Today 19.4

$\boxed{L35}$

Today 19.4

L35

Forced Vibrations

<u>Today</u> 19.4

<u>Monday</u> 19.4, 19.5

$\boxed{L35}$

L35

Today 19.4

Monday 19.4, $\underbrace{19.5}_{}$

Damped Vibrations

$\boxed{L35}$

Today 19.4

Monday 19.4, 19.5

Wednesday Review

L35

Today 19.4

Monday 19.4, 19.5

Wednesday Review

Friday Nov. 27th Holiday

L35

Today 19.4

Monday 19.4, 19.5

Wednesday Review

Friday Nov. 27th Holiday

Monday Nov. 30th Exam #4

L35

Today 19.4

Monday 19.4, 19.5

Wednesday Review

Friday Nov. 27th Holiday

Monday Nov. 30th Exam #4

Wednesday Dec. 2nd Day of Reckoning

L35

Today 19.4

Monday 19.4, 19.5

Wednesday Review

Friday Nov. 27th Holiday

Monday Nov. 30th Exam #4

Wednesday Dec. 2nd Day of Reckoning

Friday Dec. 4th Final exam

Unforced vibrations

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* 1st way: Use kinematics to get equation of motion into the form $\ddot{x} = -\omega_n^2 x$

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* 1ST way: Use kinematics to get equation of motion into the form $\ddot{x} = -\omega_n^2 x$
* 2nd way:

Unforced vibrations

So far we have been concerned about unforced vibrations, where we displace a system from equilibrium and determine the natural frequency or natural period. Typically we proceed in one of two ways:

* 1st way: Use kinematics to get equation of motion into the form $\ddot{x} = -\omega_n^2 x$
* 2nd way: Use conservation of energy to get equation of motion into the form

$$\dot{x}^2 + \omega^2 x^2 = \text{const.}$$

Solution : $x = x_m \sin(\omega_m t + \phi)$

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$\dot{x}_m = \omega_n x_m$

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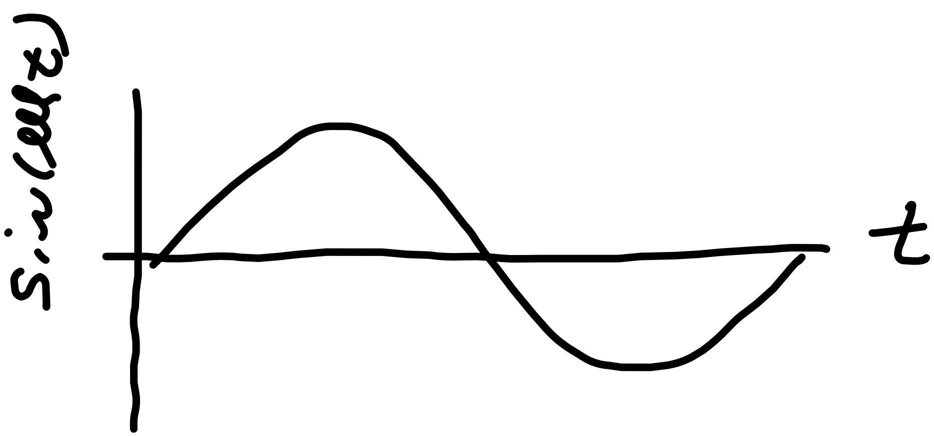

Solution : $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

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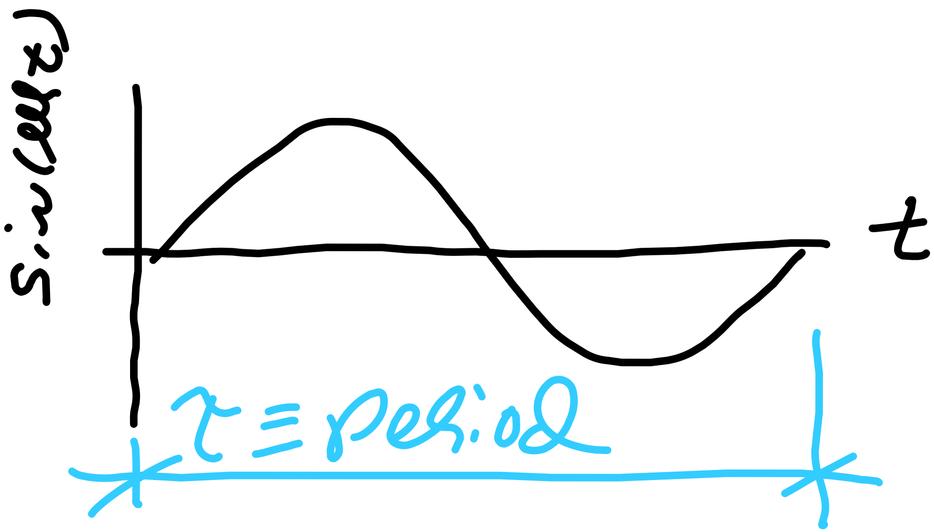

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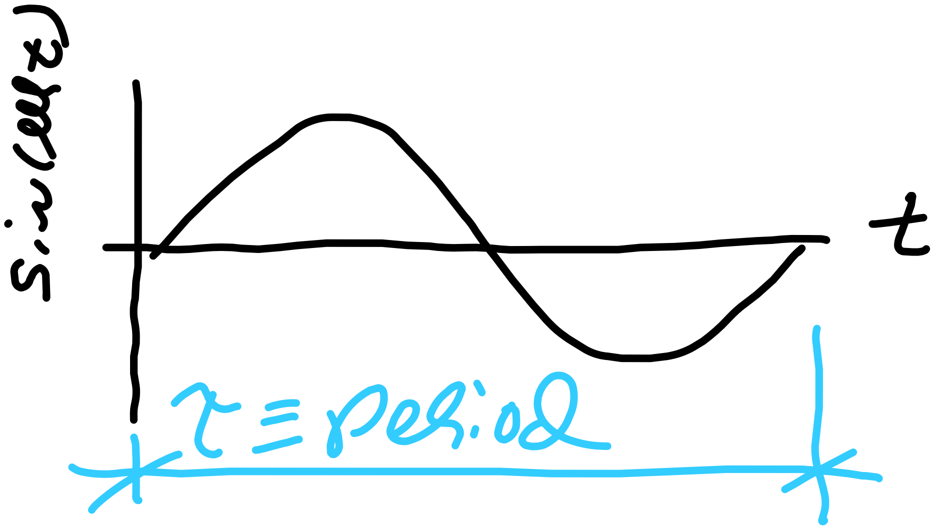
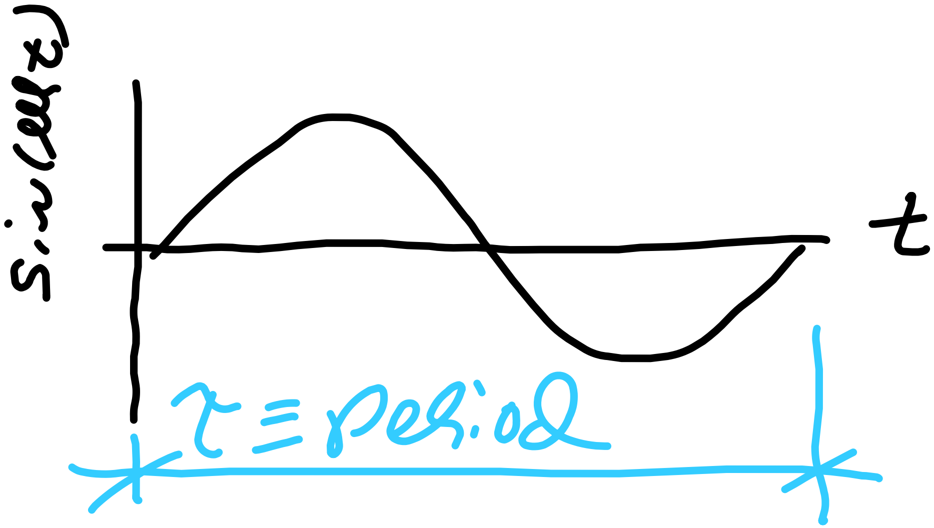


$\omega\tau = 2\pi$

Solution: $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

$\dot{x}_m = \omega_n x_m$ & $\ddot{x}_m = \omega_n^2 x_m$



$\omega \tau = 2\pi \Rightarrow \tau = \dfrac{2\pi}{\omega}$

Solution : $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

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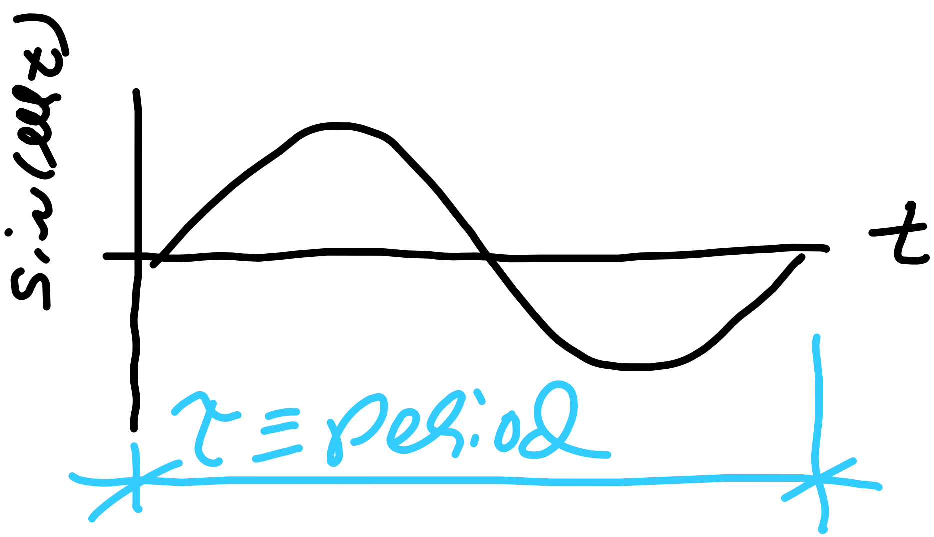

$\omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$.

