Today Review

L37

L37

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Friday Holiday

L37

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Monday Exam #4

L37

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Friday Holiday

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Wednesday Dec. 2nd Day of Reckoning

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Wednesday Dec. 2nd Day of Reckoning

Friday Dec. 4th Final Exam

Free vibrations:

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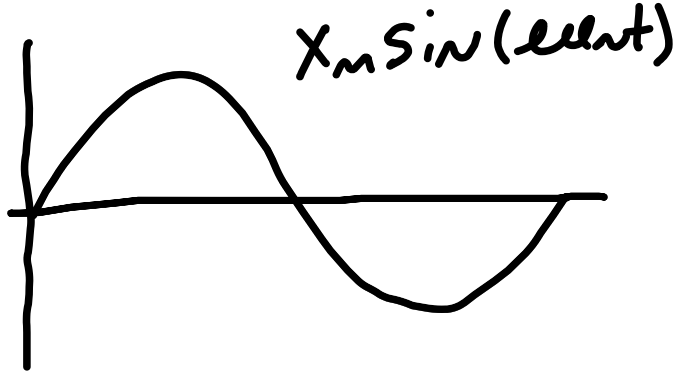

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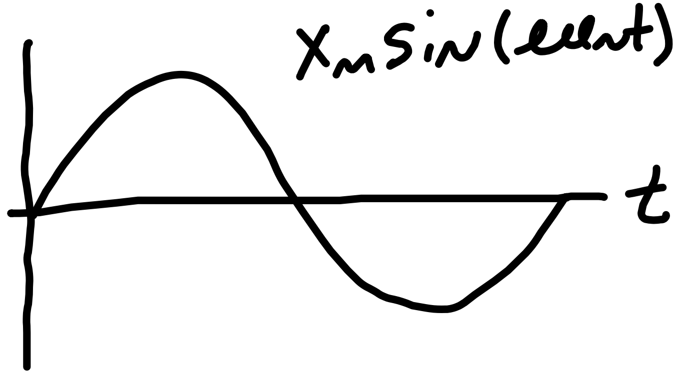

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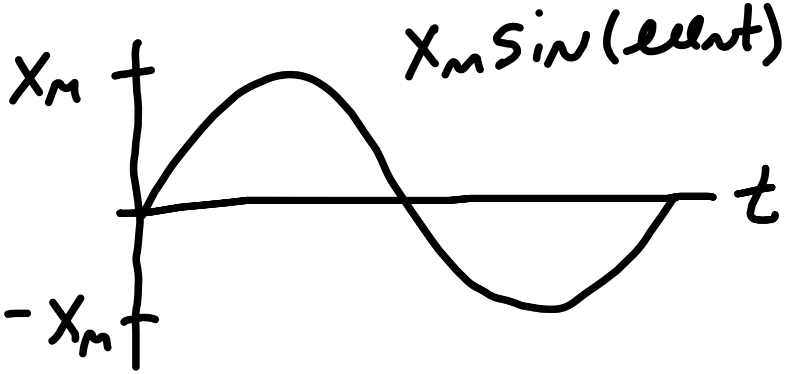

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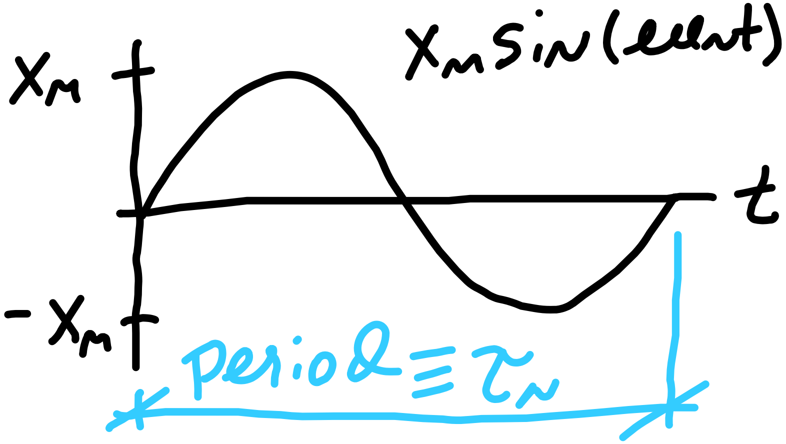

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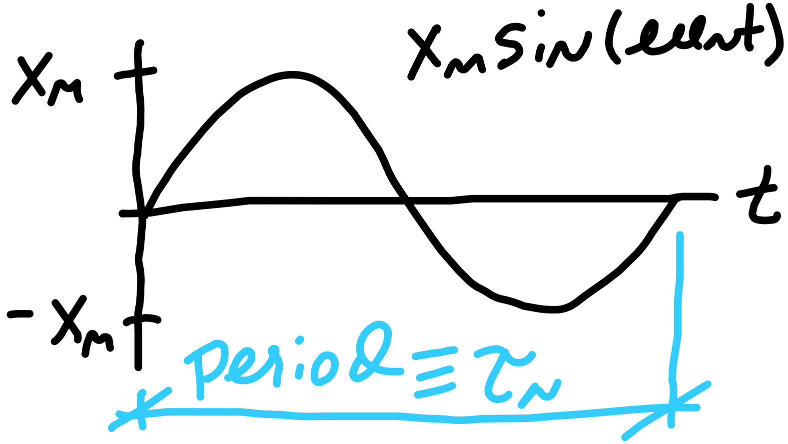

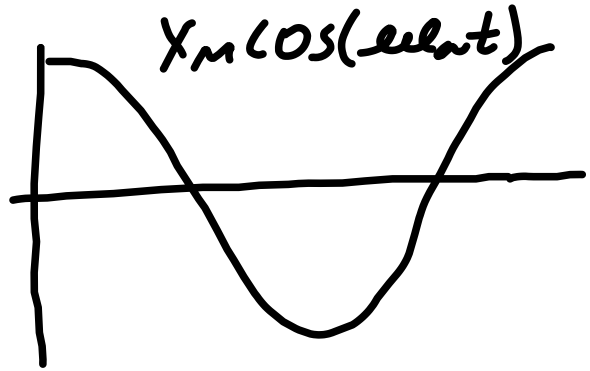

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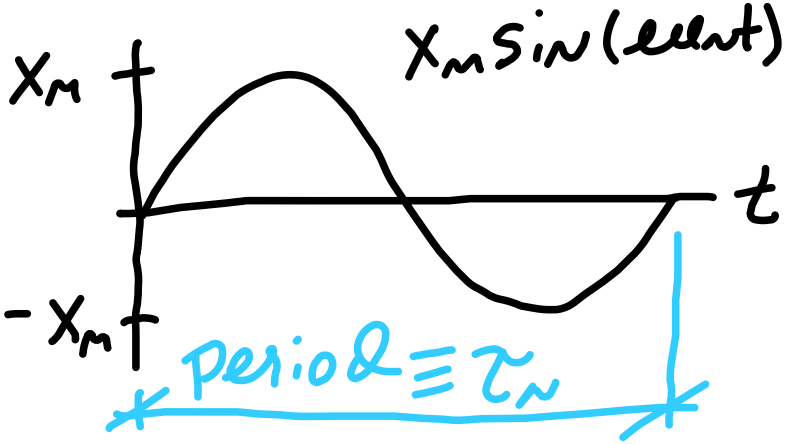

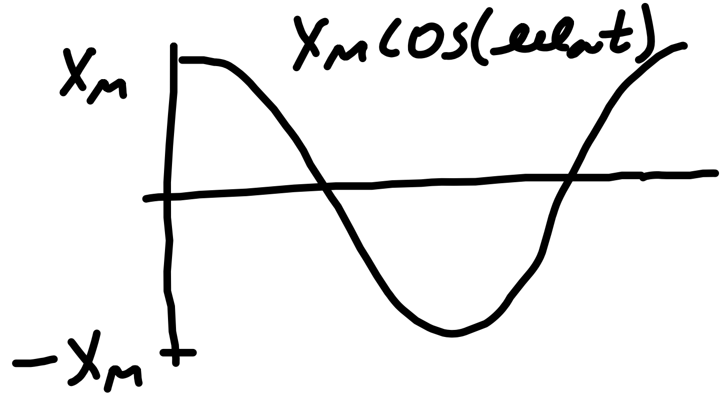

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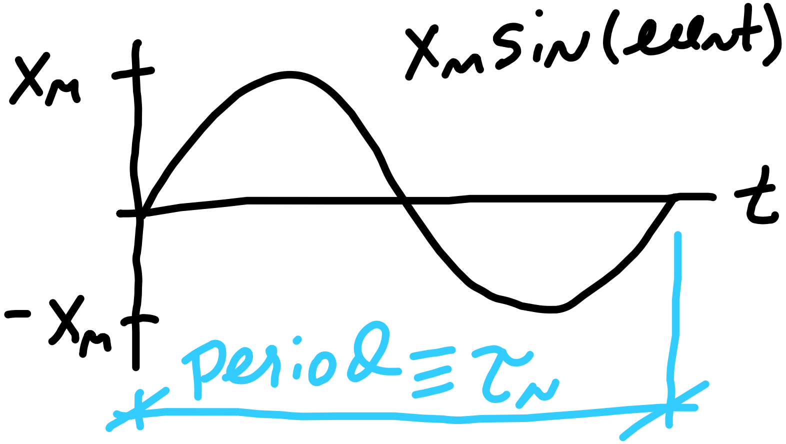

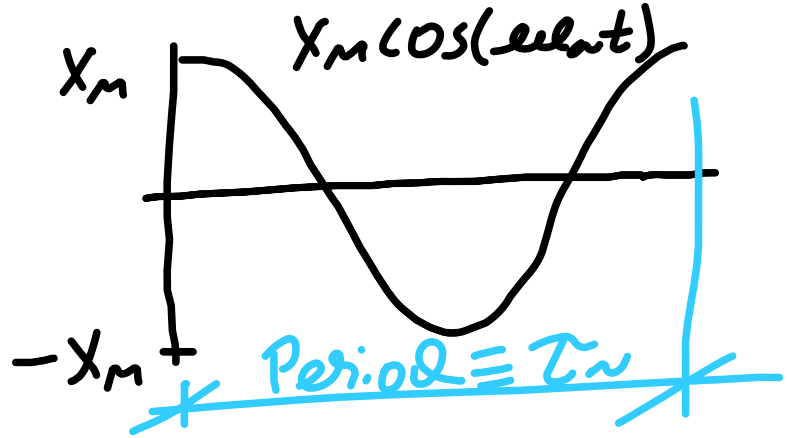

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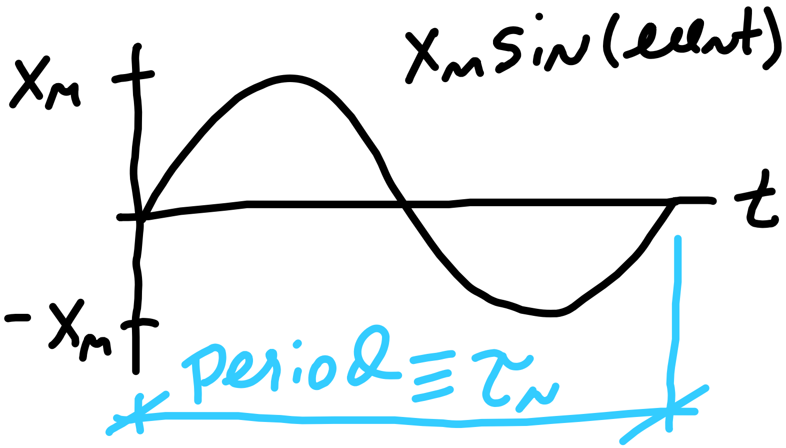

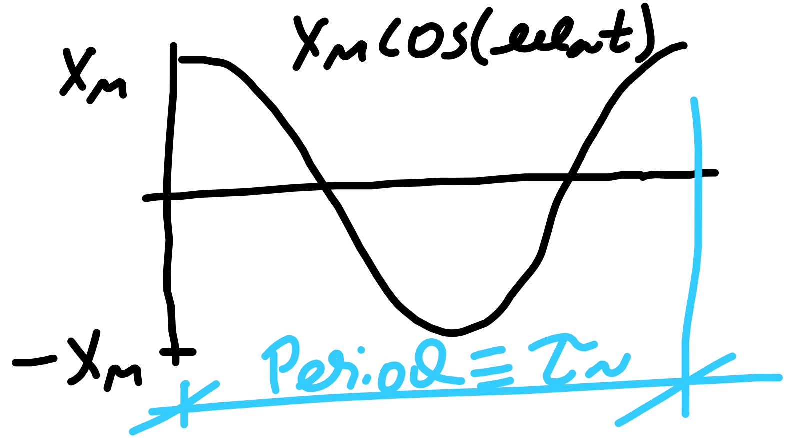

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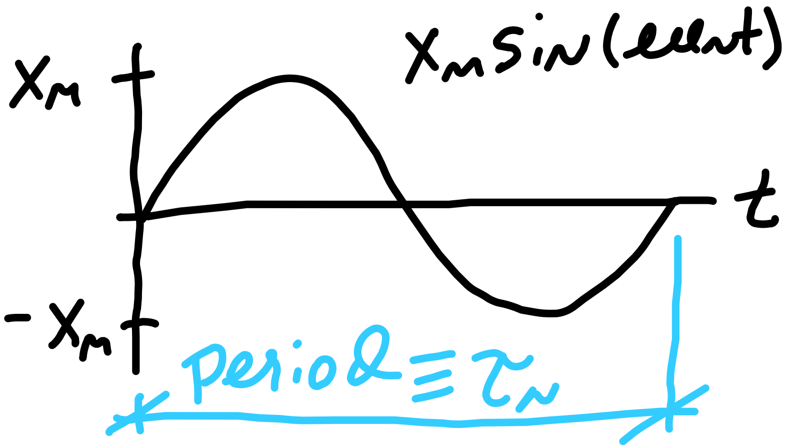

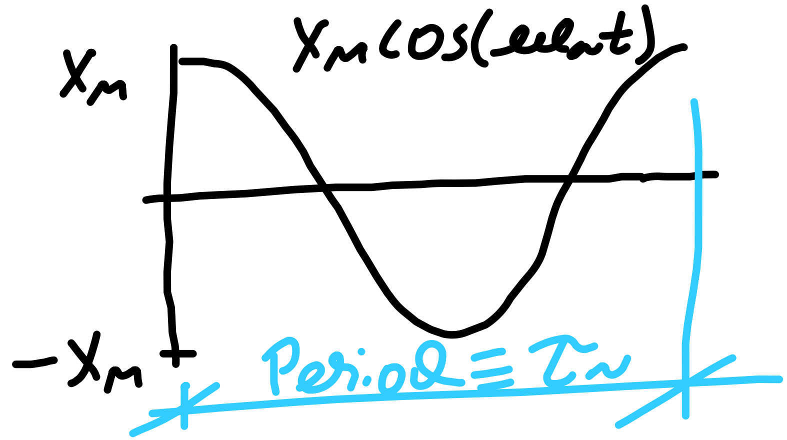

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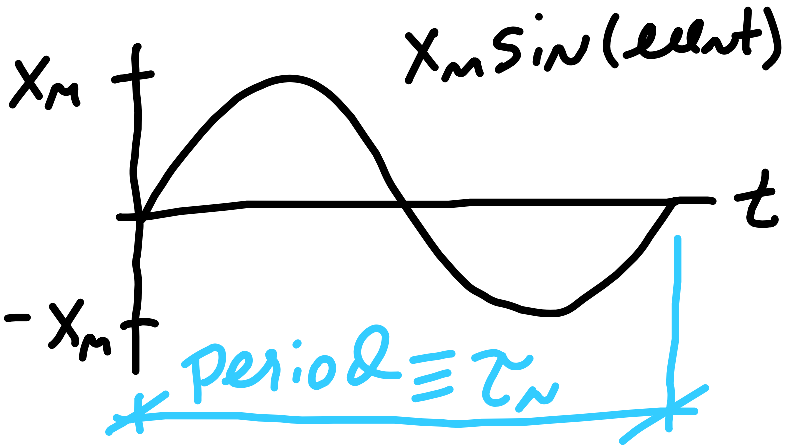

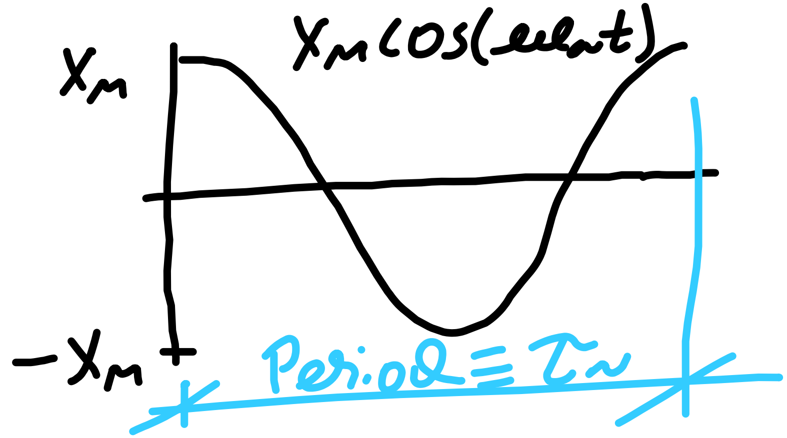

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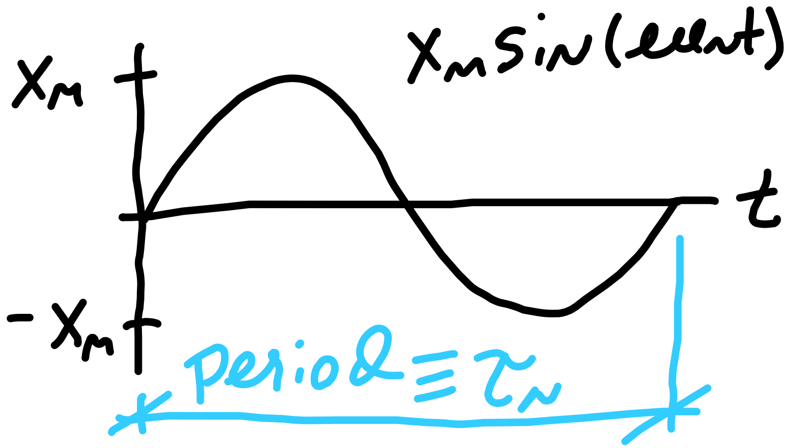

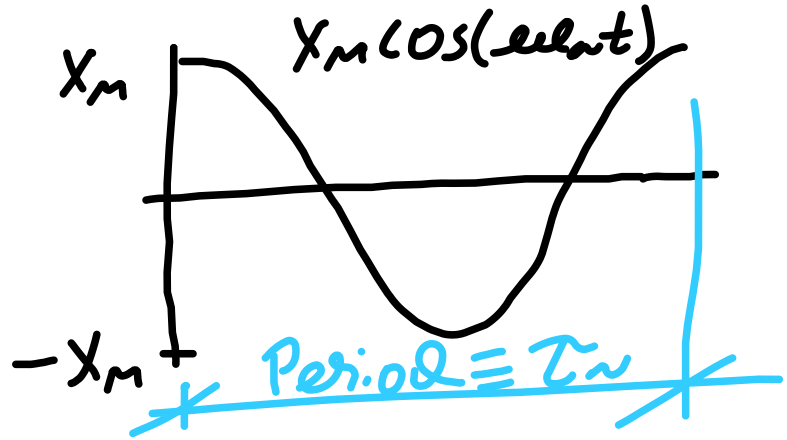

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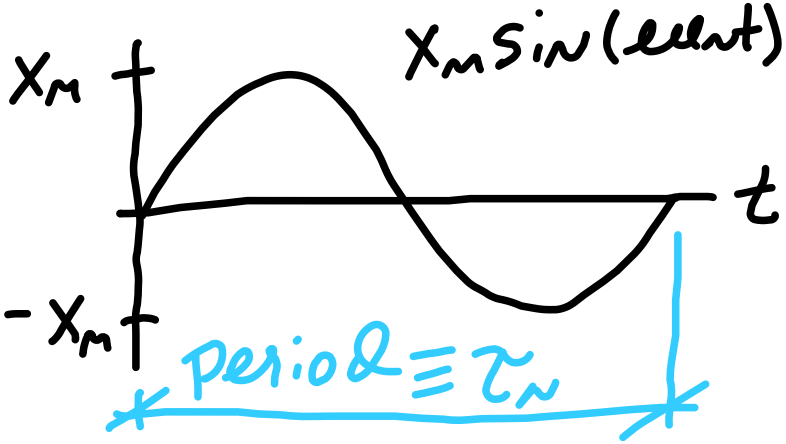

