Today 19.2, 19.3

$\boxed{L33}$

Today 19.2, 19.3

$\boxed{L33}$

Free Vibrations of rigid bodies

Today 19.2, 19.3

L33

Free Vibrations of rigid bodies

Energy methods for vibrations

Today 19.2, 19.3

Wednesday 19.3

$\boxed{L33}$

Today 19.2, 19.3

Wednesday 19.3

$\boxed{L33}$

Important dates

Today 19.2, 19.3

Wednesday 19.3

L33

Important dates

* Friday Nov 27^{th} no class

Today 19.2, 19.3

Wednesday 19.3

L33

Important dates

* Friday Nov 27th no class
* Monday Nov 30th Exam #4

Today 19.2, 19.3 L33

Wednesday 19.3

Important dates

* Friday Nov 27th no class
* Monday Nov 30th Exam #4
* Wednesday Dec 2nd Day of reckoning

Today 19.2, 19.3

L33

Wednesday 19.3

Important dates

* Friday Nov 27th no class
* Monday Nov 30th Exam #4
* Wednesday Dec 2nd Day of reckoning
* Friday Dec 4th Final exam

From last time: If $\ddot{J} = -\omega^2 J$

From last time: If $\ddot{x} = -\omega^2 x$,

then $x = A\cos(\omega t) + B\sin(\omega t)$

From last time: If $\ddot{y} = -\omega^2 y$,

then $$y = A\cos(\omega t) + B\sin(\omega t)$$

or $$y = y_{max}\sin(\omega t + \varphi)$$

From last time: If $\ddot{x} = -\omega^2 x$,

then $x = A\cos(\omega t) + B\sin(\omega t)$

or $x = x_{max}\sin(\omega t + \phi)$

or $x = x_{max}\cos(\omega t + \psi)$

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{max}\sin(\omega t + \varphi)$

or $y = y_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$,

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{max}\sin(\omega t + \varphi)$

or $y = y_{max}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

From last time: If $\ddot{J} = -\omega^2 J$,

then $J = A\cos(\omega t) + B\sin(\omega t)$

or $J = J_{MAX}\sin(\omega t + \varphi)$

or $J = J_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$.

From last time: If $\ddot{J} = -\omega^2 J$,

then $$J = A\cos(\omega t) + B\sin(\omega t)$$

or $$J = J_{MAX}\sin(\omega t + \varphi)$$

or $$J = J_{MAX}\cos(\omega t + \psi)$$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

<u>From last time</u>: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

<u>or</u> $y = y_{MAX}\sin(\omega t + \varphi)$

<u>or</u> $y = y_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

Also $V_{MAX} = \omega X_{MAX}$

From last time: If $\ddot{y} = -\omega^2 y$,

then $$y = A\cos(\omega t) + B\sin(\omega t)$$

or $$y = y_{MAX}\sin(\omega t + \varphi)$$

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So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

Also $V_{MAX} = \omega X_{MAX}$ &

$$a_{MAX} = \omega V_{MAX}$$

From last time: If $\ddot{y} = -\omega^2 y$,

then $y = A\cos(\omega t) + B\sin(\omega t)$

or $y = y_{MAX}\sin(\omega t + \varphi)$

or $y = y_{MAX}\cos(\omega t + \psi)$

So, if $\ddot{x} = -\omega^2 x$, then $x = x_m \sin(\omega t + \varphi)$

& if $\ddot{\theta} = -\omega^2 \theta$, then $\theta = \theta_m \sin(\omega t + \varphi)$

