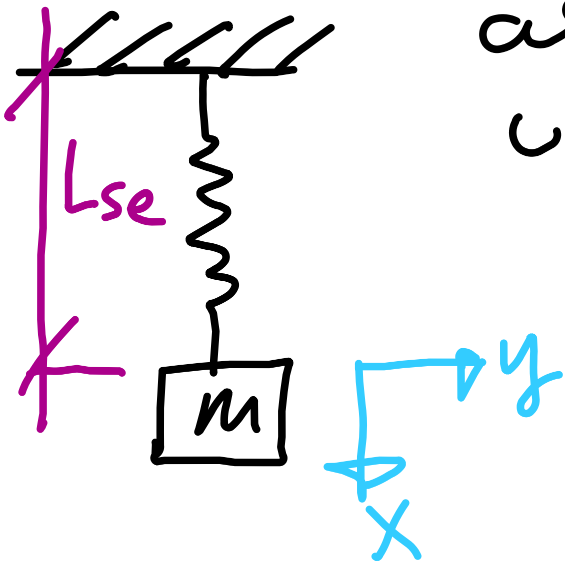

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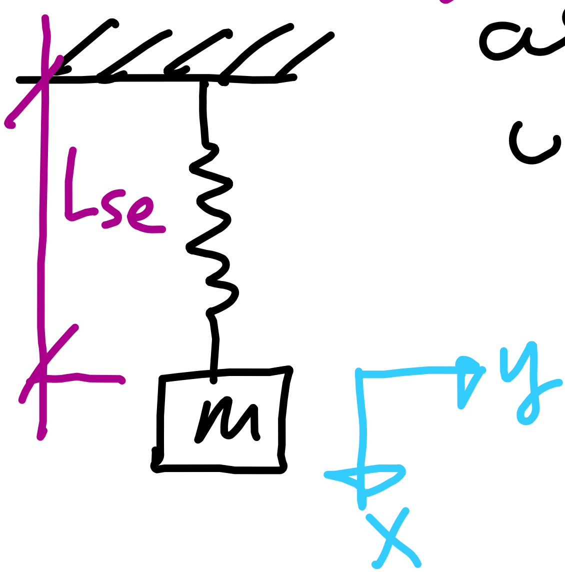

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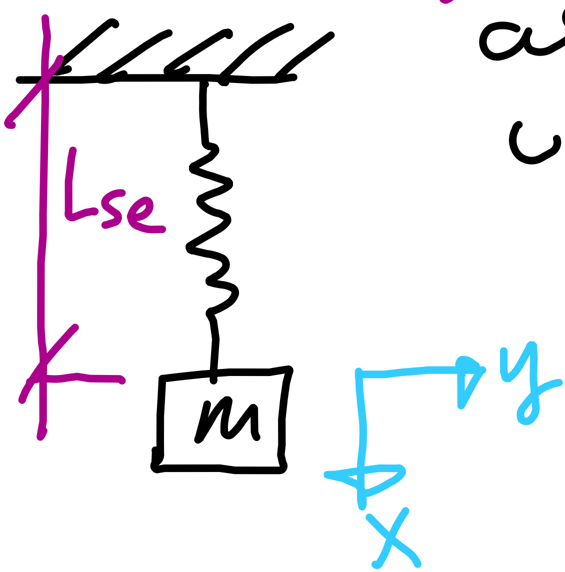

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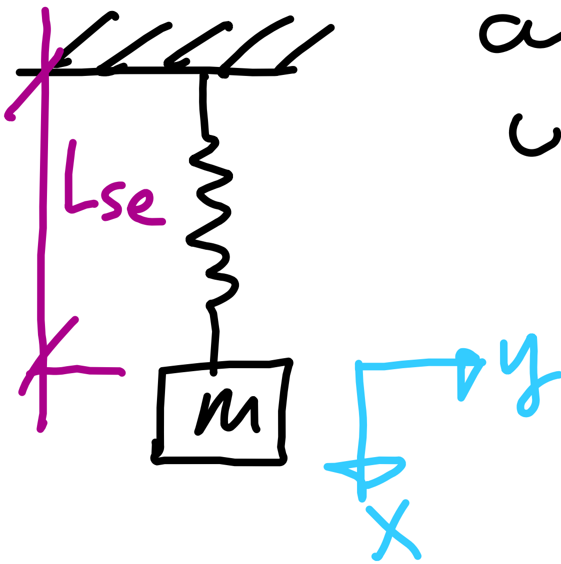

Let δ be displacement of spring from natural length L_0. This means $L_{se} = L_0 + \delta_0$, where $L_{se} \equiv$ length of spring when in equilibrium. We are going to set our coordinate system such that $x = 0$ at equilibrium

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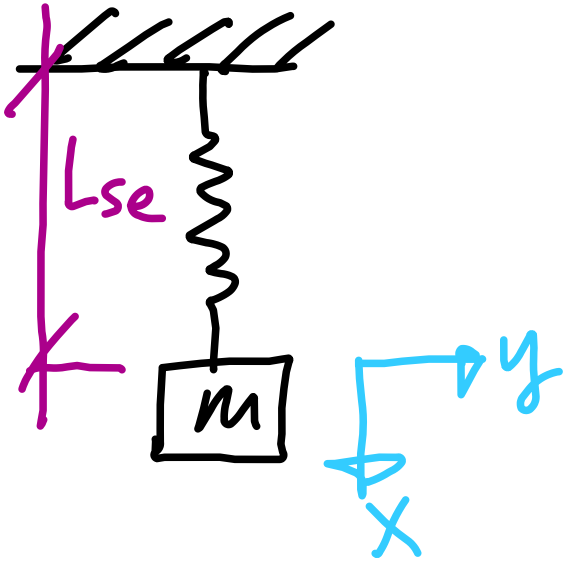

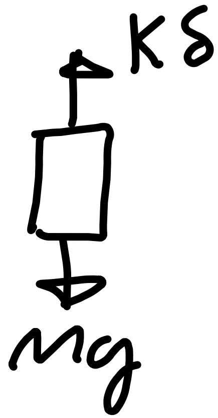

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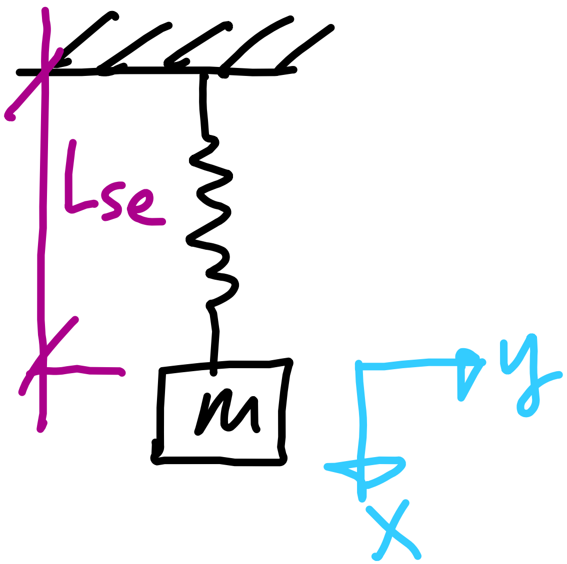

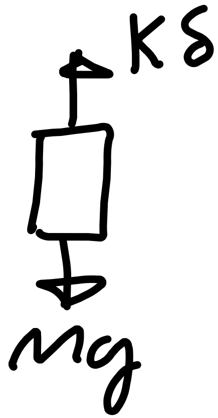

So $\sum F_x = 0 \Rightarrow$

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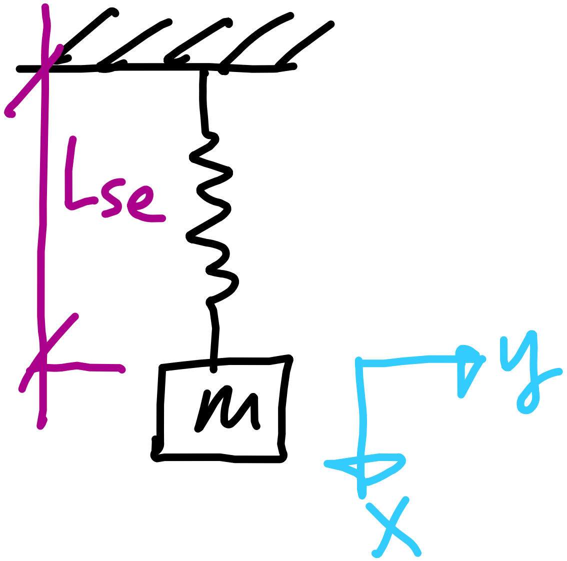

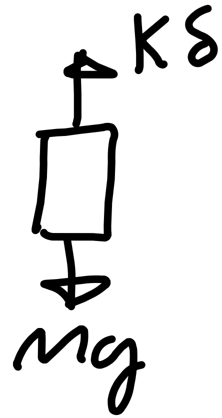

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$$-k\delta + mg = 0$$

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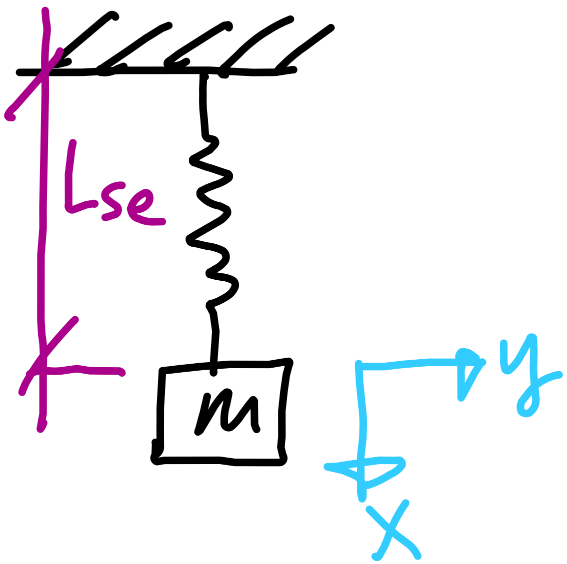