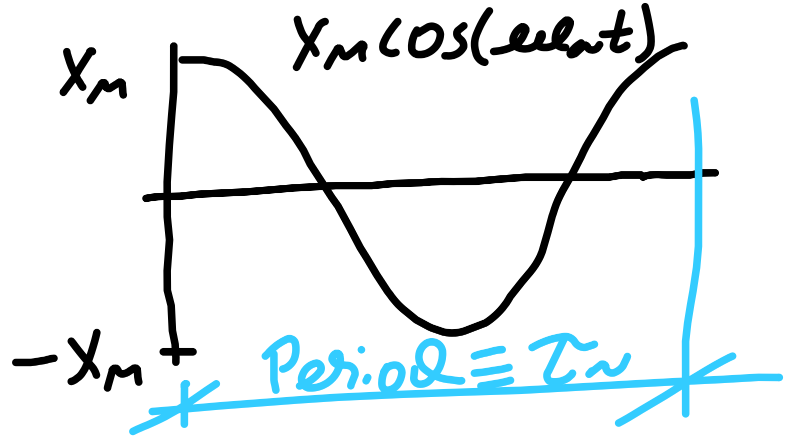

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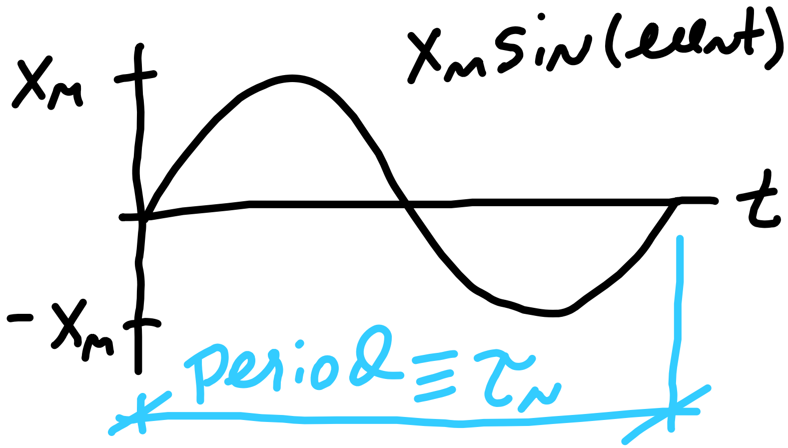

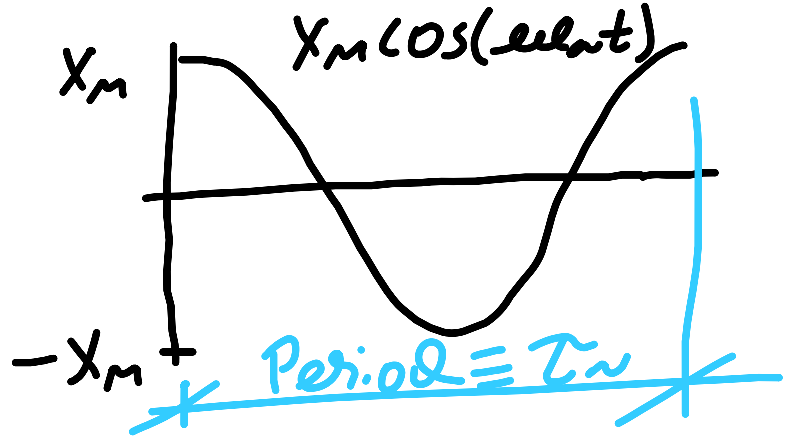

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Example: Spring

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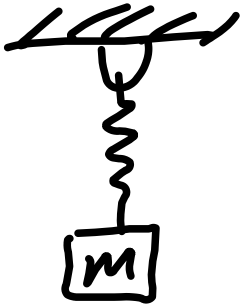

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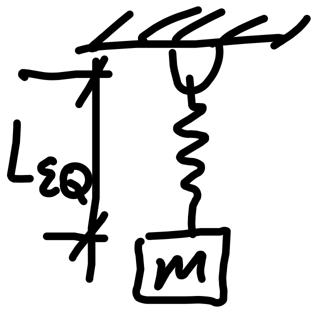

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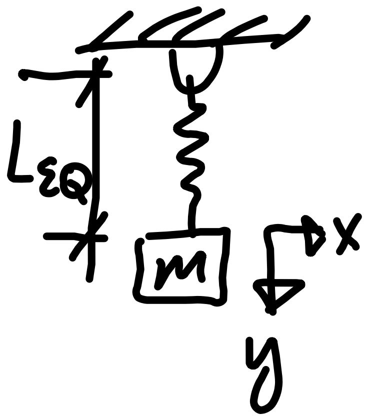
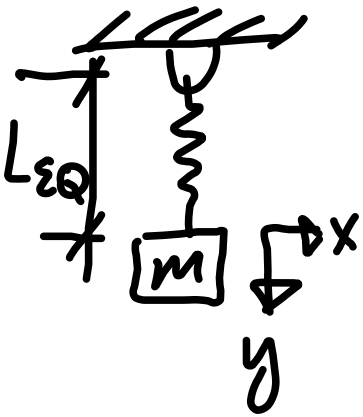


Example: Spring Equilibrium



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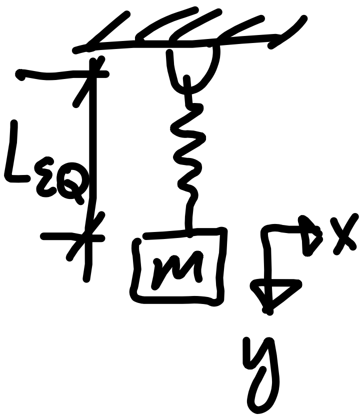

Equilibrium: $L_{EQ} = L + \delta$

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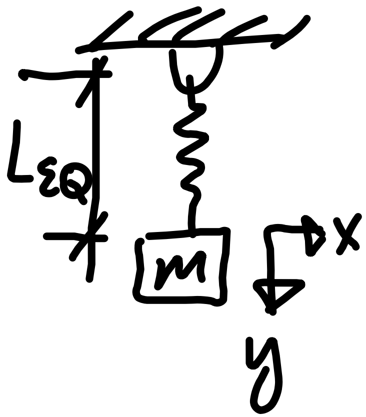

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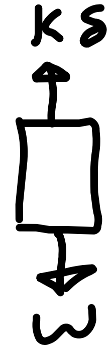

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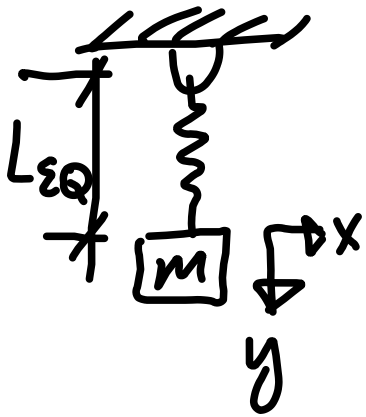
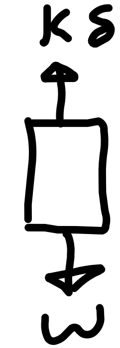


Equilibrium: $L_{EQ} = L + \delta$

$\Sigma F = 0$



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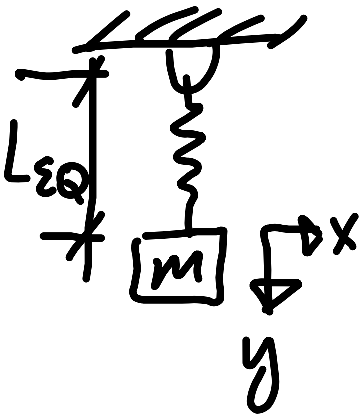

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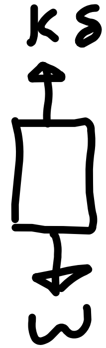
$\Sigma F = 0 \Rightarrow W - k\delta = 0$



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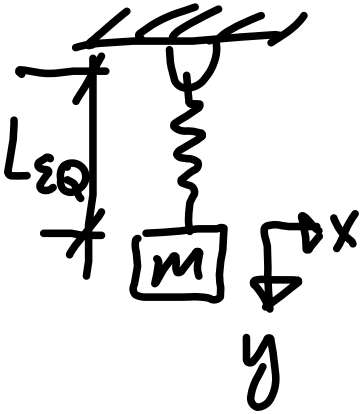

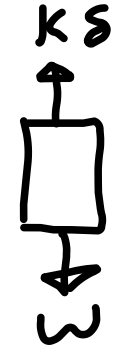

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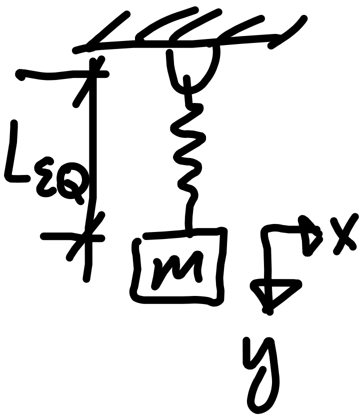

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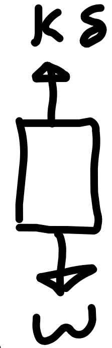

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Non-equilibrium: push mass down amount y & let go.

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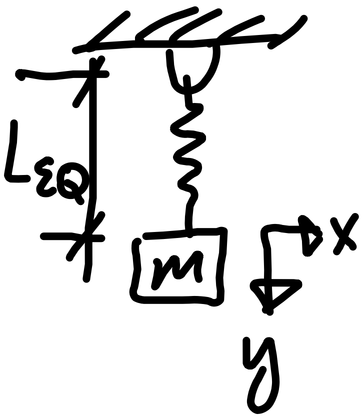

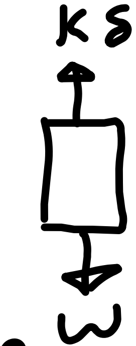

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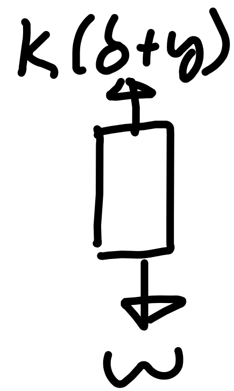

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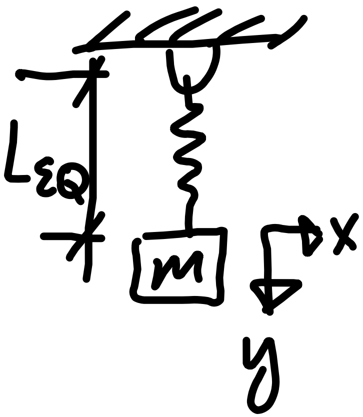

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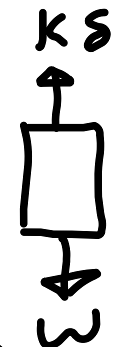

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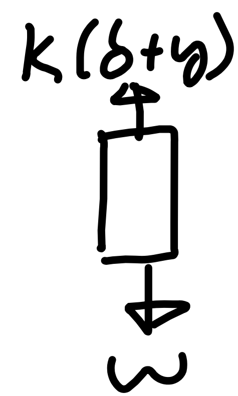

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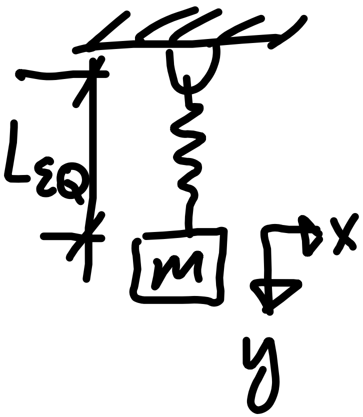

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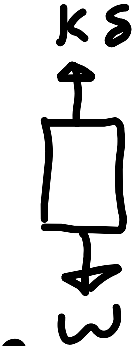

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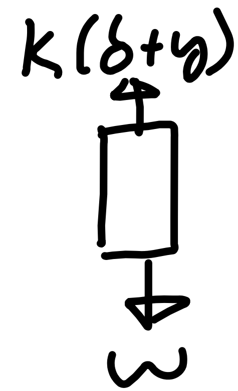

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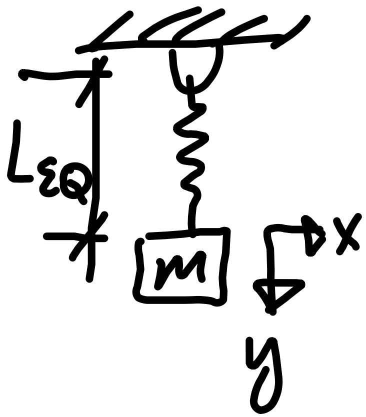

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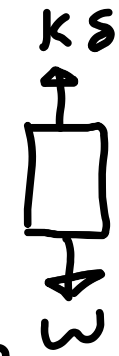

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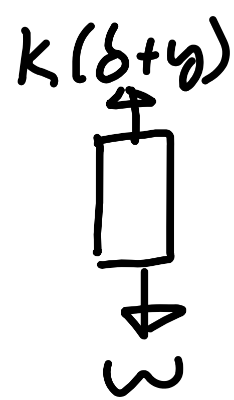
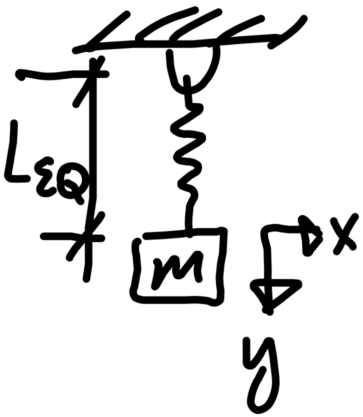
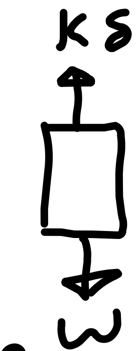
$$\Sigma F_y = m\ddot{y} \Rightarrow W - k\delta - ky = m\ddot{y}$$



Example: Spring





Equilibrium: $L_{EQ} = L + \delta$

$$\Sigma F = 0 \Rightarrow \underbrace{W - k\delta = 0}$$

Non-equilibrium: push mass down amount y & let go. Here $L = L_{EQ} + y$

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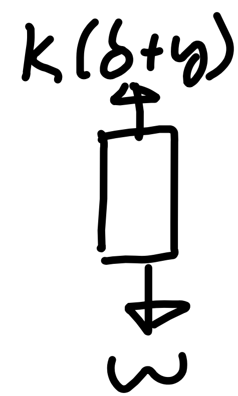
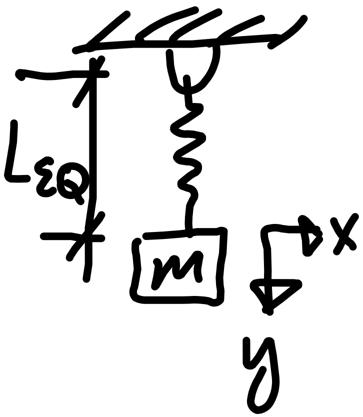


Example: Spring



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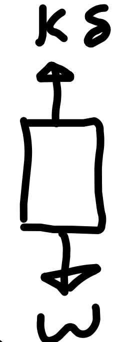

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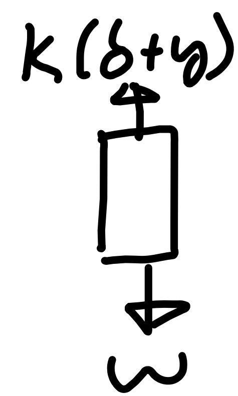

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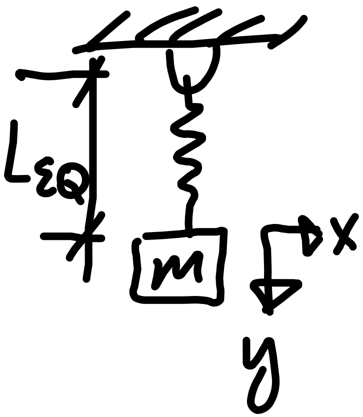

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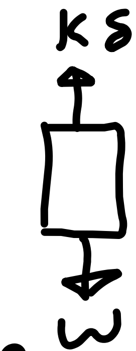

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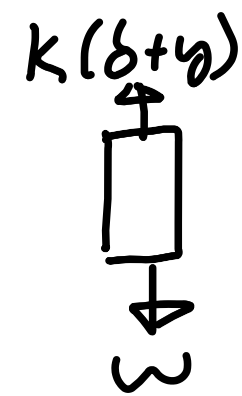
or $\ddot{y} = -\omega_n^2 y$



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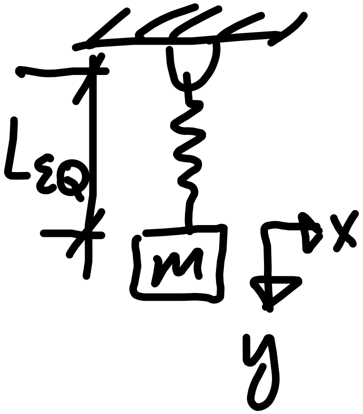

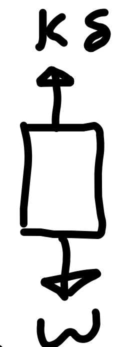

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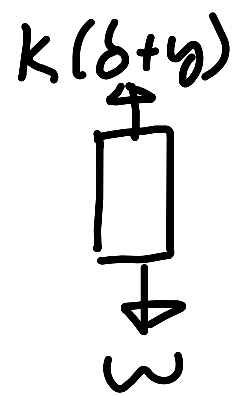

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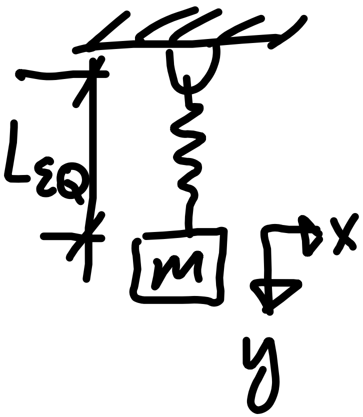

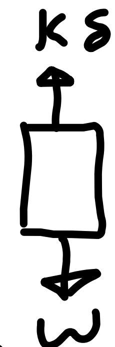

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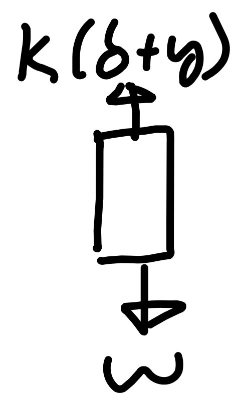
$$\tau_n = 2\pi/\omega_n$$



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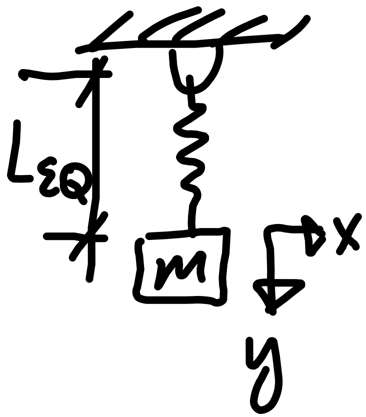

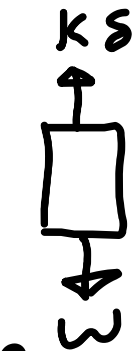

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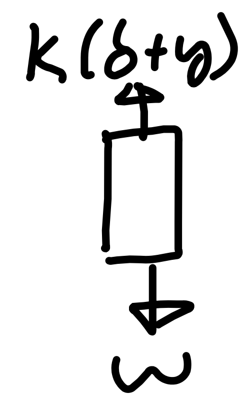