Also $V_{MAX} = \omega X_{MAX}$ &

$a_{MAX} = \omega V_{MAX} = \omega^2 X_{MAX}$

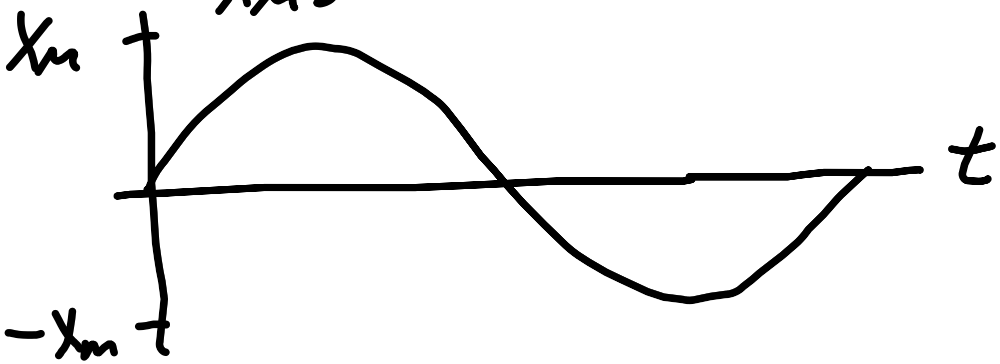
Wave anatomy

Wave anatomy

$x_m \sin(\omega t)$

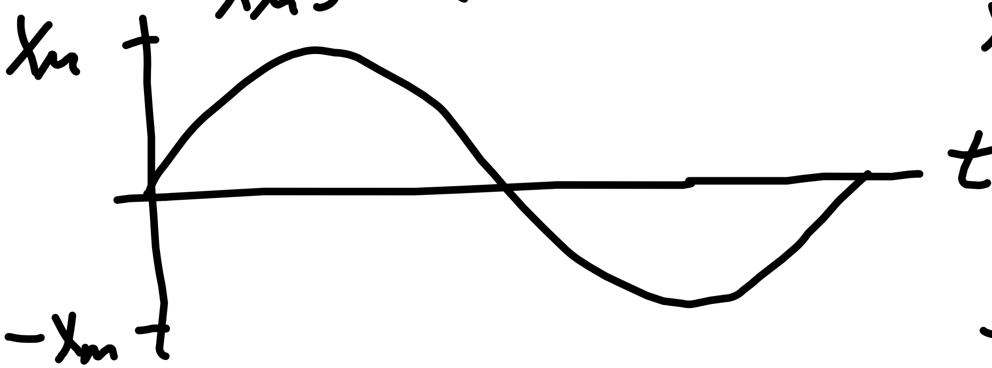
Wave anatomy

$x_m \sin(\omega t)$



Wave anatomy

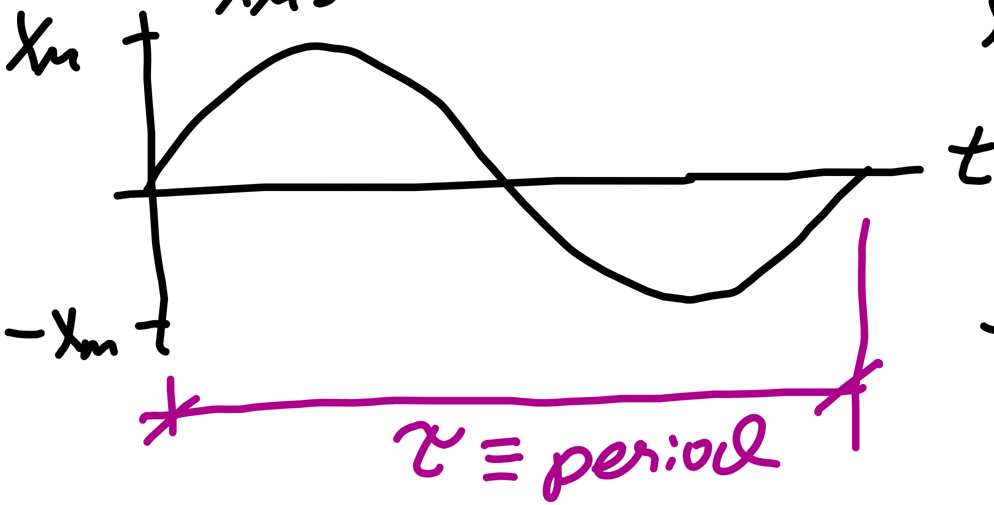
$x_m \sin(\omega t)$



$x_m \cos(\omega t)$

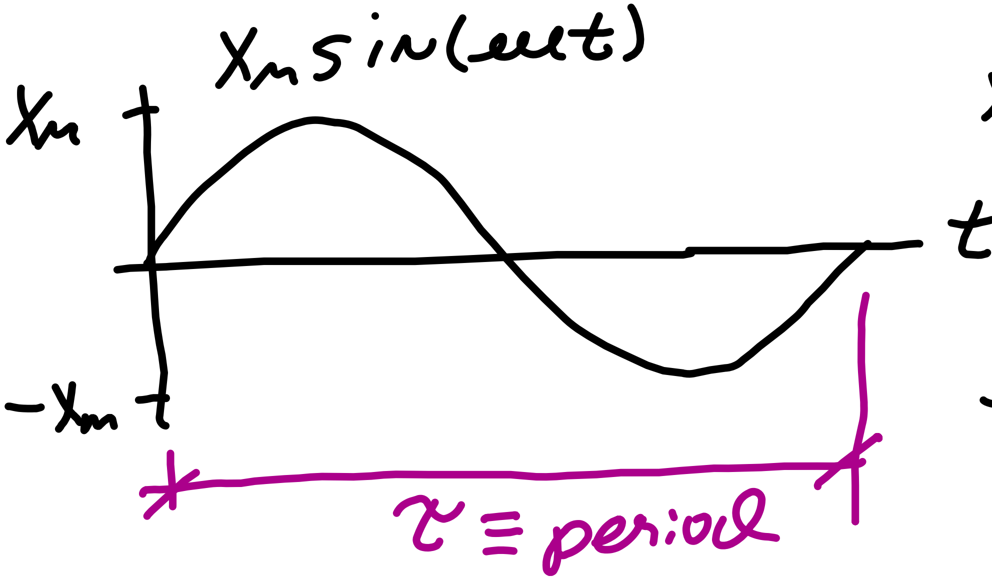
Wave anatomy

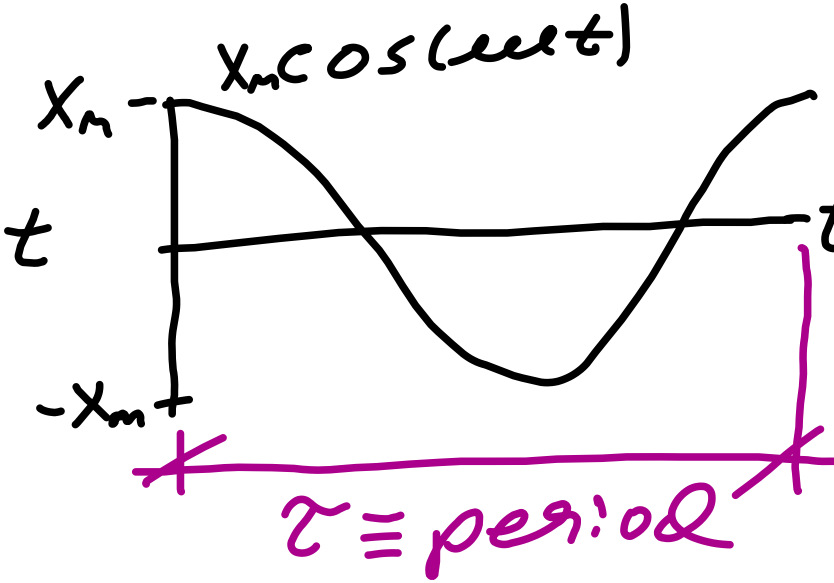
$x_m \sin(\omega t)$



$x_m \cos(\omega t)$

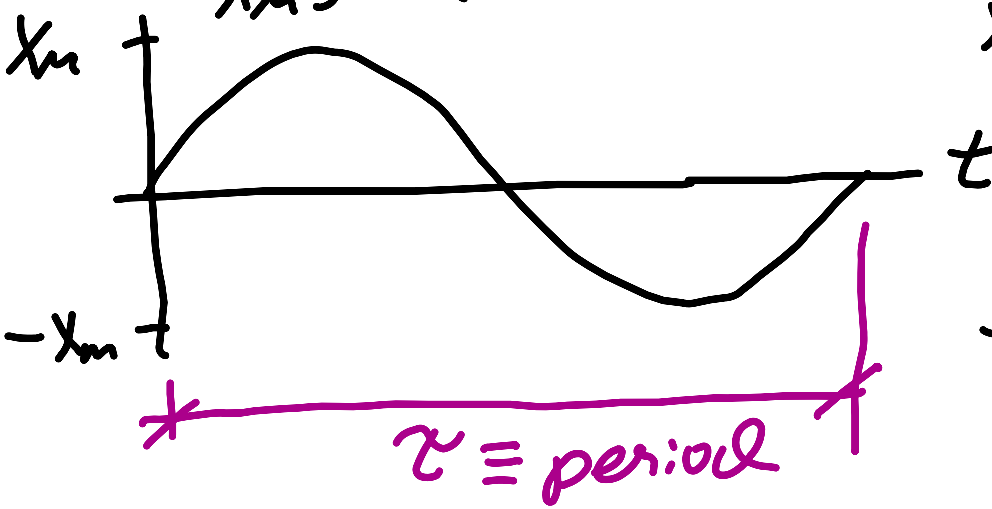
Wave anatomy



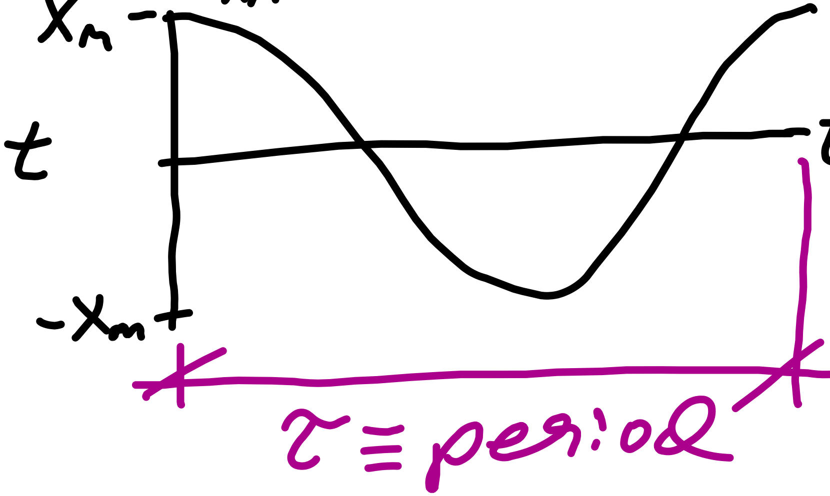


Wave anatomy

$x_m \sin(\omega t)$

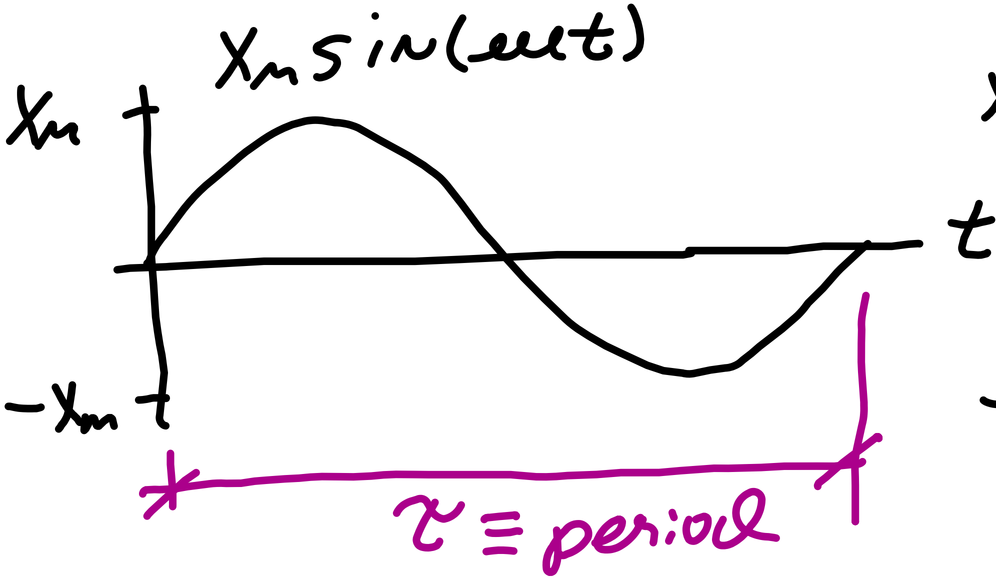


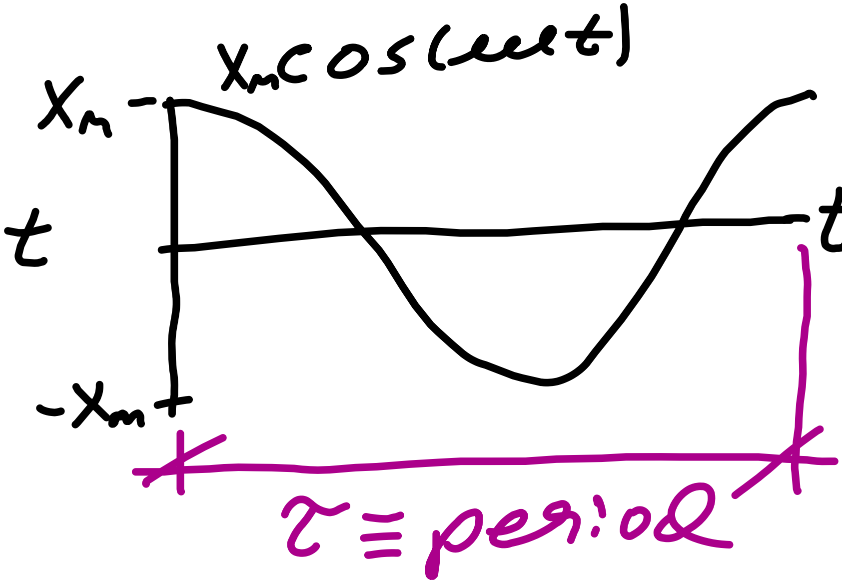
$x_m \cos(\omega t)$



So when $t =$ period

Wave anatomy

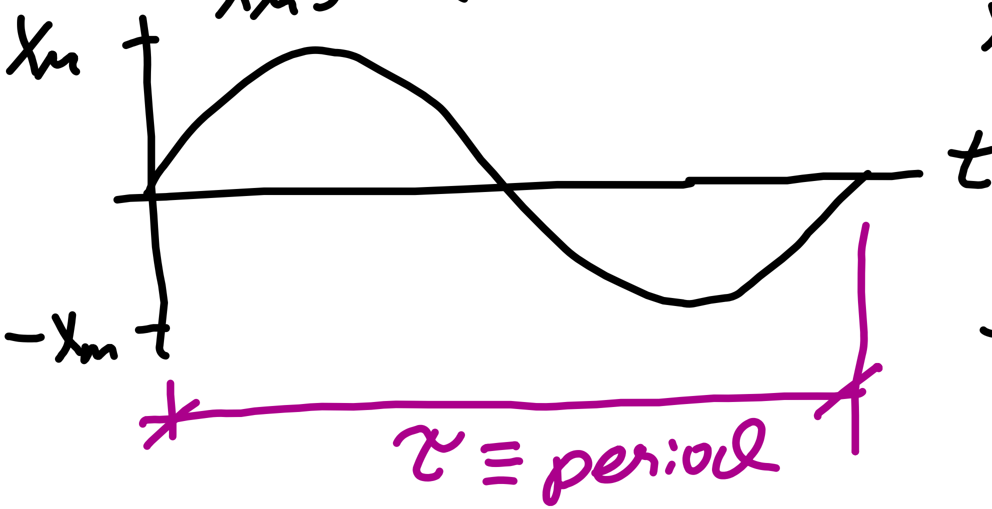




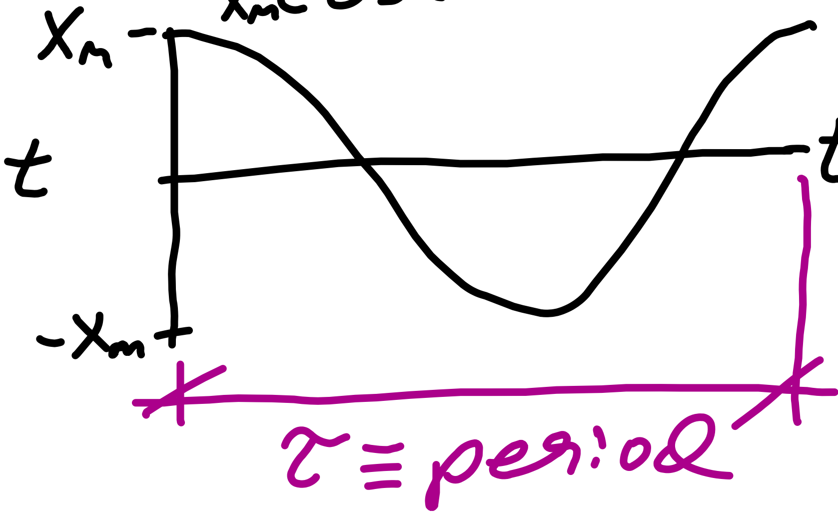
So when $t = \text{period} = \tau$

Wave anatomy

$x_m \sin(\omega t)$



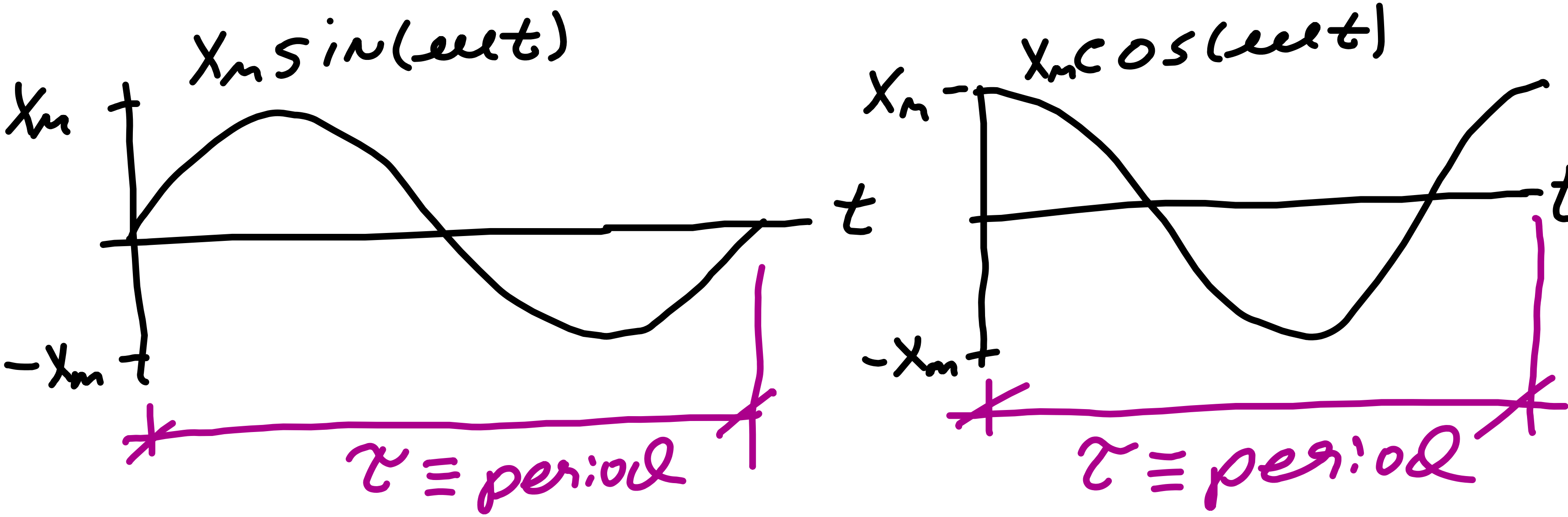
$x_m \cos(\omega t)$



so when $t = \text{period} = \tau$ then

$\omega t = 2\pi$

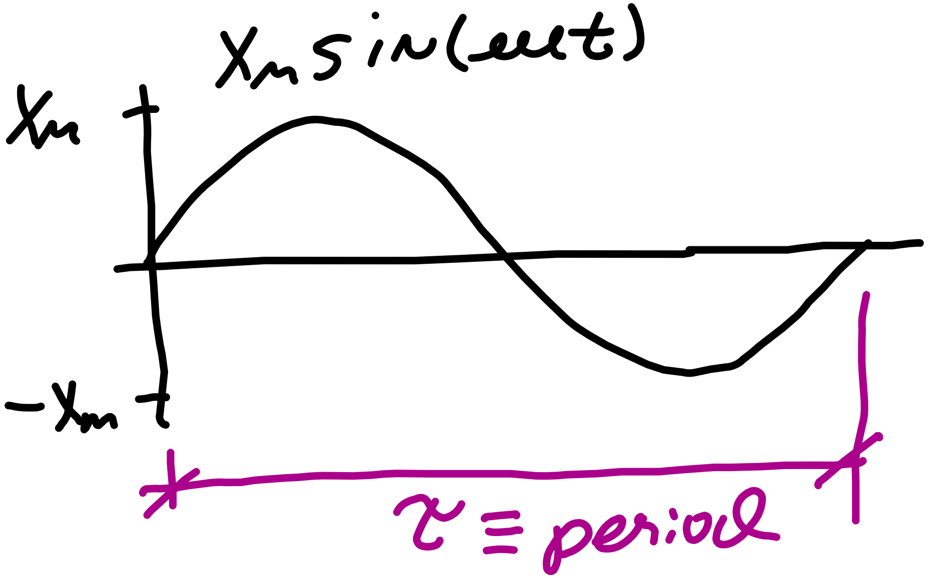
Wave anatomy

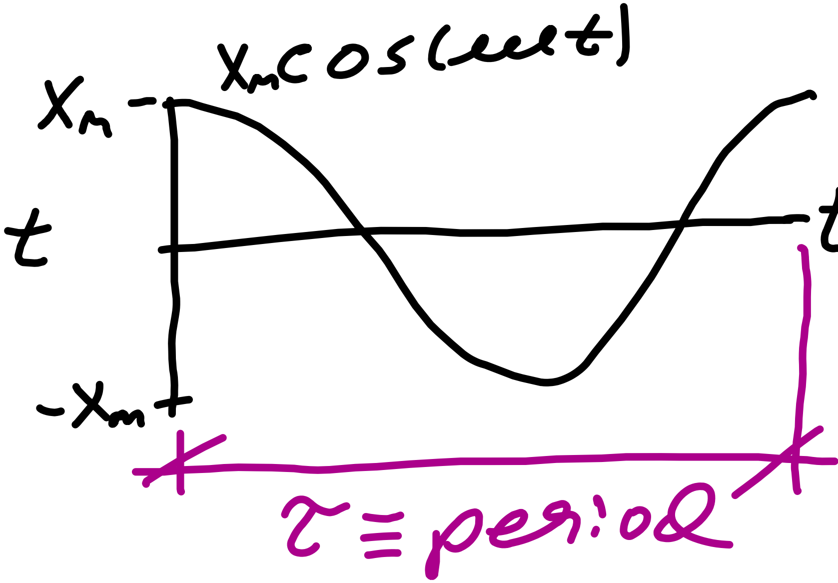


So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi$$

Wave anatomy

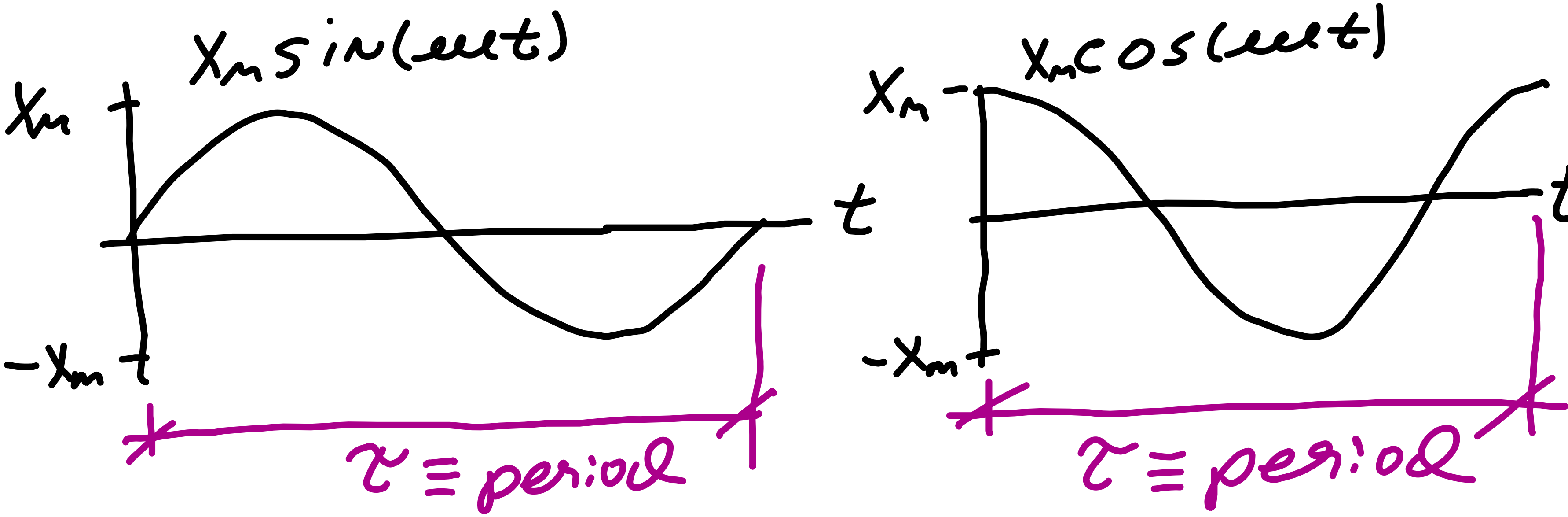




So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

Wave anatomy

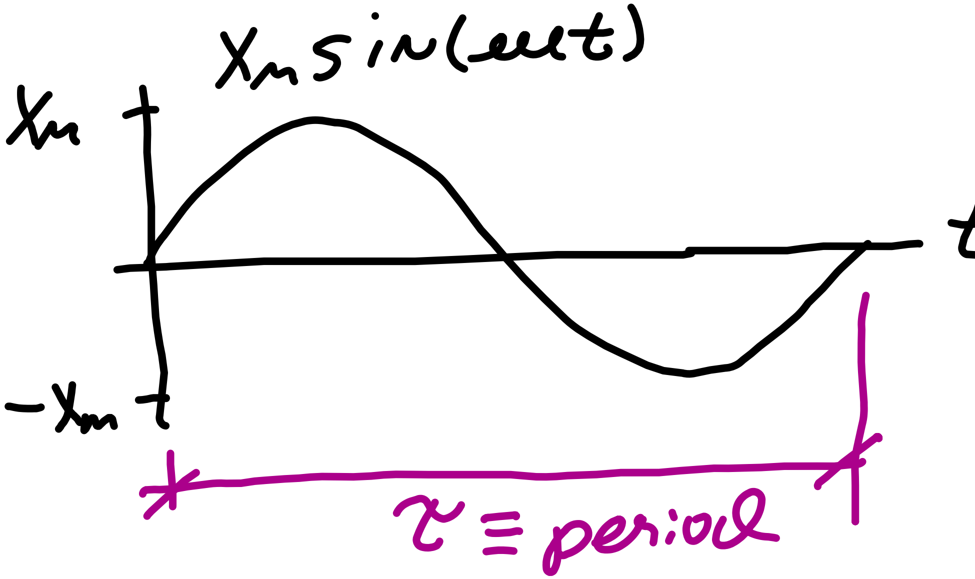


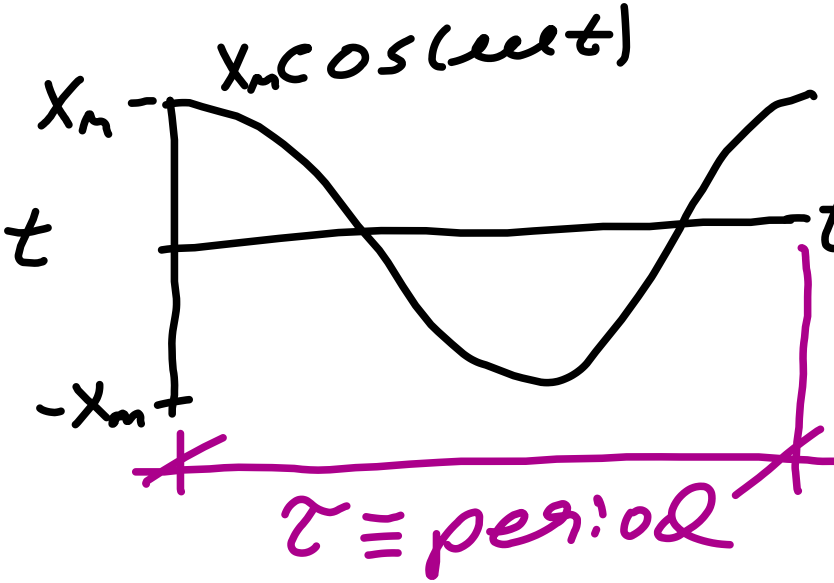
So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

