Today 19.3

$\boxed{L34}$

Today 19.3 Energy methods $\boxed{L34}$

Today 19.3

Friday 19.4

$\boxed{L\,34}$

Today 19.3

Friday 19.4

$\boxed{L\,34}$

Forced vibrations

ASU

$\boxed{L34}$

Today 19.3

Friday 19.4

Monday 19.4, 19.5

L34

Today 19.3

Friday 19.4

Monday 19.4, $\underbrace{19.5}_{\text{Damped Vibrations}}$

L 34

Today 19.3

Friday 19.4

Monday 19.4, 19.5

Wednesday Nov. 25th Review

L34

Today 19.3

Friday 19.4

Monday 19.4, 19.5

Wednesday Nov. 25th Review

Friday Nov. 27th Holiday

L34

Today 19.3

Friday 19.4

Monday 19.4, 19.5

Wednesday Nov. 25th Review

Friday Nov. 27th Holiday

Monday Nov. 30th Exam #4

L34

Today 19.3

Friday 19.4

Monday 19.4, 19.5

Wednesday Nov. 25th Review

Friday Nov. 27th Holiday

Monday Nov. 30th Exam #4

Wednesday Dec. 2nd Day of Reckoning

L34

Today 19.3

Friday 19.4

Monday 19.4, 19.5

Wednesday Nov. 25th Review

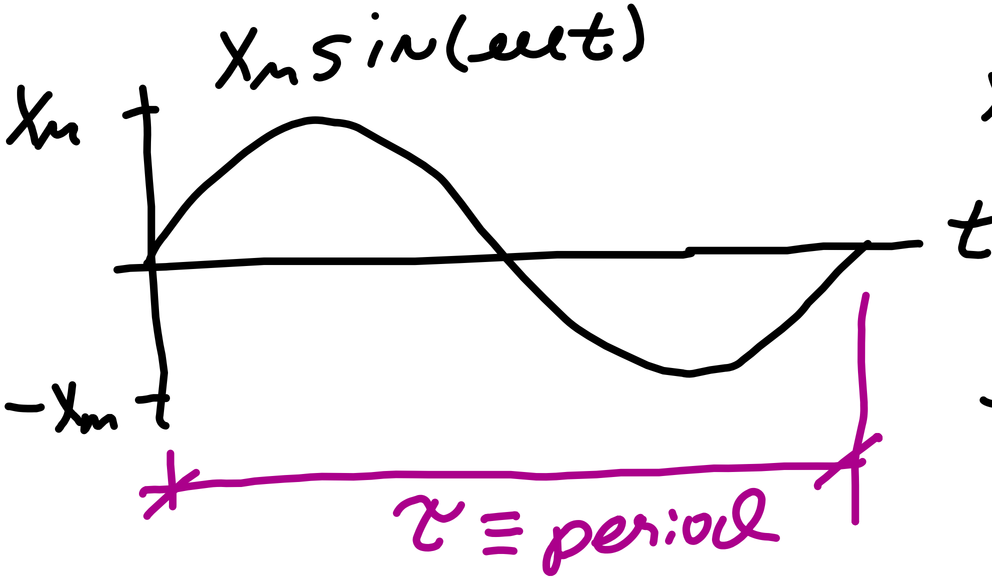
Friday Nov. 27th Holiday

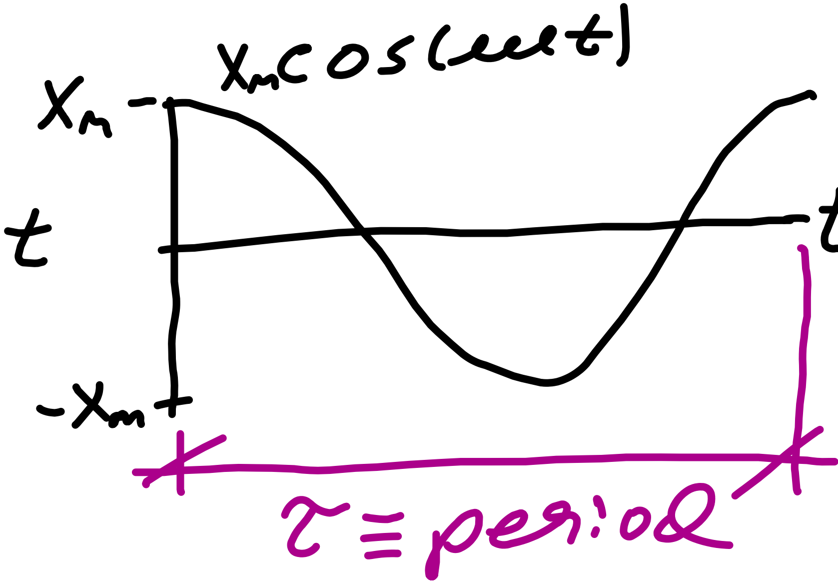
Monday Nov. 30th Exam #4

Wednesday Dec. 2nd Day of Reckoning

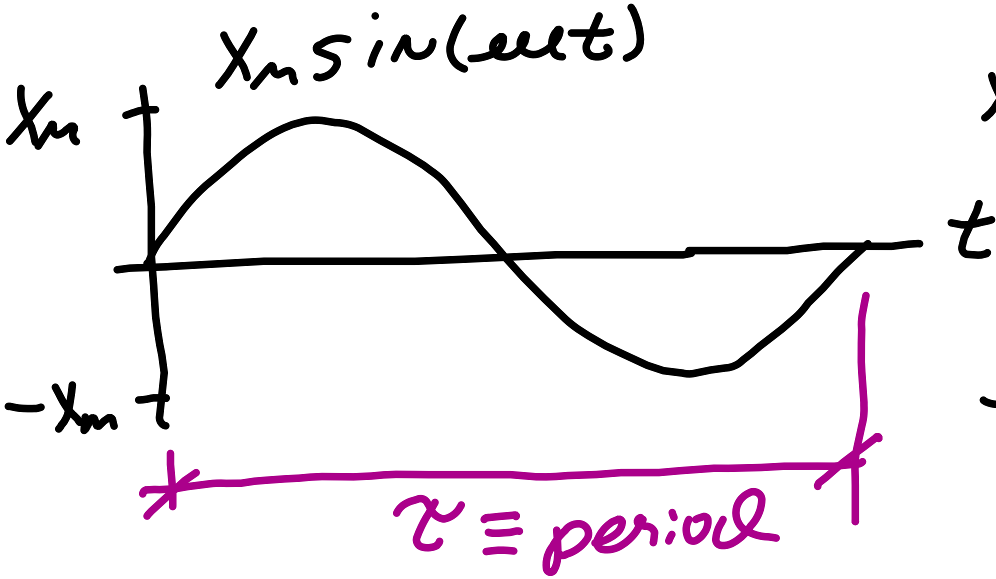
Friday Dec. 4th Final exam

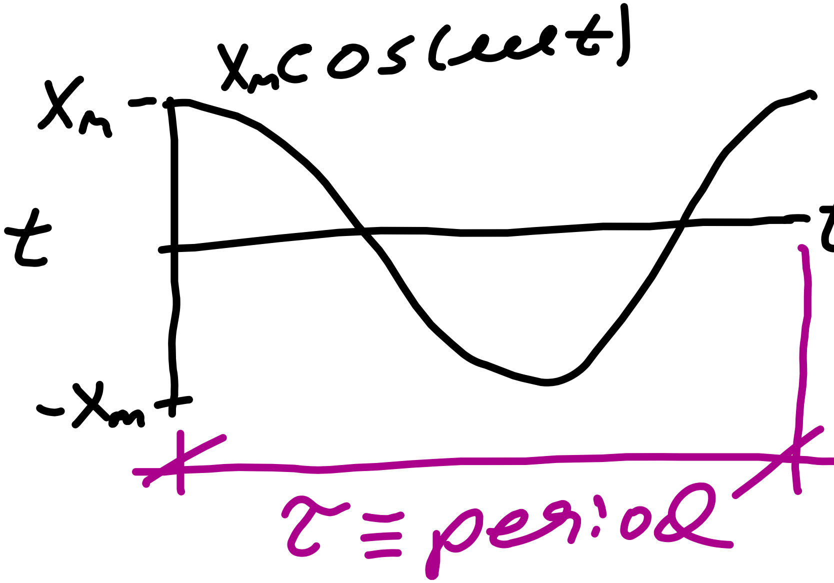
Wave anatomy





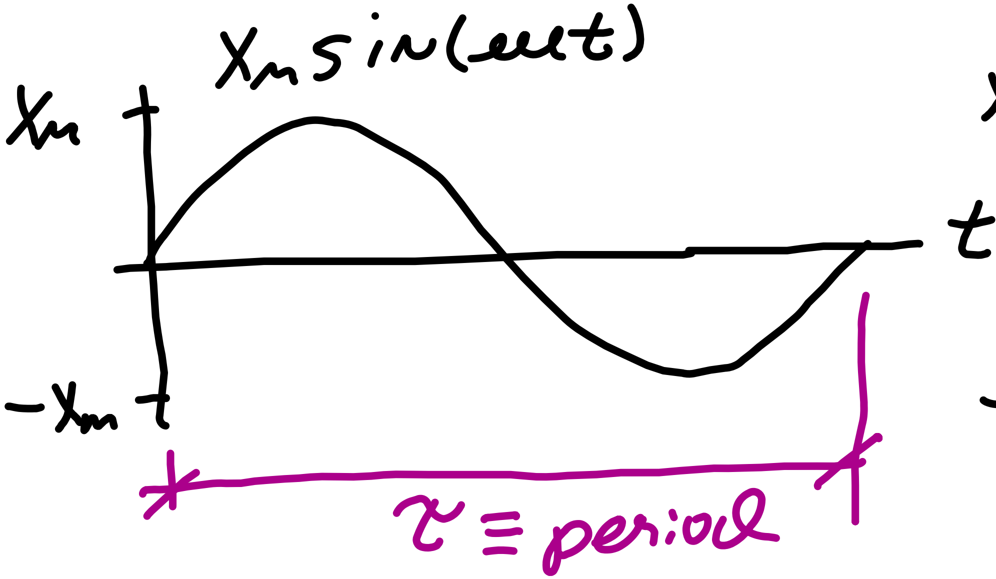
Wave anatomy

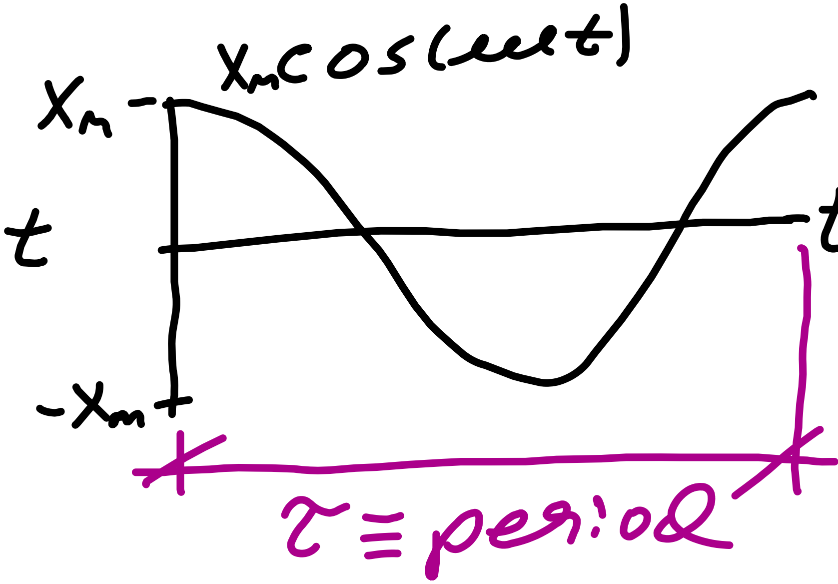




So when $t = $ period

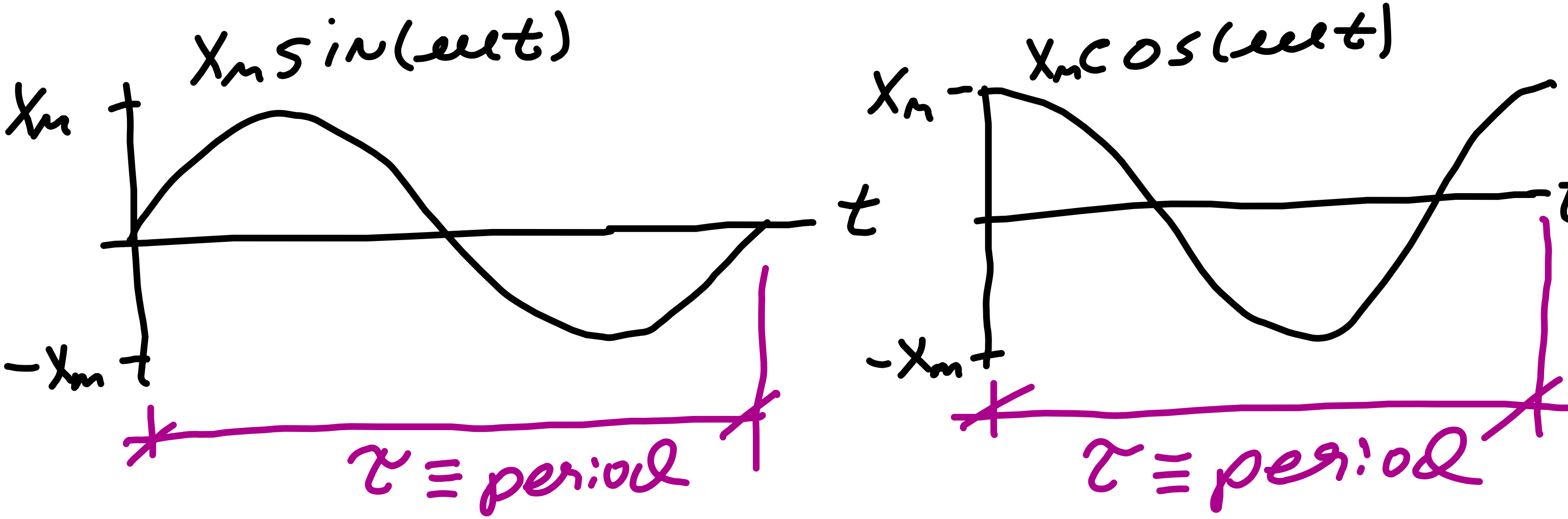
Wave anatomy





So when $t = \text{period} = \tau$

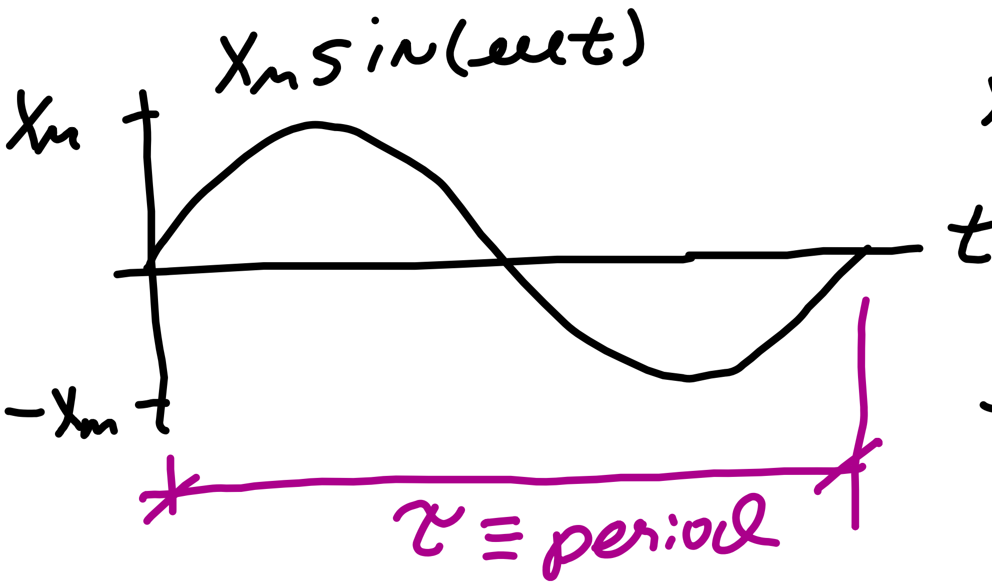
Wave anatomy

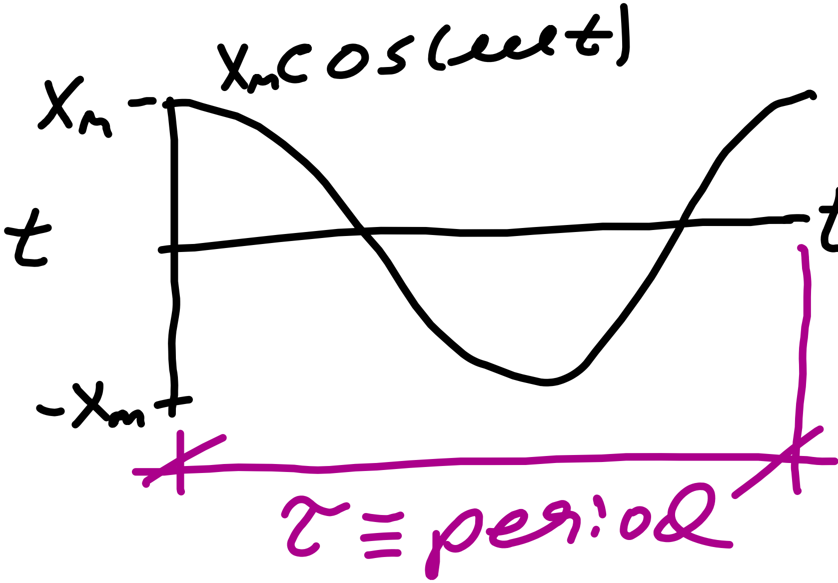


So when $t = \text{period} = \tau$ then

$\omega t = 2\pi$

Wave anatomy

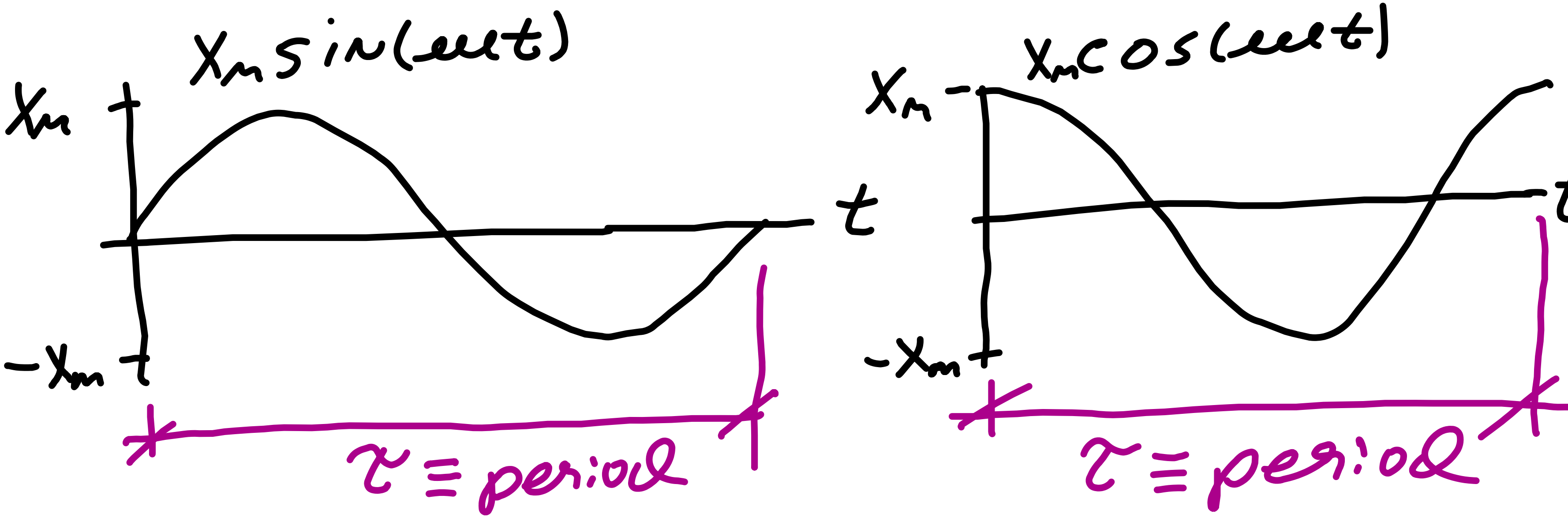




So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi$

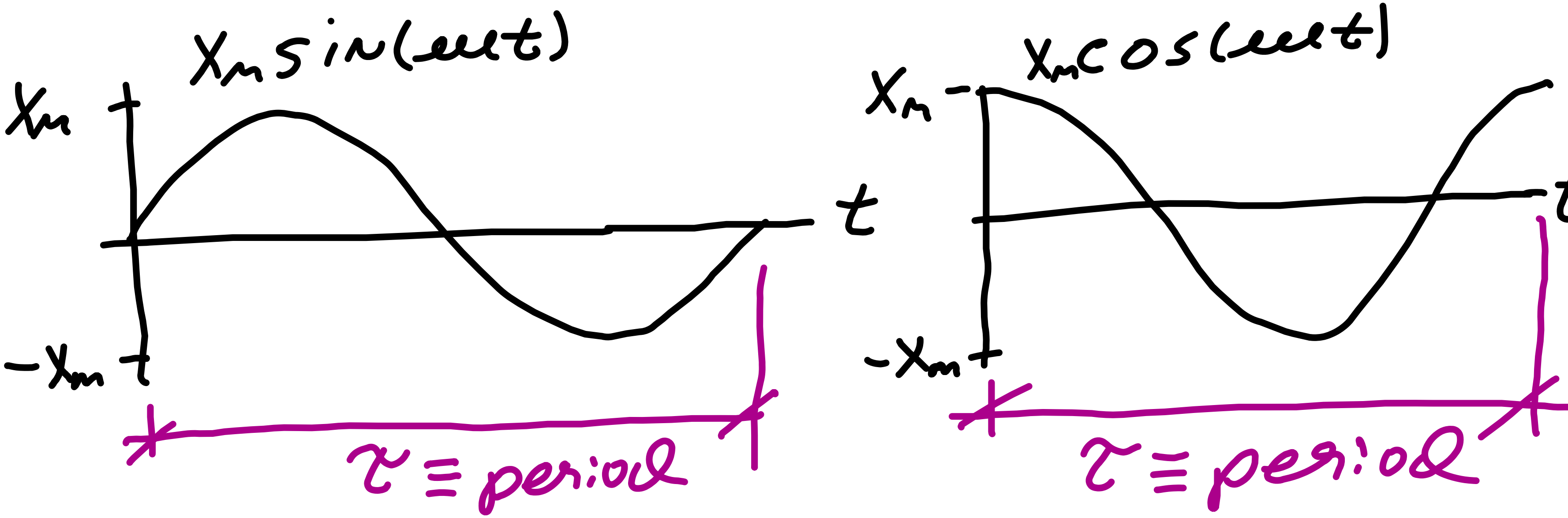
Wave anatomy



So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

Wave anatomy

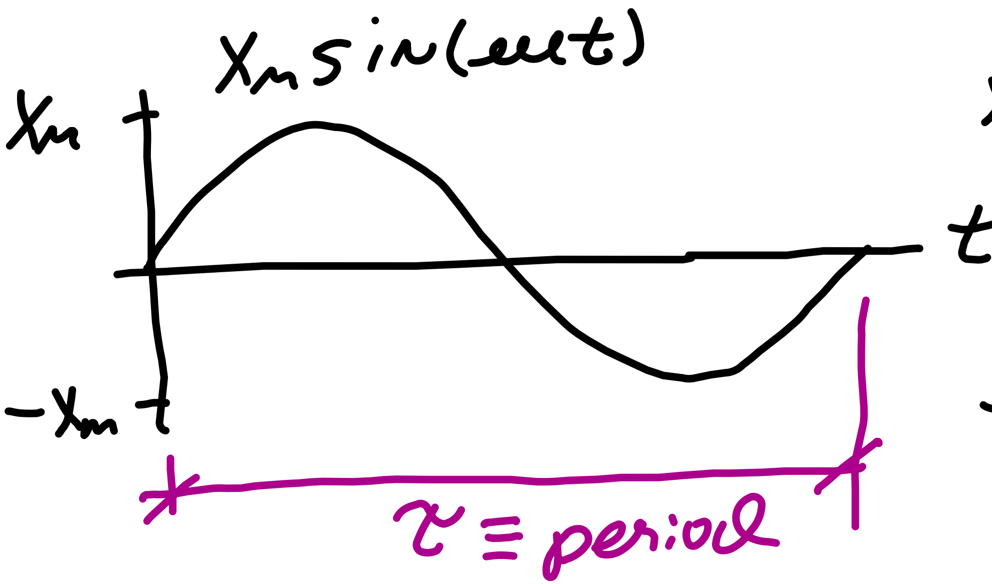


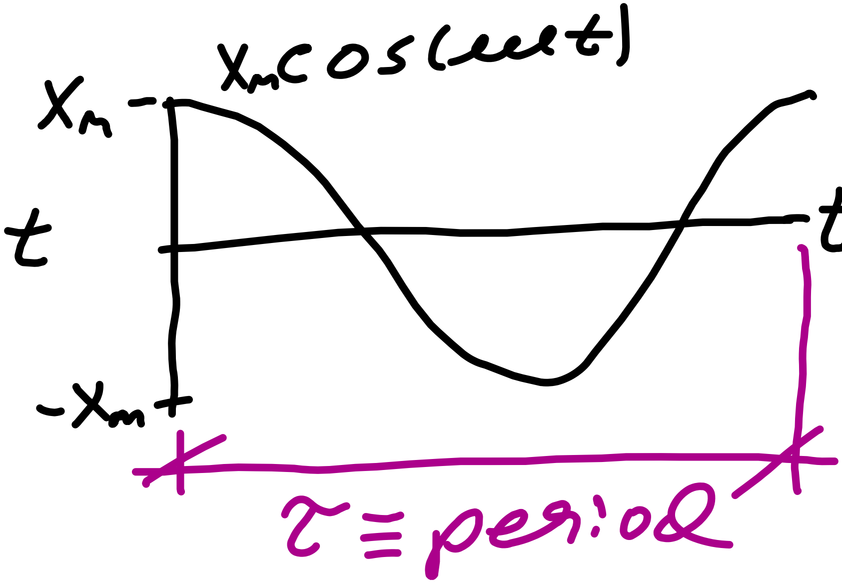
So when $t = \text{period} = \tau$ then

$$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$$

$$\& \quad \omega = \frac{2\pi}{\tau}$$

Wave anatomy