Example : Pendulum

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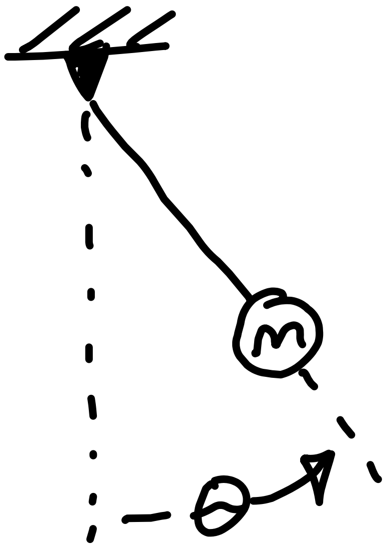
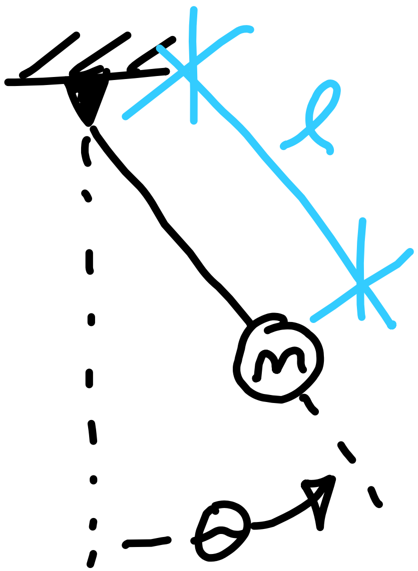
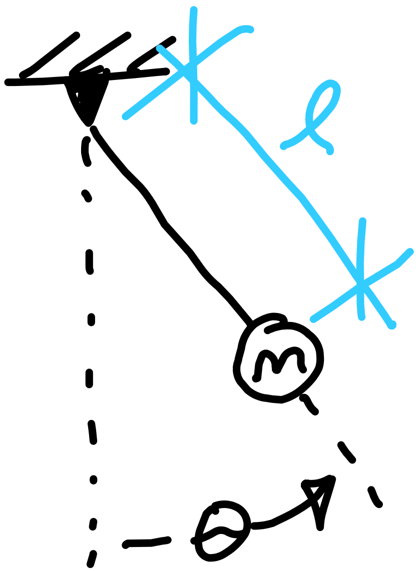
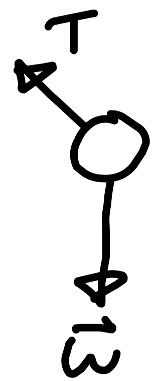


Example : Pendulum

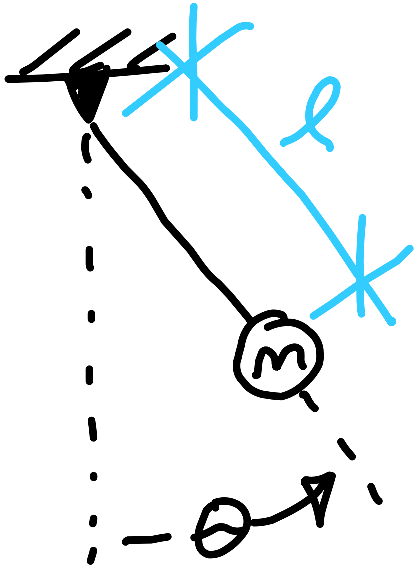


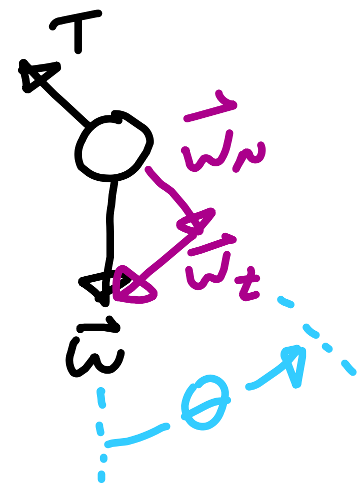
Example : Pendulum





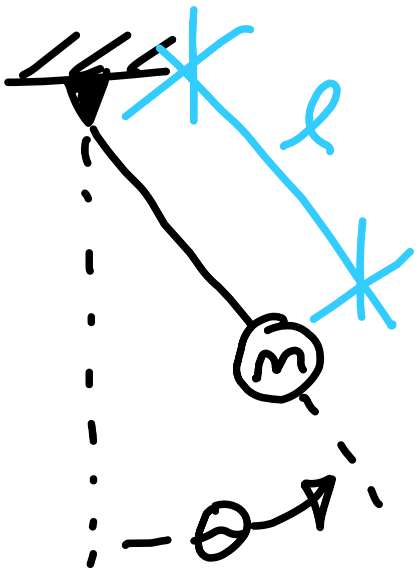
Example : Pendulum





Example: Pendulum

$$\vec{w} = \vec{w}_n + \vec{w}_t$$



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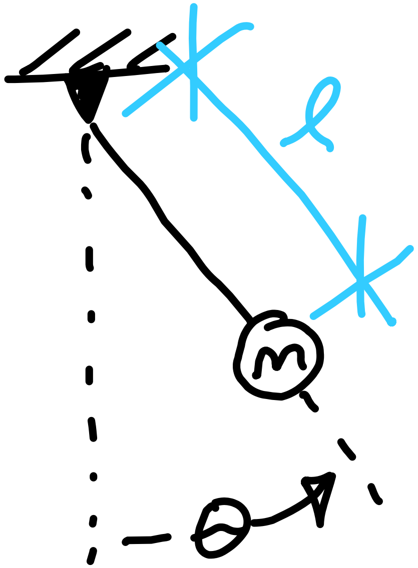
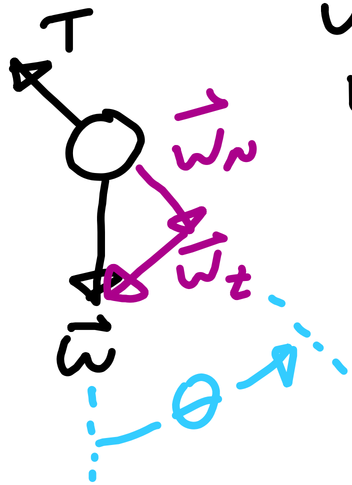
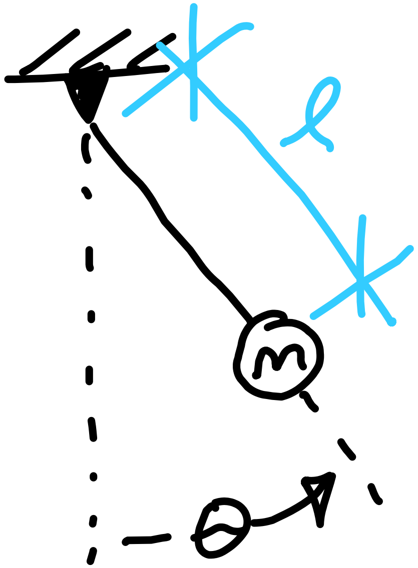


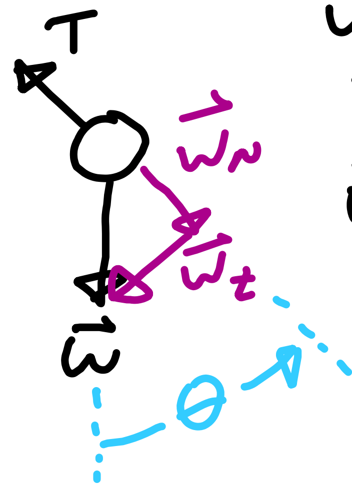


$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

Example: Pendulum



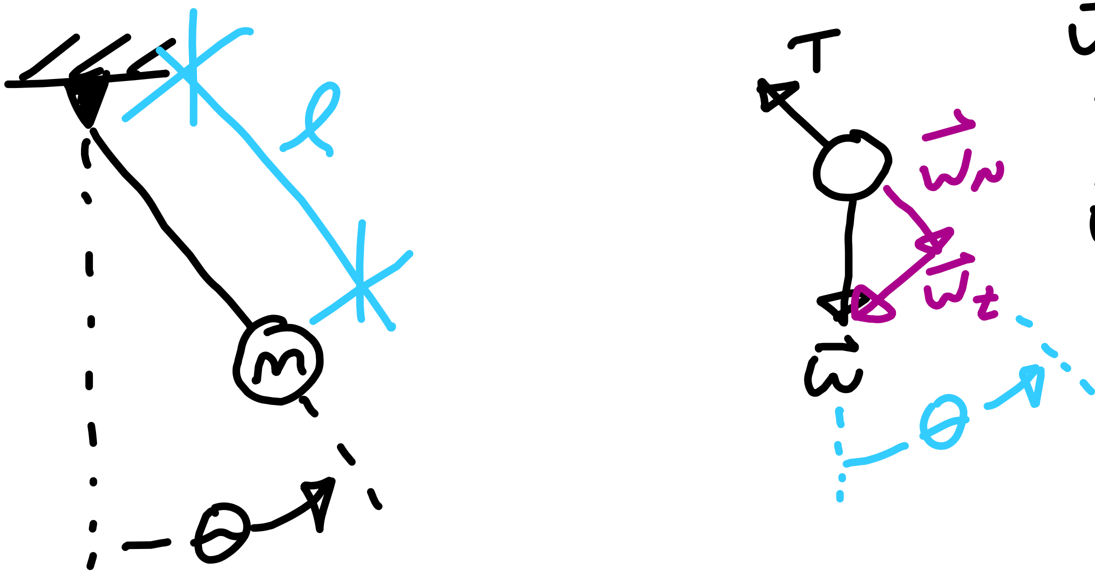


$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Example : Pendulum



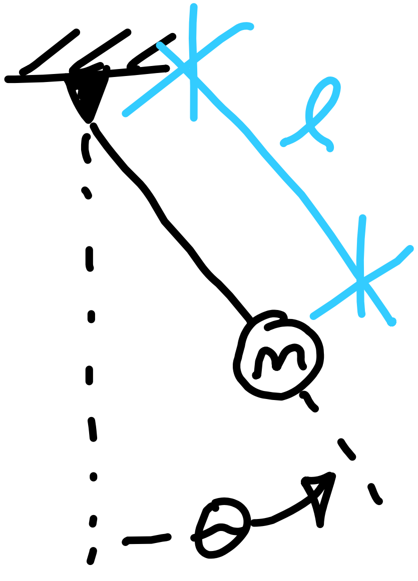
$$\vec{w} = \vec{w}_n + \vec{w}_t$$

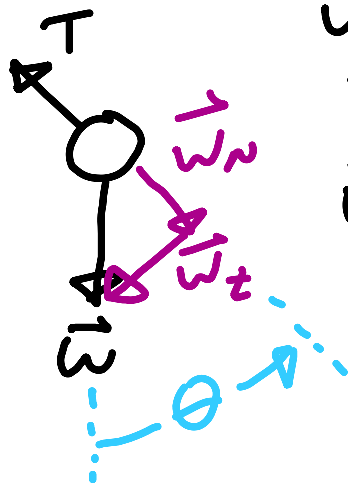
$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

Example : Pendulum





$\vec{w} = \vec{w}_n + \vec{w}_t$

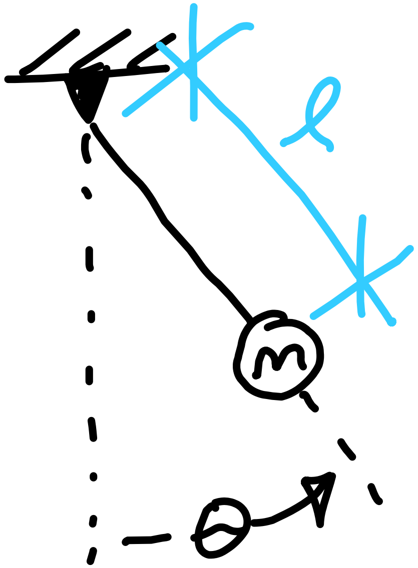
$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

Example : Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

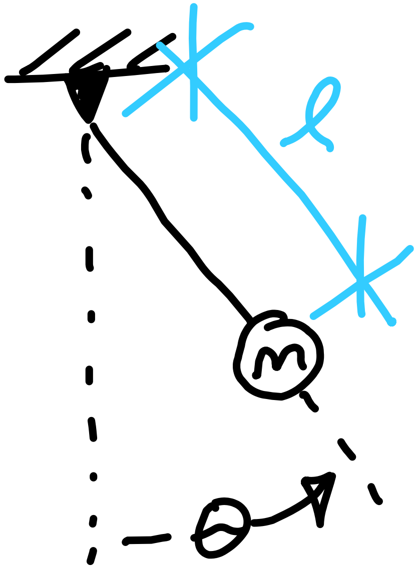
$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

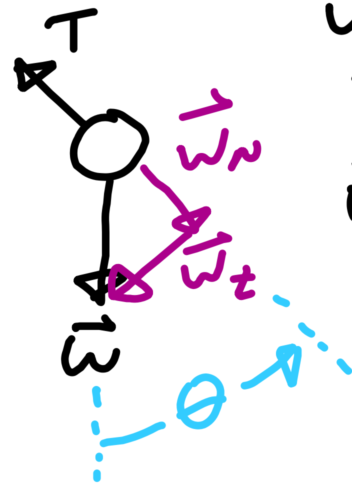
Solved 3 ways

1st way

$\Sigma F_t = m a_t$

Example : Pendulum





$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

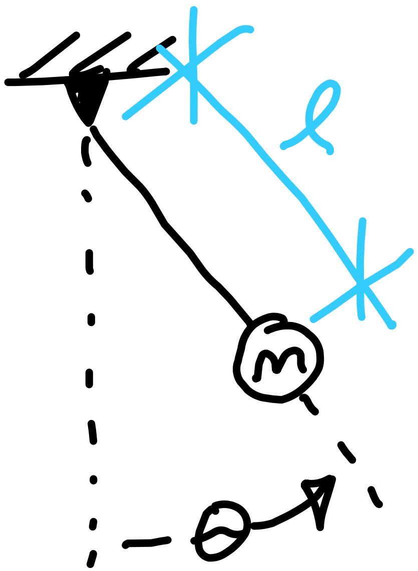
$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

1st way

$$\sum F_t = m a_t \Rightarrow -w_t = m\frac{dv}{dt}$$

Example : Pendulum



$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

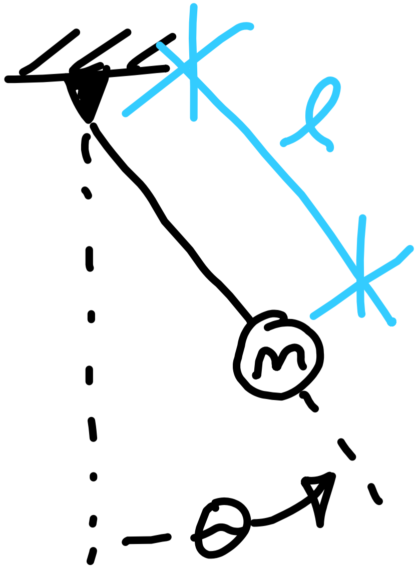
Solved 3 ways

1st way

$$\Sigma F_t = m a_t \Rightarrow -w_t = m\frac{dv}{dt}$$

but $v = \ell\dot{\theta}$

Example: Pendulum



$$\vec{w} = \vec{w}_N + \vec{w}_t$$

$$\vec{w}_N = (w\cos\theta)(-\hat{e}_N)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

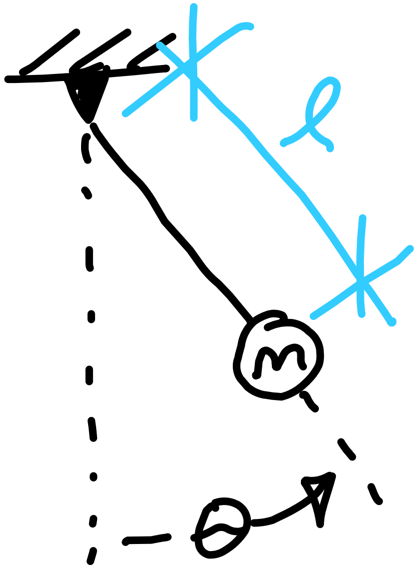
Solved 3 ways

1st way

$$\sum F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$$

but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

Example : Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

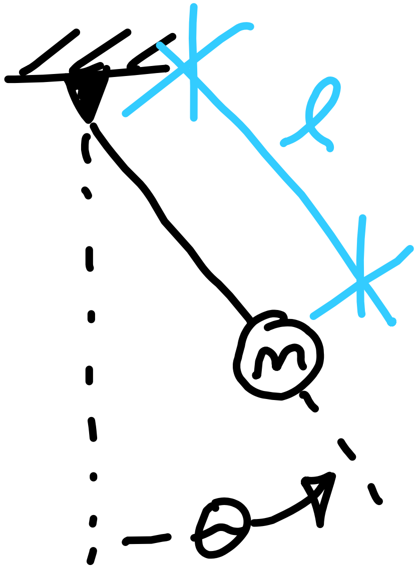
1st way

$\sum F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$

Example: Pendulum



$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

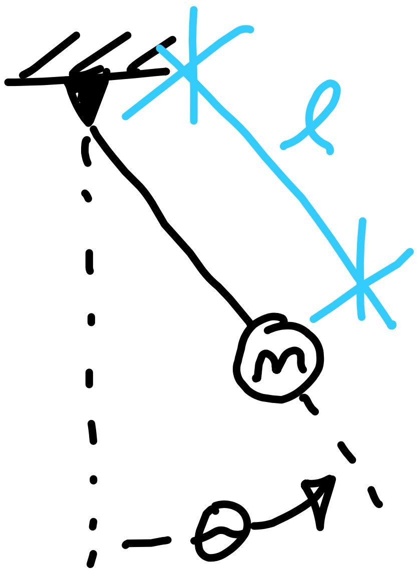
1st way

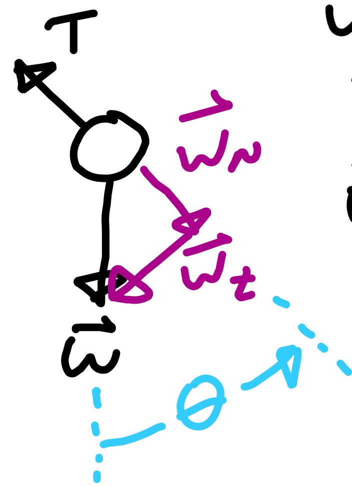
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$

Example: Pendulum





$$\vec{W} = \vec{W}_N + \vec{W}_t$$

$$\vec{W}_N = (W\cos\theta)(-\hat{e}_N)$$

$$\vec{W}_t = (W\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

1st way

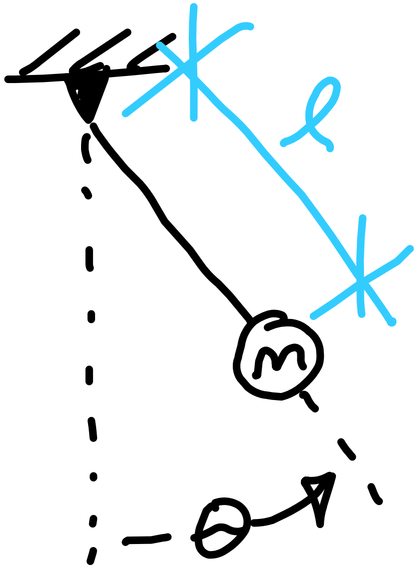
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

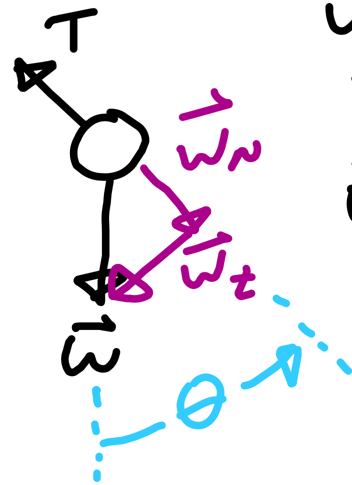
but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -W\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-W\theta \cong m\ell\ddot{\theta}$