$$\& \quad \omega = \frac{2\pi}{\tau}$$

Wave anatomy



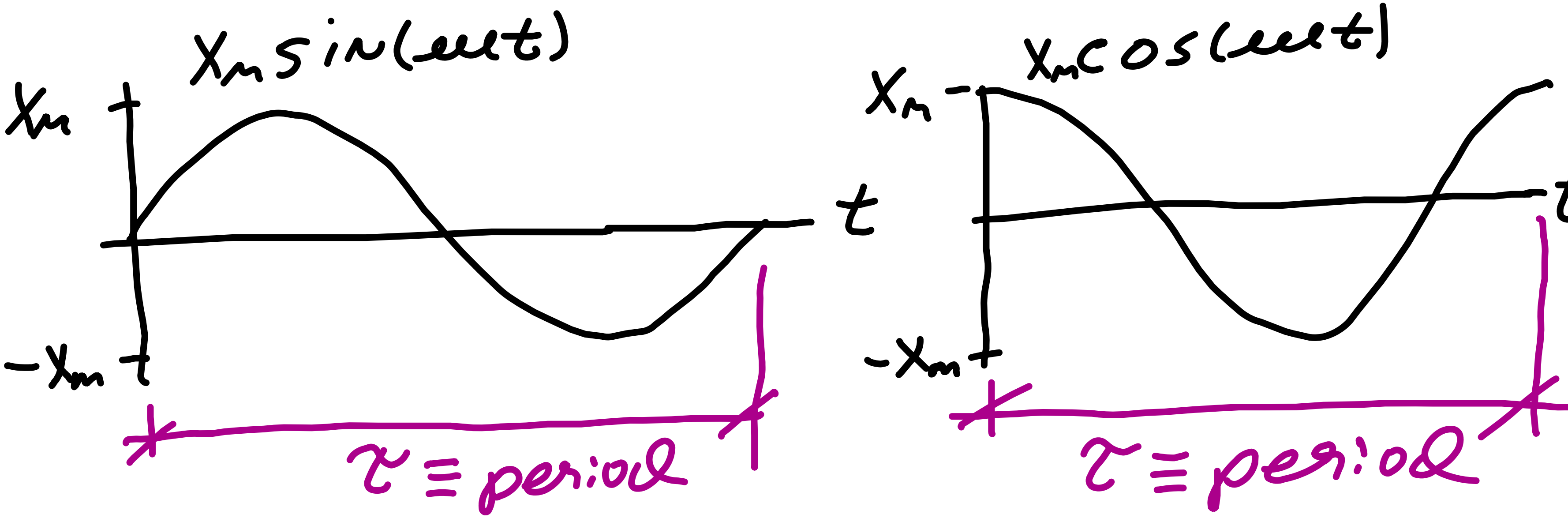


So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

& $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

Wave anatomy



So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

& $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

& $f \equiv \frac{1}{\tau}$

So far, the game has been
to analyze a given physical system

So far, the game has been to analyze a given physical system & get solution into the form

$$\ddot{x} = -\omega^2 x$$

So far, the game has been to analyze a given physical system & get solution into the form

$$\ddot{x} = -\omega^2 x$$

or

$$\ddot{\theta} = -\omega^2 \theta$$

New game

New game

Since $a = v\frac{\partial v}{\partial x}$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$,

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$v\,dv = -\omega^2 x\,dx$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$\int v\,dv = -\omega^2 \int x\,dx$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$\int v\,dv = -\omega^2 \int x\,dx \Rightarrow \frac{1}{2}v^2 + C_1 = -\omega^2 \frac{1}{2}x^2 + \omega^2 C_2$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$\int v\,dv = -\omega^2 \int x\,dx \Rightarrow \frac{1}{2}v^2 + C_1 = -\omega^2 \frac{1}{2}x^2 + \omega^2 C_2$$

or $\quad v^2 + \omega^2 x^2 = \text{const.}$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$

$\int v\,dv = -\omega^2 \int x\,dx \Rightarrow \frac{1}{2}v^2 + C_1 = -\omega^2 \frac{1}{2}x^2 + \omega^2 C_2$

<u>or</u> $v^2 + \omega^2 x^2 = \text{const.}$

<u>or</u> $\dot{x}^2 + \omega^2 x^2 = \text{const.}$

Old game :

Old game:

Get problem into the form

$$\ddot{x} = -\omega^2 x$$

Old game:

Get problem into the form

$$\ddot{x} = -\omega^2 x$$ using forces

& moments

Old game:

Get problem into the form

$\ddot{x} = -\omega^2 x$ using forces & moments

New game:

Old game:

Get problem into the form

$\ddot{x} = -\omega^2 x$ using forces & moments

New game:

Get problem into the form

$\dot{x}^2 + \omega^2 x^2 = \text{constant}$

Old game:

Get problem into the form

$$\ddot{x} = -\omega^2 x$$ using forces & moments

New game:

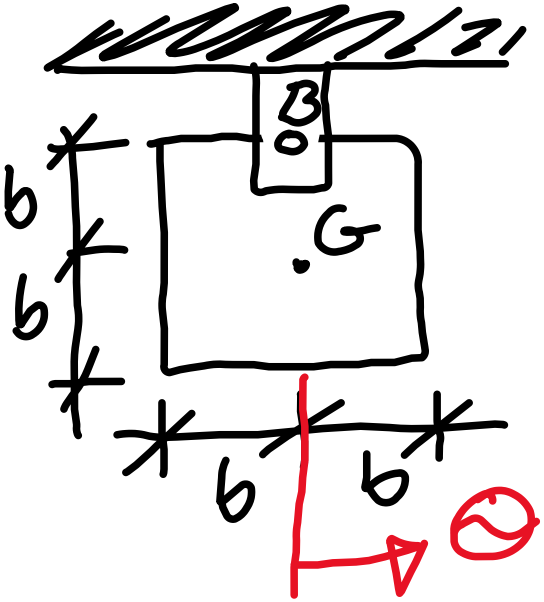
Get problem into the form

$$\dot{x}^2 + \omega^2 x^2 = \text{constant}$$

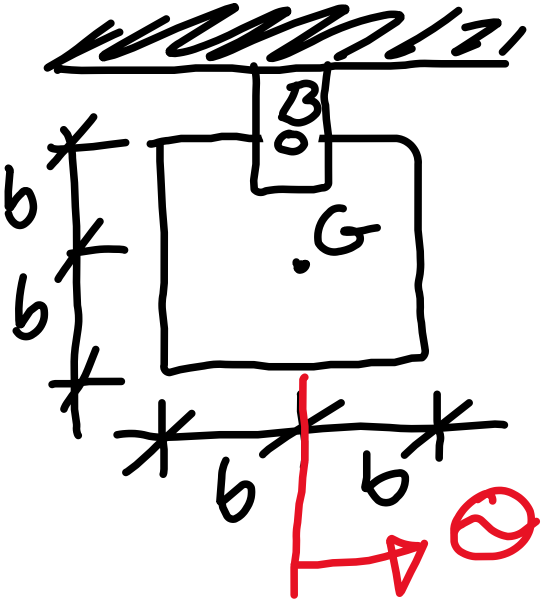
using energy conservation

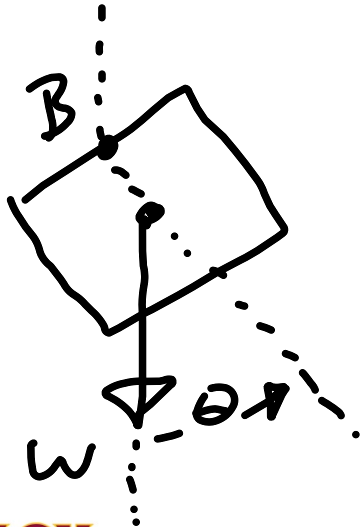
$$T + V = \text{constant}.$$

Old style

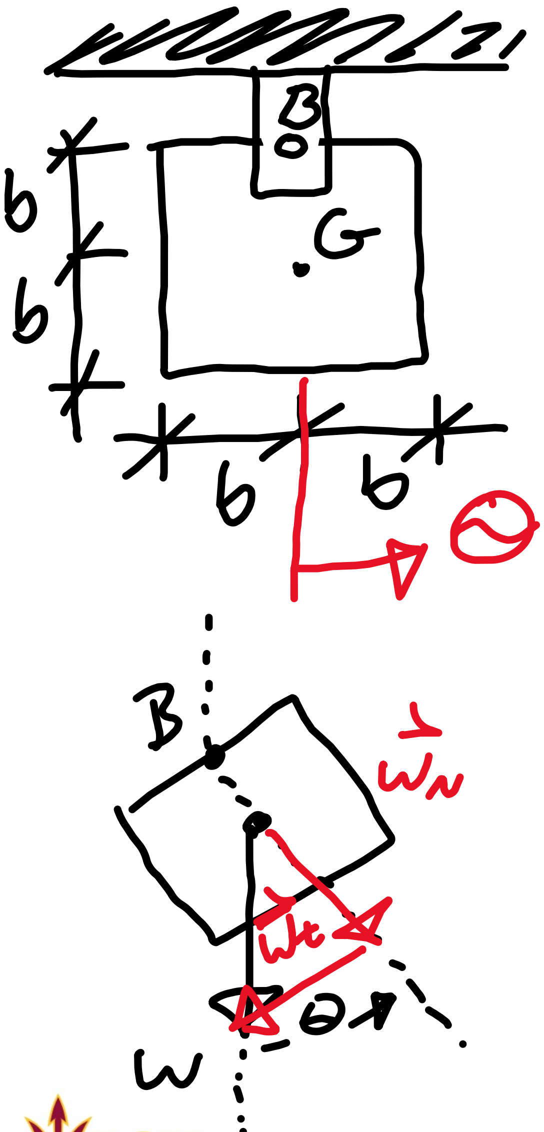


Old style



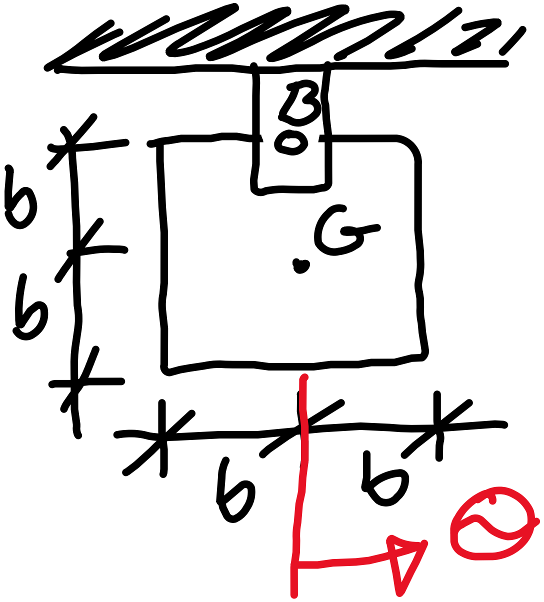


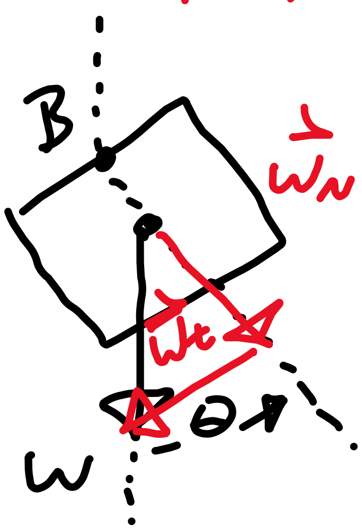
Old style



Old style

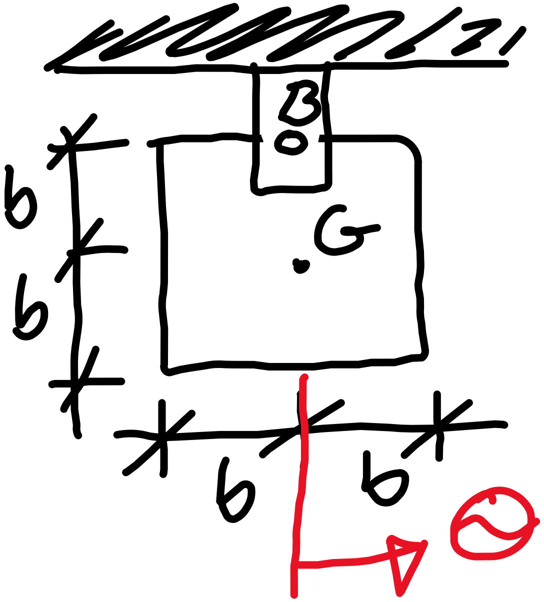
$$w_N = -W\cos\theta$$

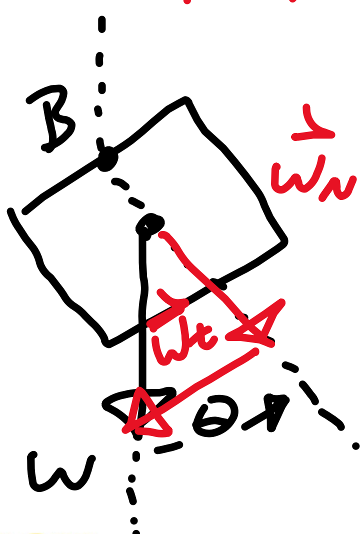




Old style

$w_N = -W\cos\theta$ & $w_t = -w\sin\theta$

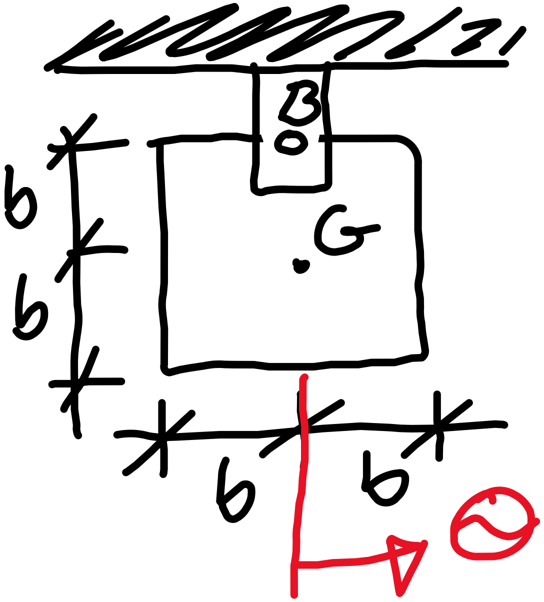


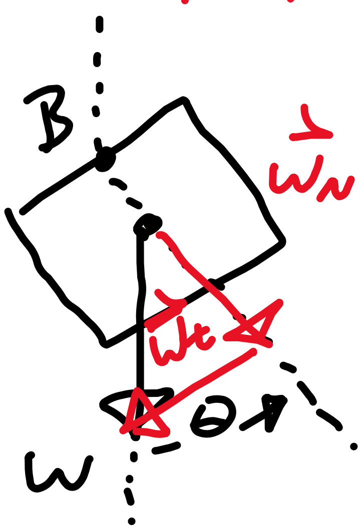


Old style

$$w_N = -W\cos\theta \;\&\; w_t = -w\sin\theta$$

$$\sum M_B = I_B\theta$$

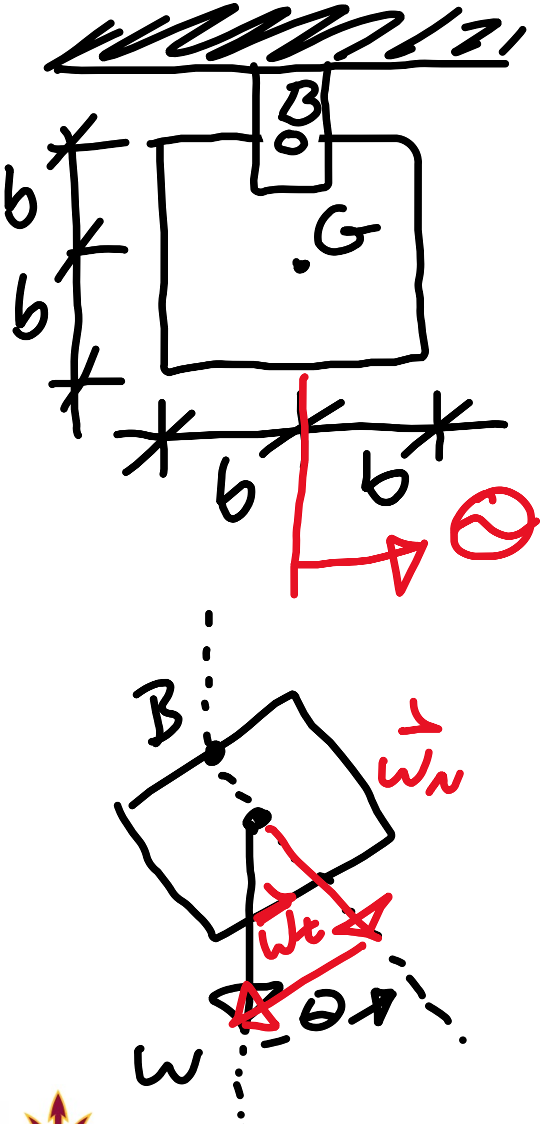




Old style

$W_N = -W\cos\theta$ & $W_t = -W\sin\theta$

$\circlearrowleft \sum M_B = I_B \theta \Rightarrow bW_t = I_B \theta$

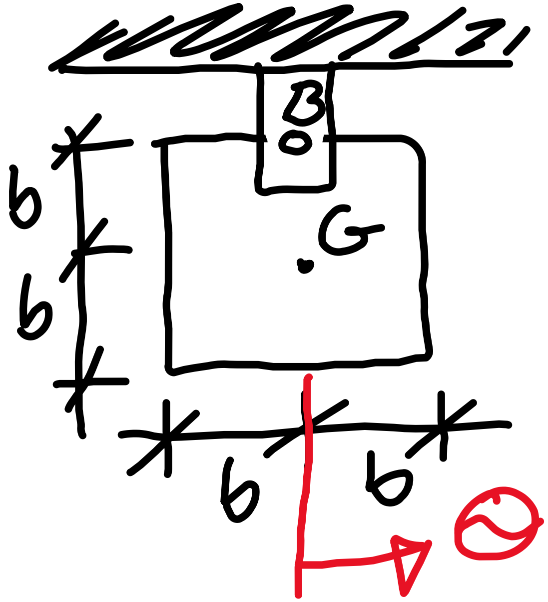


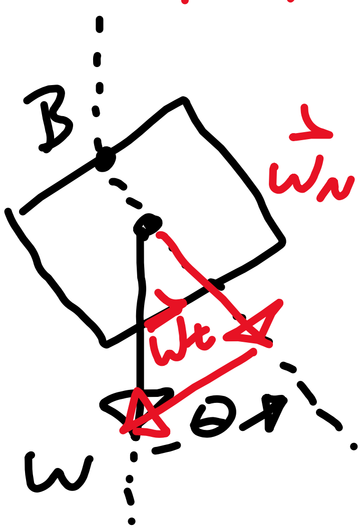
Old style

$$w_N = -W\cos\theta \;\&\; w_t = -W\sin\theta$$

$$\circlearrowleft \sum M_B = I_B\ddot{\theta} \Rightarrow b w_t = I_B\ddot{\theta}$$

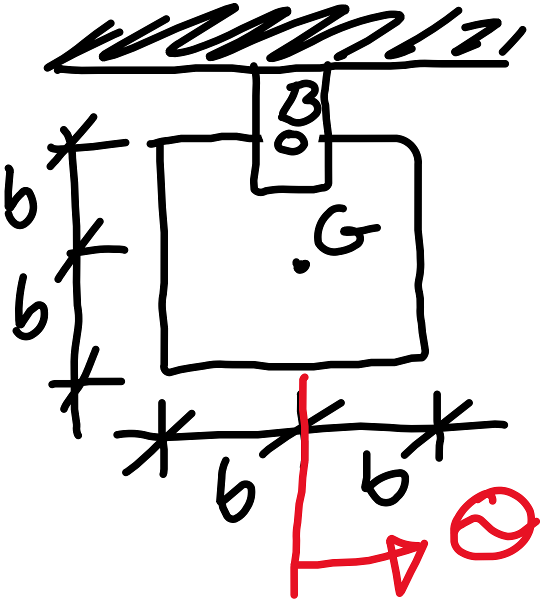
$$\Rightarrow -bW\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$$





Old style

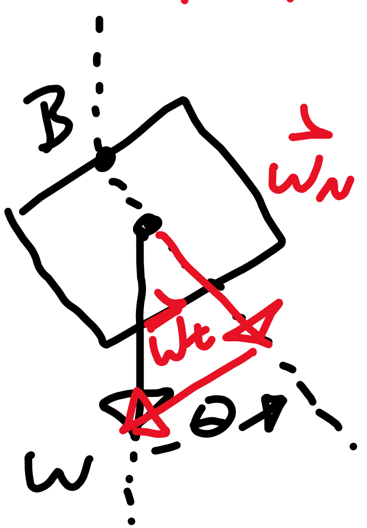
$w_N = -W\cos\theta$ & $w_t = -W\sin\theta$



$\curvearrowleft \sum M_B = I_B \ddot{\theta} \Rightarrow b w_t = I_B \ddot{\theta}$

$\Rightarrow -bW\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$

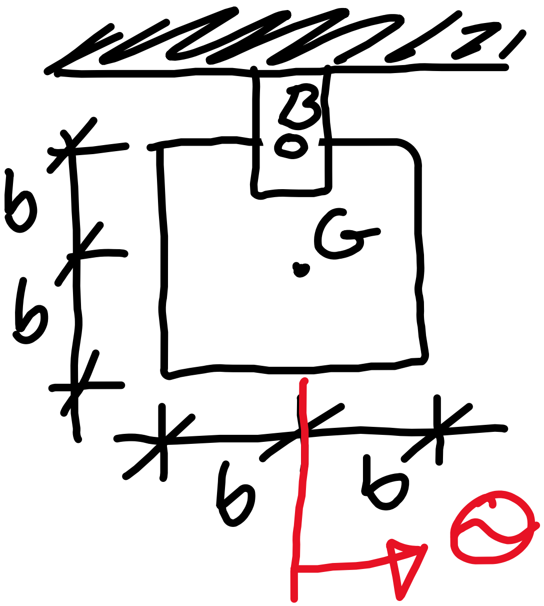
$\Rightarrow -bW\theta \cong (\bar{I} + mb^2)\ddot{\theta}$



ASU

Old style

$W_N = -W\cos\theta$ & $W_t = -W\sin\theta$

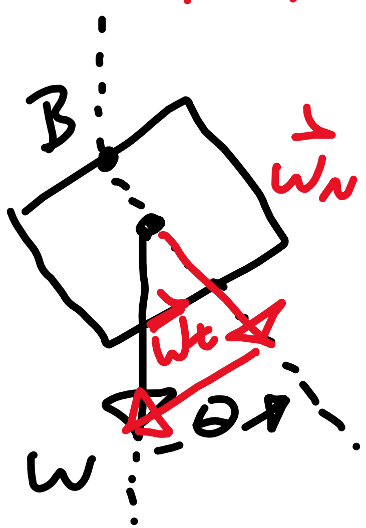


$\circlearrowleft \sum M_B = I_B\theta \Rightarrow bW_t = I_B\theta$

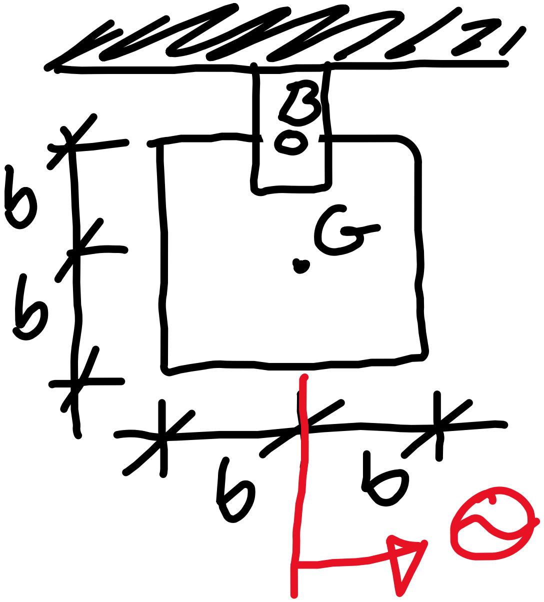
$\Rightarrow -bW\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$

$\Rightarrow -bW\theta \cong (\bar{I} + mb^2)\ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega^2\theta$



Old style

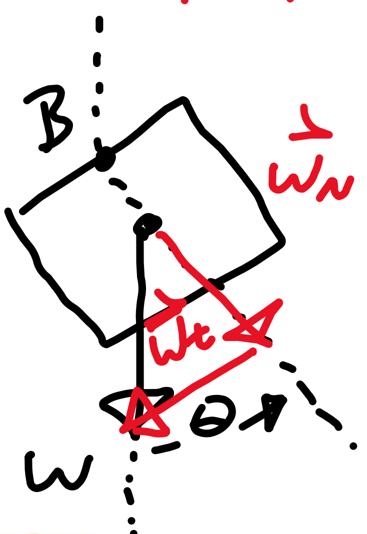


$$w_N = -W\cos\theta \ \ \& \ \ w_t = -w\sin\theta$$

$$\curvearrowleft \sum M_B = I_B\ddot{\theta} \Rightarrow b w_t = I_B\ddot{\theta}$$

$$\Rightarrow -bw\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$$

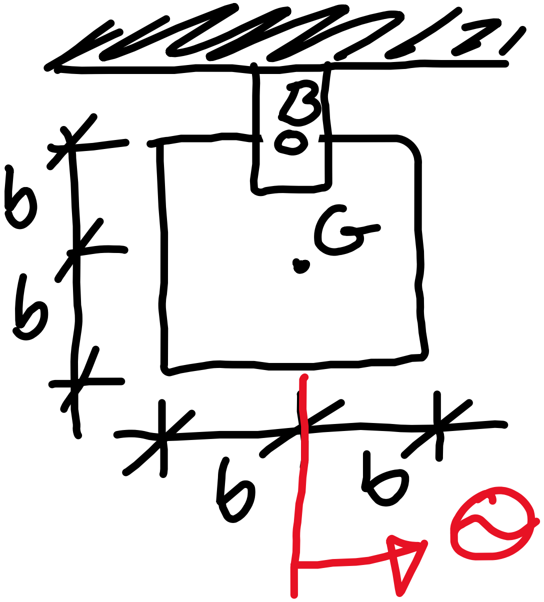
$$\Rightarrow -bw\theta \cong (\bar{I} + mb^2)\ddot{\theta}$$



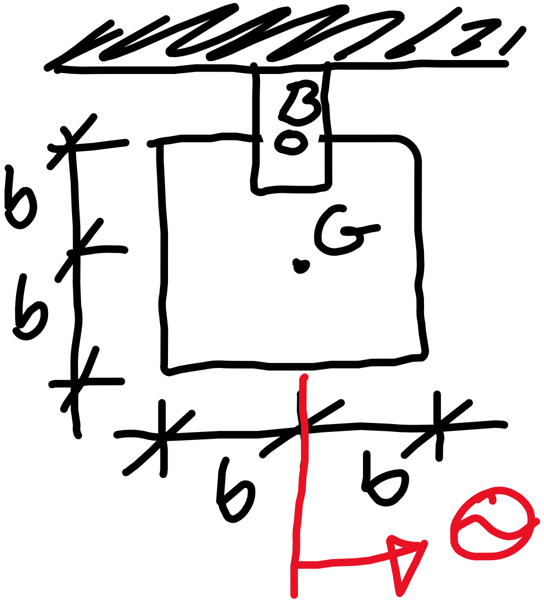
$$\Rightarrow \ddot{\theta} = -\omega^2\theta \text{, where}$$

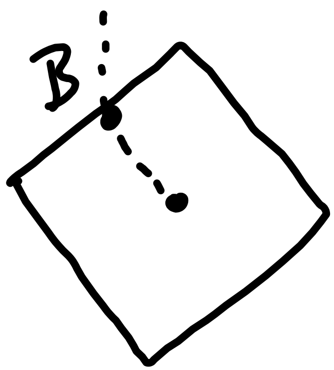
$$\omega = \sqrt{\frac{bW}{\bar{I} + mb^2}}$$