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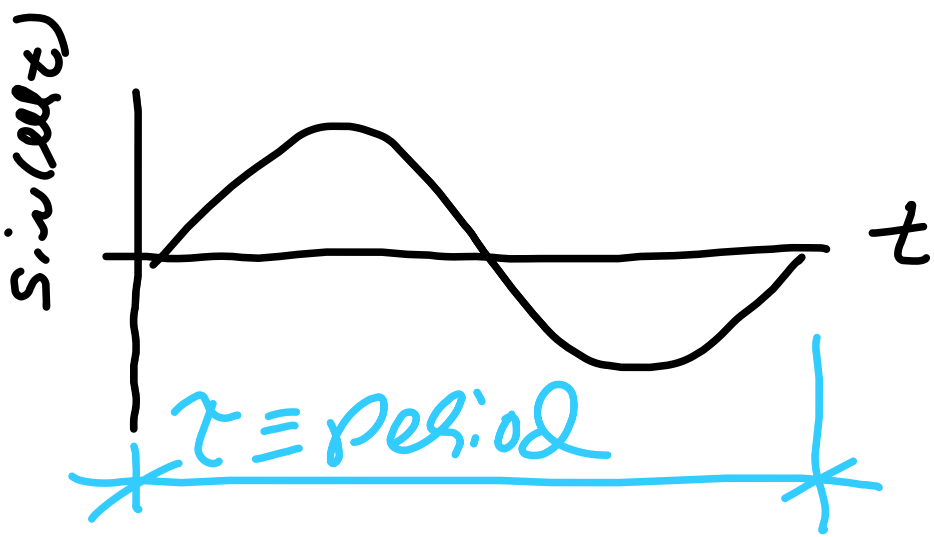

$\omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

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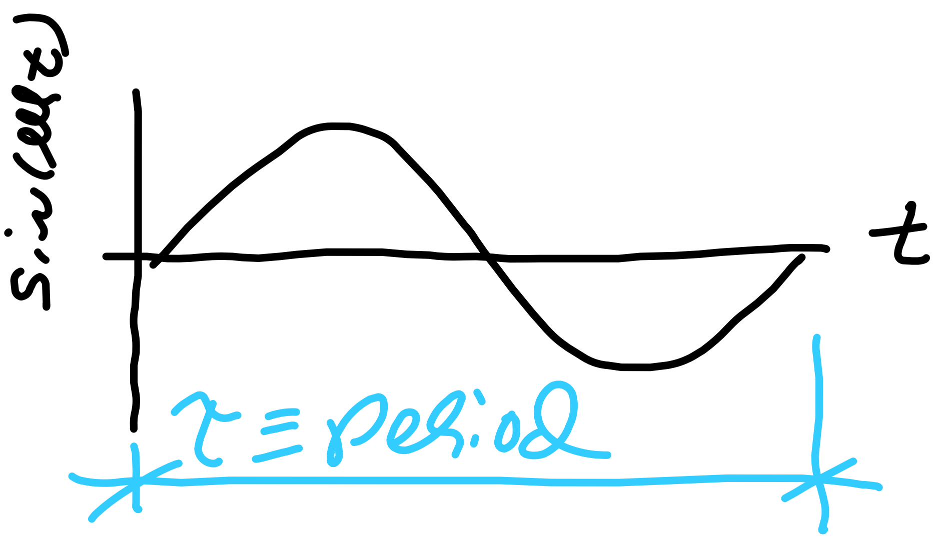

$\omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$, where

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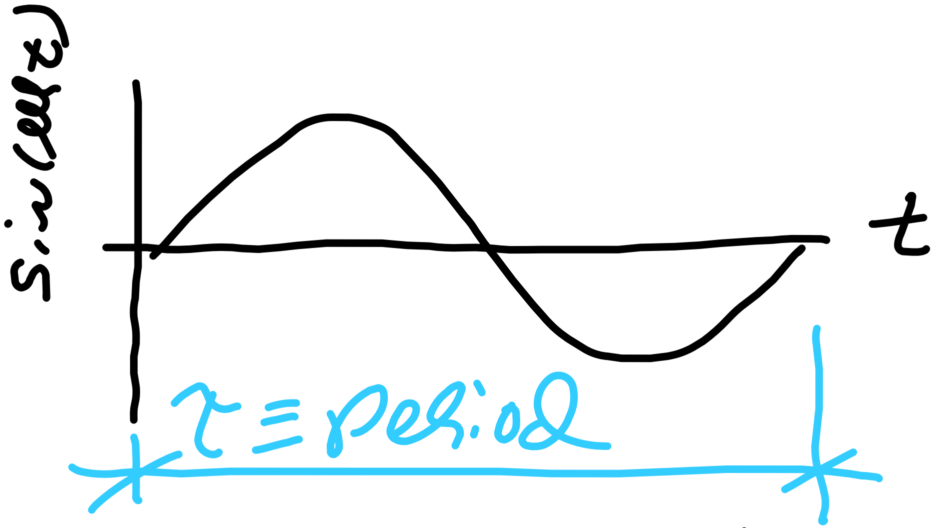
Q: What happens to x_m when the forced frequency matches the natural frequency?

<u>Solution</u>: $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

$$\dot{x}_m = \omega_n x_m \quad \& \quad \ddot{x}_m = \omega_n^2 x_m$$



$\omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$, where

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What would happen if we were to force the system at the same frequency as the natural frequency

Forced vibrations – Math

Forced vibrations – Math

If we were to analyze a physical system and obtain an equation of the form

Forced vibrations – Math

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$$A\ddot{x} + Bx = C\sin(\omega_F t)$$

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Our general solution has two parts:

Forced vibrations – Math

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* Homogeneous part

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If we were to analyze a physical system and obtain an equation of the form

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Our general solution has two parts:

* Homogeneous part (forcing term $= 0$):

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$$A\ddot{x} + Bx = 0$$

Forced vibrations – Math

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Forced vibrations – Math

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Our general solution has two parts:

* Homogeneous part (forcing term $= 0$):

$A\ddot{x} + Bx = 0 \Rightarrow \ddot{x} = -\omega_n^2 x$, where $\omega_n = \sqrt{\frac{B}{A}}$

Forced vibrations – Math

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Forced vibrations – Math

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* Particular solution: Assume the form

$$x_{part} = x_m \sin(\omega_F t)$$

Forced vibrations – Math

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* Particular solution: Assume the form

$x_{part} = X_m \sin(\omega_F t)$. Now $-A\omega_F^2 X_m + BX_m = C$

Forced vibrations – Math

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$$\Rightarrow X_m = \frac{C}{-A\omega_F^2 + B}$$

Forced vibrations – Math

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$x_{part} = X_m \sin(\omega_F t)$. Now $-A\omega_F^2 X_m + BX_m = C$

$$\Rightarrow X_m = \frac{C}{-A\omega_F^2 + B} = \frac{(C/B)}{-A\omega_F^2/B + 1}$$

Forced vibrations – Math

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Our general solution has two parts:

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$$A\ddot{x} + Bx = 0 \Rightarrow \ddot{x} = -\omega_N^2 x, \text{ where } \omega_N = \sqrt{\frac{B}{A}}$$

* Particular solution: Assume the form

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$$\Rightarrow x_m = \frac{C}{-A\omega_F^2 + B} = \frac{(C/B)}{-A\omega_F^2/B + 1}$$ But $\omega_N^2 = \frac{B}{A}$

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$\Rightarrow X_m = \dfrac{C}{-A\omega_F^2 + B} = \dfrac{(C/B)}{-A\omega_F^2/B + 1}$ But $\omega_N^2 = \frac{B}{A}$

So $\boxed{X_m = \dfrac{(C/B)}{1 - \omega_F^2/\omega_N^2}}$

From previous

$$X_m = \frac{(C/B)}{1 - \omega_F^2/\omega_N^2}$$

From previous

$$X_m = \frac{(C/B)}{1-\omega_f^2/\omega_n^2}$$

What happens to X_m when the forced frequency matches the natural frequency?

From previous

$$x_m = \frac{(C/B)}{1 - \omega_f^2/\omega_n^2}$$

• What happens to x_m when the forced frequency matches the natural frequency?

See

http://www.public.asu.edu/~dugger/wine-glass-shatter.gif
https://www.youtube.com/watch?v=0FeXjhUEXlc
https://www.youtube.com/watch?v=6IJ99phNArM&feature=youtu.be&t=137

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When $X_m \to \infty$ the system is in resonance

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When $X_m \to \infty$ the system is in resonance.
Obviously X_m can not go infinite.

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When $x_m \to \infty$ the system is in resonance.

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From previous

$X_m = \dfrac{(C/B)}{1-\omega_f^2/\omega_n^2}$ • What happens to X_m when the forced frequency matches the natural frequency?

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When $X_m \to \infty$ the system is in resonance.

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When $x_m \to \infty$ the system is in resonance.