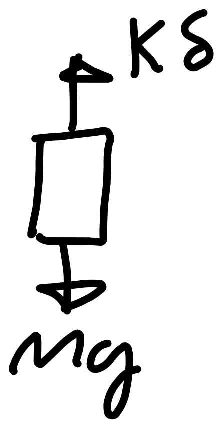

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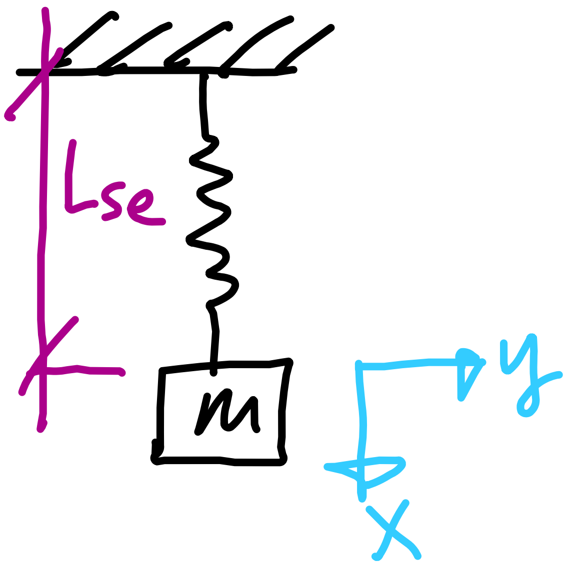

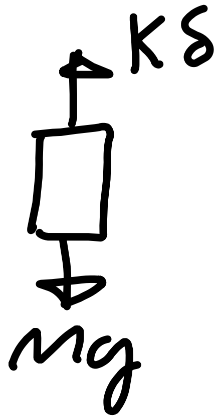

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Now move away from equilibrium by displacing mass an amount x

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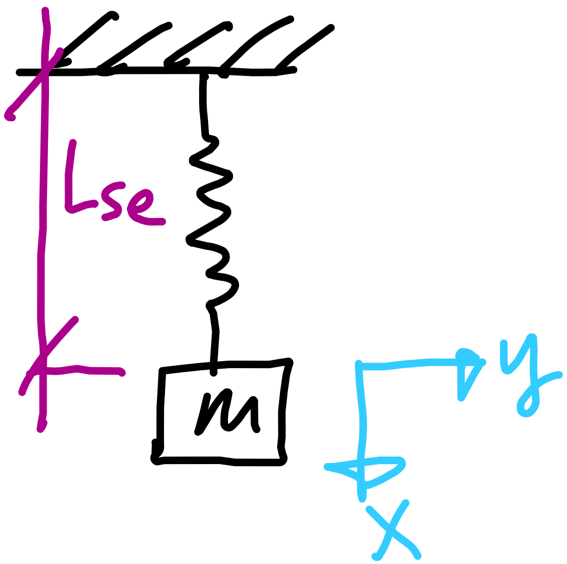
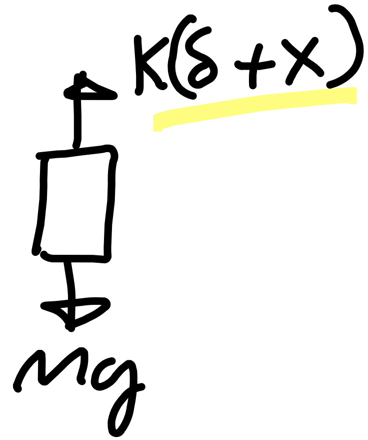
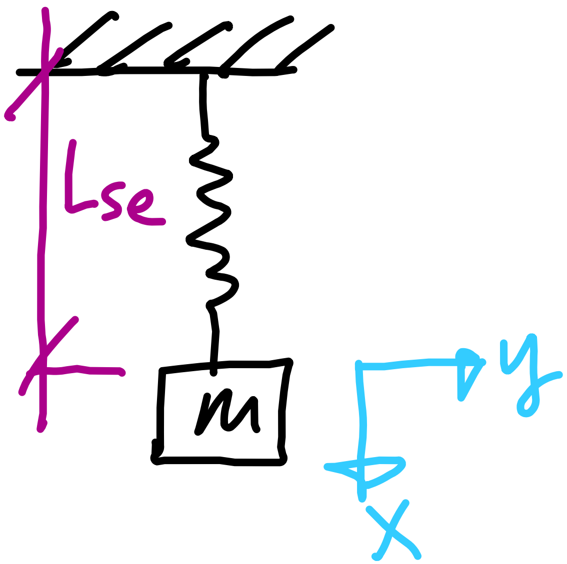
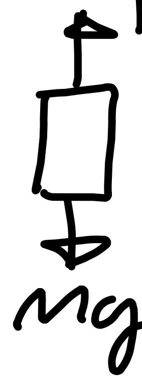




Example: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]

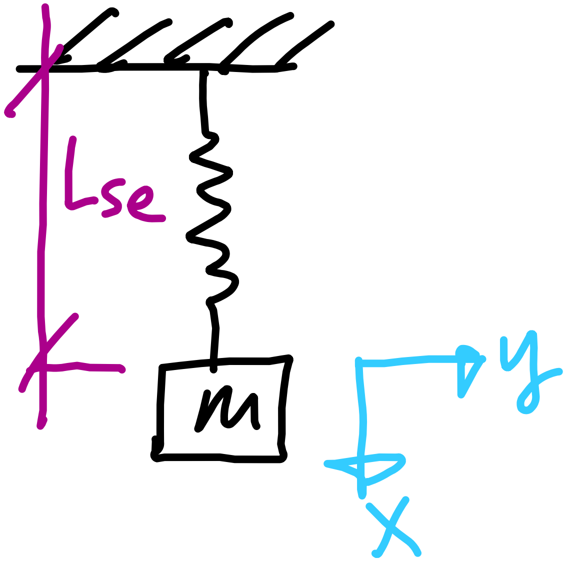


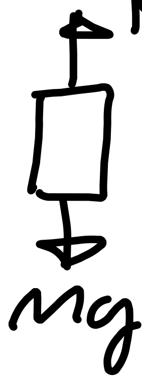


$K(\delta + x)$ So $\sum F_x = m a_x$

$\Rightarrow$

Example: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]





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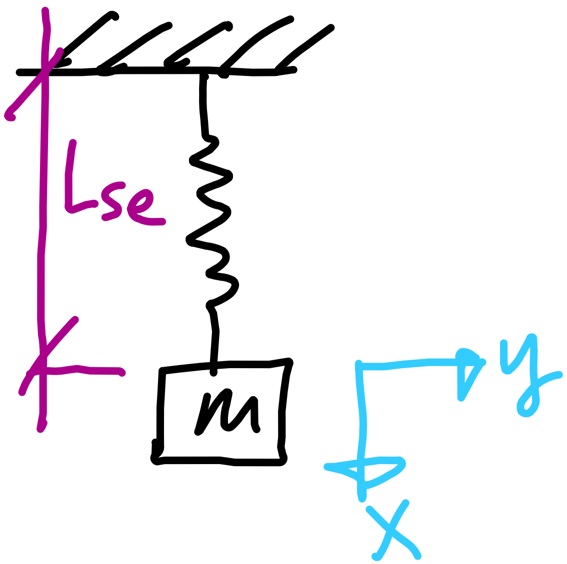

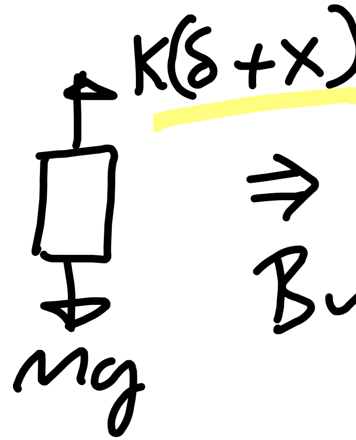

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But $-k\delta + mg = 0$

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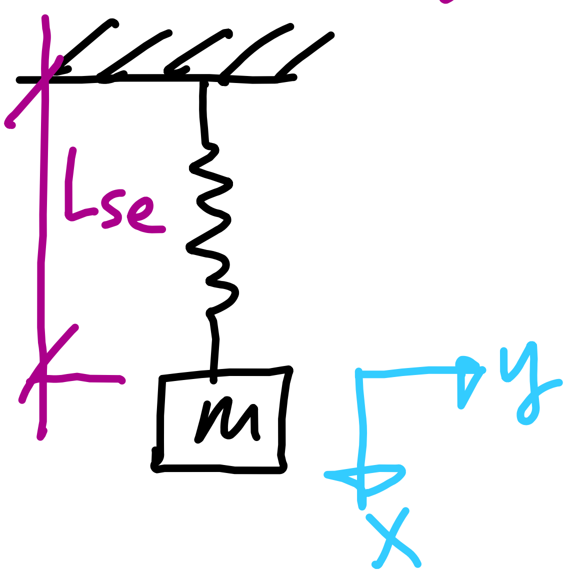

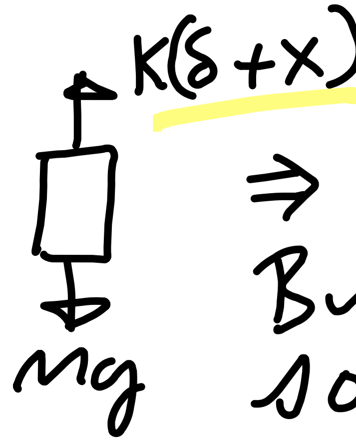

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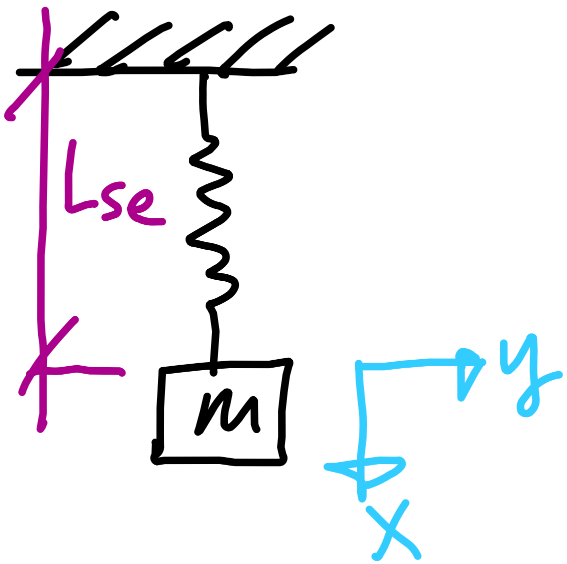