New style

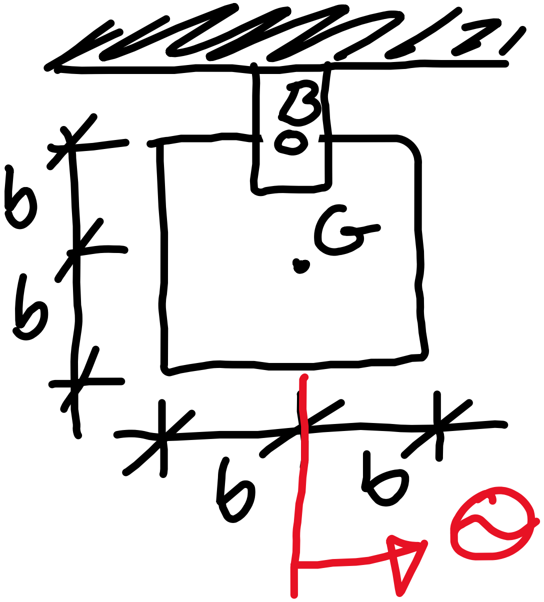


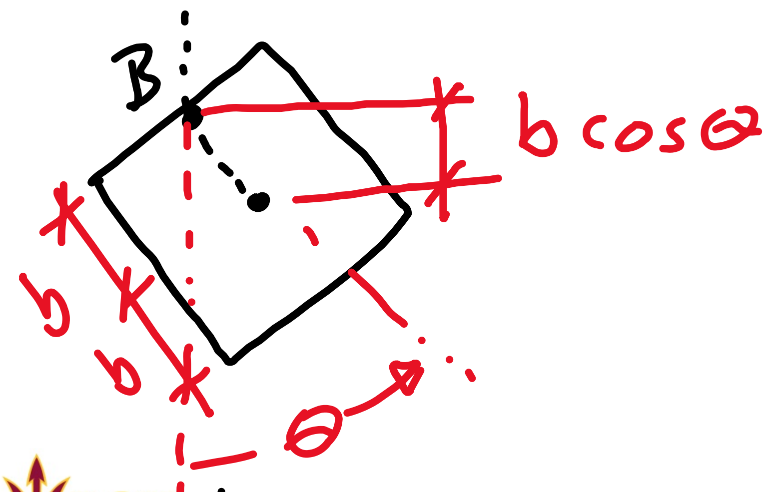
New style





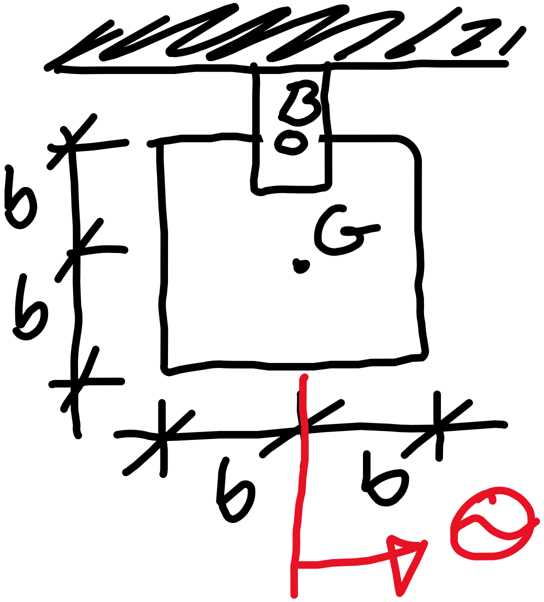
New style

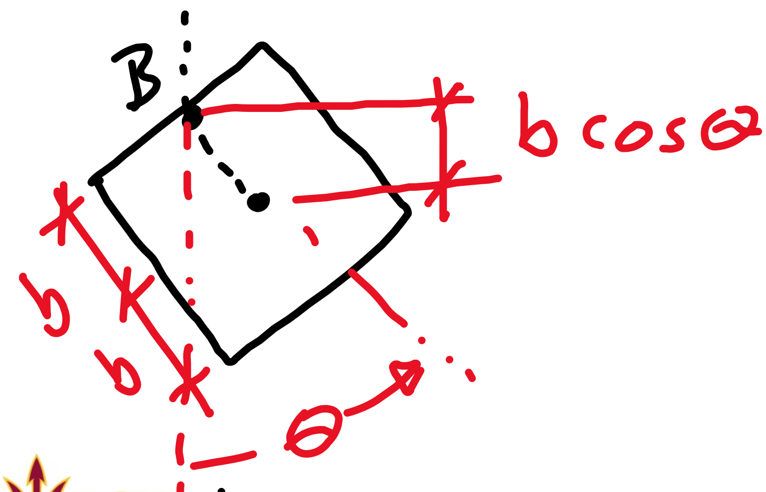




New style

$T + V = \text{const.}$

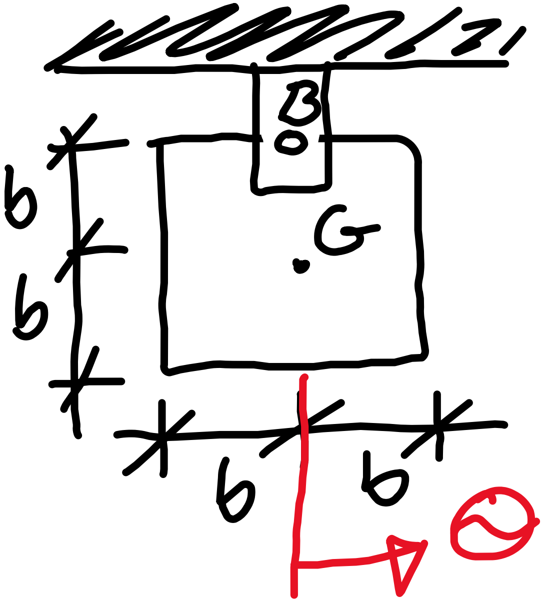


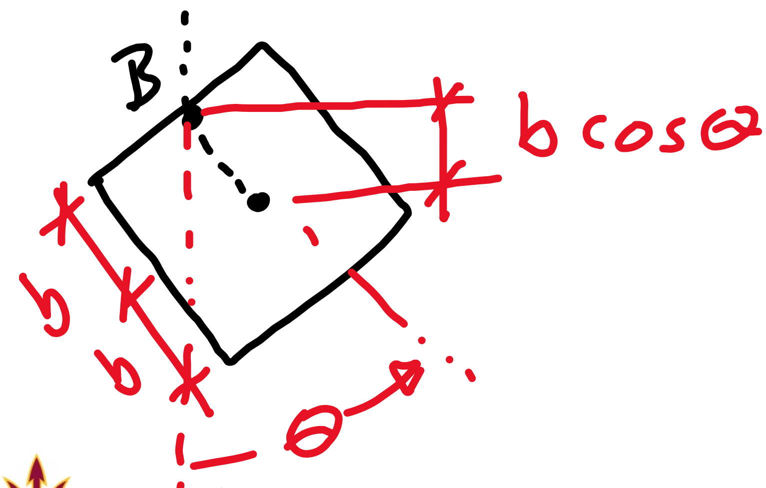


New style

$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$

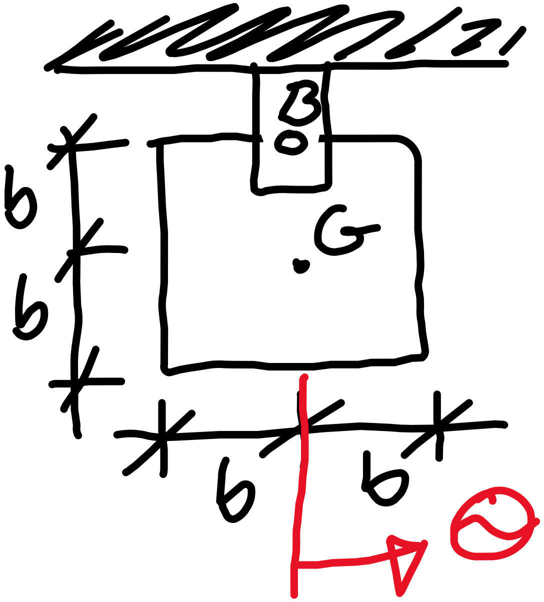


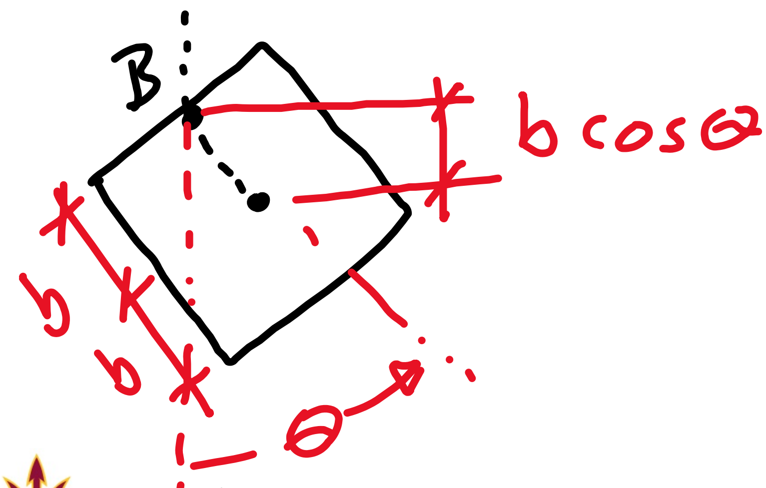


New style

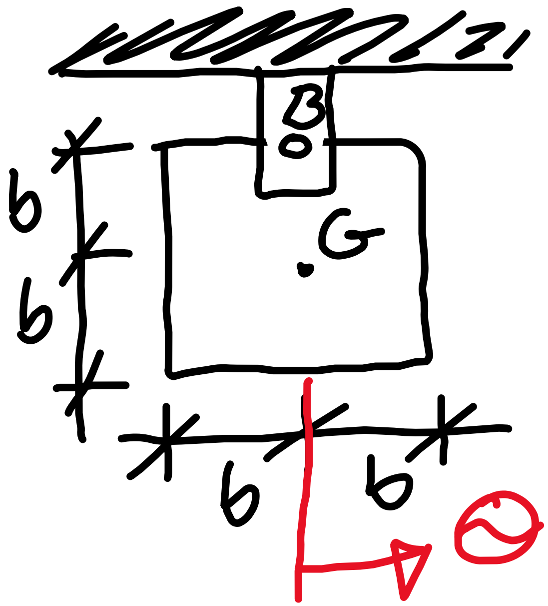
$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$





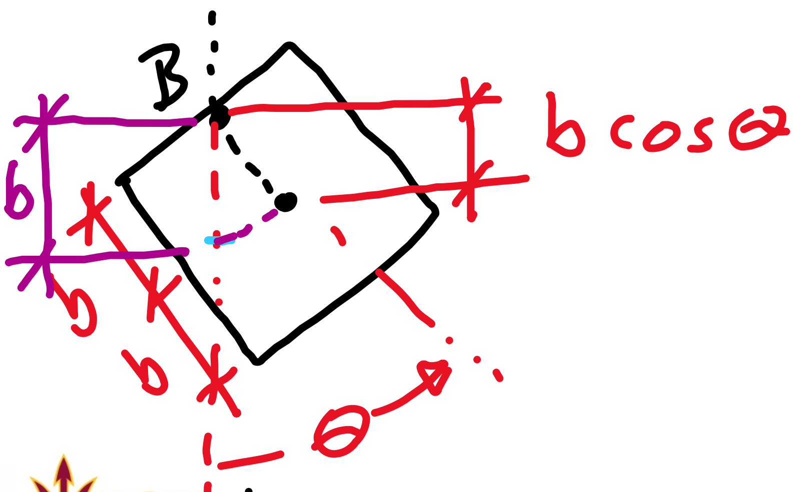
New style



$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$

but $h = b -$

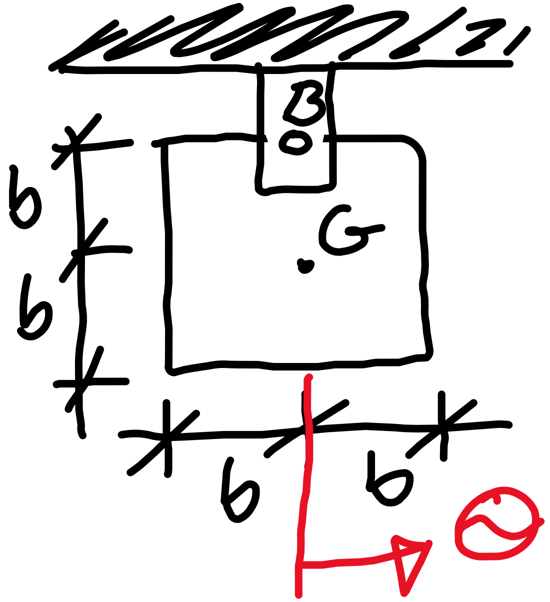


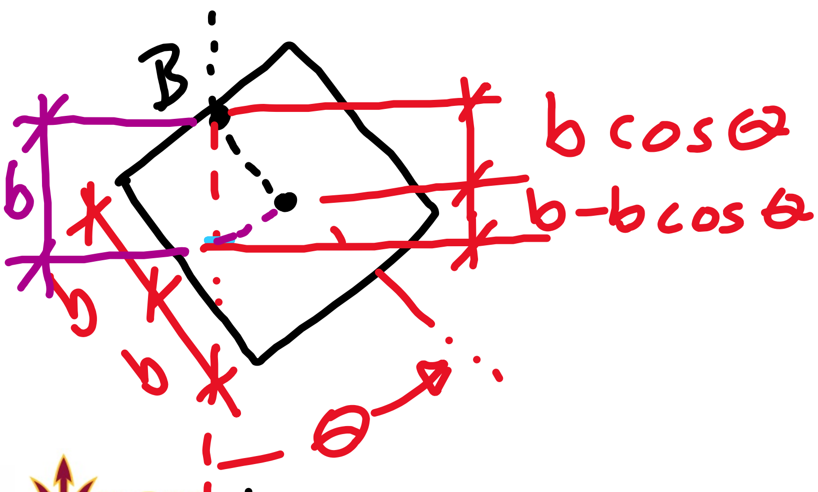
New style

$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$





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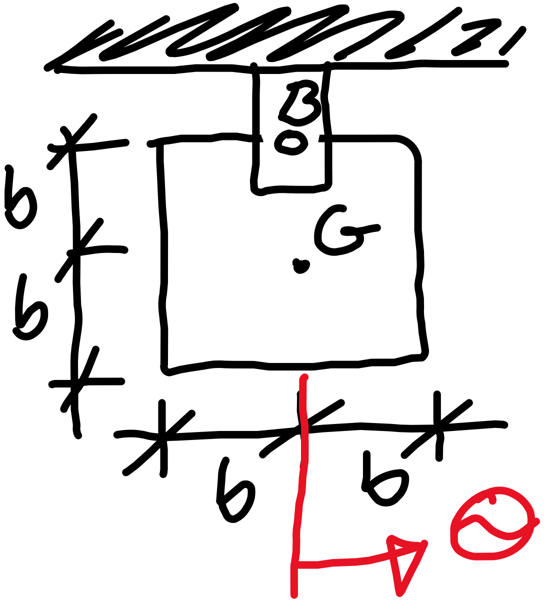
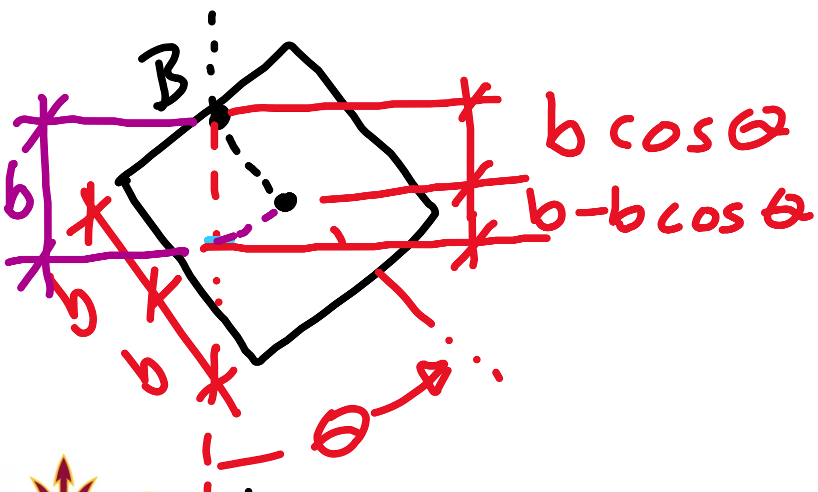