Example: Pendulum





$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

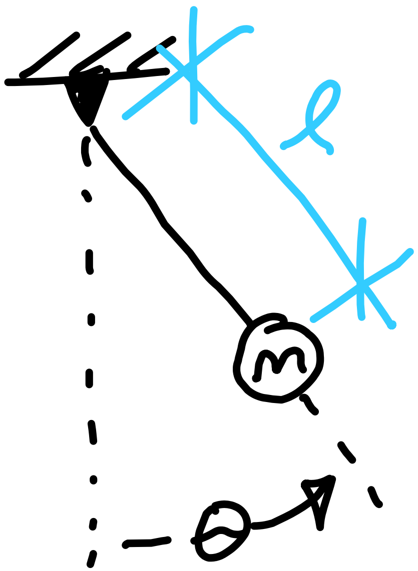
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = l\dot{\theta}$ so $-W_t = m l \ddot{\theta}$

$\Rightarrow -w\sin\theta = m l \ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-W\theta \cong m l \ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$

Example: Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

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$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

$\Sigma F_t = ma_t \Rightarrow -W_t = m\frac{dv}{dt}$

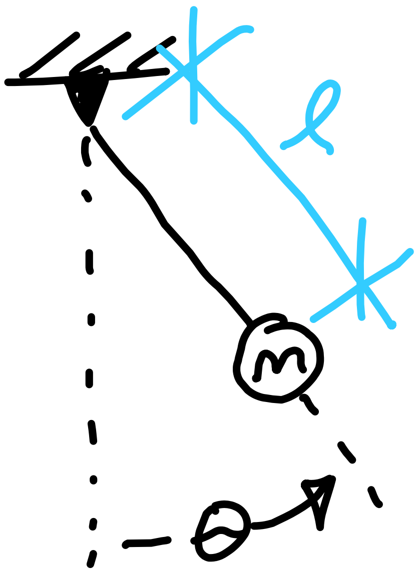
but $v = l\dot{\theta}$ so $-W_t = ml\ddot{\theta}$

$\Rightarrow -w\sin\theta = ml\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong ml\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$,

where $\omega_n = \sqrt{\frac{mg}{ml}}$

Example : Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

$\sum F_t = ma_t \Rightarrow -W_t = m\frac{dv}{dt}$

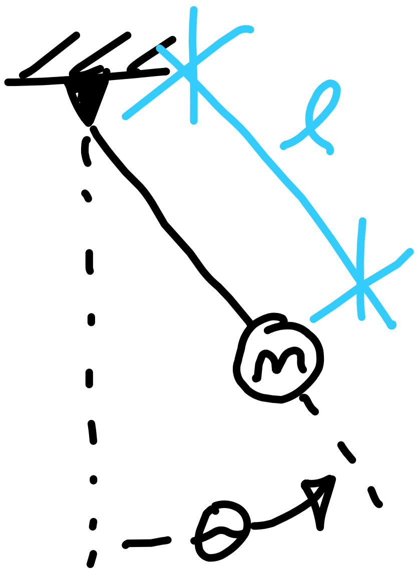
but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

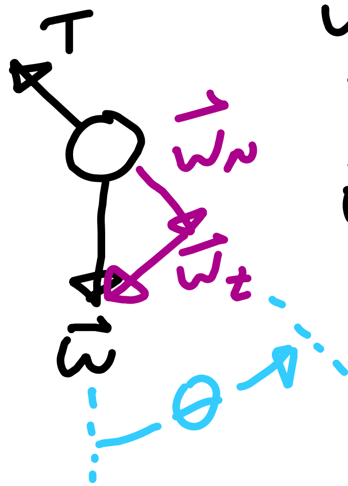
$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong m\ell\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta,$

where $\omega_n = \sqrt{\frac{mg}{m\ell}} = \sqrt{g/\ell}$

Example : Pendulum





$$\vec{w} = \vec{w}_n + \vec{w}_t$$
$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$
$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

2nd way

1st way

$\sum F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

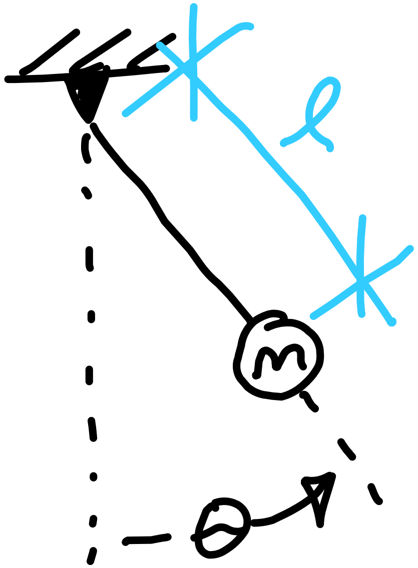
but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong m\ell\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$,

where $\omega_n = \sqrt{\frac{mg}{m\ell}} = \sqrt{g/\ell}$

Example: Pendulum



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Solved 3 ways

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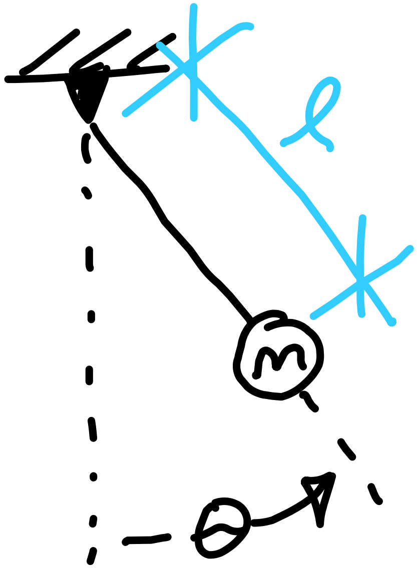

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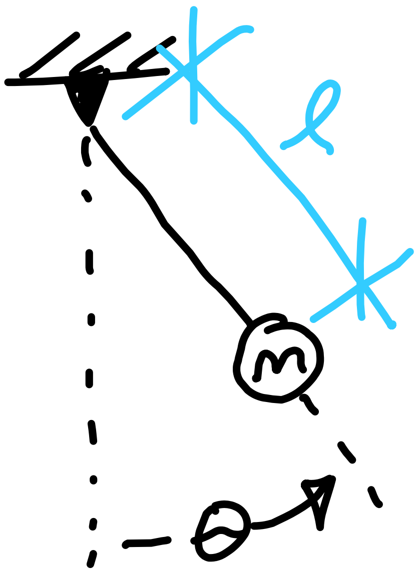

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But $I_A = m\ell^2$

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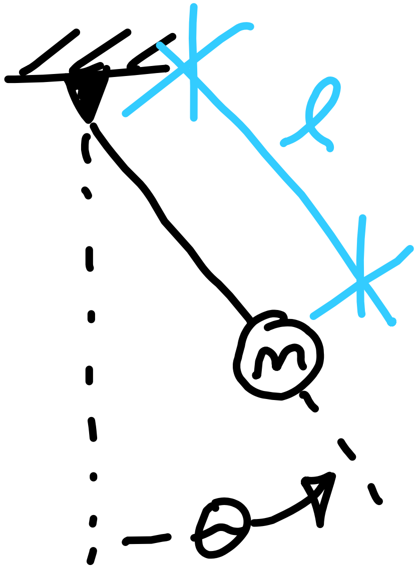

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But $I_A = ml^2$ & $W_t = W\sin\theta$

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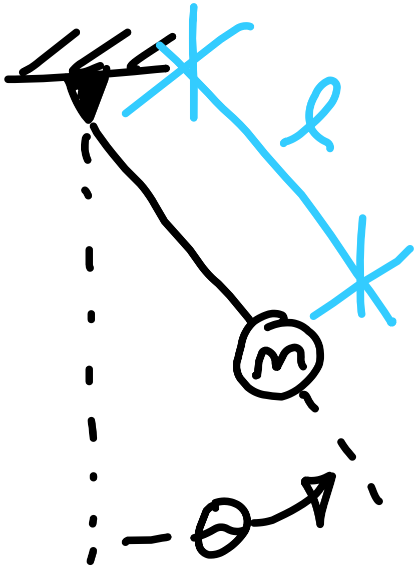

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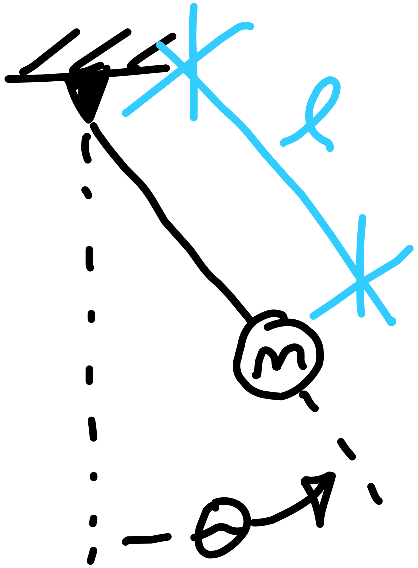

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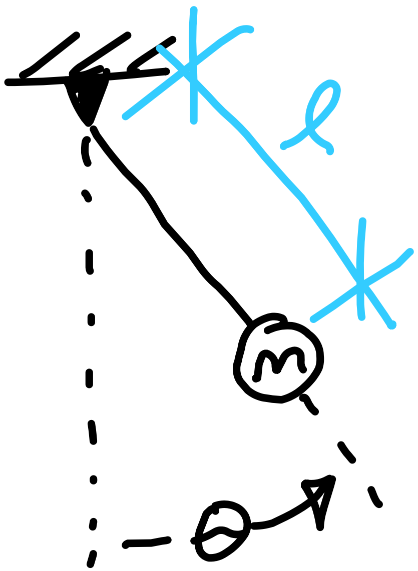

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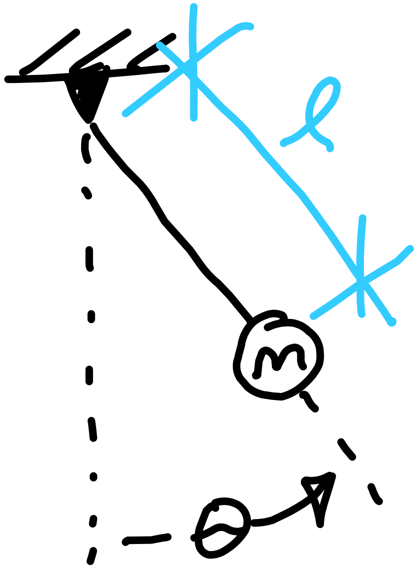

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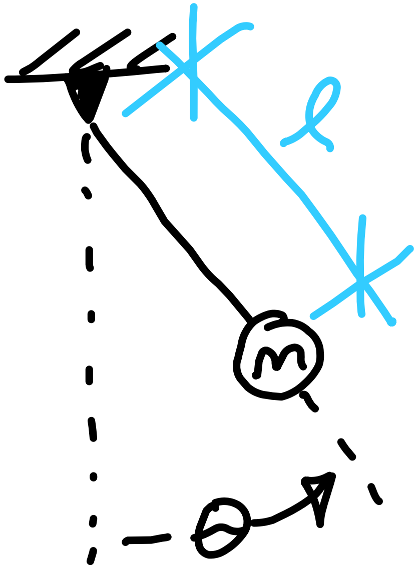

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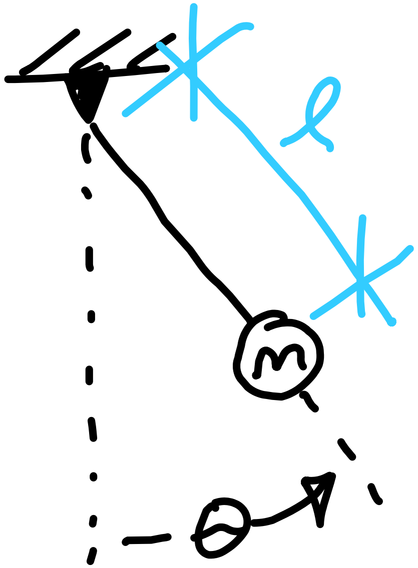

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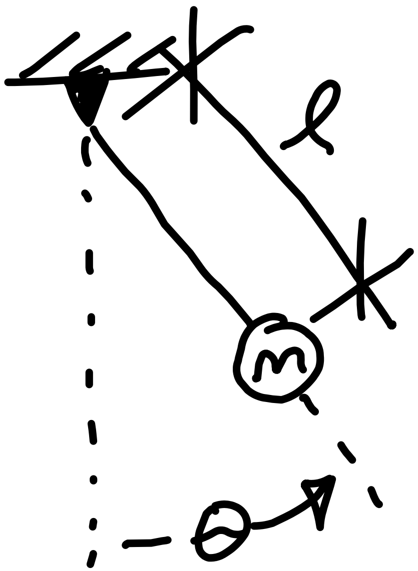
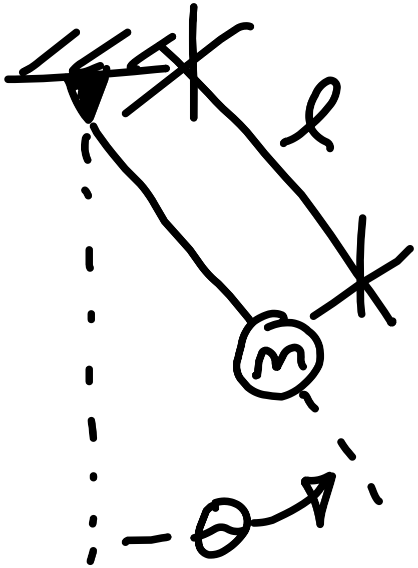
3rd way: Conservation of energy



3rd way: Conservation of energy & get into the form $\dot{x}^2 + \omega_n^2 x^2 = \text{const.}$



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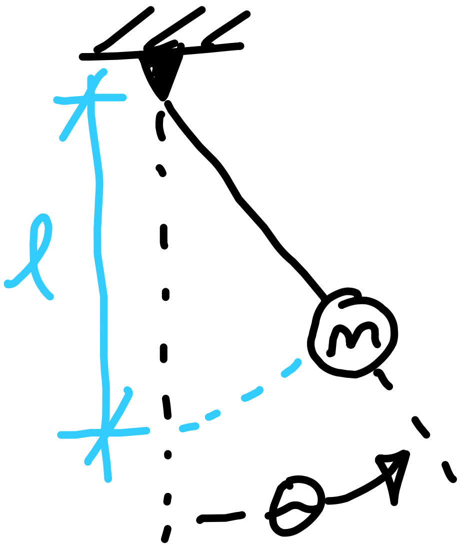

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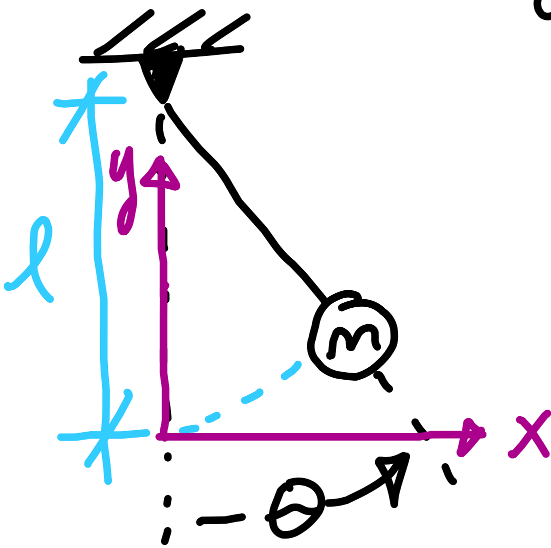

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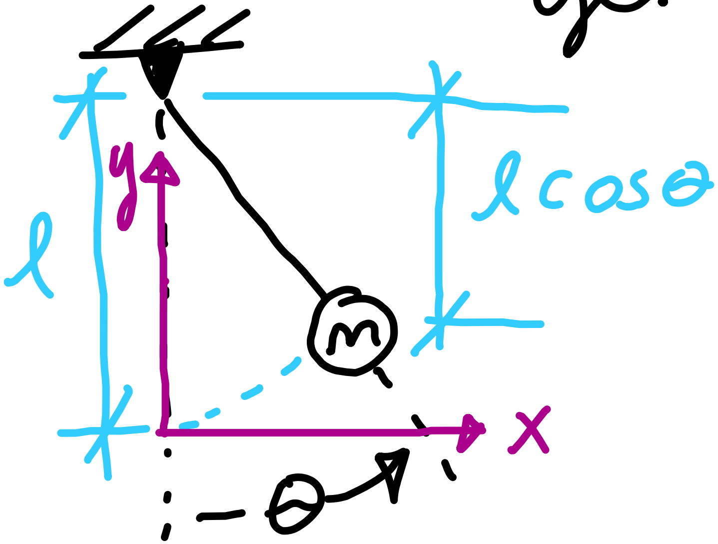

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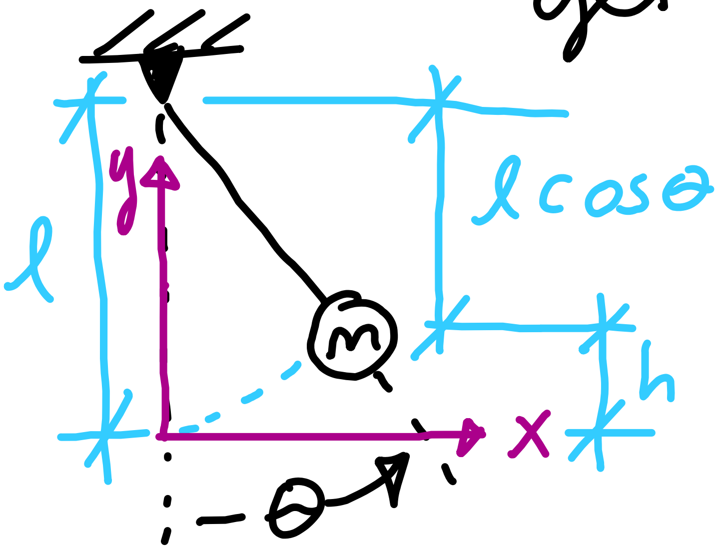

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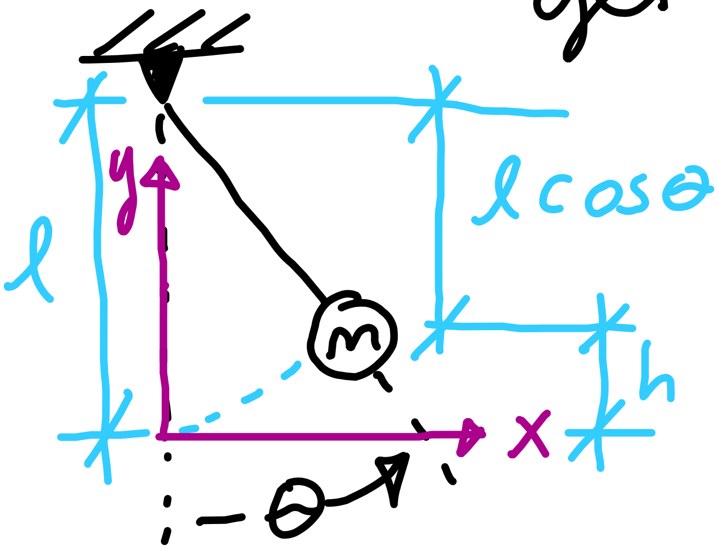

$$l = l\cos\theta + h$$

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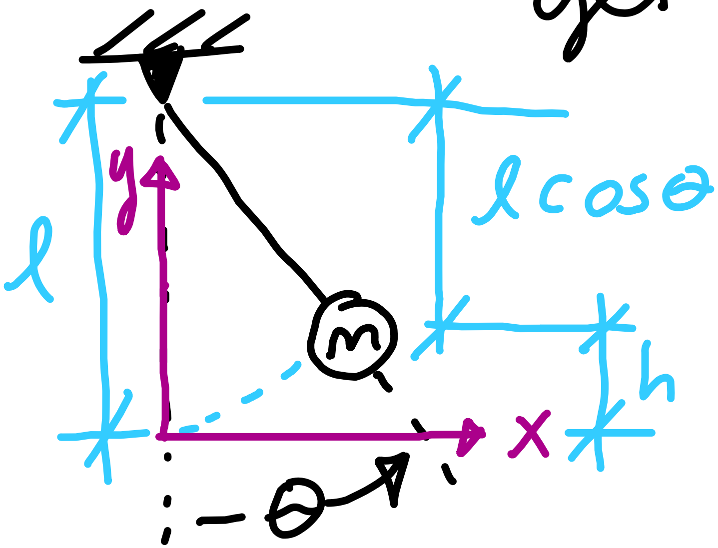