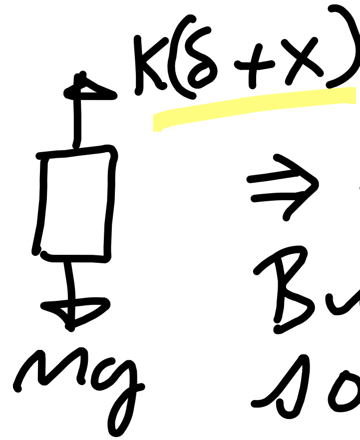

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$$\Rightarrow -kx = m\frac{d^2x}{dt^2}$$

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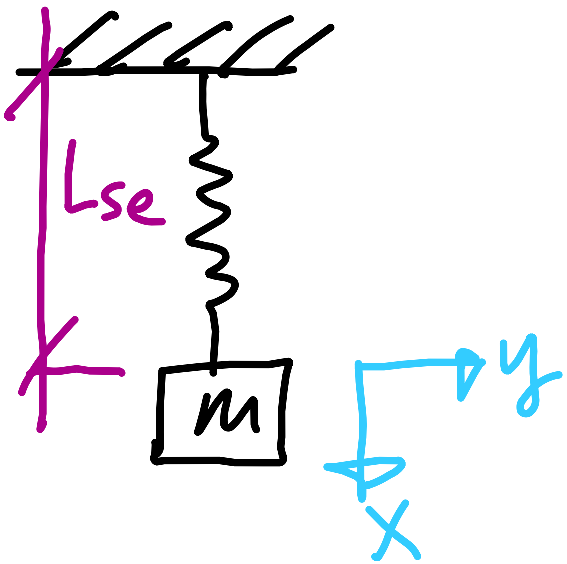

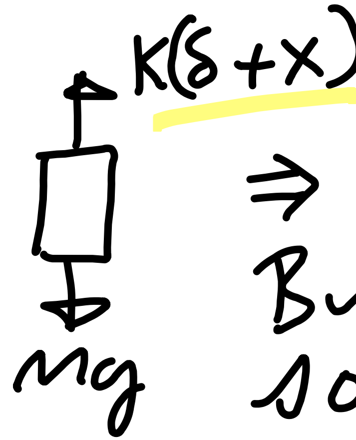

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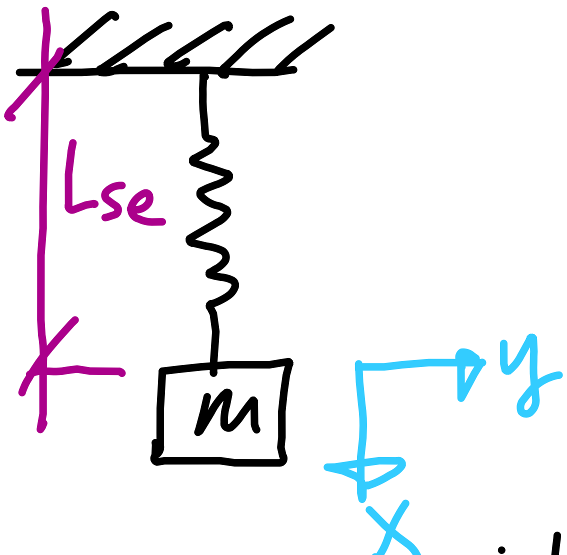

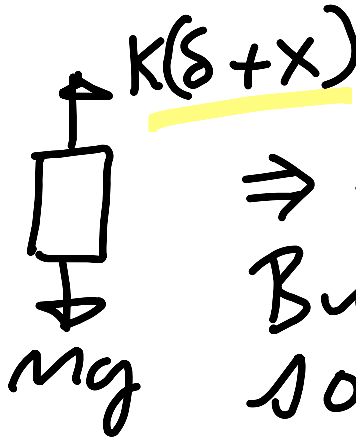

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Can rewrite as $\ddot{x} = -\omega_n^2 x$,

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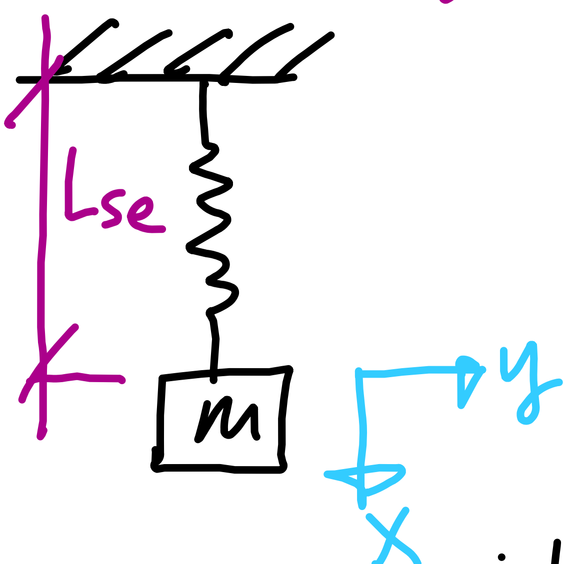

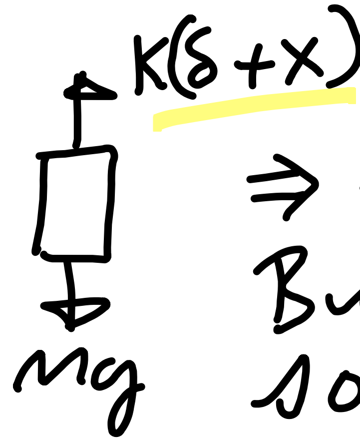

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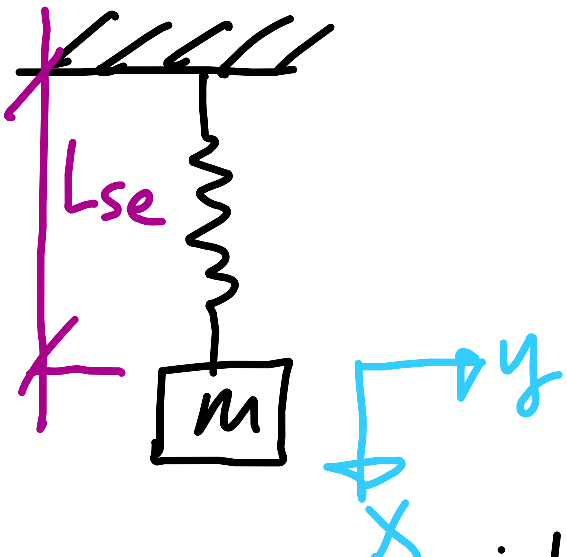

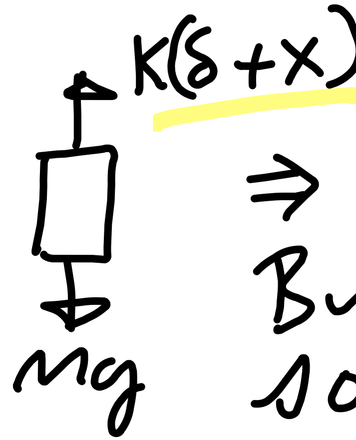

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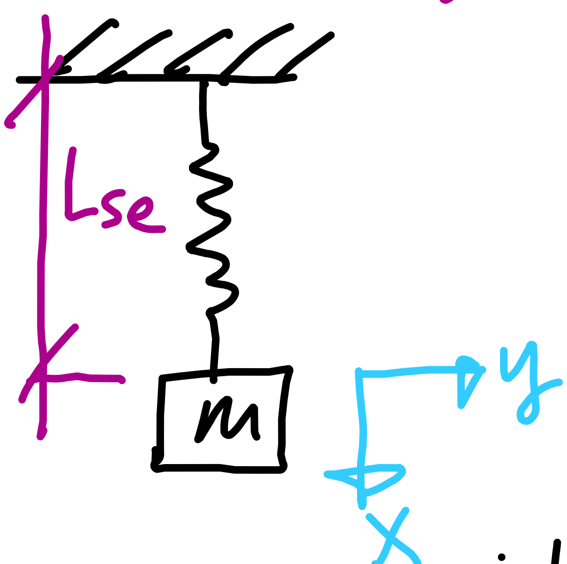

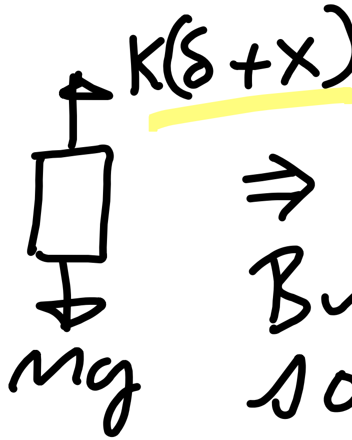

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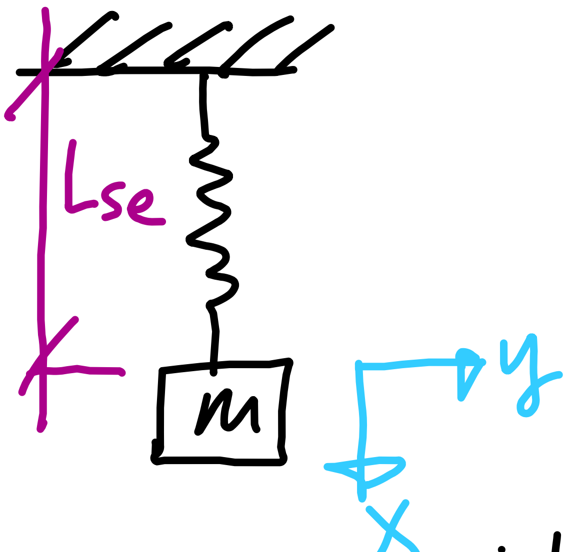

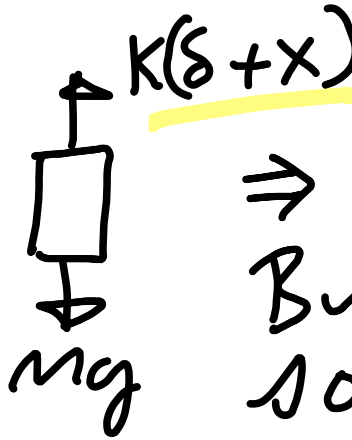

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OR

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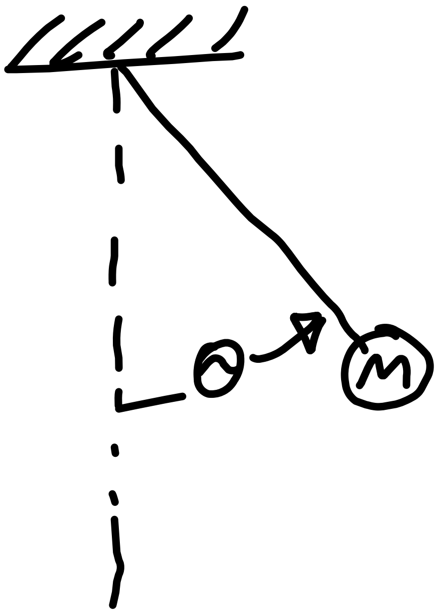
Book prefers this form