$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$



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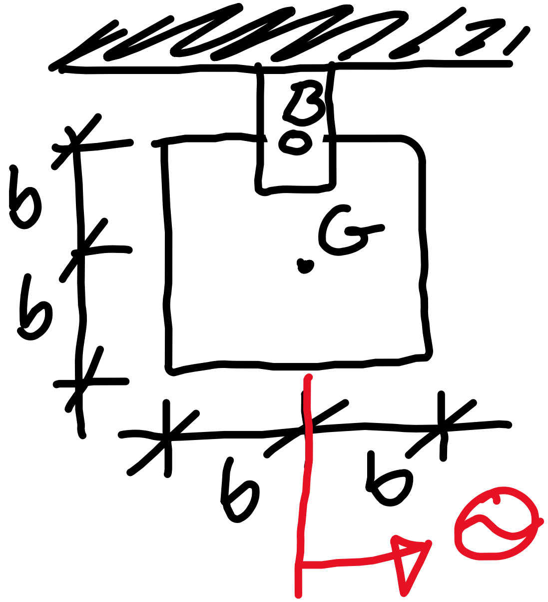
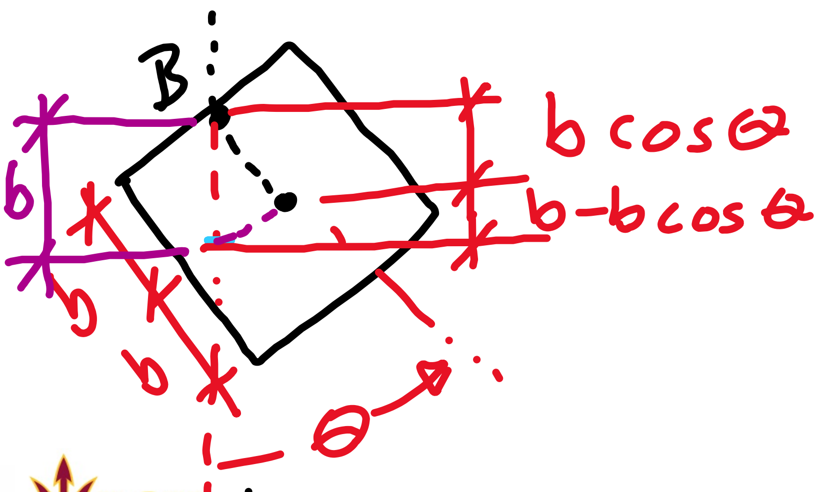


but $h = b - b\cos\theta$

$= 1 - \cos\theta$

But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$



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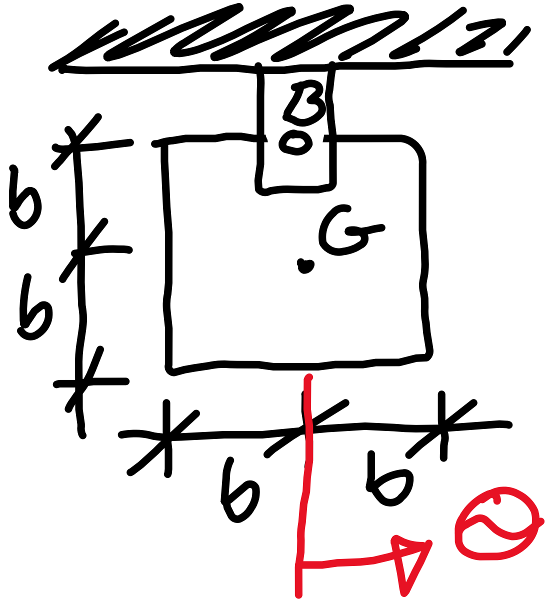
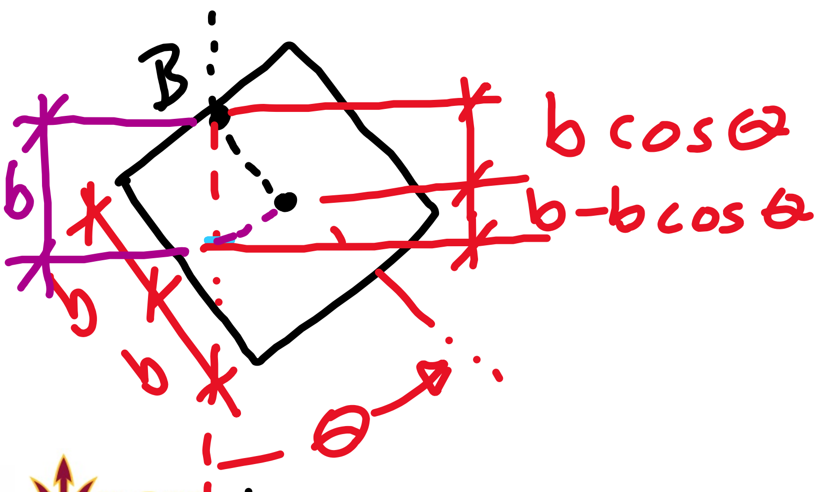
But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

so $h \cong \theta^2/2$





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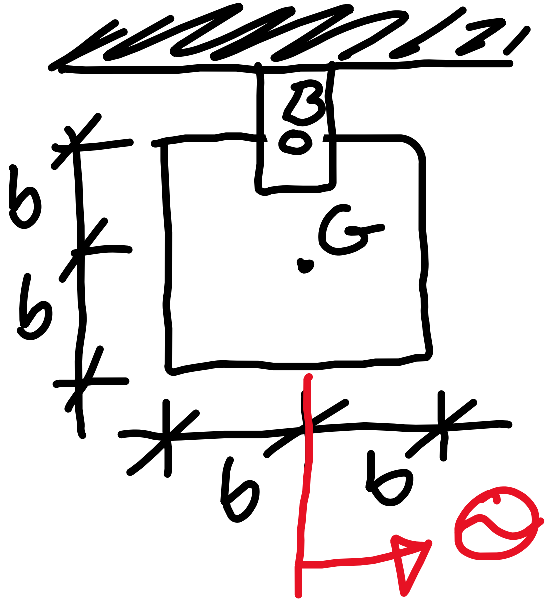

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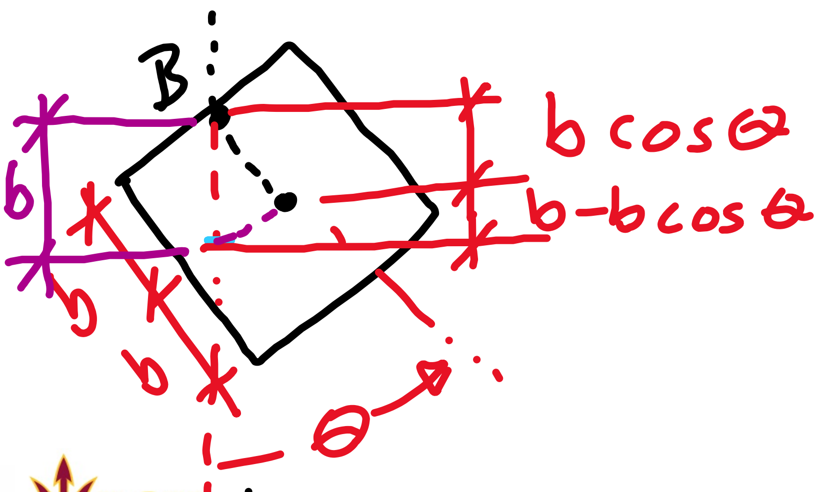
$V = mg\,\theta^2/2$



New style

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But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

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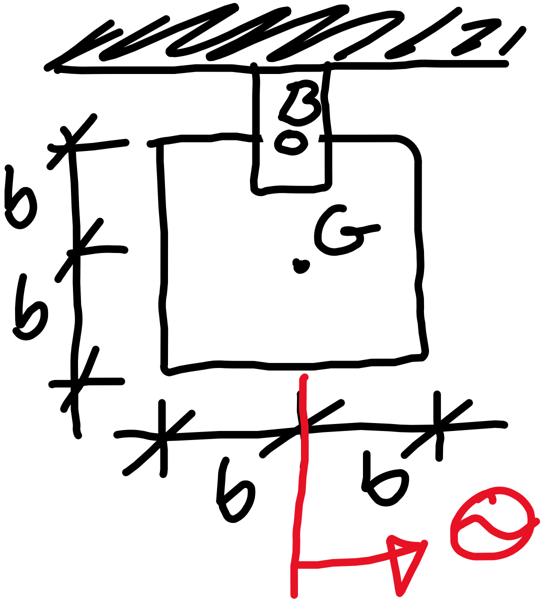
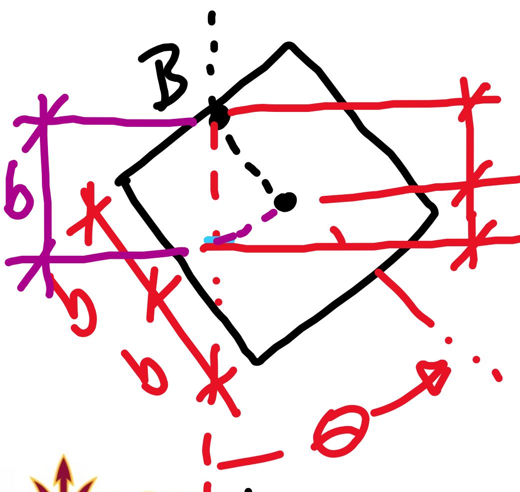
$V = mg\,\theta^2/2 \Rightarrow$

$\frac{1}{2} I_B \omega^2 + mg\,\theta^2/2 = \text{const.}$





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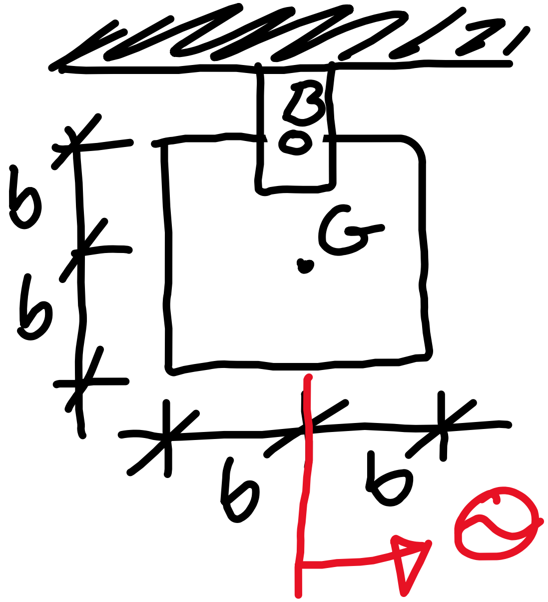

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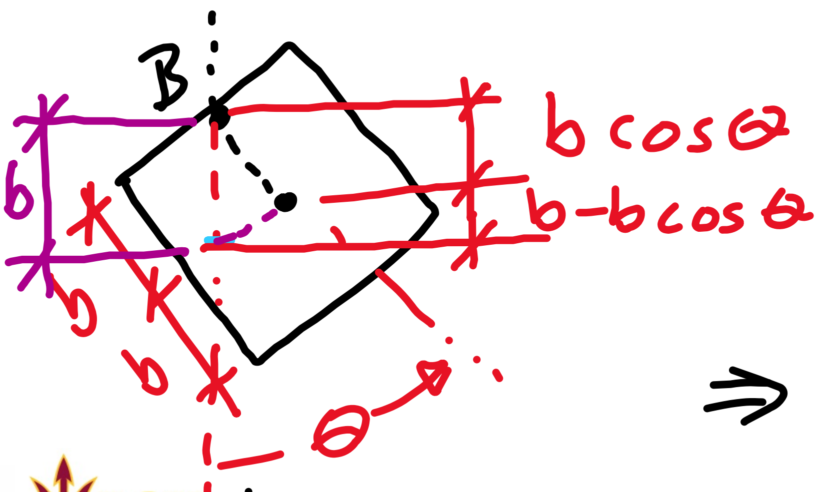
$\frac{1}{2} I_B \omega^2 + mg\,\theta^2/2 = \text{const.}$

$\Rightarrow \dot{\theta}^2 + \omega^2 \theta^2 = \text{const.}$



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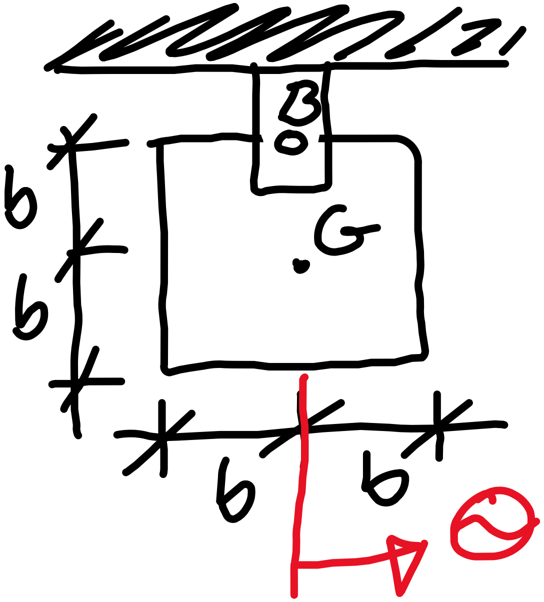

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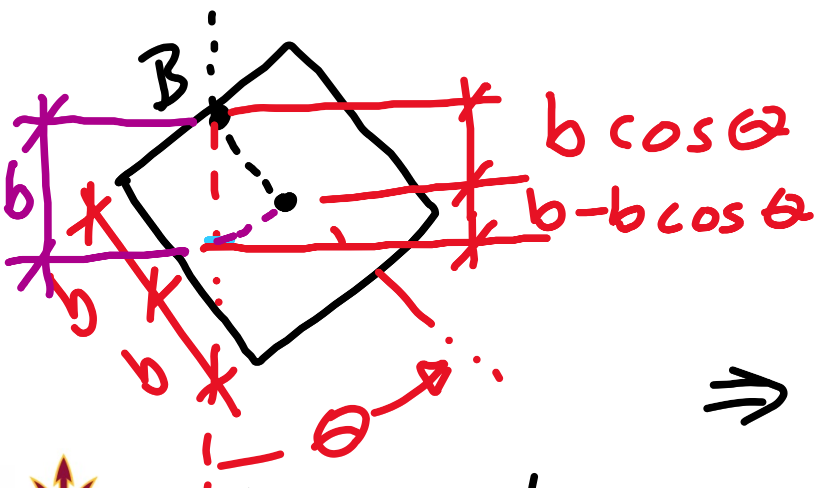
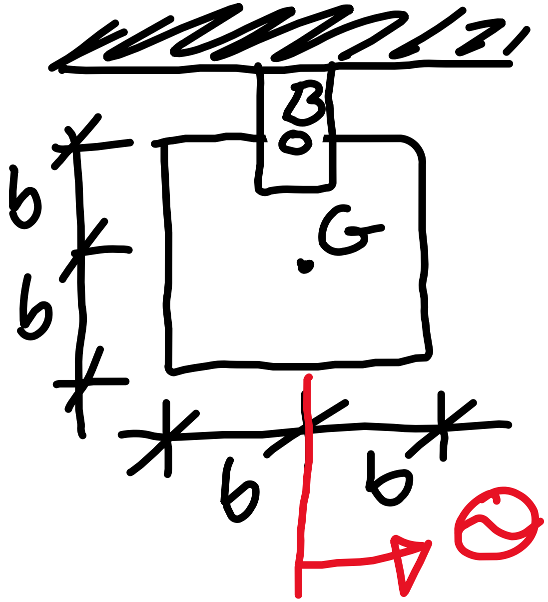
$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{const.},$



where $\omega = \sqrt{\frac{Mg}{I_B}}$

New style



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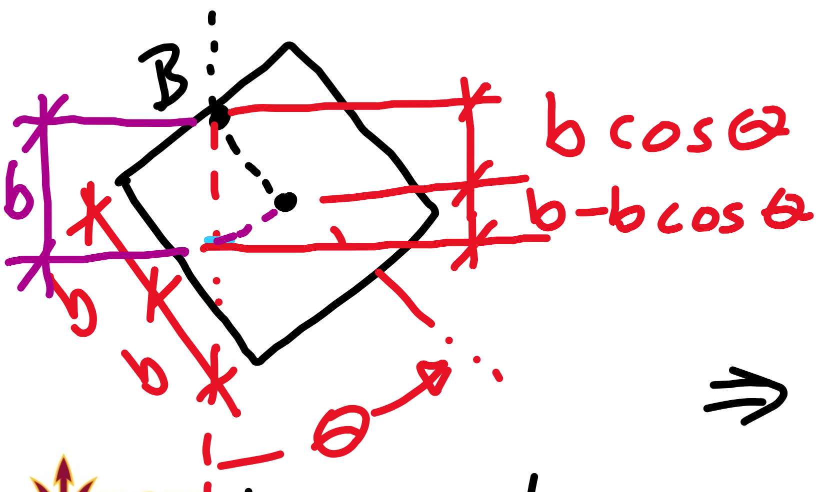
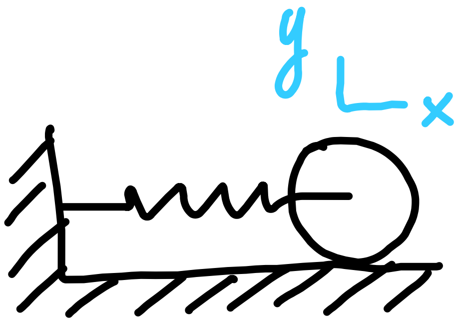
$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{const.},$



where $\omega = \sqrt{\frac{mg}{I_B}} = \sqrt{\frac{W}{I + mb^2}}$

Old style

Old style



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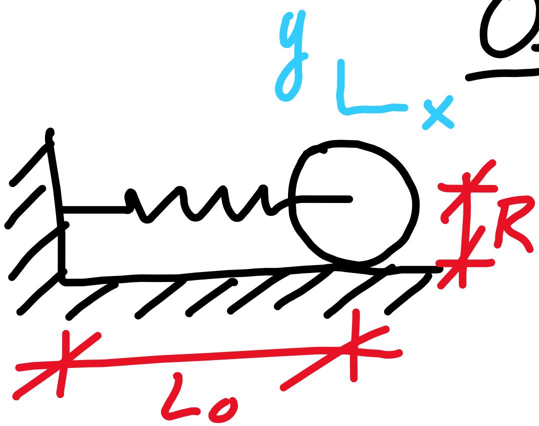

