Today 19.1, 19.2

$\boxed{L32}$

Today 19.1, 19.2

$\boxed{L32}$

$\underbrace{19.1}_{\text{Vibrations without Damping}}$

Today 19.1, 19.2

$\boxed{L32}$

19.1: Vibrations without Damping

19.2: Free vibrations of rigid bodies

ASU

Today 19.1, 19.2

Monday 19.2, 19.3

$\boxed{L32}$

Today 19.1, 19.2

$\boxed{L32}$

Monday 19.2, 19.3

Energy methods for vibrations

Today 19.1, 19.2

Monday 19.2, 19.3

$\boxed{L32}$

Important dates

Today 19.1, 19.2

Monday 19.2, 19.3

$\boxed{L32}$

Important dates

* Friday Nov 27th No class

Today 19.1, 19.2 $\boxed{L32}$

Monday 19.2, 19.3

Important dates

* Friday Nov 27^{th} No class ☺

Today 19.1, 19.2

$\boxed{L32}$

Monday 19.2, 19.3

Important dates

* Friday Nov 27th No class
* Monday Nov 30th Exam #4

Today 19.1, 19.2

L32

Monday 19.2, 19.3

Important dates

* Friday Nov 27th No class ☺
* Monday Nov 30th Exam #4
* Wednesday Dec 2nd Day of Reckoning

Today 19.1, 19.2 $\boxed{L32}$

Monday 19.2, 19.3

Important dates

* Friday Nov 27th No class ☺
* Monday Nov 30th Exam #4
* Wednesday Dec 2nd Day of Reckoning
* Friday Dec 4th Final exam

ASU

From last time: If $\ddot{J} = -\omega^2 J$

From last time: If $\ddot{f} = -\omega^2 f$,

then $f = A\cos(\omega t) + B\sin(\omega t)$

From last time: If $\ddot{J} = -\omega^2 J$,

then $J = A\cos(\omega t) + B\sin(\omega t)$

or $J = J_{max}\sin(\omega t + \varphi)$

From last time: If $\ddot{\jmath} = -\omega^2 \jmath$,

then $\jmath = A\cos(\omega t) + B\sin(\omega t)$

or $\jmath = \jmath_{MAX}\sin(\omega t + \varphi)$

or $\jmath = \jmath_{MAX}\cos(\omega t + \psi)$

From last time: If $\ddot{f} = -\omega^2 f$,

then $f = A\cos(\omega t) + B\sin(\omega t)$

or $f = f_{max}\sin(\omega t + \varphi)$

or $f = f_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$,

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{max}\sin(\omega t + \phi)$

or $y = y_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \phi)$

ASU

From last time: If $\ddot{\jmath} = -\omega^2 \jmath$,

then $\jmath = A\cos(\omega t) + B\sin(\omega t)$

or $\jmath = \jmath_{max}\sin(\omega t + \varphi)$

or $\jmath = \jmath_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$.

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{max}\sin(\omega t + \varphi)$

or $y = y_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

From last time: If $\ddot{f} = -\omega^2 f$,

then $f = A\cos(\omega t) + B\sin(\omega t)$

or $f = f_{MAX}\sin(\omega t + \varphi)$

or $f = f_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m\sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2\theta$, then $\theta = \theta_m\sin(\omega t + \varphi)$

Also $V_{MAX} = \omega X_{MAX}$

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{MAX}\sin(\omega t + \varphi)$

or $y = y_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

Also $V_{MAX} = \omega x_{MAX}$ &

$a_{MAX} = \omega V_{MAX}$

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{MAX}\sin(\omega t + \varphi)$