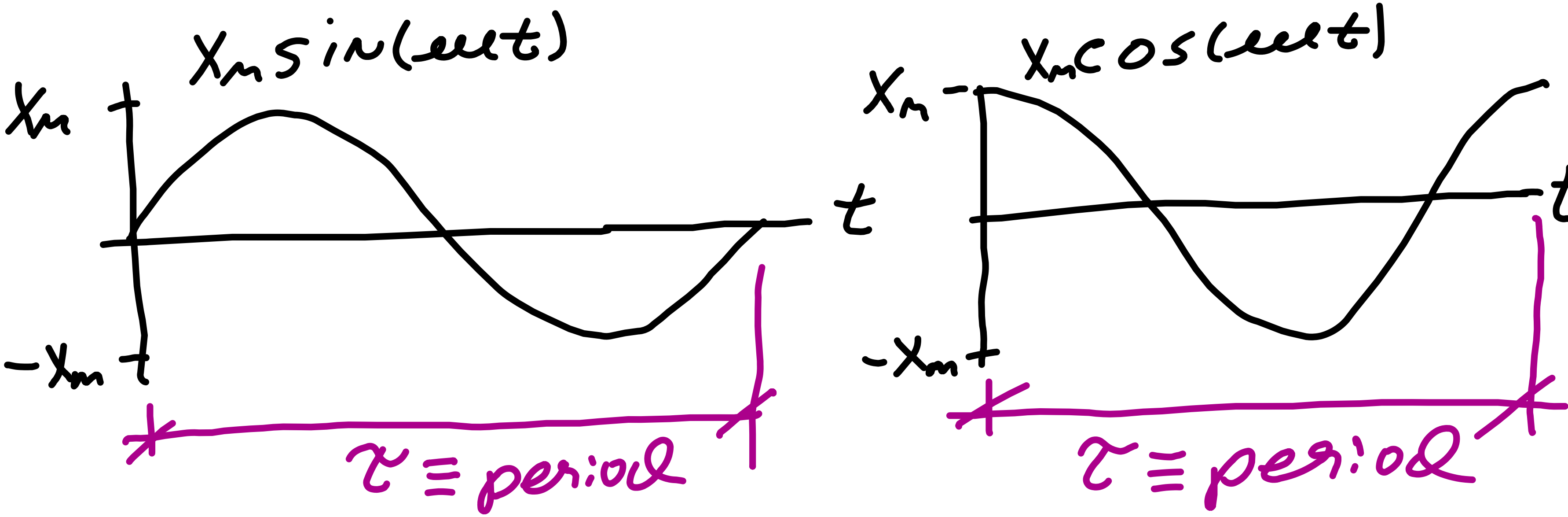


So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

Wave anatomy



So when $t = \text{period} = \tau$ then

$\omega t = 2\pi \Rightarrow \omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

$\&$ $\omega = \frac{2\pi}{\tau}$ Also Frequency $\equiv f$

$\&$ $f \equiv \frac{1}{\tau}$

We have seen that $\ddot{x} = -\omega_n^2 x$

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = X_m \sin(\omega_n t + \phi)$

We have seen that $\ddot{x} = -\omega^2 x$ has solution $x = x_m \sin(\omega t + \phi)$. And that $\ddot{x} = -\omega^2 x \Rightarrow$

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = x_m \sin(\omega_n t + \phi)$. And that $\ddot{x} = -\omega_n^2 x \Rightarrow \dot{x}^2 + \omega_n^2 x^2 = \text{const.}$

ASU

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = x_m \sin(\omega_n t + \phi)$. And that $\ddot{x} = -\omega_n^2 x \Rightarrow \dot{x}^2 + \omega_n^2 x^2 = \text{const.}$

The form $\ddot{x} = -\omega_n^2 x$ comes up for vibrating systems analyzed using $\Sigma \vec{M} = I \vec{\alpha}$

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = x_m \sin(\omega_n t + \phi)$. And that $\ddot{x} = -\omega_n^2 x \Rightarrow \dot{x}^2 + \omega_n^2 x^2 = \text{const.}$

The form $\ddot{x} = -\omega_n^2 x$ comes up for vibrating systems analyzed using $\Sigma \vec{M} = I\vec{\alpha}$ & $\Sigma \vec{F} = m\vec{a}$.

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = x_m \sin(\omega_n t + \phi)$. And that $\ddot{x} = -\omega_n^2 x \Rightarrow \dot{x}^2 + \omega_n^2 x^2 = \text{const.}$

The form $\ddot{x} = -\omega_n^2 x$ comes up for vibrating systems analyzed using $\sum \vec{M} = I\vec{\alpha}$ & $\sum \vec{F} = m\vec{a}$. The form $\dot{x}^2 + \omega^2 x^2 = \text{const.}$ shows up for vibrating systems analyzed using energy conservation:

We have seen that $\ddot{x} = -\omega_n^2 x$ has solution $x = x_m \sin(\omega_n t + \phi)$. And that $\ddot{x} = -\omega_n^2 x \Rightarrow \dot{x}^2 + \omega_n^2 x^2 = \text{const}$.