Example: Simple pendulum



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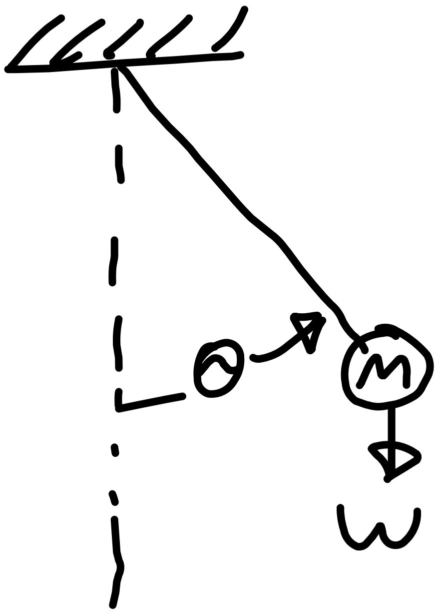

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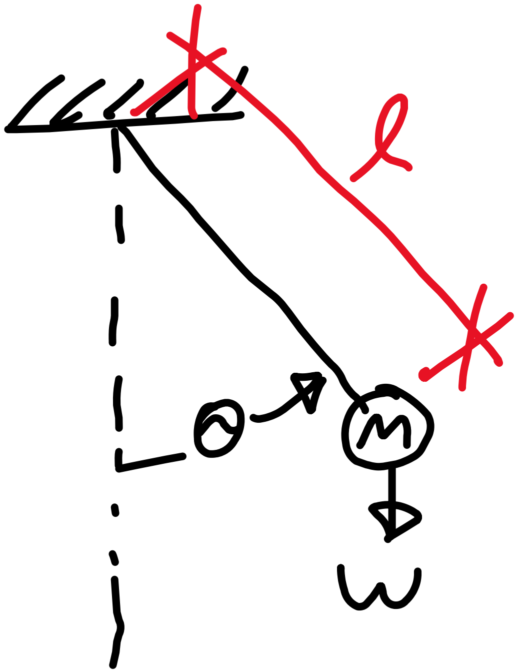

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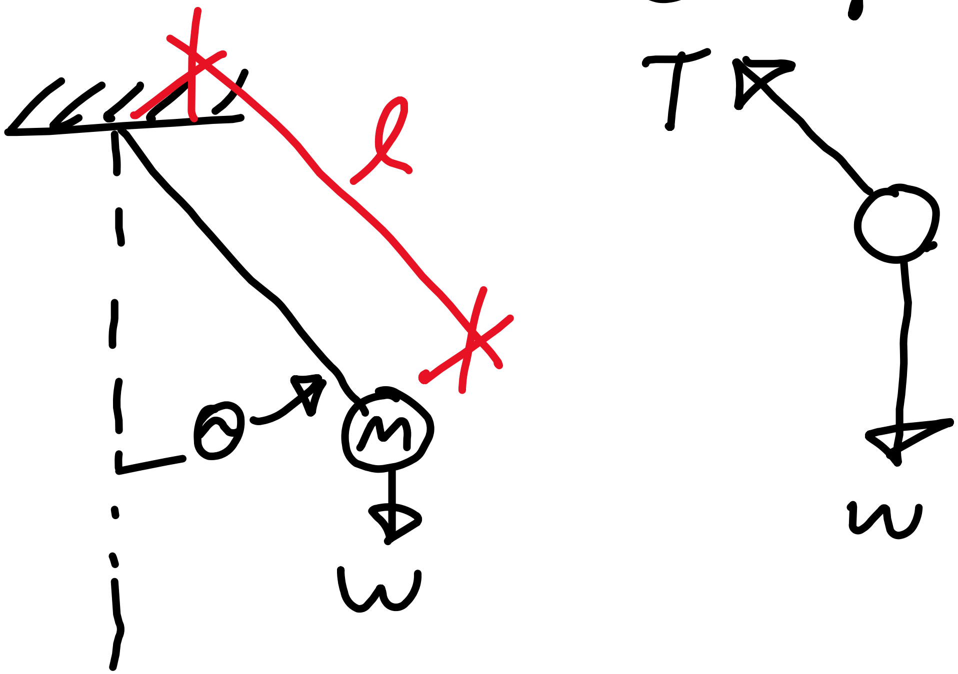

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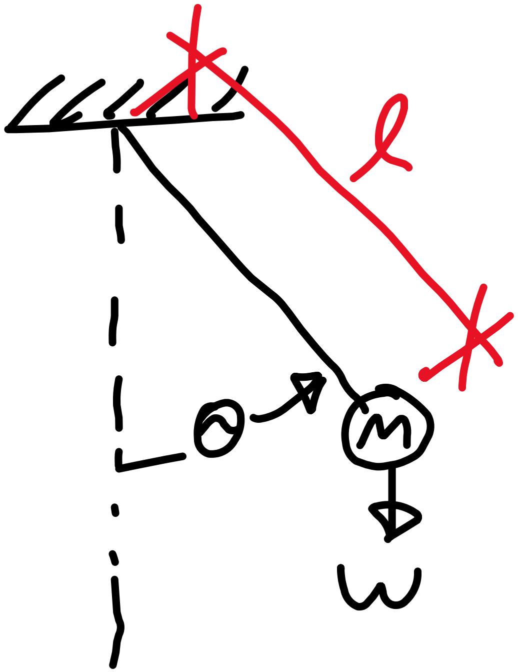

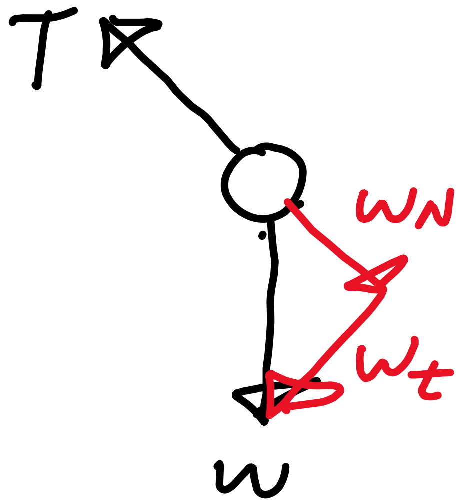

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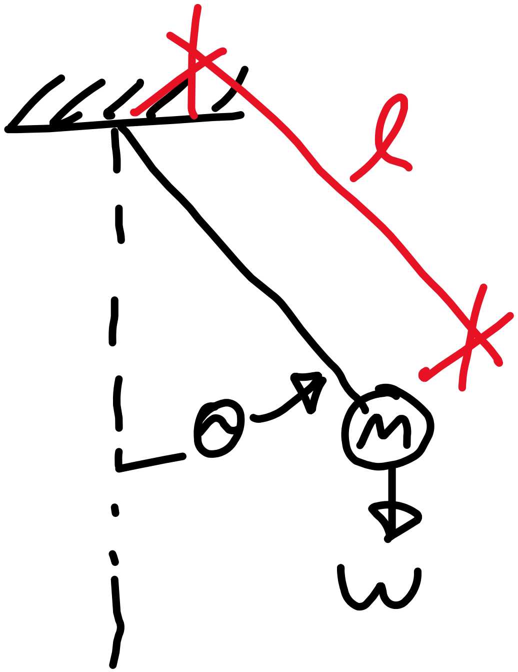
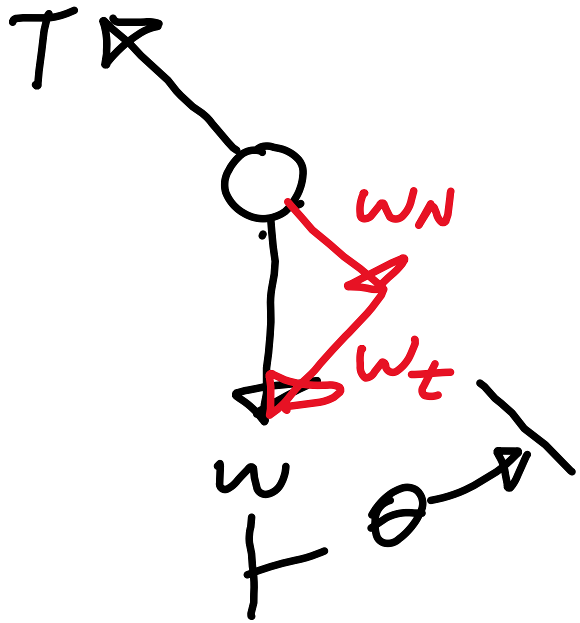
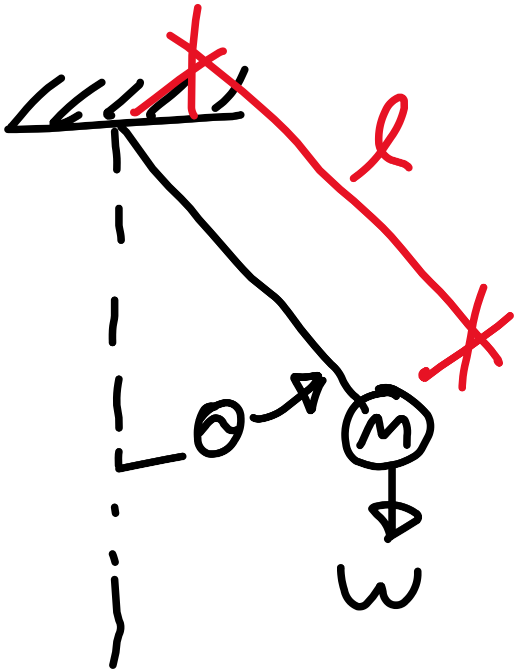
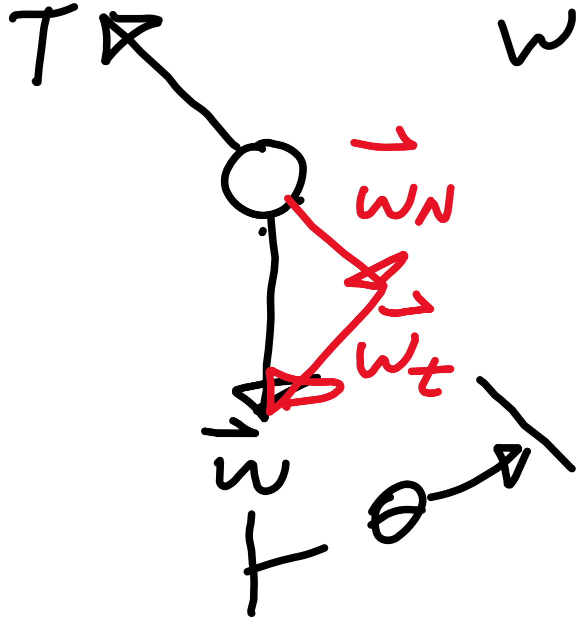