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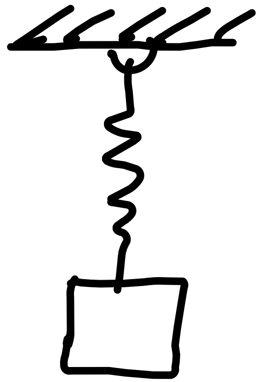
When $x_m \to \infty$ the system is in resonance. Obviously x_m can not go infinite. Instead, a system in resonance will eventually reach a new state that has a different natural frequency or simply destruct [as seen in videos]

Example



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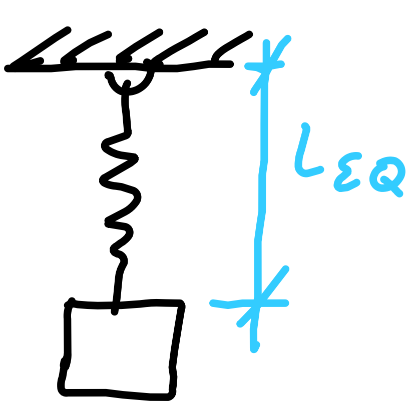

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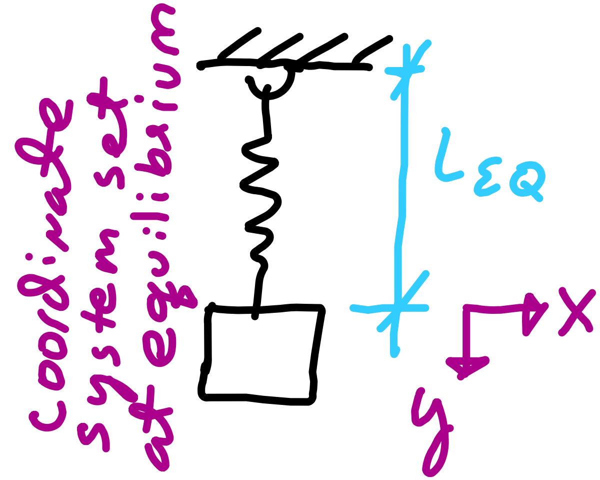

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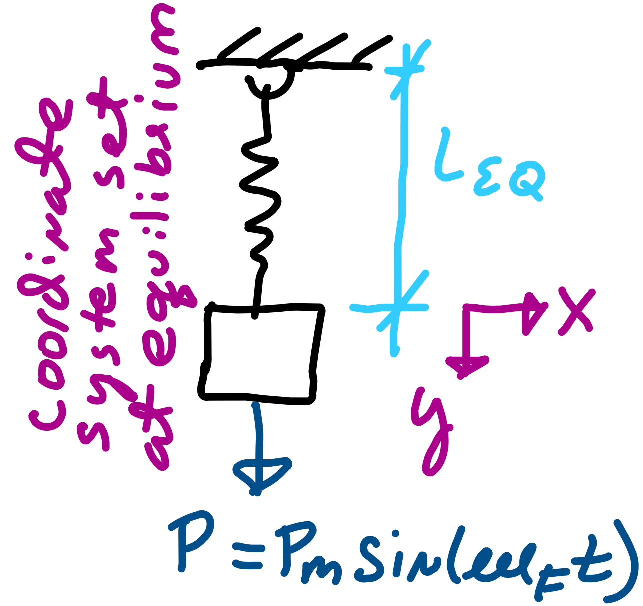
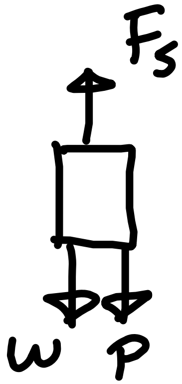
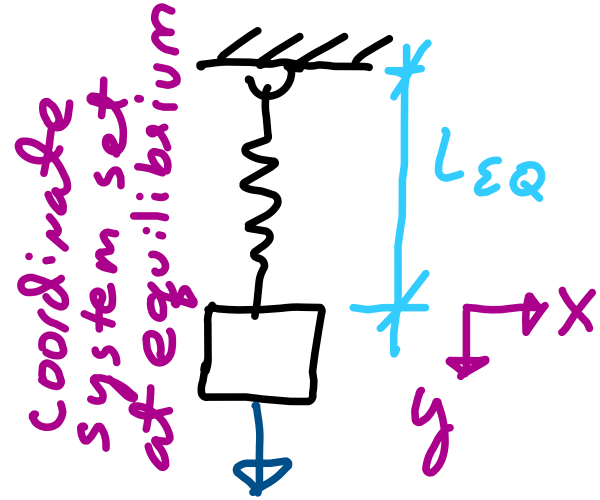


coordinate system set at equilibrium
L_{EQ}
x
y
$P = P_m \sin(\omega_f t)$

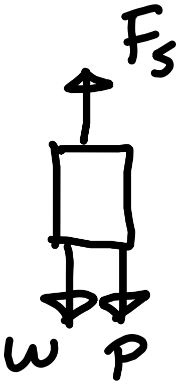
Example



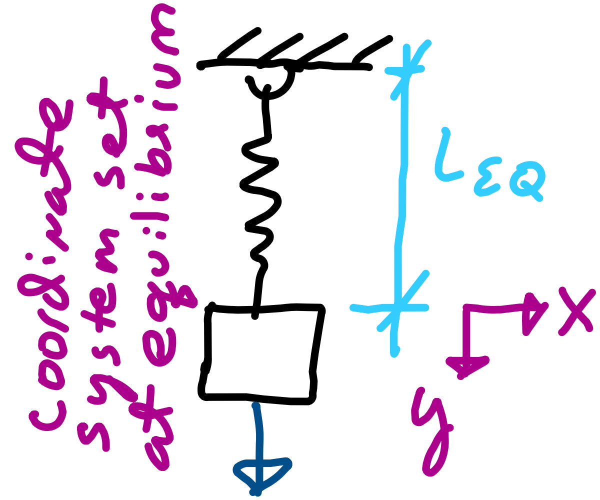


$$P = P_m \sin(\omega_F t)$$

Example



$$\Sigma F_y = m\ddot{y}$$



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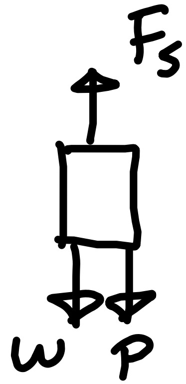
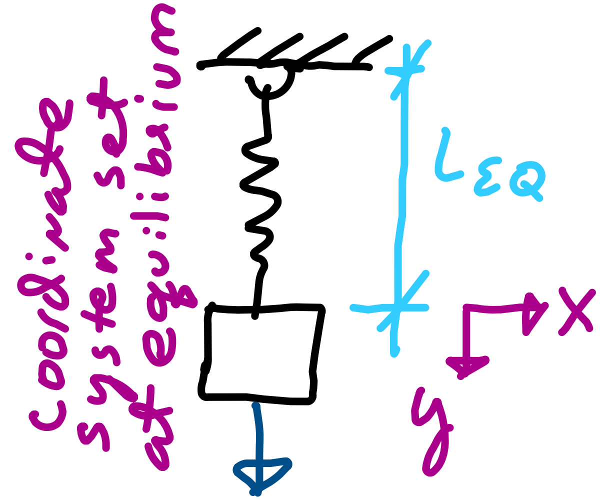


$$\Sigma F_y = m\ddot{y} \Rightarrow$$

$$-K(L - L_0) + W + P = m\ddot{y}$$



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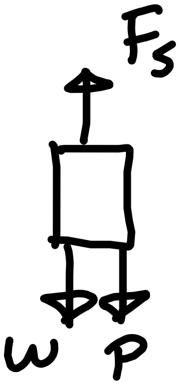

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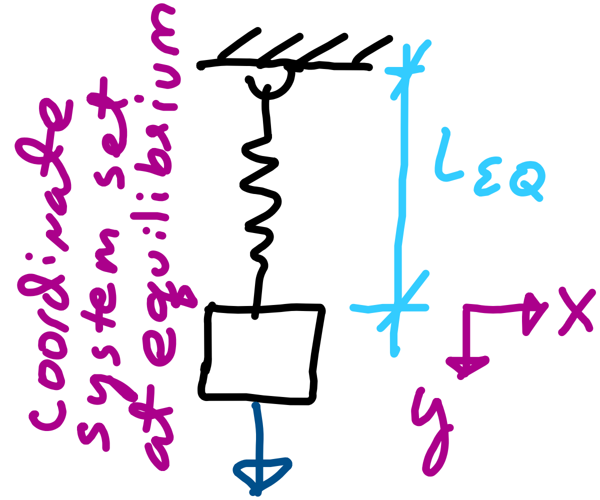
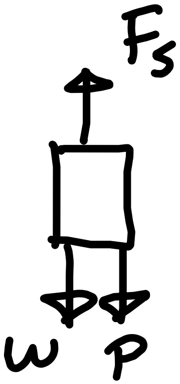
At equilibrium

$$-k(L_{EQ}-L_0)+W=0$$



$$P=P_m\sin(\omega_f t)$$

Example



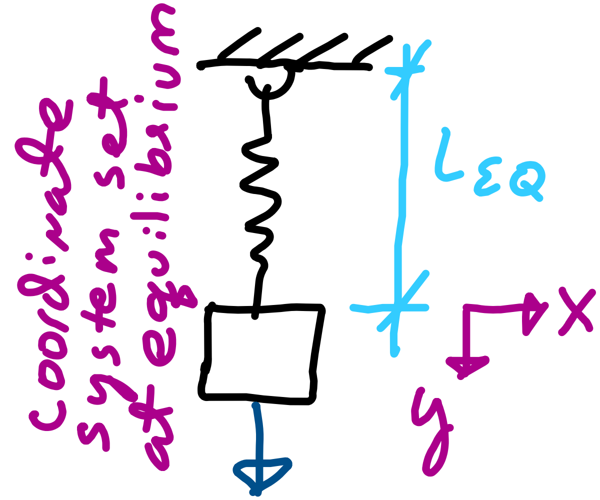
$\Sigma F_y = m\ddot{y} \Rightarrow$

$-K(L-L_0)+W+P=m\ddot{y}$

At equilibrium

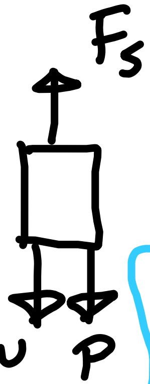
$-K(L_{EQ}-L_0)+W=0$, where

$L_{EQ}=L_0+\delta$



$P = P_m \sin(\omega_F t)$

Example



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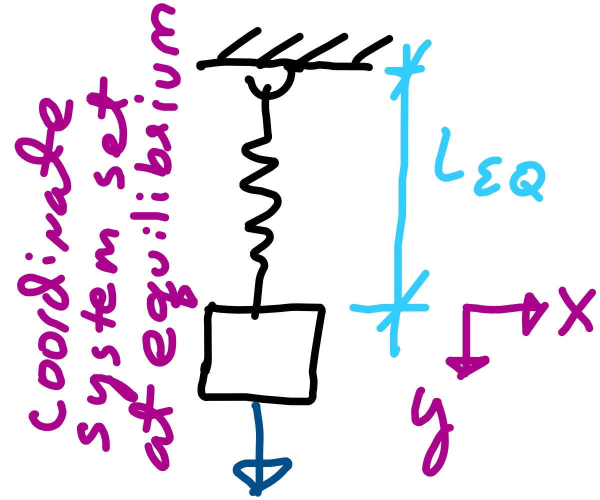
At equilibrium