The form $\ddot{x} = -\omega_n^2 x$ comes up for vibrating systems analyzed using $\sum \vec{M} = I\vec{\alpha}$ & $\sum \vec{F} = m\vec{a}$. The form $\dot{x}^2 + \omega^2 x^2 = \text{const}$. shows up for vibrating systems analyzed using energy conservation:

$$T + V = \text{const}.$$

In what follows, we want to remember that

In what follows, we want to remember that

$$x = x_m \sin(\omega t + \phi)$$

In what follows, we want to remember that

$$x = x_m \sin(\omega t + \varphi)$$

$$\Rightarrow \dot{x} = x_m \omega \cos(\omega t + \varphi)$$

In what follows, we want to remember that

$$x = x_m \sin(\omega t + \varphi)$$

$$\Rightarrow \dot{x} = x_m \omega \cos(\omega t + \varphi)$$

$$\Rightarrow \dot{x}_m = x_m \omega$$

In what follows, we want to remember that

$$x = x_m \sin(\omega t + \varphi)$$

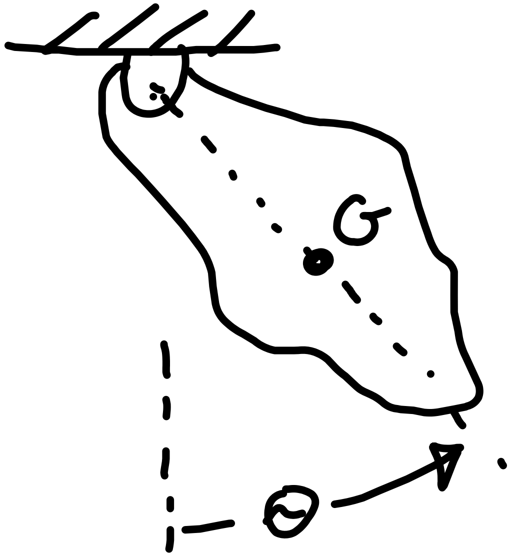
$$\Rightarrow \dot{x} = x_m \omega \cos(\omega t + \varphi)$$

$$\Rightarrow \dot{x}_m = x_m \omega$$

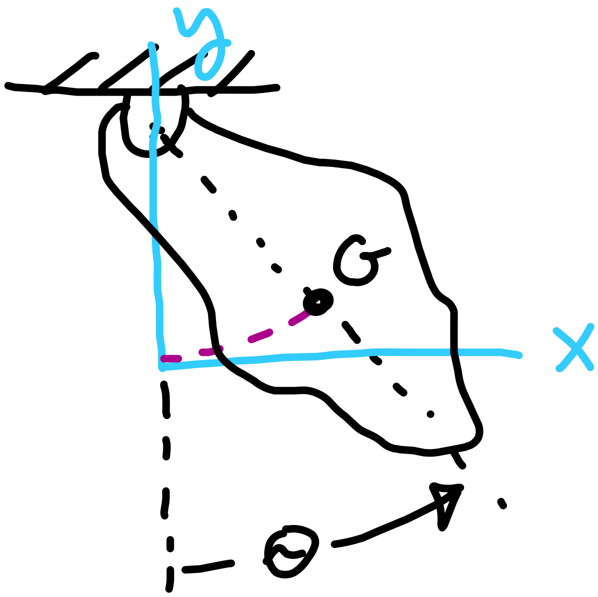
& similarly

$$\dot{\theta}_m = \theta_m \omega$$

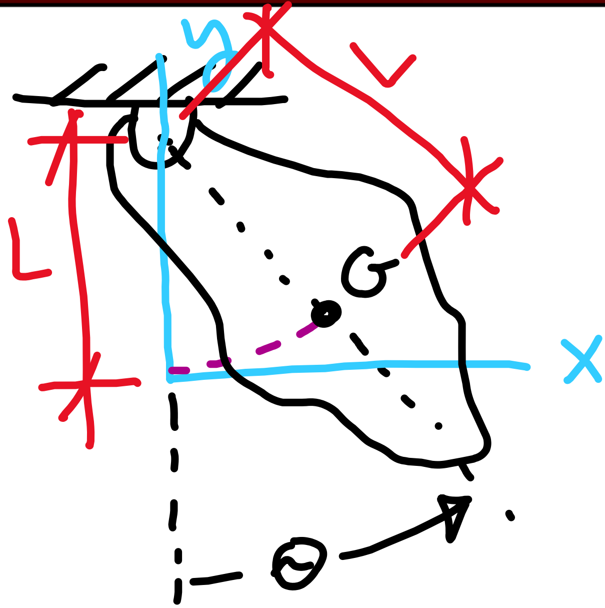
For the system shown



For the system shown

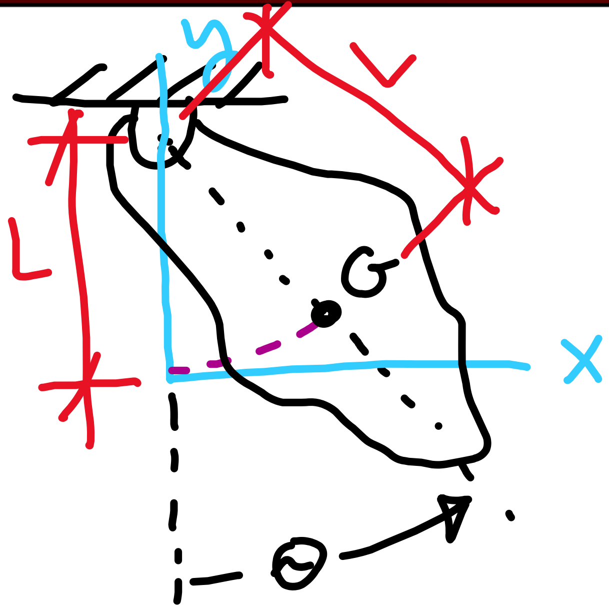


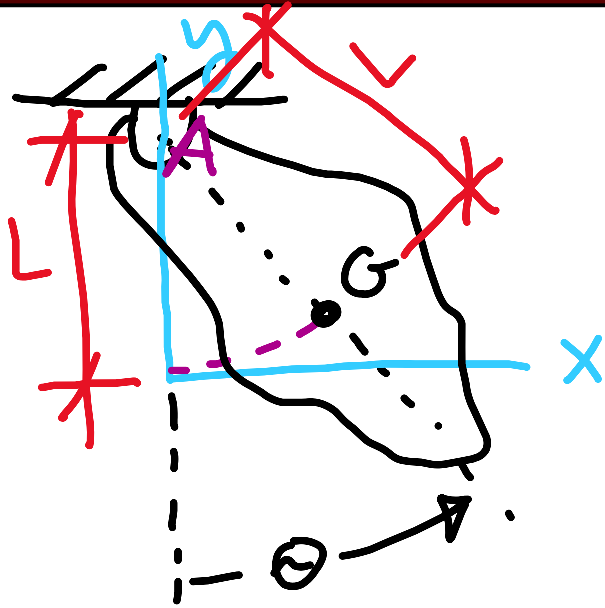
For the system shown



For the system shown

Find eom 3 ways:





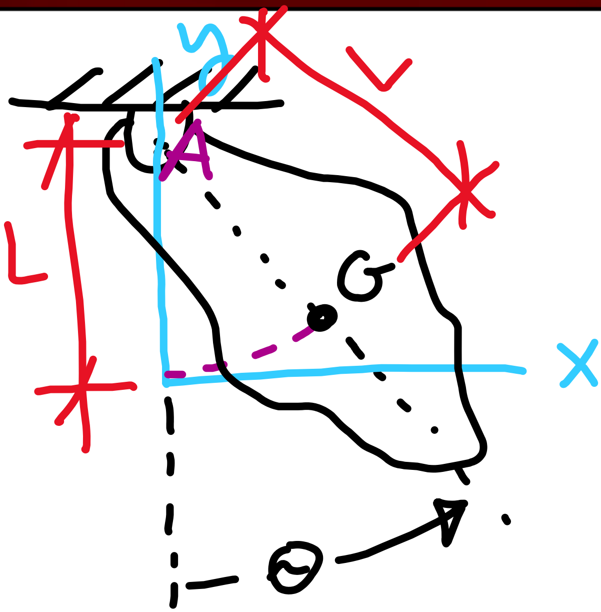
For the system shown

Find eqn 3 ways:

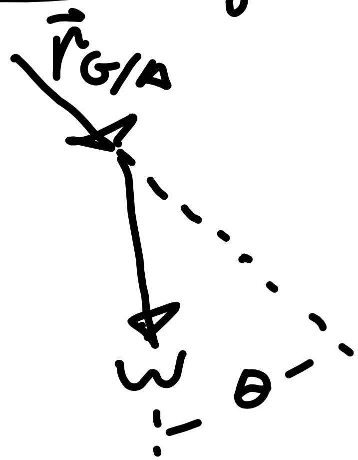
1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta}$

For the system shown

Find eom 3 ways:



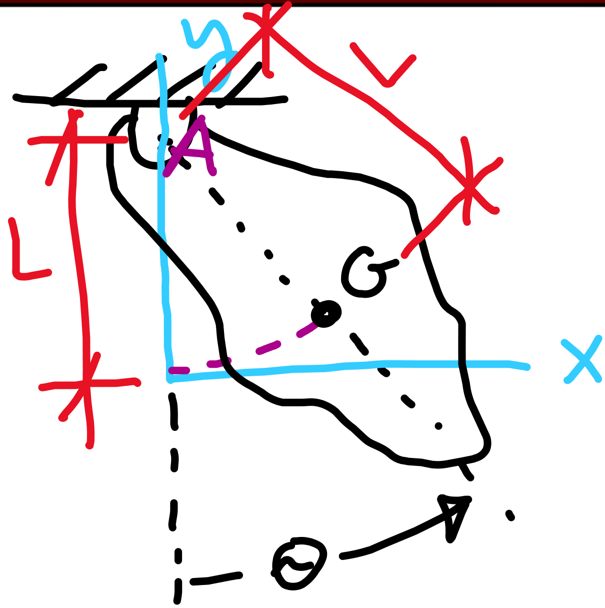
1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta}$

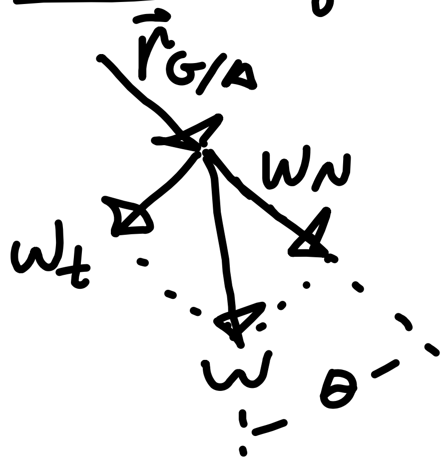


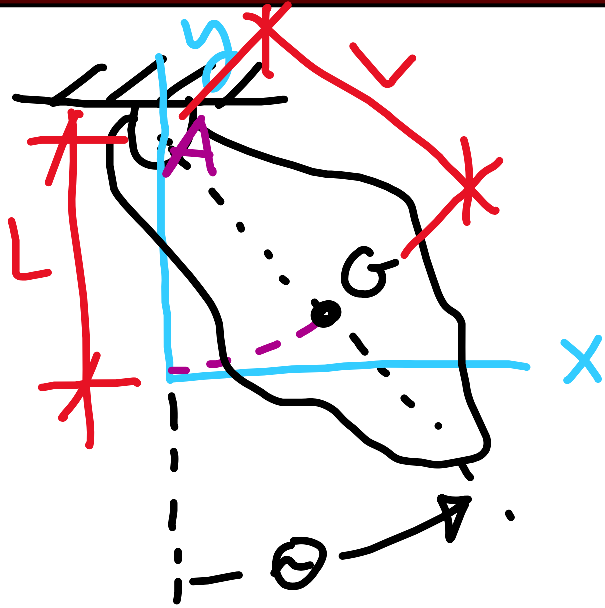
For the system shown

Find eqn 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta}$





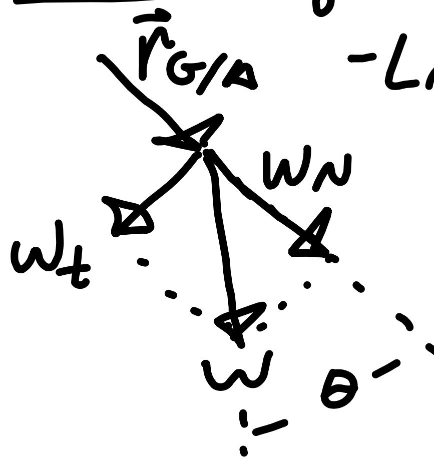


For the system shown

Find eqn 3 ways:

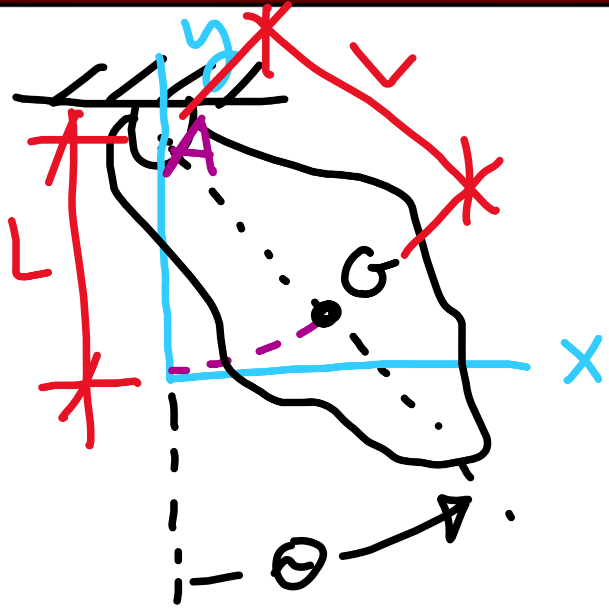
1st way: $\circlearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow$

$-Lmg\sin\theta = I_A\ddot{\theta}$

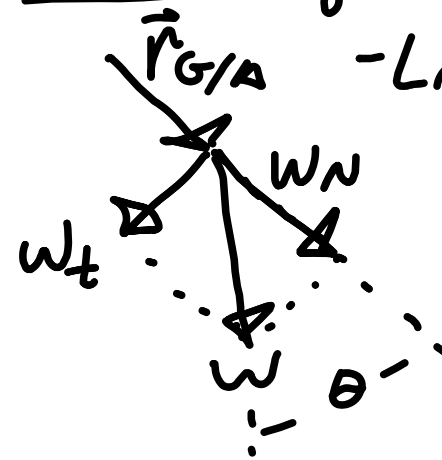


For the system shown

Find eqn 3 ways:

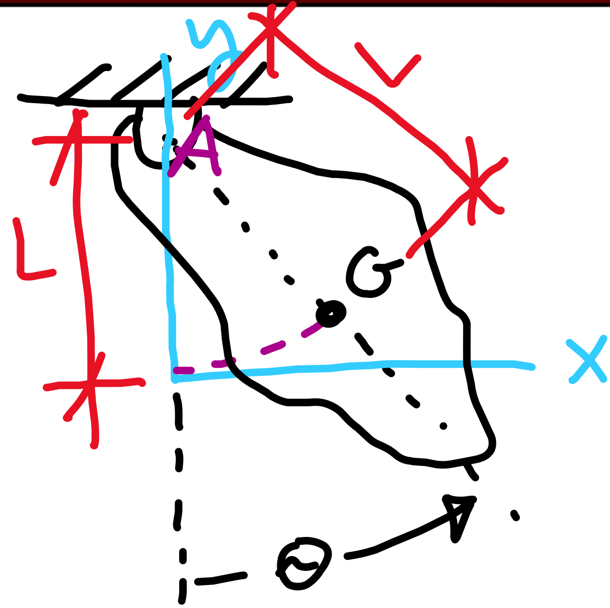


1st way: $\curvearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A\ddot{\theta}$

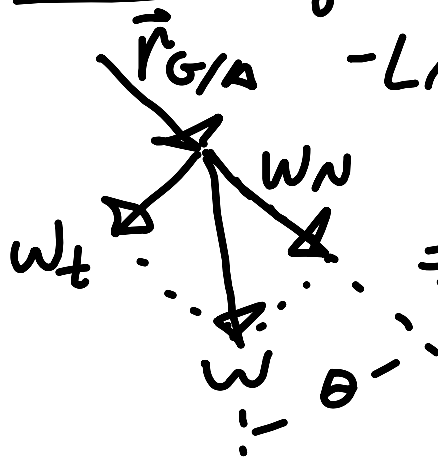
$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