Example: Simple pendulum

$\vec{W} = \vec{W_N} + \vec{W_t}$





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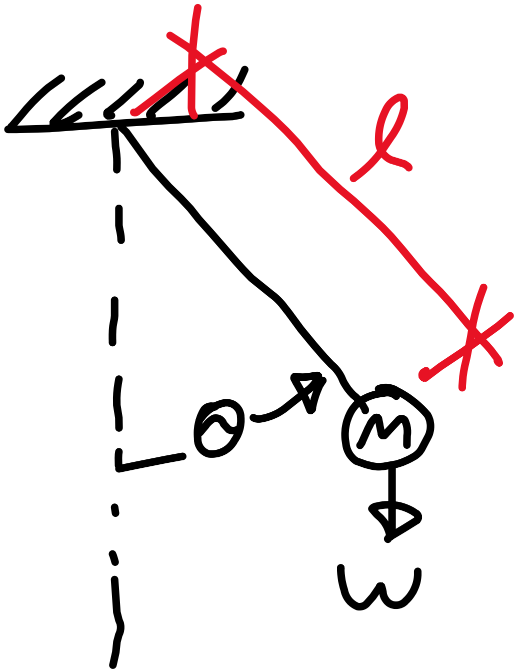

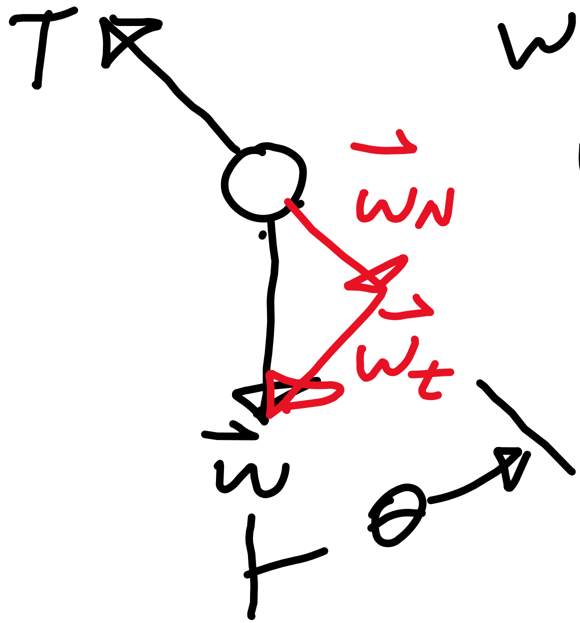

$$\vec{W} = \vec{W}_N + \vec{W}_t$$

$$W_N = -W\cos\theta$$

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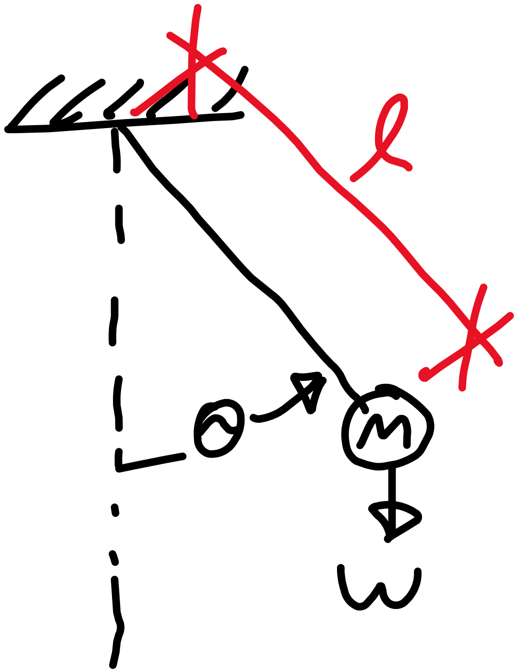

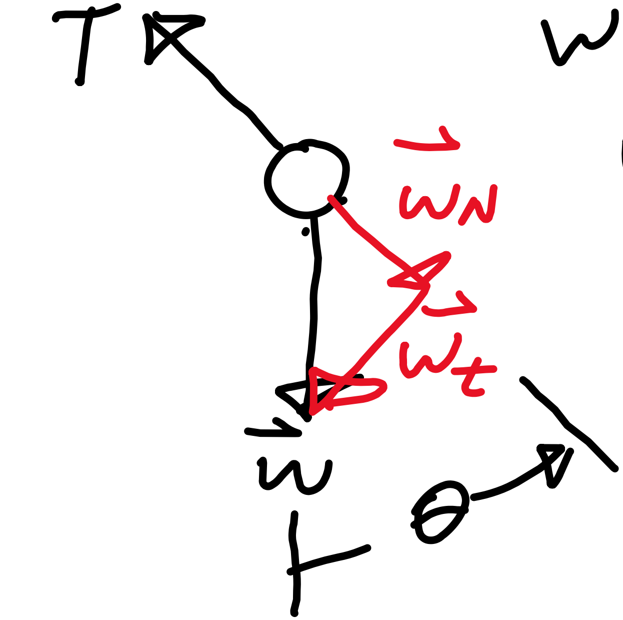

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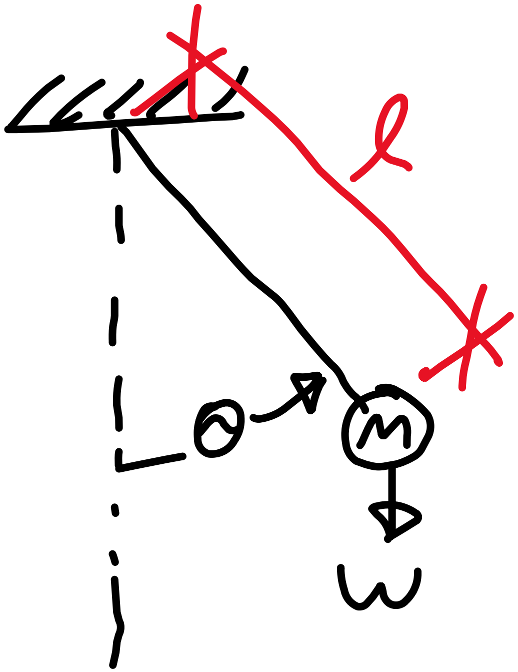

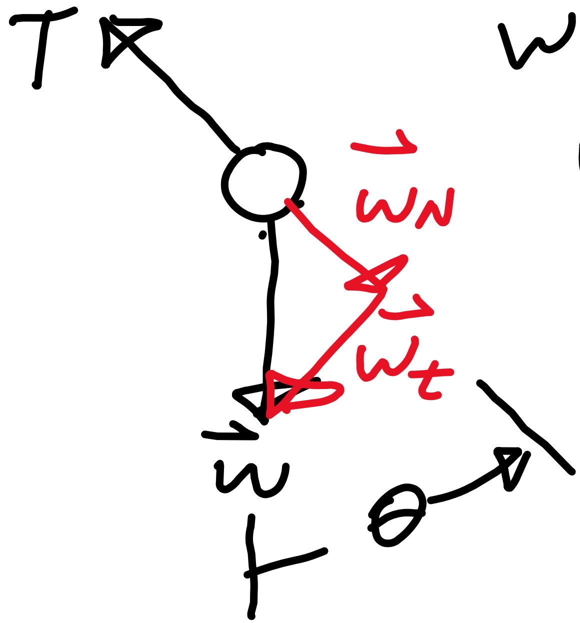

$\vec{W} = \vec{W}_N + \vec{W}_t$

$W_N = -W\cos\theta$

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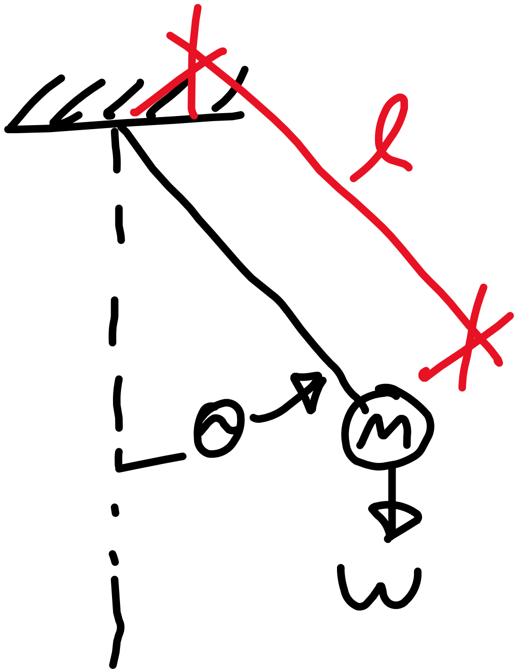
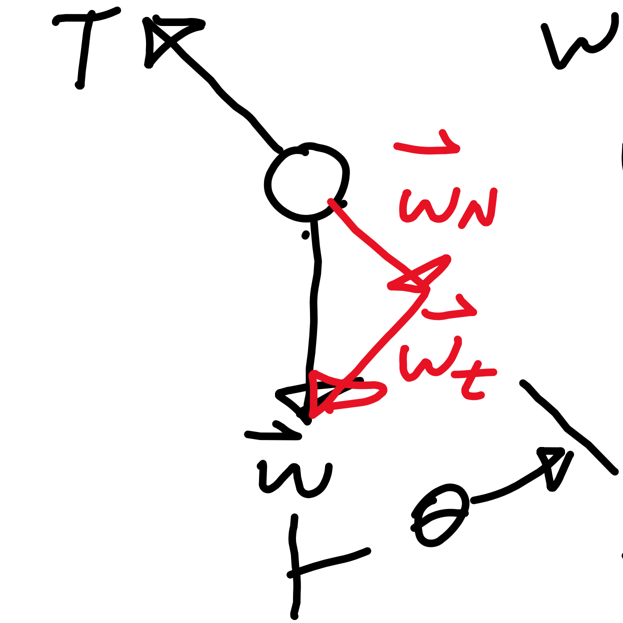
$\Sigma F_N = \cancel{0} \Rightarrow$