$-K(L_{EQ}-L_0)+W=0$, where

$L_{EQ}=L_0+\delta$ so $-K\delta+W=0$



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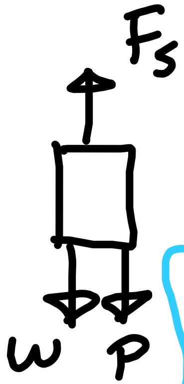

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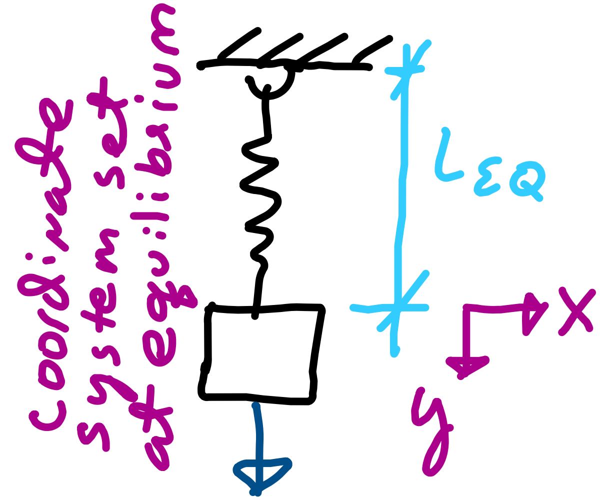
$$-K(L-L_0)+W+P = m\ddot{y}$$

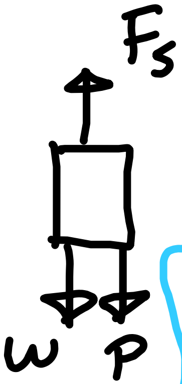
$$\boxed{\begin{array}{l}\underline{\text{At equilibrium}}\\ -K(L_{EQ}-L_0)+W=0, \text{ where}\\ L_{EQ}=L_0+\delta \text{ so } -K\delta+W=0\end{array}}$$



$$P = P_m \sin(\omega_F t)$$

Now $L = L_{EQ} + y =$

Example



$$\Sigma F_y = m\ddot{y} \Rightarrow$$

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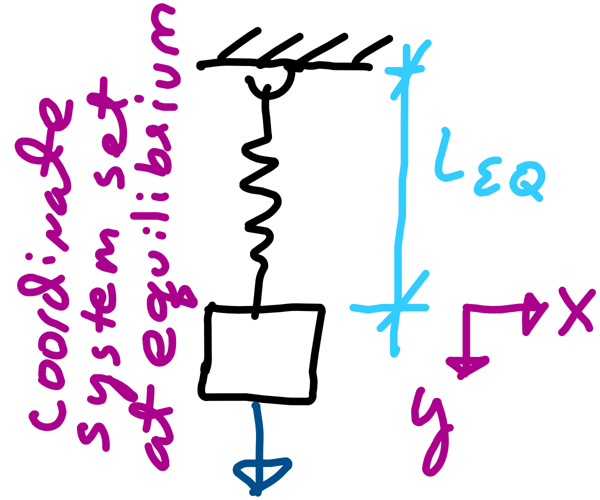
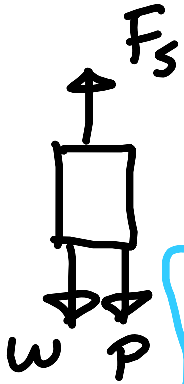


$P = P_m \sin(\omega_F t)$

Example



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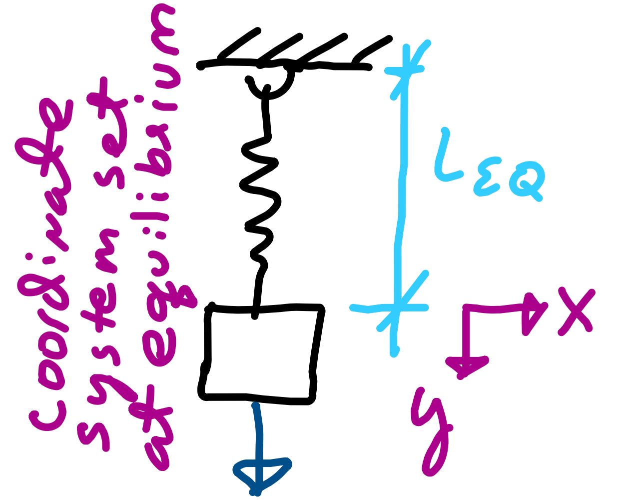

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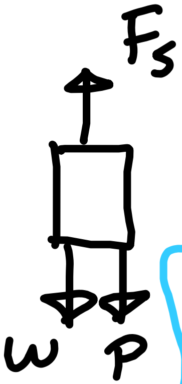
Example



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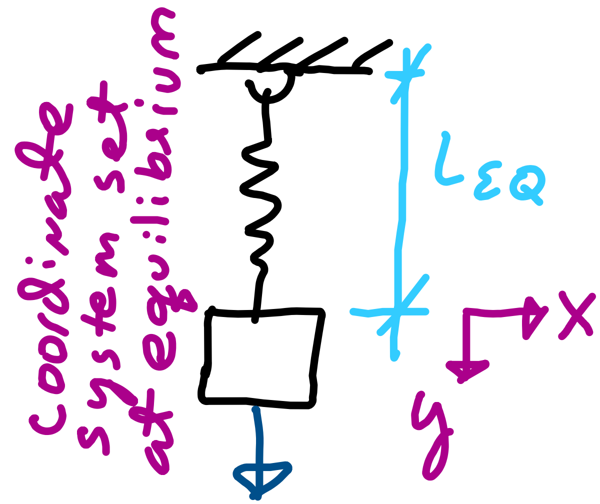

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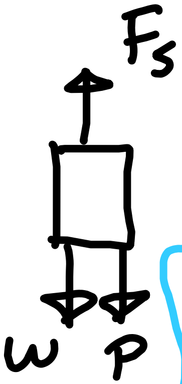

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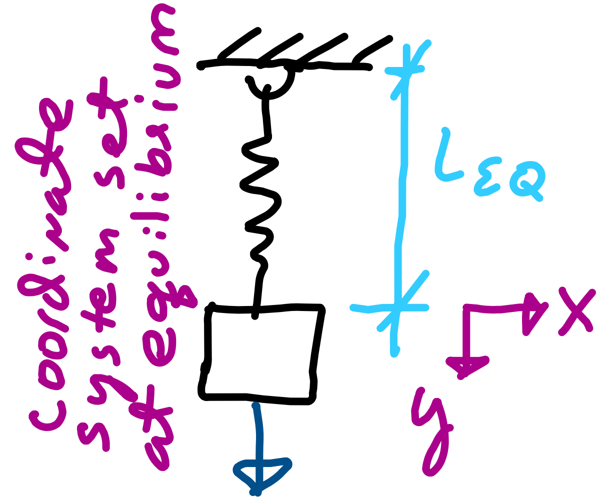

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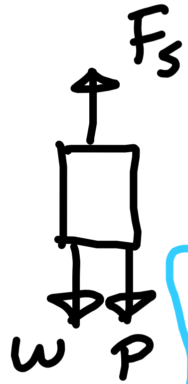
Example



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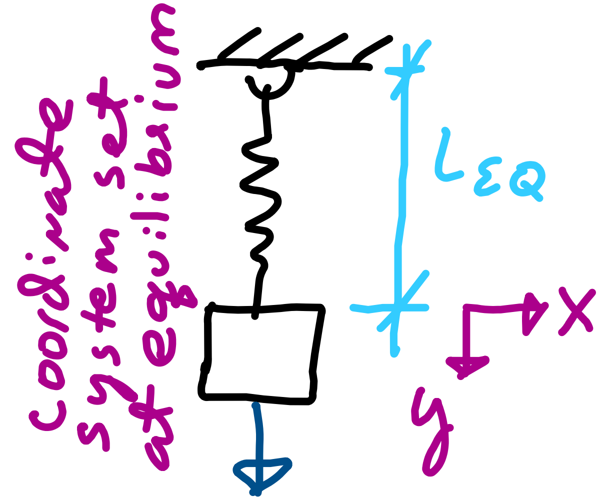

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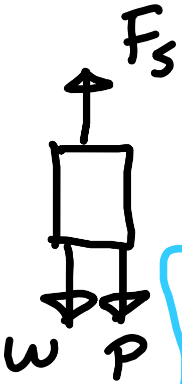

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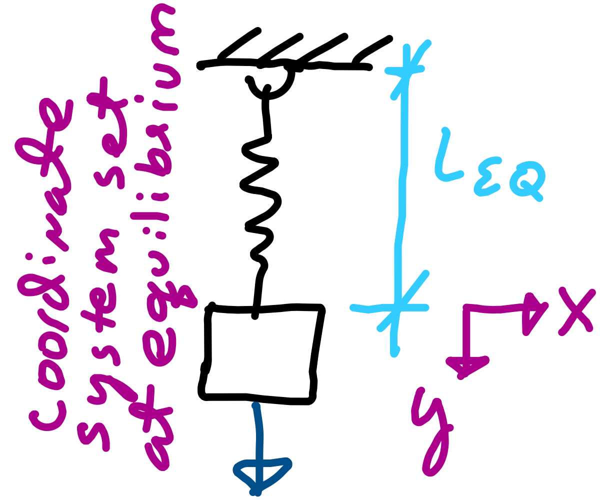