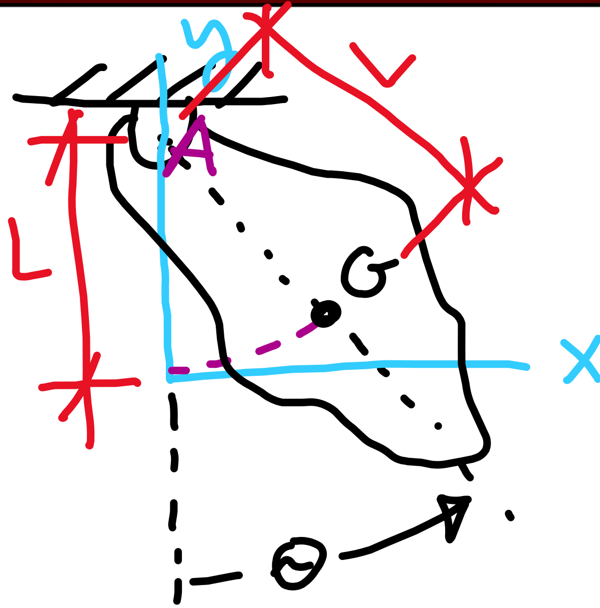
$-Lmg\sin\theta = I_A \ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$

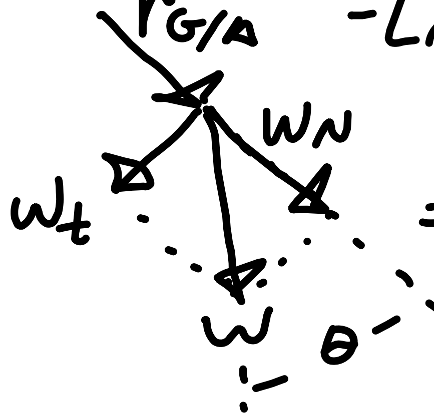
$\Rightarrow \ddot{\theta} = -\omega_n^2\theta$

For the system shown

Find ω_n 3 ways:



1st way: $\circlearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow$

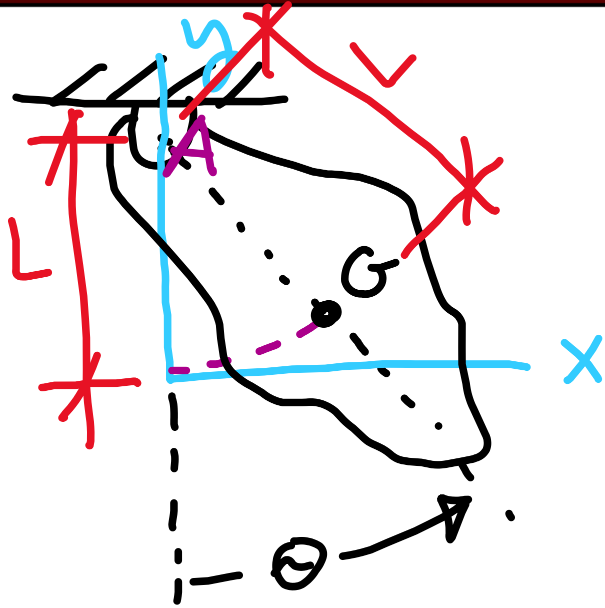


$-Lmg\sin\theta = I_A \ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta$, where

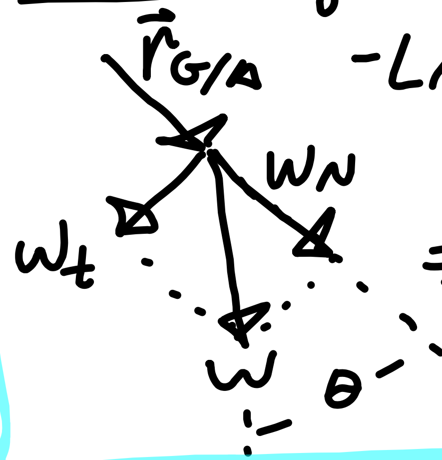
$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A \ddot{\theta}$$

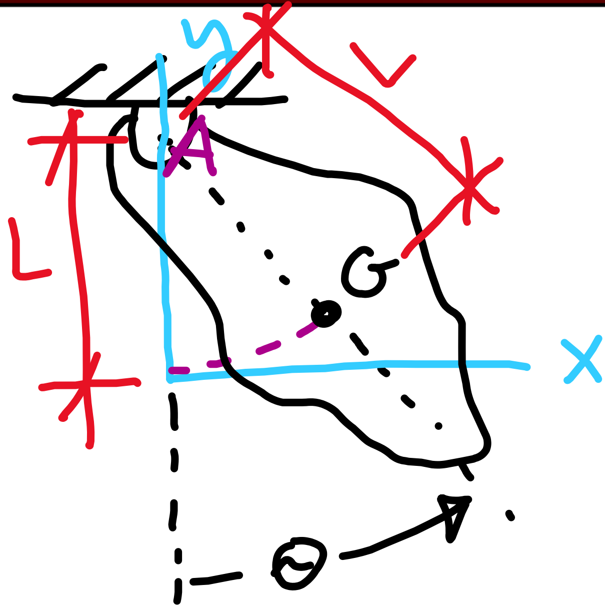
$$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta, \text{ where}$$

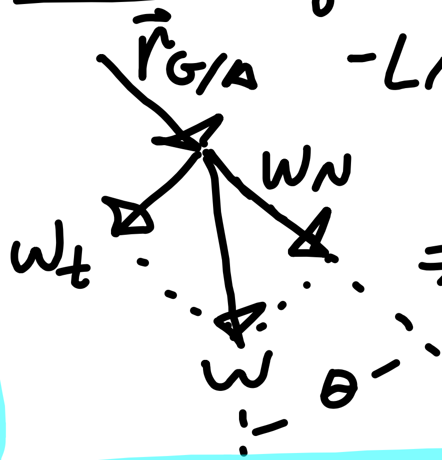
$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

For the system shown

Find ω_n 3 ways:



1st way: $\circlearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A\ddot{\theta}$$

$$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$$

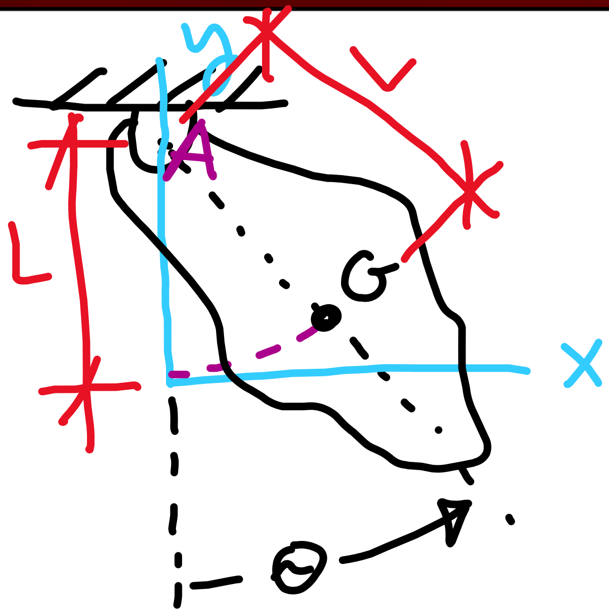
$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta, \text{ where}$$

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

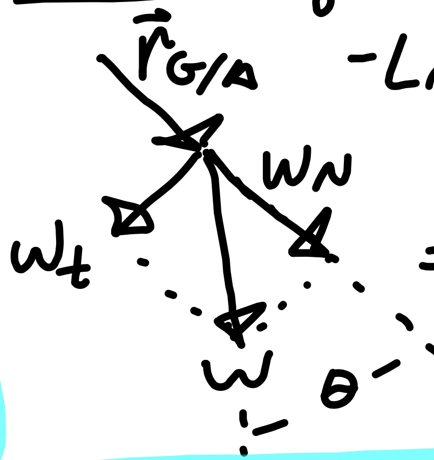
2nd way:

For the system shown

Find ω_n 3 ways:



1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A\ddot{\theta}$$

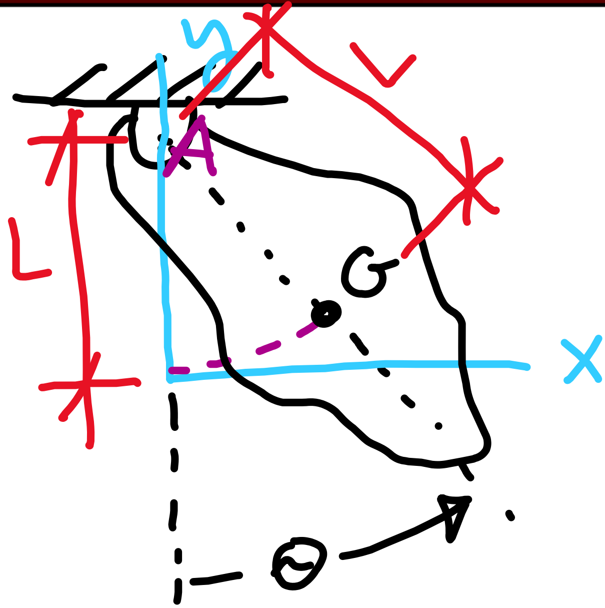
$$\Rightarrow -Lmg\,\theta = I_A\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta, \text{ where}$$

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2nd way:

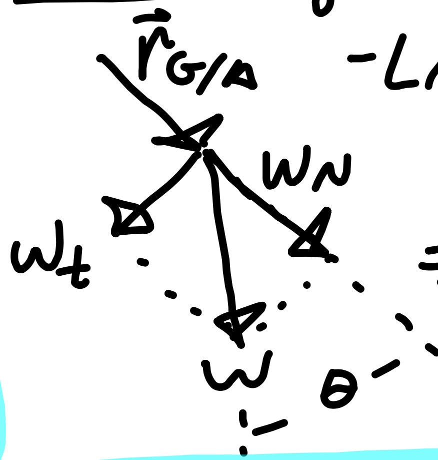
$$T + V = \text{const.}$$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A \ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta$, where

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

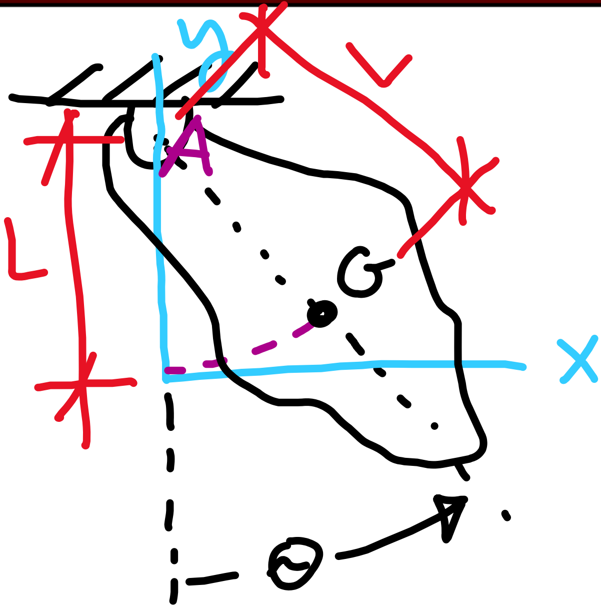
2nd way:

$T + V = \text{const.}$, where

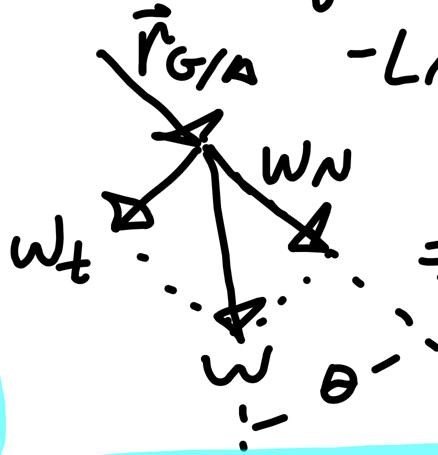
$T = \frac{1}{2} I_A \dot{\theta}^2$

For the system shown

Find ω_n 3 ways:



1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A\ddot{\theta}$$

$$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta \text{, where}$$

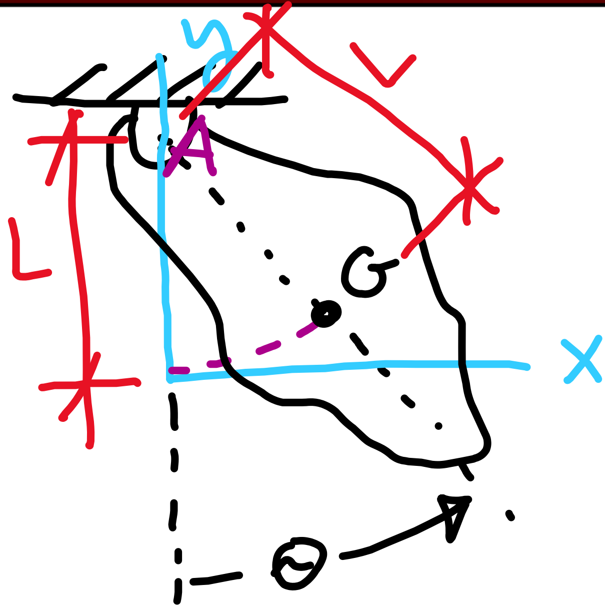
$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2nd way:

$T + V = \text{const.}$, where

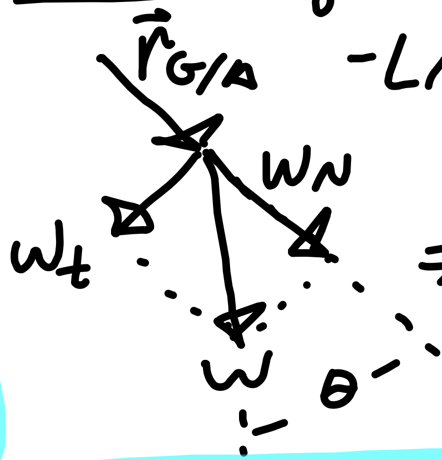
$T = \frac{1}{2} I_A \dot{\theta}^2$ & $V = mg(L - L\cos\theta)$

For the system shown



Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A \ddot{\theta}$

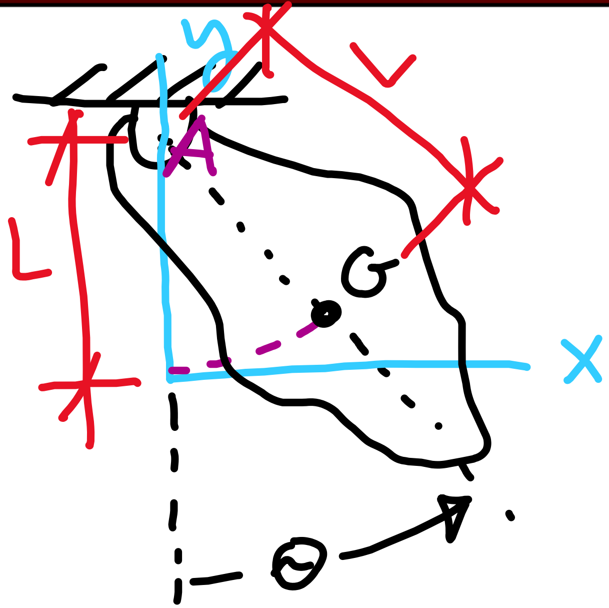
$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta$, where $\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$

2nd way:

$T + V = \text{const.}$, where

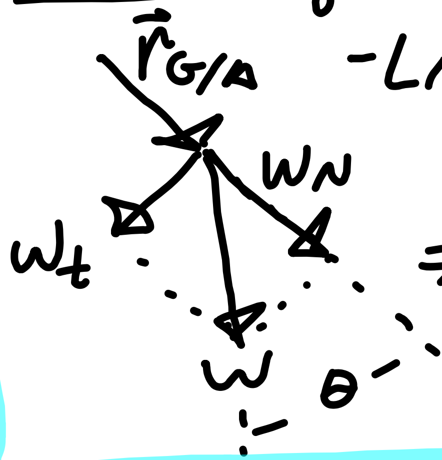
$T = \frac{1}{2} I_A \dot{\theta}^2$ & $V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1 - \frac{\theta^2}{2}\right)\right)$



For the system shown

Find ω_n 3 ways:

1st way: $\curvearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A\ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2\theta$, where

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2nd way:

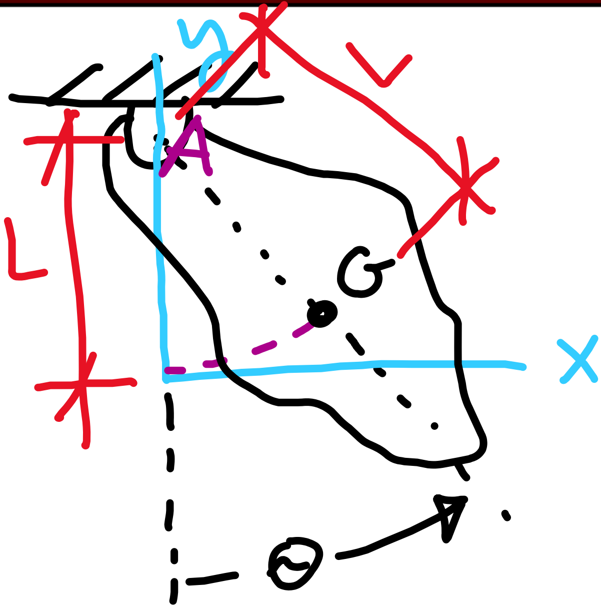
$T+V=\text{const.}$, where

$T=\frac{1}{2}I_A\dot{\theta}^2$ & $V = mg(L-L\cos\theta) \cong mg\left(L-L\left(1-\frac{\theta^2}{2}\right)\right)$

$\Rightarrow V = mgL\,\theta^2/2$

For the system shown

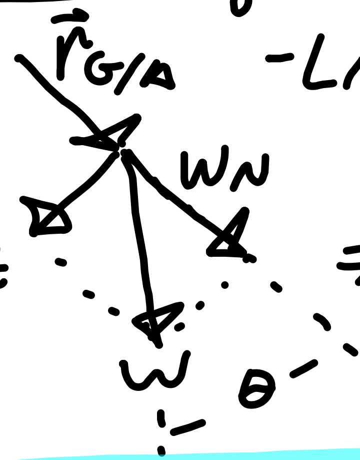
Find ω_n 3 ways:



1^{st} way:

$$\curvearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$$

$$-Lmg\sin\theta = I_A\ddot{\theta}$$



$$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta, \text{ where}$$

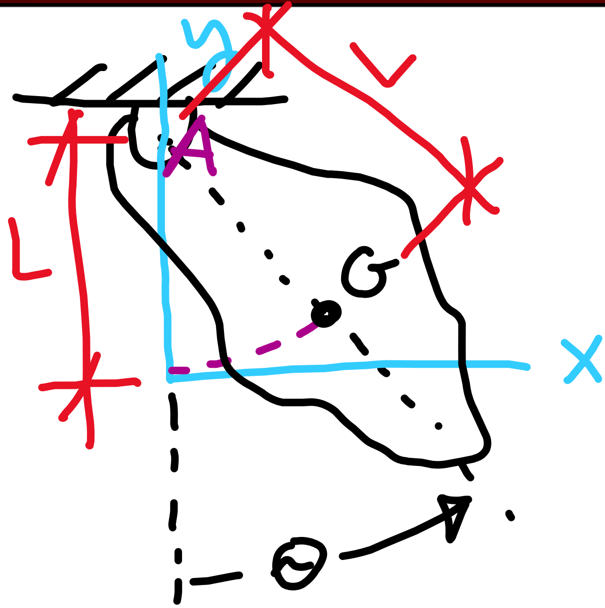
$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2^{nd} way:

$T+V = \text{const.}$, where

$$T = \tfrac{1}{2} I_A \dot{\theta}^2 \quad \& \quad V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1 - \frac{\theta^2}{2}\right)\right)$$

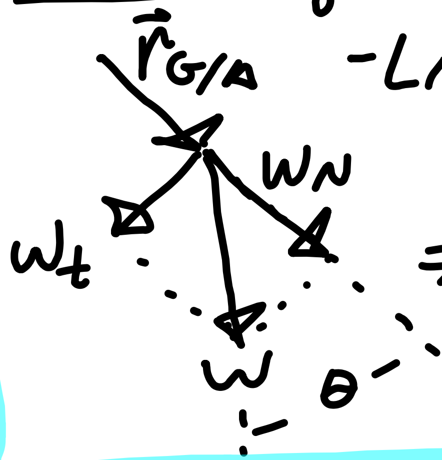
$$\Rightarrow V = mgL\theta^2/2 \Rightarrow \tfrac{1}{2} I_A \dot{\theta}^2 + mgL\theta^2/2 = \text{const.}$$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A\ddot{\theta}$$

$$\Rightarrow -Lmg\theta = I_A\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta, \text{ where}$$

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

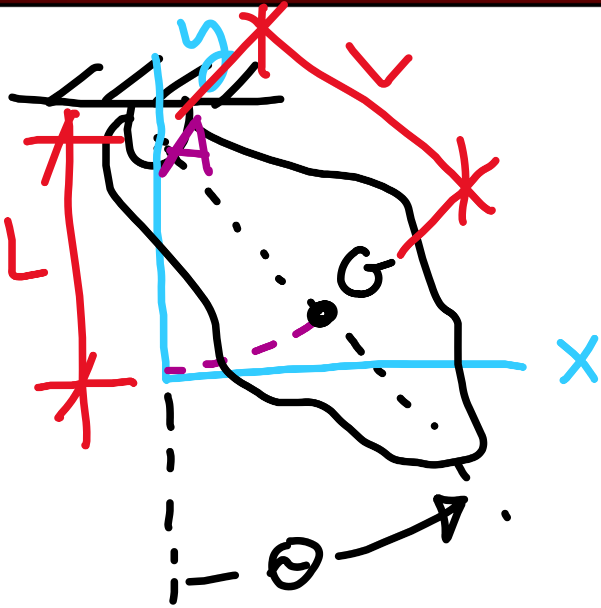
2nd way:

$T+V=\text{const.}$, where

$$T = \tfrac{1}{2} I_A \dot{\theta}^2 \ \ \& \ \ V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1-\frac{\theta^2}{2}\right)\right)$$

$$\Rightarrow V = mgL\theta^2/2 \Rightarrow \tfrac{1}{2} I_A\dot{\theta}^2 + mgL\theta^2/2 = \text{const.} \Rightarrow$$

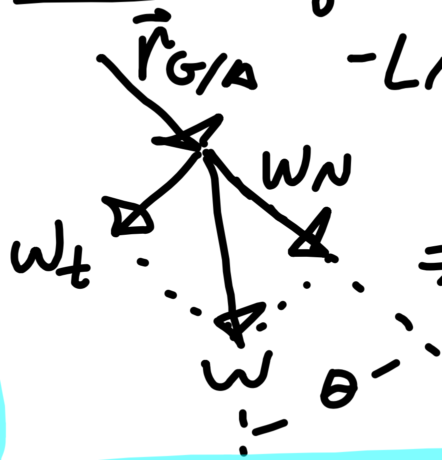
$$\dot{\theta}^2 + \omega^2\theta^2 = \text{const.}$$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A \ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta$, where

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2nd way:

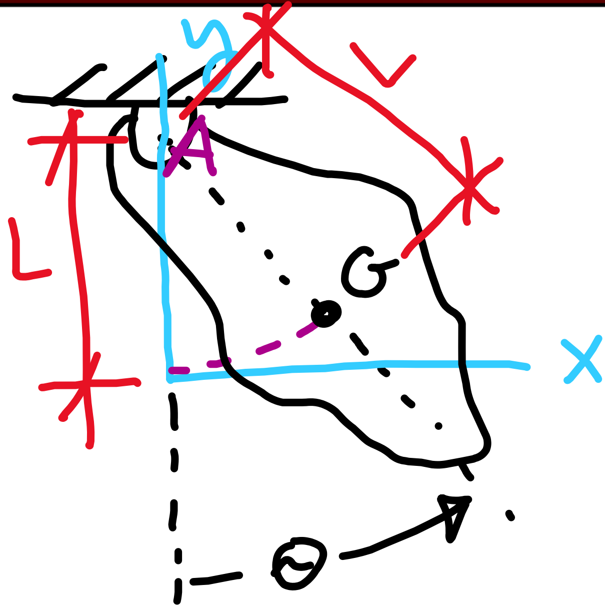
$T + V = \text{const.}$, where

$T = \frac{1}{2} I_A \dot{\theta}^2$ & $V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1 - \frac{\theta^2}{2}\right)\right)$

$\Rightarrow V = mgL\theta^2/2 \Rightarrow \frac{1}{2} I_A \dot{\theta}^2 + mgL\theta^2/2 = \text{const.} \Rightarrow$

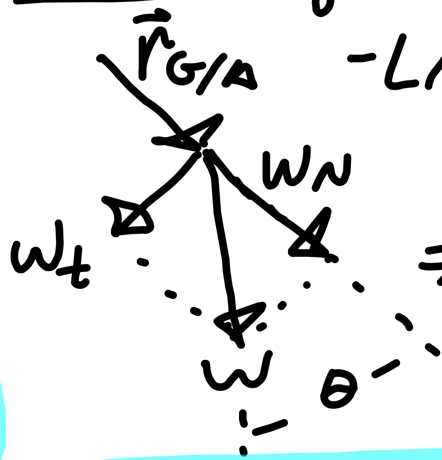
$\dot{\theta}^2 + \omega_n^2 \theta^2 = \text{const.}$, where $\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$

For the system shown



Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow$



$-Lmg\sin\theta = I_A \ddot{\theta}$

$\Rightarrow -Lmg\theta = I_A \ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\omega_n^2 \theta$, where

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

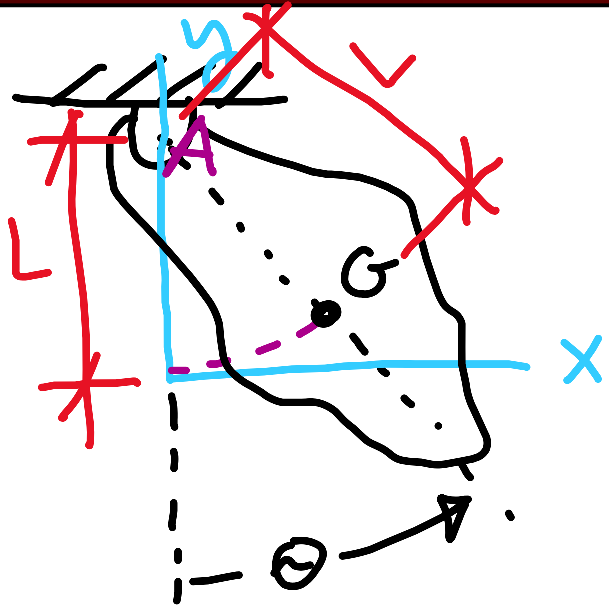
2nd way:

$T+V=\text{const.}$, where

$T = \frac{1}{2} I_A \dot{\theta}^2$ & $V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1-\frac{\theta^2}{2}\right)\right)$

$\Rightarrow V = mgL\theta^2/2 \Rightarrow \frac{1}{2} I_A \dot{\theta}^2 + mgL\theta^2/2 = \text{const.} \Rightarrow$

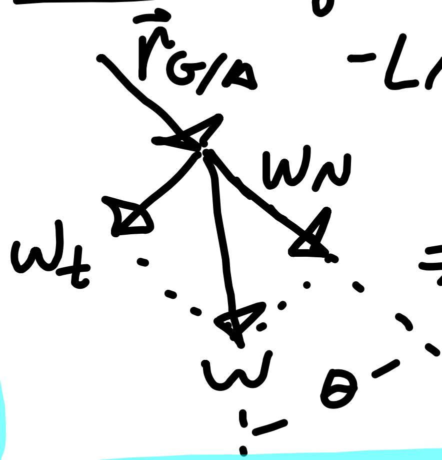
$\dot{\theta}^2 + \omega_n^2 \theta^2 = \text{const.}$, where $\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$



For the system shown

Find ω_n 3 ways:

1st way: $\circlearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow$



$$-Lmg\sin\theta = I_A\ddot{\theta}$$

$$\Rightarrow -Lmg\,\theta = I_A\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\omega_n^2\theta, \text{ where}$$

$$\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$$

2nd way:

$T+V = \text{const.}$, where

$T = \frac{1}{2}I_A\dot{\theta}^2$ & $V = mg(L - L\cos\theta) \cong mg\left(L - L\left(1-\frac{\theta^2}{2}\right)\right)$

$\Rightarrow V = mgL\theta^2/2 \Rightarrow \frac{1}{2}I_A\dot{\theta}^2 + mgL\theta^2/2 = \text{const.} \Rightarrow$

$\dot{\theta}^2 + \omega_n^2\theta^2 = \text{const.}$, where $\boxed{\omega_n = \sqrt{\frac{Lmg}{I_A}}}$

3rd way $\longrightarrow$

3rd way:

3rd way: Let $t = t_1$ when $T = 0$

<u>3rd way</u>: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $T_1 + V_1 = T_2 + V_2$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\overset{0}{T_1} + V_1 = T_2 + V_2$

$\underline{3^{rd}\text{ way}}$: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2} \Rightarrow$

$$V_1 = T_2$$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2} \Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max

$\underline{3^{rd}\text{ way}}$: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancel{T_1} + V_1 = T_2 + \cancel{V_2} \Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$

<u>3rd way</u>: Let $t=t_1$ when $T=0$ & $t=t_2$ when $V=0$ so $\cancelto{0}{T_1}+V_1 = T_2+\cancelto{0}{V_2} \Rightarrow$ $V_1 = T_2$ at $t=t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t=t_2$ T is max

__3rd way__: Let $t=t_1$ when $T=0$ & $t=t_2$ when $V=0$ so $\cancelto{0}{T_1}+V_1 = T_2+\cancelto{0}{V_2} \Rightarrow$

$V_1 = T_2$ at $t=t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t=t_2$ T is max, so

$T_2 = \frac{1}{2}I_A\dot{\theta}_m^2$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancel{T_1} + V_1 = T_2 + \cancel{V_2} \Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t = t_2$ T is max, so

$T_2 = \frac{1}{2} I_A \dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2} I_A \dot{\theta}_m^2$

3rd way: Let $t=t_1$ when $T=0$ & $t=t_2$ when $V=0$ so $\cancel{T_1}^{0}+V_1 = T_2+\cancel{V_2}^{0}$ $\Rightarrow$

$V_1 = T_2$ at $t=t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t=t_2$ T is max, so

$T_2 = \frac{1}{2}I_A\dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2}I_A\dot{\theta}_m^2$, but

$\dot{\theta}_m = \theta_m \omega_n$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2} \Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t = t_2$ T is max, so

$T_2 = \frac{1}{2}I_A\dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2}I_A\dot{\theta}_m^2$, but

$\dot{\theta}_m = \theta_m \omega_n$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2}I_A\omega_n^2\theta_m^2$

3rd way: Let $t=t_1$ when $T=0$ &

$t=t_2$ when $V=0$ so $\cancel{T_1}+V_1=T_2+\cancel{V_2} \Rightarrow$

$V_1=T_2$ at $t=t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t=t_2$ T is max, so

$T_2=\frac{1}{2}I_A\dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2=\frac{1}{2}I_A\dot{\theta}_m^2$, but

$\dot{\theta}_m=\theta_m\omega_n$ so $mg\frac{L}{\cancel{2}}\theta_m^2=\frac{1}{\cancel{2}}I_A\omega_n^2\theta_m^2$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2} \Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t = t_2$ T is max, so

$T_2 = \frac{1}{2} I_A \dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2} I_A \dot{\theta}_m^2$, but

$\dot{\theta}_m = \theta_m \omega_n$ so $mg\frac{L}{\cancel{2}}\cancel{\theta_m^2} = \frac{1}{\cancel{2}} I_A \omega_n^2 \cancel{\theta_m^2}$

3rd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ so $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$ $\Rightarrow$

$V_1 = T_2$ at $t = t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t = t_2$ T is max, so

$T_2 = \frac{1}{2} I_A \dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2} I_A \dot{\theta}_m^2$, but

$\dot{\theta}_m = \theta_m \omega_n$ so $mg\frac{L}{\cancel{2}}\cancel{\theta_m^2} = \frac{1}{\cancel{2}} I_A \omega_n^2 \cancel{\theta_m^2}$

$$\Rightarrow \quad \frac{mgL}{I_A} = \omega_n^2$$

<u>3rd way</u>: Let $t=t_1$ when $T=0$ &

$t=t_2$ when $V=0$ so $\cancel{T_1}+V_1 = T_2+\cancel{V_2}$ $\Rightarrow$

$V_1 = T_2$ at $t=t_1$ V is max, so

$V_1 \cong mg\frac{L}{2}\theta_m^2$ & at $t=t_2$ T is max, so

$T_2 = \frac{1}{2}I_A\dot{\theta}_m^2$ so $mg\frac{L}{2}\theta_m^2 = \frac{1}{2}I_A\dot{\theta}_m^2$, but