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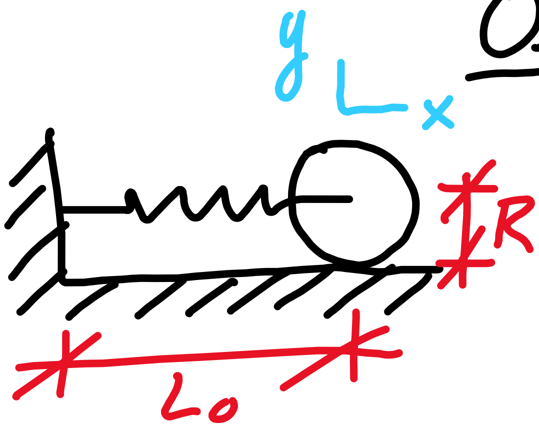
move forward



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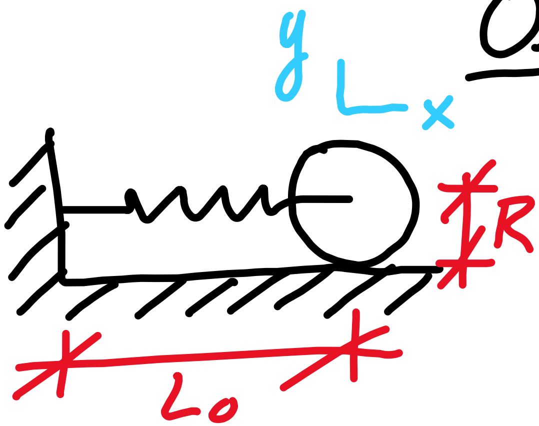

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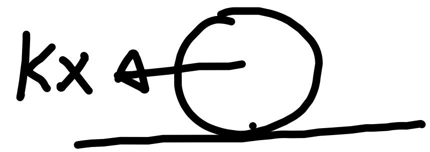

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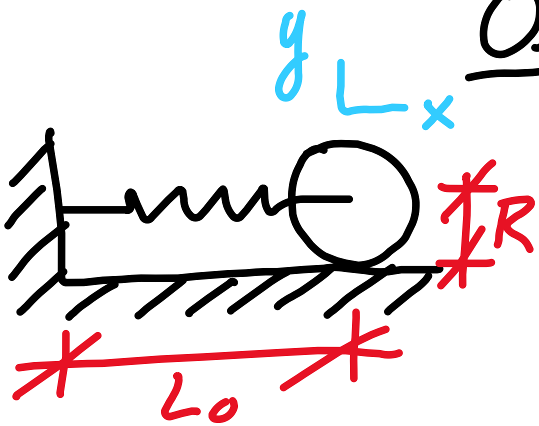

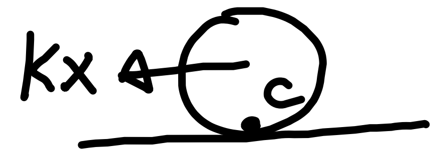

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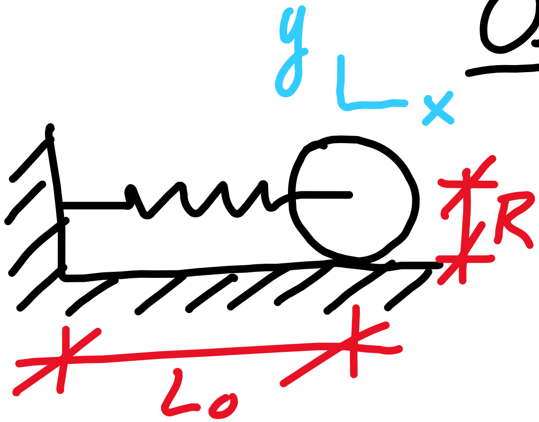

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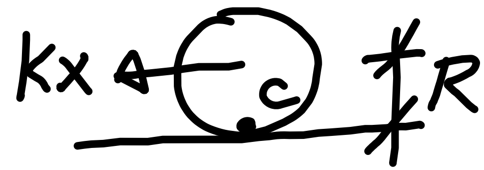

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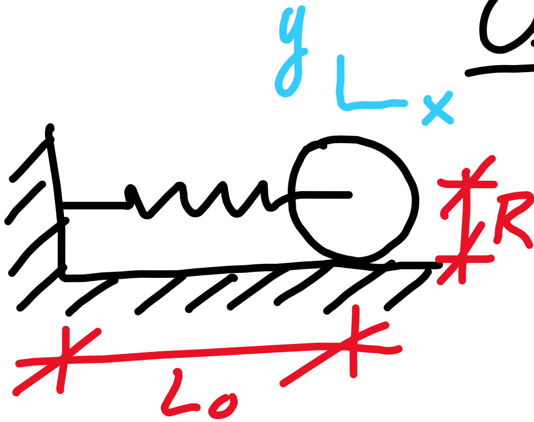

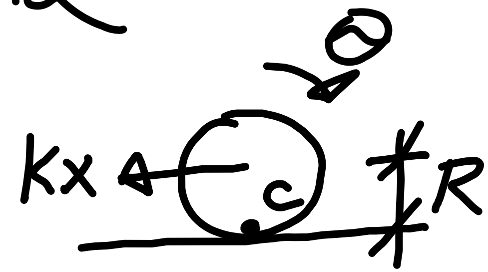

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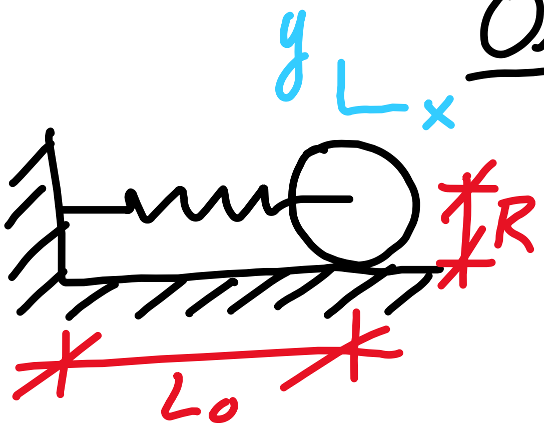
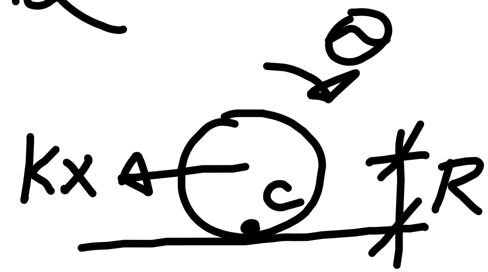
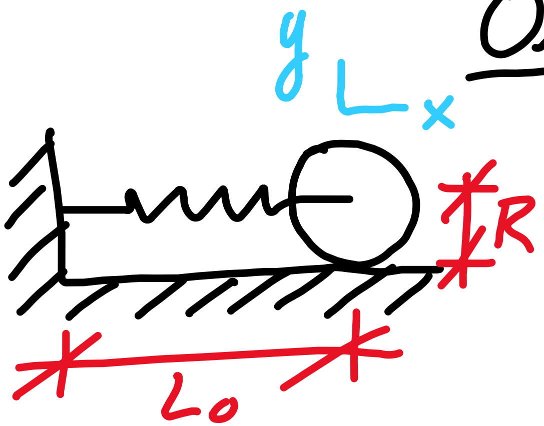


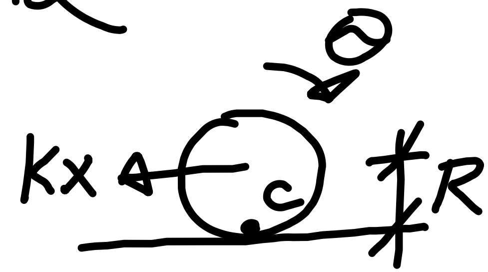


$$\curvearrowleft \sum M_c = I_c \ddot{\theta}$$

Old style

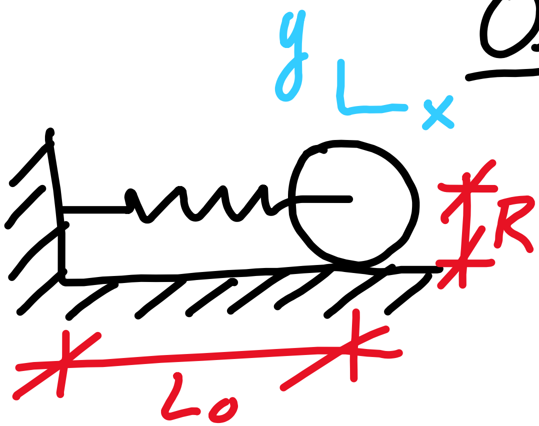
move forward





$$\circlearrowleft \sum M_c = I_c \ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta}$$

Old style



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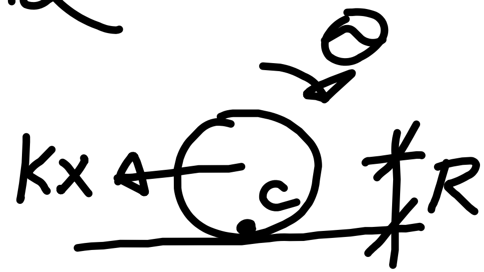
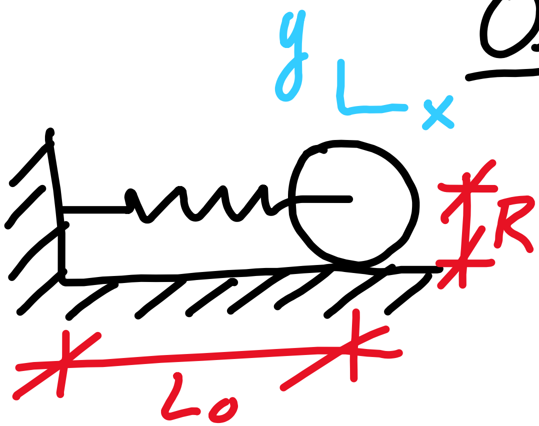


$$\sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + mR^2)\ddot{\theta}$$ But

$$x = R\theta$$

Old style



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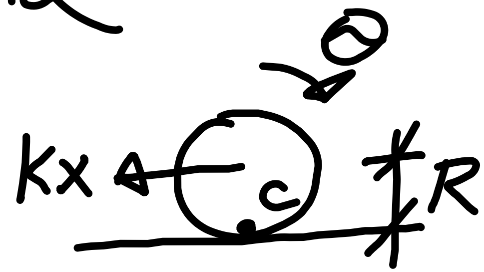
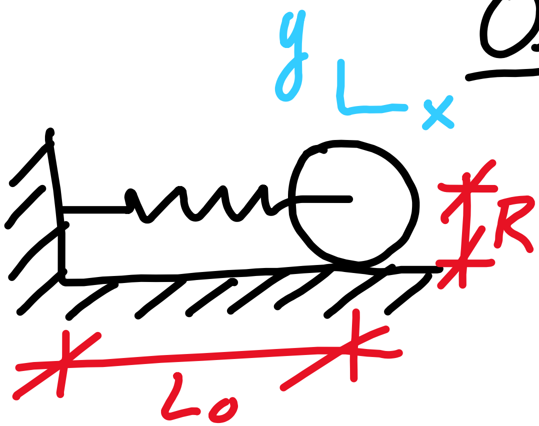


$$\sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

$$x = R\theta \quad \text{so} \quad -KR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$$

Old style



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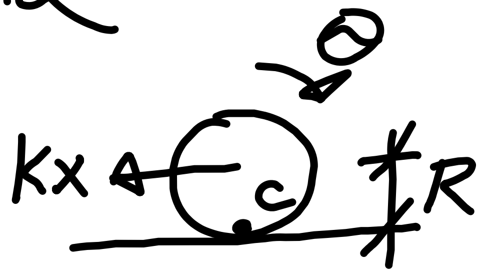
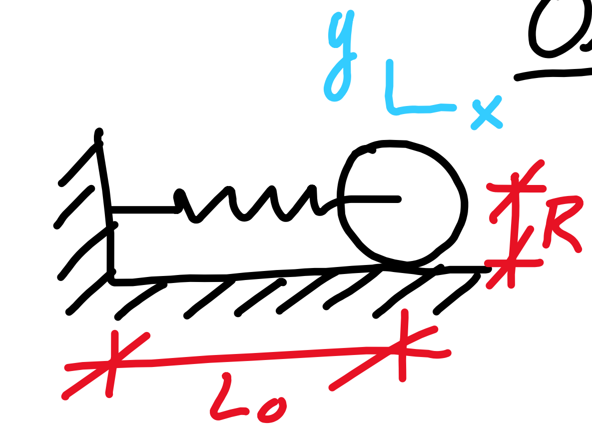


$$\sum M_c = I_c \ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta} \quad \underline{\text{or}}$$

$$\ddot{\theta} = -\omega^2\theta ,$$

Old style



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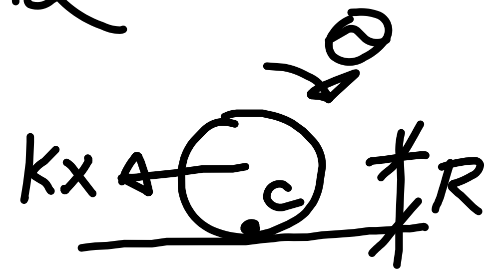
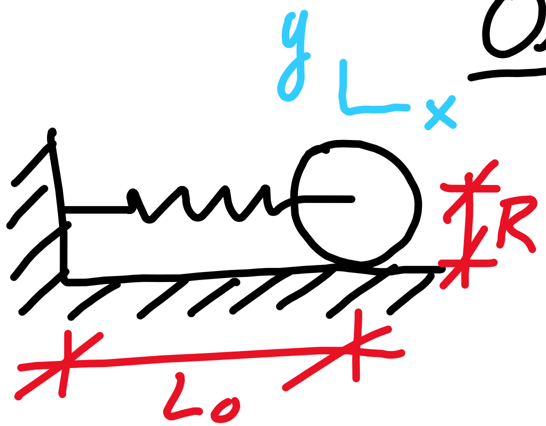


$$\sum M_c = I_c\ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$$ But

$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$$ or

$$\ddot{\theta} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{kR^2}{\bar{I} + mR^2}}$$

Old style



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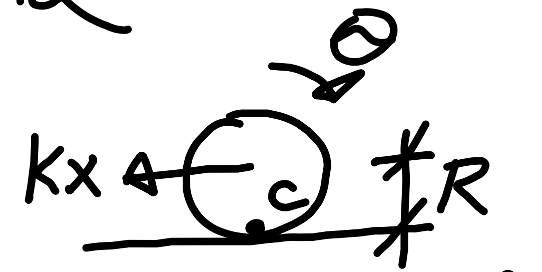

$$\circlearrowleft \sum M_c = I_c\ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