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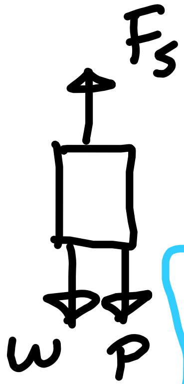
$\Rightarrow m\ddot{y}+Ky=P$

Example



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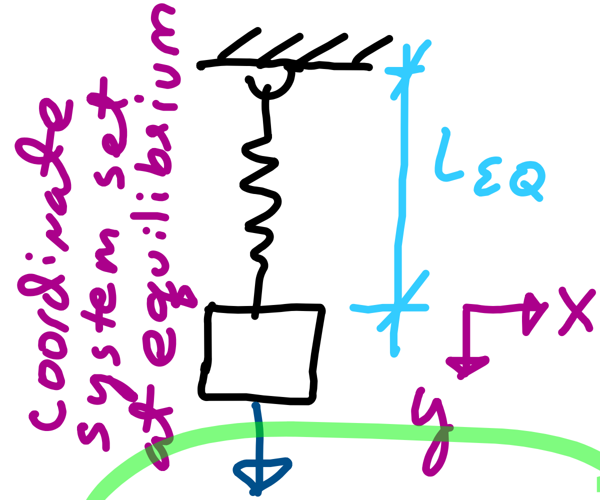

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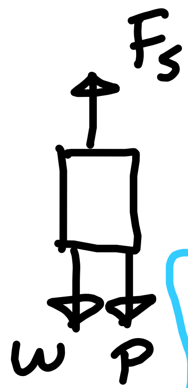

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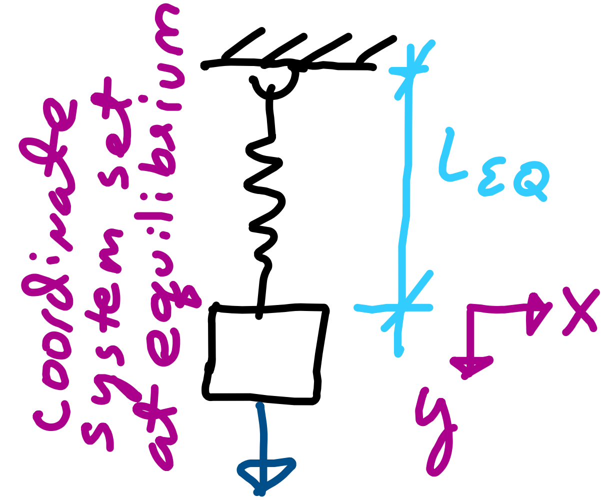

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From previous slide

$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

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Homogeneous equation:

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From previous slide

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From previous slide

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<u>Homogeneous equation</u>: $m\ddot{y} + ky = 0 \Rightarrow$

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<u>Particular solution</u>:

From previous slide

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Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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Particular solution: Assume

$y_{part} = y_m (\sin \omega_F t)$

From previous slide

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$y_{part} = y_m (\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

$$\Rightarrow \quad y_m = \frac{P_m}{k - m\omega_F^2}$$

From previous slide

$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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Particular solution: Assume

$$y_{part} = y_m(\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$$

$$\Rightarrow \qquad y_m = \frac{P_m}{k - m\omega_F^2} = \frac{(P_m/k)}{1 - m\omega_F^2/k}$$

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$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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Particular solution: Assume

$y_{part} = y_m(\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

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From previous slide

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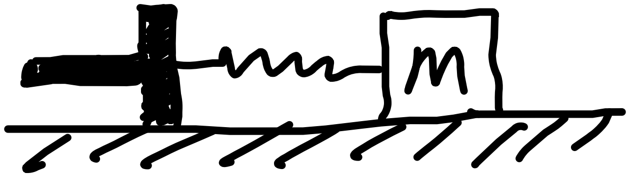
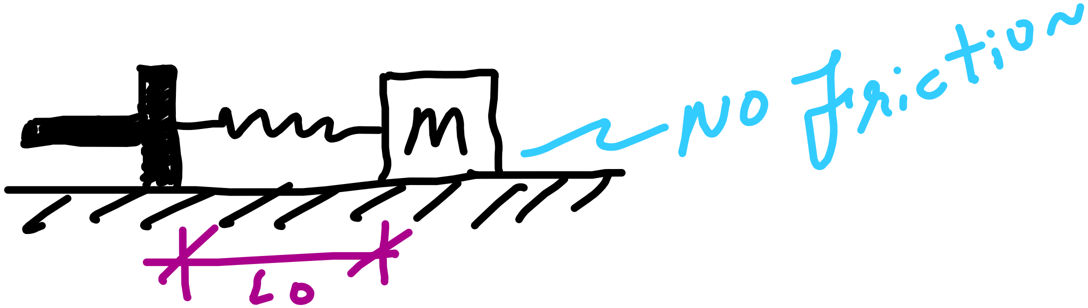
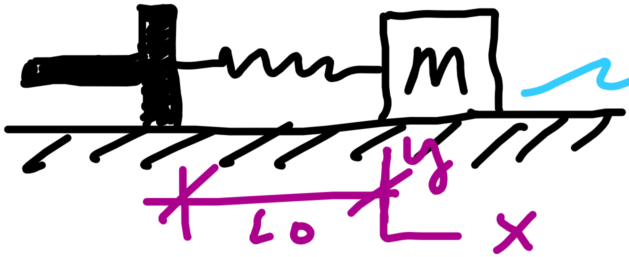
$\frac{k}{m} = \omega_n$ so $\boxed{y_m = \dfrac{(P_m/k)}{1 - \omega_F^2/\omega_n^2}}$

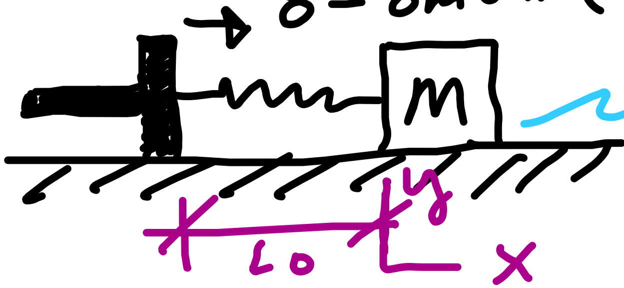

m


M
No friction
L_0



No friction

$\delta = \delta_m \sin(\omega_f t)$



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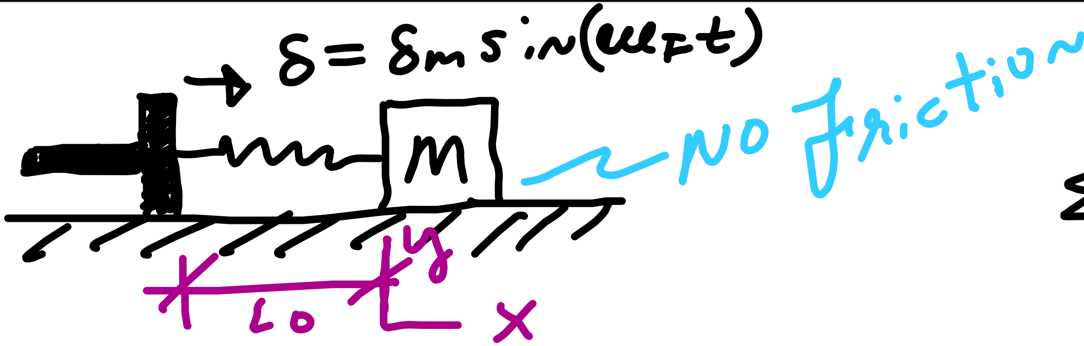
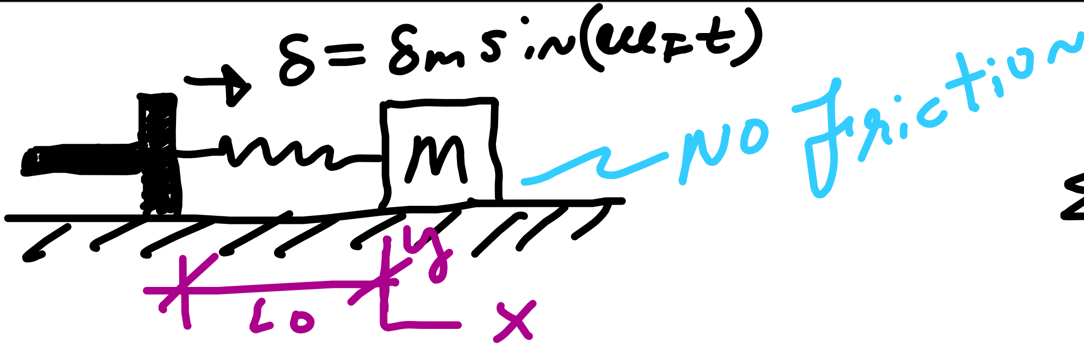


At equilibrium

$\sum \vec{F}_x = 0$



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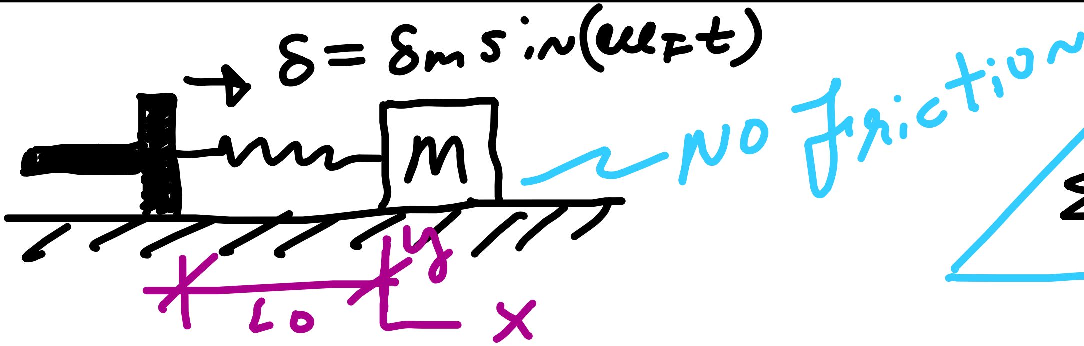