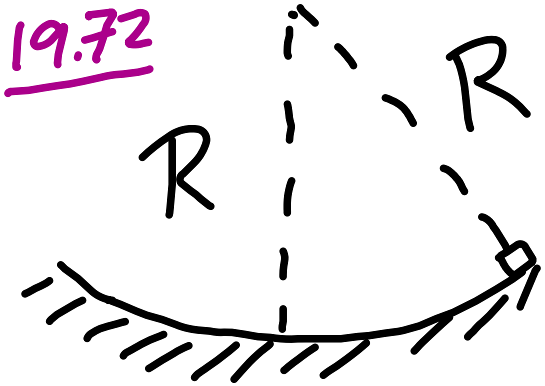
$\dot{\theta}_m = \theta_m\omega_n$ so $mg\frac{L}{\cancel{2}}\cancel{\theta_m^2} = \frac{1}{\cancel{2}}I_A\omega_n^2\cancel{\theta_m^2}$

$\Rightarrow$ $\frac{mg}{I_A} = \omega_n^2$ $\Rightarrow$ $\boxed{\omega_n = \sqrt{\frac{mg}{I_A}}}$

Another example

Another example [done two ways]

19.72



Another example [done two ways]

19.72

Another example [done two ways]

19.72

Another example [done two ways]

19.72

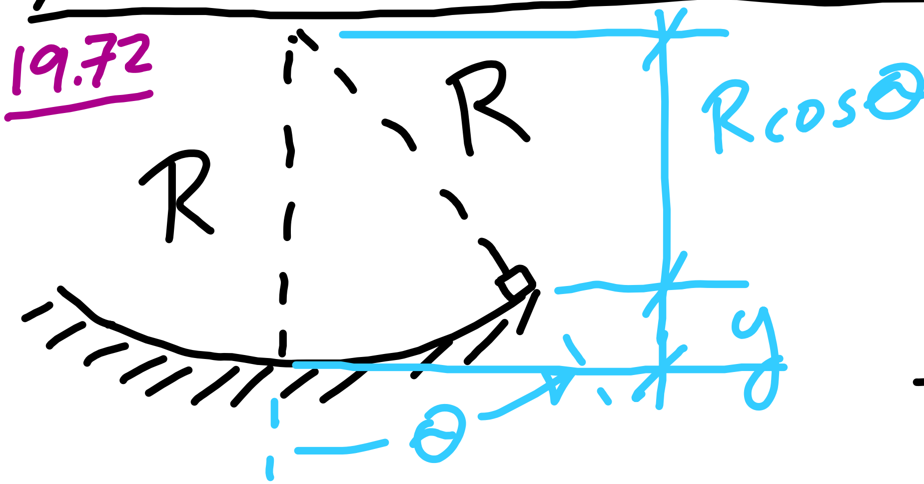
Another example [done two ways]

19.72

$$\Rightarrow R = R\cos\theta + y$$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$

Another example [done two ways]

19.72

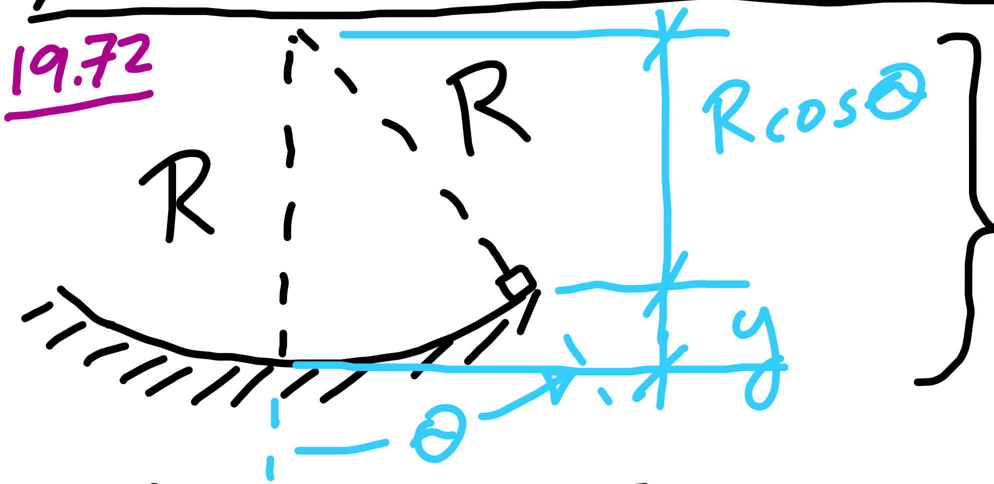
$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$ if

$\theta = \text{small}$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if

θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})]$

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
$\theta =$ small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow \quad y = R\theta^2/2$

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$ if

θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

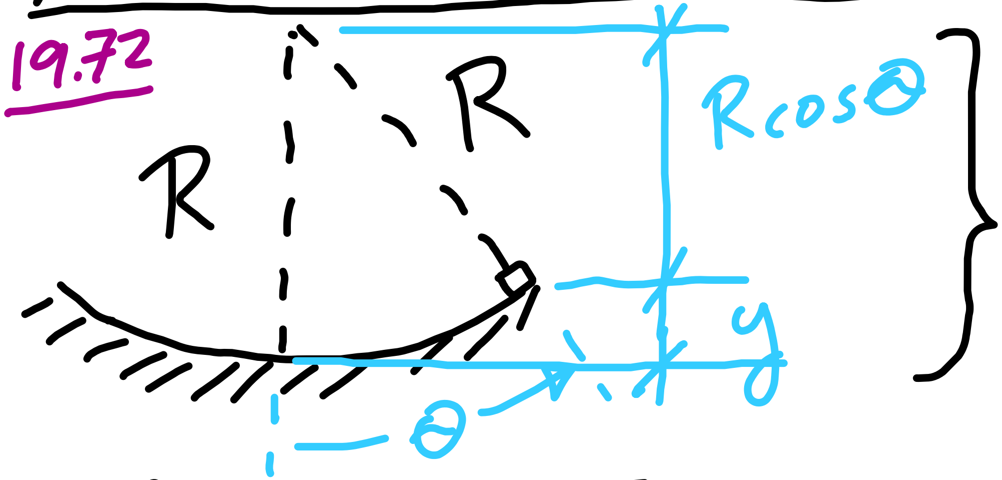
$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

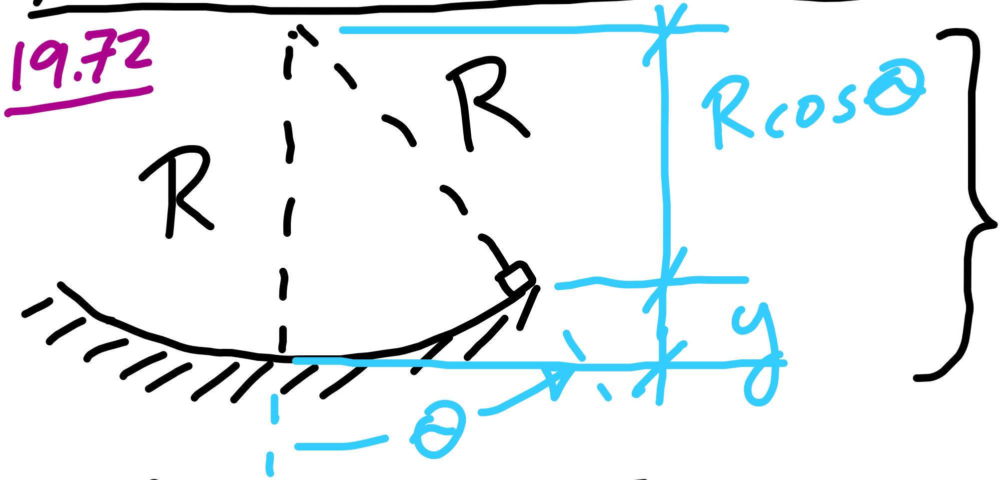
$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$ if θ = small then

$$y \cong R\left[1-\left(1-\frac{\theta^2}{2}\right)\right] \Rightarrow y = R\theta^2/2$$

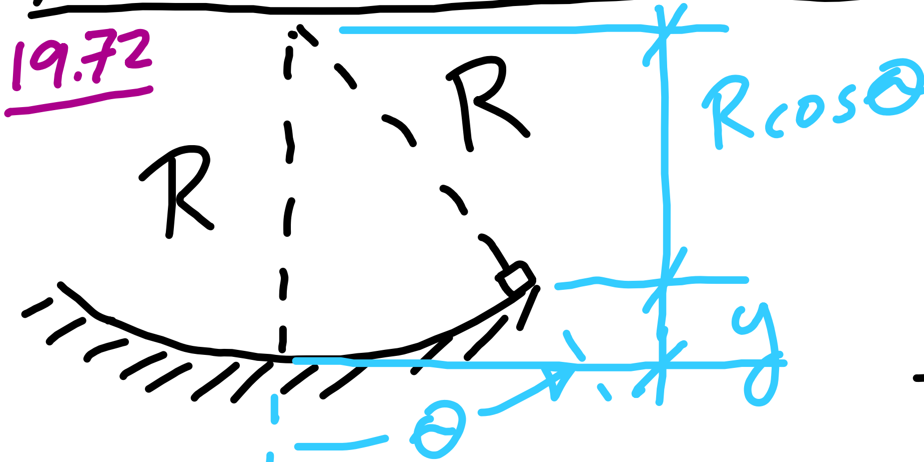
1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow \quad y = R\theta^2/2$ **1st way:**

$T+V = $ const., where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$ & $y = R\frac{\theta^2}{2}$

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$ & $y = R\frac{\theta^2}{2}$ so $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{mg}{2}R\theta^2 = C$

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1 - \cos\theta)$ if
θ = small then

$y \cong R[1 - (1 - \frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

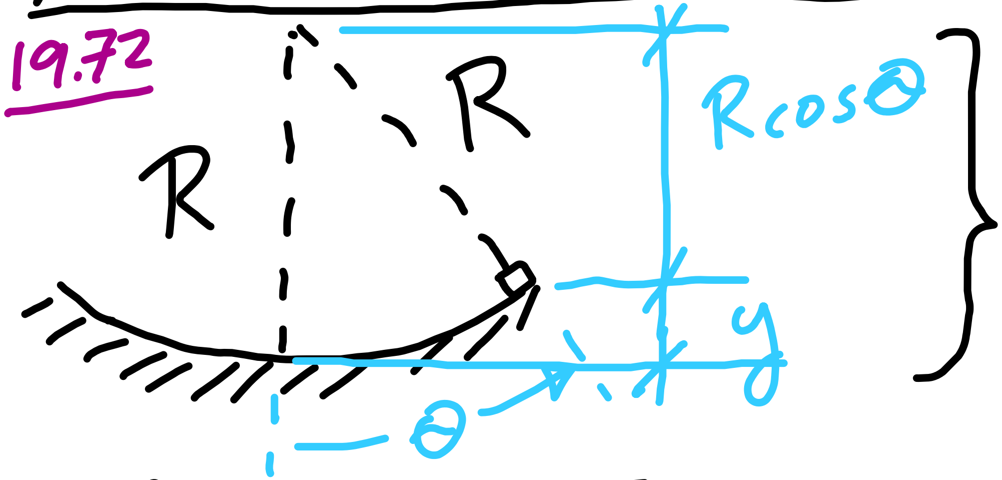
$T + V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$ & $y = R\frac{\theta^2}{2}$ so $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{mg}{2}R\theta^2 = C$

$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{const.}$

Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$ if

θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$ & $y = R\frac{\theta^2}{2}$ so $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{mg}{2}R\theta^2 = C$

$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{const.}$, where $\boxed{\omega = \sqrt{g/R}}$

ASU

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

$y \simeq R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V = \text{const.}$, where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$ & $y = R\frac{\theta^2}{2}$ so $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{mg}{2}R\theta^2 = C$

$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{CONST.}$, where $\boxed{\omega = \sqrt{g/R}}$

2nd way $\longrightarrow$

2^{nd} way :

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = \theta$

$\underline{2^{nd} \text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$

2nd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ $\Rightarrow$ $T_1 + V_1 = T_2 + V_2$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $T_1 + V_1 = T_2 + V_2$

$\underline{2^{nd} \text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $T_1 + V_1 = T_2 + V_2$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $T_1 + V_1 = T_2 + V_2$

$\Rightarrow \quad V_1 = T_2$

<u>2nd way</u>: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancel{T_1} + V_1 = T_2 + \cancel{V_2}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} m R^2 \dot{\theta}_m^2$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0 \Rightarrow \cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} m R^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega \theta_m$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0 \Rightarrow \cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2}MR^2\dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\theta_m$

so $T_2 = \frac{1}{2}MR^2\omega^2\theta_m^2$

2nd way: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} M R^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega \theta_m$

So $T_2 = \frac{1}{2} M R^2 \omega^2 \theta_m^2$ Now

$V_1 = T_2$

__2nd way__: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0$ $\Rightarrow$ $\overset{0}{\cancel{T_1}} + V_1 = T_2 + \overset{0}{\cancel{V_2}}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} MR^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\theta_m$

so $T_2 = \frac{1}{2} MR^2 \omega^2 \theta_m^2$ Now

$V_1 = T_2$ $\Rightarrow$ $\frac{1}{2} mgR\theta_m^2 = \frac{1}{2} MR^2\omega^2\theta_m^2$

2nd way: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} M R^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega \theta_m$

so $T_2 = \frac{1}{2} M R^2 \omega^2 \theta_m^2$ Now

$V_1 = T_2$ $\Rightarrow$ $\cancel{\tfrac{1}{2}} mgR\theta_m^2 = \cancel{\tfrac{1}{2}} M R^2 \omega^2 \theta_m^2$

<u>2nd way:</u> Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} M R^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega \theta_m$

so $T_2 = \frac{1}{2} M R^2 \omega^2 \theta_m^2$ Now

$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\, mgR\theta_m^2 = \cancel{\tfrac{1}{2}}\, M R^2 \omega^2 \theta_m^2$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0 \Rightarrow \cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2} M R^2 \dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\theta_m$

so $T_2 = \frac{1}{2} M R^2 \omega^2 \theta_m^2$ Now

$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\cancel{m}g\cancel{R}\theta_m^2 = \cancel{\tfrac{1}{2}}\cancel{M}R^{\cancel{2}}\omega^2\theta_m^2$

<u>2nd way</u>: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0 \Rightarrow \overset{0}{\cancel{T_1}} + V_1 = T_2 + \overset{0}{\cancel{V_2}}$

$\Rightarrow V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2}MR^2\dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\theta_m$

so $T_2 = \frac{1}{2}MR^2\omega^2\theta_m^2$ Now

$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\cancel{m}g\cancel{R}\cancel{\theta_m^2} = \cancel{\tfrac{1}{2}}\cancel{M}R^{\cancel{2}}\omega^2\cancel{\theta_m^2}$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$ $\Rightarrow$ $\cancel{T_1}^{0} + V_1 = T_2 + \cancel{V_2}^{0}$