or $y = y_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

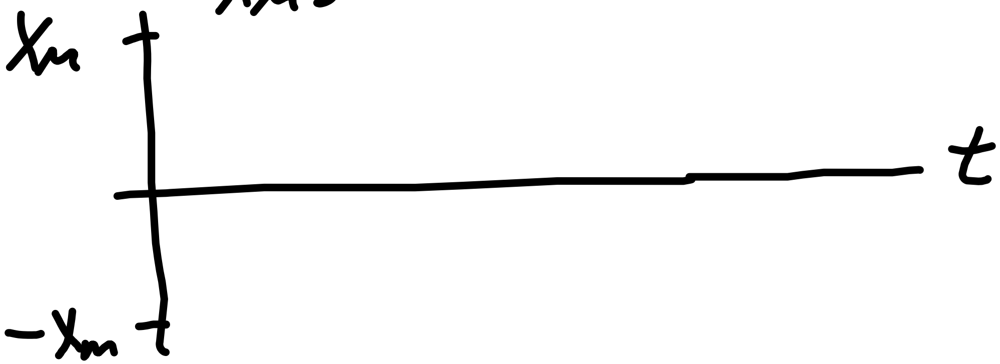
Also $V_{MAX} = \omega X_{MAX}$ &

$a_{MAX} = \omega V_{MAX} = \omega^2 X_{MAX}$

Wave anatomy

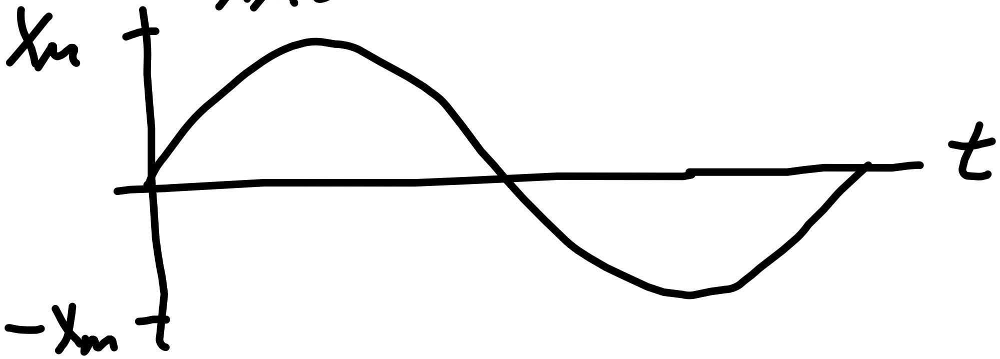
Wave anatomy

$x_m \sin(\omega t)$



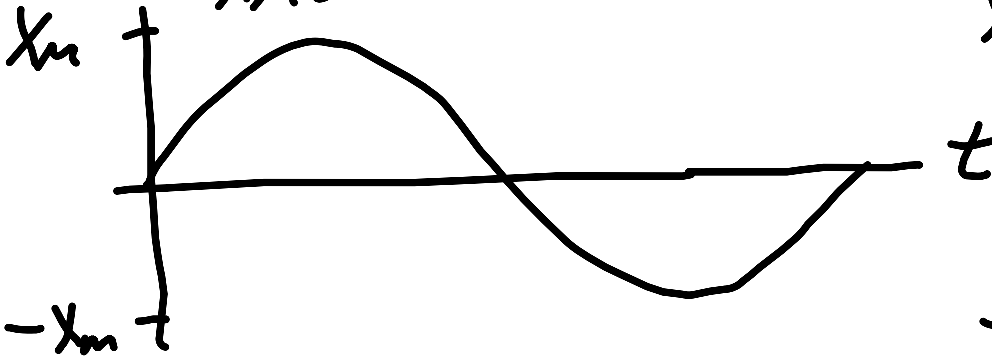
Wave anatomy

$x_m \sin(\omega t)$



Wave anatomy

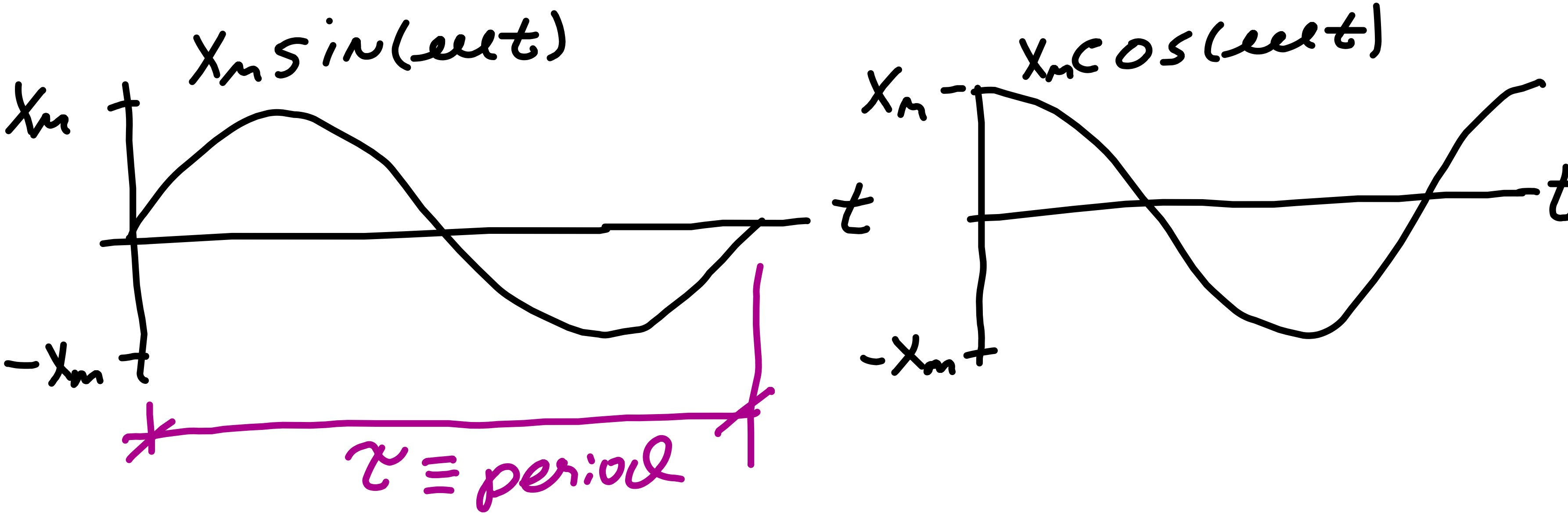
$x_m \sin(\omega t)$



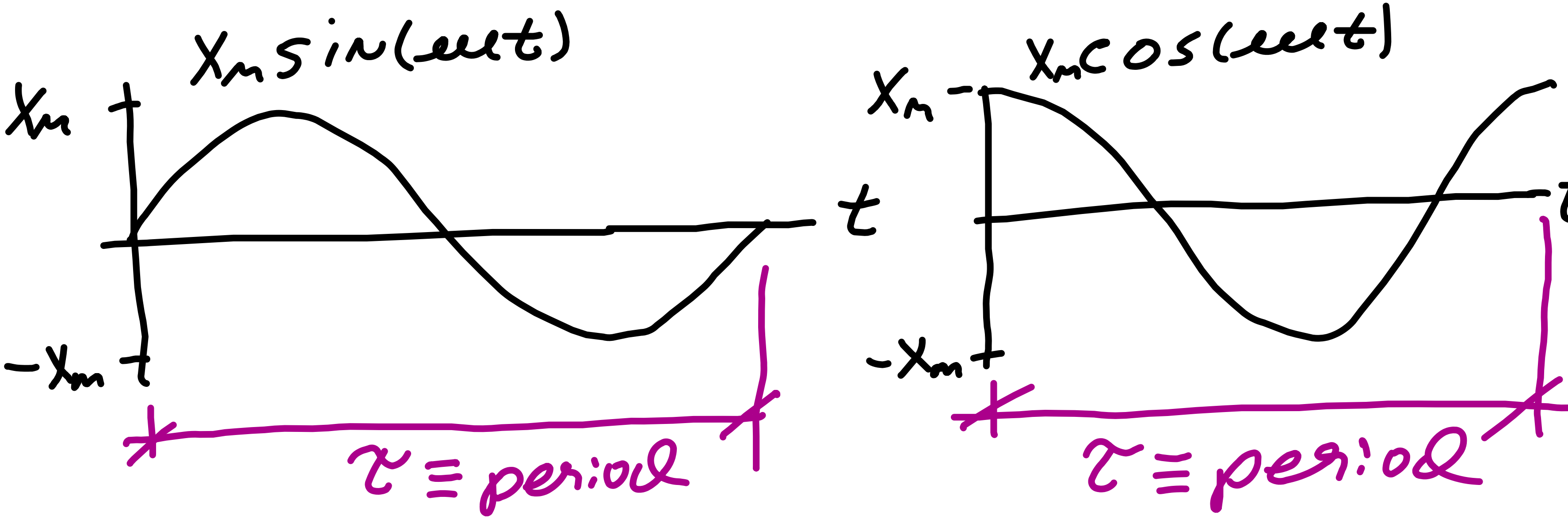
$x_m \cos(\omega t)$

ASU

Wave anatomy

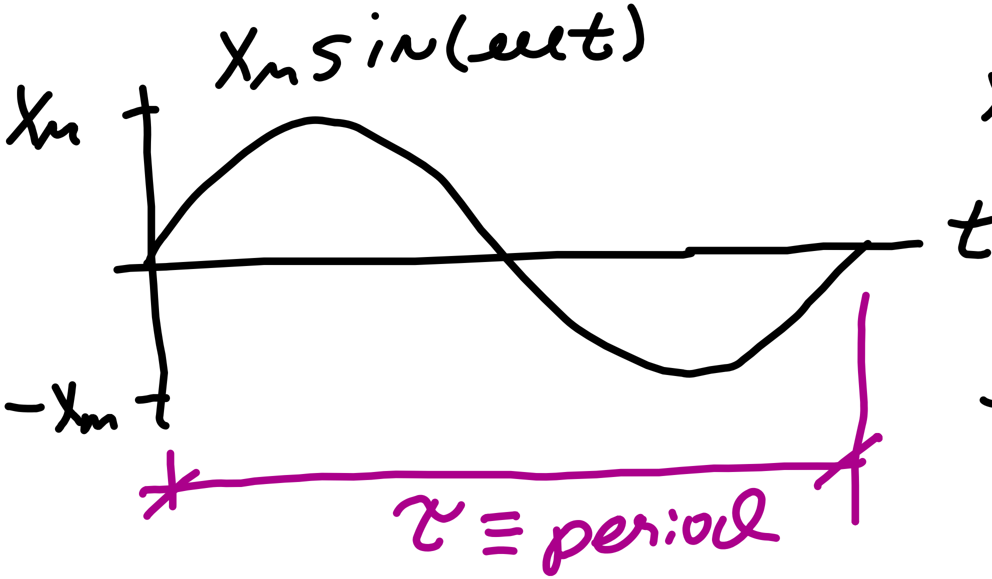


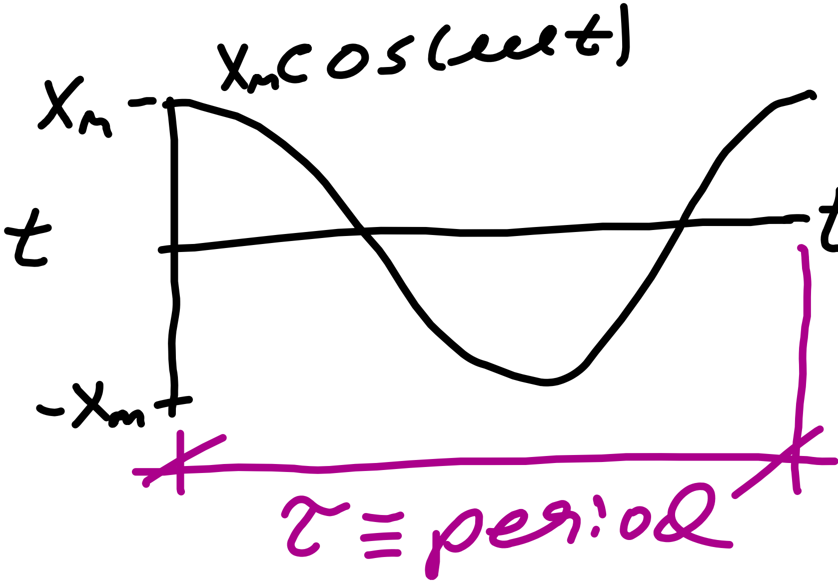
Wave anatomy



ASU

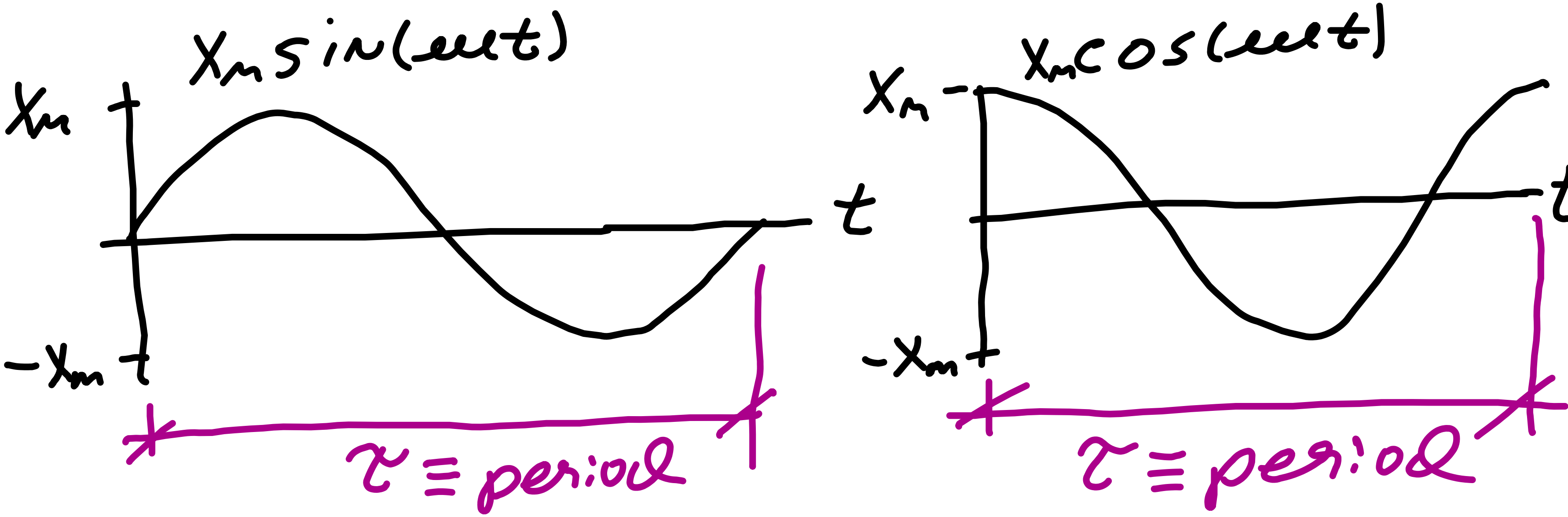
Wave anatomy





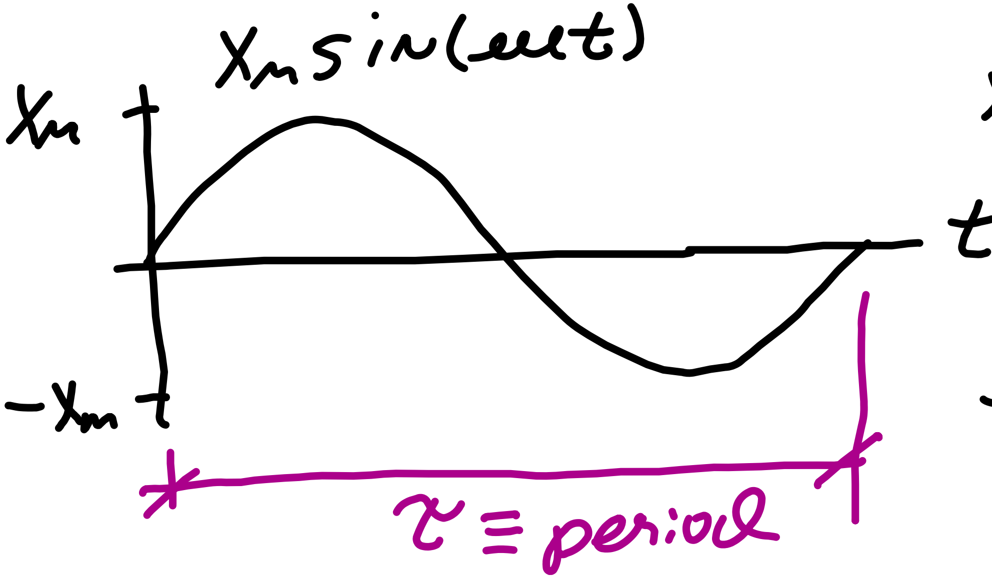
So when $t =$ period

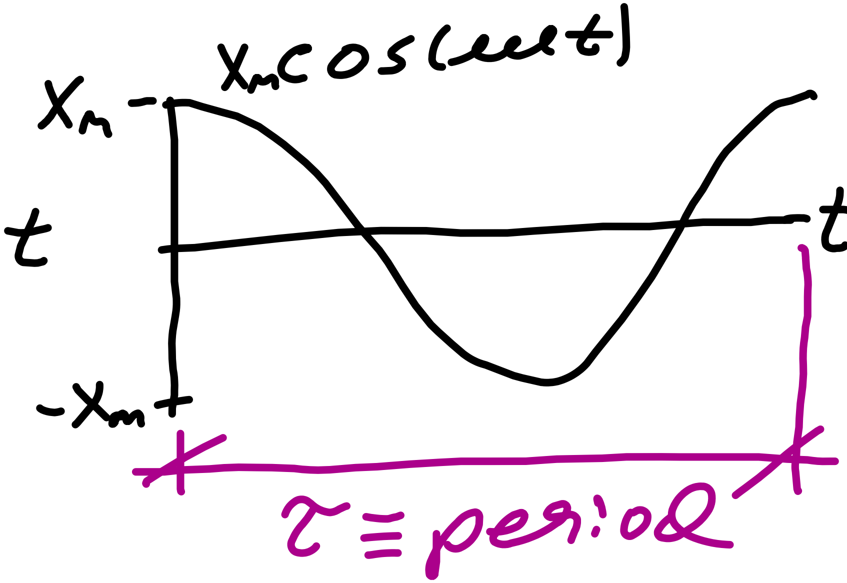
Wave anatomy



So when $t =$ period $= \tau$

Wave anatomy

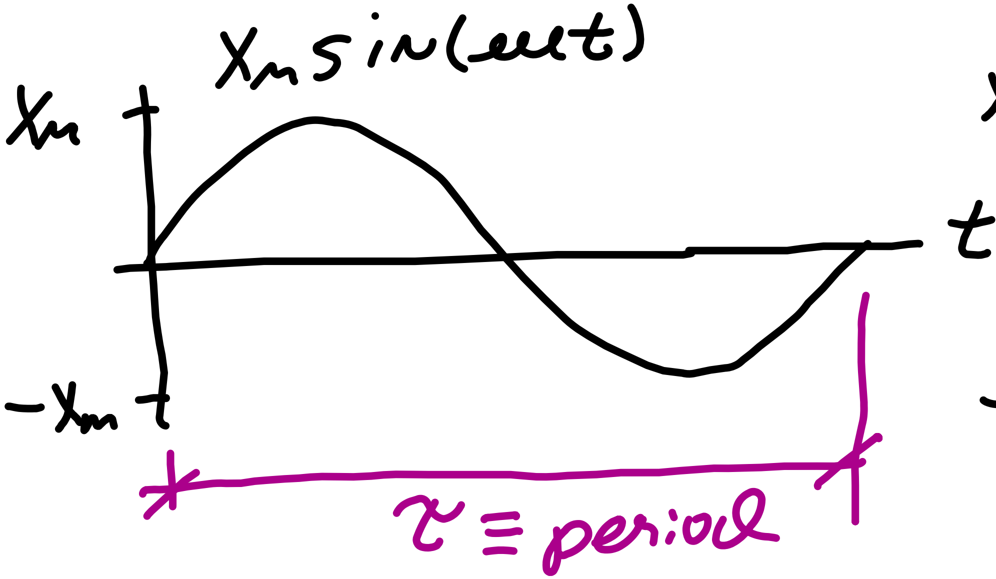


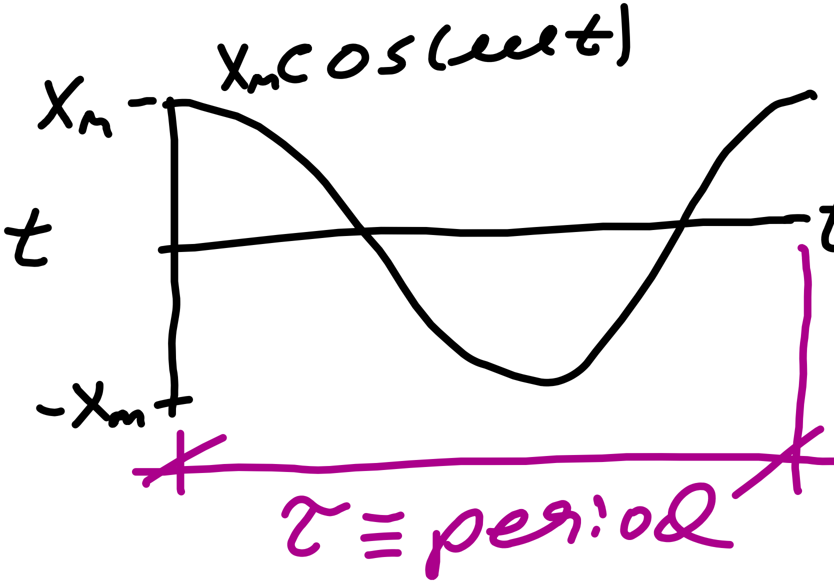


So when $t = \text{period} = \tau$ then

$\omega t = 2\pi$

Wave anatomy

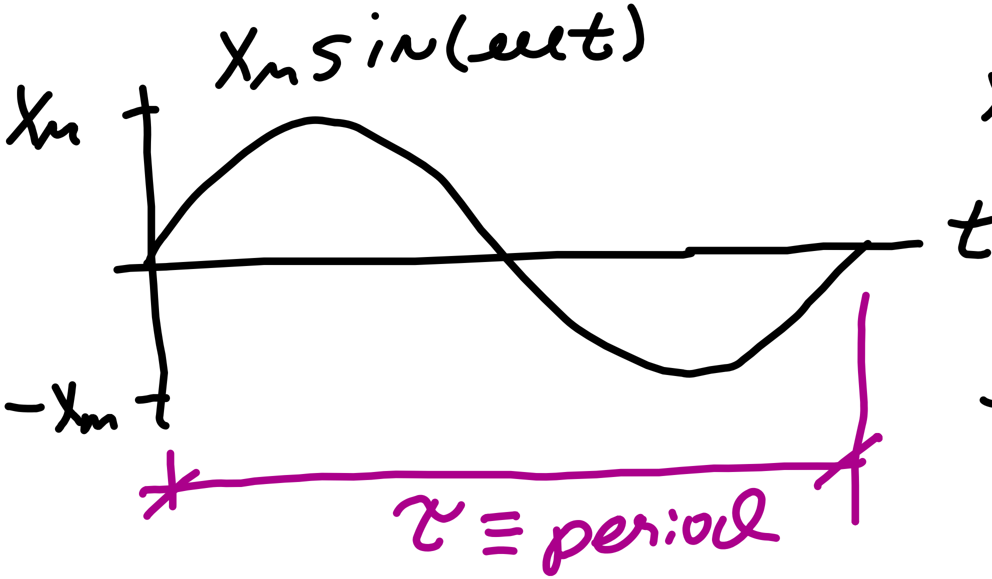


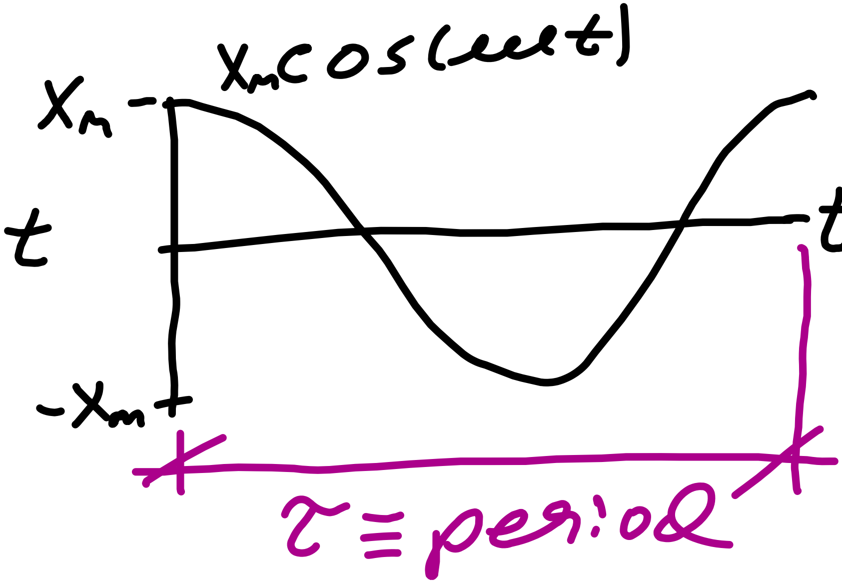


So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi$

Wave anatomy

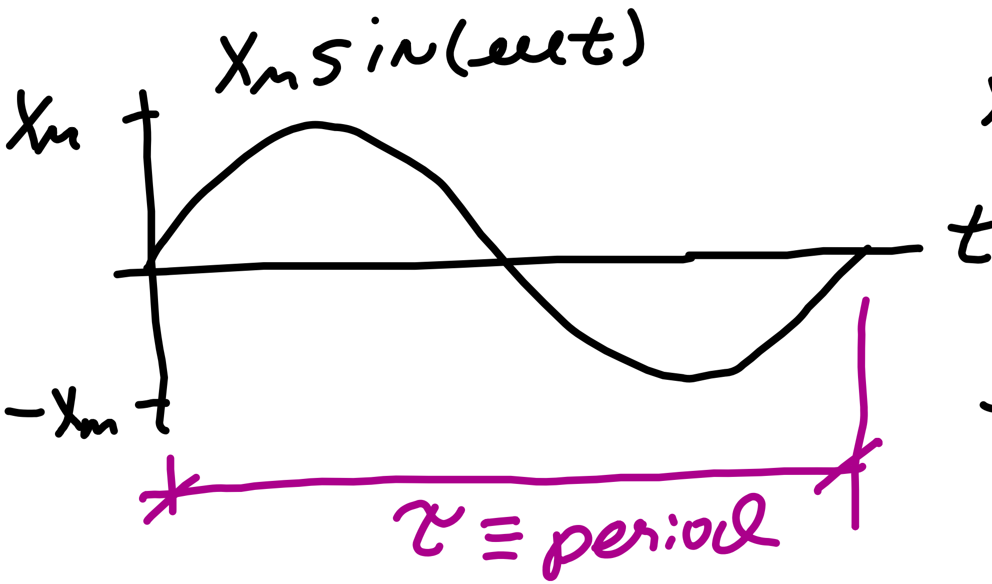


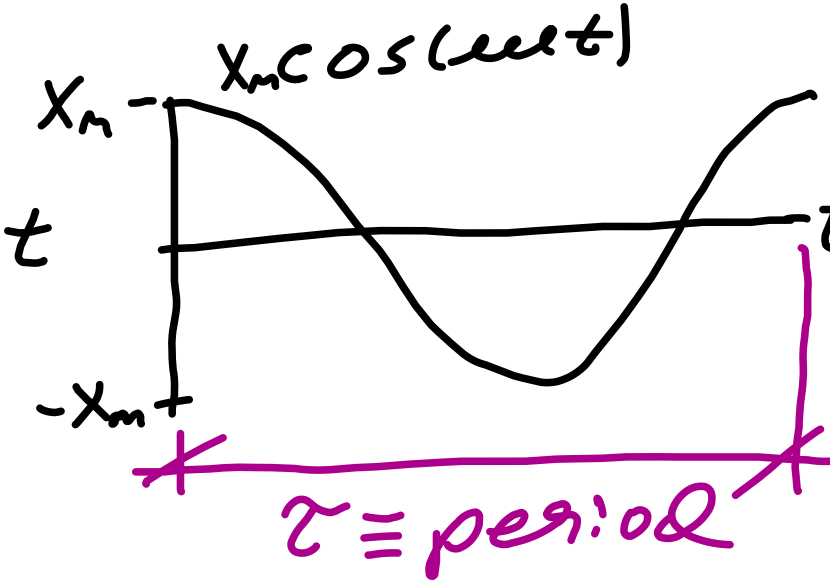


So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

Wave anatomy



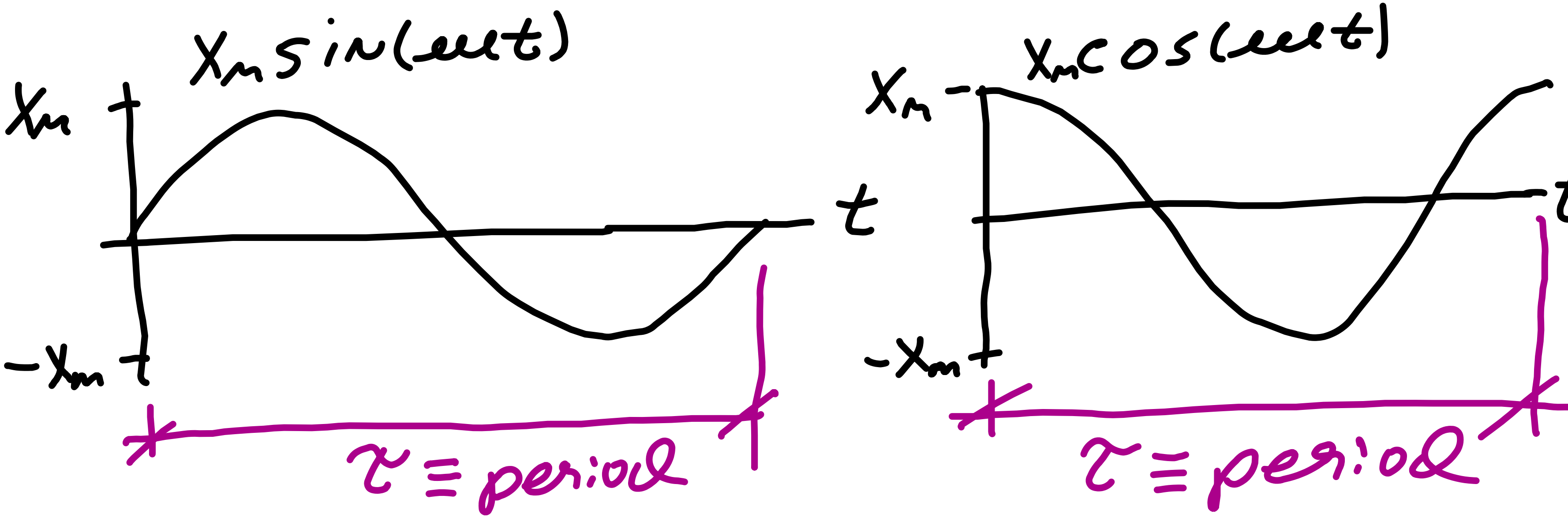


So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

$\&\ \omega = \frac{2\pi}{\tau}$

Wave anatomy



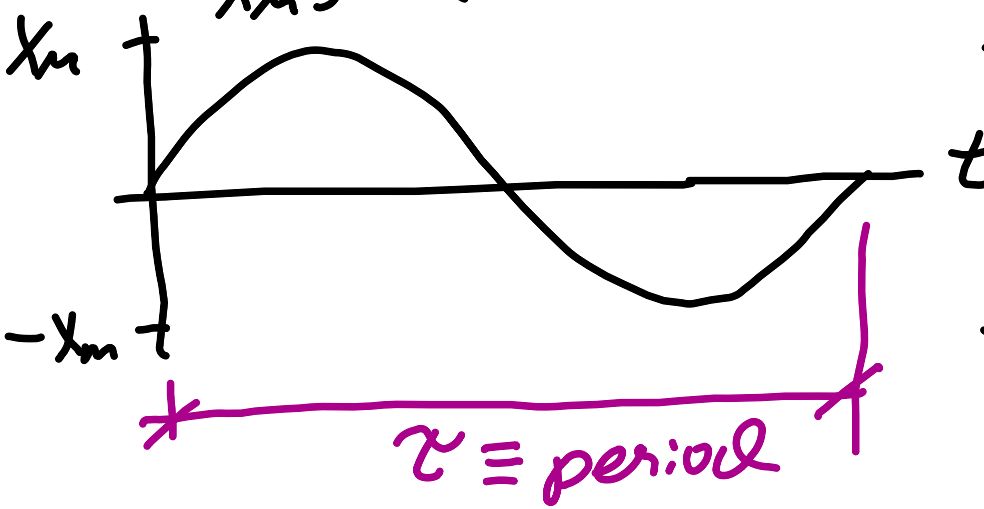
So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

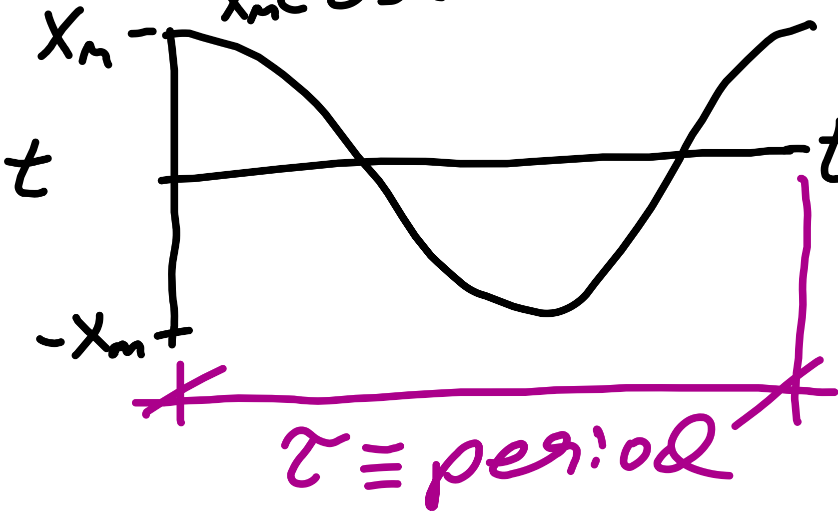
& $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

Wave anatomy

$x_m \sin(\omega t)$



$x_m \cos(\omega t)$



So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

& $f \equiv \frac{1}{\tau}$

For small angles

For small angles

$$\sin\theta \cong \theta$$

For small angles

$\sin\theta \cong \theta$ **Must** use radians

For small angles

$\sin\theta \cong \theta$ **Must** use radians

Is 5° small enough for above approximation?

For small angles

$\sin\theta \cong \theta$ **Must** use radians

Is 5° small enough for above approximation? x

$2\pi \text{ rad} = 360^\circ$

For small angles

$\sin\theta \cong \theta$ **Must** use radians

Is 5° small enough for above approximation?

$2\pi \text{ rad} = 360^\circ \Rightarrow$

$$\left(\frac{2\pi \text{ rad}}{360^\circ}\right) = 1$$

For small angles

$\sin\theta \cong \theta$ Must use radians

Is 5° small enough for above approximation?

$2\pi\,\text{rad} = 360^\circ \Rightarrow$

$\left(\frac{2\pi\,\text{rad}}{360^\circ}\right) = 1$ Now $\theta = 5^\circ = 5^\circ\left(\frac{2\pi}{360}\right)\,\text{rad}$

For small angles

$\sin\theta \cong \theta$ Must use radians

Is 5° small enough for above approximation?

$2\pi\,\text{rad} = 360^\circ \Rightarrow$

$\left(\frac{2\pi\,\text{rad}}{360^\circ}\right) = 1$ Now $\theta = 5^\circ = 5^\circ\left(\frac{2\pi}{360^\circ}\right)\text{rad}$

$\Rightarrow \quad \theta = 0.08727$

For small angles

$\sin\theta \cong \theta$ **Must** use radians

Is 5° small enough for above approximation? $2\pi\,\text{rad} = 360^\circ \Rightarrow$

$\left(\frac{2\pi\,\text{rad}}{360^\circ}\right) = 1$ Now $\theta = 5^\circ = 5^\circ\left(\frac{2\pi}{360}\right)\text{rad}$

$\Rightarrow \theta = 0.08727$ But

$\sin(5^\circ) = 0.08716$

For small angles

$\sin\theta \cong \theta$ Must use radians

Is 5° small enough for above approximation? $2\pi\,\text{rad} = 360^\circ \Rightarrow$

$\left(\frac{2\pi\,\text{rad}}{360^\circ}\right) = 1$ Now $\theta = 5^\circ = 5^\circ\left(\frac{2\pi}{360}\right)\,\text{rad}$

$\Rightarrow \theta = 0.08727$ But

$\sin(5^\circ) = 0.08716 \Rightarrow \frac{\theta}{\sin\theta} = 1.0013$

For small angles

$\sin\theta \cong \theta$ Must use radians

Is 5° small enough for above approximation?

$2\pi \text{ rad} = 360^\circ \Rightarrow$

$\left(\frac{2\pi \text{ rad}}{360^\circ}\right) = 1$ Now $\theta = 5^\circ = 5^\circ\left(\frac{2\pi}{360}\right) \text{ rad}$

$\Rightarrow \theta = 0.08727$ But

$\sin(5^\circ) = 0.08716 \Rightarrow \frac{\theta}{\sin\theta} = 1.0013$

So $\sin\theta \cong \theta$ for $\theta = 5^\circ$ if we need $\sin\theta$ to within about 0.1%

For many of the problems,

For many of the problems, the name of the game is to analyze a given physical system

For many of the problems, the name of the game is to analyze a given physical system & get solution into the form

For many of the problems, the name of the game is to analyze a given physical system & get solution into the form

$$\ddot{x} = -\omega^2 x$$

For many of the problems, the name of the game is to analyze a given physical system & get solution into the form

$$\ddot{X} = -\omega^2 X$$

or

For many of the problems, the name of the game is to analyze a given physical system & get solution into the form

$$\ddot{x} = -\omega^2 x$$

or

$$\ddot{\Theta} = -\omega^2 \Theta$$

ASU

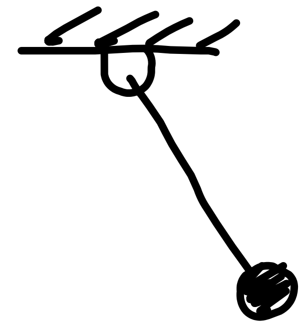
Notes on problems

Notes on problems

19.13: pendulum problem

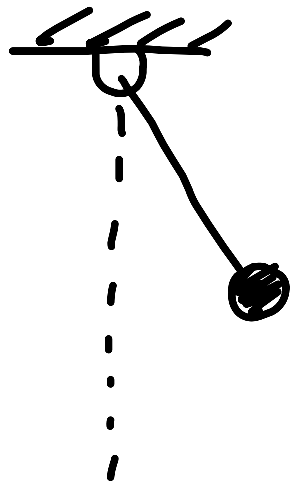
Notes on problems

19.13: pendulum problem



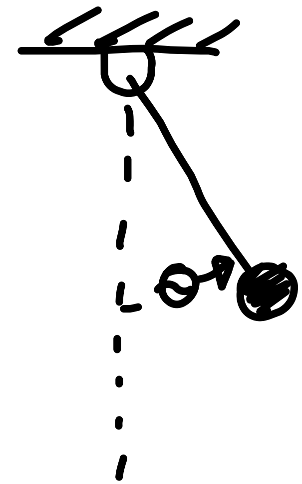
Notes on problems

19.13: pendulum problem



Notes on problems

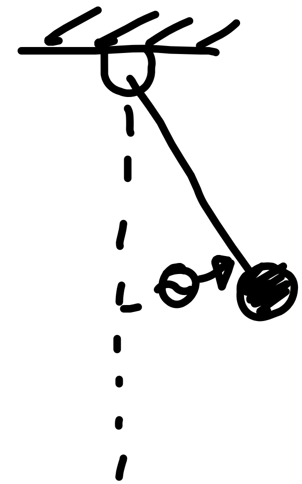
19.13: pendulum problem



Notes on problems

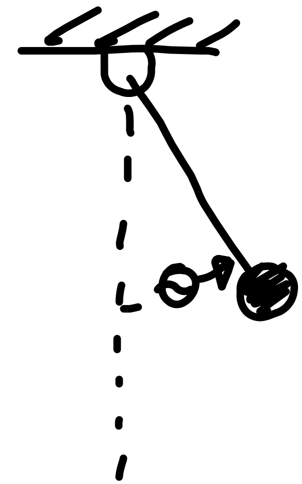
19.13: pendulum problem

In part b you are asked to find the acceleration of the bob



Notes on problems

19.13: pendulum problem



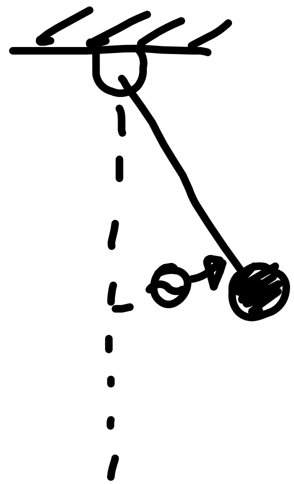
In part b you are asked to find the acceleration of the bob: Remember to use the normal & transverse components in obtaining the total

Notes on problems

19.13: pendulum problem

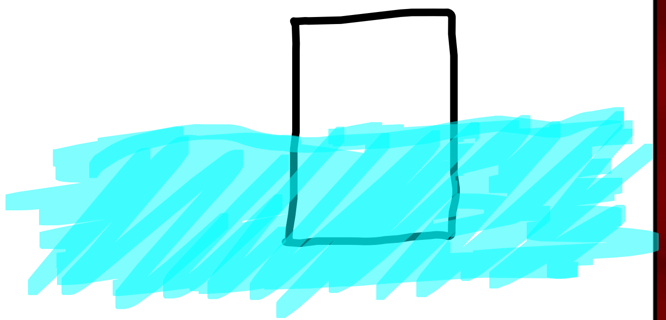
In part b you are asked to find the acceleration of the bob: Remember to use the normal & transverse components in obtaining the total

i.e. $a = [a_n^2 + a_t^2]^{1/2}$



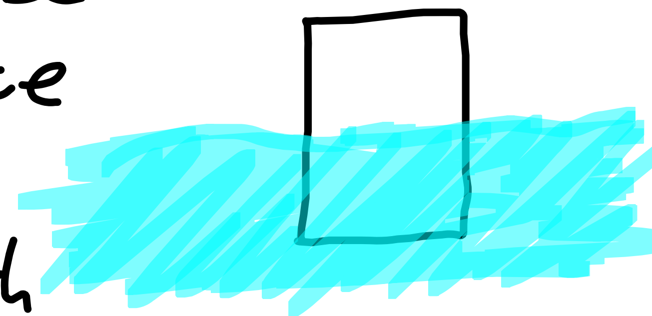
Notes on problems

19.26:



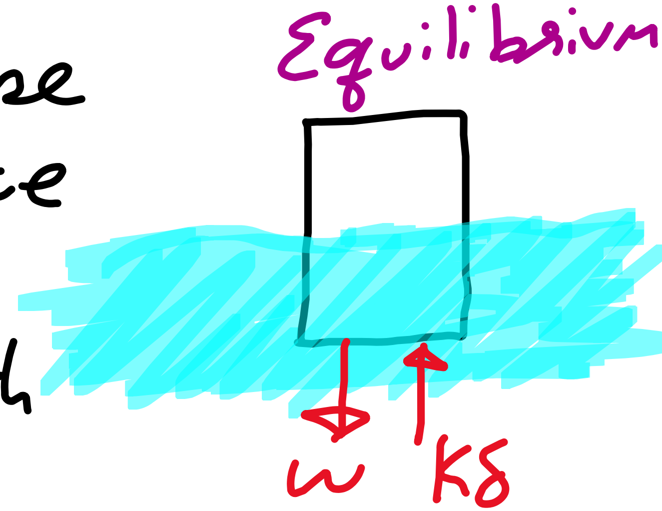
Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



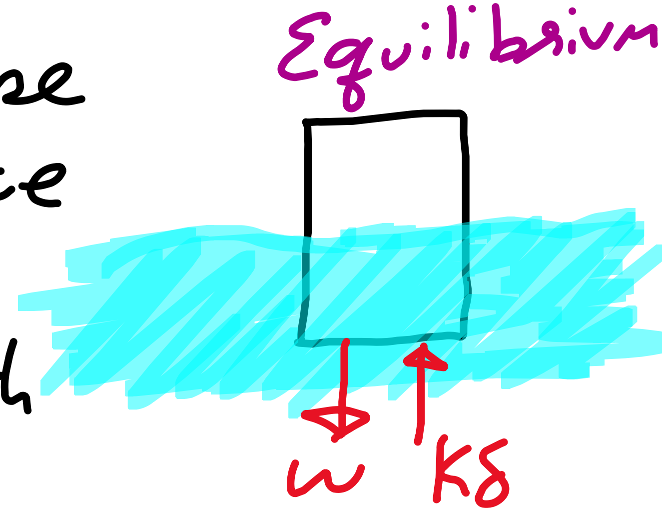
Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



Notes on problems

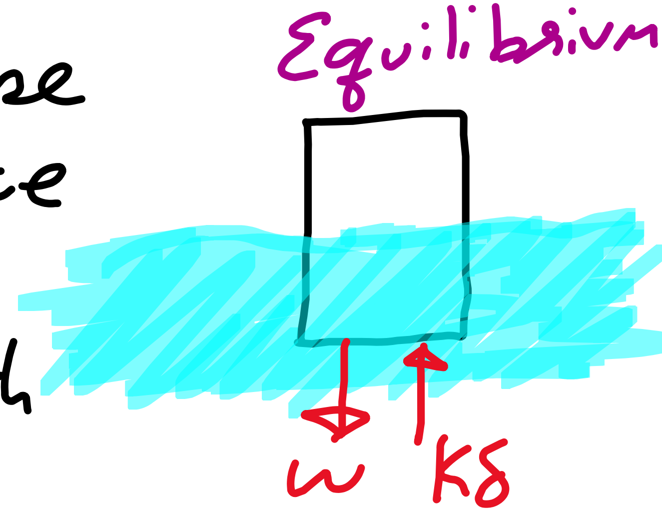
19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



This is just a spring problem with a strange looking expression for k.

Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$

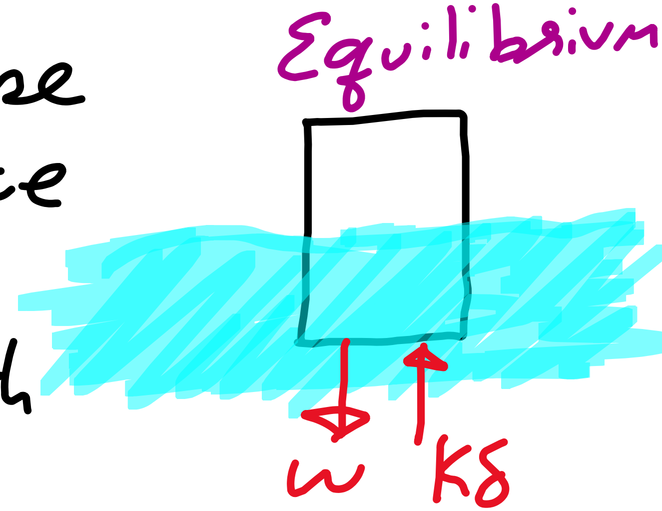


This is just a spring problem with a strange looking expression for k.

They give you

τ_{NE}

Notes on problems



19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$

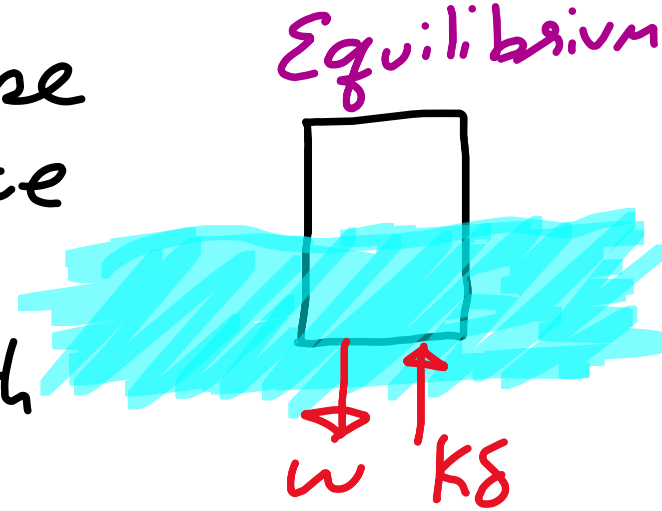
This is just a spring problem with a strange looking expression for k.

They give you

$\tau_{NE} \equiv$ period of motion for empty barrel

Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



This is just a spring problem with a strange looking expression for k.

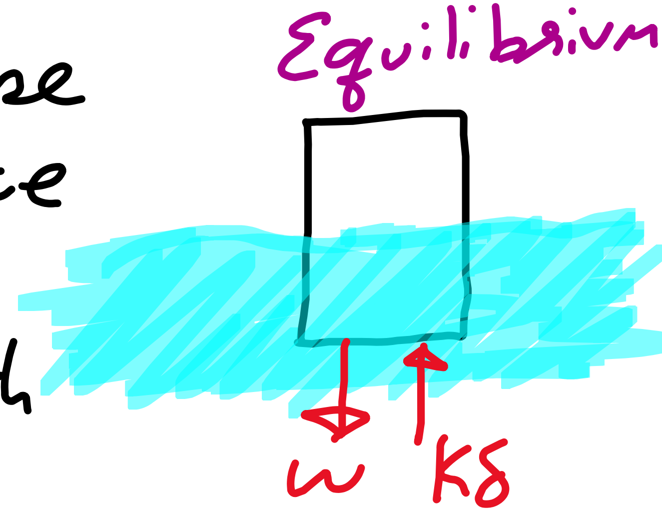
They give you

$\tau_{NE} \equiv$ period of motion for empty barrel

& τ_{NF}

Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



This is just a spring problem with a strange looking expression for k.