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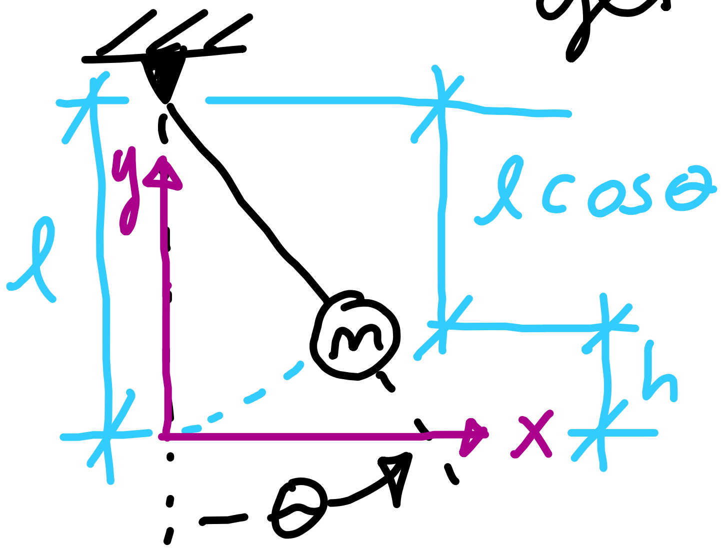

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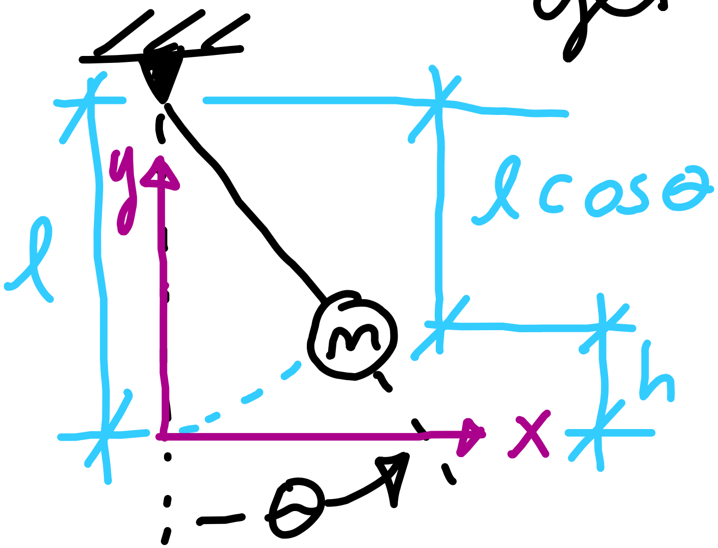

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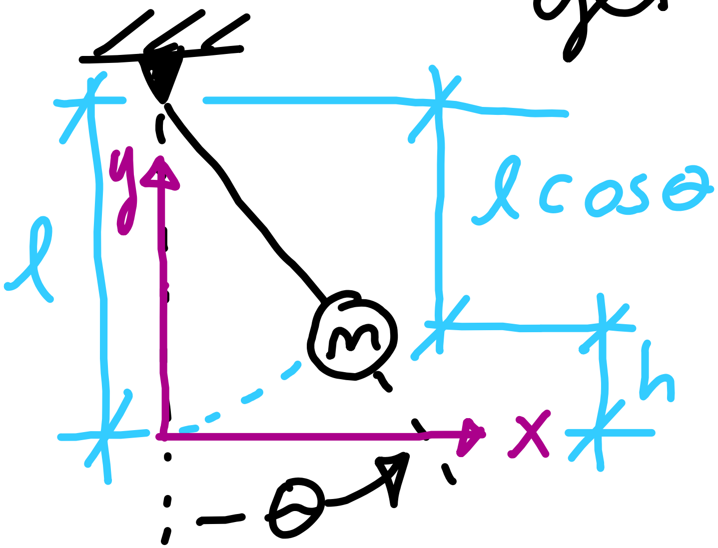

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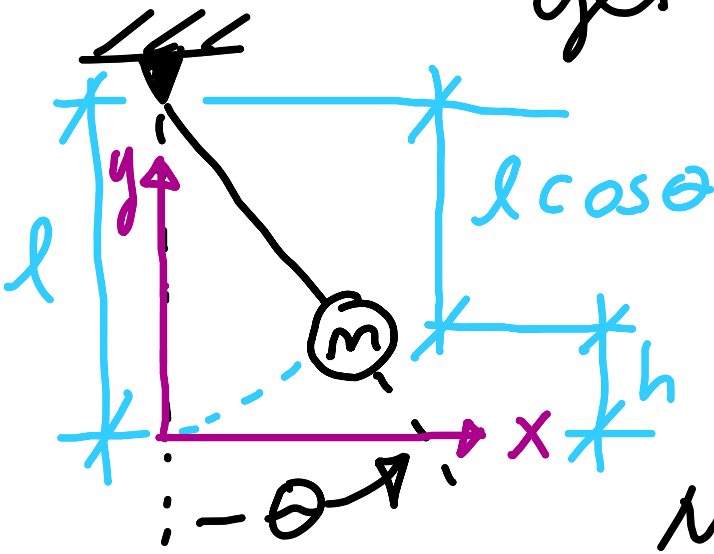

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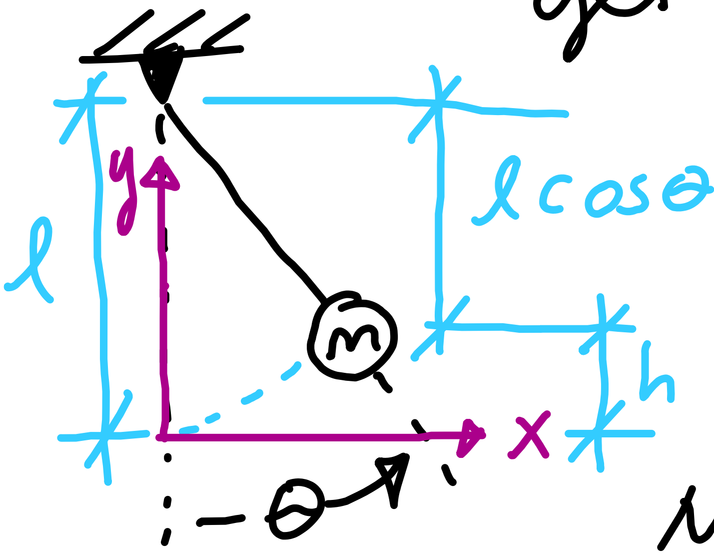

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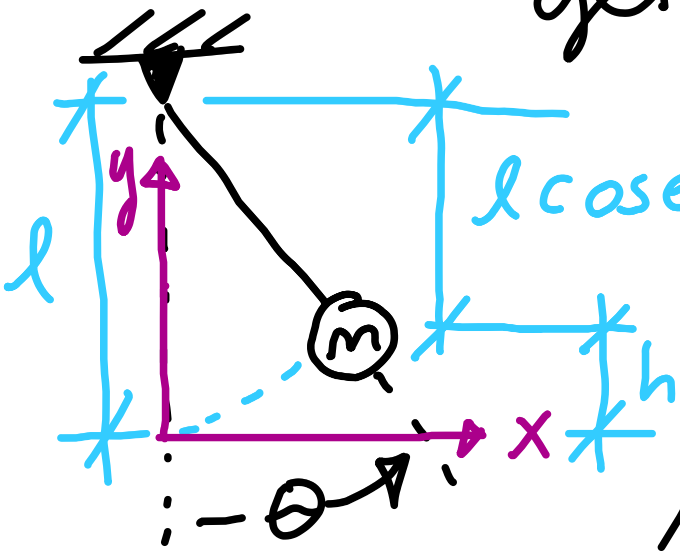

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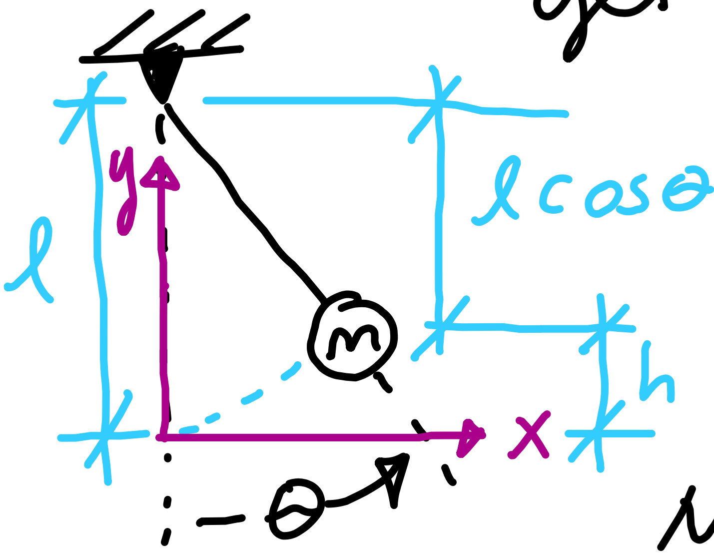

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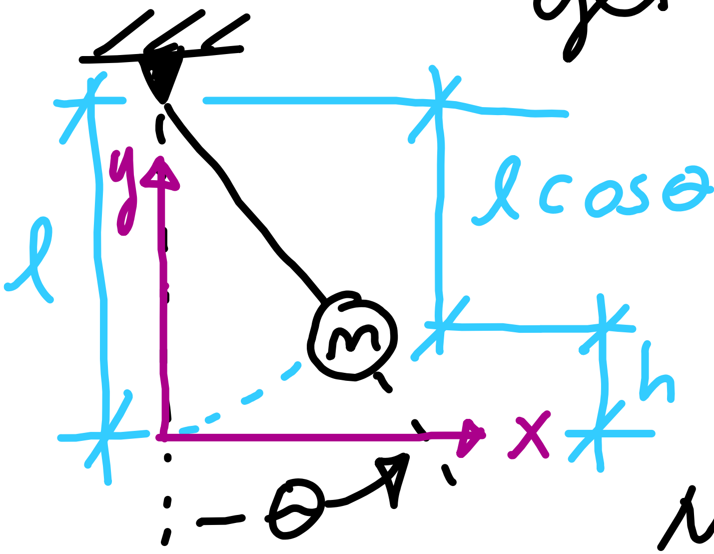

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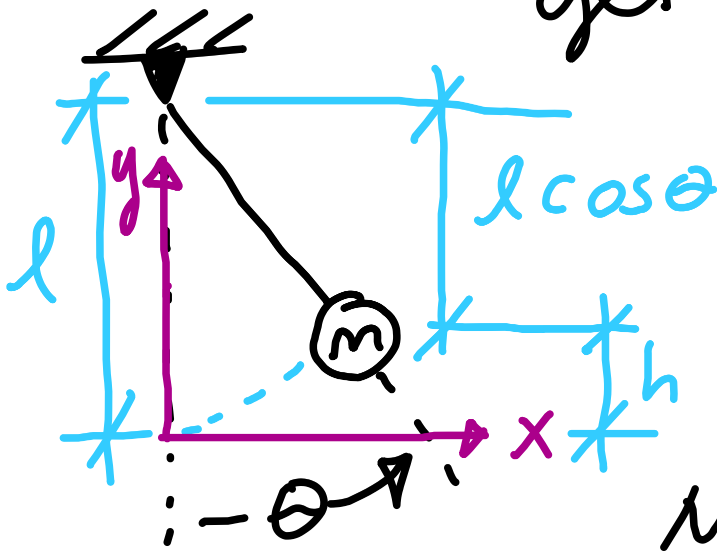

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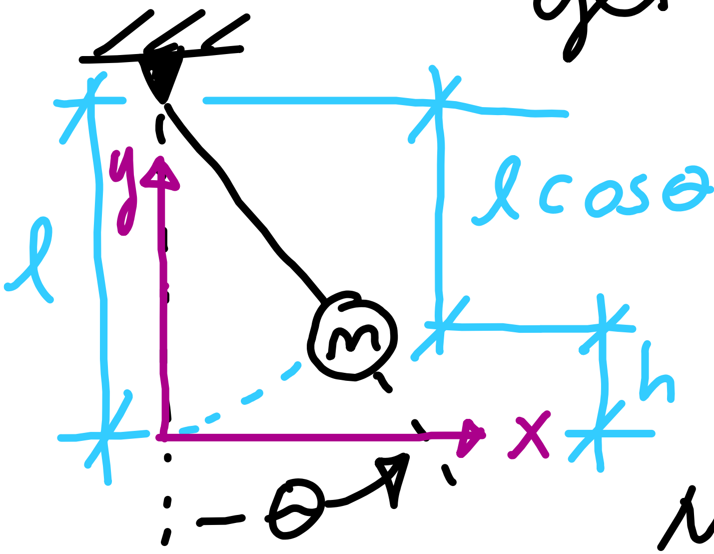

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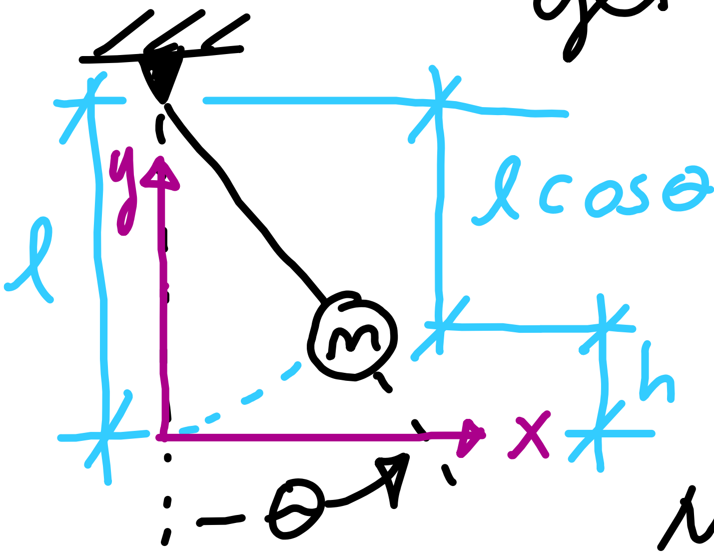
$\frac{1}{2}m\ell^2\dot{\theta}^2+\frac{1}{2}mg\ell\theta^2=\text{const.}$ Divide both sides by $\frac{1}{2}m\ell^2$

3rd way: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2 x^2 = \text{const.}$



$\ell = \ell\cos\theta + h$ so

$h = \ell(1-\cos\theta)$ small θ

$\Rightarrow \cos\theta \cong 1 - \frac{\theta^2}{2} \Rightarrow$

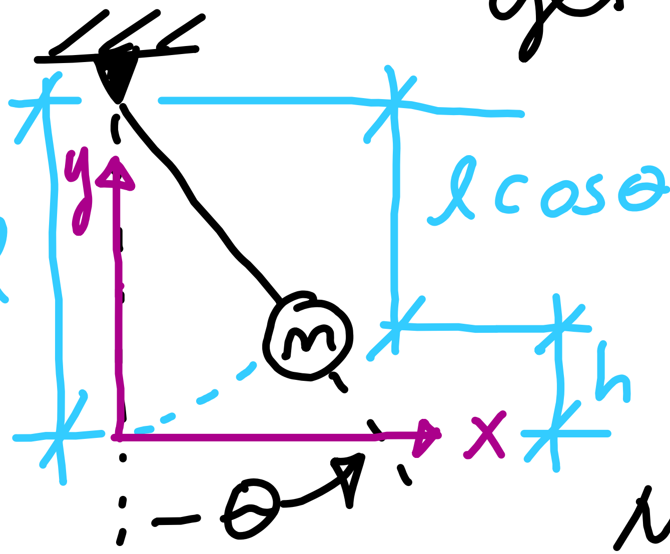
$h \cong \ell(1-1+\frac{\theta^2}{2}) = \ell\theta^2/2$

Now $T+V = \text{const.}$ & $T = \frac{1}{2}mv^2$

& $V = mgh$ But $v = \ell\dot{\theta}$ & $h = \ell\theta^2/2$ so

$\frac{1}{2}m\ell^2\dot{\theta}^2 + \frac{1}{2}mg\ell\theta^2 = \text{const.}$ Divide both sides by $\frac{1}{2}m\ell^2$ to get $\dot{\theta}^2 + (\frac{g}{\ell})\theta^2 = \text{Const.}$

3rd way: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2 x^2=\text{const.}$



$l = l\cos\theta + h$ so

$h = l(1-\cos\theta)$ small θ

$\Rightarrow \cos\theta \cong 1-\frac{\theta^2}{2} \Rightarrow$

$h \cong l(1-1+\frac{\theta^2}{2}) = l\theta^2/2$

Now $T+V=\text{const.}$ & $T=\frac{1}{2}mv^2$

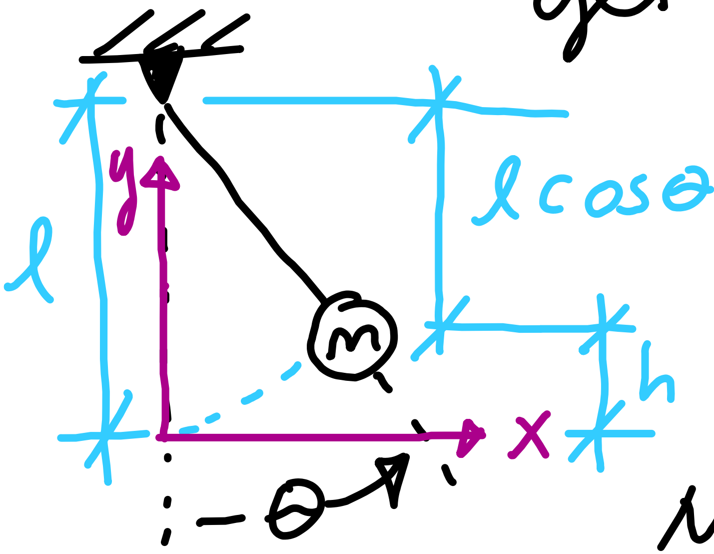
& $V=mgh$ But $v=l\dot{\theta}$ & $h=l\theta^2/2$ so

$\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mgl\theta^2=\text{const.}$ Divide both

sides by $\frac{1}{2}ml^2$ to get $\dot{\theta}^2+\left(\frac{g}{l}\right)\theta^2=\text{Const.}$

or $\dot{\theta}^2+\omega_n^2\theta^2=\text{const.}$

$\underline{3^{rd}\text{ way}}$: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2x^2=\text{const.}$



$l = l\cos\theta + h$ so

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Now $T+V=\text{const.}$ & $T=\frac{1}{2}mv^2$

& $V=mgh$ But $v=l\dot{\theta}$ & $h=l\theta^2/2$ so

$\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mgl\theta^2=\text{const.}$ Divide both