$\Rightarrow$ $V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

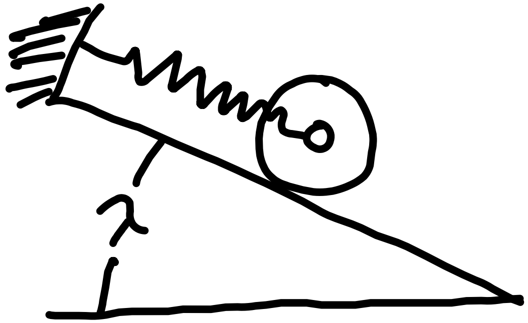
$T_2 = \frac{1}{2}MR^2\dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\,\theta_m$

so $T_2 = \frac{1}{2}MR^2\omega^2\theta_m^2$ Now

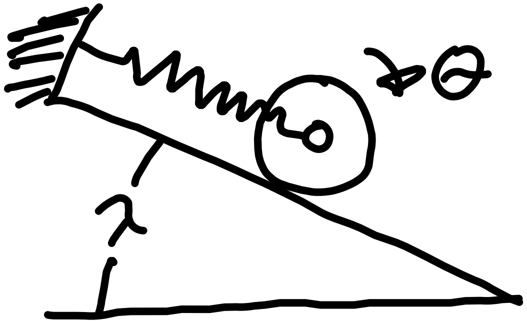
$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\cancel{m}gR\cancel{\theta_m^2} = \cancel{\tfrac{1}{2}}\cancel{M}R^{\cancel{2}}\omega^2\cancel{\theta_m^2}$

$$\Rightarrow \boxed{\sqrt{\frac{g}{R}} = \omega}$$

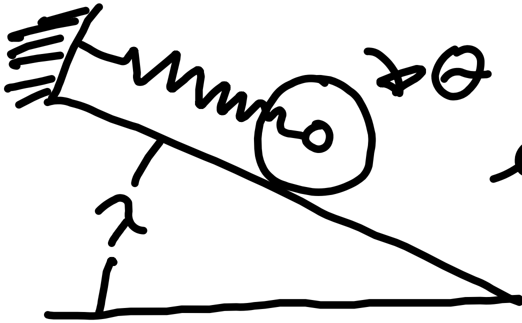
Another example :



Another example : push forward θ

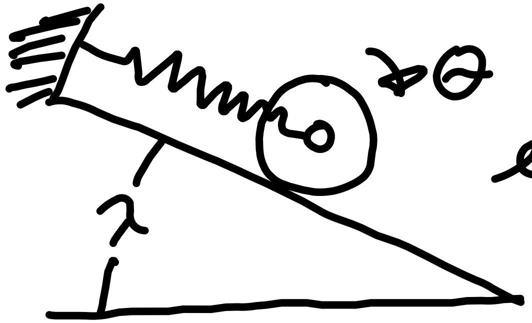


Another example: push forward θ. In lecture 32 we found that



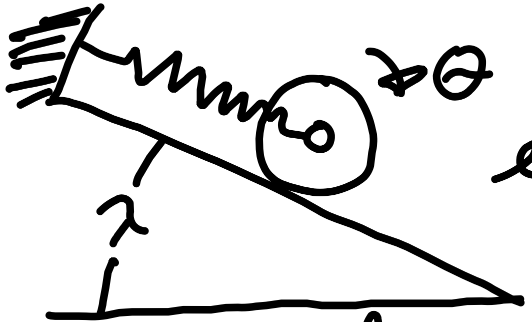
$$\omega_n = \sqrt{\frac{2k}{3M}}$$

Another example : push forward @. In lecture 32 we found that



$\omega_n = \sqrt{\dfrac{2k}{3M}}$ by using $\sum \vec{M} = I \vec{\alpha}$

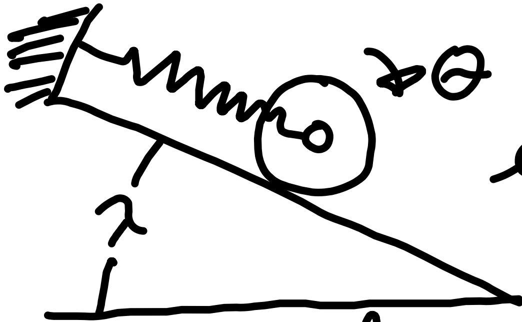
Another example: push forward θ. In lecture 32 we found that



$\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\sum \vec{M} = I\vec{\alpha}$

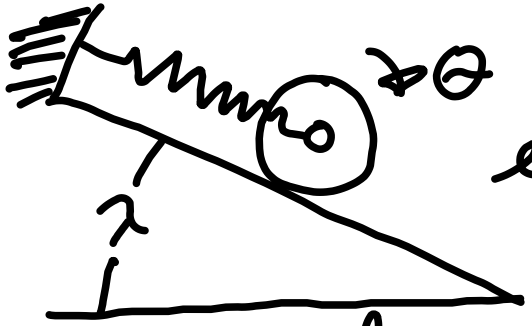
we want to analyze using energy conservation:

Another example: push forward θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\sum \vec{M} = I\vec{\alpha}$



we want to analyze using energy conservation: $T+V = \text{const.}$

Another example: push forward θ. In lecture 32 we found that

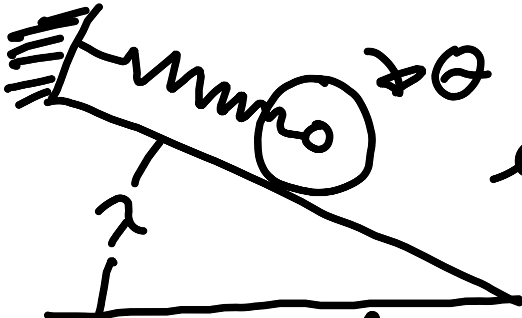


$$\omega_n = \sqrt{\frac{2k}{3M}}$$ by using $\sum \vec{M} = I\vec{\alpha}$

We want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

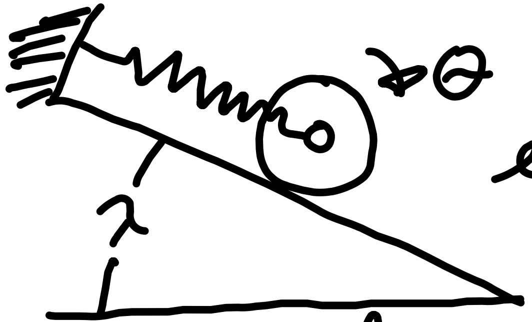
Another example: push forward θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I\vec{\alpha}$



we want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2}M\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2$$

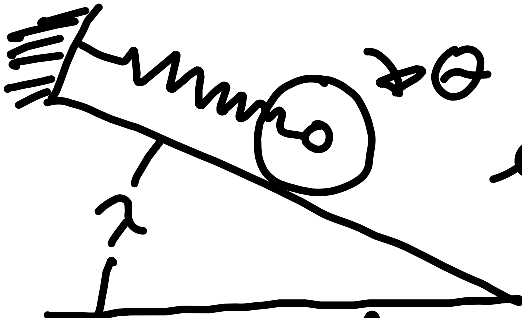
Another example: push forward θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\sum \vec{M} = I\vec{\alpha}$



we want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2}M\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 = \frac{3}{4}MR^2\dot{\theta}^2$$

Another example: push forward θ. In lecture 32 we found that

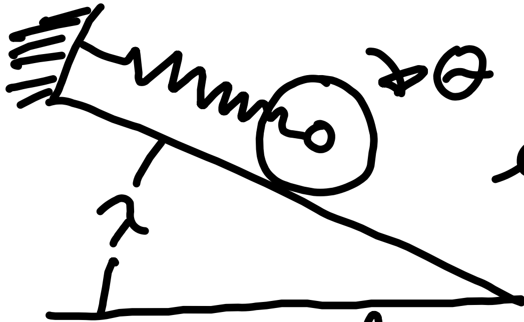


$\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\sum \vec{M} = I\vec{\alpha}$

we want to analyze using energy conservation: $T + V = \text{const.}$, where

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 = \frac{3}{4}MR^2\dot{\theta}^2$

& $V = \frac{1}{2}k(L - L_0)^2 + mgh$



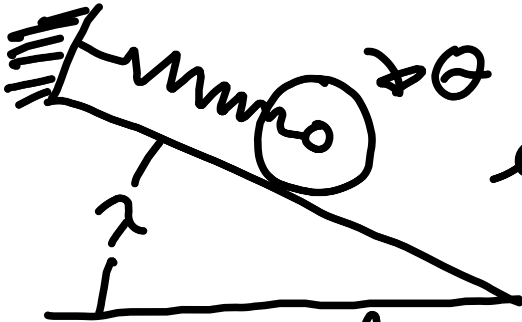
Another example: push forward θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I \vec{\alpha}$

We want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} MR^2 \dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\dot{\theta}^2 = \frac{3}{4} MR^2 \dot{\theta}^2$$

& $V = \frac{1}{2} k (L - L_0)^2 + mgh$ but $h = -(\sin\lambda) R\theta$

Another example: push forward θ. In lecture 32 we found that



$\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I \vec{\alpha}$

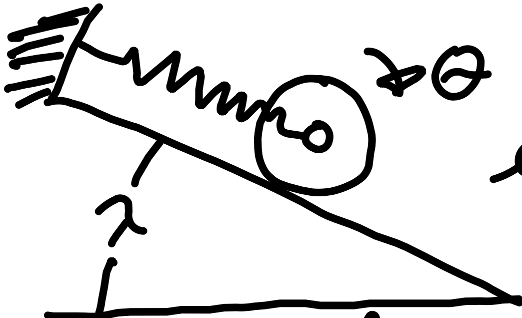
we want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} MR^2 \dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\dot{\theta}^2 = \frac{3}{4} MR^2 \dot{\theta}^2$$

$$\& \; V = \frac{1}{2} k (L - L_0)^2 + mgh \quad \text{but} \quad h = -(\sin\lambda) R\theta$$

Only 1st order in θ.

Another example: push forward Θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = \bar{I}\vec{\alpha}$



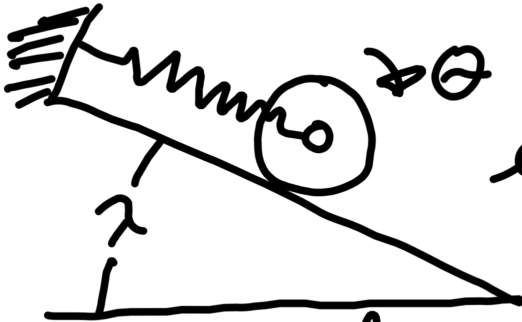
we want to analyze using energy conservation: $T+V = \text{const.}$, where

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}MR^2\dot{\Theta}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\dot{\Theta}^2 = \tfrac{3}{4}MR^2\dot{\Theta}^2$$

$$\& \; V = \tfrac{1}{2}k(L-L_0)^2 + mgh \quad \text{but } h = -(\sin\lambda)R\Theta$$

Only 1st order in Θ. Could cause us some trouble

Another example: push forward θ. In lecture 32 we found that



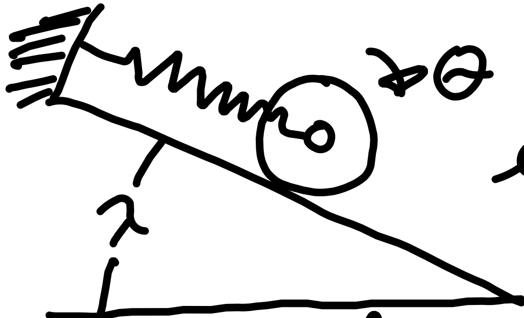
$$\omega_n = \sqrt{\frac{2k}{3M}}$$ by using $\Sigma \vec{M} = I\vec{\alpha}$

We want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}MR^2\dot{\theta}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\dot{\theta}^2 = \tfrac{3}{4}MR^2\dot{\theta}^2$$

& $V = \tfrac{1}{2}k(L - L_0)^2 + mgh$ but $h = -(\sin\lambda)R\theta$

Only 1st order in θ. Could cause us some trouble ↳ ☹



Another example: push forward θ. In lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I \vec{\alpha}$

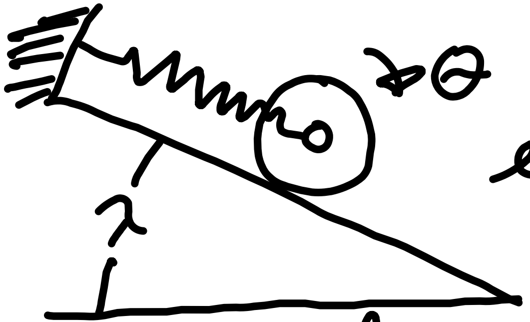
We want to analyze using energy conservation: $T + V = \text{const.}$, where

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} MR^2 \dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\dot{\theta}^2 = \frac{3}{4} MR^2 \dot{\theta}^2$$

& $V = \frac{1}{2} k (L - L_0)^2 + mgh$ but $h = -(\sin\lambda) R\theta$

This is where it gets tricky:

Another example: push forward Θ. In



lecture 32 we found that $\omega_n = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I \vec{\alpha}$

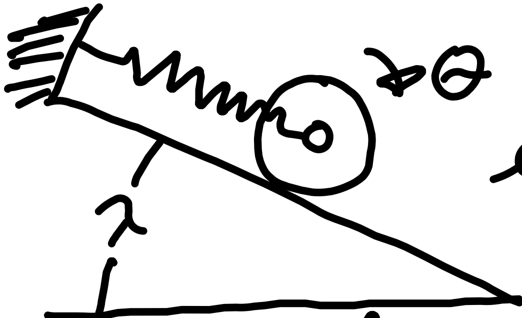
We want to analyze using energy conservation: $T+V = \text{const.}$, where

$$T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} MR^2 \dot{\Theta}^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right) \dot{\Theta}^2 = \frac{3}{4} MR^2 \dot{\Theta}^2$$

$\&$ $V = \frac{1}{2} k (L - L_0)^2 + mgh$ but $h = -(\sin\lambda) R\Theta$

This is where it gets tricky: We have potential energy in terms of L

Another example: push forward θ. In lecture 32 we found that $\omega = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I\vec{\alpha}$



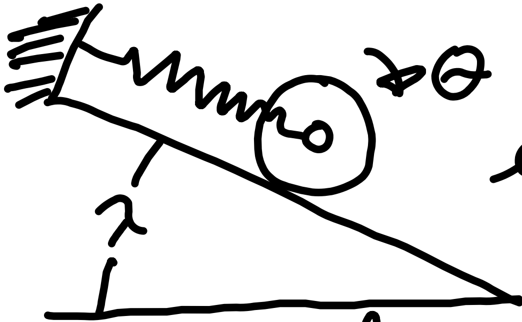
We want to analyze using energy conservation: $T+V = \text{const.}$, where

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 = \frac{3}{4}MR^2\dot{\theta}^2$$

& $V = \frac{1}{2}k(L-L_0)^2 + mgh$ but $h = -(\sin\lambda)R\theta$

This is where it gets tricky: We have potential energy in terms of L & want to get everything in terms of θ.

Another example: push forward Θ. In lecture 32 we found that $\omega = \sqrt{\frac{2k}{3M}}$ by using $\Sigma \vec{M} = I\vec{\alpha}$



We want to analyze using energy conservation: $T+V = \text{const.}$, where

$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}MR^2\dot{\Theta}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\dot{\Theta}^2 = \tfrac{3}{4}MR^2\dot{\Theta}^2$$

& $V = \frac{1}{2}k(L-L_0)^2 + mgh$ but $h = -(\sin\lambda)R\Theta$

<u>This is where it gets tricky</u>: We have potential energy in terms of L & want to get everything in terms of Θ. We have to be careful in how we go about this.

From previous slide

$$V = \frac{1}{2}k(L-L_0)^2 + mgh$$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

From previous slide

$V = \frac{1}{2}k(L - L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

<u>Note</u>: We can <u>NOT</u> take $L - L_0 = x$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We MUST measure x from equilibrium.

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We MUST measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$.