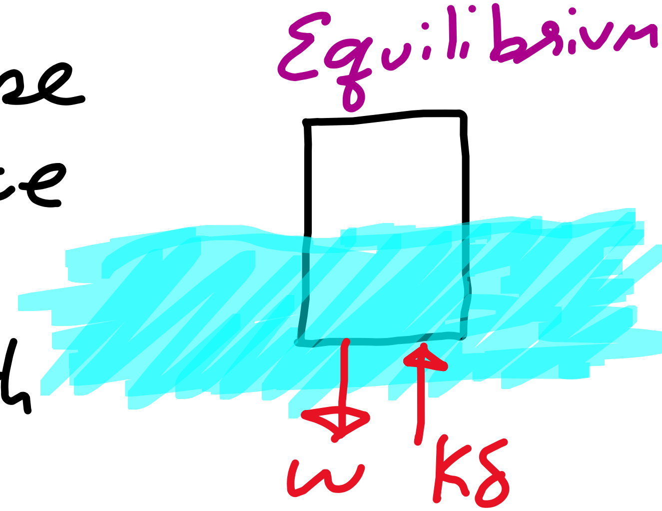
They give you

$\tau_{NE} \equiv$ period of motion for empty barrel

& $\tau_{NF} \equiv$ period of motion for full barrel

Notes on problems

19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



This is just a spring problem with a strange looking expression for k.

They give you

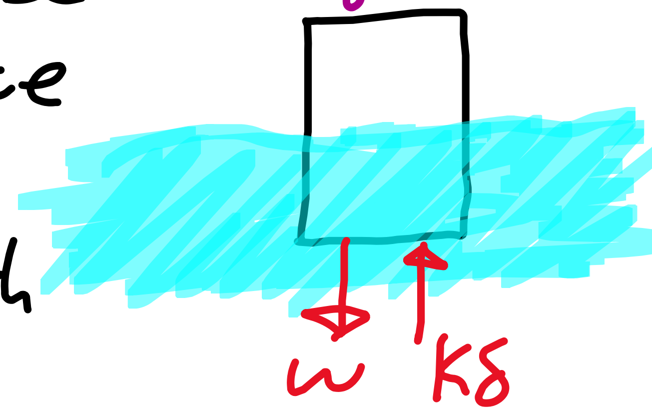
$\tau_{NE} \equiv$ period of motion for empty barrel

& $\tau_{NF} \equiv$ period of motion for full barrel

Note: If you divide the two

Notes on problems

Equilibrium



19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$

This is just a spring problem with a strange looking expression for k.

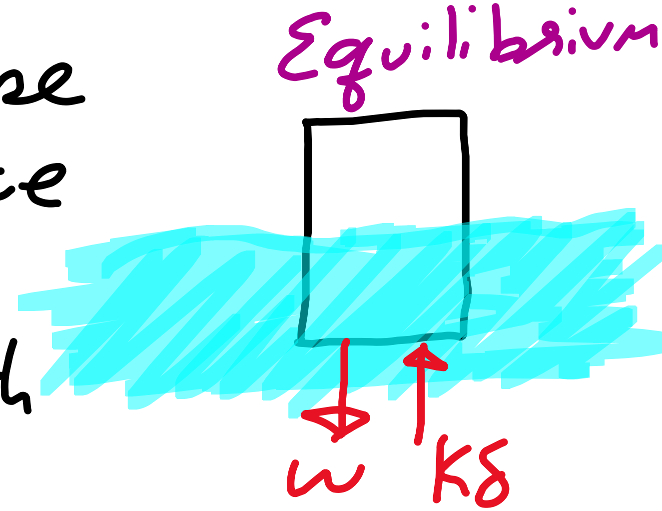
They give you

$\tau_{NE} \equiv$ period of motion for empty barrel

& $\tau_{NF} \equiv$ period of motion for full barrel

Note: If you divide the two $[\tau_{NF}/\tau_{NE}]$

Notes on problems



19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$

This is just a spring problem with a strange looking expression for k.

They give you

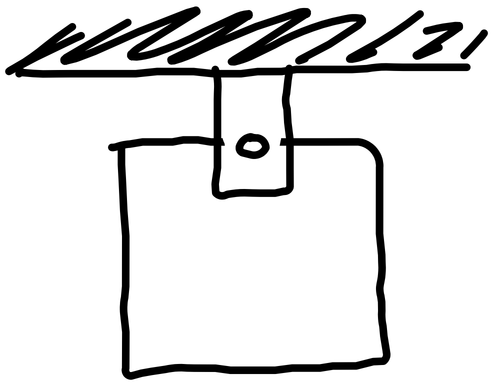
$\tau_{NE} \equiv$ period of motion for empty barrel

& $\tau_{NF} \equiv$ period of motion for full barrel

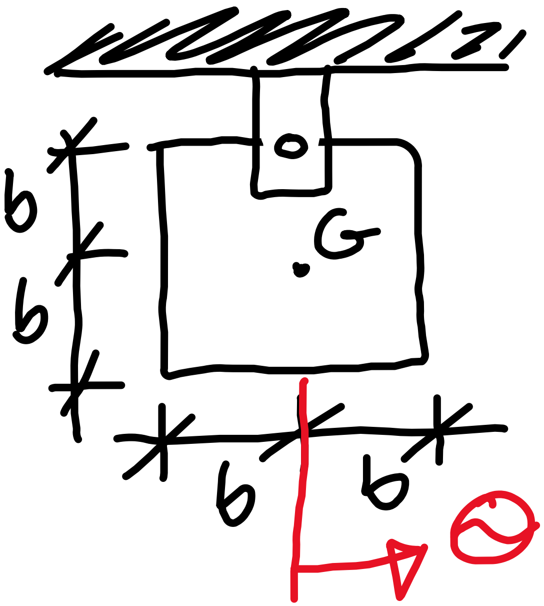
Note: If you divide the two $[\tau_{NF}/\tau_{NE}]$ you can get nice expression relating M_{Full} to M_{Empty}

Rigid body example

Rigid body example

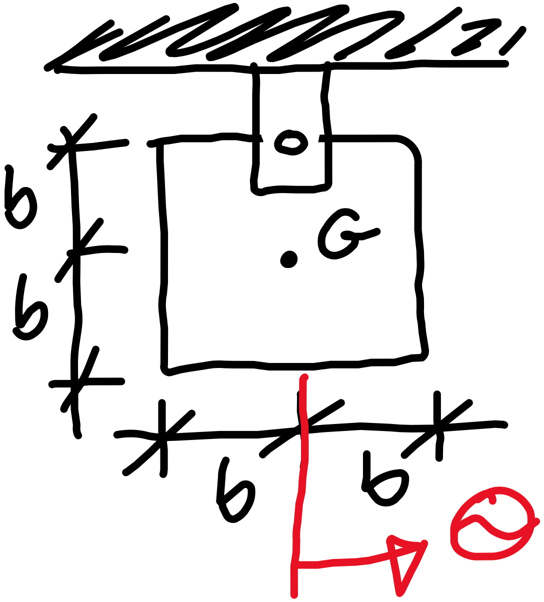


Rigid body example



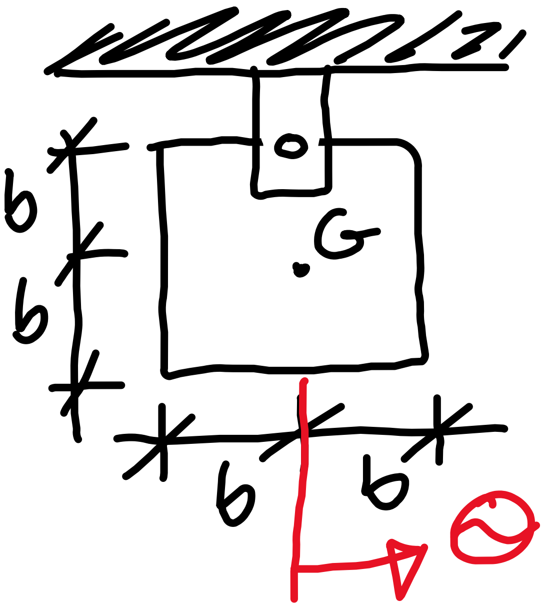
Rigid body example

Displace small amount θ & find period



Rigid body example

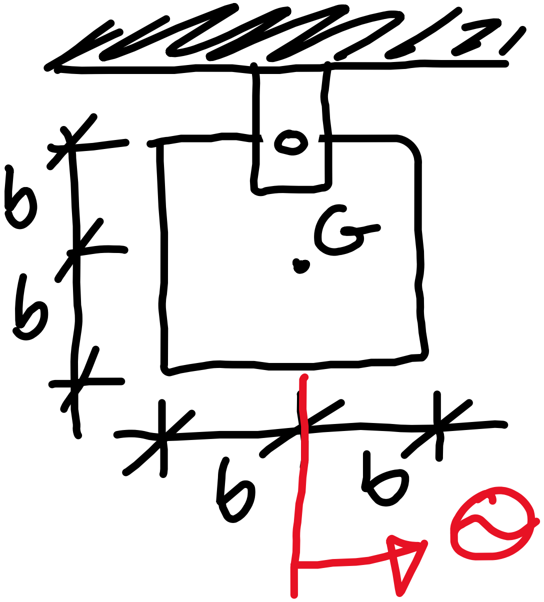
Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right]$$

Rigid body example

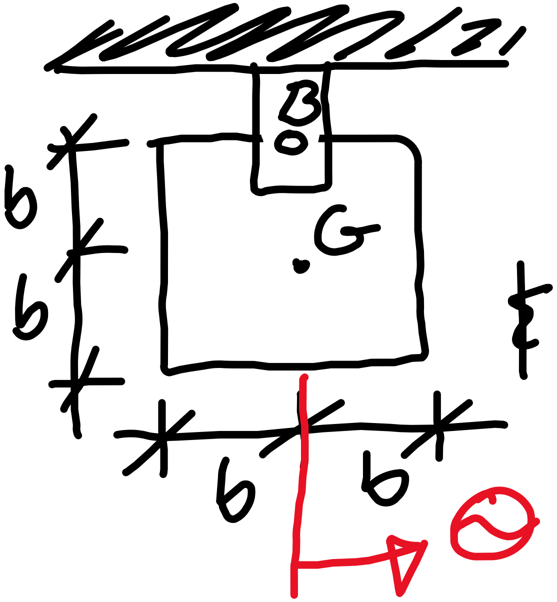
Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

Rigid body example

Displace small amount θ & find period

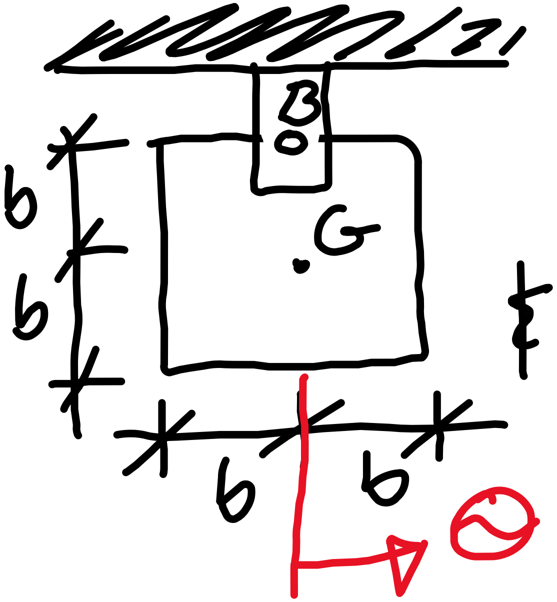


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2$$

Rigid body example

Displace small amount θ & find period

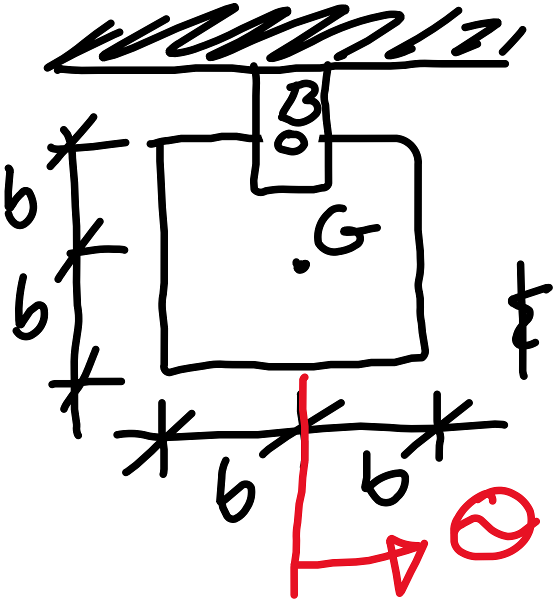


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 +$$

Rigid body example

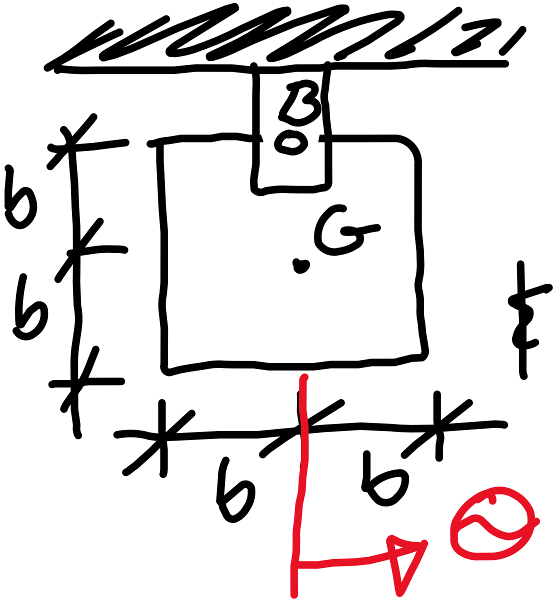
Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

Rigid body example



Displace small amount θ & find period

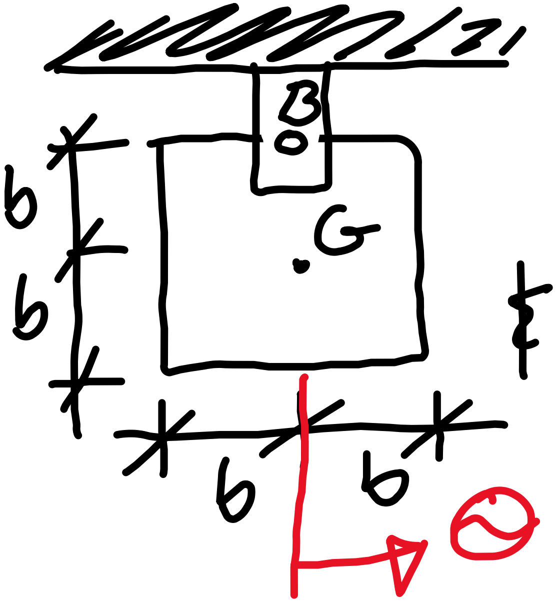
$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12}mb^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right)mb^2$$

Rigid body example

Displace small amount θ & find period



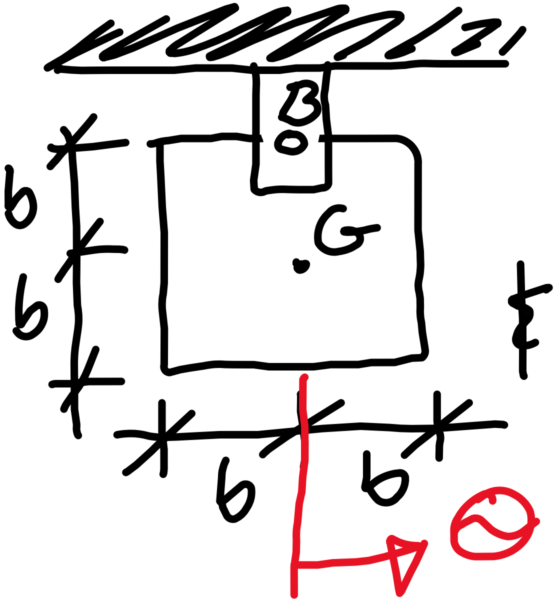
$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2$$

Rigid body example

Displace small amount θ & find period



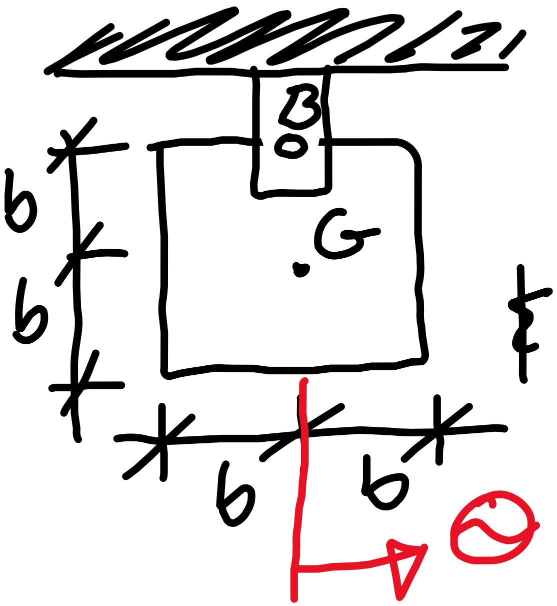
$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Rigid body example

Displace small amount θ & find period



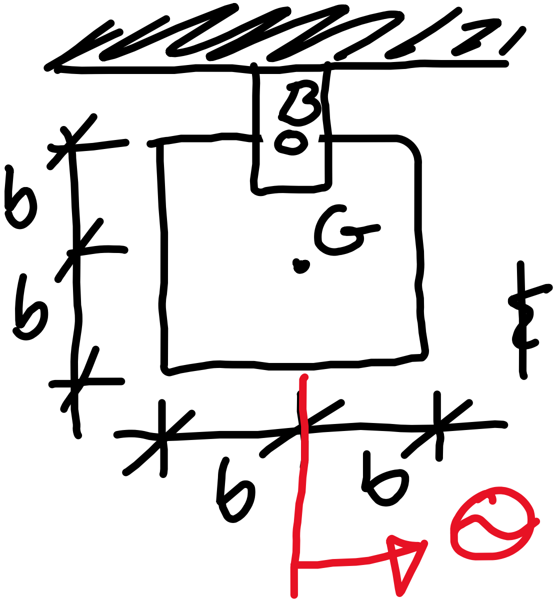
$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

At equilibrium

Rigid body example



Displace small amount θ & find period

$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

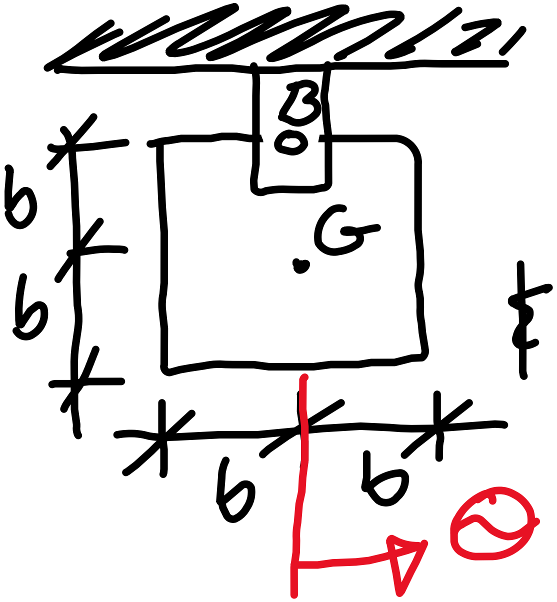
$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

At equilibrium $\circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha}$

ASU

Rigid body example

Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

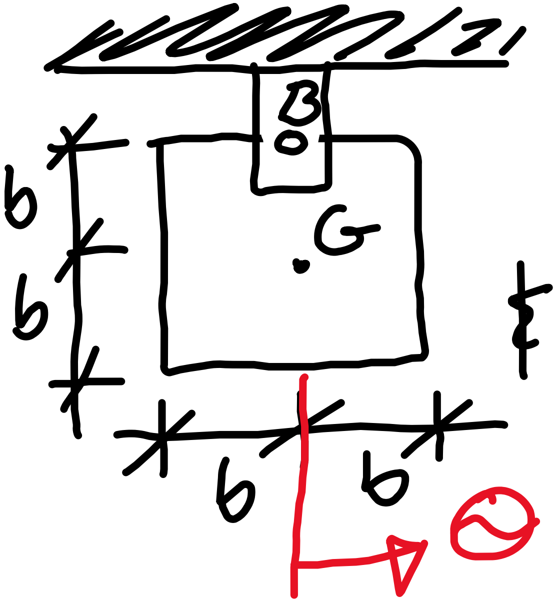
$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

At equilibrium $\circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha} = \cancel{0}$

Rigid body example

Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

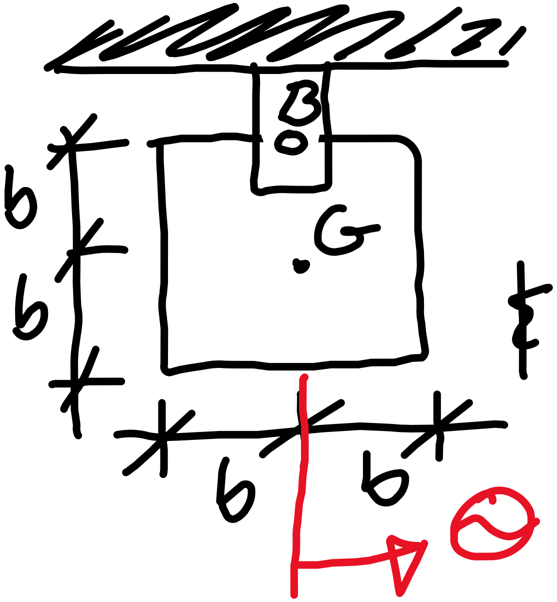
$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

At equilibrium $\circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha} = \cancel{0}$

Nothing interesting going on at equilibrium

Rigid body example



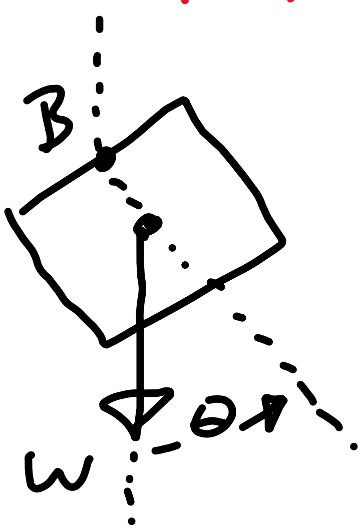
Displace small amount θ & find period

$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\text{\&}\ I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

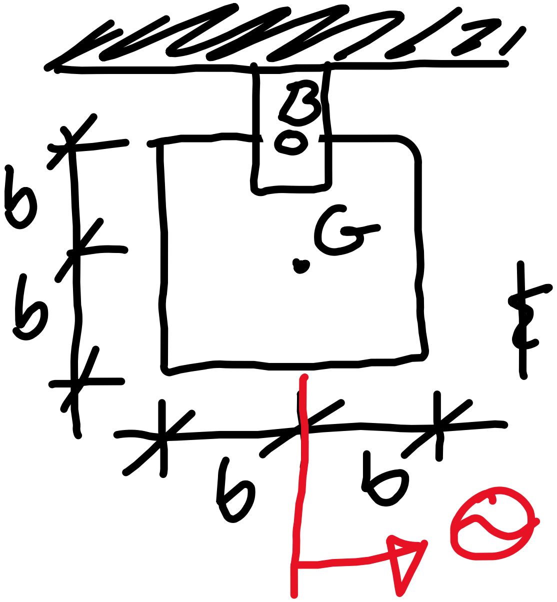
$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Displace amount θ:



Rigid body example

Displace small amount θ & find period

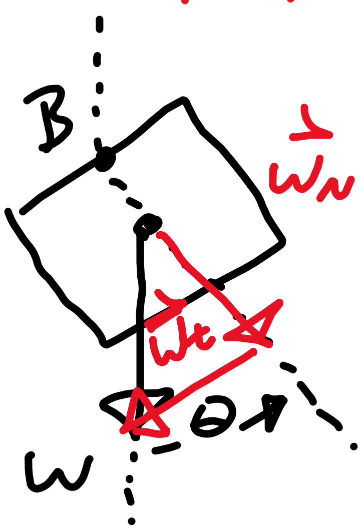


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

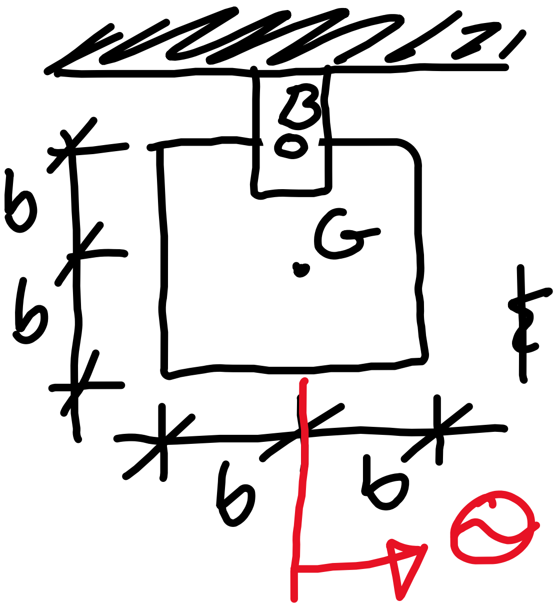
$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Displace amount θ:



Rigid body example

Displace small amount θ & find period

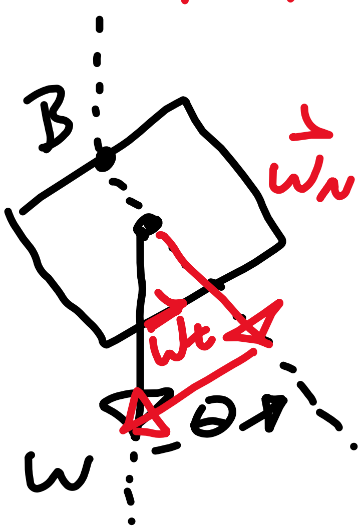


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

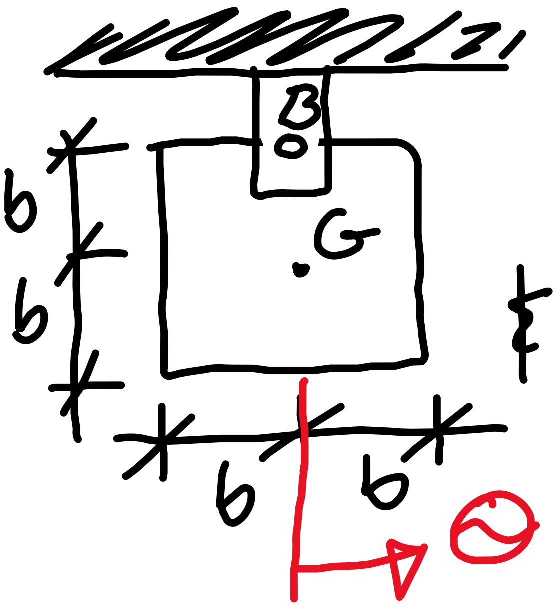
Displace amount θ:



$$w_N = -W\cos\theta$$

Rigid body example

Displace small amount θ & find period



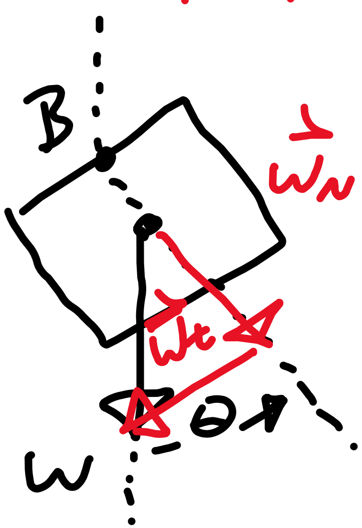
$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12} m b^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right) mb^2 = \frac{20}{12} mb^2 = \frac{5}{3} mb^2$$