Example: Simple pendulum





$$\vec{W} = \vec{W}_N + \vec{W}_t$$

$$W_N = -W\cos\theta$$

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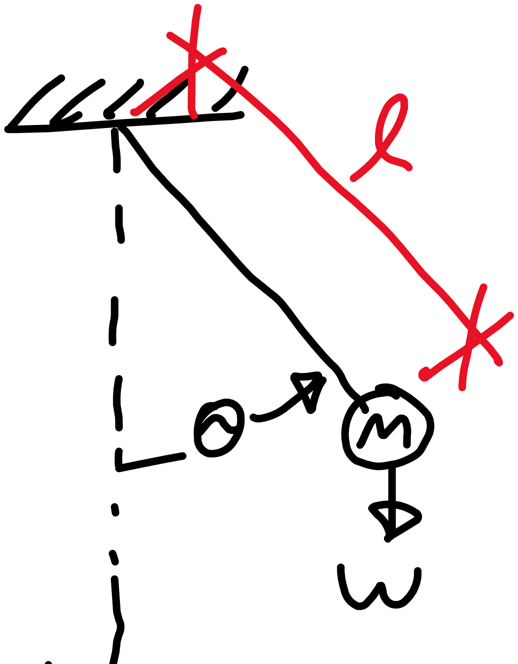
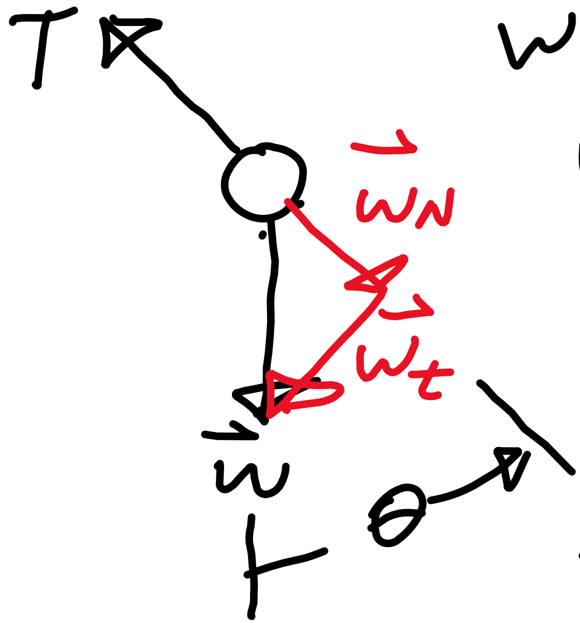
$$\Sigma F_N = 0 \Rightarrow$$

$$T = W\cos\theta$$

Example: Simple pendulum





$$\vec{W} = \vec{W}_N + \vec{W}_t$$

$$W_N = -W\cos\theta$$

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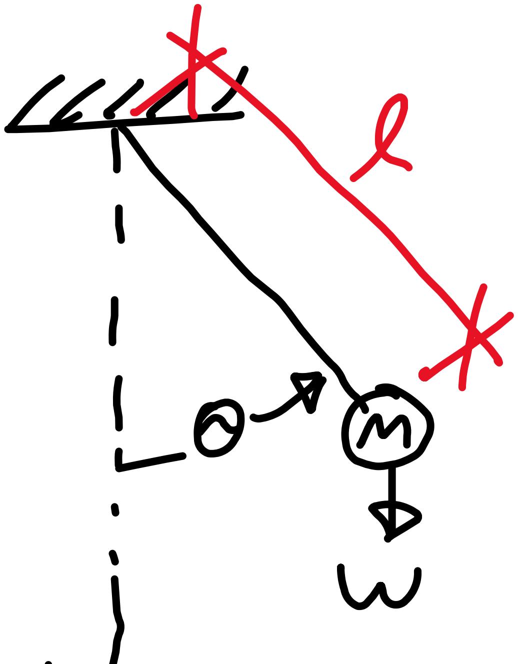
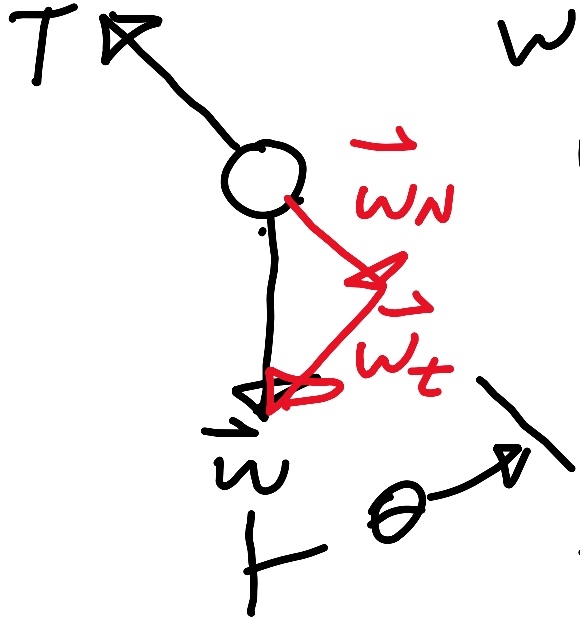
$$\sum F_N = 0 \Rightarrow$$

$$T = W\cos\theta$$

$$\sum F_t = M a_t$$

Example: Simple pendulum





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$$\Sigma F_N = 0 \Rightarrow$$

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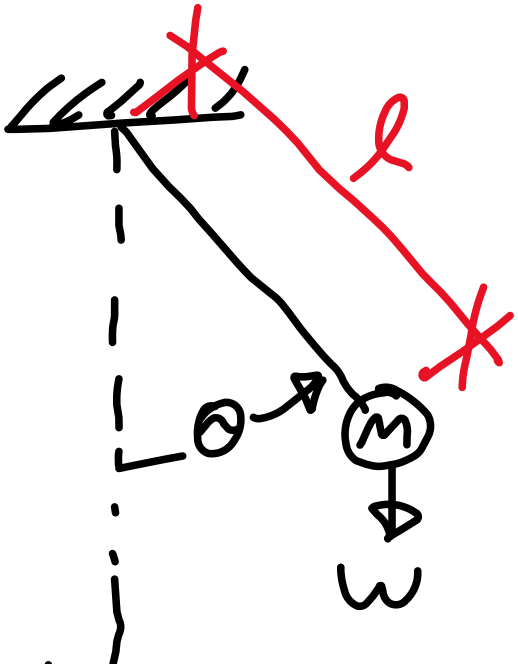
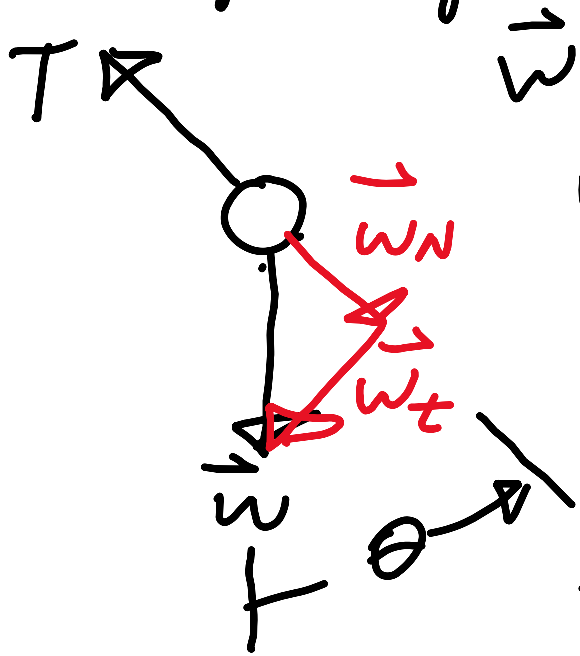
$$\Sigma F_t = m a_t \Rightarrow -mg\sin\theta = m\ell\alpha$$

Example: Simple pendulum





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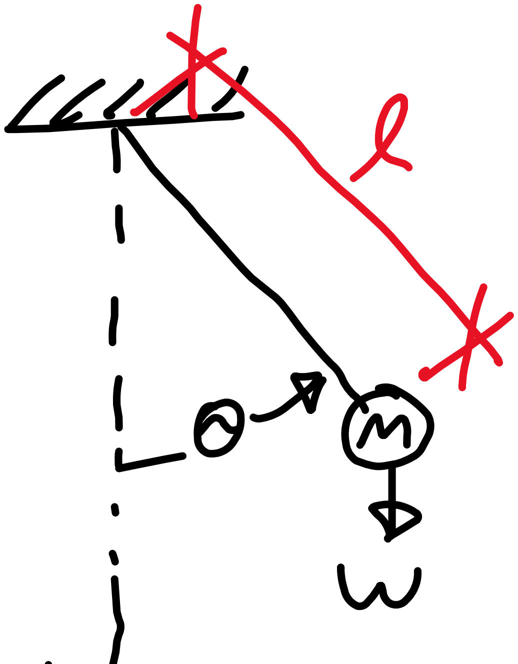
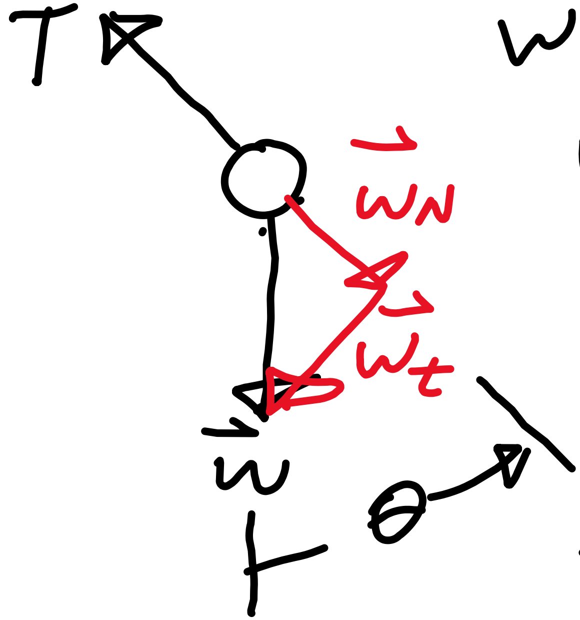
$$-g\sin\theta = l\ddot{\theta}$$