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

<u>Note</u>: We can <u>NOT</u> take $L - L_0 = x$. We <u>must</u> measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We must measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can **NOT** take $L - L_0 = x$. We **must** measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium $L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

<u>Note</u>: We can <u>NOT</u> take $L - L_0 = x$. We <u>must</u> measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can **NOT** take $L - L_0 = x$. We **must** measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\theta$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We MUST measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\theta$, then

$V = \frac{1}{2}k(R\theta + \delta_{ST})^2 - mgR\theta\sin\beta$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\Theta$

Note: We can NOT take $L - L_0 = x$. We MUST measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\Theta$, then

$V = \frac{1}{2}k(R\Theta + \delta_{ST})^2 - mgR\Theta\sin\beta \Rightarrow$

$V = \frac{1}{2}kR^2\Theta^2 +$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\Theta$

<u>Note</u>: We can <u>NOT</u> take $L - L_0 = x$. We <u>must</u> measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\Theta$, then

$V = \frac{1}{2}k(R\Theta + \delta_{ST})^2 - mgR\Theta\sin\beta \Rightarrow$

$V = \frac{1}{2}kR^2\Theta^2 + kR\Theta\delta_{ST} +$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\Theta$

<u>Note</u>: We can <u>NOT</u> take $L - L_0 = x$. We <u>must</u> measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\Theta$, then

$V = \frac{1}{2}k(R\Theta + \delta_{ST})^2 - mgR\Theta\sin\beta \Rightarrow$

$V = \frac{1}{2}kR^2\Theta^2 + kR\Theta\delta_{ST} + \frac{1}{2}k\delta_{ST}^2 -$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We must measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\theta$, then

$V = \frac{1}{2}k(R\theta + \delta_{ST})^2 - mgR\theta\sin\beta \Rightarrow$

$V = \frac{1}{2}kR^2\theta^2 + kR\theta\delta_{ST} + \frac{1}{2}k\delta_{ST}^2 - mgR\theta\sin\beta$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can **NOT** take $L - L_0 = x$. We **must** measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\theta$, then

$$V = \frac{1}{2}k(R\theta + \delta_{ST})^2 - mgR\theta\sin\beta \Rightarrow$$

$$V = \frac{1}{2}kR^2\theta^2 + kR\theta\delta_{ST} + \underbrace{\frac{1}{2}k\delta_{ST}^2}_{\text{const.}} - mgR\theta\sin\beta$$

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\theta$

Note: We can NOT take $L - L_0 = x$. We must measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\theta$, then

$$V = \frac{1}{2}k(R\theta + \delta_{ST})^2 - mgR\theta\sin\beta \Rightarrow$$

$$V = \frac{1}{2}kR^2\theta^2 + kR\theta\delta_{ST} + \underbrace{\frac{1}{2}k\delta_{ST}^2}_{\text{const.}} - mgR\theta\sin\beta$$

1^{ST} order in θ

From previous slide

$V = \frac{1}{2}k(L-L_0)^2 + mgh$ & $h = -(\sin\beta)R\Theta$

Note: We can NOT take $L - L_0 = x$. We must measure x from equilibrium. At equilibrium $L_{EQ} = L_0 + \delta_{ST}$. So, away from equilibrium

$L = L_{EQ} + x \Rightarrow L - L_0 = L_{EQ} + x - L_0 = L_0 + \delta_{ST} + x - L_0$

$\Rightarrow L - L_0 = x + \delta_{ST}$ & since $x = R\Theta$, then

$$V = \frac{1}{2}k(R\Theta + \delta_{ST})^2 - mgR\Theta\sin\beta \Rightarrow$$

$$V = \frac{1}{2}kR^2\Theta^2 + \underbrace{kR\Theta\delta_{ST}} + \underbrace{\frac{1}{2}k\delta_{ST}^2}_{\text{const.}} - \underbrace{mgR\Theta\sin\beta}$$

1^{st} order in Θ

1^{st} order in Θ not expected ☹

We can rewrite as

$$V = \frac{1}{2}kR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2 + R\theta\left[k\delta_{ST} - mg\sin\beta\right]$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}kR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + R\theta\left[k\delta_{ST} - mg\sin\beta\right]$$

We can rewrite as

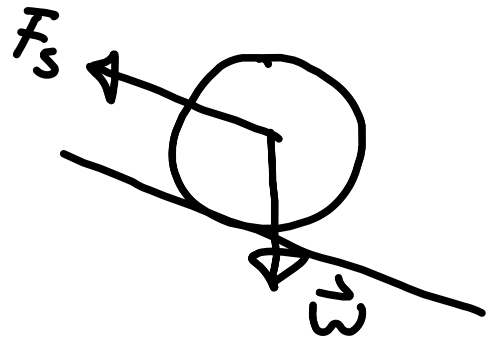
$$V = \underbrace{\tfrac{1}{2} k R^2 \theta^2 + \tfrac{1}{2} k \delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underline{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}$$

part we do not like

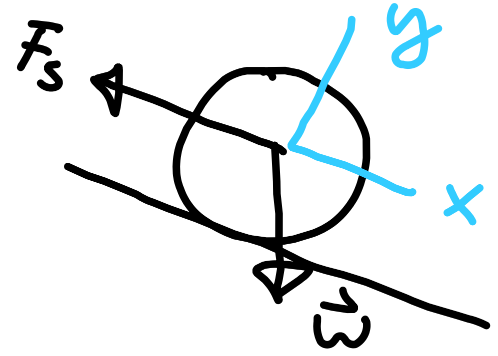
Note: At equilibrium



We can rewrite as

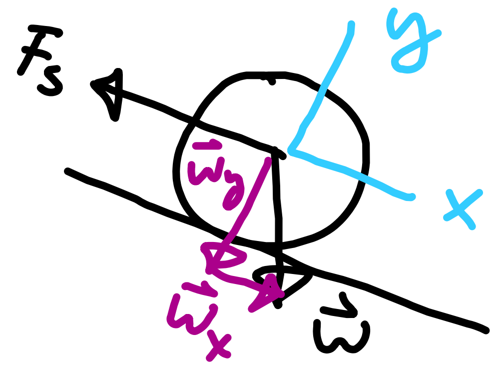
$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}K\delta_{ST}^2}_{\text{part we like ☺}} + \underbrace{R\theta\left[K\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like ☹}}$$

<u>Note</u>: At equilibrium



We can rewrite as

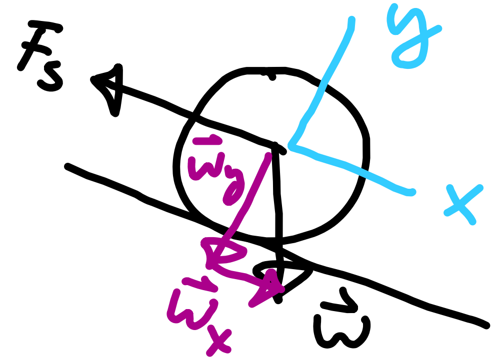
$$V = \underbrace{\tfrac{1}{2}kR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like ☺}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like ☹}}$$



Note: At equilibrium

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like ☺}} + \underbrace{R\theta[k\delta_{ST} - mg\sin\beta]}_{\text{part we do not like ☹}}$$

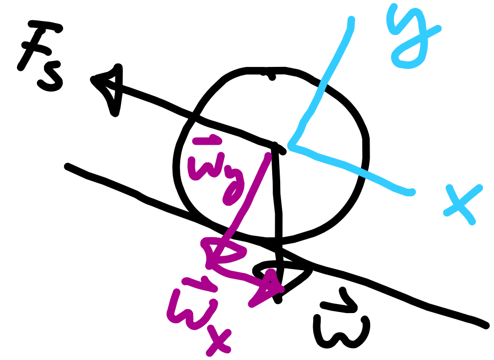


Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like ☺}} + \underbrace{R\theta[k\delta_{ST} - mg\sin\beta]}_{\text{part we do not like ☹}}$$



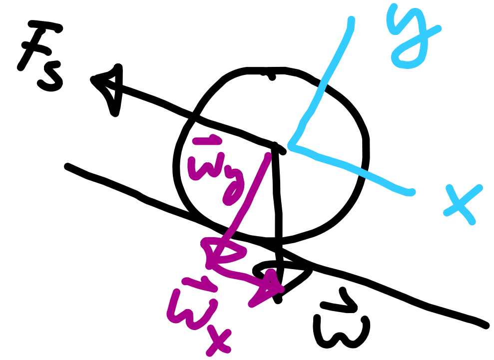
Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

$$\Rightarrow -k\delta_{ST} + mg\sin\beta = 0$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



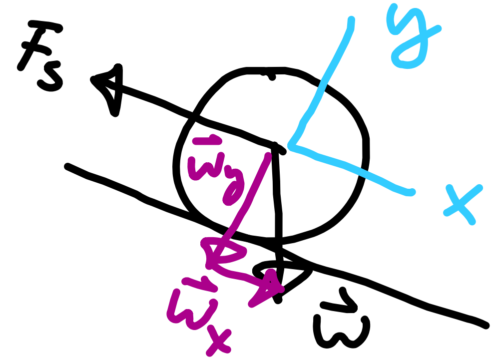
Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{W}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

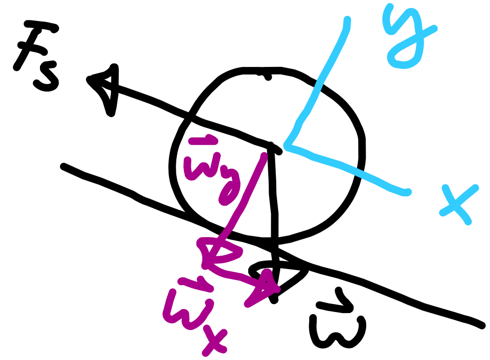
$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{W}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

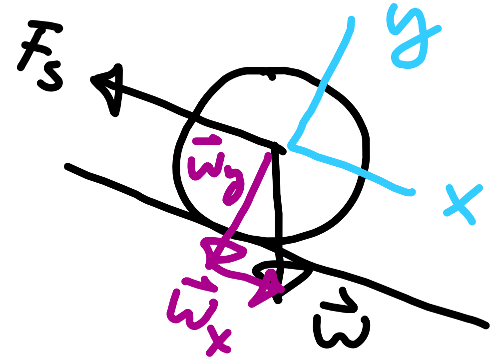
$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{W}_x = 0$$

$$\Rightarrow -k\delta_{ST} + mg\sin\beta = 0$$

Now $V = \frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2$

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

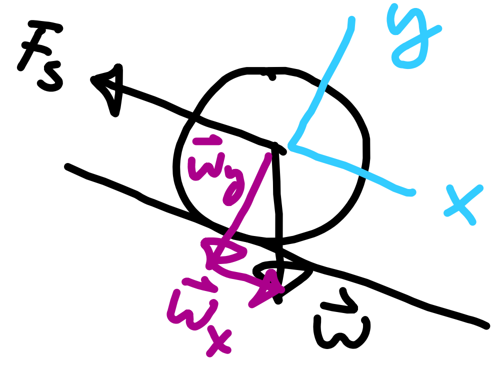
$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}$ So $T+V = \text{const.}$

$$\Rightarrow \tfrac{3}{4}MR^2\dot{\theta}^2 + \tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2 = \text{const}$$

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

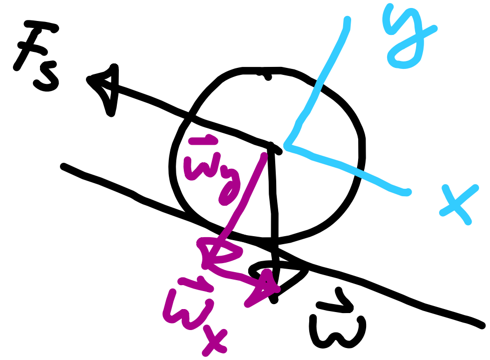
$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\tfrac{1}{2}KR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}$ So $T+V=\text{const.}$

$$\Rightarrow \tfrac{3}{4}MR^2\dot{\theta}^2 + \tfrac{1}{2}KR^2\theta^2 + \underbrace{\tfrac{1}{2}k\delta_{ST}^2}_{\text{const}} = \text{const}$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

$$\sum F_x = 0 \Rightarrow \vec{F}_s + \vec{W}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}$ So $T+V = \text{const.}$

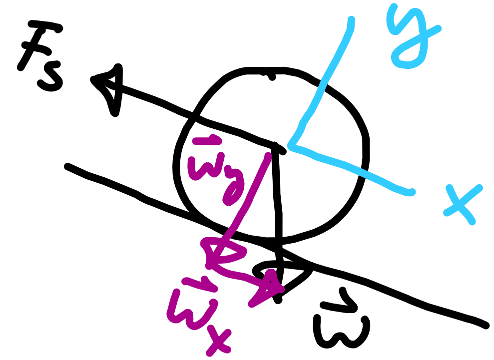
$$\Rightarrow \frac{3}{4}MR^2\dot{\theta}^2 + \frac{1}{2}KR^2\theta^2 + \underbrace{\frac{1}{2}k\delta_{ST}^2}_{\text{const}} = \text{const} \Rightarrow$$

$$\frac{3}{4}MR^2\dot{\theta}^2 + \frac{1}{2}KR^2\theta^2 = \text{const.}$$

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}kR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$

Note: At equilibrium



$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\tfrac{1}{2}kR^2\theta^2 + \tfrac{1}{2}k\delta_{ST}^2}$ $\therefore$ $T+V = \text{const.}$

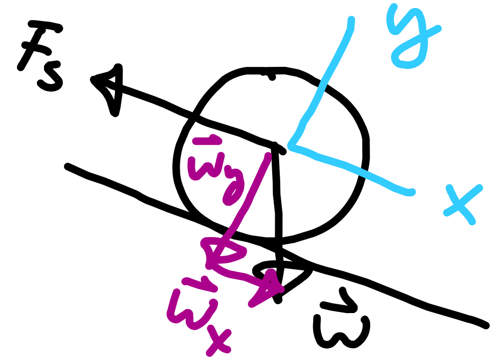
$$\Rightarrow \tfrac{3}{4}MR^2\dot{\theta}^2 + \tfrac{1}{2}kR^2\theta^2 + \underbrace{\tfrac{1}{2}k\delta_{ST}^2}_{\text{const}} = \text{const} \Rightarrow$$

$$\tfrac{3}{4}MR^2\dot{\theta}^2 + \tfrac{1}{2}kR^2\theta^2 = \text{const.} \Rightarrow$$

$$\dot{\theta}^2 + \omega^2\theta^2 = \text{const},$$

We can rewrite as

$$V = \underbrace{\tfrac{1}{2}KR^2\Theta^2 + \tfrac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\Theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{w}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\tfrac{1}{2}KR^2\Theta^2 + \tfrac{1}{2}k\delta_{ST}^2}$. So $T+V = \text{const}$

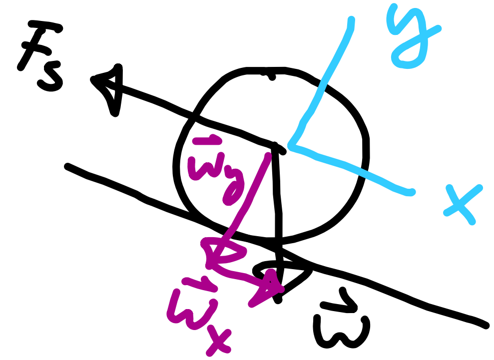
$$\Rightarrow \tfrac{3}{4}MR^2\dot{\Theta}^2 + \tfrac{1}{2}KR^2\Theta^2 + \underbrace{\tfrac{1}{2}k\delta_{ST}^2}_{\text{const}} = \text{const} \Rightarrow$$

$$\tfrac{3}{4}MR^2\dot{\Theta}^2 + \tfrac{1}{2}KR^2\Theta^2 = \text{const}. \Rightarrow$$

$$\dot{\Theta}^2 + \omega^2\Theta^2 = \text{const}, \text{ where } \omega = \sqrt{\frac{2K}{3M}}$$

We can rewrite as

$$V = \underbrace{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}_{\text{part we like}} + \underbrace{R\theta\left[k\delta_{ST} - mg\sin\beta\right]}_{\text{part we do not like}}$$



Note: At equilibrium

$$\Sigma F_x = 0 \Rightarrow \vec{F}_s + \vec{W}_x = 0$$

$$\Rightarrow \underline{-k\delta_{ST} + mg\sin\beta = 0}$$

Now $V = \underline{\frac{1}{2}KR^2\theta^2 + \frac{1}{2}k\delta_{ST}^2}$ So $T+V=\text{const.}$

$$\Rightarrow \frac{3}{4}MR^2\dot{\theta}^2 + \frac{1}{2}KR^2\theta^2 + \underbrace{\frac{1}{2}k\delta_{ST}^2}_{\text{const}} = \text{const} \Rightarrow$$

$$\frac{3}{4}MR^2\dot{\theta}^2 + \frac{1}{2}KR^2\theta^2 = \text{const.} \Rightarrow$$

$$\dot{\theta}^2 + \omega^2\theta^2 = \text{const}, \text{ where } \omega = \sqrt{\frac{2K}{3M}}$$

As before

The last problem can serve as a cautionary tale

The last problem can serve as a cautionary tale:

At first glance, a problem may look easy to do using energy conservation

<u>The last problem can serve as a cautionary tale</u>:

At first glance, a problem may look easy to do using energy conservation, but may end up more difficult than if kinematics were used instead