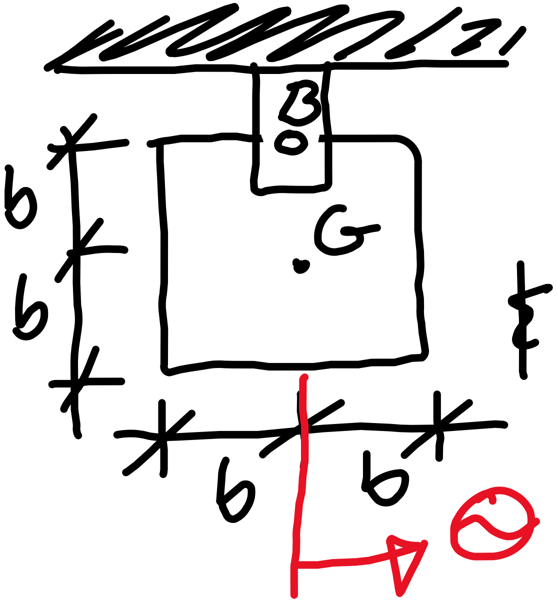
Displace amount θ:

$w_n = -W\cos\theta$ & $w_t = -W\sin\theta$



Rigid body example

Displace small amount θ & find period

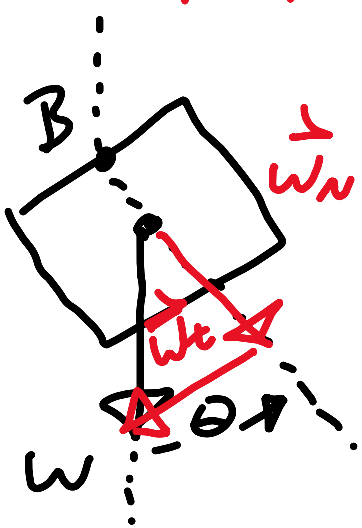


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:

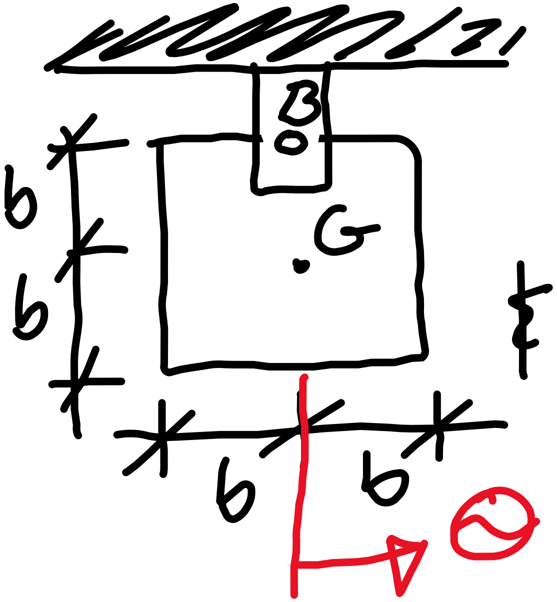


$w_n = -w\cos\theta$ & $w_t = -w\sin\theta$

[just like for simple pendulum]

Rigid body example

Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12}mb^2$$

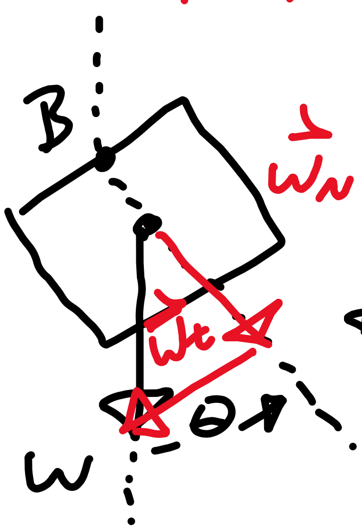
$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:

$w_N = -W\cos\theta$ & $w_t = -W\sin\theta$

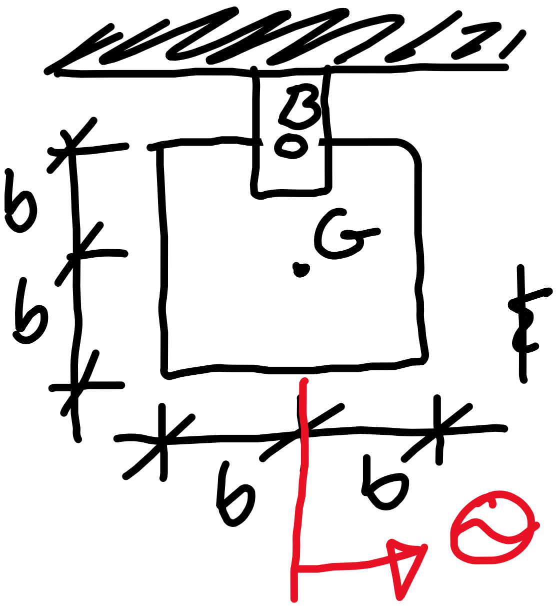
[just like for simple pendulum]



$$\circlearrowleft \sum M_B = I_B \alpha$$

Rigid body example

Displace small amount θ & find period

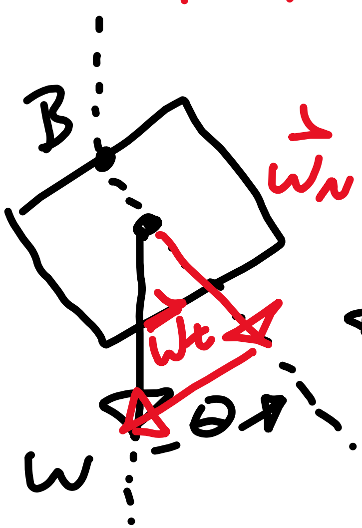


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Displace amount θ:



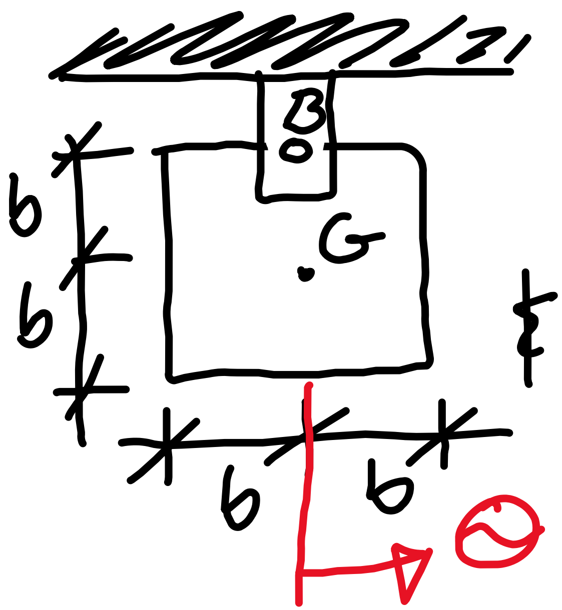
$w_N = -w\cos\theta$ & $w_t = -w\sin\theta$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B \alpha \Rightarrow -bmg\sin\theta = I_B \ddot{\theta}$$

Rigid body example

Displace small amount θ & find period

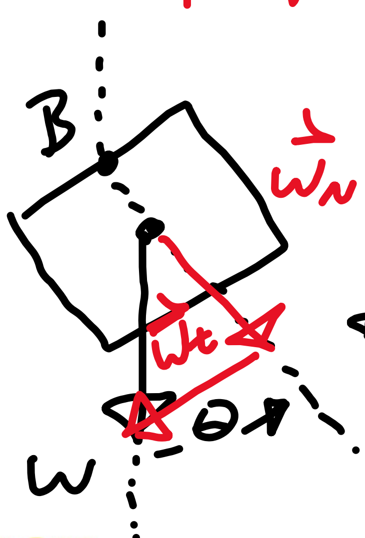


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:



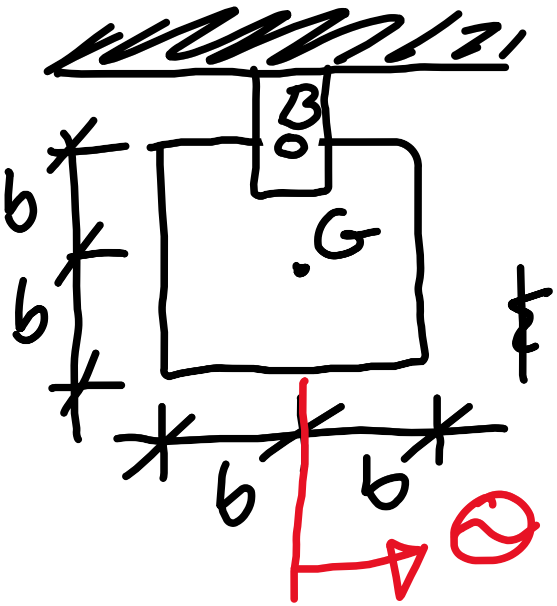
$w_N = -W\cos\theta$ & $w_t = -W\sin\theta$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

Take θ small such that $\sin\theta \cong \theta$

Rigid body example



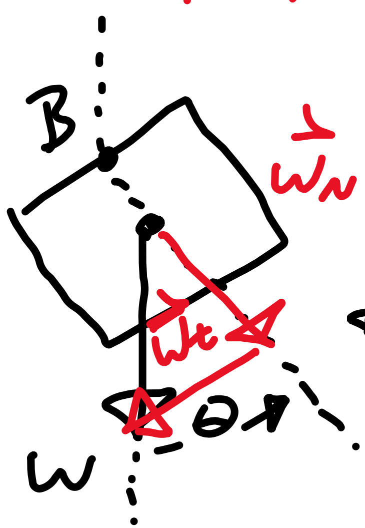
Displace small amount Θ & find period

$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Displace amount Θ:



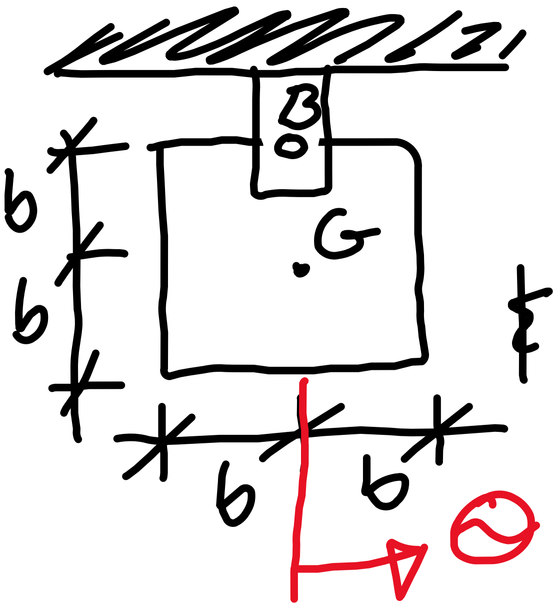
$$w_n = -w\cos\Theta \quad \& \quad w_t = -w\sin\Theta$$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B \alpha \Rightarrow -bmg\sin\Theta = I_B\ddot{\Theta}$$

$$\Rightarrow \quad -bmg\Theta \cong I_B\ddot{\Theta}$$

Rigid body example



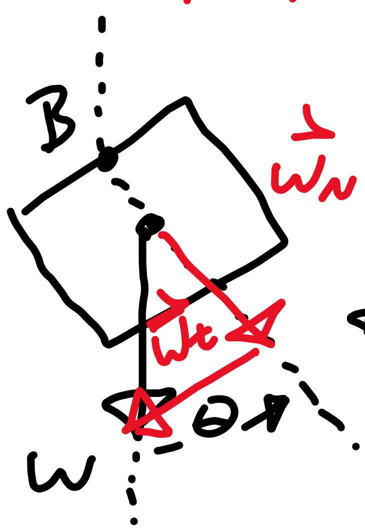
Displace small amount θ & find period

$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 + m b^2$$

$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Displace amount θ:



$$w_N = -W\cos\theta \;\; \& \;\; w_t = -W\sin\theta$$

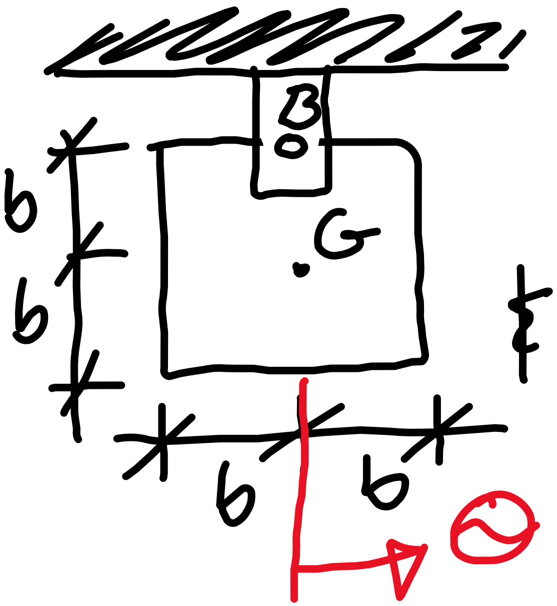
[just like for simple pendulum]

$$\circlearrowleft \Sigma M_B = I_B \alpha \Rightarrow -bmg\sin\theta = I_B \ddot{\theta}$$

$$\Rightarrow -bmg\,\theta \cong I_B \ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2 \theta,$$

Rigid body example

Displace small amount θ & find period

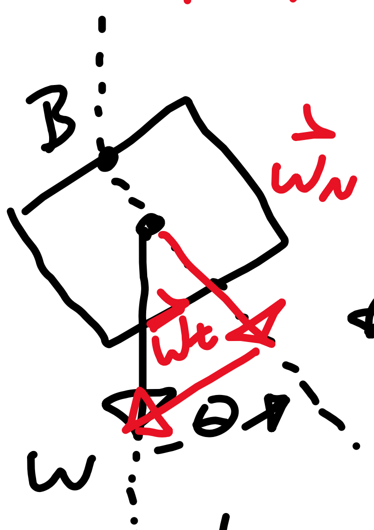


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:



$$w_N = -w\cos\theta \quad \& \quad w_t = -w\sin\theta$$

[just like for simple pendulum]

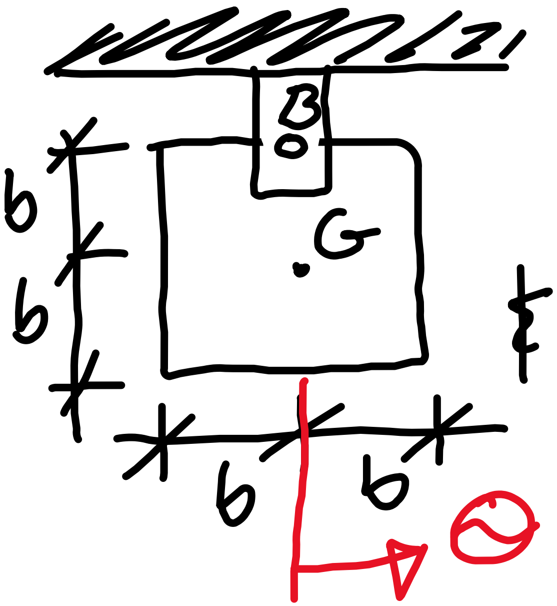
$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $\omega = \sqrt{\dfrac{bmg}{I_B}}$

Rigid body example

Displace small amount θ & find period

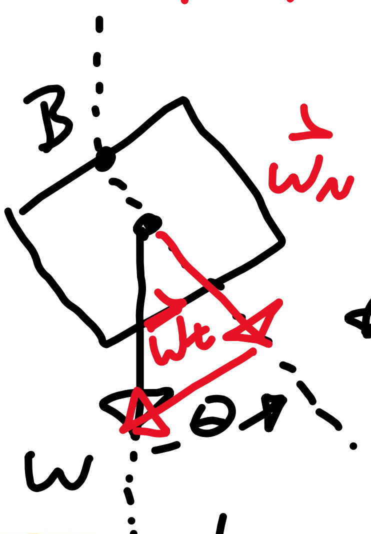


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

& $$I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:



$w_n = -w\cos\theta$ & $w_t = -w\sin\theta$

[just like for simple pendulum]

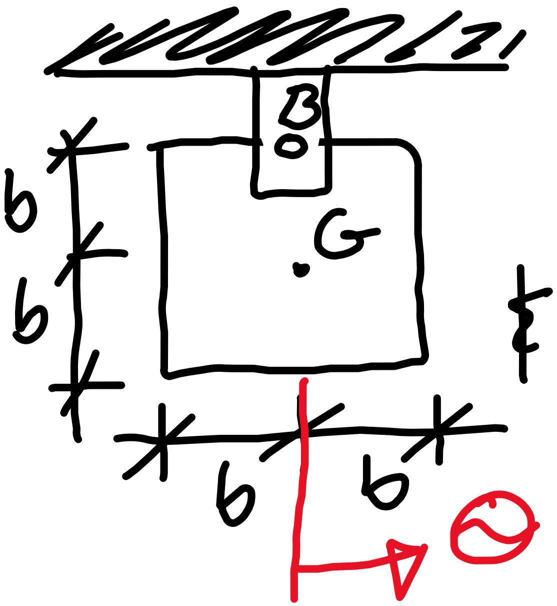
$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $$\omega = \sqrt{\frac{bmg}{I_B}} = \sqrt{\frac{bmg}{5mb^2/3}}$$

Rigid body example

Displace small amount θ & find period

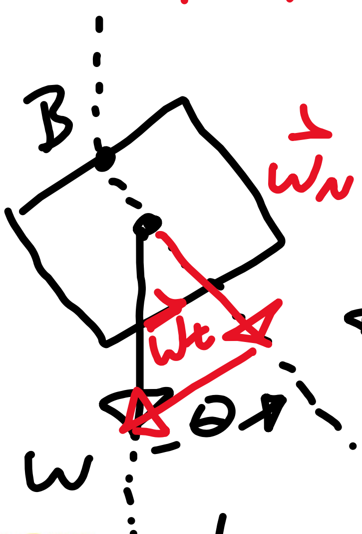


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right) mb^2 = \frac{20}{12} mb^2 = \frac{5}{3} mb^2$$

Displace amount θ:



$w_N = -w\cos\theta$ & $w_t = -w\sin\theta$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $\omega = \sqrt{\frac{bmg}{I_B}} = \sqrt{\frac{bmg}{5mb^2/3}} = \sqrt{\frac{3g}{5b}}$

From previous slide

$$\omega = \sqrt{\frac{3g}{5b}}$$

From previous slide

$$\omega = \sqrt{\frac{3g}{5b}} \quad \& \text{ since } \omega\tau = 2\pi$$

From previous slide

$\omega = \sqrt{\dfrac{3g}{5b}}$ & since $\omega\tau = 2\pi$

then $\tau = \dfrac{2\pi}{\omega}$

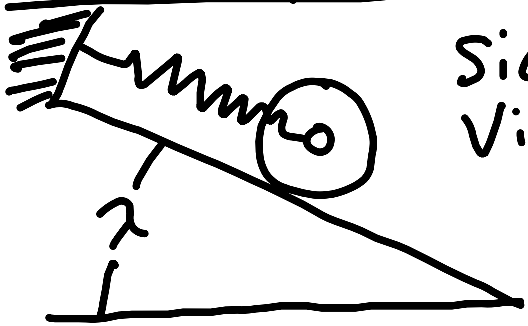
From previous slide

$$\omega = \sqrt{\frac{3g}{5b}} \quad \& \text{ since } \omega\tau = 2\pi$$

then $\tau = \dfrac{2\pi}{\omega} \Rightarrow$

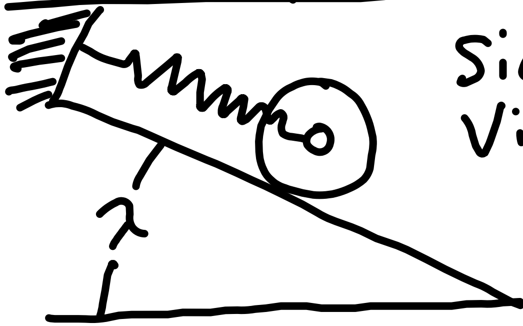
$$\boxed{\tau = 2\pi\sqrt{\frac{5b}{3g}}}$$

Example



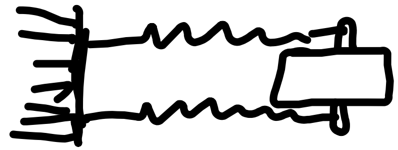
Side View

Example

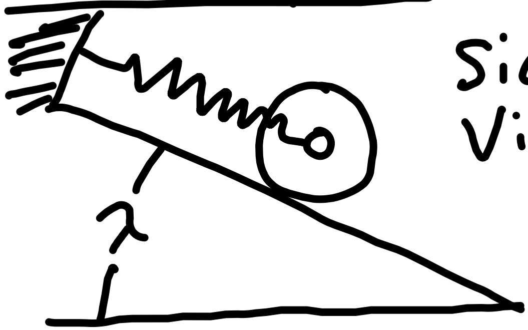


Side View

Top View

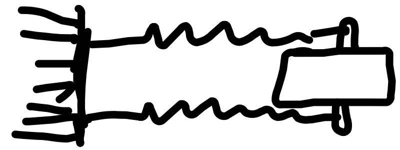


Example



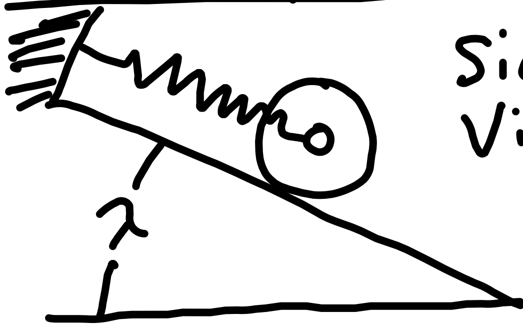
Side View

Top View



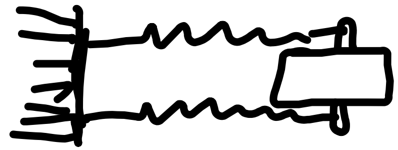
push disk down hill an amount d,

Example



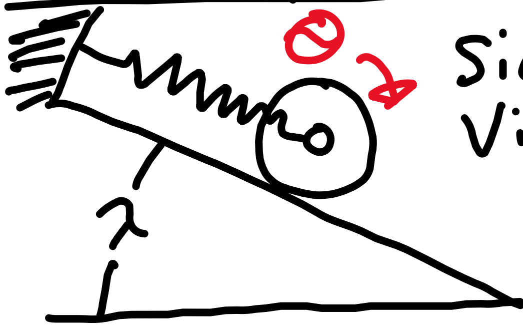
Side View

Top View



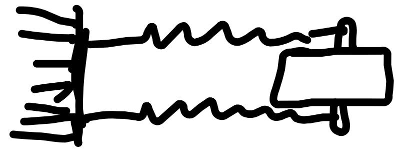
push disk down hill an amount d, find period

Example



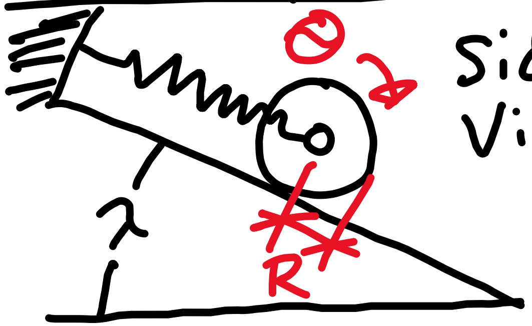
Side View

Top View



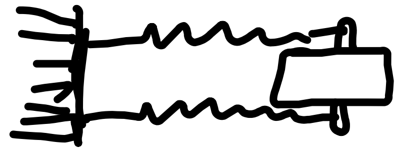
push disk down hill an amount d, find period

Example



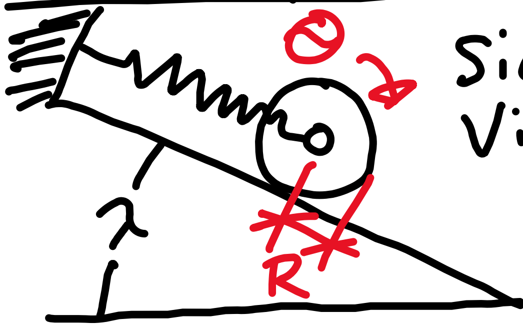
Side View

Top View



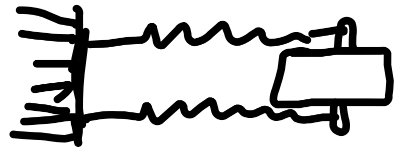
push disk down hill an amount d, find period

Example



Side View

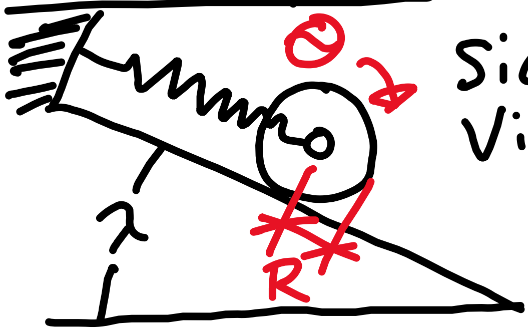
Top View



push disk down hill an amount d, find period

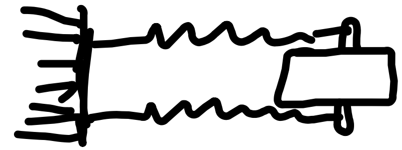
Equilibrium:

Example



Side View

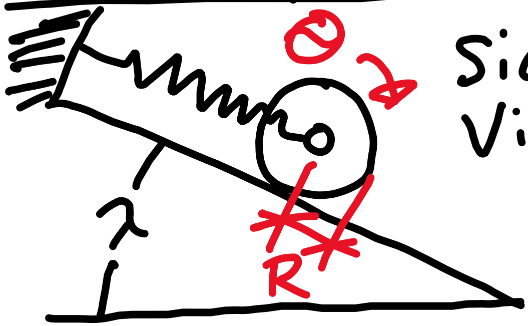
Top View



push disk down hill an amount d, find period

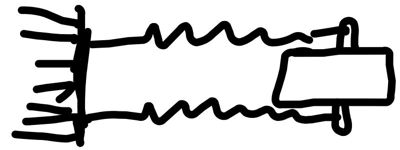
Equilibrium:

Example



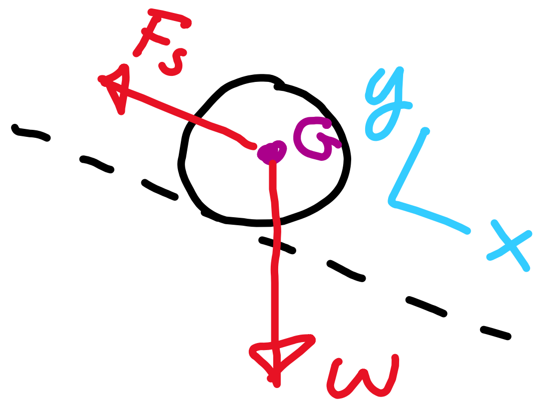
Side View

Top View

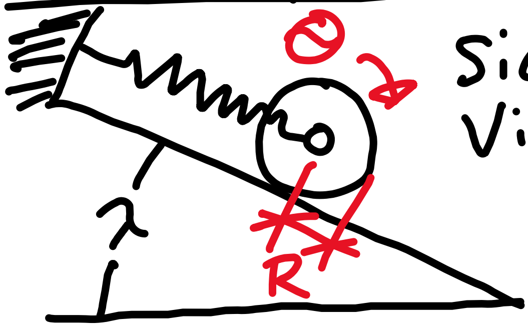


push disk down hill an amount d, find period

Equilibrium:

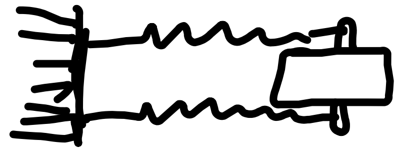


Example



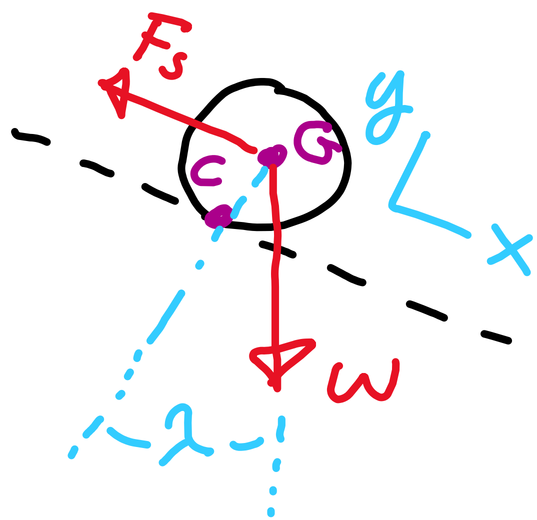
Side View

Top View

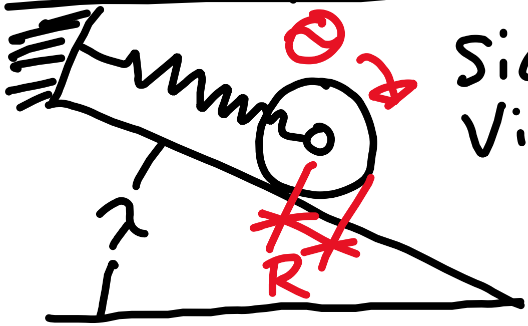


push disk down hill an amount d, find period

Equilibrium:

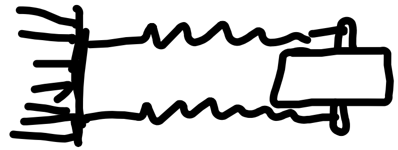


Example



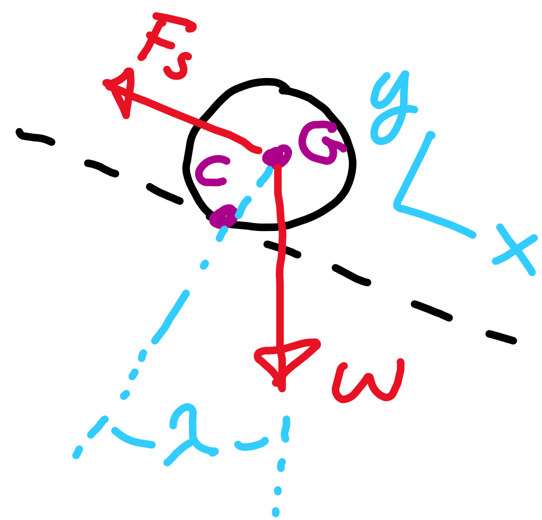
Side View

Top View



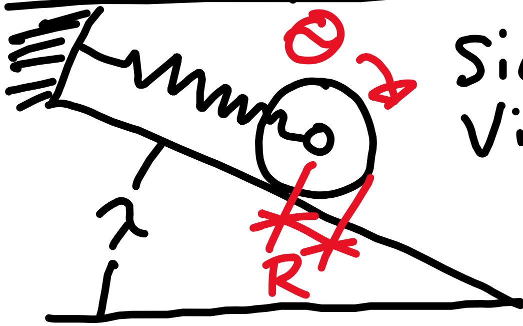
push disk down hill an amount d, find period

Equilibrium:



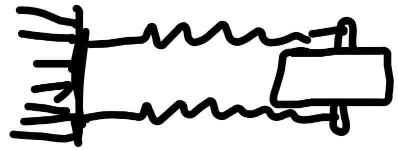
$$\vec{w} = \hat{x}\, w \sin\lambda + \hat{y}\, w \cos\lambda$$

Example



Side View

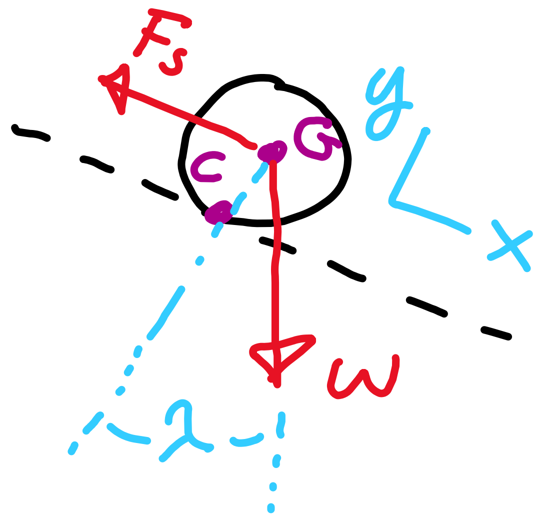
Top View



push disk down hill an amount d, find period

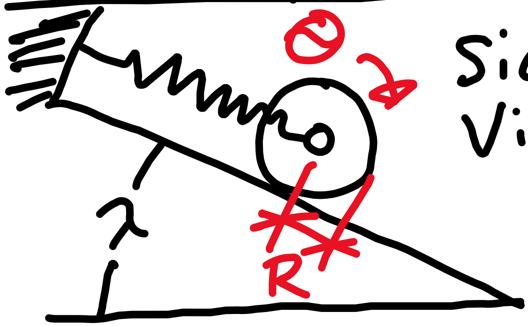
Equilibrium:

$\sum \vec{M}_c = 0$



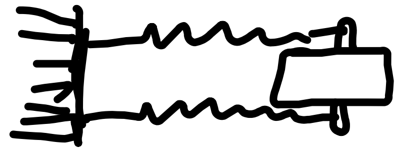
$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

Example



Side View

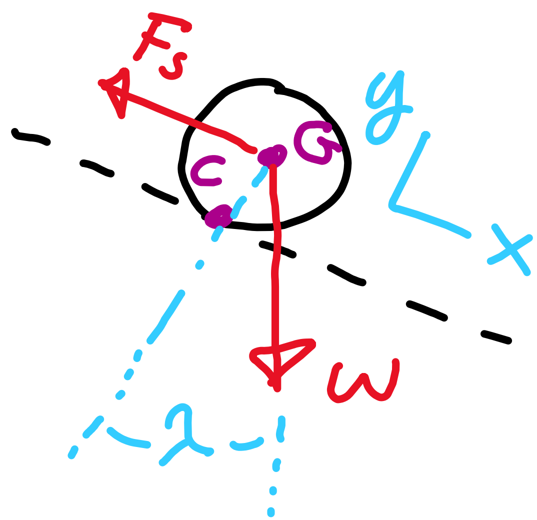
Top View



push disk down hill an amount θ, find period

Equilibrium:

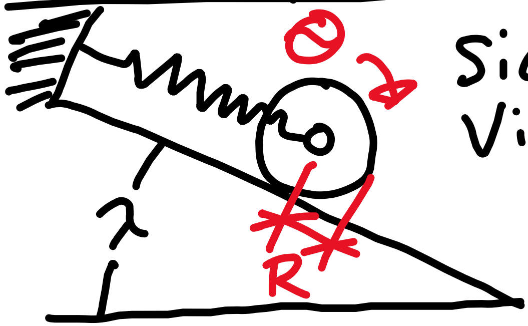
$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$



$\vec{w} = \hat{x} W \sin\lambda + \hat{y} W \cos\lambda$

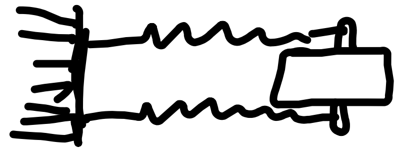
$R W \sin\lambda - R F_s = 0$

Example



Side View

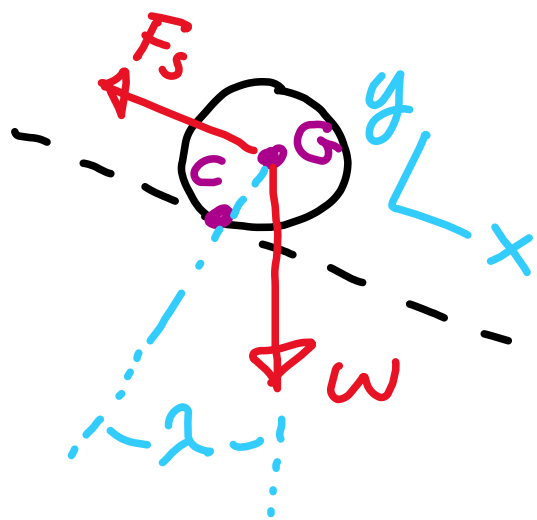
Top View



push disk down hill an amount δ, find period

Equilibrium:

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$

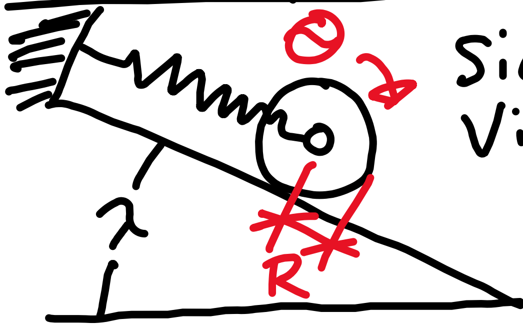


$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

$R W \sin\lambda - R F_s = 0$

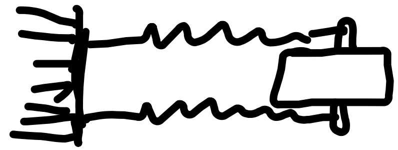
$\Rightarrow R w \sin\lambda = R k \delta$

Example



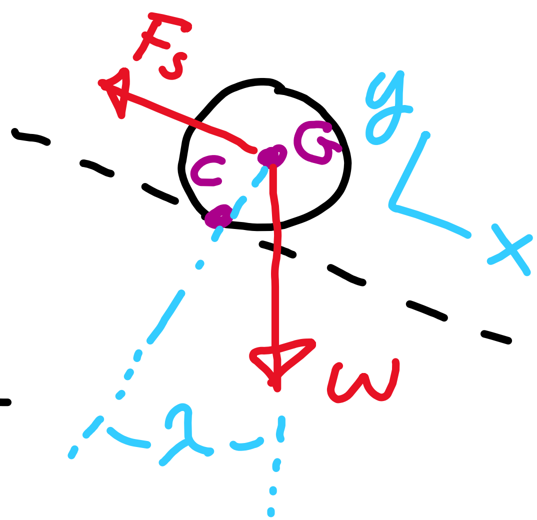
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\sum \vec{M}_c = 0 \Rightarrow$

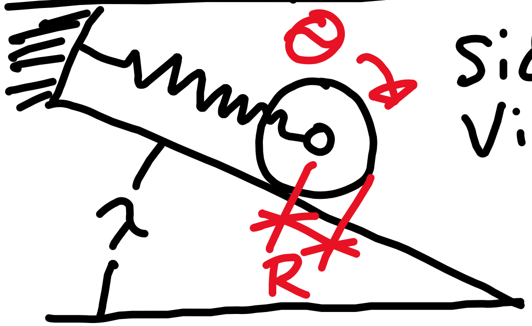
$\Rightarrow Rw\sin\lambda = Rk\delta$

$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$Rw\sin\lambda - RF_s = 0$

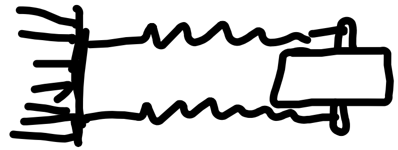
push down amount d:

Example



Side View

Top View

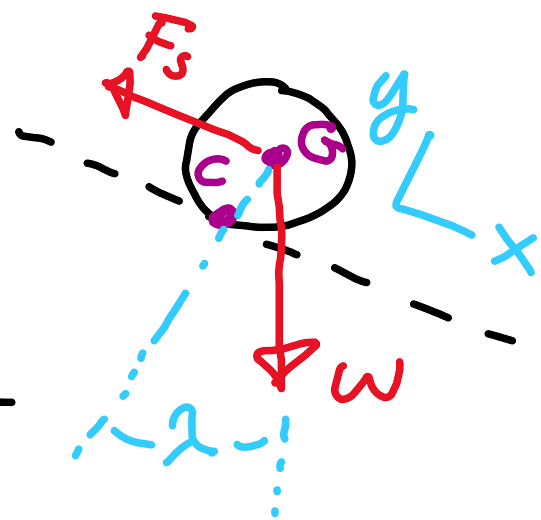


push disk down hill an amount δ, find period

Equilibrium:

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow R w \sin\lambda = R k \delta$



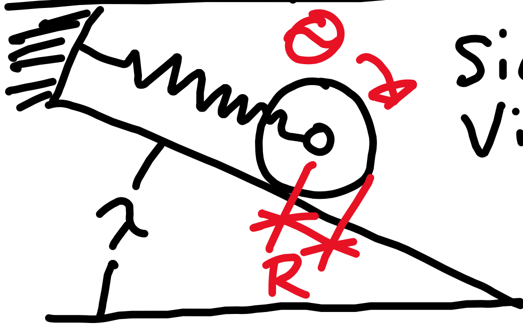
$\vec{w} = \hat{x} w\sin\lambda + \hat{y} w\cos\lambda$

$R W \sin\lambda - R F_s = 0$

push down amount δ:

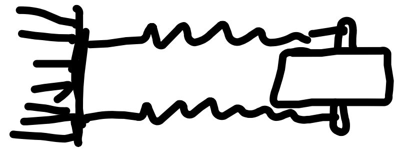
$\circlearrowleft \sum \vec{M}_c = I_c \ddot{\theta}$

Example



Side View

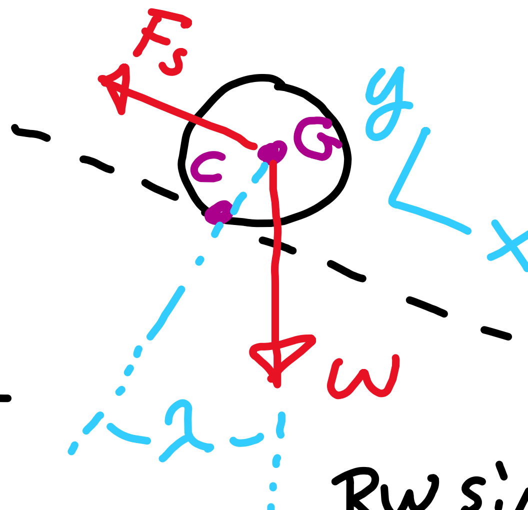
Top View



push disk down hill an amount d, find period

Equilibrium:

$\sum \vec{M}_c = 0 \Rightarrow$



$\vec{W} = \hat{x} W\sin\lambda + \hat{y} W\cos\lambda$

$RW\sin\lambda - RF_s = 0$

$\Rightarrow RW\sin\lambda = RK\delta$

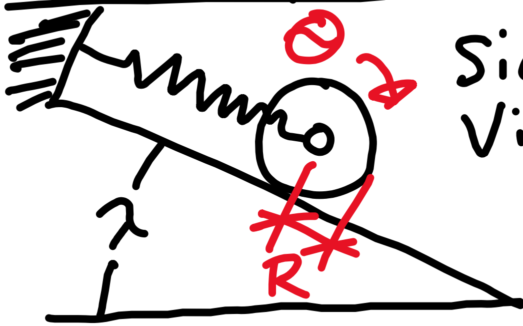
push down amount d:

$\sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$RW\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

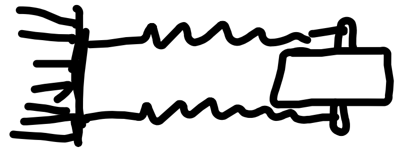
But $RW\sin\lambda = RK\delta$

Example



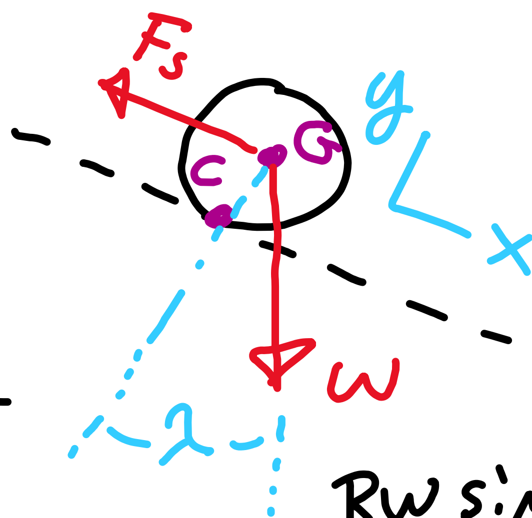
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

$\circlearrowright \sum \vec{M}_c = 0 \Rightarrow$

$R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R K \delta$

push down amount d:

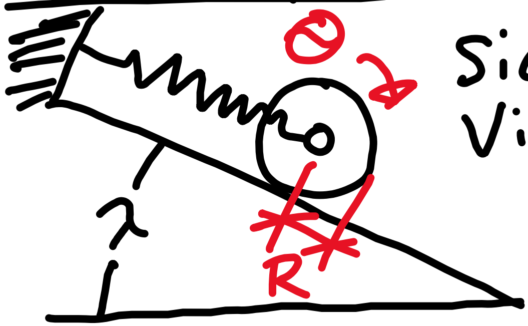
$\circlearrowright \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$R w \sin\lambda - R K(\delta + d) = I_c \ddot{\theta}$

But $R w \sin\lambda = R K \delta$ so

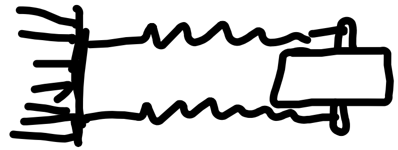
$- R K d = I_c \ddot{\theta}$

Example



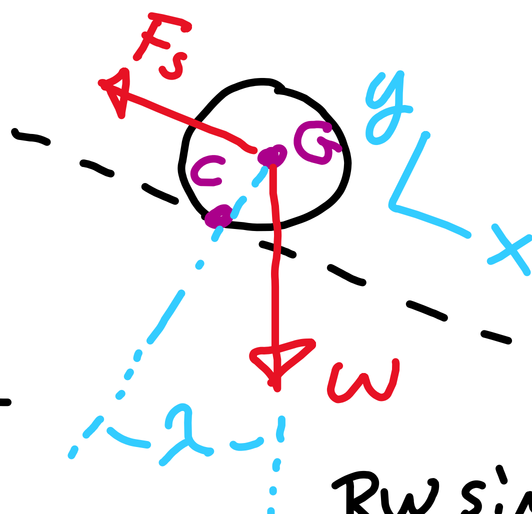
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$

$RW\sin\lambda - RF_s = 0$

$\Rightarrow Rw\sin\lambda = Rk\delta$

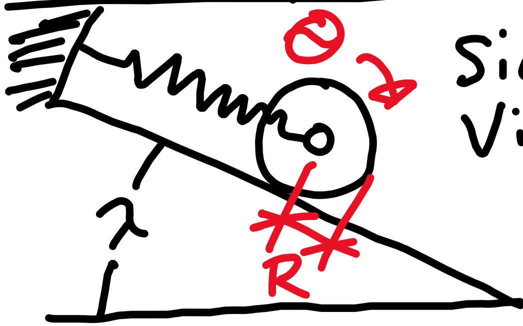
push down amount d:

$\circlearrowleft \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - Rk(\delta + d) = I_c\ddot{\theta}$

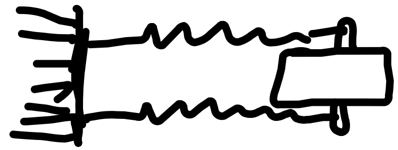
But $Rw\sin\lambda = Rk\delta$ so $-Rkd = I_c\ddot{\theta}$ & since $d = R\theta$

Example



Side View

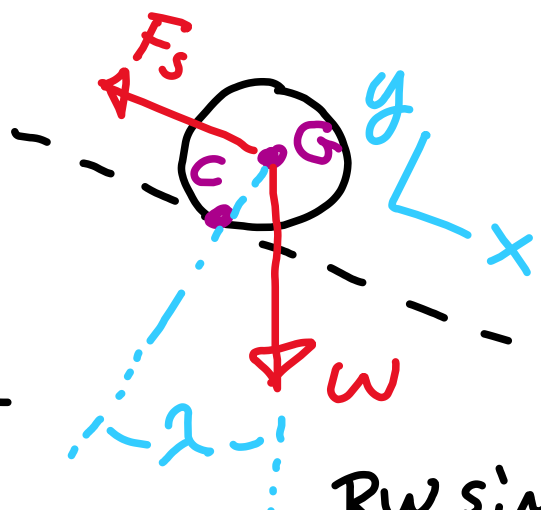
Top View



push disk down hill an amount d, find period

Equilibrium:

$\curvearrowright \sum \vec{M}_c = 0 \Rightarrow$



$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

$R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R k \delta$

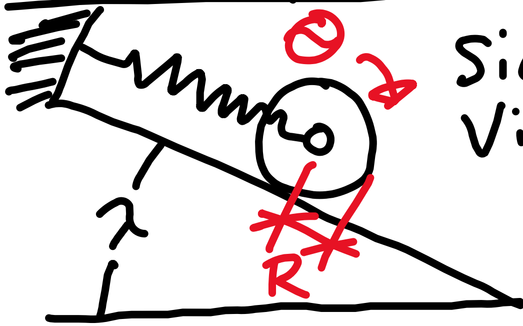
push down amount d:

$\curvearrowright \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$R w \sin\lambda - R k(\delta + d) = I_c \ddot{\theta}$

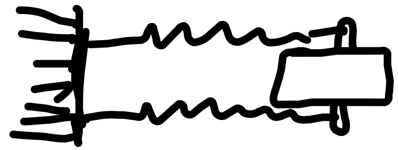
But $R w \sin\lambda = R k \delta$ so $- R k d = I_c \ddot{\theta}$ & since $d = R\theta$ then $- R^2 k \theta = I_c \ddot{\theta}$

Example



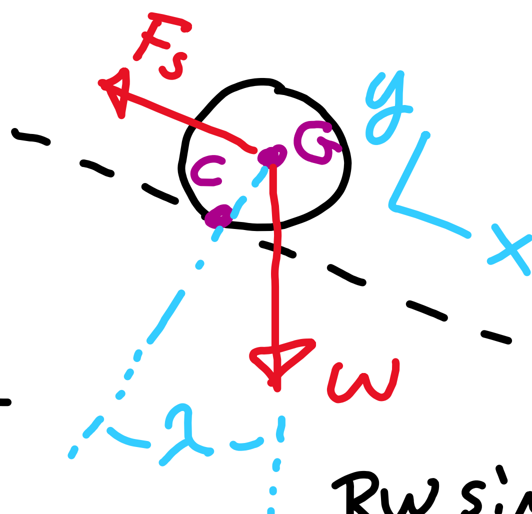
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x} w\sin\lambda + \hat{y} w\cos\lambda$

$\curvearrowright \sum \vec{M}_c = 0 \Rightarrow$ $R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R k \delta$

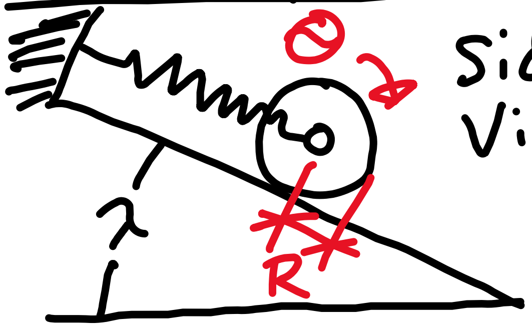
push down amount d:

$\curvearrowright \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$ $R w \sin\lambda - R k(\delta + d) = I_c \ddot{\theta}$

But $R w \sin\lambda = R k \delta$ so $-R k d = I_c \ddot{\theta}$ & since $d = R\theta$ then $-R^2 k \theta = I_c \ddot{\theta}$, where $I_c = \bar{I} + mR^2$

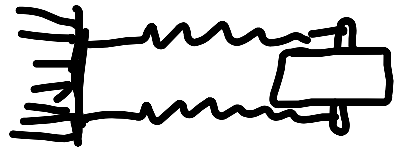
ASU

Example



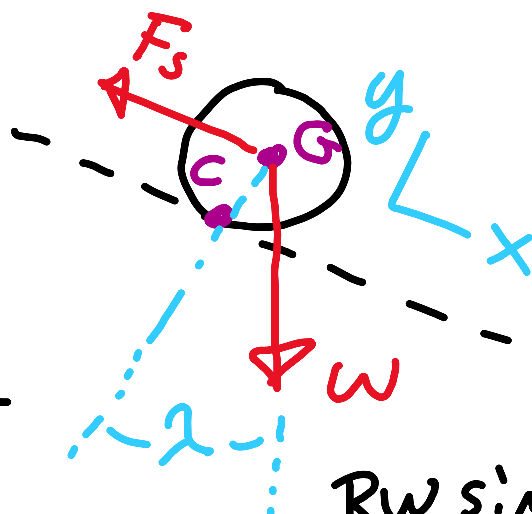
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$ $RW\sin\lambda - RF_s = 0$

$\Rightarrow Rw\sin\lambda = Rk\delta$

push down amount d:

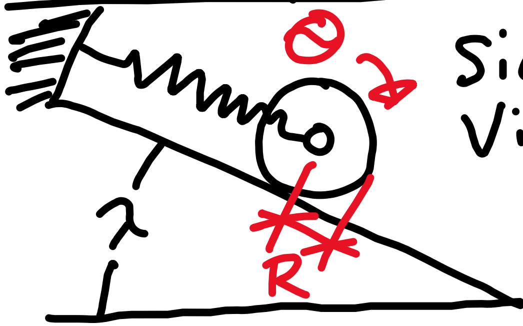
$\circlearrowleft \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