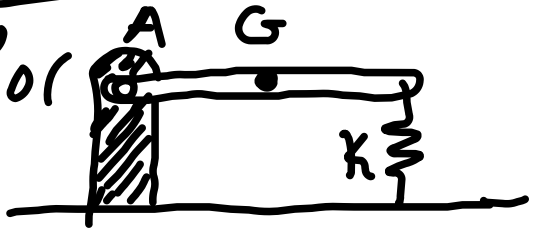
sides by $\frac{1}{2}ml^2$ to get $\dot{\theta}^2+\left(\frac{g}{l}\right)\theta^2=\text{Const.}$

or $\quad\dot{\theta}^2+\omega_n^2\theta^2=\text{const.}$ where

$\omega_n=\sqrt{\frac{g}{l}}$

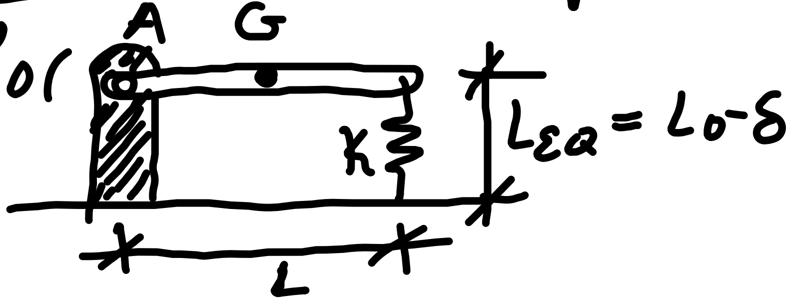
A more complicated example

A more complicated example Find τ_n for



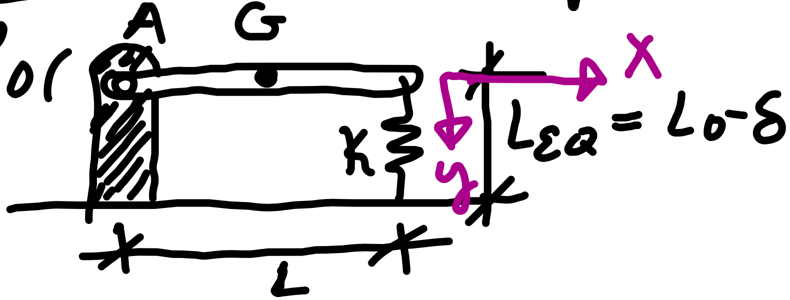
A more complicated example

Find τ_n for



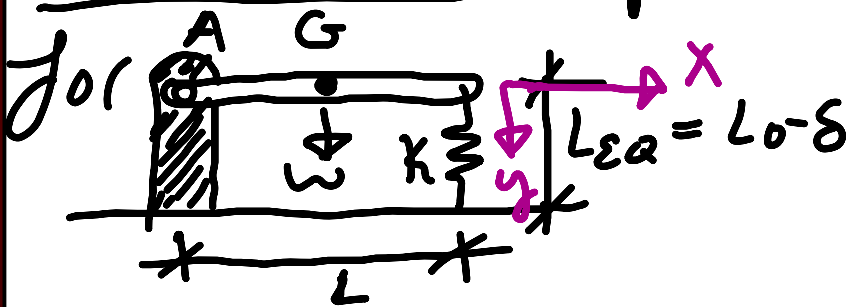
A more complicated example

Find τ_n for

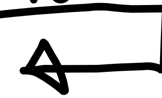


A more complicated example

Find τ_n for

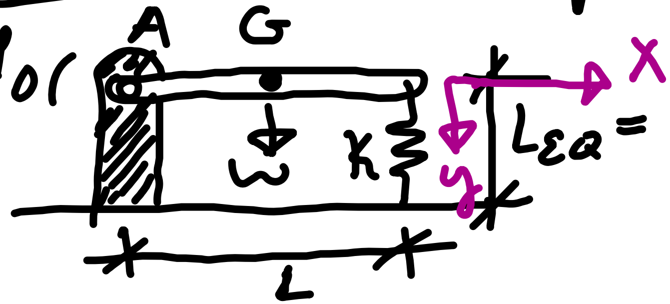


Equilibrium:



A more complicated example

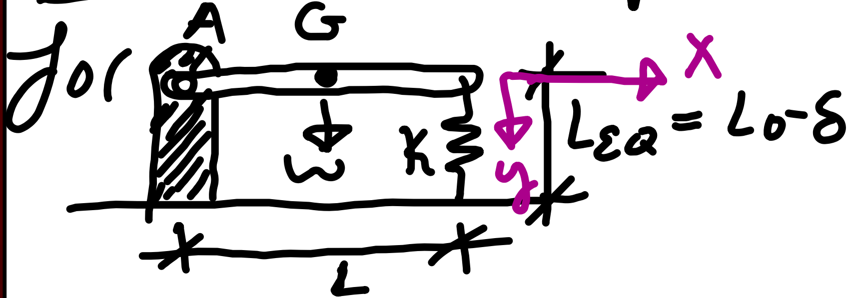
Find τ_n for



$L_{EQ} = L_0 - \delta$

Equilibrium: $\sum M_A = 0$

A more complicated example Find τ_n for

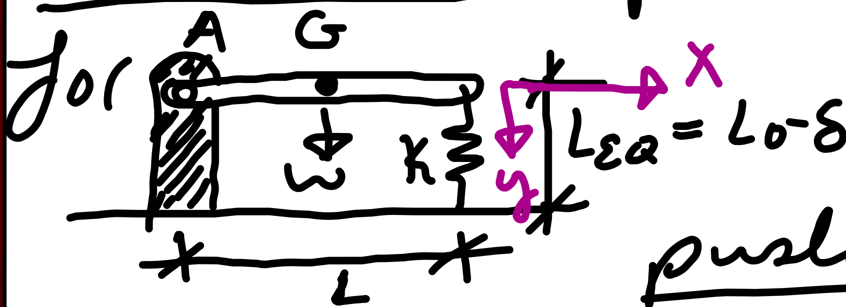


Equilibrium: $\curvearrowleft \sum M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

A more complicated example

Find τ_n for

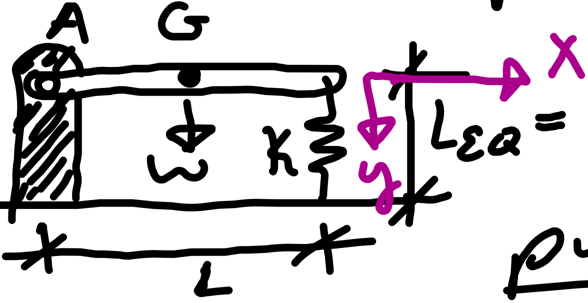


Equilibrium: $\curvearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

A more complicated example Find τ_n for



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push down small amount y:

$\sum M_A = I_A \ddot{\theta}$

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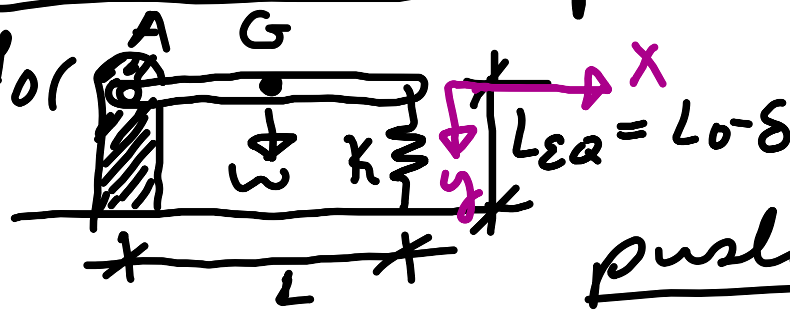


Equilibrium: $\circlearrowleft \sum M_A = 0$

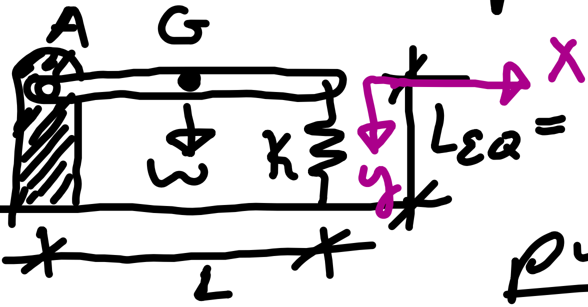
$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

$\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow W\frac{L}{2} - K\delta - Ky = I_A \ddot{\theta}$

A more complicated example

Find τ_n for



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Equilibrium: $\circlearrowleft \sum M_A = 0$

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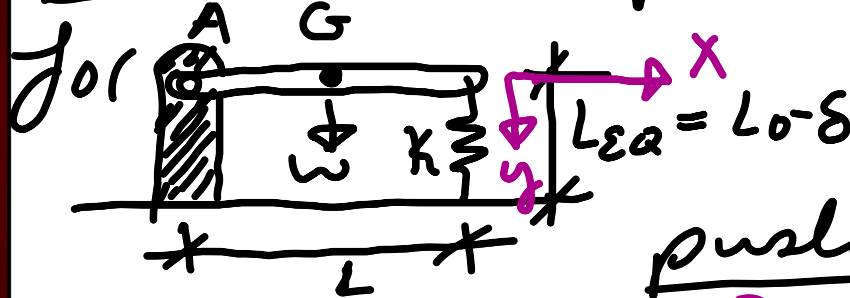
A more complicated example

Find τ_n for



Equilibrium: $\curvearrowleft \Sigma M_A = 0$

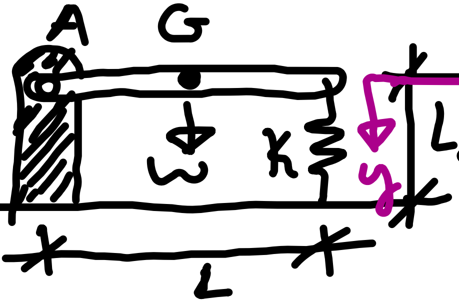
$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\curvearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{=0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

A more complicated example

Find τ_n for



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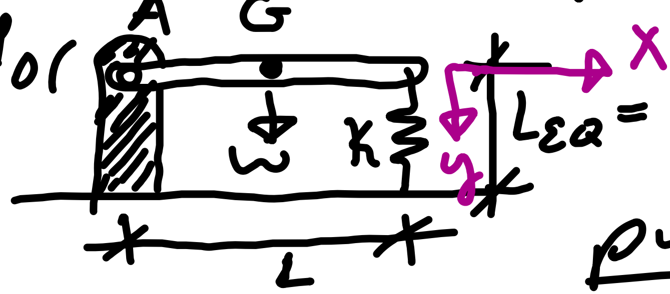
But $y = L\sin\theta$

A more complicated example Find τ_n for



Equilibrium: $\curvearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

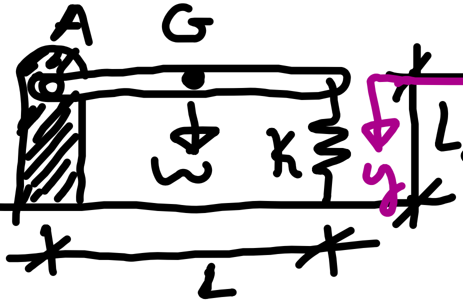
push down small amount y:

$\curvearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow W\frac{L}{2} - k\delta - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A \ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta$

A more complicated example

Find τ_n for



Equilibrium: $\circlearrowright \Sigma M_A = 0$

$$\Rightarrow W\frac{L}{2} - k\delta = 0$$

push down small amount y:

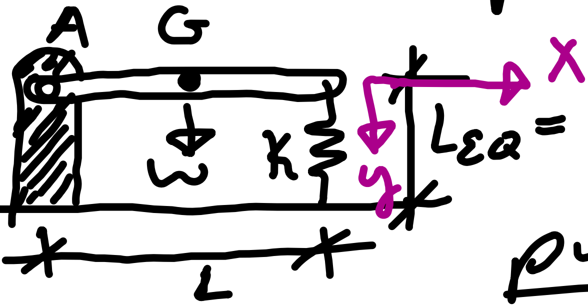
$$\circlearrowright \Sigma M_A = I_A\ddot{\theta} \Rightarrow \overbrace{W\frac{L}{2} - k\delta}^{0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$$-kL\theta = I_A\ddot{\theta}$$

A more complicated example

Find ω_n for



Equilibrium: $\circlearrowright \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

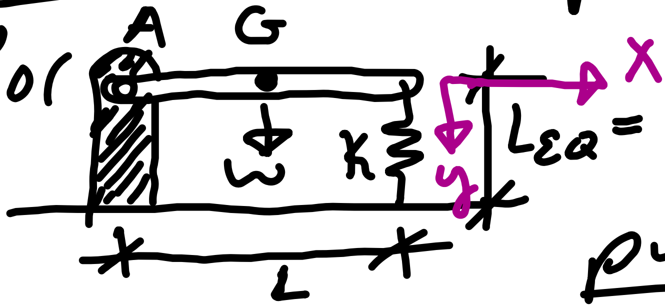
$\circlearrowright \Sigma M_A = I_A\ddot{\theta} \Rightarrow \cancel{W\frac{L}{2} - K\delta} - Ky = I_A\ddot{\theta} \Rightarrow -Ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$

A more complicated example

Find τ_n for



$L_{EQ} = L_0 - \delta$

Equilibrium: $\circlearrowleft \sum M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

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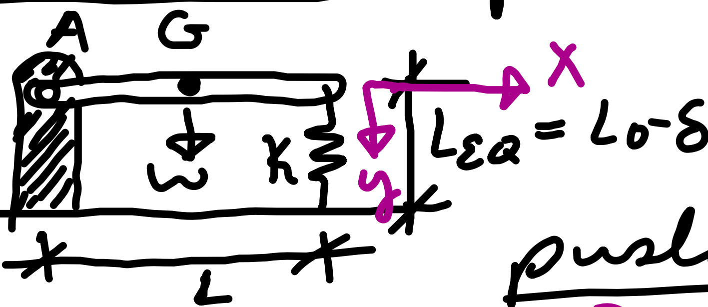
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$$\omega_n = \sqrt{\frac{KL^2}{I_A}}$$

A more complicated example Find ω_n

For



Equilibrium: $\circlearrowright \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

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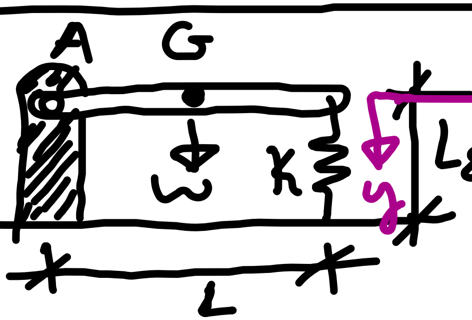
$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2$

A more complicated example

Find τ_n for



Equilibrium: $\circlearrowleft \Sigma M_A = 0$

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push down small amount y:

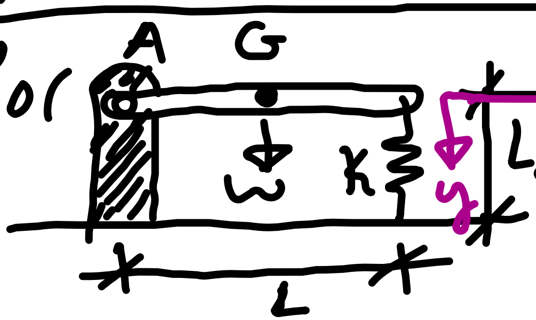
$\circlearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \cancel{W\frac{L}{2} - K\delta} - Ky = I_A\ddot{\theta} \Rightarrow \quad -Ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4}$

A more complicated example Find τ_n

For 

$L_{EQ} = L_0 - \delta$

Equilibrium: $\Sigma M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$\Sigma M_A = I_A\ddot{\theta} \Rightarrow \overset{0}{\cancel{w\frac{L}{2} - k\delta}} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

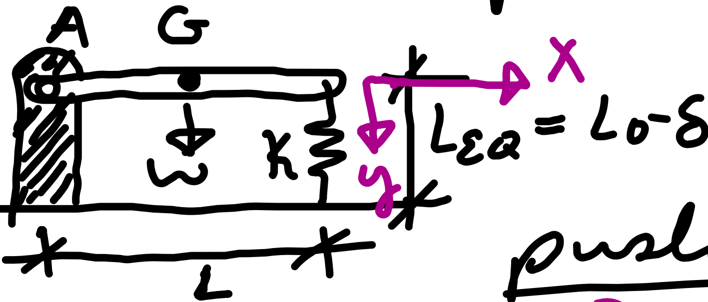
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4}$

$\Rightarrow I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right)$

A more complicated example Find τ_n

for 

Equilibrium: $\curvearrowleft \sum M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

$\curvearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow \overbrace{\cancel{W\frac{L}{2} - K\delta}}^{0} - Ky = I_A\ddot{\theta} \Rightarrow \quad -ky = I_A\ddot{\theta}$

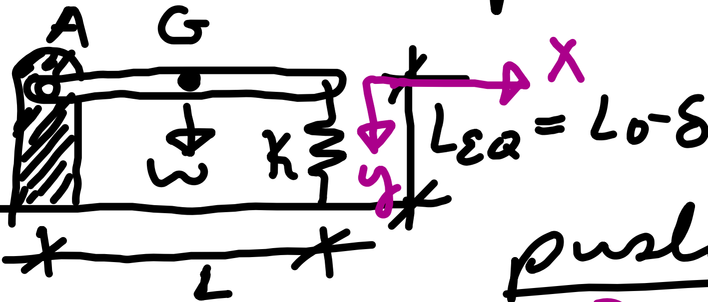
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\quad \ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow \quad I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$

A more complicated example Find ω_n for



Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$$\circlearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow \underbrace{\cancel{w\frac{L}{2} - k\delta}}_{} - ky = I_A\ddot{\theta} \Rightarrow \quad -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

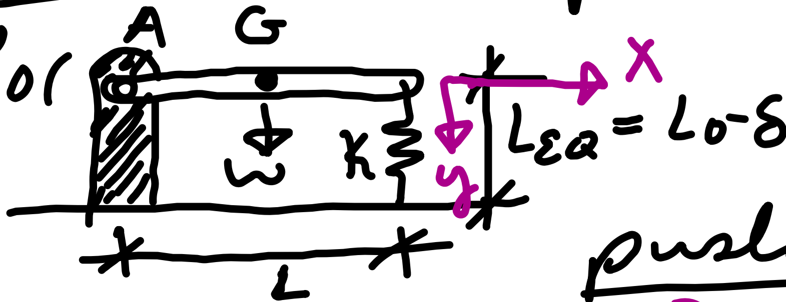
$\omega_n = \sqrt{\dfrac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \dfrac{mL^2}{12} + \dfrac{mL^2}{4}$

$\Rightarrow \quad I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right) = mL^2/3$. Now

$$\omega_n = \sqrt{\frac{kL^2}{mL^2/3}}$$

A more complicated example: Find ω_n

For



Equilibrium: $\circlearrowleft \sum M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$\circlearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow \cancel{w\frac{L}{2} - k\delta}^{\,0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

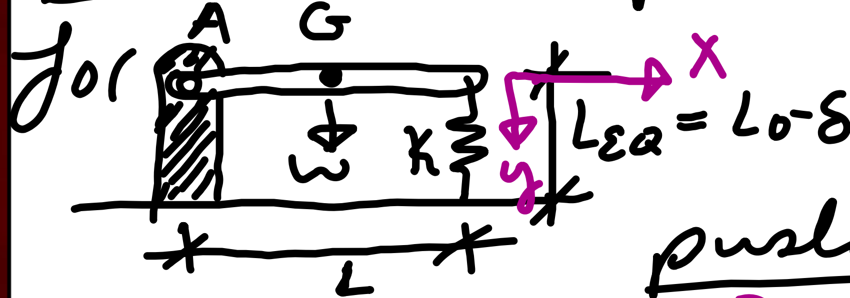
$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4}$

$\Rightarrow I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right) = mL^2/3$. Now

$\omega_n = \sqrt{\frac{kL^2}{mL^2/3}} = \sqrt{\frac{3k}{m}}$

A more complicated example. Find τ_N for



Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$$\Rightarrow W\frac{L}{2} - k\delta = 0$$

push down small amount y:

$$\circlearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \cancel{W\frac{L}{2} - k\delta} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

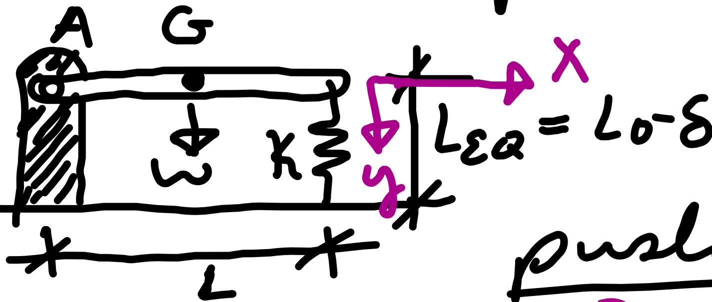
$\omega_n = \sqrt{\dfrac{kL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$. Now