Example: Simple pendulum





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$W_N = -W\cos\theta$

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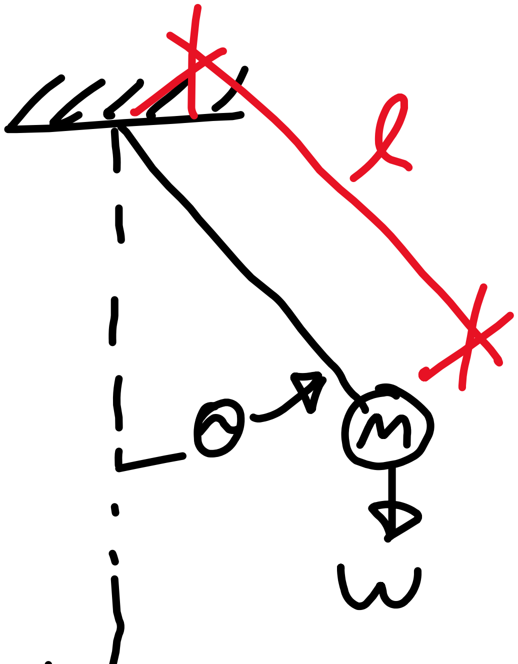
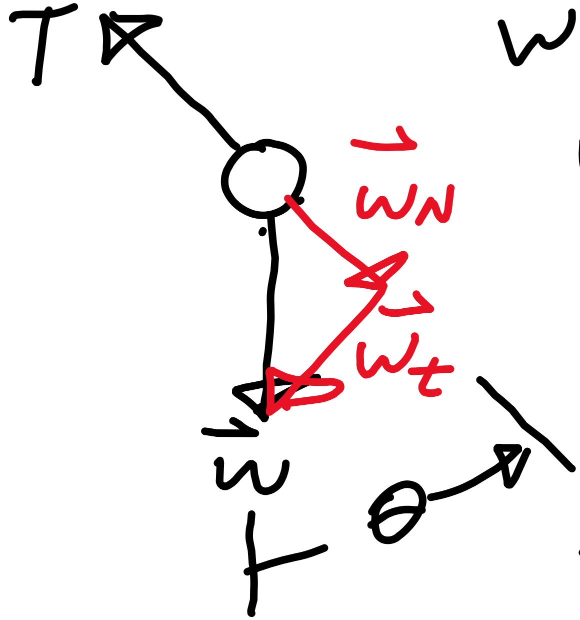
$\Sigma F_N = 0 \Rightarrow$

$T = W\cos\theta$

& $\Sigma F_t = ma_t \Rightarrow -mg\sin\theta = m\ell\alpha \Rightarrow$
$-g\sin\theta = \ell\ddot{\theta}$ Not quite the right form ☹

Example: Simple pendulum





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$W_N = -W\cos\theta$

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$\Sigma F_N = 0 \Rightarrow$

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$\Sigma F_t = M a_t \Rightarrow -mg\sin\theta = m\ell\alpha \Rightarrow$

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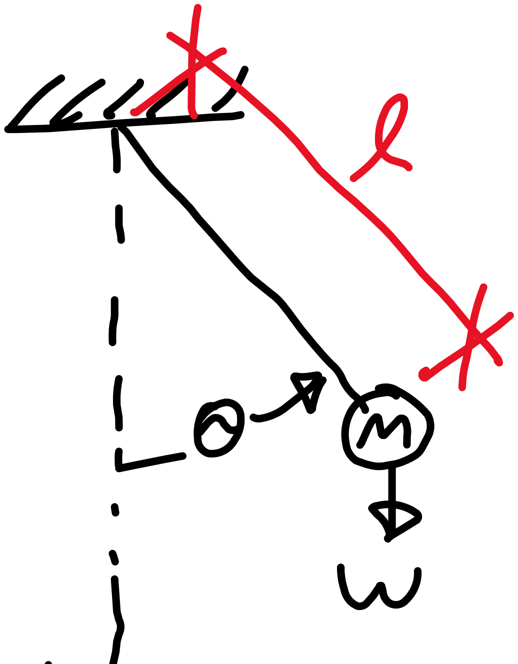
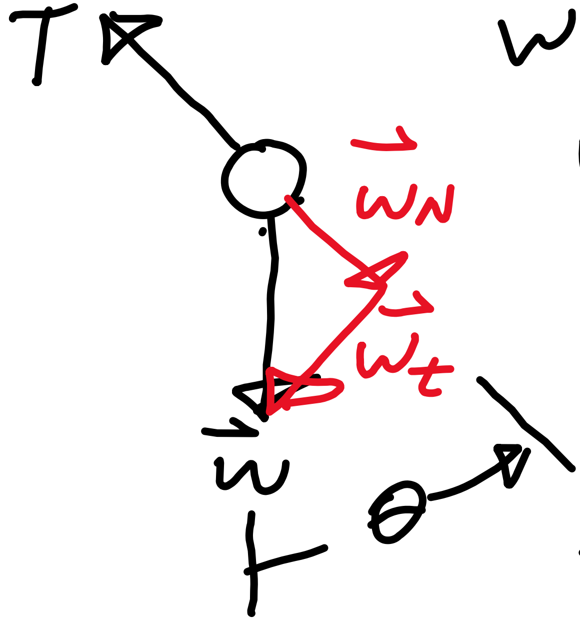
$$\ddot{\theta} = -(\text{const})\theta$$

Example: Simple pendulum





$\vec{W} = \vec{W}_N + \vec{W}_t$

$W_N = -W\cos\theta$

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$\Sigma F_N = 0 \Rightarrow$

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$\circlearrowright\ \Sigma F_t = Ma_t \Rightarrow -mg\sin\theta = m\ell\alpha \Rightarrow$

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$\ddot{\theta} = -(\text{const})\theta$, but have

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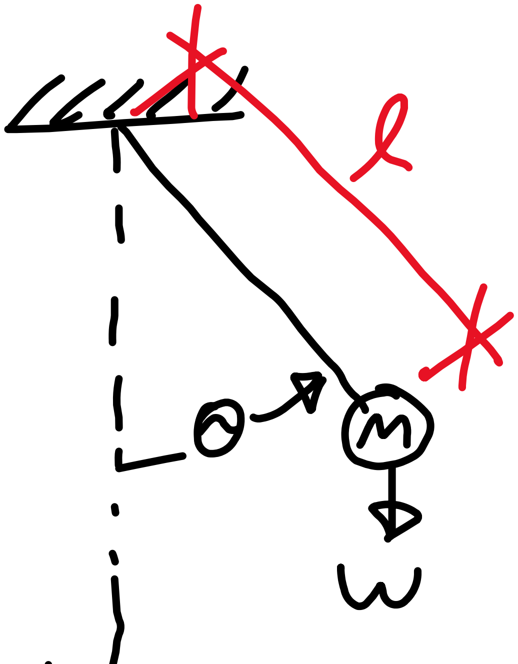
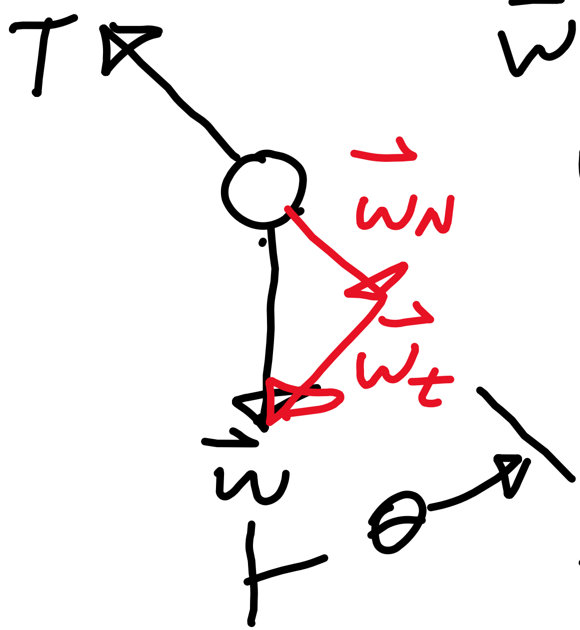
Example: Simple pendulum





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$\Sigma F_N = 0 \Rightarrow$

$T = W\cos\theta$

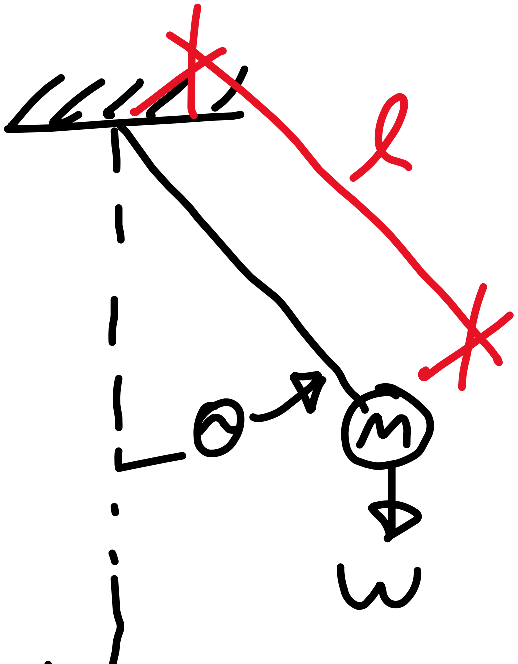
$\Sigma F_t = Ma_t \Rightarrow -mg\sin\theta = m\ell\alpha \Rightarrow$

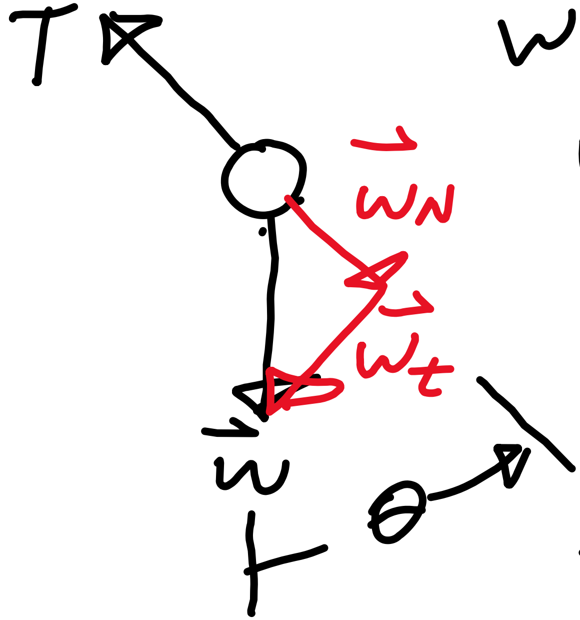
$-g\sin\theta = \ell\ddot{\theta}$ Not quite the right form ☹, we want the form