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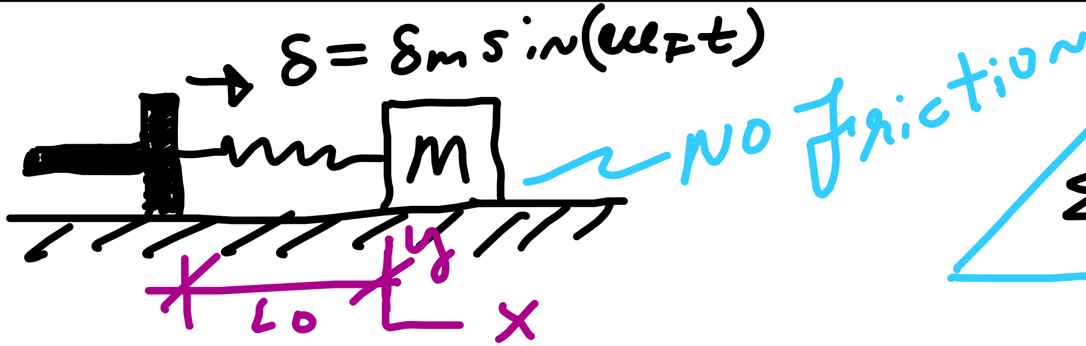

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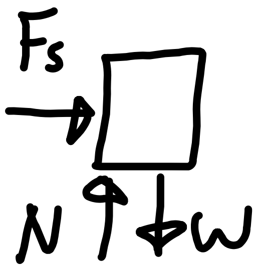
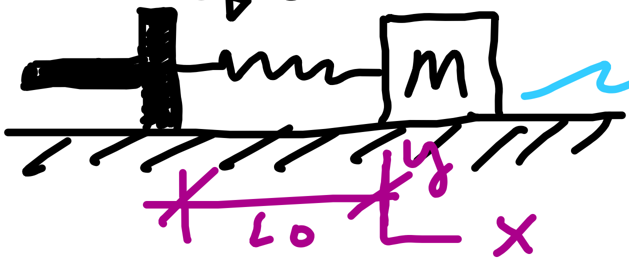


$\delta = \delta_m \sin(\omega_F t)$



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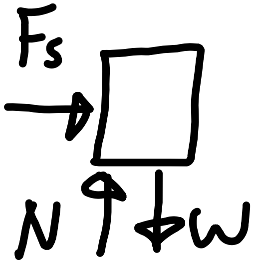

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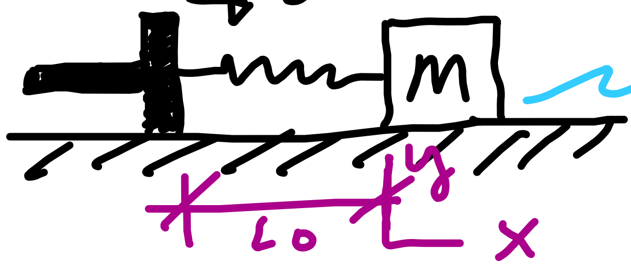

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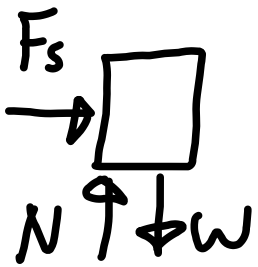

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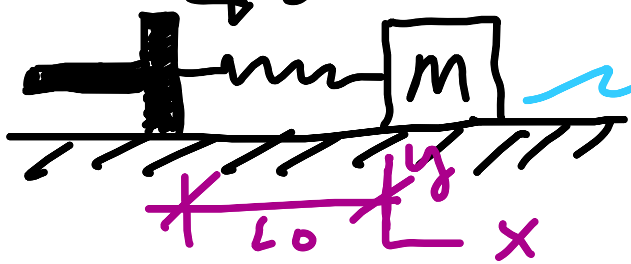

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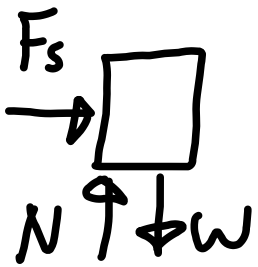

$$\sum F_x = m\ddot{x} \Rightarrow \quad F_s = m\ddot{x} \quad \& \quad F_s = -k(l - l_0)$$

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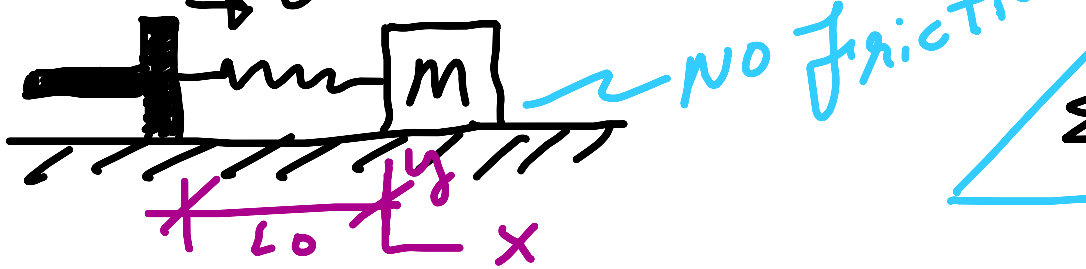

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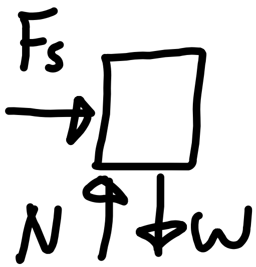
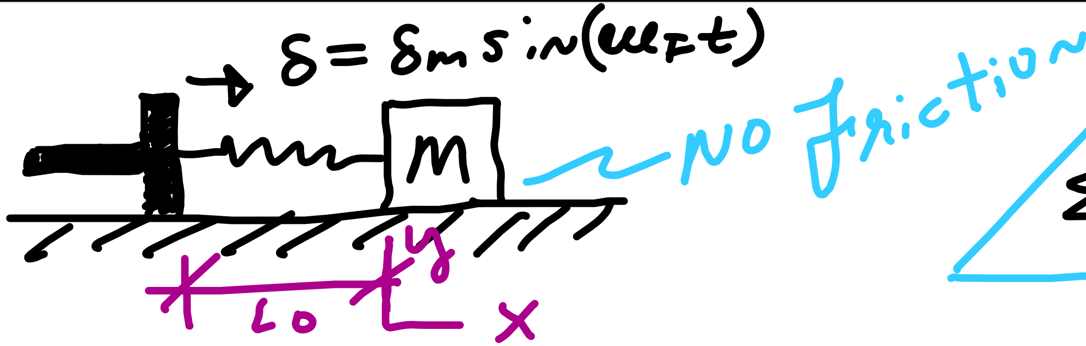


$$\sum F_x = m\ddot{x} \Rightarrow F_s = m\ddot{x} \ \& \ F_s = -k(L - L_0)$$

$$\& \ L = L_0 - \delta + x$$



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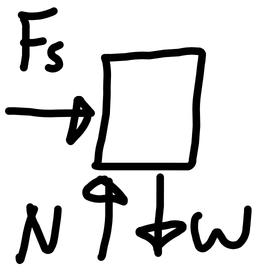

$\Sigma F_x = m\ddot{x} \Rightarrow F_s = m\ddot{x}$ & $F_s = -k(L - L_0)$

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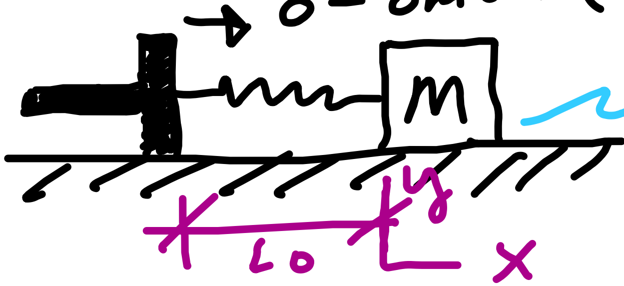

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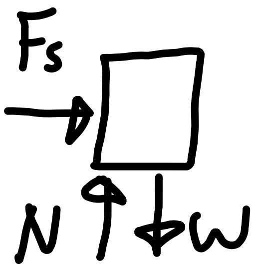

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Now $-k(x - \delta) = m\ddot{x}$

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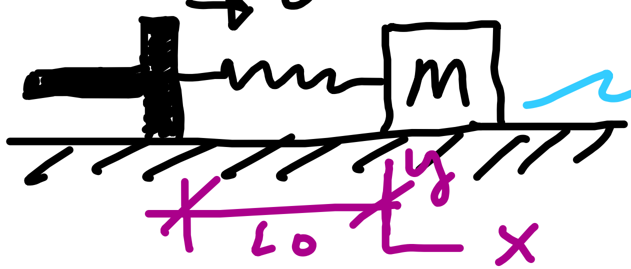

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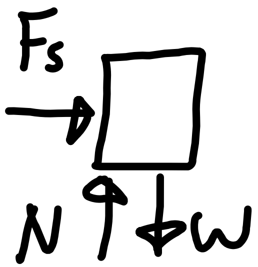

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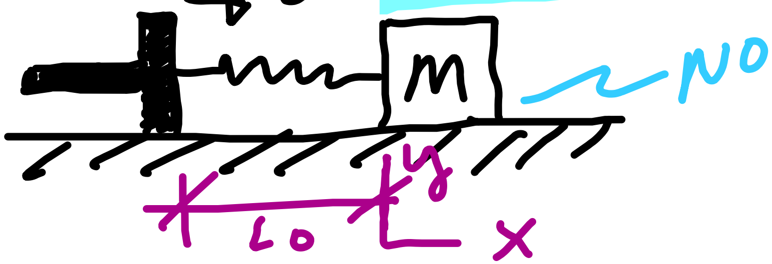
& $L = L_0 - \delta + x$ So $L - L_0 = -\delta + x$