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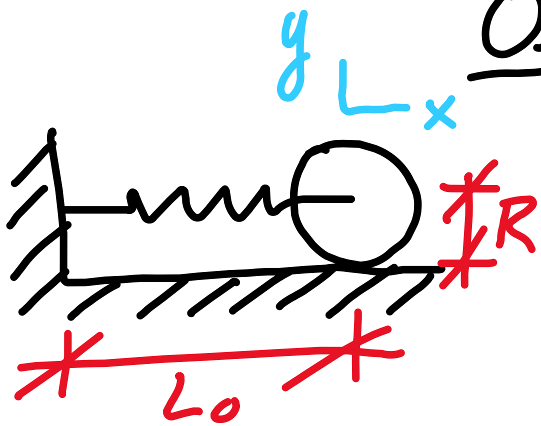
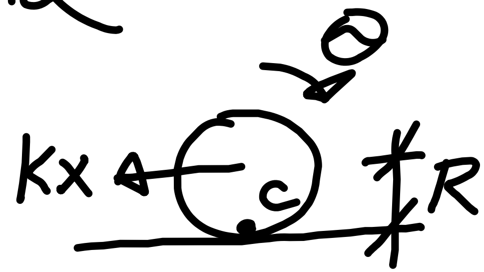
New style

Old style

move forward





$$\sum M_c = I_c \ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$$ But

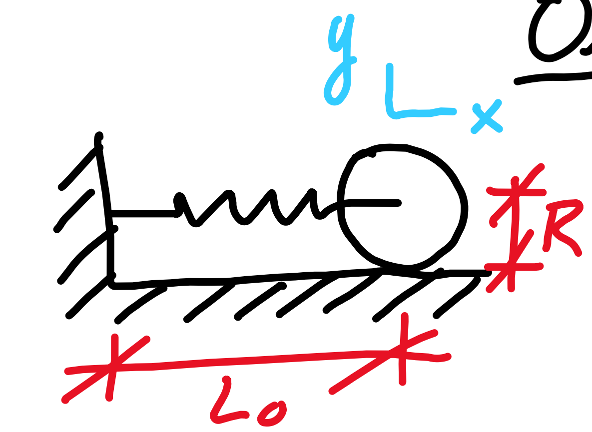
$x = R\theta$ so $-kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$$\ddot{\theta} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{kR^2}{\bar{I} + mR^2}}$$

New style

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

Old style



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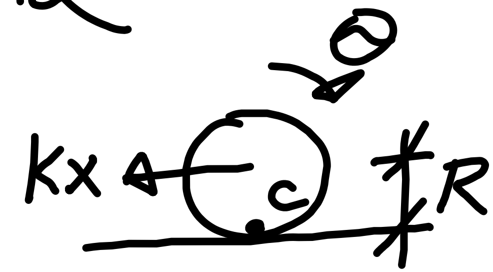


$$\sum M_c = I_c \ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$$ But

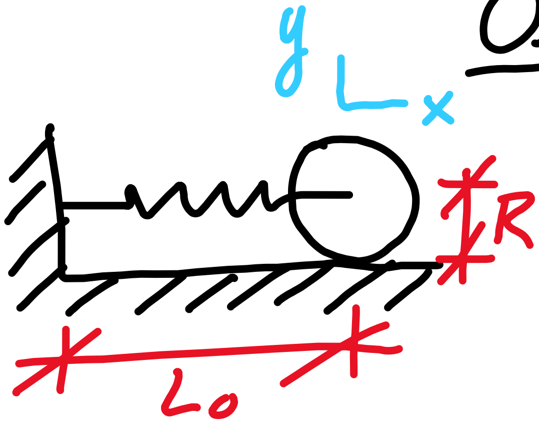
$x = R\theta$ so $-kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$$\ddot{\theta} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{kR^2}{\bar{I} + mR^2}}$$

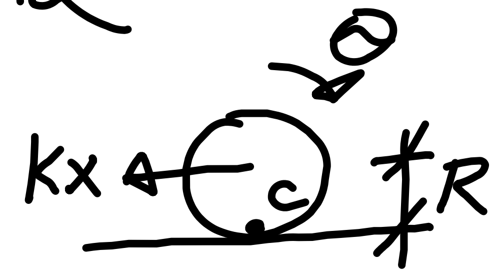
New style

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2$$

Old style



move forward



$$\sum M_c = I_c\ddot{\theta} \Rightarrow -KxR = (\bar{I}+MR^2)\ddot{\theta}$$ But

$$x = R\theta$$ so $$-KR^2\theta = (\bar{I}+MR^2)\ddot{\theta}$$ or

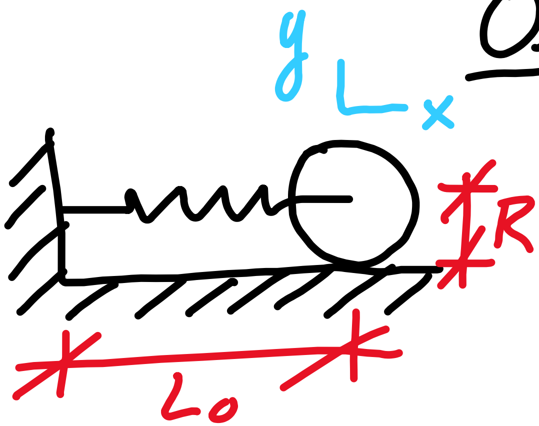
$$\ddot{\theta} = -\omega^2\theta, \quad \text{where } \omega = \sqrt{\frac{KR^2}{\bar{I}+MR^2}}$$

New style

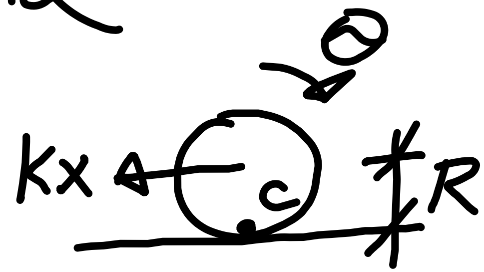
$$T = \frac{1}{2}M\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(MR^2+\bar{I})\dot{\theta}^2$$ &

$$V = \frac{1}{2}Kx^2$$

Old style



move forward



$\sum M_c = I_c \ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$ But

$x = R\theta$ so $-kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

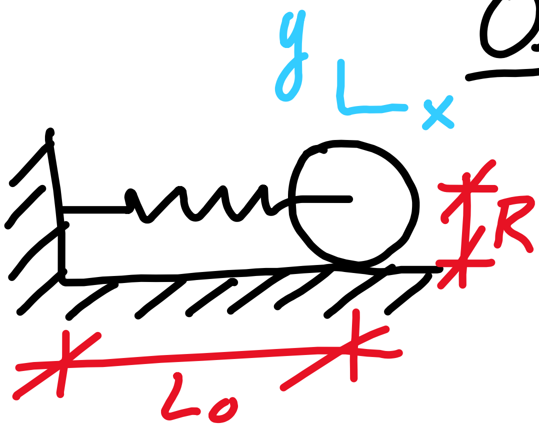
$\ddot{\theta} = -\omega^2\theta$, where $\omega = \sqrt{\dfrac{kR^2}{\bar{I} + mR^2}}$

New style

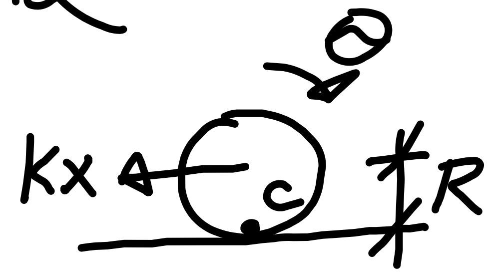
$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2$ &

$V = \frac{1}{2} kx^2 = \frac{1}{2} kR^2\theta^2$

Old style



move forward



$\sum M_c = I_c \ddot{\Theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\Theta}$ But

$x = R\Theta$ so $-kR^2\Theta = (\bar{I} + mR^2)\ddot{\Theta}$ or

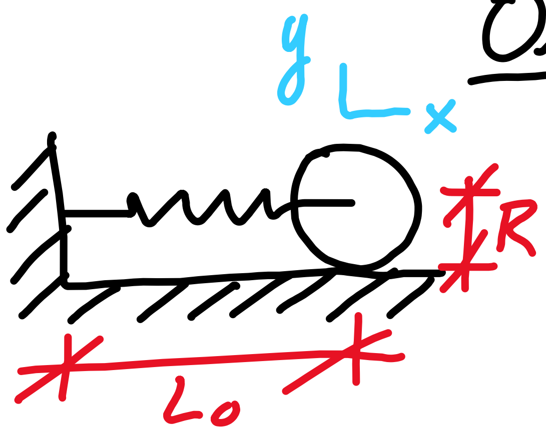
$\ddot{\Theta} = -\omega^2\Theta$, where $\omega = \sqrt{\dfrac{kR^2}{\bar{I} + mR^2}}$

New style

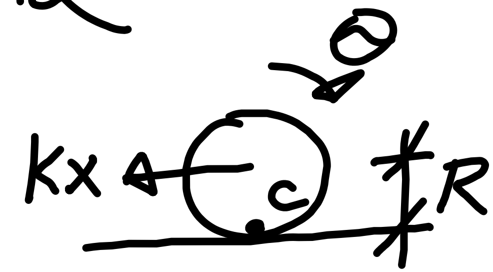
$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(mR^2 + \bar{I})\dot{\Theta}^2$ &

$V = \frac{1}{2}kx^2 = \frac{1}{2}kR^2\Theta^2$ so $T + V = \text{const.}$

Old style



move forward



$\circlearrowleft \sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + MR^2)\ddot{\theta}$ But

$x = R\theta$ so $-KR^2\theta = (\bar{I} + MR^2)\ddot{\theta}$ or

$\ddot{\theta} = -\omega^2 \theta$, where $\omega = \sqrt{\dfrac{KR^2}{\bar{I} + MR^2}}$

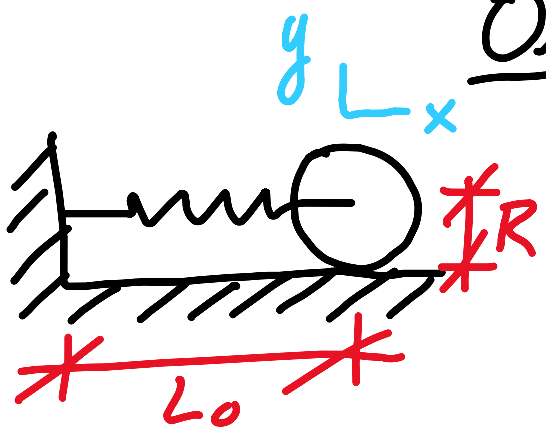
New style

$T = \frac{1}{2} M \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2}(MR^2 + \bar{I})\dot{\theta}^2$ &

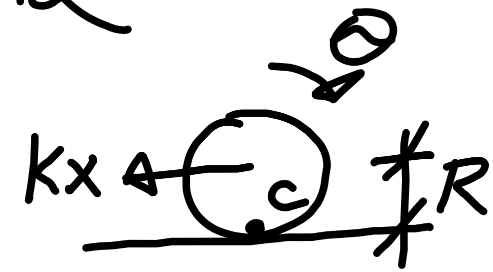
$V = \frac{1}{2} Kx^2 = \frac{1}{2} KR^2 \theta^2$ so $T + V = \text{const.} \Rightarrow$

$\frac{1}{2}(MR^2 + \bar{I})\dot{\theta}^2 + \frac{1}{2} KR^2 \theta^2$

Old style



move forward



$$\circlearrowleft \sum M_c = I_c\ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta} \quad \underline{\text{or}}$$

$$\ddot{\theta} = -\omega^2\theta, \quad \text{where} \quad \omega = \sqrt{\frac{kR^2}{\bar{I}+mR^2}}$$

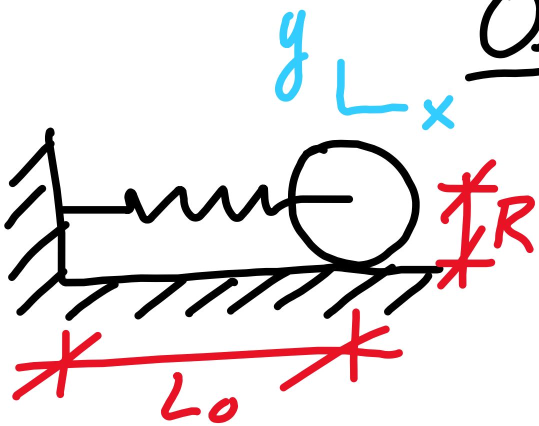
New style

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2 \quad \&$$

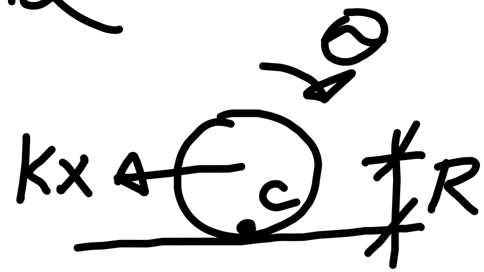
$$V = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kR^2\theta^2 \quad \text{so} \quad T + V = \text{const.} \Rightarrow$$

$$\tfrac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2 + \tfrac{1}{2}kR^2\theta^2 \quad \text{or} \quad \dot{\theta}^2 + \omega^2\theta^2 = C$$

Old style



move forward



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$x = R\theta$ so $-KR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$\ddot{\theta} = -\omega^2\theta$, where $\omega = \sqrt{\frac{KR^2}{\bar{I} + mR^2}}$

New style

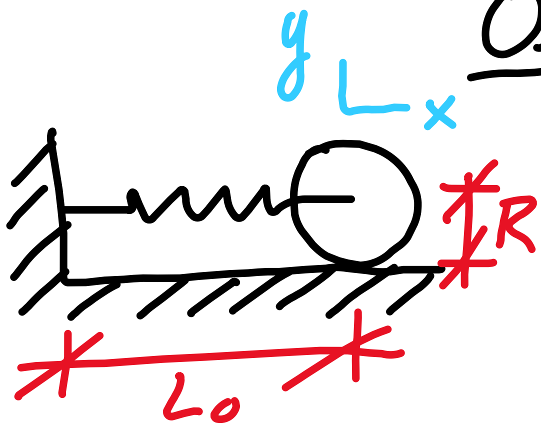
$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2$ &

$V = \frac{1}{2}Kx^2 = \frac{1}{2}KR^2\theta^2$ so $T + V = \text{const.} \Rightarrow$

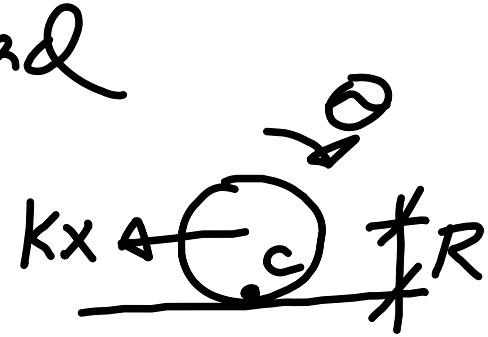
$\frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2 + \frac{1}{2}KR^2\theta^2$ or $\dot{\theta}^2 + \omega^2\theta^2 = C$,

where $\omega = \sqrt{\frac{KR^2}{\bar{I} + mb^2}}$

Old style



move forward



$\sum M_c = I_c \ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta}$ But

$x = R\theta$ so $-kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$\ddot{\theta} = -\omega^2\theta$, where $\omega = \sqrt{\dfrac{kR^2}{\bar{I} + mR^2}}$

New style

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2$ &

$V = \frac{1}{2}kx^2 = \frac{1}{2}kR^2\theta^2$ so $T + V = \text{const.} \Rightarrow$

$\frac{1}{2}(mR^2 + \bar{I})\dot{\theta}^2 + \frac{1}{2}kR^2\theta^2$ or $\dot{\theta}^2 + \omega^2\theta^2 = C_1$

where $\omega = \sqrt{\dfrac{kR^2}{\bar{I} + mb^2}}$