$\ddot{\theta} = -(\text{const})\theta$, but have

$\ddot{\theta} = -(\text{const})\sin\theta$ Need small angle approx

Example: Simple pendulum





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Note: $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$

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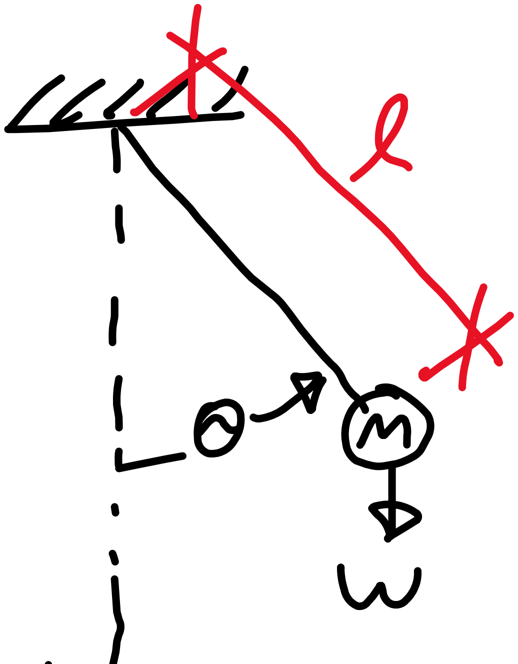

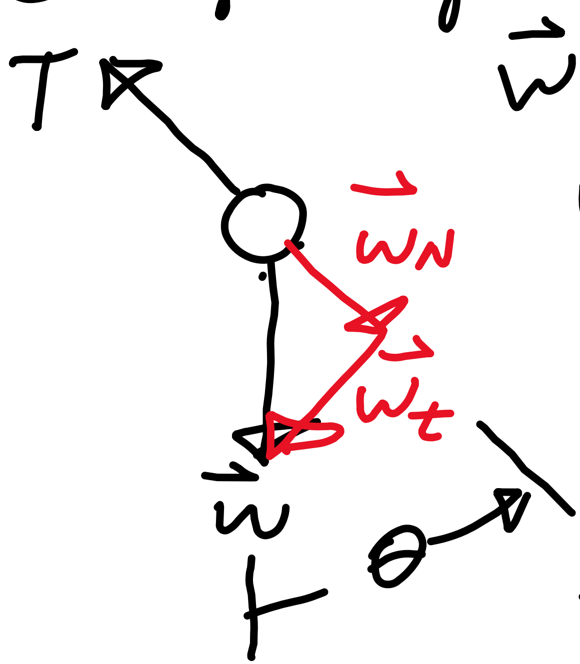

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So, for small angles $\sin\theta \cong \theta$

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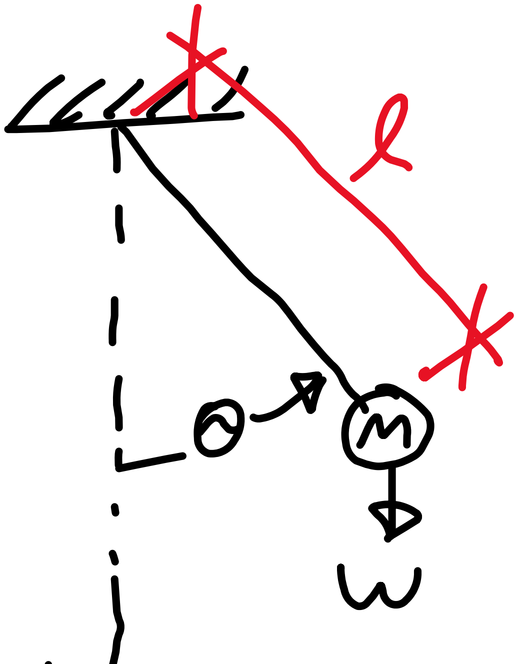

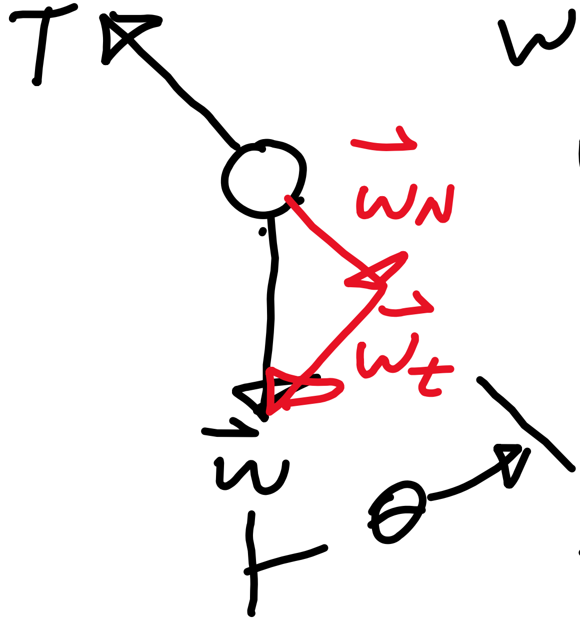

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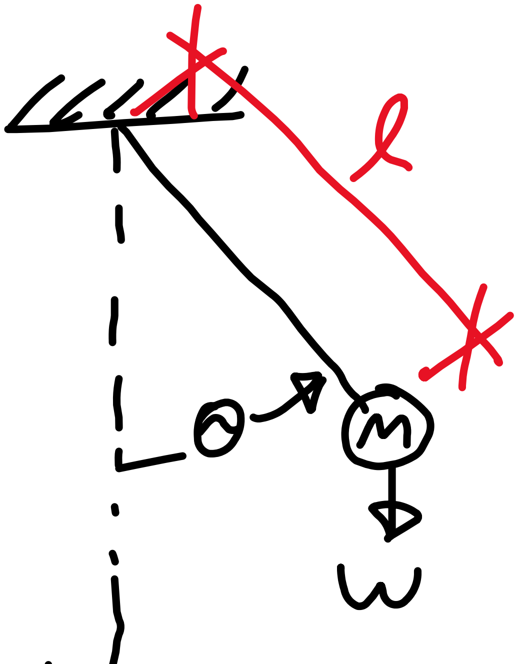

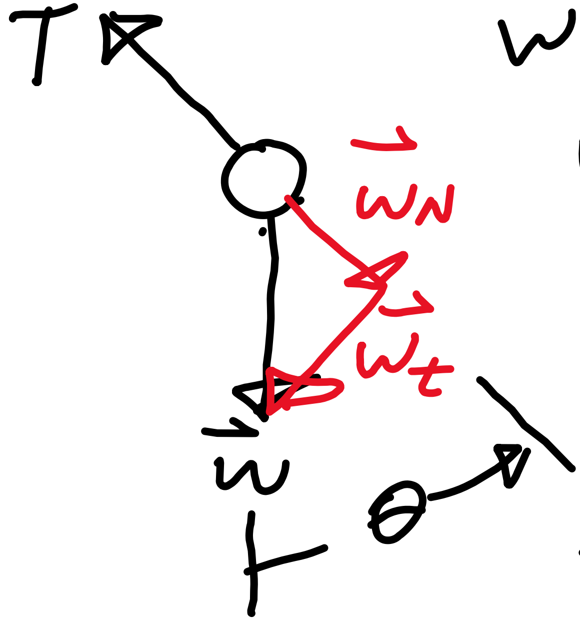

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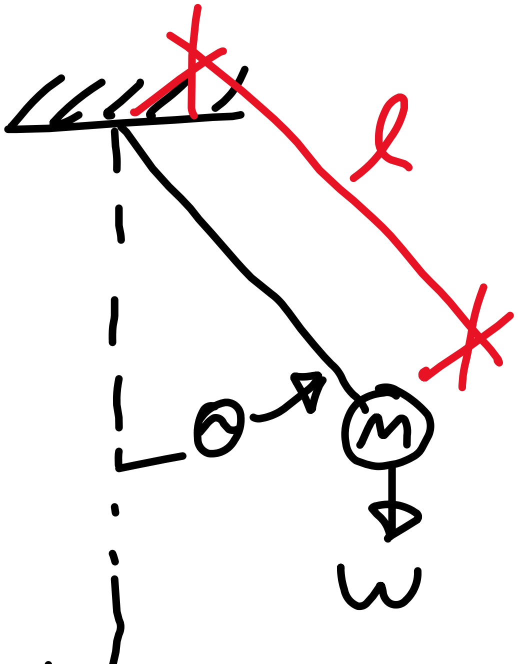

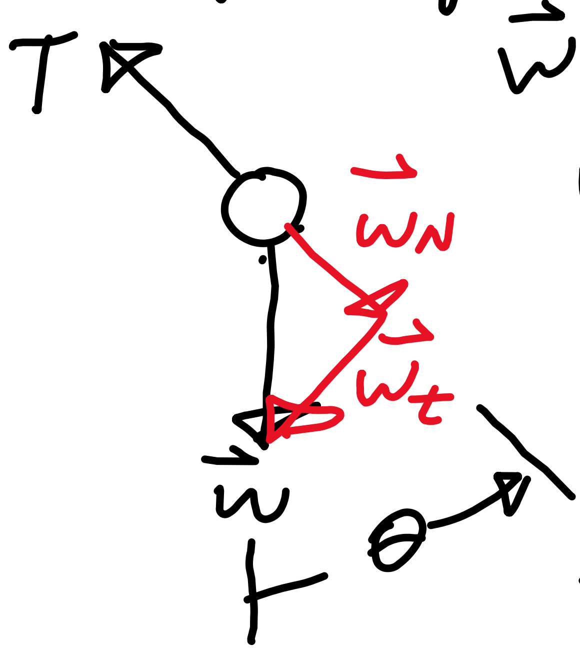

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Now $\ddot{\theta} = -\omega_n^2\theta$, where $\omega_n = \sqrt{\frac{g}{\ell}} \Rightarrow$

$\theta = \theta_{MAX}\sin(\omega_n t + \phi)$