Now $-k(x-\delta) = m\ddot{x} \Rightarrow m\ddot{x} + kx = k\delta$

$\delta = \delta_m \sin(\omega_f t)$



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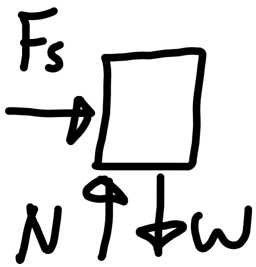

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& $L = L_0 - \delta + x$ so $L - L_0 = -\delta + x$

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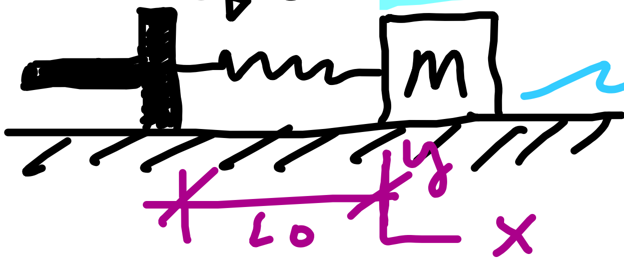

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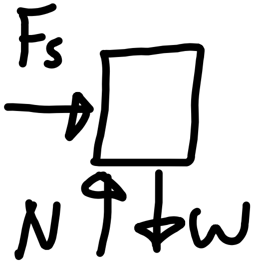

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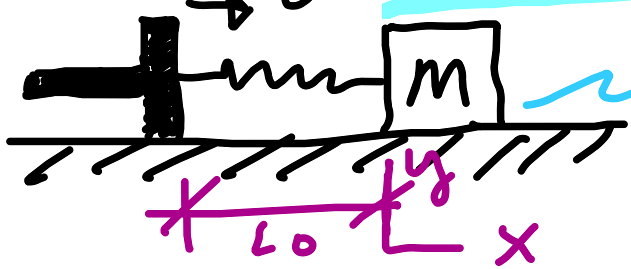

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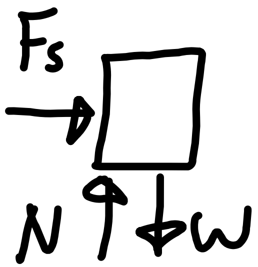

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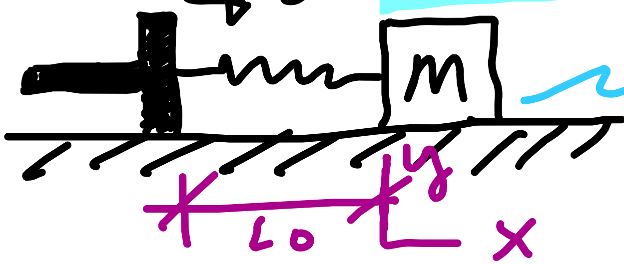

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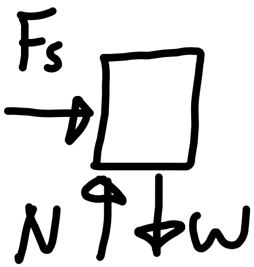

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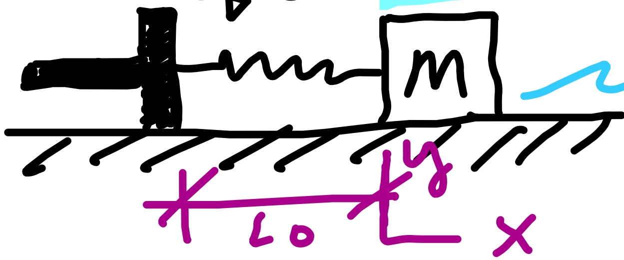

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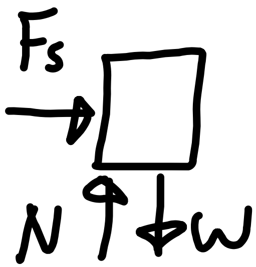

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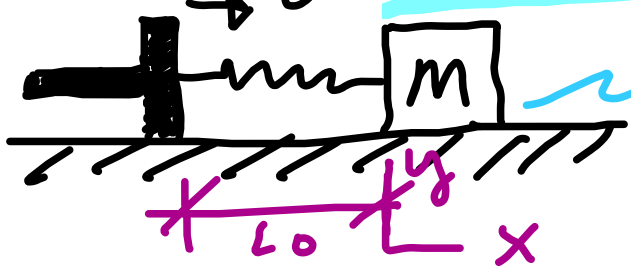

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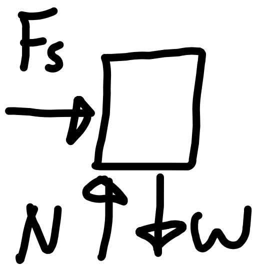

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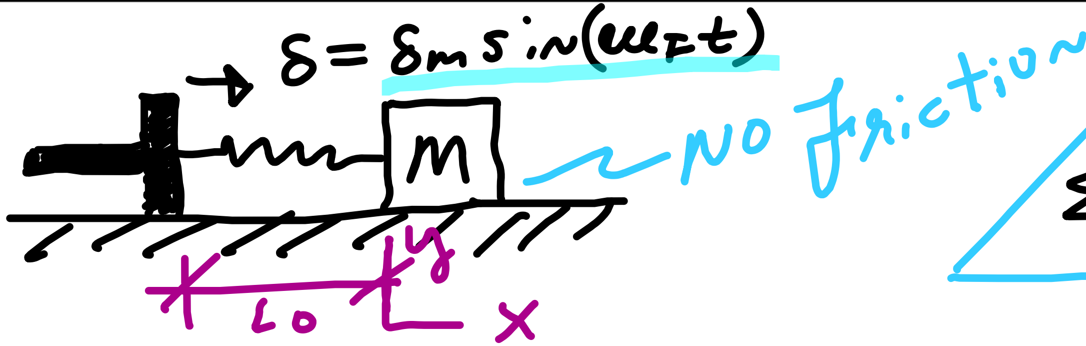

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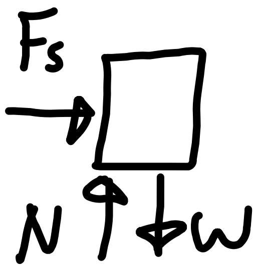

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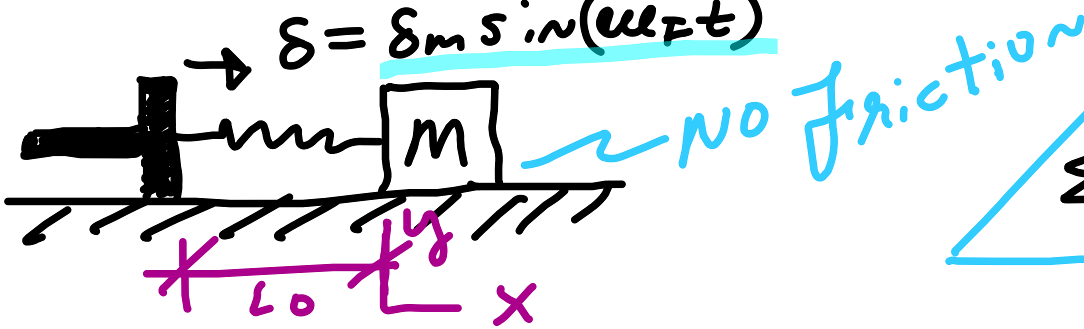

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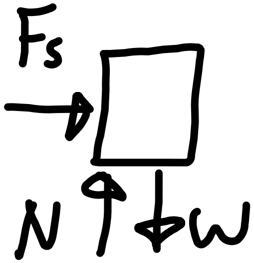

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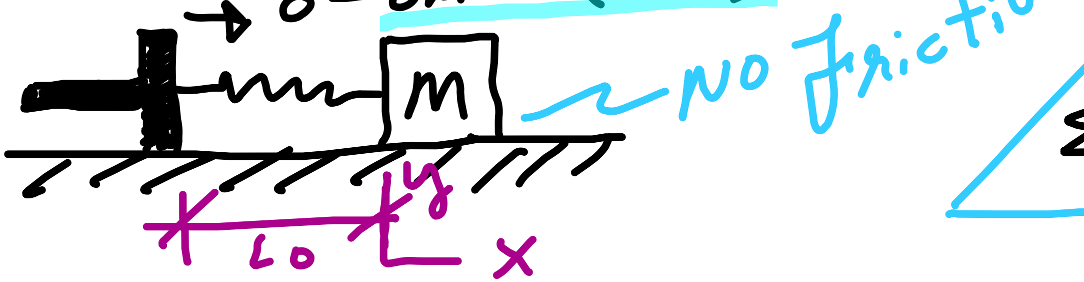

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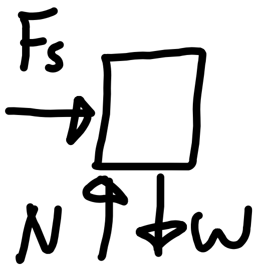

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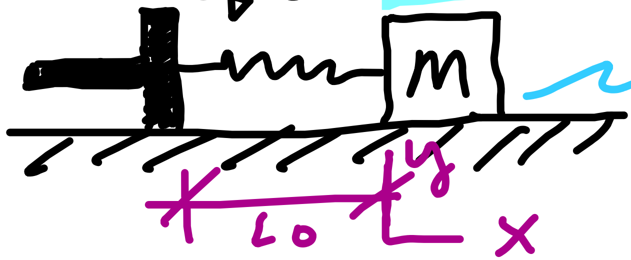

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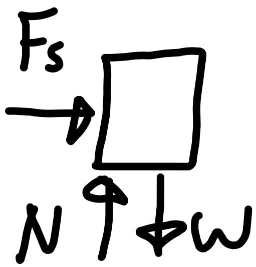

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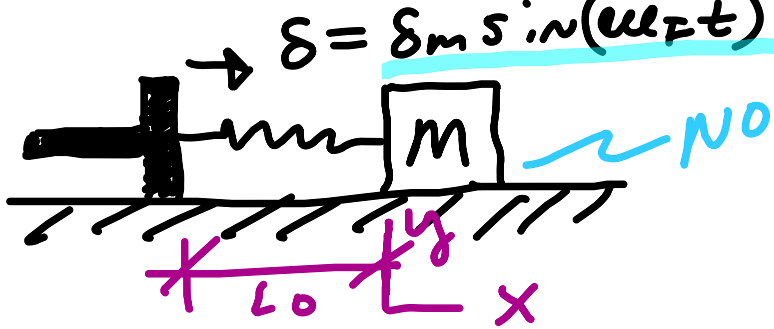
ASU



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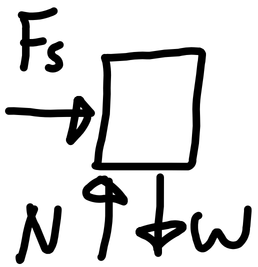

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What would happen to real spring when $\omega_F = \omega_n$?