$\omega_N = \sqrt{\dfrac{kL^2}{ML^2/3}} = \sqrt{\dfrac{3k}{M}}$ & since $\tau_N = \dfrac{2\pi}{\omega_N}$

A more complicated example

Find τ_N for



Equilibrium: $\curvearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\curvearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \underbrace{\cancel{W\frac{L}{2} - k\delta}}_{0} - ky = I_A\ddot{\theta} \Rightarrow \quad -ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta \cong I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_N^2\theta$, where

$\omega_N = \sqrt{\dfrac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow \quad I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$. Now

$\omega_N = \sqrt{\dfrac{KL^2}{ML^2/3}} = \sqrt{\dfrac{3k}{M}}$ & since $\tau_N = \dfrac{2\pi}{\omega_N}$

then $\boxed{\tau_N = 2\pi\sqrt{\dfrac{M}{3k}}}$

Forced vibrations: Get into the form $A\ddot{x} + Bx = C\sin(\omega_F t)$

Forced vibrations: Get into the form $A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

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* Homogeneous solution when $C = 0$

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* Homogeneous solution when $C=0$

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Forced vibrations: Get into the

Form $\quad A\ddot{x}+Bx=\underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

* Homogeneous solution when $C=0$

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Example:

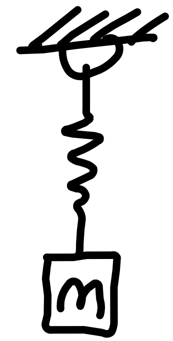
Forced vibrations: Get into the form

$$A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$$

* Homogeneous solution when $C = 0$

* Particular solution: Assume $x_p = X_m \sin(\omega_F t)$

Example:



Forced vibrations: Get into the

Form $A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

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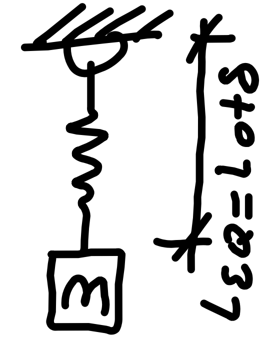

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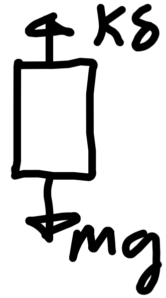

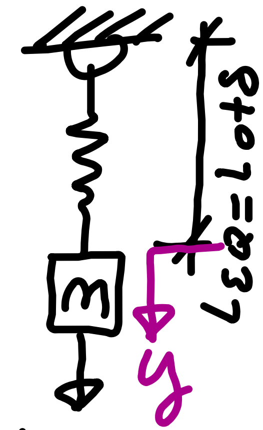

$P = P_m \sin(\omega_F t)$

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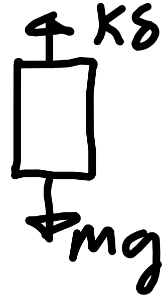
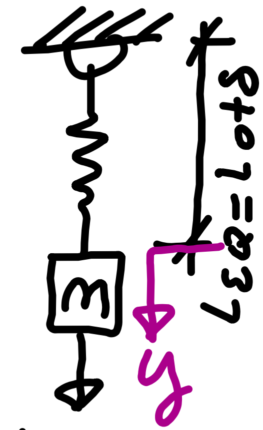


$\Sigma F_y = 0$



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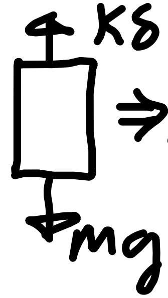

$\Sigma F_y = 0$

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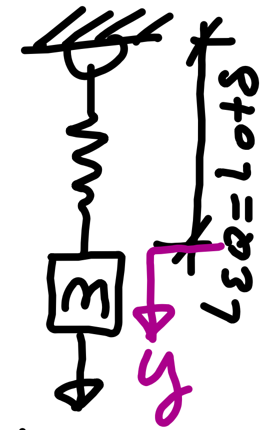

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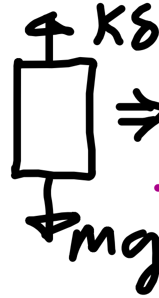

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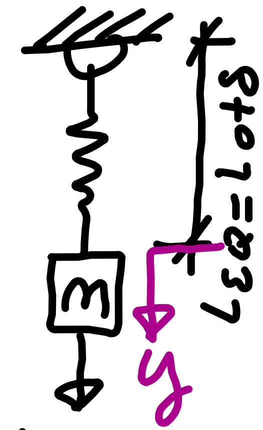

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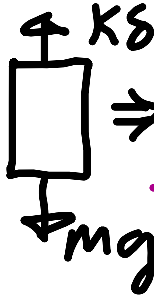

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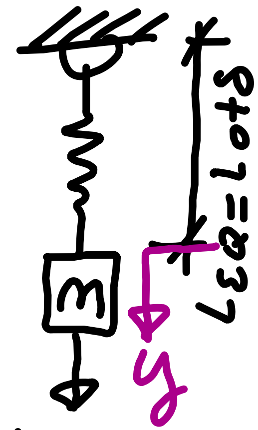

$P = P_m \sin(\omega_F t)$

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Forced vibrations: Get into the form

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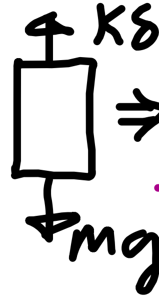

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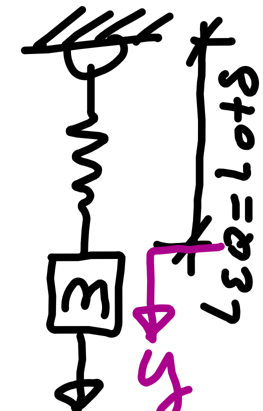

Non-Equilibrium: $\Sigma F_y = m\ddot{y}$

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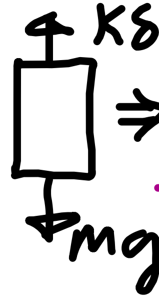

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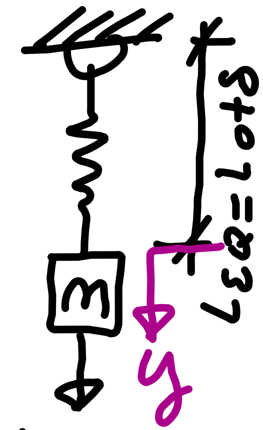

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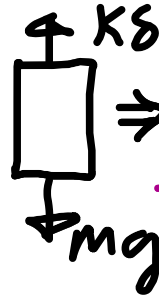

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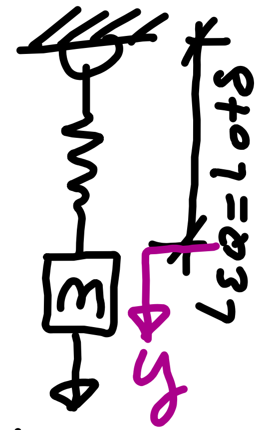

$P = P_m \sin(\omega_F t)$

Non-Equilibrium: $\Sigma F_y = m\ddot{y}$

$\Rightarrow mg - k\delta - ky + P = m\ddot{y} \Rightarrow m\ddot{y} + ky = P$

Forced vibrations: Get into the form

$$A\ddot{x}+Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$$

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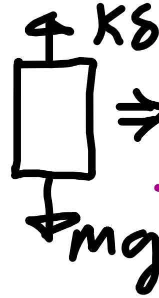

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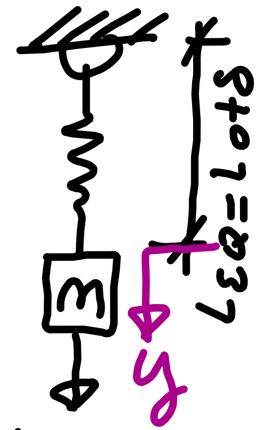

Non-Equilibrium: $\Sigma F_y = m\ddot{y}$

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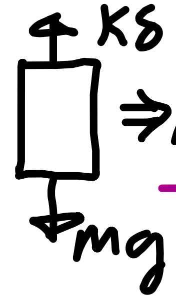

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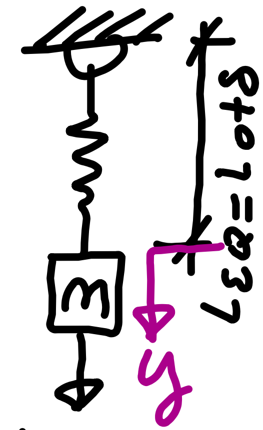

$\Rightarrow mg - k\delta - ky + P = m\ddot{y} \Rightarrow m\ddot{y} + ky = P_m \sin(\omega_F t)$

$P = P_m \sin(\omega_F t)$

Homogeneous Part:

Forced vibrations: Get into the form $A\ddot{x}+Bx=\underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

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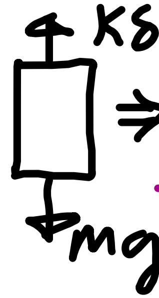

$\Sigma F_y = 0$

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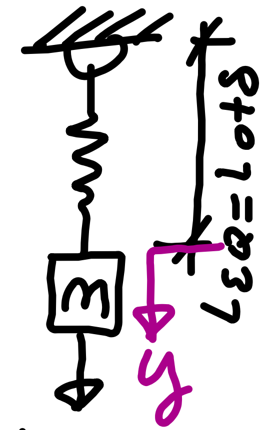

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Homogeneous part: Take $P_m = 0$ $\qquad P = P_m \sin(\omega_F t)$

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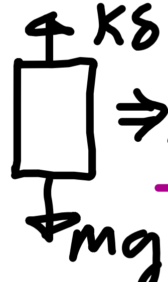

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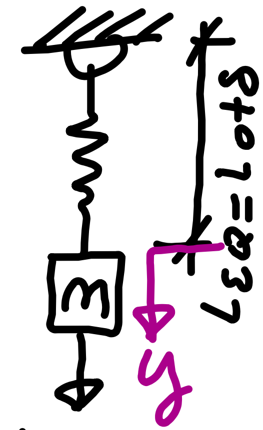

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$P = P_m \sin(\omega_F t)$

Homogeneous part: Take $P_m = 0$

$\Rightarrow m\ddot{y} = -ky$

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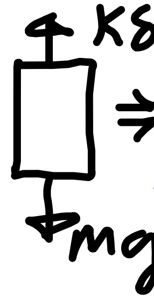
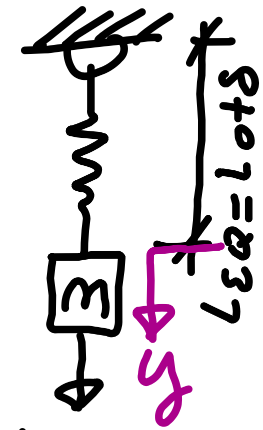
Example:

Equilibrium: $\Sigma F_y = 0 \Rightarrow \underline{mg - k\delta = 0}$





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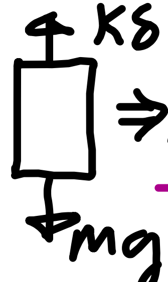
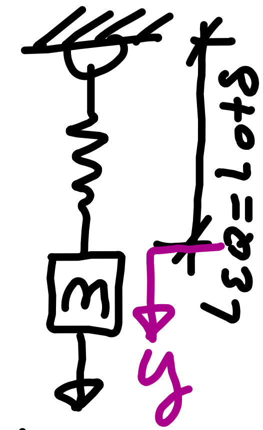


$\sum F_y = 0$

$\Rightarrow mg - k\delta = 0$



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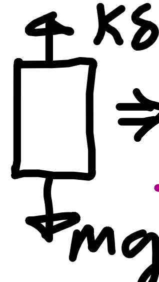

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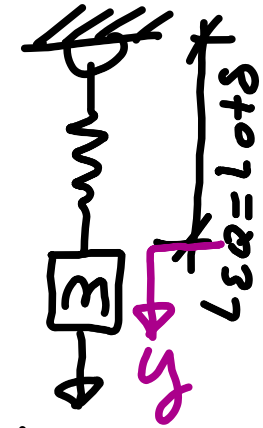

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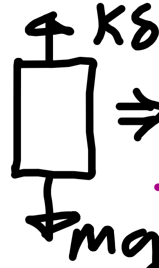

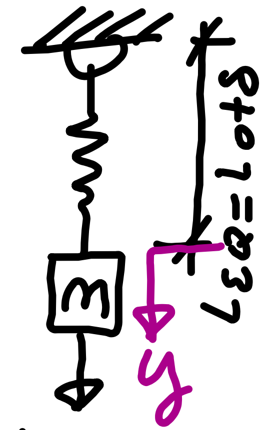

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From previous $-m y_n \omega_F^2 + k y_n = P_n$

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$\Rightarrow \quad y_n (K - m\omega_F^2) = P_n$

From previous $\quad -m y_n \omega_F^2 + k y_n = P_n$

$$\Rightarrow \quad y_n(K - m\omega_F^2) = P_n \Rightarrow y_n = \frac{P_n}{K - m\omega_F^2}$$

ASU

From previous $-m y_m \omega_F^2 + k y_m = P_m$

$$\Rightarrow \quad y_m(k - m\omega_F^2) = P_m \Rightarrow y_m = \left[\frac{P_m}{k - m\omega_F^2}\right]\left(\frac{1/k}{1/k}\right)$$

From previous $-m y_n \omega_F^2 + k y_n = P_n$

$\Rightarrow \quad y_n (k - m\omega_F^2) = P_n \Rightarrow y_n = \left[\dfrac{P_n}{k - m\omega_F^2}\right]\left(\dfrac{1/k}{1/k}\right)$

$$\Rightarrow \quad y_n = \frac{(P_n/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$$

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$\Rightarrow \quad y_n = \frac{(P_m/K)}{1 - \left(\frac{m}{K}\right)\omega_F^2}$ But $\omega_N^2 = \frac{K}{m}$

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$$\Rightarrow \quad y_m = \frac{(P_m/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$$ But $\omega_N^2 = \frac{k}{m}$ So

$$y_m = \frac{(P_m/k)}{1 - \omega_F^2/\omega_N^2}$$

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But $\omega_N^2 = \frac{k}{m}$ so

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& resonance when $\omega_F = \omega_N$

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$$\Rightarrow \quad y_m (k - m\omega_F^2) = P_m \Rightarrow y_m = \left[\frac{P_m}{k - m\omega_F^2}\right]\left(\frac{1/k}{1/k}\right)$$

$$\Rightarrow \quad y_m = \frac{(P_m/k)}{1-\left(\frac{m}{k}\right)\omega_F^2} \quad \text{But } \omega_N^2 = \frac{k}{m} \text{ so}$$

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& resonance when $\omega_F = \omega_N$

Unforced damped vibrations:

Unforced damped vibrations: Form is

$$m\ddot{x} + c\dot{x} + kx = 0$$

Unforced damped vibrations: Form is

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x\,e^{\lambda t}$

ASU

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$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t} \Rightarrow$

$\lambda^2 m + \lambda c + k = 0$

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$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t} \Rightarrow$

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Unforced damped vibrations: Form is

$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t}$ $\Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

Unforced damped vibrations: Form is

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x e^{\lambda t}$ $\Rightarrow$

$$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}} \quad \Rightarrow \quad \lambda = \frac{c \pm \sqrt{c^2-4mk}}{2m}$$

* Critically damped when $c^2-4mk=0$

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* Underdamped when $c^2 - 4mk < 0$

Unforced damped vibrations:

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x\,e^{\lambda t}$ (Form is) $\Rightarrow$

$$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}} \Rightarrow \lambda=\frac{c\pm\sqrt{c^2-4mk}}{2m}$$

* Critically damped when $c^2-4mk=0$

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* Underdamped when $c^2-4mk<0$

{vibrates}

Unforced damped vibrations:

$m\ddot{x} + c\dot{x} + kx = 0$, Assume x Form is $e^{\lambda t}$ $\Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

* Critically damped when $c^2 - 4mk = 0$

  {No vibrations, gets to equilibrium fastest}

* Underdamped when $c^2 - 4mk < 0$

  {vibrates}

* Overdamped when $c^2 - 4mk > 0$

Unforced damped vibrations: Form is

$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x = e^{\lambda t}$ $\Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

* Critically damped when $c^2 - 4mk = 0$

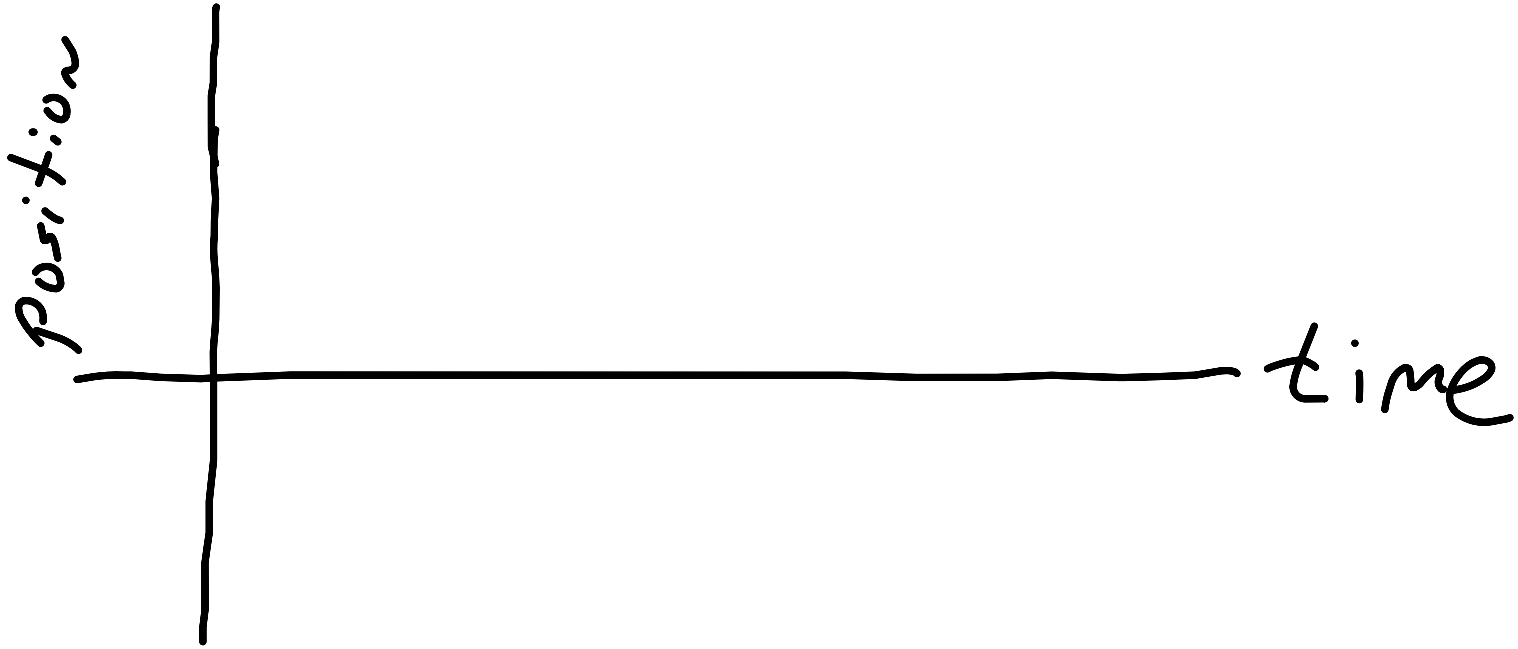
{No vibrations, gets to equilibrium fastest}