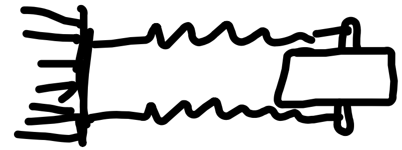
$\Rightarrow I_c = \frac{1}{2}mR^2 + mR^2$

Example



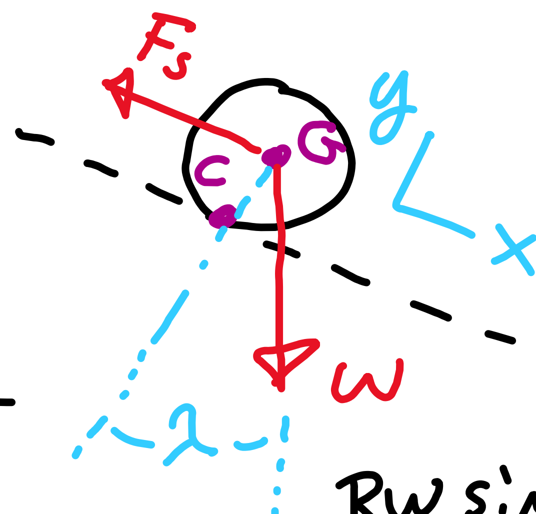
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$$

$$\vec{w} = \hat{x}\, w \sin\lambda + \hat{y}\, w\cos\lambda$$

$$R W \sin\lambda - R F_s = 0$$

$$\Rightarrow R w \sin\lambda = R k \delta$$

$$\circlearrowleft \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$$

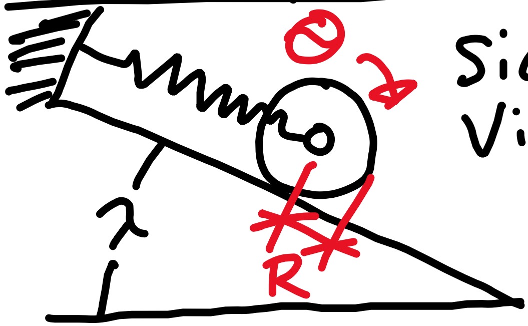
push down amount d:

$$R w \sin\lambda - R k(\delta + d) = I_c \ddot{\theta}$$

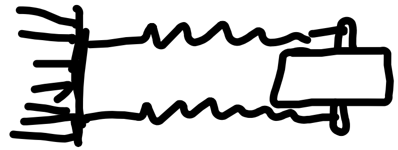
But $R w \sin\lambda = R k \delta$ so $-R k d = I_c \ddot{\theta}$ & since $d = R\theta$ then $-R^2 k \theta = I_c \ddot{\theta}$, where $I_c = \bar{I} + mR^2$

$$\Rightarrow I_c = \tfrac{1}{2} mR^2 + mR^2$$

Example

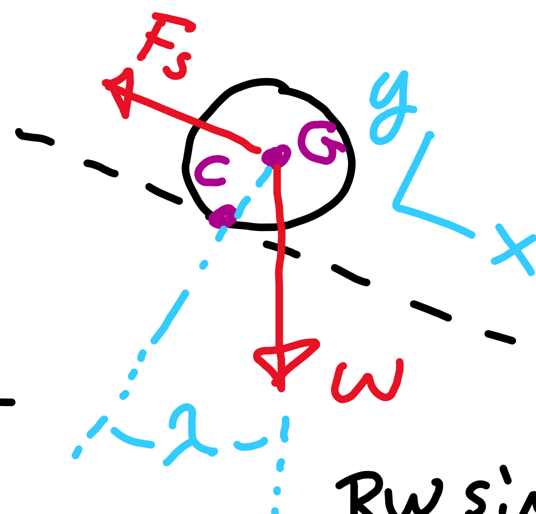


Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{W} = \hat{x} W \sin\lambda + \hat{y} W \cos\lambda$

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$ $\quad R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R K \delta$

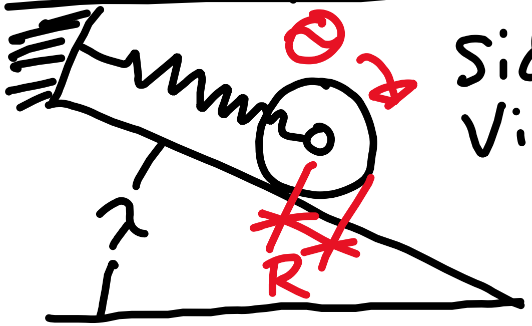
push down amount d:

$\circlearrowleft \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$ $\quad R w \sin\lambda - RK(\delta + d) = I_c \ddot{\theta}$

But $R w \sin\lambda = RK\delta$ so $-RKd = I_c \ddot{\theta}$ & since $d = R\theta$ then $-R^2 K \theta = I_c \ddot{\theta}$, where $I_c = \bar{I} + mR^2$

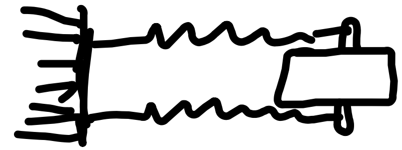
$\Rightarrow I_c = \frac{1}{2} mR^2 + mR^2 = \frac{3}{2} mR^2$

Example



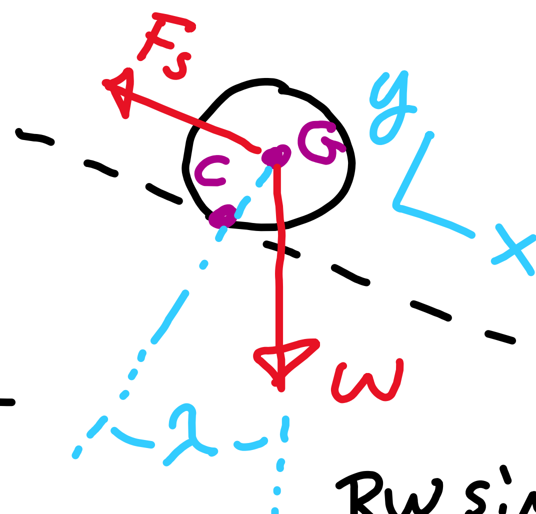
side view

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x}\, w \sin\lambda + \hat{y}\, w\cos\lambda$

$\circlearrowright \Sigma \vec{M}_c = 0 \Rightarrow$

$RW\sin\lambda - RF_s = 0$

$\Rightarrow Rw\sin\lambda = RK\delta$

push down amount d:

$\circlearrowright \Sigma \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

$\Rightarrow I_c = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$ so

$R^2K\theta = \frac{3}{2}MR^2\ddot{\theta}$

From previous slide

$$-R^2 K \Theta = \frac{3}{2} M R^2 \ddot{\Theta}$$

From previous slide

$$-R^2 K\Theta = \frac{3}{2} M R^2 \ddot{\Theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{m}\right)\Theta = \ddot{\Theta}$$

From previous slide

$$-R^2K\Theta = \frac{3}{2}MR^2\ddot{\Theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2\Theta$,

From previous slide

$$-R^2 K \Theta = \frac{3}{2} M R^2 \ddot{\Theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2 \Theta$, where $\omega = \sqrt{\frac{2K}{3M}}$

From previous slide

$$-R^2 K\theta = \frac{3}{2} M R^2 \ddot{\theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{M}\right)\theta = \ddot{\theta}$$

or $\ddot{\theta} = -\omega^2 \theta$, where $\omega = \sqrt{\frac{2K}{3M}}$

& since $\omega\tau = 2\pi$

From previous slide

$$-R^2 K\Theta = \frac{3}{2} M R^2 \ddot{\Theta} \implies -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2 \Theta$, where $\omega = \sqrt{\frac{2K}{3M}}$

& since $\omega\tau = 2\pi$

$$\tau = \frac{2\pi}{\omega}$$

From previous slide

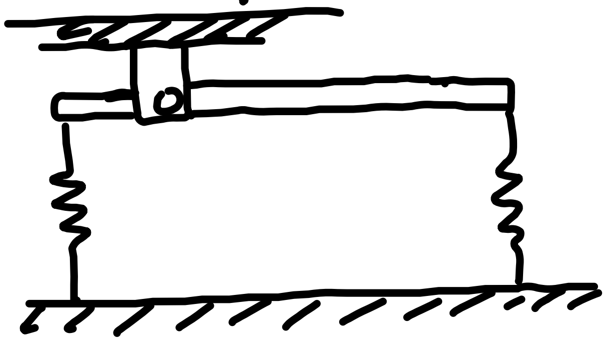
$$-R^2 K\Theta = \frac{3}{2} M R^2 \ddot{\Theta} \implies -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2 \Theta$, where $\omega = \sqrt{\frac{2K}{3M}}$

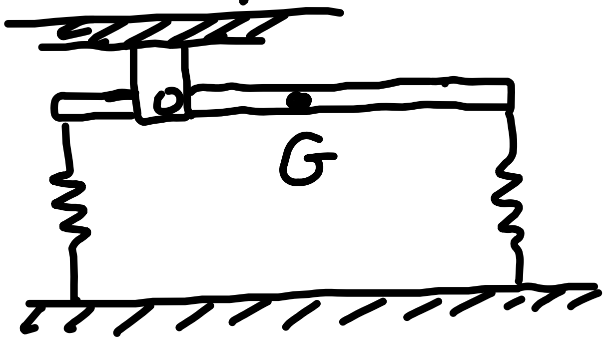
& since $\omega\tau = 2\pi$

$$\boxed{\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2K}}}$$

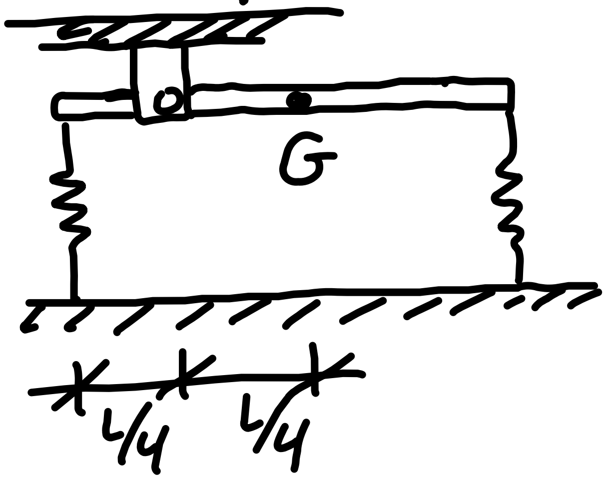
Example



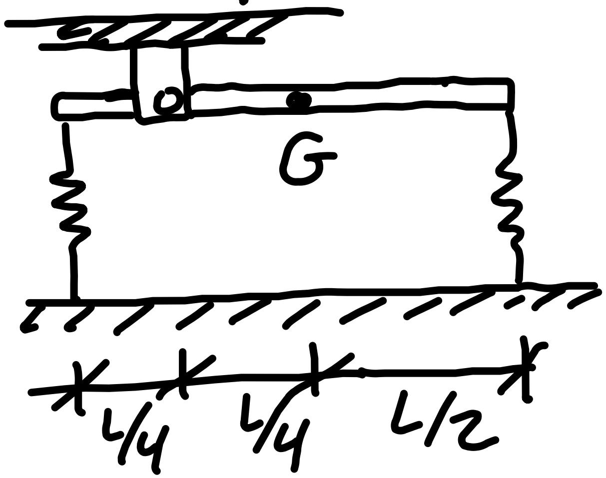
Example



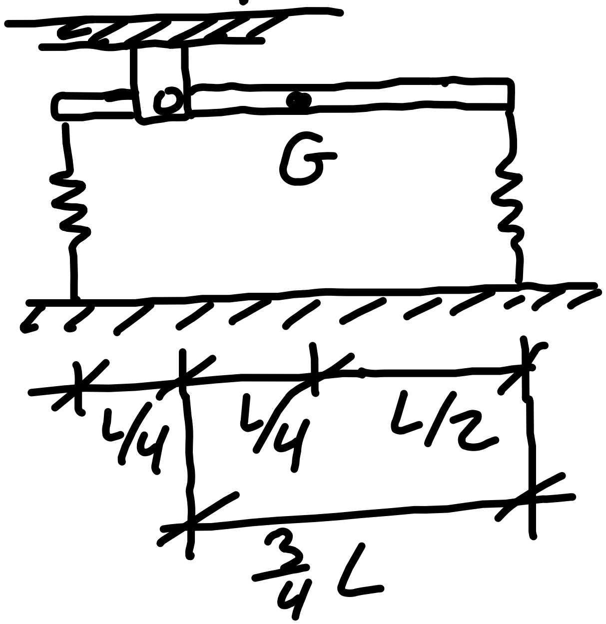
Example



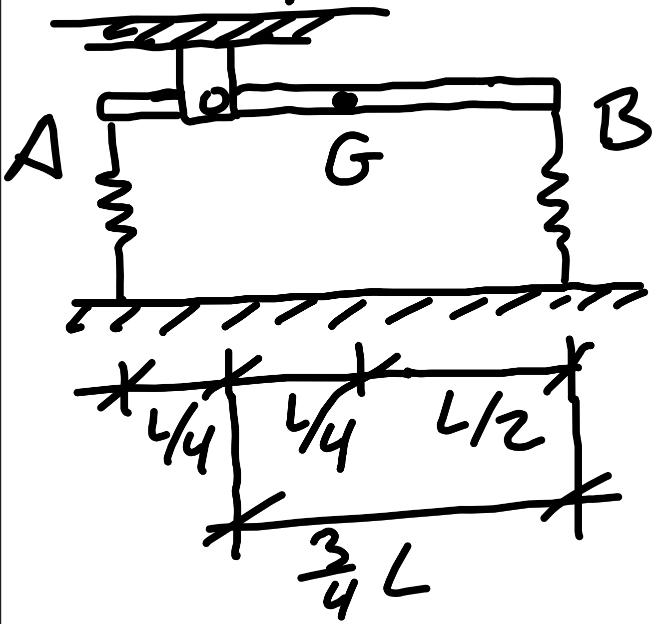
Example



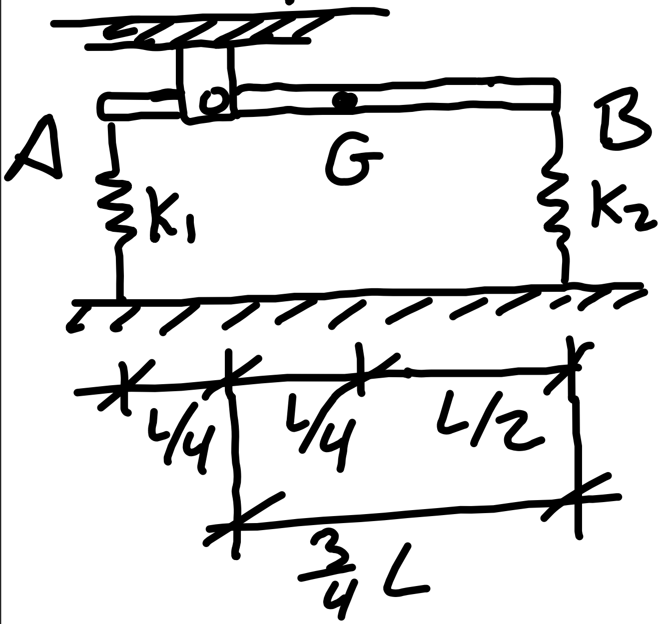
Example



Example

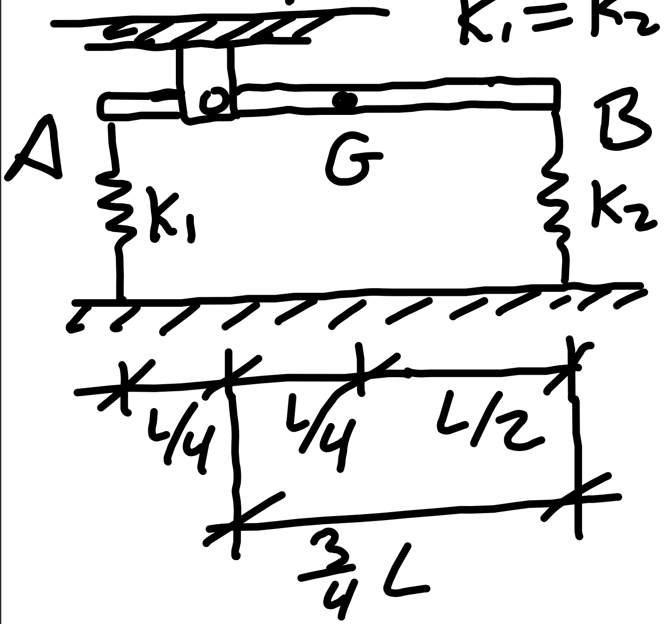


Example

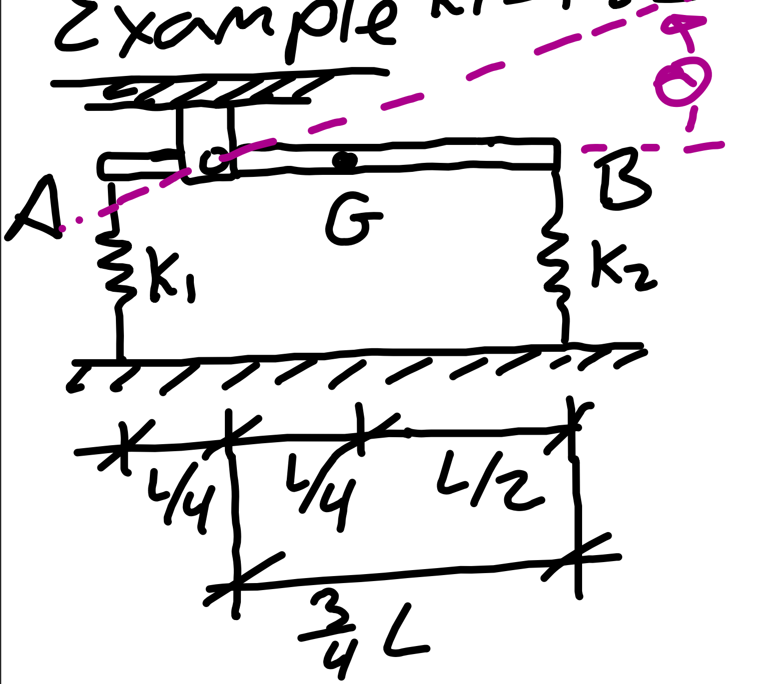


Example

$k_1 = k_2 = k$

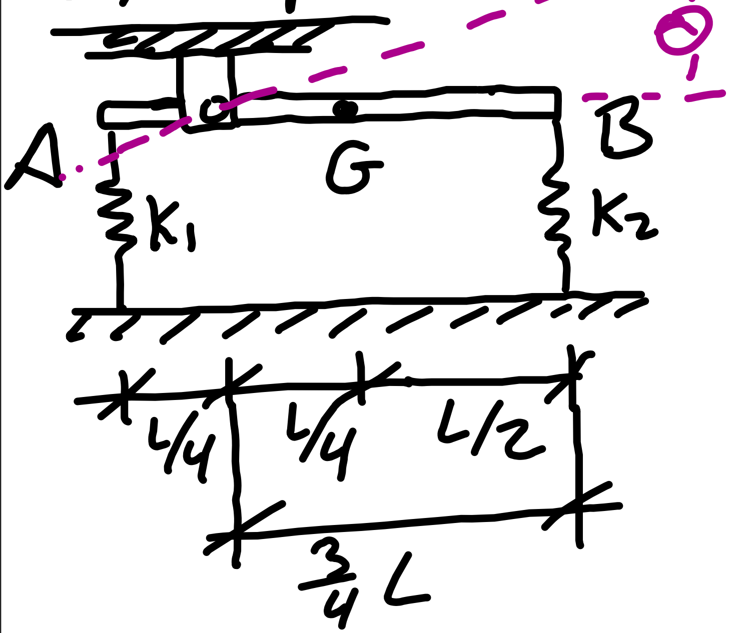


Example $k_1 = k_2 = k$



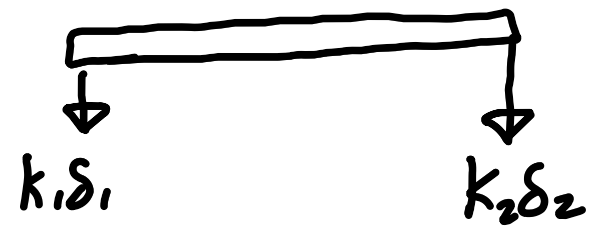
Push side B up a small distance δ & find ω_n

Example $k_1 = k_2 = k$

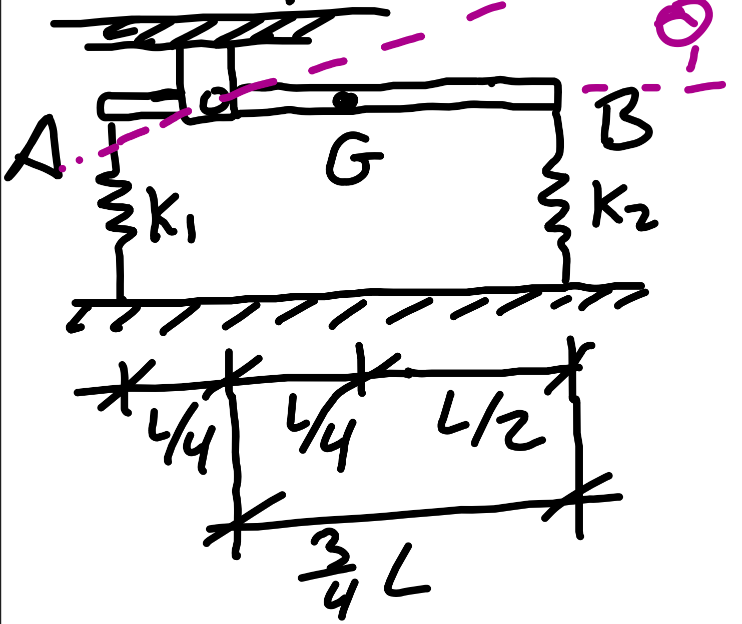


Push side B up a small distance δ & Find ω_n:

Equilibrium

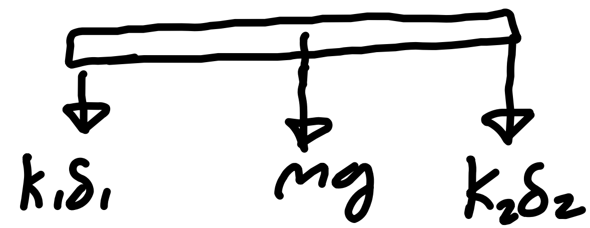


Example $k_1 = k_2 = k$

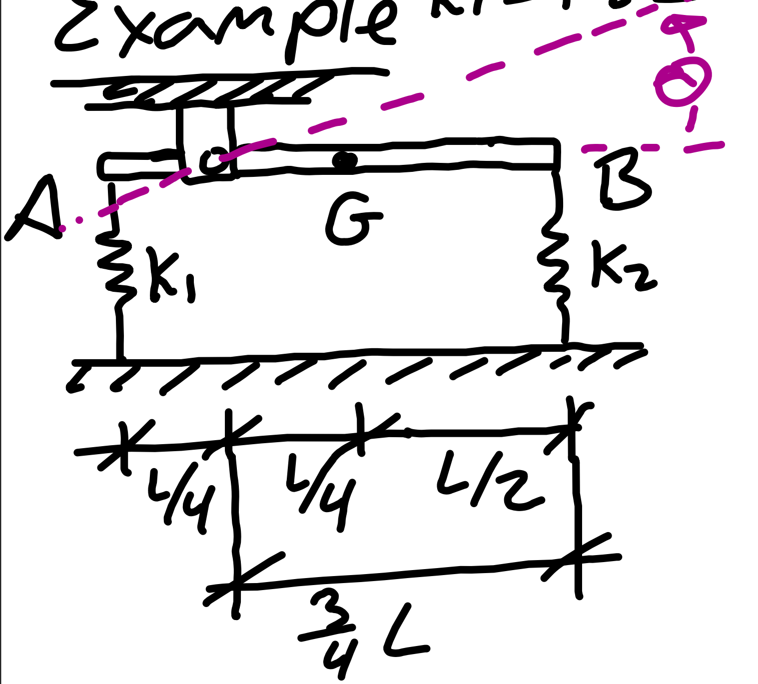


Push side B up a small distance δ & Find ω_n:

Equilibrium

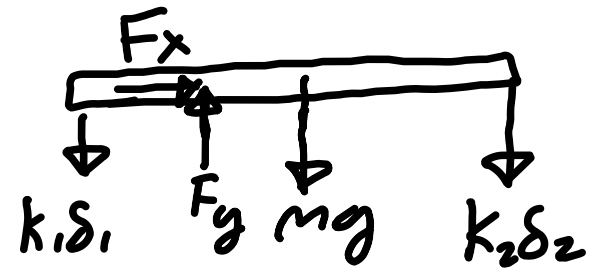


Example $k_1 = k_2 = k$

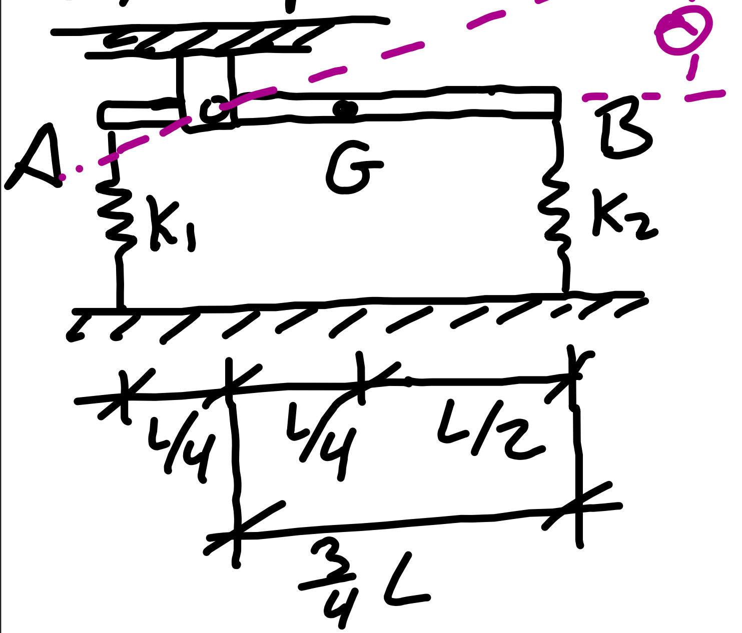


Push side B up a small distance δ & Find eqn:

Equilibrium



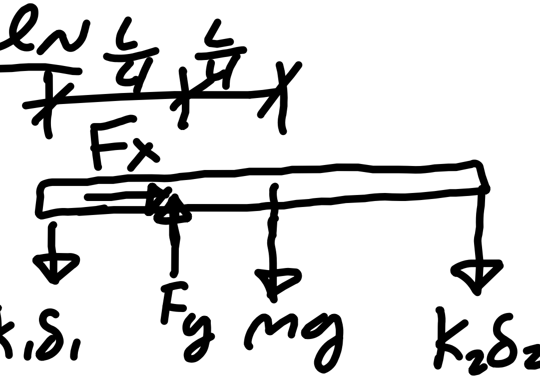
Example $k_1 = k_2 = k$



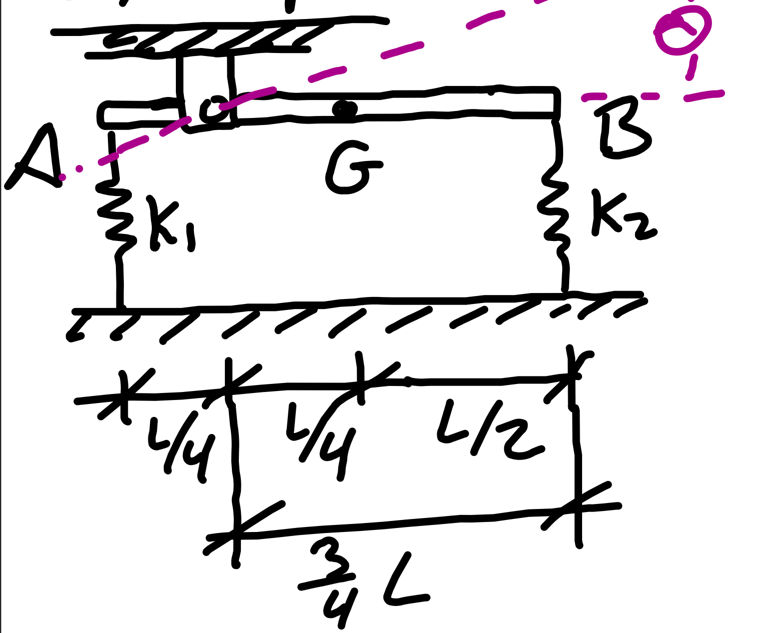
Push side B up a small distance δ &

Find ω_n

Equilibrium

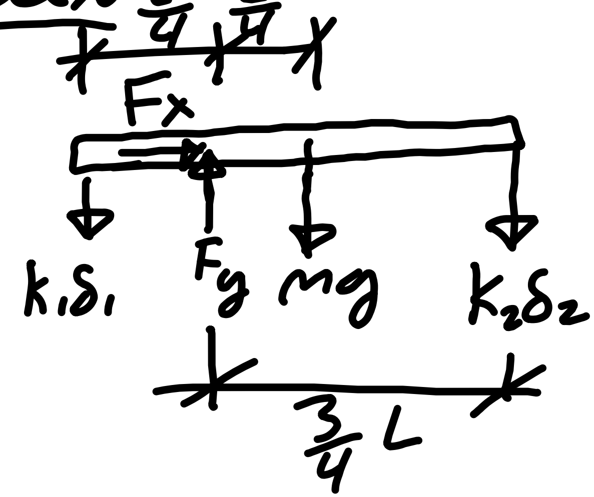


Example $k_1 = k_2 = k$

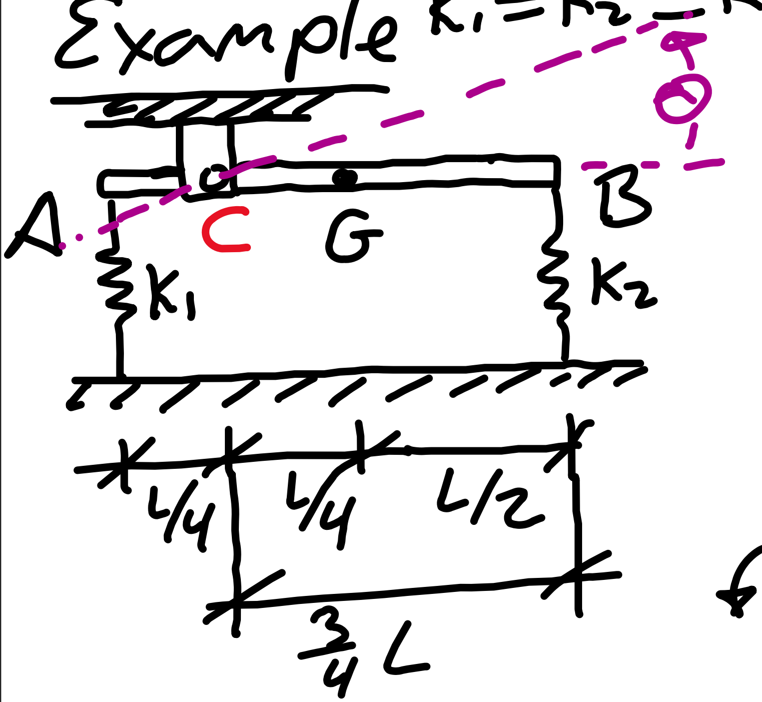


Push side B up a small distance δ & Find ω_n

Equilibrium

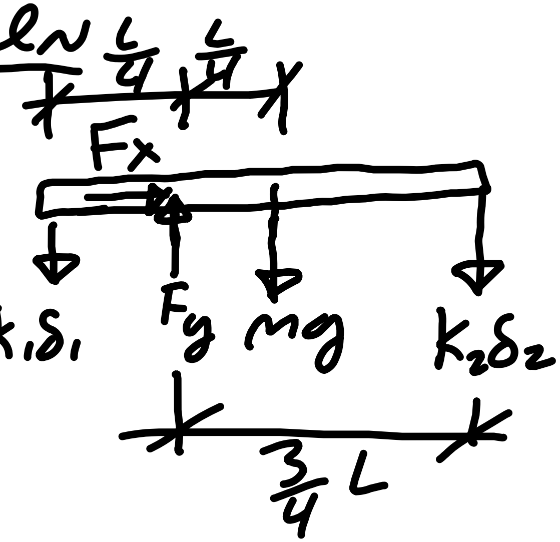


Example $k_1 = k_2 = k$



Push side B up a small distance δ &

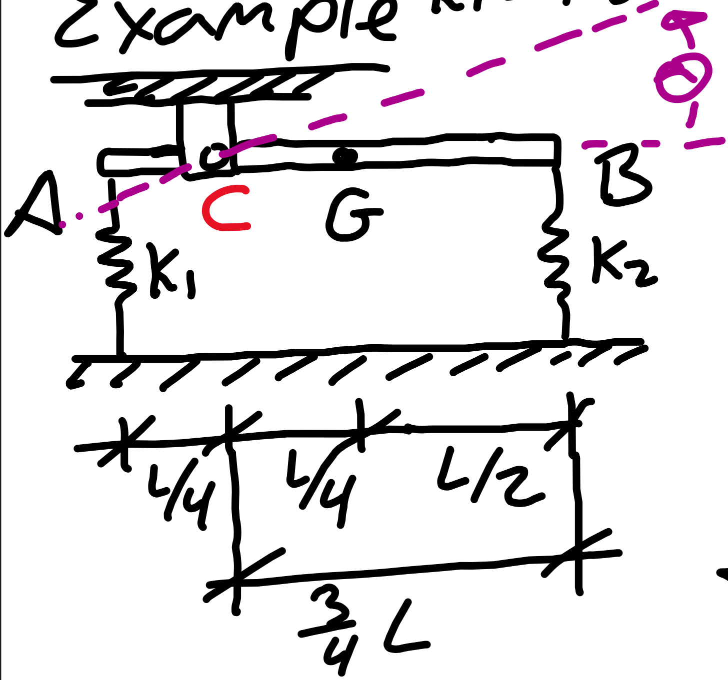
Find eqn



Equilibrium

$$\circlearrowleft \sum M_C = I_C \alpha$$

Example $k_1 = k_2 = k$

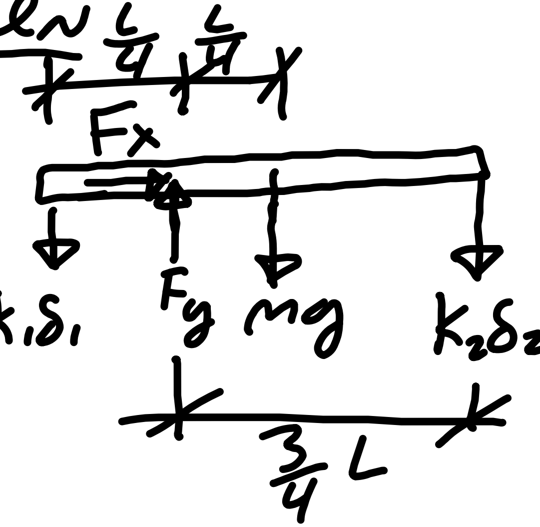


Push side B up a small distance δ & Find ω_n

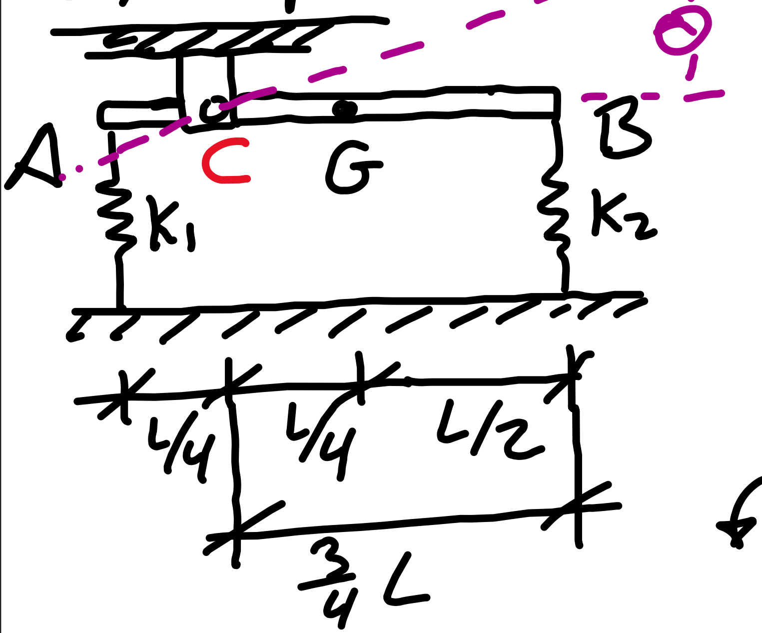
Equilibrium

$$\circlearrowleft \sum M_C = I_C \alpha$$

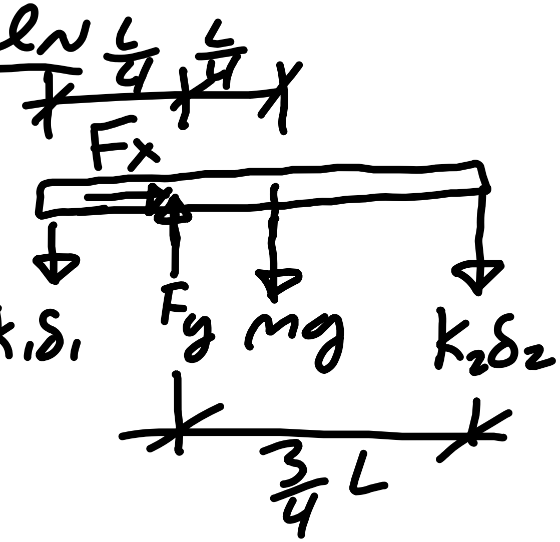
θ



Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find eqn

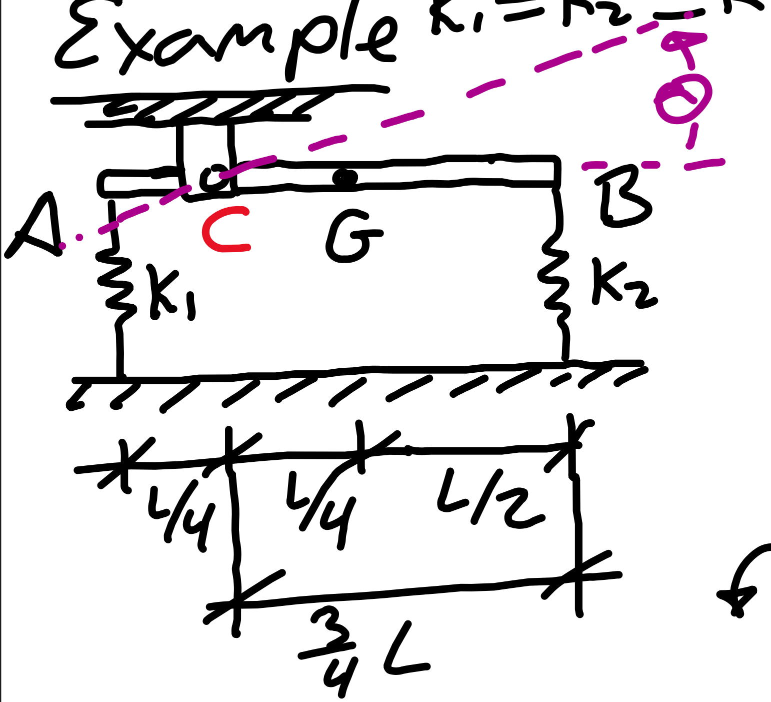


Equilibrium

$\circlearrowleft \sum M_C = I_C\alpha$ $\quad 0$

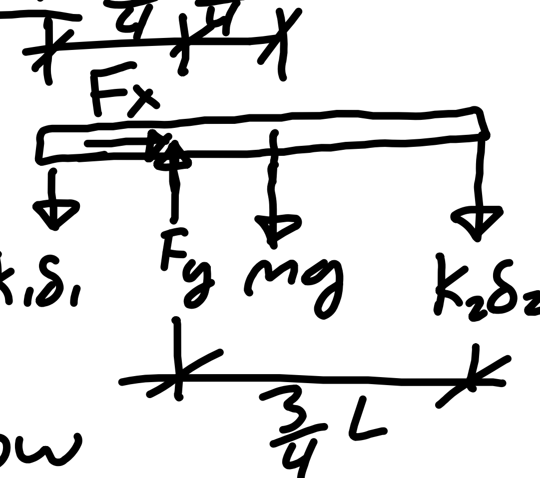
so $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$

Example $k_1 = k_2 = k$



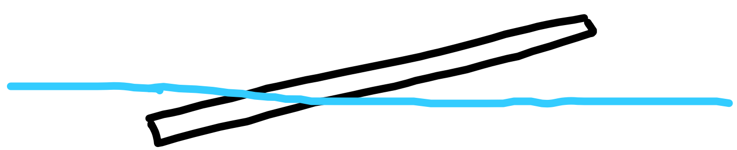
Push side B up a small distance d & Find eqn

Equilibrium

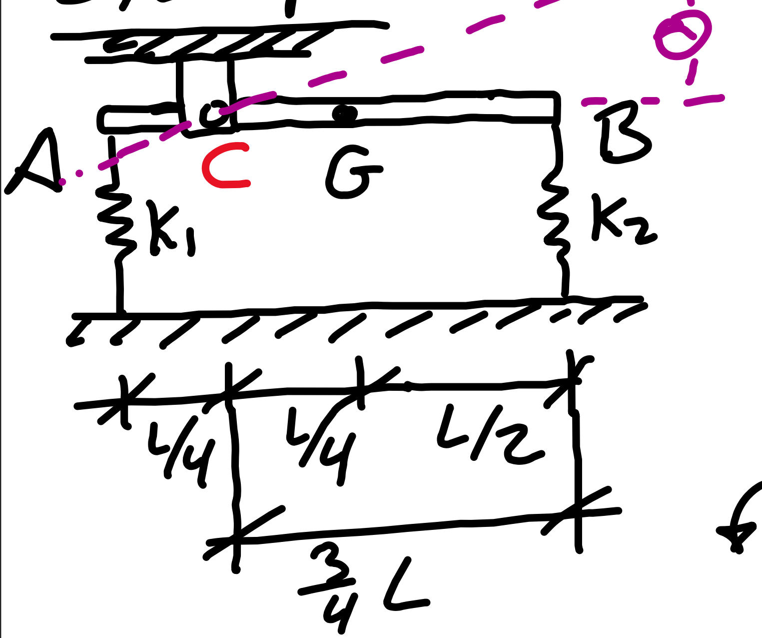


$$\circlearrowleft \sum M_C = \cancel{I_C\alpha} \; 0$$

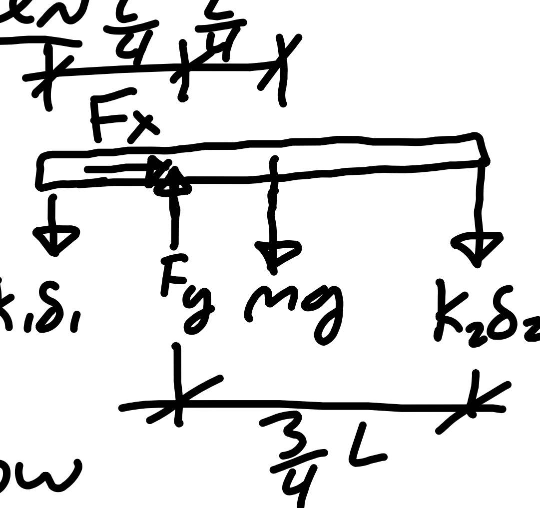
So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now push up side B a distance d



Example $k_1 = k_2 = k$



Push side B up a small distance d & Find ω_n

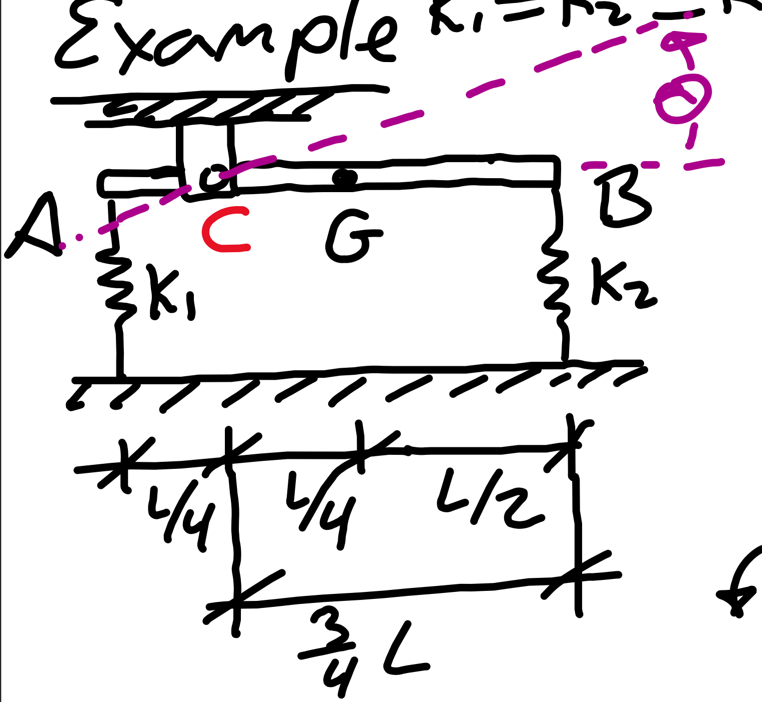


Equilibrium

$$\circlearrowleft \sum M_C = \cancel{I_C \alpha}\ 0$$

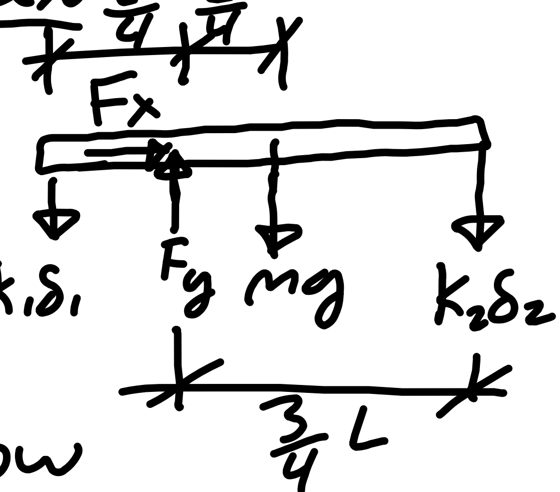
So $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now push up side B a distance d

Example $k_1 = k_2 = k$



Push side B up a small distance d &

Find eqn

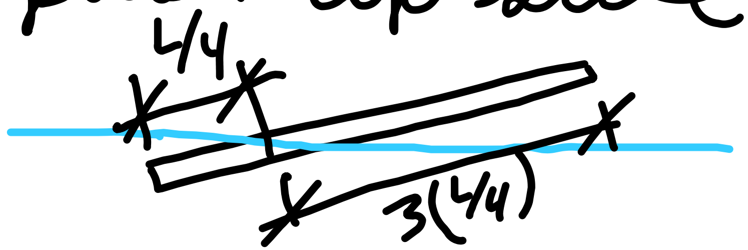


Equilibrium

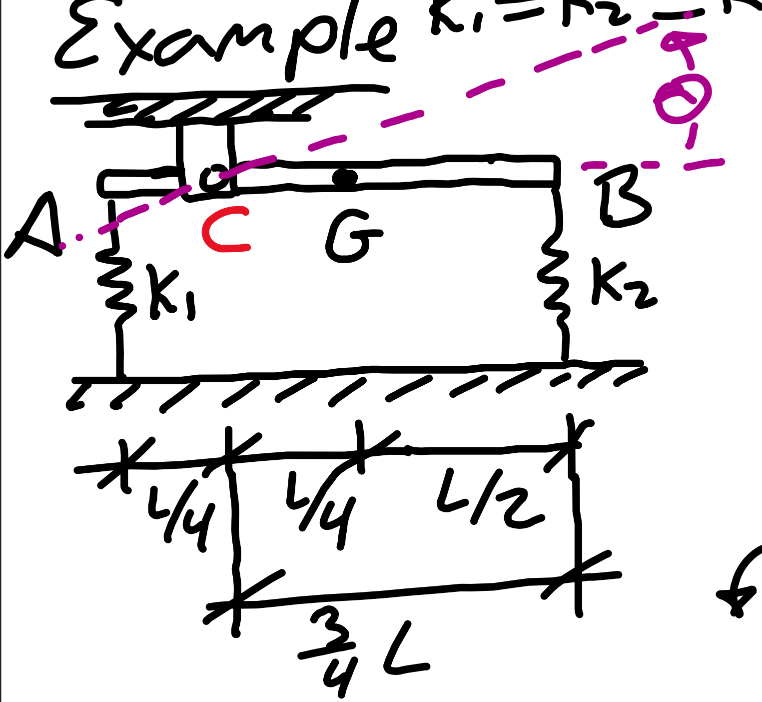
$\circlearrowleft \sum M_C = I_C\alpha$ 0

so $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now

push up side B a distance d



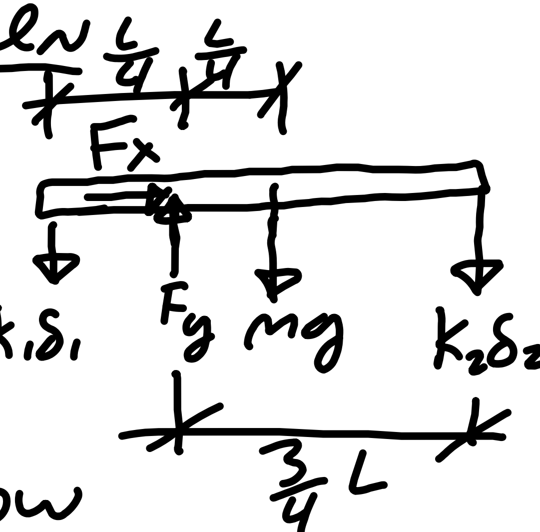
Example $k_1 = k_2 = k$



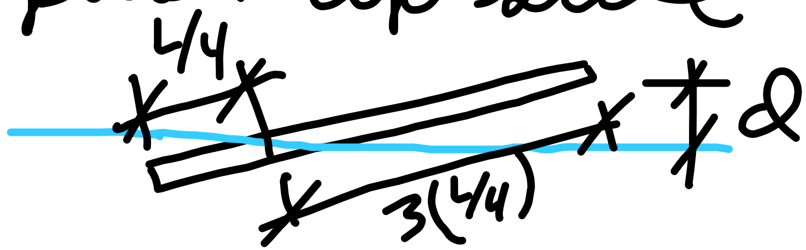
Push side B up a small distance d & Find eqn

Equilibrium

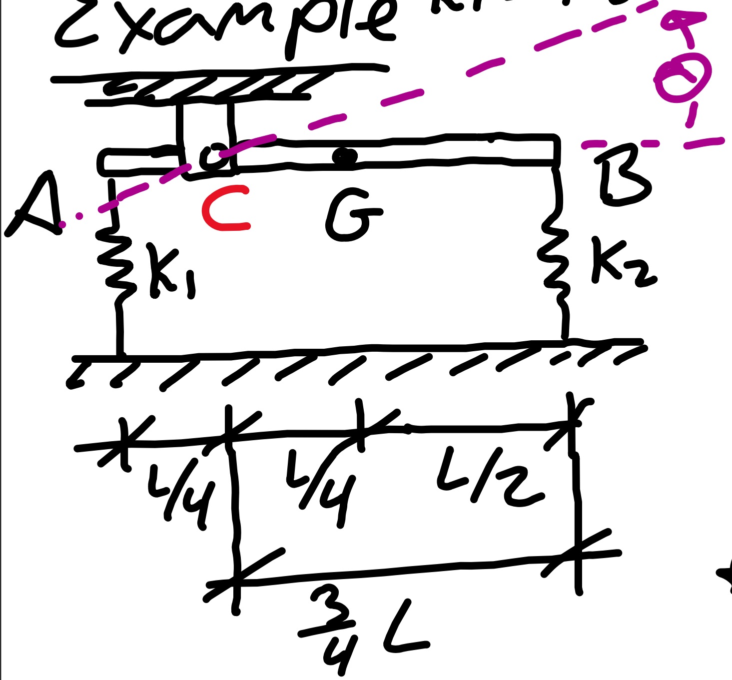
$\curvearrowleft \sum M_C = I_C\alpha = 0$



So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now push up side B a distance d