Notes on problem 19.125

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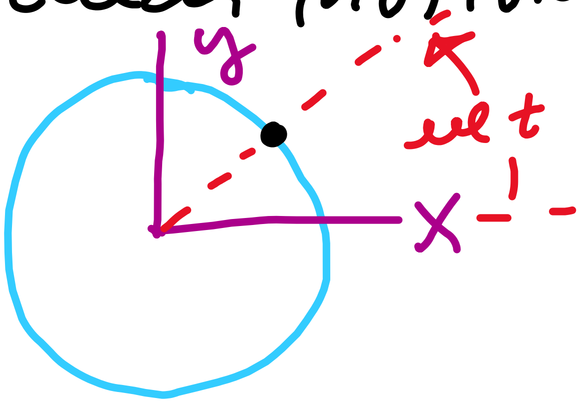

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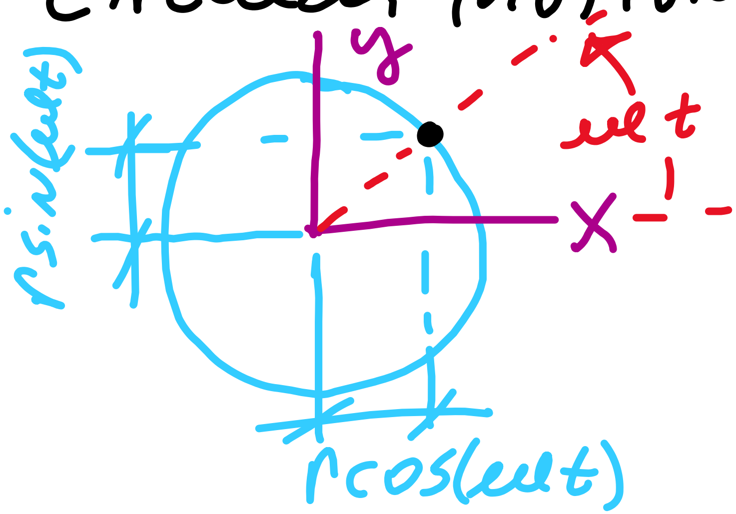

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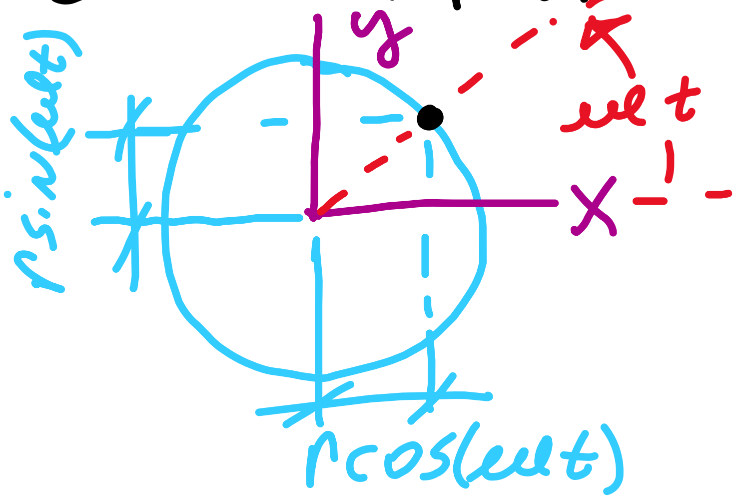

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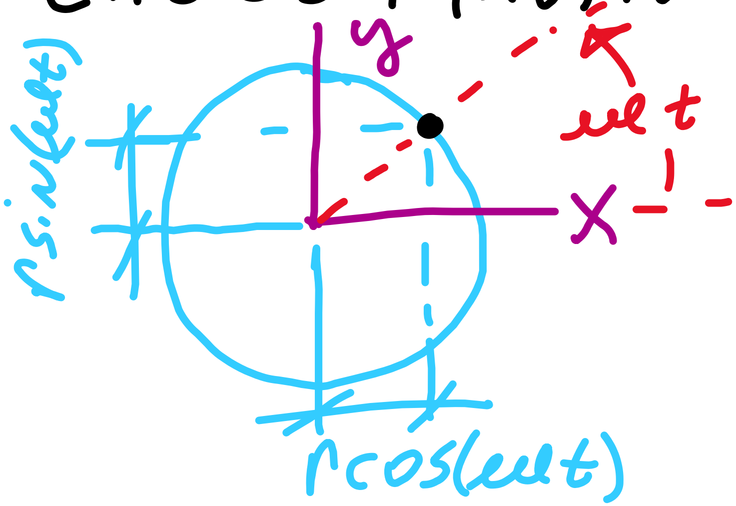

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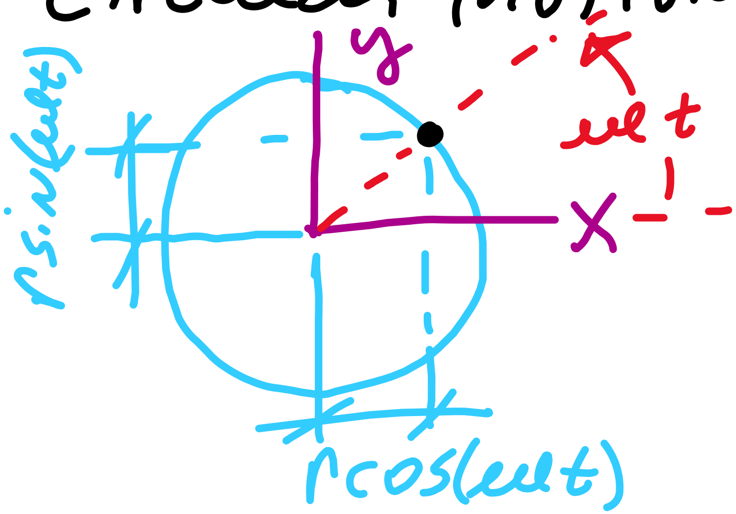

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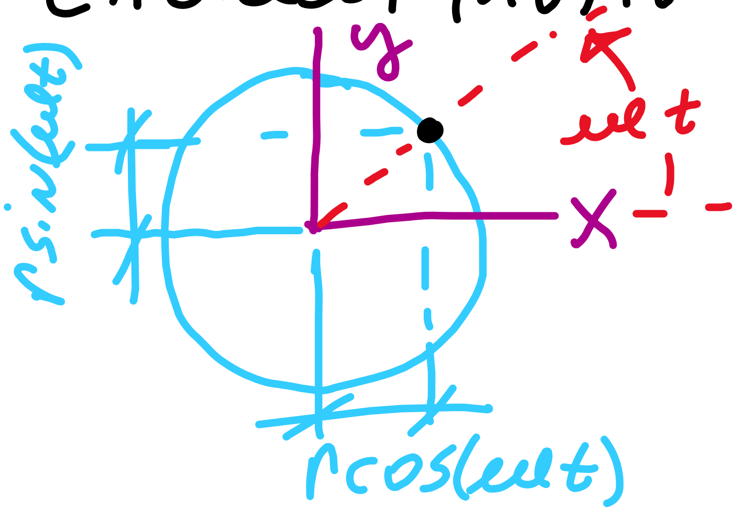

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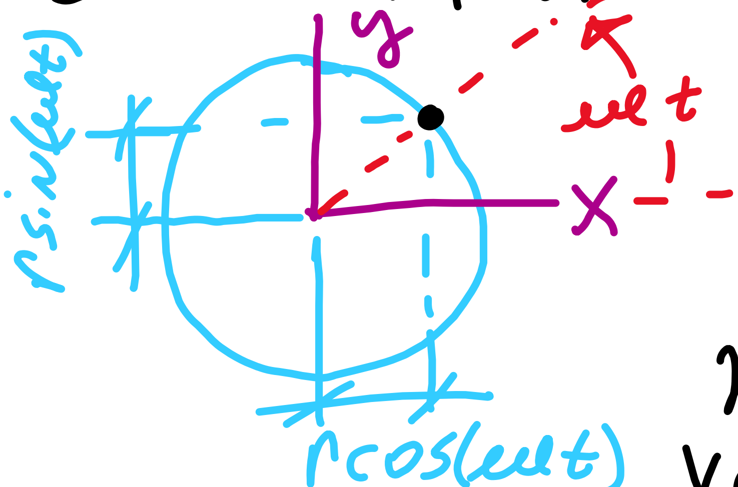

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$\Rightarrow \quad x = x_m \cos(\omega t)$ & $y = y_m \sin(\omega t)$

$x_m = y_m = r$. Circular motion looks like a very particular vibration in x & y.

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When you set $e = 0.006in$, it is probably going to be easiest to work in normal and transverse components instead of cartesian coordinates