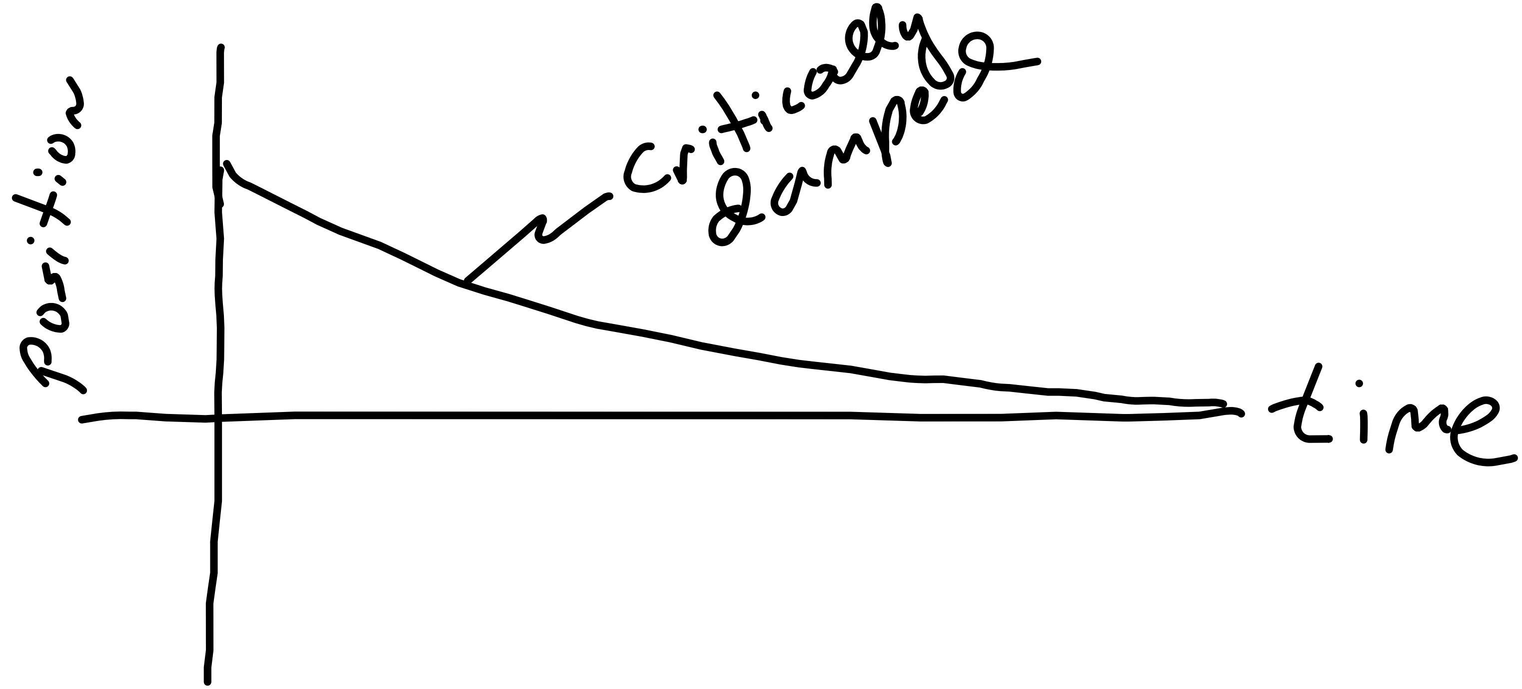
* Underdamped when $c^2 - 4mk < 0$

{vibrates}

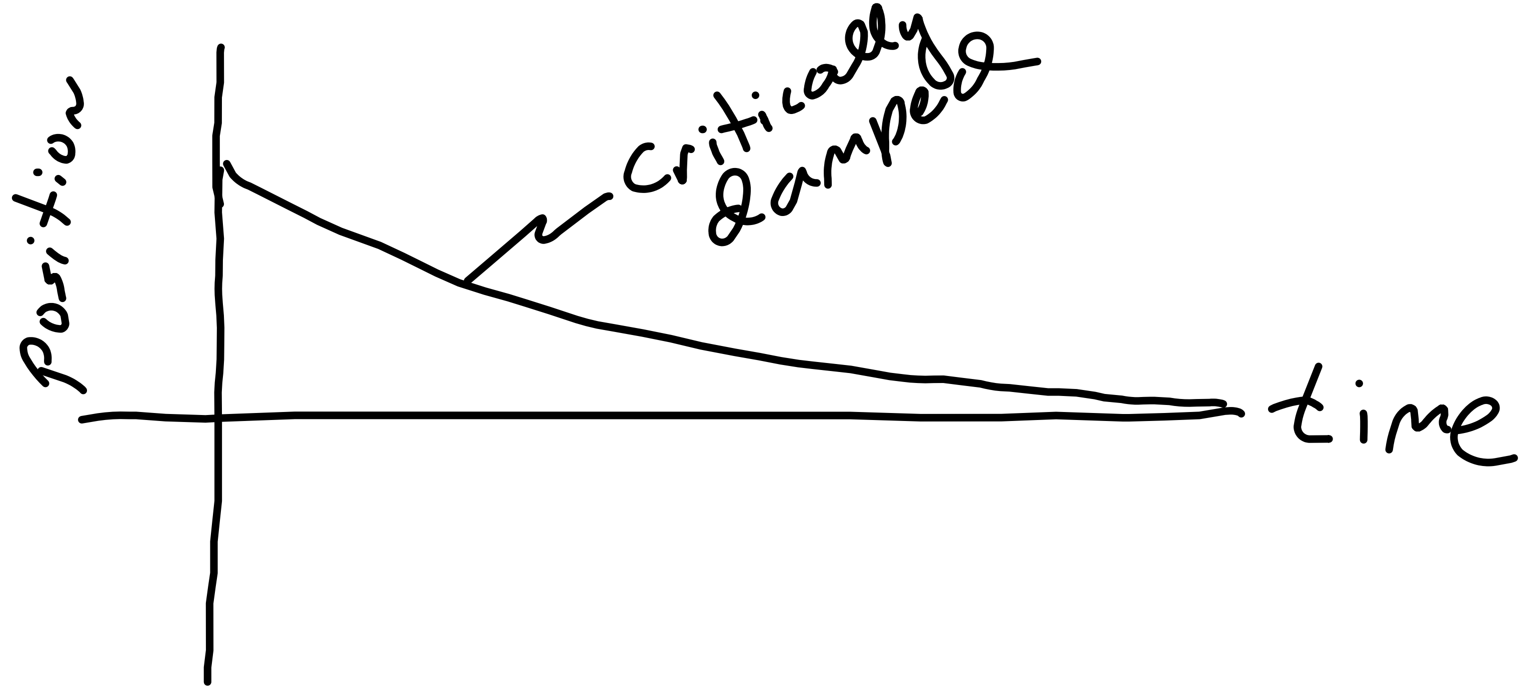
* Overdamped when $c^2 - 4mk > 0$

{No Vibration}

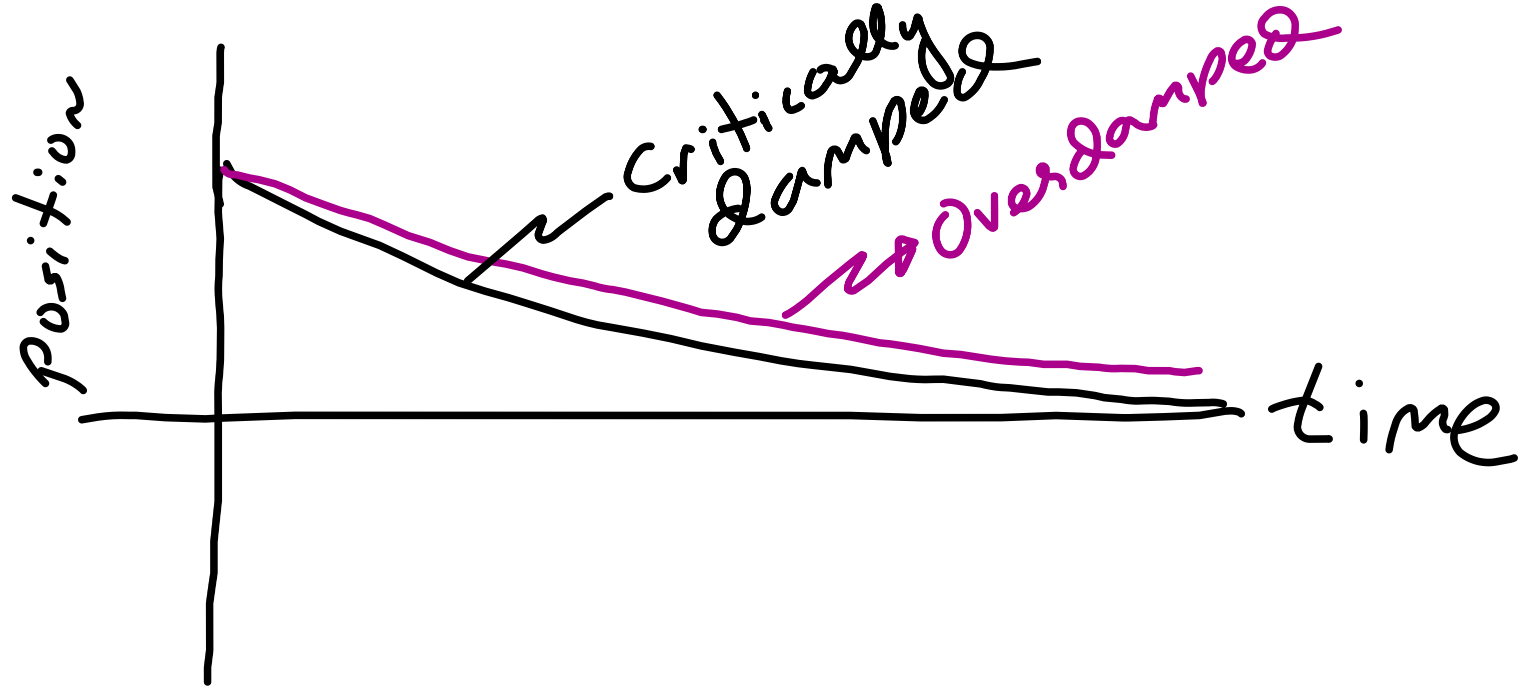

Position
time

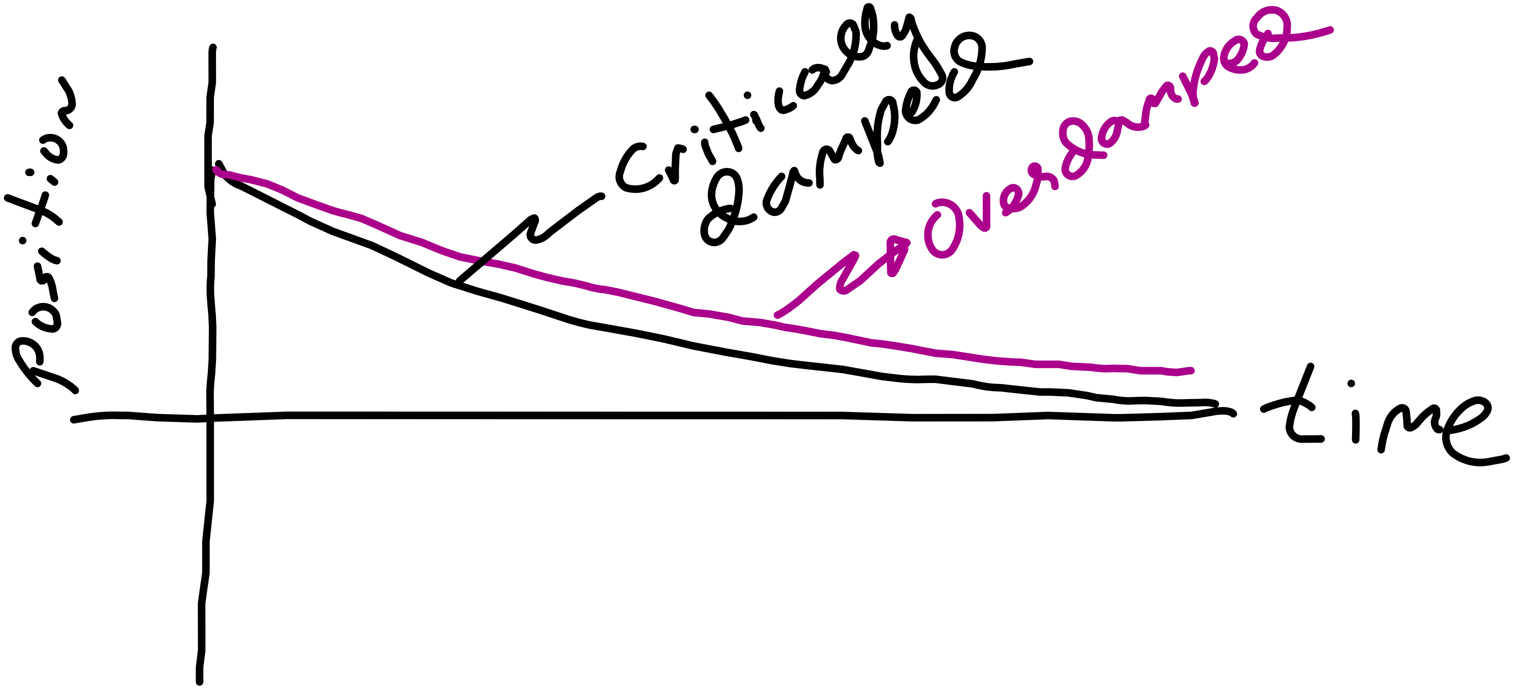


* No Vibration

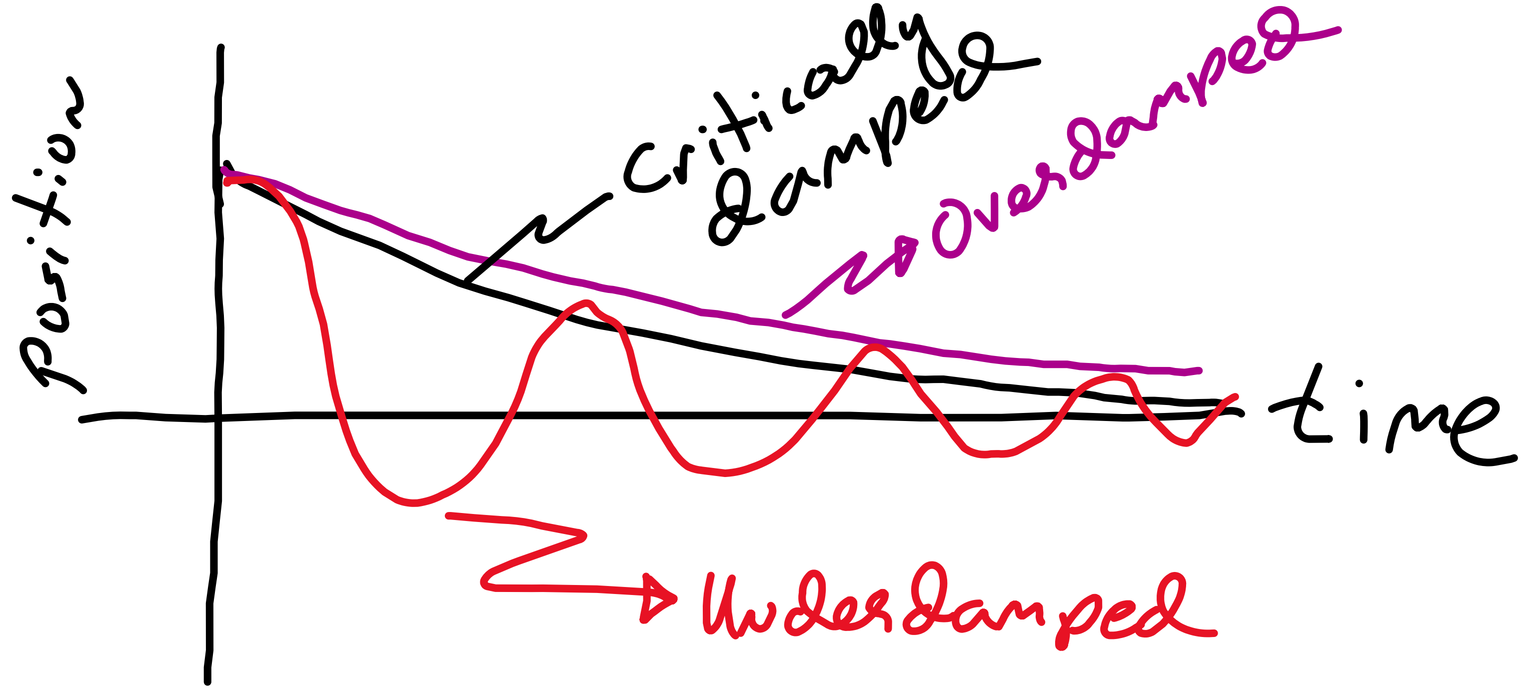


* No Vibration
* Gets to equilibrium fastest

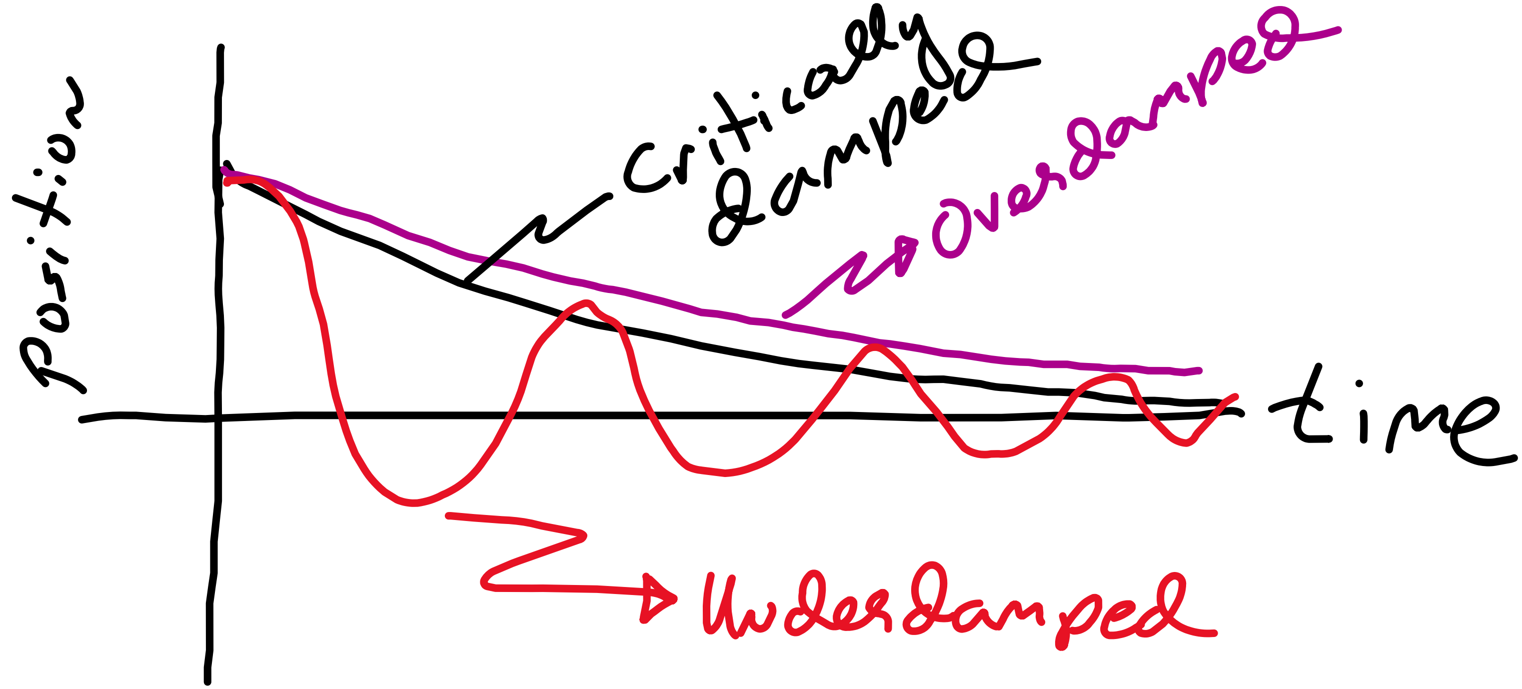

Position
time
Critically damped
Overdamped



* No vibration



Position
Critically damped
Overdamped
time
Underdamped



* Vibrates

The End