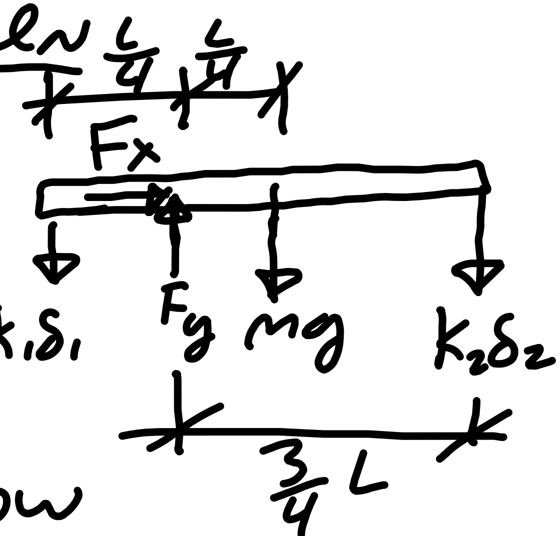


Example $k_1 = k_2 = k$



Push side B up a small distance d &

Find ω_n

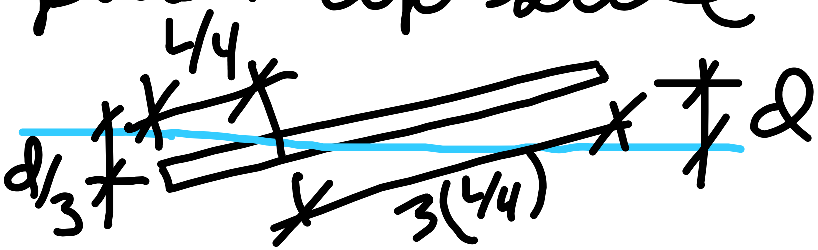


Equilibrium

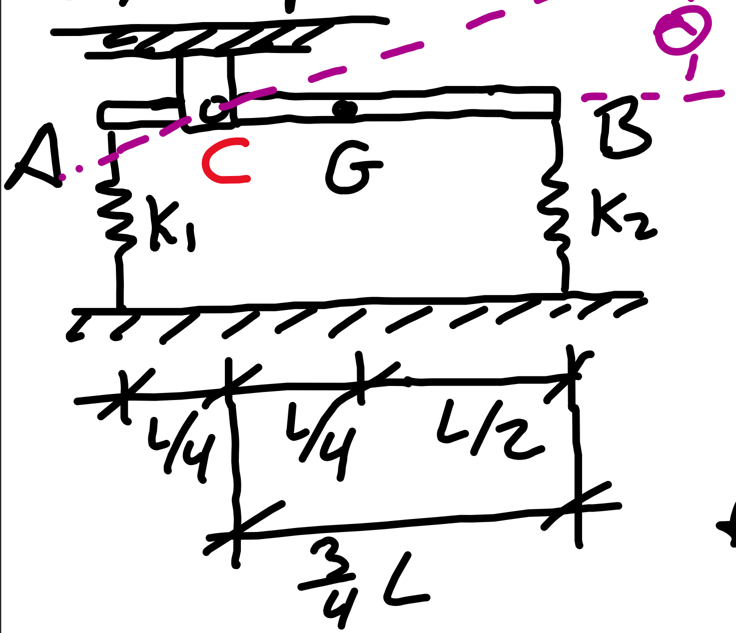
$$\circlearrowleft \sum M_C = \cancel{I_C \alpha}\ 0$$

so $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now

push up side B a distance d



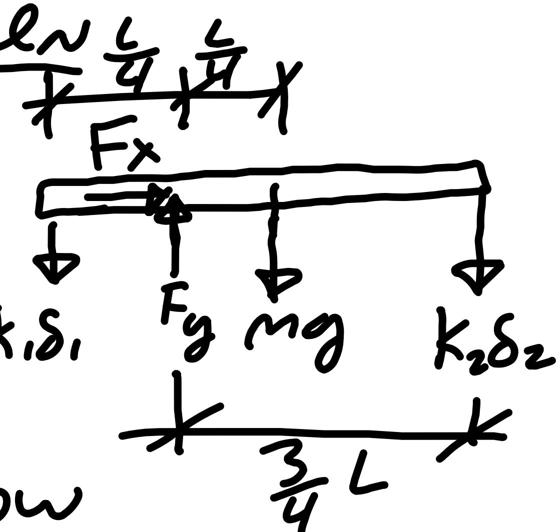
Example $k_1 = k_2 = k$



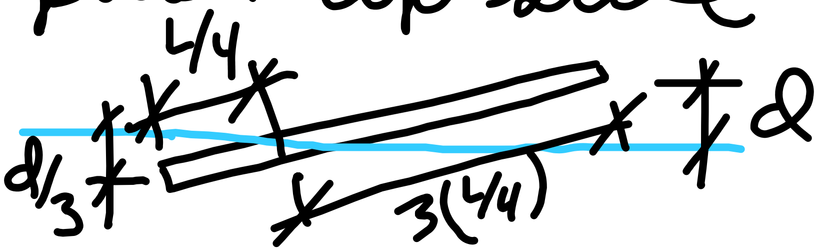
Push side B up a small distance d & Find eqn

Equilibrium

$\circlearrowleft \sum M_c = \cancel{I_c \alpha}$ 0

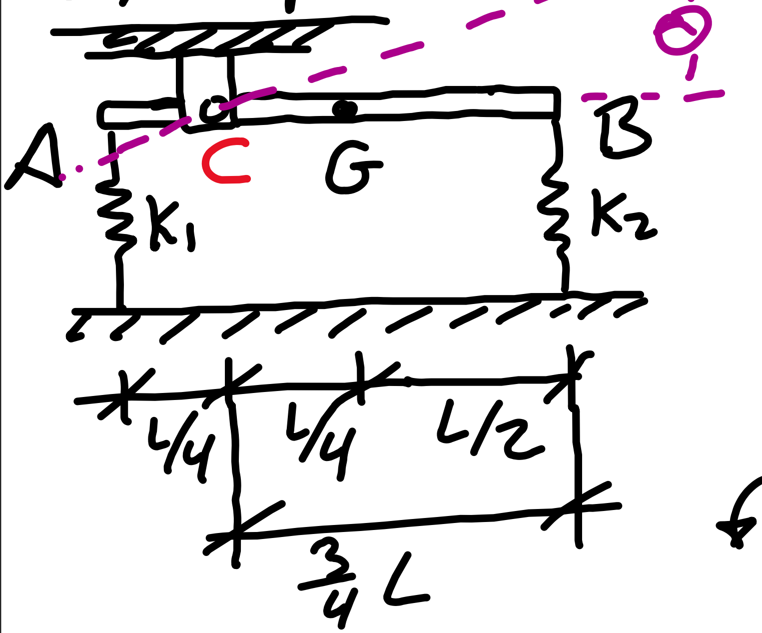


So $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now push up side B a distance d

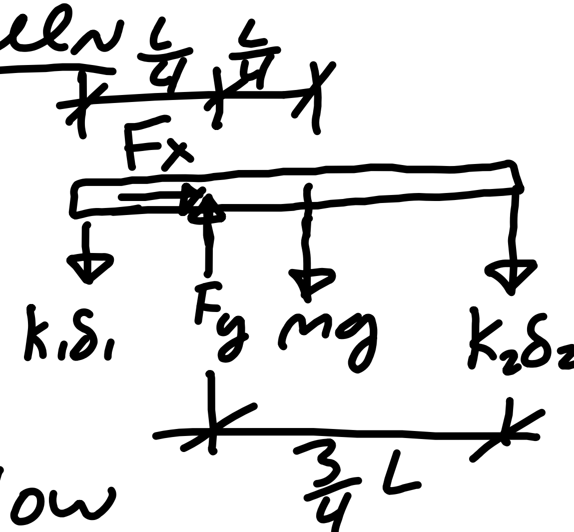


So $\circlearrowleft \sum M_c = I_c \alpha$

Example $k_1 = k_2 = k$



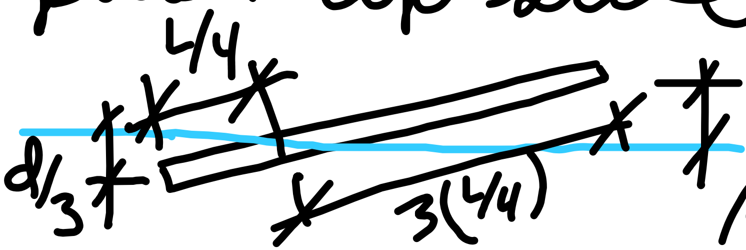
Push side B up a small distance d & Find ω_n



Equilibrium

$\circlearrowleft \sum M_C = I_C\alpha = 0$

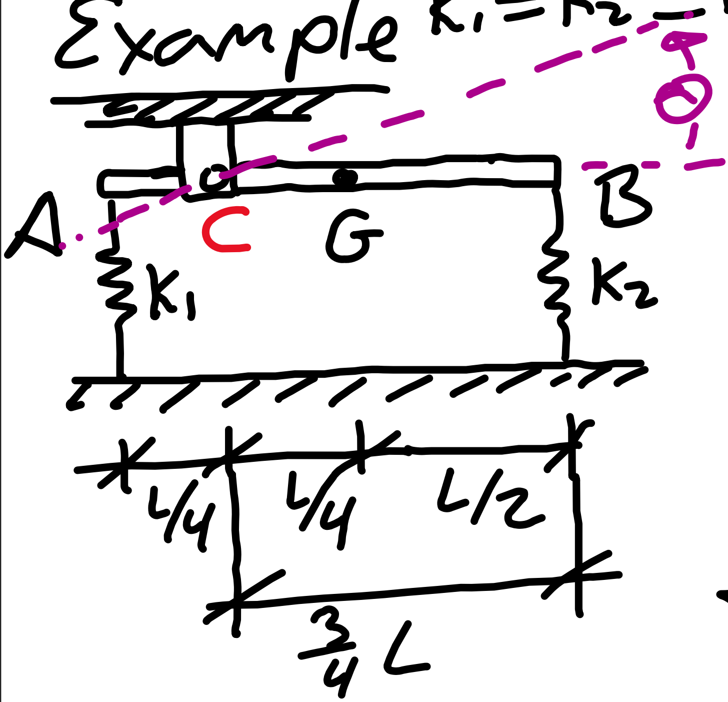
So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now push up side B a distance d



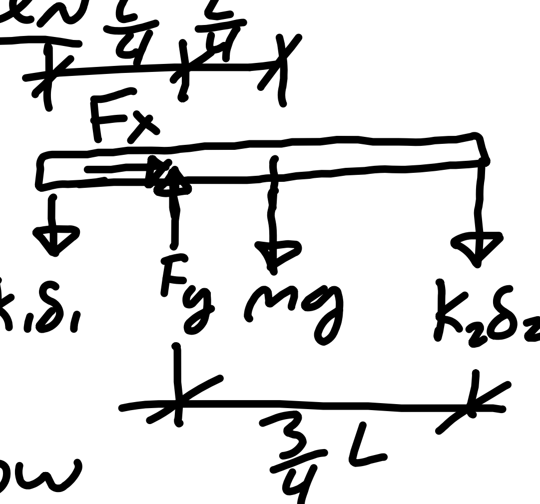
d so $\circlearrowleft \sum M_C = I_C\alpha \Rightarrow$

$M_C = k(\delta - d/3)\frac{L}{4} -$

Example $k_1 = k_2 = k$



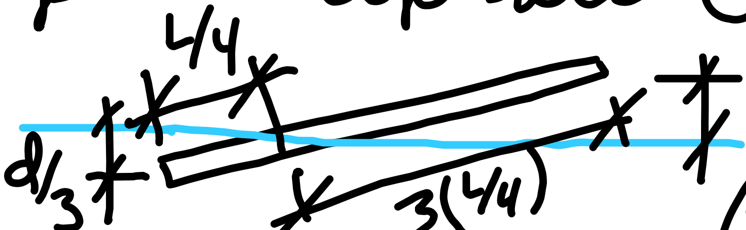
Push side B up a small distance d & Find ω_n



Equilibrium

$$\circlearrowleft \sum M_C = \cancel{I_C\alpha}\ 0$$

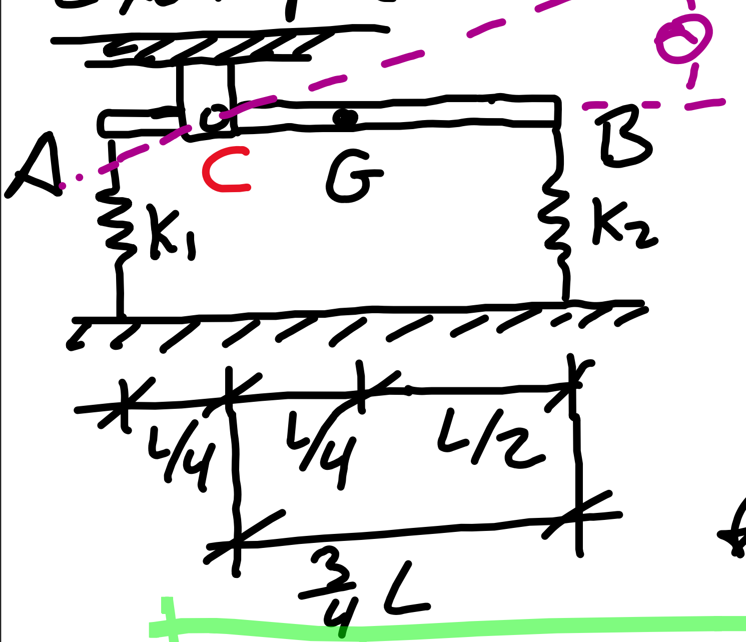
so $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now push up side B a distance d



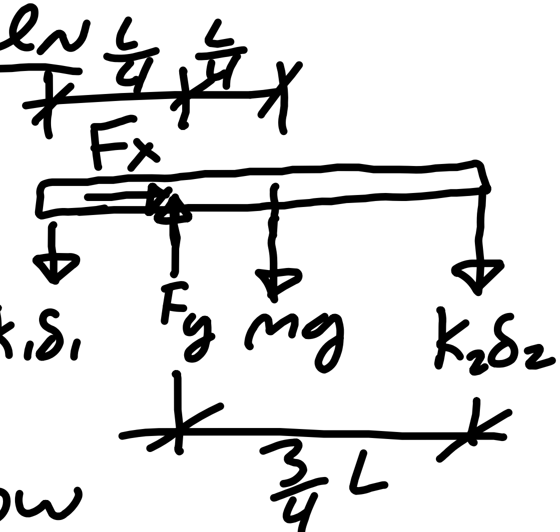
so $\circlearrowleft M_C = I_C\alpha \Rightarrow$

$$M_C = k(\delta - d/3)\frac{L}{4} - mg\frac{L}{4} -$$

Example $k_1 = k_2 = k$



Push side B up a small distance d & find ω_n

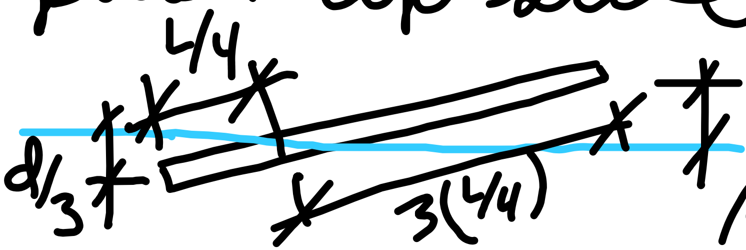


Equilibrium

$$\circlearrowleft \sum M_C = I_C\alpha \qquad \cancel{0}$$

So $\boxed{k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = \cancel{0}}$ Now

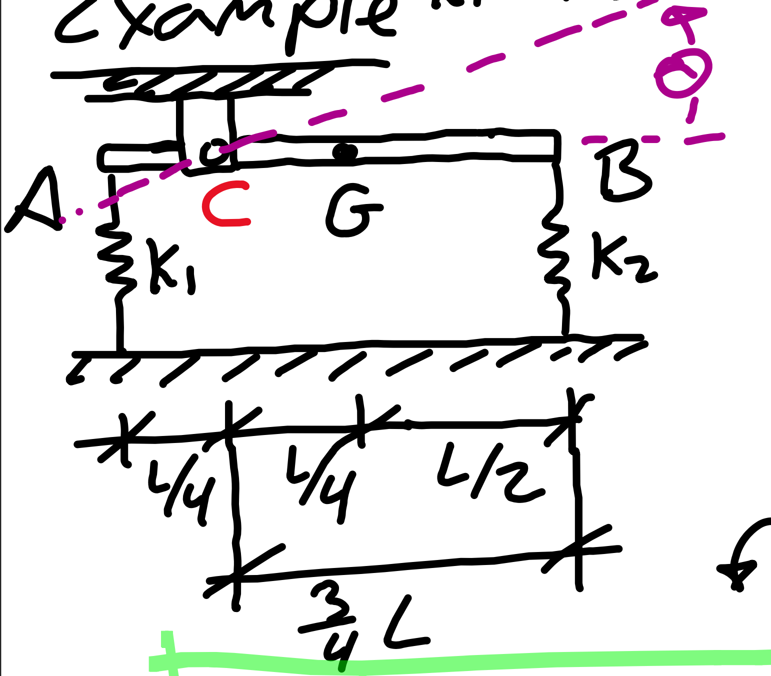
push up side B a distance d



So $\circlearrowleft M_c = I_c\alpha \Rightarrow$

$$M_c = k(\delta_1 - d/3)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + d)\frac{3L}{4}$$

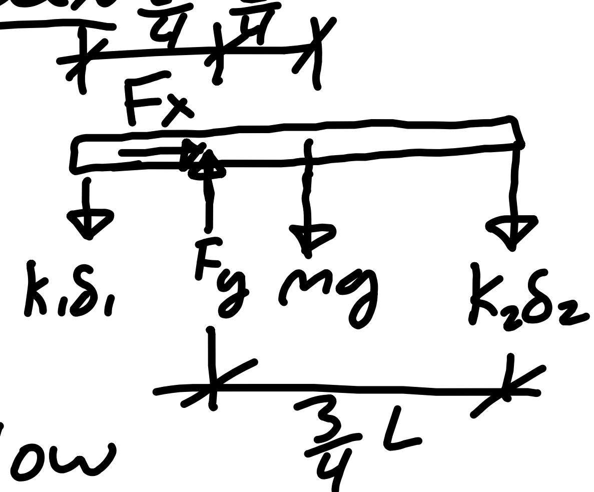
Example $k_1 = k_2 = k$



Push side B up a small distance δ & find ω_n

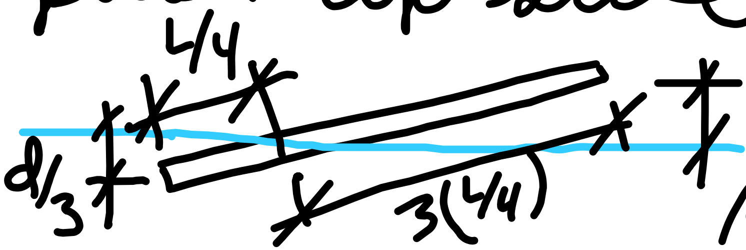
Equilibrium

$$\circlearrowleft \Sigma M_C = I_C \alpha = 0$$



so $\boxed{k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0}$ Now

push up side B a distance δ



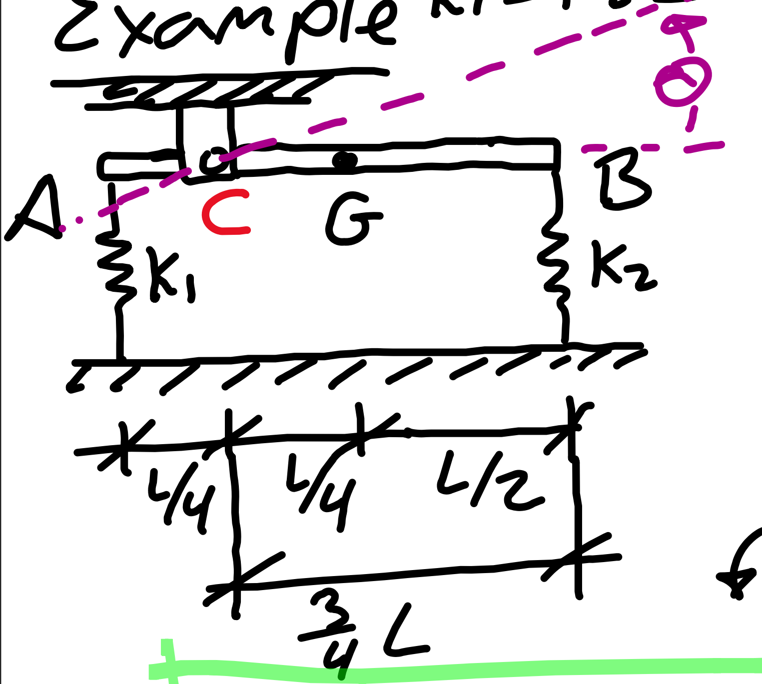
so $\circlearrowleft M_c = I_c \alpha \Rightarrow$

$$M_c = k(\delta_1 - \delta/3)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + \delta)\frac{3L}{4}$$

$$\Rightarrow \quad -k\frac{\delta L}{12} - k\frac{3\delta L}{4} = I_c\ddot{\theta}$$

ASU

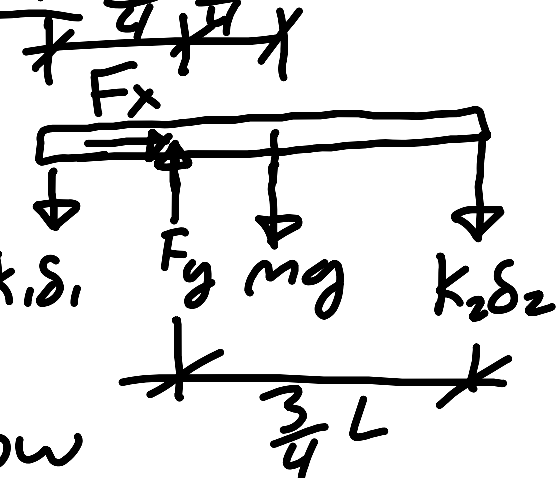
Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find ω_n

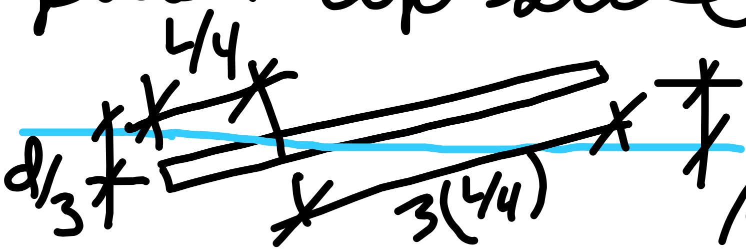
Equilibrium

$$\curvearrowleft \sum M_C = I_C \alpha$$



So $\boxed{k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0}$

Now push up side B a distance δ

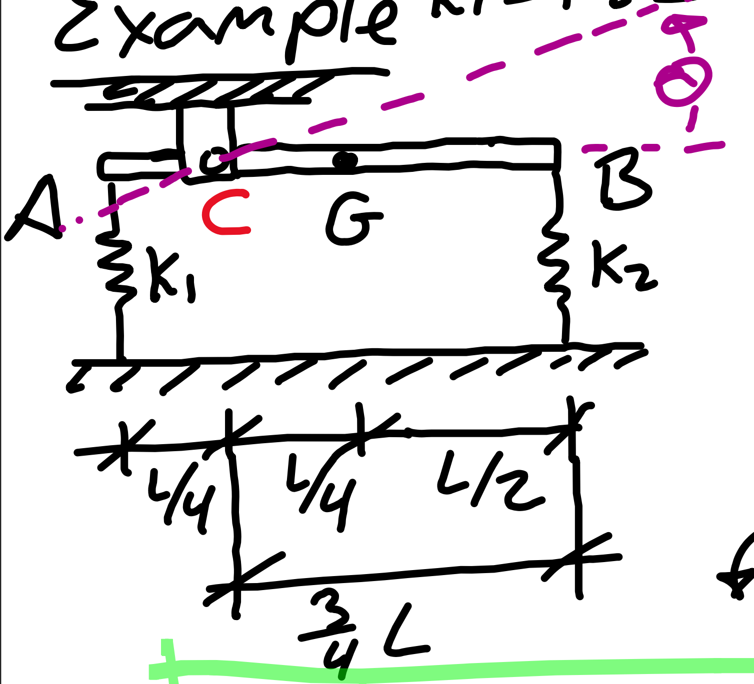


So $\curvearrowleft M_C = I_C\alpha \Rightarrow$

$$M_C = k\left(\delta_1 - \delta/3\right)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + \delta)\frac{3L}{4}$$

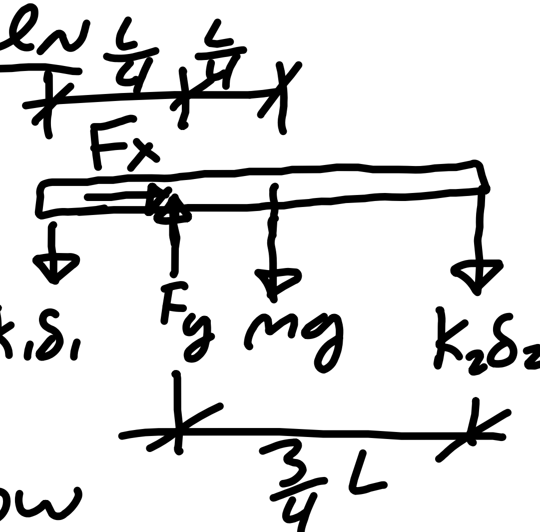
$$\Rightarrow -k\frac{\delta L}{12} - k\frac{3\delta L}{4} = I_C\ddot{\theta} \Rightarrow -\frac{k\delta L}{12}(1+9) = I_C\ddot{\theta}$$

Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find eqn

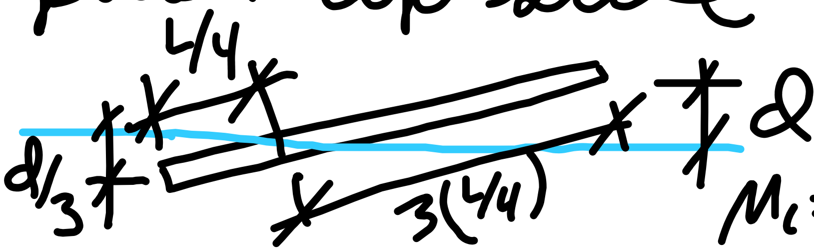
Equilibrium



$\circlearrowleft \sum M_C = I_C \alpha$ θ

so $\boxed{k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0}$

Now push up side B a distance δ



so $\circlearrowleft M_C = I_C\alpha \Rightarrow$

$$M_C = k\left(\delta_1 - \frac{\delta}{3}\right)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + \delta)\frac{3L}{4}$$

$$\Rightarrow -k\frac{\delta L}{12} - k\frac{3\delta L}{4} = I_C\ddot{\theta} \Rightarrow -\frac{k\delta L}{12}(1+9) = I_C\ddot{\theta}$$

From previous slide

From previous slide

$$-\frac{k\ell L}{12}(1+9) = I_c\ddot{\theta}$$

From previous slide

$$-\frac{kdL}{12}(1+9) = I_c\ddot{\theta} \quad \text{But} \quad d = L\sin\theta$$

From previous slide

$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta}$ But $d = L\sin\theta \cong L\theta$

From previous slide

$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta}$ But $d = L\sin\theta \cong L\theta$

So $-kL^2\left(\frac{5}{6}\right) = I_c \ddot{\theta}$

From previous slide

$$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta} \quad \text{But} \quad d = L\sin\theta \cong L\theta$$

So $$-kL^2\left(\frac{5}{6}\right) = I_c \ddot{\theta} \quad \underline{\underline{\text{or}}}$$

$$\ddot{\theta} = -\omega^2 \theta$$

From previous slide

$$-\frac{kdL}{12}(1+9) = I_c\ddot{\theta} \quad \text{But} \quad d = L\sin\theta \cong L\theta$$

So $-kL^2\left(\frac{5}{6}\right) = I_c\ddot{\theta}$ or

$$\ddot{\theta} = -\omega^2\theta, \text{ where } \boxed{\omega = L\sqrt{\frac{5k}{6I_c}}}$$